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**Working Paper**

## The Implications of Heterogeneous Resource Intensities on Technical Change and Growth

Economics Working Paper Series, No. 09/120

**Provided in Cooperation with:**

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*Suggested Citation:* Pittel, Karen; Bretschger, Lucas (2009) : The Implications of Heterogeneous Resource Intensities on Technical Change and Growth, Economics Working Paper Series, No. 09/120, ETH Zurich, CER-ETH - Center of Economic Research, Zurich, <https://doi.org/10.3929/ethz-a-005907386>

This Version is available at:

<https://hdl.handle.net/10419/171563>

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October 2009

Economics Working Paper Series



Eidgenössische Technische Hochschule Zürich  
Swiss Federal Institute of Technology Zurich

# The Implications of Heterogeneous Resource Intensities on Technical Change and Growth

KAREN PITTEL\* and LUCAS BRETSCHGER\*\*

*October 2010*

## Abstract

We analyze an economy in which sectors are heterogeneous with respect to the intensity of natural resource use. Long-term dynamics are driven by resource prices, sectoral composition, and directed technical change. We study the balanced growth path and determine stability conditions. Technical change is found to be biased towards the resource-intensive sector. Resource taxes have no impact on dynamics except when the tax rate varies over time. Constant research subsidies raise the growth rate while increasing subsidies have the opposite effect. We also find that supporting sectors by providing them with productivity enhancing public goods can raise the growth rate of the economy and additionally provide an effective tool for structural policy.

Keywords: sustainable development, sectoral heterogeneity, directed technical change

JEL Classification: O4 (economic growth), O41 (multi-sector growth models), Q01 (sustainable development), Q3 (non-renewable resources)

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# 1 Introduction

The last years have witnessed a remarkable rise in interest in natural resource scarcity and its long-run economic consequences. There are widespread concerns that diminishing resource input will have negative effects on our long-run living-standards. If economic growth were indeed impaired by lower resource use, this would particularly affect recently developed economies, which have grown heavily in energy-intensive sectors. Moreover, resource-intensive sectors in the leading economies would be expected to suffer most and possibly to vanish in the long run.

To counteract resource scarcity, improvements in technology are the most powerful mechanism. Several recent papers have shown that long-run growth may be compatible with the essential use of non-renewable resources, once we assume endogenous technological change, see Barbier (1999), Scholz and Ziemes (1999), and Grimaud and Rougé (2003, 2005). This literature uses a single final output framework, which is convenient but sidesteps the relationship between sectoral structure and aggregate growth. This is a limitation, since the economies are specialized in different sectors and aggregate development depends on sectoral growth. In particular, one could expect that technological progress is faster in resource-extensive sectors, that sectors exhibit diverging growth rates, and that economies specialized in resource-intensive sectors achieve lower development paths.

The paper shows that, contrary to the common beliefs, resource-intensive sectors conduct more research and increase efficiency faster than the rest of the economy. Second, we demonstrate that the resource-intensive sectors are able to sustain output in the long run, because the declining input of natural resources is compensated by sufficiently rising efficiency. We show the existence of a balanced growth path and provide conditions for saddle-path stability of the system. This holds true even when learning externalities in the research sector are purely sector-specific. In addition, we demonstrate that the share of resource-intensive sectors can be constant in the long run, as profit incentives induce a more-than-proportional research effort in these sectors. Finally, we confirm that increasing resource scarcity need not hamper economic growth, even when sectors have large differences in resource use.

The model assumptions are based on empirical regularities. In reality, sectors differ substantially in terms of input intensities, specifically with regard to knowledge intensity and natural resource use, which is crucial for the kind of disaggregation used. Second, sectors offer significantly different investment opportunities and innovations are often sector-specific. Third, policies directed at specific sectors are very popular and often implemented in practice.

We show that under the these empirically relevant assumptions, incentives arise

from the rising scarcity of resource-intensive goods to invest relatively more in R&D in the resource-intensive sector. Consequently, the composition of consumption in terms of productivity-weighted sectoral goods remains constant along a balanced growth path. Our results are in line with predictions of international organizations and recent empirical observations. The International Energy Agency (IEA) emphasizes that the largest potential for improving future energy efficiency lies in the energy-intensive sectors (IEA 2008, p. 112). Moreover, it sees good development perspectives for emerging economies despite their increasing shares of energy-intensive sectors (IEA 2008, p. 115). Along the same lines, Demailly and Quirion (2008) find that energy-intensive industries have performed much better economically under strict climate policies than previously expected.<sup>1</sup>

We study the implications of various policies such as resource taxes, labor and research subsidies and the sectoral, productivity enhancing provision of public goods. We look at the effectiveness of these policies in lowering resource use and raising growth, i.e. promoting ‘sustainable’ development. Interestingly, the results are mixed. In particular, research subsidies have positive growth effects in both sectors but resource taxes affect dynamics only when the tax rate varies over time. It is also shown that sectoral policies might not induce the desired effects, and that the sectors targeted by policies may not matter for the actual policy effects. For example, a sector-specific provision of public goods that aims at increasing the share of the targeted sector, will not raise but lower the share of this sector. And, although the growth effect of public good provision is positive, this result is independent of the sector in which the goods are provided.

The paper is related to recent literature on innovation, growth, and resource use. The basic technology assumptions for the different sectors in the model are based on Romer (1990).<sup>2</sup> By stressing the role of sectoral research activities and directed technological change, we apply the theory of factor-induced technical change, as introduced by Hicks (1932) and applied by Acemoglu (2002), to economic sectors and determine the conditions for sector-induced research. Smulders and de Nooij (2003) and Di Maria and Valente (2008) are closest to our approach. However, these papers do not assume that natural resources and labor are employed in all the different sectors of the economy. Moreover, we introduce labor reallocation between the different pro-

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<sup>1</sup>For an extensive overview of the literature dealing with applied approaches to technological change and environmental policy, see e.g. Jaffe et al. (2002) and references within.

<sup>2</sup>This is similar to Bretschger and Pittel (2005) who consider a multi-sector economy with sector-specific natural resource use but without directed technological change, as the substitution elasticity between sectoral outputs is assumed to be unity. Withagen (1999), Pittel (2002), and Xepapadeas (2002) provide surveys on the impact of natural resource use on economic growth. The impact of natural resource use in dynamic multi-sector models is also treated by Peretto (2008) and Bretschger (2008).

duction and research sectors, which realistically allows for more flexible adjustments in the economy. Due to this endogeneity, policies can affect the speed of resource extraction as well as aggregate research activities - both of which are crucial for the dynamics of the economy.<sup>3</sup> Our approach is also close to papers where heterogeneous sectors cause ongoing structural change, see Kuznets (1957), Kongsamut et al. (2001), López et al. (2007), and Acemoglu and Guerrieri (2008). Compared to this literature we introduce a new kind of multi-sector economy suited to discuss the direction of technical change and development under natural resource constraints. We show that despite the heterogeneity of sectors, long-run growth might not be accompanied by structural change. In extension of the literature we also study implications of different types of policies that aim at supporting sustainability.

The remainder of the paper is organized as follows. Section 2 describes the model in detail. The short and long-run dynamics of the model are analyzed in Section 3. Section 4 deals with the effects of policies striving at increasing the share of resource extensive sectors and fostering sustainability, i.e. raising growth and lowering resource extraction. Finally, Section 5 concludes.

## 2 The Model

In our economy, horizontally differentiated goods are produced in two sectors – a resource-intensive and a resource-extensive sector. In each sector, the differentiated goods are assembled to sectoral outputs which are consumed by the households. Blueprints for new products are developed by sector specific research activities and sold to monopolistic producers in each sector. Besides natural resources, labor constitutes the second primary input, which is employed in research as well as in intermediate production. Sectors differ with respect to resource intensity of production. We consider infinitely living households that maximize lifetime utility. Savings are either in the form of investment in bonds or in R&D.

### 2.1 Production

**Sectoral output** The outputs of the two sectors,  $\tilde{X}$  and  $\tilde{Z}$ , each consist of a continuum of horizontally differentiated intermediate goods,  $x_i$ ,  $i \in [0, n]$ , and  $z_j$ ,  $j \in [0, m]$ ,

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<sup>3</sup>Smulders and de Nooij (2003) take the supply of energy as exogenously given. We extend their approach by endogenizing the dynamics associated with the input of non-renewable natural resources. Di Maria and Valente (2008) endogenize the supply of inputs, capital and resources, but again do not assume that all inputs are used in the different sectors. They conclude that long-run development is characterized by resource-augmenting technological progress only. For the case that the economic sectors employ all the inputs and only differ with respect to input intensities, we are able to show that every sector conducts R&D in the long run. The direction of technological change is endogenous and depends on the degree of heterogeneity with respect to resource intensities.

where  $n$  and  $m$  denote the number of varieties in the respective sectors.<sup>4</sup> Gains from specialization arise, i.e. the larger the variety of goods, the more productive the aggregate:<sup>5</sup>

$$\tilde{X} = \left( \int_0^n x_i^\beta di \right)^{1/\beta} \quad \text{and} \quad \tilde{Z} = \left( \int_0^m z_j^\beta dj \right)^{1/\beta}. \quad (1)$$

where  $0 < \beta < 1$ .

The index of consumption  $C$  reflects households' preferences for sectoral output. Alternatively,  $C$  could be interpreted as aggregate output of the economy as, for example, done in Section 4.2.  $C$  depends on  $\tilde{X}$  and  $\tilde{Z}$ , according to the following CES function:

$$C_t = \left( \tilde{X}_t^{\frac{\nu-1}{\nu}} + \tilde{Z}_t^{\frac{\nu-1}{\nu}} \right)^{\frac{\nu}{\nu-1}}, \quad \nu > 0, \nu \neq 1 \quad (2)$$

where  $\nu$  denotes the elasticity of substitution between  $\tilde{X}$  and  $\tilde{Z}$ . For  $\nu < 1$ ,  $\tilde{X}$  and  $\tilde{Z}$  are complements while they constitute substitutes for  $\nu > 1$ . This implies that for  $\nu < 1$  both sectoral outputs are essential.<sup>6</sup> We abstract from Cobb-Douglas preferences, i.e.  $\nu = 1$ , as in this case sector shares would not be endogenously determined.

To facilitate calculations without loss of generality, we choose the consumption good to be the numeraire of the system so that its price is unity, i.e.  $p_C \equiv 1$ . At each point in time, utility maximization results in:

$$\frac{\tilde{X}}{\tilde{Z}} = \left( \frac{p_{\tilde{X}}}{p_{\tilde{Z}}} \right)^{-\nu} \quad \Leftrightarrow \quad \frac{p_{\tilde{X}} \tilde{X}}{p_{\tilde{Z}} \tilde{Z}} = \left( \frac{\tilde{X}}{\tilde{Z}} \right)^{-\frac{1-\nu}{\nu}} = \frac{\phi}{1-\phi} = \tilde{\phi} \quad (3)$$

with  $\phi = \frac{p_{\tilde{X}} \tilde{X}}{C}$  and  $1 - \phi = \frac{p_{\tilde{Z}} \tilde{Z}}{C}$  denoting the expenditure shares of  $\tilde{X}$  and  $\tilde{Z}$ , such that the relative sector share of  $x$ -goods is given by  $\tilde{\phi}$ , which will prove to be a very useful variable below.

Competition in  $x$ - and  $z$ -production is monopolistic. Each type of good is produced by only one firm that has to acquire the according patent first.  $x$ - as well as  $z$ -intermediates are produced from labor  $L$  and non-renewable resources  $R$  using the following Cobb-Douglas production technologies:

$$x_i = (L_{x_i})^\alpha (R_{x_i})^{1-\alpha} \quad \text{and} \quad z_j = (L_{z_j})^\delta (R_{z_j})^{1-\delta} \quad (4)$$

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<sup>4</sup>For notational convenience the time index will be suppressed whenever no ambiguity arises.

<sup>5</sup>In contrast to the productivity adjusted aggregates,  $\tilde{X}$  and  $\tilde{Z}$ , we denote aggregate physical amounts of  $x_i$  and  $z_i$  by  $X = \int_0^n x_i di$  and  $Z = \int_0^m z_j dj$ . The prices for  $\tilde{X}$  and  $\tilde{Z}$  are  $p_{\tilde{X}}$  and  $p_{\tilde{Z}}$ .  $p_{x_i}$  and  $p_{z_i}$  on the other hand denote prices for individual goods.

<sup>6</sup>Please note that resources are essential in our model for  $\nu < 1$  as well as for  $\nu > 1$  since they are essential inputs in the production of  $\tilde{X}$  as well as  $\tilde{Z}$ .

with  $0 < \alpha, \delta < 1$ .  $L_k$  and  $R_k$ ,  $k = x_i, z_j$ , denote the input of labor and resources in the production of  $x_i$  and  $z_j$ . It is assumed that sectors differ with respect to their resource intensities, i.e.  $\alpha \neq \delta$ .<sup>7</sup>

Maximization of profits gives the first-order conditions for the input of labor and resources in the two sectors. Considering that  $x_i = x$  and  $z_j = z$  in the symmetric equilibrium gives the sectoral demands for labor and resources in terms of  $\tilde{\phi}$  and  $C$ :

$$\begin{aligned} L_X &= \alpha\beta \frac{\tilde{\phi}}{1+\tilde{\phi}} \frac{C}{w} & \text{and} & & L_Z &= \delta\beta \frac{1}{1+\tilde{\phi}} \frac{C}{w} \\ R_X &= (1-\alpha)\beta \frac{\tilde{\phi}}{1+\tilde{\phi}} \frac{C}{p_R} & \text{and} & & R_Z &= (1-\delta)\beta \frac{1}{1+\tilde{\phi}} \frac{C}{p_R} \end{aligned} \quad (5)$$

with  $R_K = \int_0^l R_k di$  and  $L_K = \int_0^l L_k dj$ ,  $(K, l, k) \in (X, n, x_i), (Z, m, z_j)$ .  $w$  and  $p_R$  denote the wage rate and the price of resources. Individual firms' demands are obtained by dividing the respective sectoral demands by the 'number' of intermediates in each sector, i.e.  $n$  and  $m$  respectively. Summing up the resource demands of the two sectors in (5) gives the aggregate extraction of resources at each point in time:

$$R = R_X + R_Z = ((1-\alpha)\tilde{\phi} + (1-\delta)) \frac{\beta}{1+\tilde{\phi}} \frac{C}{p_R}. \quad (6)$$

From (5) and the production functions for  $x$  and  $z$ , (4), sectoral equilibrium profits from intermediates production can be derived:

$$\Pi_X = (1-\beta) \frac{\tilde{\phi}}{1+\tilde{\phi}} C \quad \text{and} \quad \Pi_Z = (1-\beta) \frac{1}{1+\tilde{\phi}} C. \quad (7)$$

**R&D** Blueprints for new types of goods are generated in two separate R&D sectors. The only rival input to research is labor, yet production also profits from past research activities which give rise to positive sector specific spill-overs. Production is linear in labor as well as in research experience which equals the 'number' of blueprints generated in the respective sector in the past ( $n$  for the  $x$ -sector and  $m$  for the  $z$ -sector):

$$\dot{n} = \frac{dn}{dt} = \frac{L_n}{a} n \quad \text{and} \quad \dot{m} = \frac{dm}{dt} = \frac{L_m}{a} m \quad (8)$$

with  $L_n$  and  $L_m$  denoting the input of labor to sectoral research.  $a$  represents the unit input coefficient of labor in research which is assumed to be equal in the two

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<sup>7</sup> We abstract from  $\alpha = \delta$  as this case does not provide much insight beyond the existing literature. In the long-run, the relative sector share,  $\tilde{\phi}$ , is equal to unity and the model collapses to a model with only one intermediates sector (as e.g. analyzed by Scholz/Ziemes 1999, Schou 2002 and Grimaud et al. 2009). The only difference to this type of model arises with respect to transitional dynamics as, in case the initial number of intermediate goods differs in the two sectors, sectoral R&D activities are not identical outside the balanced growth path.



sectors. In order not to predetermine sectoral convergence by the model assumptions we exclude spill-overs between the sectors.<sup>8</sup>

Given the four different uses of labor, equilibrium in the labor market requires

$$L_X + L_Z + L_n + L_m = 1 \quad (9)$$

where, for simplicity, the size of the labor force is normalized to unity.

Research markets are assumed to be perfectly competitive, such that in equilibrium patent values  $V_n$ , resp.  $V_m$ , are equal to marginal production costs

$$V_n = \frac{aw}{n} \quad \text{and} \quad V_m = \frac{aw}{m}. \quad (10)$$

Furthermore, equilibrium on the patent market requires the value of a patent to be equal to the discounted stream of profits generated by the production of the respective intermediate good which implies the following no-arbitrage conditions to hold:

$$\dot{V}_n = rV_n - \Pi_x \quad \text{and} \quad \dot{V}_m = rV_m - \Pi_z \quad (11)$$

where  $r$  is the interest rate on all assets.  $\Pi_x = \frac{\Pi_X}{n}$  and  $\Pi_z = \frac{\Pi_Z}{m}$  stand for individual intermediate firms' profits;  $\Pi_X$  and  $\Pi_Z$  are given in (7).

**Resources** Natural resources are non-renewable. The resource stock  $S$  is depleted by the extraction of resources  $R$  for production, such that the dynamics of the resource stock are

$$\dot{S} = -R. \quad (12)$$

It is assumed that resources are extracted at no cost. Resource firms maximize intertemporal profits

$$\max_R \int_0^\infty p_R(t)R(t)e^{-\int_0^t r(s)ds} dt \quad (13)$$

subject to (12) which yields the familiar Hotelling rule:<sup>9</sup>

$$g_{p_R} = \frac{\dot{p}_R}{p_R} = r. \quad (14)$$

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<sup>8</sup>Labor is taken to be the only rival input in R&D as assuming that resources were also an input in research would affect the quality of long run growth (semi-endogenous instead of fully endogenous growth, see Groth/Schou 2007) but not the results on sectoral behavior which is the focus here.

<sup>9</sup> $g_b$  denotes the growth rate of variable  $b$ , i.e.  $g_b = \frac{\dot{b}}{b}$ , where  $\dot{b}$  is the time derivative of  $b$ .

## 2.2 Consumers

Households derive utility from consumption  $C$ . The representative household maximizes discounted lifetime utility with respect to its intertemporal budget constraint:

$$\begin{aligned} \max_c \quad & \int_0^\infty \log C(t) e^{-\rho t} dt \\ \text{s.t.} \quad & \dot{W} = rW + w - C \end{aligned} \quad (15)$$

where  $W = nV_n + mV_m + p_R S$  denotes total household asset holdings. We assume utility to be logarithmic in order to simplify the exposition (see Smulders and de Nooij 2003 and Di Maria and Valente 2008 for similar set-ups). Households supply labor inelastically. From the first-order conditions of household maximization we get the familiar Keynes-Ramsey rule

$$g_C = r - \rho. \quad (16)$$

## 3 System Dynamics

To analyze the dynamics of the economy we reduce the system to two first-order differential equations which are functions of the relative sector share,  $\tilde{\phi}$ , and the input of labor in intermediates production of sector  $z$ , i.e.  $L_Z$ .

**Lemma 1.** *The dynamics of the economic system are given by*

$$\dot{L}_Z = \left( \frac{1}{2a} \left( \delta + \tilde{\phi}\alpha + \frac{1-\beta}{\beta}(1+\tilde{\phi}) \right) L_Z - \rho - \frac{1}{2a} - \frac{\tilde{\phi}}{1+\tilde{\phi}} g_{\tilde{\phi}} \right) L_Z \quad (17)$$

$$\dot{\tilde{\phi}} = \left( \frac{\left[ \frac{1}{2a} \frac{\alpha-\delta}{\delta} \left( \delta + \tilde{\phi}\alpha + \frac{1-\beta}{\beta}(1+\tilde{\phi}) \right) - \frac{1}{\delta a} \left( \frac{1-\beta}{\beta} \right)^2 (1-\tilde{\phi}) \right] L_Z - \frac{1}{2a}(\alpha-\delta)}{\frac{1}{\nu-1}} \right) \tilde{\phi}. \quad (18)$$

*Proof.* see Appendix A1. □

These two equations describe the dynamics of the system along the balanced growth path (BGP) as well as during the transition to the BGP. Inspection of (18) reveals immediately that the elasticity of substitution between the two sectoral outputs,  $\nu$ , is crucial for the dynamics of the system. As will be shown later in this section, the switch in the sign of  $\dot{\tilde{\phi}}$  that arises for  $\nu \leq 1$  causes instability of the system for  $\nu > 1$  while the economy is saddle-path stable for  $\nu < 1$ .

With respect to long-term growth, a path will be called a BGP if all variables grow at constant – possibly zero or negative – rates. This implies that along a BGP

(i) aggregate production and production in both intermediate sectors grow at the same rate and (ii) expenditure shares, sectoral labor inputs and the interest rate are constant over time ( $\dot{\tilde{\phi}} = \dot{L}_Z = \dot{r} = 0$ ).

For the BGP values of  $L_Z$  and  $\tilde{\phi}$  we get (see Appendix A2):

$$L_Z^* = \frac{\delta(1+2a\rho)}{\delta + \tilde{\phi}^*\alpha + \frac{1-\beta}{\beta}(1+\tilde{\phi}^*)} \quad (19)$$

$$\tilde{\phi}^* = \frac{A}{B} \quad (20)$$

with

$$A = \left( (1-\beta)^2(1+2a\rho) - a\alpha\beta^2\delta\rho \right) - a(\alpha-\delta)\beta(1-\beta)\rho + a\delta^2\beta^2\rho \quad (21)$$

$$B = \left( (1-\beta)^2(1+2a\rho) - a\alpha\beta^2\delta\rho \right) + a(\alpha-\delta)\beta(1-\beta)\rho + a\alpha^2\beta^2\rho \quad (22)$$

where asterisks indicate variable values or growth rates along the BGP.<sup>10</sup>

As to be expected, the direct effect of labor productivity in the  $z$ -sector,  $\delta$ , on  $L_Z^*$  is positive while the direct effect of  $\alpha$  is negative. Also the discount rate,  $\rho$ , exerts a positive direct effect as higher impatience induces households to allocate less labor to research and more to today's goods production. Equally, the reaction of  $L_Z^*$  to  $\beta$  and  $\tilde{\phi}^*$  follows intuition: higher gains of specialization, i.e. a lower  $\beta$ , and a higher relative sector share of  $x$ -products,  $\tilde{\phi}^*$ , lead to a lower input of labor into  $z$ -production.

With respect to  $\tilde{\phi}^*$ , (20) reflects that the gains of specialization, the productivity of R&D as well as the discount rate affect both sectors symmetrically.  $\tilde{\phi}^*$  only deviates from unity due to  $\alpha \neq \delta$ . Yet, whether  $\tilde{\phi}^*$  reacts positive or negative to a rise in either  $\alpha$  or  $\delta$  depends crucially on the parametrization of the model. A rise in  $\alpha$ , for example, increases on the one hand labor productivity in the  $x$ -sector, which ceteris paribus affects  $\tilde{\phi}^*$  positively. On the other hand the rise in  $\alpha$  decreases the productivity of resources which lowers  $\tilde{\phi}^*$ . Yet, less productive resources also render  $z$ -products scarcer, therefore increase  $z$ -prices and induce higher incentives to invest in  $n$ -R&D both of which increase  $\tilde{\phi}^*$ . Which effect dominates, depends crucially on the parametrization.

The equilibrium input of labor into  $x$ -intermediates can be derived from (5):

$$L_X^* = \tilde{\phi}^* \frac{\alpha}{\delta} L_Z^*. \quad (23)$$

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<sup>10</sup>In contrast to Acemoglu/Guerrieri (2008), optimization on markets leads to long-run balanced growth in our model. Acemoglu and Guerrieri assume R&D to be subject to decreasing or negative spill-overs from knowledge in which case non-balanced growth arises. In our paper, however, R&D is linear in knowledge in which case the two sectors grow at the same rate.

(23) shows that the share of labor allocated to  $x$ - rather than  $z$ -intermediates rises with the relative sector share and labor productivity in the  $x$ -sector.

From the no-arbitrage conditions for the patent market follows that along the BGP the following relations hold (see Appendix A2):

$$L_n^* = \tilde{\phi}^* \frac{1-\beta}{\beta} \frac{L_Z^*}{\delta} - a\rho \quad (24)$$

$$L_m^* = \frac{1-\beta}{\beta} \frac{L_Z^*}{\delta} - a\rho. \quad (25)$$

Complementing the results on  $L_X$  and  $L_Z$ , we see that a *c.p.* higher  $\tilde{\phi}^*$  induces an allocation of labor towards R&D that develops new patents for the  $x$ -sector while the input of labor into both research sectors rises with higher gains of specialization.

The growth rate of resource extraction along the BGP can be determined by expressing the aggregate demand for resources, (6), in growth rates and considering the Keynes-Ramsey rule, (16). This gives

$$g_R = -\rho. \quad (26)$$

Differentiation of (2) confirms that balanced growth requires consumption and the production of  $x$ - and  $z$ -aggregates to grow at the same rate, i.e.

$$g_C^* = g_{\tilde{X}}^* = g_{\tilde{Z}}^*. \quad (27)$$

Identical constant growth rates of  $\tilde{X}$  and  $\tilde{Z}$  together with (3) imply that the sectoral expenditure shares,  $\phi$  and  $1-\phi$ , as well as the relative expenditure share,  $\tilde{\phi}$ , are constant over time.

The condition that along the BGP aggregate production in both sectors has to grow at the same rate carries important implications for research efforts in equilibrium. Considering the production technologies for  $x$  and  $z$ , (4), as well as (26), the growth rates of  $\tilde{X}$  and  $\tilde{Z}$  along the BGP are given by

$$g_{\tilde{X}}^* = \frac{1-\beta}{\beta} g_n^* - (1-\alpha)\rho \quad (28)$$

$$g_{\tilde{Z}}^* = \frac{1-\beta}{\beta} g_m^* - (1-\delta)\rho. \quad (29)$$

**Proposition 1.** *Along the balanced growth path, research is biased towards the resource intensive sector. If, e.g., the  $z$ -sector is more resource intensive ( $\alpha > \delta$ ),  $g_m^* > g_n^*$  holds.*

*Proof.* From (28), (29) and  $g_{\tilde{X}}^* = g_{\tilde{Z}}^*$  along the BGP follows straightforwardly that the following relation holds:

$$g_m^* = (\alpha - \delta)\rho \frac{\beta}{1 - \beta} + g_n^*. \quad (30)$$

□

This condition states that for balanced growth to be feasible, differences in resource intensities between sectors have to be compensated by research. It can also easily be seen that in case that sectors are identical, innovation rates along the BGP are the same in the two sectors. If, however, sectors differ with respect to resource intensities, more research will be conducted in the sector that is more resource intensive and thus subject to a stronger drag from declining resource inputs.

While the aggregate productivity weighted amounts of goods produced in both sectors,  $\tilde{X}$  and  $\tilde{Z}$ , grow at the same, potentially positive rate in equilibrium, the physical amounts individual intermediates produced in either sector,  $x$  and  $z$ , decrease over time. Taking into account that labor shares are constant along the BGP, it follows from (4) and  $g_{R_i} = g_R = -\rho$  that

$$g_x^* = -(1 - \alpha)\rho < 0 \quad (31)$$

$$g_z^* = -(1 - \delta)\rho < 0. \quad (32)$$

The reduction in the produced amounts is due to the decreasing input of natural resources. If the  $z$ -sector is more resource intensive than the  $x$ -sector,  $z$  falls faster than  $x$ . As economic intuition suggests, it follows from (3) that the price ratio follows a time path that is inverse to the development of quantities, i.e. prices in the more resource intensive sector rise faster due to increasing resource prices.

From (8), (28) and (29) we can express the equilibrium growth rate of consumption as

$$\begin{aligned} g_C^* &= \frac{1 - \beta}{\beta} \frac{L_n^*}{a} - (1 - \alpha)\rho \\ &= \frac{1 - \beta}{\beta} \frac{L_m^*}{a} - (1 - \delta)\rho. \end{aligned} \quad (33)$$

with  $L_n^*$  and  $L_m^*$  being specified in (24) and (25). Overall, the sign of  $g_C$  in (33) is ambiguous. Two forces determine whether long-term development is sustainable ( $g_C > 0$ ):  $-(1 - \alpha)\rho$  and  $-(1 - \delta)\rho$  represent the negative growth effects stemming from the declining input of natural resources and impatience while  $\frac{1 - \beta}{\beta} \frac{L_n^*}{a}$  and  $\frac{1 - \beta}{\beta} \frac{L_m^*}{a}$  reflect the growth stimulating effects of research.

After substituting (19), (24) and (25),  $g_C^*$  can be rewritten in terms of the model parameters only:

$$g_C^* = \frac{\frac{(1-\beta)^2}{a\beta} - \beta(\alpha^2 + \delta^2)\rho}{2 - \beta(2 - \alpha - \delta)} - \rho. \quad (34)$$

The resulting expression shows that consumption growth along the BGP can be negative if research is not sufficiently productive (high  $a$ ), such that the drag on growth from resources overcompensates improvements in productivity from research. Similarly, growth might be negative if agents are highly impatient. With respect to changes in resource intensities, effects on growth are ambiguous. A decrease in resource intensity (increase in  $\alpha$ , resp.  $\delta$ ) induces on the one hand a less severe drag on growth, but on the other hand causes a reallocation of labor away from research.

Let us finally consider the transitional dynamics of this economy.

**Lemma 2.** *The system given by (17) and (18) is locally saddle-path stable for  $\nu < 1$ , i.e. when  $\tilde{X}$  and  $\tilde{Z}$  are complements, and unstable for  $\nu > 1$ , i.e. when  $\tilde{X}$  and  $\tilde{Z}$  are substitutes.*

*Proof.* see Appendix A3. □

The result that for  $\nu > 1$  the system is unstable corresponds to the recent literature, see e.g. Acemoglu (2002) who also provides an intuitive explanation of which we only provide the gist. If  $\tilde{X}$  and  $\tilde{Z}$  are complements ( $\nu < 1$ ), both goods are essential such that R&D will be biased towards the scarcer, i.e. more expensive, product. Consequently, if the initial share of a product is lower than its equilibrium share, it will rise over time towards the BGP. If, however,  $\tilde{X}$  and  $\tilde{Z}$  are substitutes ( $\nu > 1$ ), R&D will be biased towards the product with the larger than equilibrium share, inducing a development away from the BGP. In this paper we are able to show that the stability properties described by Acemoglu also hold in the presence of sectoral heterogeneity with an essential non-renewable resource.<sup>11</sup>

## 4 Policy analysis

In Subsection 3 we have derived that growth depends on research effort and resource use. Growth effects of policy can therefore stem from either higher innovation rates and/or slower resource extraction. Yet, alternative policies not only differ with respect

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<sup>11</sup> It can be shown that our results, regarding the existence of a BGP as well as the stability properties, can be extended to economies in which sectors differ with respect to research productivity ( $a_n \neq a_m$ ) and/or gains of specialization ( $\beta_x \neq \beta_z$ ). Higher gains of specialization in sector  $x$  (i.e.  $\beta_z > \beta_x$ ) would, for example, imply an even stronger bias towards  $z$ -research. For more details, see Appendix B.

to the channels through which they affect growth, but also with respect to their impact on the sectoral structure.

In the following we consider different types of policies that might constitute alternatives for a policy maker.<sup>12</sup> In this section we assume throughout that  $\alpha > \delta$ , i.e. that the  $x$ -sector is less resource intensive. We check different policies with respect to their ability to foster growth, to slow down resource extraction and to affect the sectoral structure of the economy. For those policy variables for which the tax or subsidy level has no impact on the dynamics of the economy, we check for possible effects of time-varying tax/subsidy schedules. Specifically we focus on

- resource taxation (tax rate  $\tau$ )
- labor subsidization (subsidy rate  $s_w$ )
- differentiated research subsidization ( $s_n$  and  $s_m$ )
- differentiated provision of productive public goods (shares  $\mu_x$ , resp.  $\mu_z$ , of consumption).

In case of time-varying policy instruments,  $g_i$  denotes the growth rate of the respective policy variable  $i$ .

The analysis of the policy instruments is conducted in two steps. First, the traditional instruments, i.e. taxes and subsidies, are analyzed in Subsection 4.1 while the provision of public goods is treated in Subsection 4.2.

## 4.1 Policy analysis 1: Taxes and Subsidies

In the following we consider ad valorem taxes on the input of resources as well as uniform subsidies on labor and differentiated subsidies on research. Research subsidies are in the form of wage subsidies.

To clearly distinguish the effects of each instrument, we assume in the following that the government can balance its budget via lump-sum taxation or subsidization of households. In this case, policies are not tied together by budgetary requirements and each instrument can be analyzed independently.

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<sup>12</sup>We focus on policy instruments and targets that are currently discussed in the field of energy and climate policies rather than on optimal policies. The optimum is quite simple to detect because the only market failures actually included in the model are monopolistic competition in the intermediate sector and the externality from the research sectors which are well studied in the endogenous growth literature, see Romer (1990). Here we concentrate on the question whether the measures to save on resource use and to support resource-extensive sectors work as commonly assumed or are ineffective or even counter-productive.

Due to the policy interventions, the BGP values of  $\tilde{\phi}$  and  $L_Z$  modify to:<sup>13</sup>

$$L_Z^{p1*} = \frac{\delta(1+2a\rho)}{\delta + \phi^{p1*}\alpha + \frac{1-\beta}{\beta}\left(\frac{1}{\bar{s}_m} + \frac{\tilde{\phi}^{p1*}}{\bar{s}_n}\right)} \quad (35)$$

$$\tilde{\phi}^{p1*} = \frac{\bar{s}_n}{\bar{s}_m} \frac{A - aD_1}{B - aD_2} \quad (36)$$

with

$$D_1 = \beta(\alpha - \delta)(\rho\delta\beta(\bar{s}_m - 1) + g_{\bar{\tau}}(1 - \beta + \bar{s}_m\beta\delta)) \quad (37)$$

$$D_2 = -\beta(\alpha - \delta)(\rho\alpha\beta(\bar{s}_n - 1) + g_{\bar{\tau}}(1 - \beta + \bar{s}_n\beta\alpha)). \quad (38)$$

For notational convenience we denote  $\bar{\tau} = 1 + \tau$ ,  $\bar{s}_m = 1 - s_m$  and  $\bar{s}_n = 1 - s_{r_n}$ . We retrieve the no-policy BGP values of the two variables by setting  $\bar{s}_m = \bar{s}_n = 1$  and  $g_{\bar{\tau}} = 0$ .

By proceeding as in the no-policy section, it can furthermore be shown that

$$g_C^{p1*} = \frac{1-\beta}{\beta} \frac{L_m^{p1*}}{a} - (1-\delta)(\rho + g_{\bar{\tau}}) \quad (39)$$

$$L_m^{p1*} = \frac{1}{s_m\delta} \frac{1-\beta}{\beta} L_Z^{p1*} - a\rho. \quad (40)$$

For the BGP rate of resource extraction we get

$$g_R^{p1*} = -(\rho + g_{\bar{\tau}}). \quad (41)$$

By employing the above BGP relations we can now derive the comparative statics of the different policy instruments.

**Proposition 2.** *If resource tax rates,  $\tau$ , and labor subsidies,  $s_w$ , are constant over time, they have no impact on long-run growth, resource extraction and the relative sector share. Research subsidies,  $s_m$  and  $s_n$ , affect growth as well as the relative sector share even if their rates are constant over time:*

$$\begin{aligned} \frac{d\tilde{\phi}^{p1*}}{ds_n} &< 0, & \frac{dg_C^{p1*}}{ds_n} &> 0, \\ \frac{d\tilde{\phi}^{p1*}}{ds_m} &> 0, & \frac{dg_C^{p1*}}{ds_m} &> 0. \end{aligned} \quad (42)$$

*Tax rates that change over time affect long-run growth, resource extraction and the relative sector share as follows:*<sup>14</sup>

$$\frac{d\tilde{\phi}^{p1*}}{dg_{\tau}} < 0, \quad \frac{dg_R^{p1*}}{dg_{\tau}} < 0, \quad \frac{dg_C^{p1*}}{dg_{\tau}} < 0. \quad (43)$$

<sup>13</sup> For the derivation of the underlying dynamic system, see Appendix C1.

<sup>14</sup> As resource taxes are assumed to be ad valorem taxes on the input of resources, tax rates which are continuously increasing and at some point exceed unity are feasible.



*Labor subsidies do not effect long-run growth, resource extraction and the relative sector share, even if they are time-varying:*

$$\frac{d\tilde{\phi}^{p1*}}{dg_{sw}} = \frac{dg_R^{p1*}}{dg_{sw}} = \frac{dg_C^{p1*}}{dg_{sw}} = 0, \quad (44)$$

*Proof.* Taking the derivatives of  $\tilde{\phi}^{p1*}$ ,  $g_R^{p1*}$  and  $g_C^{p1*}$  as determined by (35), (36), (39), (40) and (41) with respect to the policy variables yields either zero or the above signs (see Appendix C2).  $\square$

Resource taxation only affects long-run growth and the sectoral structure if the rate of taxation changes over time. If  $\tau$  is constant, resource taxation lowers resource demand permanently by a constant factor while intertemporal arbitrage of resource owners remains unaffected. As a consequence, the producer price of resources declines leaving the price that intermediate's producers have to pay unchanged. Thus constant taxation exerts neither an effect on resource and labor allocation nor on the speed of resource extraction. A rising rate of resource taxation, however, alters growth via two channels. An increase in taxation induces the speed of resource extraction to rise as resource owners foresee the future decrease in the non-taxed share of resource revenues. The resulting negative effect on growth is naturally stronger in the more resource intensive sector. To compensate for this stronger resource drag, labor is allocated towards research in this sector. However, the resource extraction effect dominates such that the overall effect remains negative. These findings are in line with resource models that comprise single final goods sectors, see e.g. Groth and Schou (2007), which reveals that they continue to hold in the case of heterogeneous sectors. We additionally show how the adjustment mechanisms work with multiple sectors. In intermediates production, labor is reallocated towards the more resource intensive sector. Due to the tax induced faster increase of intermediates' prices in  $z$ -production, the value share of the  $z$ -sector rises which raises profitability and thereby attracts labor from the  $x$ -sector and lowers  $\tilde{\phi}$ .<sup>15</sup>

Labor subsidization has no effect on growth and sector structure - neither for constant nor for rising subsidy rates. The intuition is, that as labor inputs in all sectors are equally affected by the subsidy, no labor reallocation is induced.<sup>16</sup> The level of research subsidy rates affects the allocation of labor in our model as it distorts the production cost ratio between intermediates production and research. This effect corresponds to the standard results of endogenous growth theory, see Romer (1990).

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<sup>15</sup>Empirical evidence that taxes on oil have been rising considerably during the last decades is presented by Daubanes (2009). The current Swiss example of an announced rise in CO<sub>2</sub> -taxes is an example for anticipated tax increases.

<sup>16</sup>This neutrality result depends of course on the assumption that there is no labor-leisure choice in our model.

We add to the literature by analysing the direction in which research subsidies change the relative market shares in the multi-sector economy. This depends on whether the more or the less resource intensive sector is subsidized. Subsidies to research in the less resource intensive sector ( $s_n$ ) induce the relative sector size of this sector to decrease - and vice versa for the more resource intensive sector. The line of reasoning is equivalent to the case of resource taxation presented above. Research subsidization exerts no effect on resource extraction in our model. Although the interest rate and therefore the growth rate of the resource price change due to subsidization, the rate of extraction remains unaltered as income and substitution effects of interest rate changes on the savings decision of households cancel.

## 4.2 Policy analysis 2: Productive public goods

The present framework is especially suited to study sector specific policies. As a second policy option we thus consider financing activities that enhance the productivity of resources in either one or both sectors. The productivity improvement is assumed to result from investing in the public provision of sector specific infrastructure, which requires a sector-specific formulation of the approach of Barro (1990) and Barro and Sala-i-Martin (1992).

For simplicity we again assume that the financial revenues necessary are generated via lump-sum taxation. It is further assumed that the share of consumption – resp. aggregate output – used for public good provision is equal to the amount of public goods  $G_k$ ,  $k = x, z$ , produced from this share, i.e.  $G_k = \mu_k C$ ,  $\mu_x + \mu_z < 1$ . In this case, the production functions for  $x_i$  and  $z_j$  modify to<sup>17</sup>

$$x_i = L_{x_i}^\alpha (G_x R_{x_i})^{1-\alpha} \quad \text{and} \quad z_j = L_{z_j}^\delta (G_z R_{z_j})^{1-\delta}. \quad (45)$$

such that in equilibrium  $\tilde{X}$  and  $\tilde{Z}$  read

$$\tilde{X} = n^{\frac{1-\beta}{\beta}} L_X^\alpha (\mu_x C R_X)^{1-\alpha} \quad \text{and} \quad \tilde{Z} = m^{\frac{1-\beta}{\beta}} L_Z^\delta (\mu_z C R_Z)^{1-\delta}. \quad (46)$$

The new equilibrium values of the relative sector share and labor input in  $z$ -intermediates are given by<sup>18</sup>

$$L_Z^{p2*} = \frac{\delta (1 + 2a\rho)}{\delta + \phi^{\tilde{p}2*} \alpha + \frac{1-\beta}{\beta} (1 + \phi^{\tilde{p}2*})} \quad (47)$$

$$\tilde{\phi}^{p2*} = \frac{\alpha A + aE_1}{\delta B - aE_2} \quad (48)$$

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<sup>17</sup> For (45) to be compatible with (4), one may think of (4) as a specific case of (45) with  $G_k = 1$ , i.e. with a constant provision of public goods that is set equal to unity.

<sup>18</sup> For the derivation of the underlying dynamic system, see Appendix D1.

with

$$\begin{aligned} E_1 &= (1 - \beta(1 - \delta))[\beta((1 - \delta)\alpha g_{\mu_z} - (1 - \alpha)\delta g_{\mu_z}) + (\alpha - \delta)(\alpha\beta - 1)\rho] \\ E_2 &= (1 - \beta(1 - \alpha))[\beta((1 - \delta)\alpha g_{\mu_z} - (1 - \alpha)\delta g_{\mu_z}) + (\alpha - \delta)(\delta\beta - 1)\rho]. \end{aligned}$$

Note that setting  $g_\mu = 0$  does not replicate the no-policy equilibrium in this case. It can furthermore be shown by proceeding as in the no-policy section that

$$g_C^{p2*} = \frac{1}{\delta} \left( \frac{1 - \beta}{\beta} \frac{L_m^{p2*}}{a} - (1 - \delta)\rho \right) + \frac{1 - \delta}{\delta} g_{\mu_z} \quad (49)$$

$$L_m^{p2*} = \frac{1}{\delta} \frac{1 - \beta}{\beta} L_Z^{p2*} - a\rho \quad (50)$$

where the functional forms of (49) and (50) are identical to the equilibrium conditions for  $g_C^*$  and  $L_m^*$  in the no policy scenario. For the BGP rate of resource extraction we get

$$g_R^{p2*} = -\rho. \quad (51)$$

**Proposition 3.** *The provision of public goods raises growth independently of the level of the consumption share devoted to productive public spending,  $\mu_k$ ,  $k = x, z$ .*

*Proof.* For the positive effect of public good provision on  $g_C^{p2*}$  see Appendix E. This positive effect is independent of the level of  $\mu_k$  for  $g_{\mu_k} = 0$  as follows from (47) to (50).  $\square$

For the economic intuition behind this result, consider the case in which the policy maker provides public goods to the less resource intensive sector only. In this case, the feed-back effect of the provision of public goods on  $x$ -production is equivalent to a rise in  $x$ -sector productivity. This increase in productivity induces a slower increase of intermediates' prices in  $x$ -production which lowers profitability and leads to a reallocation of labor from  $x$ - to  $z$ -sector research. Due to the increase in  $z$ -sector research, growth rises. In the  $x$ -sector, the reallocation of labor reduces research which affects growth negatively. But, in the aggregate this negative effect is overcompensated by the productivity increase due to public good provision.

For no-policy balanced growth (Section 3) we showed that in equilibrium the difference in research activities between the two sectors is determined by (30). This relation remained unperturbed by the taxes and subsidies considered in the previous subsection as neither affect production technologies directly. In the case of public good provision, however, the productivity of intermediate goods' production increases

due to policy. The new equilibrium allocation of research efforts is determined by:

$$\frac{1-\beta}{\beta} \left[ \frac{g_m^{p2*}}{\delta} - \frac{g_n^{p2*}}{\alpha} \right] = \left( \frac{1-\delta}{\delta} - \frac{1-\alpha}{\alpha} \right) \rho - \frac{1-\alpha}{\alpha} g_{\mu_x} + \frac{1-\delta}{\delta} g_{\mu_z} \quad (52)$$

Comparing (30) and (52) shows that the gap between research in the two sectors might in- or decrease due to productive public spending, depending on the model calibration and policy rule.

Employing the BGP relations, (47) to (51), we get the comparative statics of the different policy instruments.

**Proposition 4.** *As the growth effect of productive public spending is independent of the level of  $\mu_k$ ,  $k = x, z$  (see Proposition 3), a one-time increase in  $\mu_k$  has no impact on long-run growth, resource extraction and the relative sector share. If, however, the growth rate of  $\mu_k$  changes, the BGP values of  $\tilde{\phi}$ ,  $g_R$  and  $g_C$  are affected as follows:*

$$\begin{aligned} \frac{d\tilde{\phi}^{p2*}}{dg_{\mu_x}} &< 0, & \frac{dg_R^{p2*}}{dg_{\mu_x}} &= 0, & \frac{dg_C^{p2*}}{dg_{\mu_x}} &> 0, \\ \frac{d\tilde{\phi}^{p2*}}{dg_{\mu_z}} &> 0, & \frac{dg_R^{p2*}}{dg_{\mu_z}} &= 0, & \frac{dg_C^{p2*}}{dg_{\mu_z}} &> 0. \end{aligned} \quad (53)$$

*Proof.* Taking the derivatives of  $\tilde{\phi}^{p2*}$ ,  $g_R^{p2*}$  and  $g_C^{p2*}$ , as given by (47) to (51), with respect to  $\mu_k$  and  $g_{\mu_k}$  yields either zero or the signs above (see Appendix D2).  $\square$

A rising share of public goods provision lowers profitability in the respective sector which leads to a reallocation of labor to the other sector.<sup>19</sup> Due to the increase in the research of this sector, growth rises. In the other sector, research efforts decline, but in the aggregate the induced negative growth effect is again overcompensated by continuing productivity increases.

## 5 Conclusions

The paper derives the long-run consequences of sectoral heterogeneity when sectors differ with respect to resource use. We have shown that sector-specific research activities and induced innovations are crucial for the dynamic behavior of the economy. Research has to overcome the drag on growth that arises from rising resource scarcity. Moreover, resource intensive sectors can only stay competitive if they succeed to achieve faster research growth. According to our results, unconstrained markets provide sufficient incentives to investors that this indeed happens. Most importantly,

<sup>19</sup>The increase of the share is of course limited as  $\mu_x + \mu_z < 1$  has to hold. Positive growth effects which are triggered by a rising share can therefore only be temporary.

we find that research is biased towards resource intensive sectors and that an economy develops along a balanced growth path despite sectoral heterogeneity. By focusing on input substitution in a multi-sector setting the paper adds to recent advances in growth economics. Given the empirical fact of large differences in resource intensity between the sectors and taking into account the predictions of increasing scarcities of natural resources these results are relevant to predict the further development of living standards. Our findings are in line with empirical results on the competitiveness of energy-intensive industries under tight carbon policies, see Demailly and Quirion (2008).

In the second part of the paper we analyzed the consequences of different policies aiming at fostering sectoral change and sustainability, i.e. raising growth and lowering resource extraction. First, we considered the implications of traditional policy instruments: subsidies and taxes. It was shown that resource taxes only raise growth and lower resource extraction if the tax rate decreases over time. Labor subsidies, however, are allocation neutral in our model and do not generate any real effects. Subsidies on research activities proved to be more effective, with the level of subsidy rates affecting growth positively – independent of which sector receives the subsidies. Structural effects of policy arise as the effect of research subsidization on market shares depends on which sector is subsidized. Subsidies to research in one sector induce the relative sector share of this sector to decrease.

Secondly, we considered the provision of productive public goods as a possible means to raise sectoral and overall growth. We showed that the introduction of public goods affects growth directly when public good provision is tied to overall consumption. In this case, productivity in the sector in which public goods are provided rises and thereby affects growth as well as sector shares. Increasing the share of consumption devoted to public goods over time, induces a further positive effect on growth. The rising share lowers profitability in the respective sector which leads to a reallocation of labor to the other sector. Due to the increase in the research of this sector, growth rises. In the other sector, research efforts decline, but in the aggregate the induced negative growth effect is again overcompensated by continuing productivity increases. The provision of public goods proves to be an effective tool to enhance growth and simultaneously induce sectoral change.

The present analysis could be extended by realistically assuming that research in the two sectors is subject to different technology risks. In this case the asymmetry in research returns could explain and justify the disproportionate investment in resource extensive sectors as it is required from institutional investors in some countries (see also Bretschger and Pittel 2005 on this topic). In the present set-up, the overproportional investment in the resource extensive sector would simply be crowded out by an

adjustment of non-regulated investment towards the resource intensive sector. This analysis, however, is left for future research.

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## 6 Appendix

### A. No policy scenario

#### A1. Derivation of dynamic system

To derive the equation of motion for  $L_Z$ , (17), substitute intermediates profits, (7), into the no-arbitrage condition for the patent market, (11), which gives

$$g_{V_n} = r - (1 - \beta) \frac{\tilde{\phi}}{1 + \tilde{\phi}} \frac{C}{V_n n} \quad (54)$$

for sector  $x$ .

The equilibrium condition for the research sector, (10), implies  $g_{V_n} = g_w - g_n$ . Substituting the latter as well as (10) into (54) and considering furthermore that from (5) we know that  $\frac{C}{w} \frac{1 + \tilde{\phi}}{\tilde{\phi}} = \frac{L_X}{\alpha \beta}$  gives

$$g_w - g_n = r - \frac{1 - \beta}{a} \frac{\tilde{\phi}}{1 + \tilde{\phi}} \frac{C}{w} = r - \frac{1 - \beta}{a \beta} \frac{L_X}{\alpha}. \quad (55)$$

As (5) implies  $g_w = g_C - g_{L_X} + \frac{1}{1 + \tilde{\phi}} g_{\tilde{\phi}}$  and we have (16) from consumer optimization, (55) can be rewritten as

$$g_C - g_{L_X} + \frac{1}{1 + \tilde{\phi}} g_{\tilde{\phi}} - g_n = g_C + \rho - \frac{1 - \beta}{a \beta} \frac{L_X}{\alpha}. \quad (56)$$



From (5) we also know  $L_X = \frac{\alpha}{\delta}\tilde{\phi}L_Z$  which implies  $g_{L_X} = g_{\tilde{\phi}} + g_{L_Z}$ . Employing these relations as well as (8) gives for (56)

$$\frac{L_n}{a} = \tilde{\phi} \frac{1-\beta}{a\beta} \frac{L_Z}{\delta} - \rho - \frac{\tilde{\phi}}{1+\tilde{\phi}} g_{\tilde{\phi}} - g_{L_Z}. \quad (57)$$

Proceeding equivalently, we get from the no-arbitrage condition of sector  $z$ , (11), that

$$\frac{L_m}{a} = \frac{1-\beta}{a\beta} \frac{L_Z}{\delta} - \rho - \frac{\tilde{\phi}}{1+\tilde{\phi}} g_{\tilde{\phi}} - g_{L_Z}. \quad (58)$$

Adding (57) to (58) and rearranging gives

$$2g_{L_Z} = (1+\tilde{\phi}) \frac{1-\beta}{a\beta} \frac{L_Z}{\delta} - \frac{L_m + L_n}{a} - 2 \frac{\tilde{\phi}}{1+\tilde{\phi}} g_{\tilde{\phi}} - 2\rho. \quad (59)$$

By considering that from the equilibrium condition for the labor market, (9), it follows that  $L_n + L_m = 1 - (1 + \frac{\alpha}{\delta}\tilde{\phi})L_Z$ , we finally get (17):

$$L_Z = \left[ \frac{1}{2a\delta} \left( \alpha + \delta\tilde{\phi} + (1+\tilde{\phi}) \frac{1-\beta}{\beta} \right) L_Z - \frac{1}{2a} (1 + 2a\rho) - \frac{\tilde{\phi}}{1+\tilde{\phi}} g_{\tilde{\phi}} \right] L_Z. \quad (60)$$

To get an expression for  $\dot{\tilde{\phi}}$  first consider that from (3) and the production technologies in the two sectors follows

$$\tilde{\phi}^{\frac{\nu}{\nu-1}} = \left( \frac{n}{m} \right)^{\frac{1-\beta}{\beta}} \frac{L_X^\alpha}{L_Z^\delta} \frac{R_X^{1-\alpha}}{R_Z^{1-\delta}}. \quad (61)$$

Consideration of  $L_X = \frac{\alpha}{\delta}\tilde{\phi}L_Z$  and  $R_X = \frac{1-\alpha}{1-\delta}\tilde{\phi}R_Z$  gives

$$\tilde{\phi}^{\frac{1}{\nu-1}} = \left( \frac{n}{m} \right)^{\frac{1-\beta}{\beta}} \left[ \frac{\alpha^\delta (1-\alpha)^{1-\delta}}{\delta^\delta (1-\delta)^{1-\delta}} \right] L_X^{\alpha-\delta} R_X^{\delta-\alpha}. \quad (62)$$

Differentiating (62) with respect to time and expressing the resulting expression in growth rates gives after substituting  $g_n = \frac{L_n}{a}$  and  $g_m = \frac{L_m}{a}$

$$\frac{1}{\nu-1} g_{\tilde{\phi}} = \frac{1-\beta}{a\beta} (L_n - L_m) + (\alpha - \delta)(g_{L_X} - g_{R_X}). \quad (63)$$

For the difference in the input of labor in the two types of R&D it follows from (57) and (58) that

$$L_n - L_m = (\tilde{\phi} - 1) \frac{1-\beta}{\beta} \frac{L_Z}{\delta}. \quad (64)$$

Furthermore, (5), (14) and (16) imply that  $g_{R_X} = \frac{1}{1+\tilde{\phi}} g_{\tilde{\phi}} - \rho$ . By substituting this relation as well as (64) into (63), we get

$$\frac{1}{\nu-1} g_{\tilde{\phi}} = -(1-\tilde{\phi}) \frac{1}{\delta a} \left( \frac{1-\beta}{\beta} \right)^2 L_Z + (\alpha - \delta)(g_{L_X} - \frac{\tilde{\phi}}{1+\tilde{\phi}} g_{\tilde{\phi}} + \rho). \quad (65)$$

Recalling  $g_{L_X} = g\tilde{\phi} + g_{L_Z}$  and (17) finally gives (18):

$$\dot{\tilde{\phi}} = \left[ \frac{\left( \frac{1}{2a\delta}(\alpha - \delta) \left( \alpha + \delta\tilde{\phi} + (1 + \tilde{\phi})\frac{1-\beta}{\beta} \right) - (1 - \tilde{\phi})\frac{1}{\delta a} \left( \frac{1-\beta}{\beta} \right)^2 \right) L_Z - \frac{1}{2a}(\alpha - \delta)}{\frac{1}{\nu-1}} \right] \tilde{\phi}. \quad (66)$$

## A2. Balanced growth path

From (60) and (66) the BGP values of  $L_Z$  and  $\tilde{\phi}$  can be obtained by setting  $\dot{L}_Z = \dot{\tilde{\phi}} = 0$  which gives (19) and (20). Considering furthermore that  $\dot{L}_Z = 0$ , we get the BGP labor shares in the two research sectors, (24) and (25), from (57) and (58).

## A3. Stability

To check for the stability properties of the system, we derive the Jacobian of (17) and (18) in the proximity of the steady state,  $\tilde{\phi}^*$  and  $L_Z^*$ ,

$$D = \begin{pmatrix} \left. \frac{\partial \dot{\tilde{\phi}}}{\partial \tilde{\phi}} \right|_{\tilde{\phi}^*, L_Z^*} & \left. \frac{\partial \dot{\tilde{\phi}}}{\partial L_Z} \right|_{\tilde{\phi}^*, L_Z^*} \\ \left. \frac{\partial \dot{L}_Z}{\partial \tilde{\phi}} \right|_{\tilde{\phi}^*, L_Z^*} & \left. \frac{\partial \dot{L}_Z}{\partial L_Z} \right|_{\tilde{\phi}^*, L_Z^*} \end{pmatrix}. \quad (67)$$

It can be shown that

$$\begin{aligned} \det D &= -(1 - \nu) \frac{\tilde{\phi}^{*2} L_Z^*}{2a^2 \delta^2} \left( \frac{1-\beta}{\beta} \right)^2 \left( 2 \frac{1-\beta}{\beta} + \alpha + \delta \right) \\ \text{tr} D &= \frac{1}{2a\delta} \frac{L_Z^* \left[ (1 + \tilde{\phi}^*) \left( \alpha \tilde{\phi}^* + \delta + (1 + \tilde{\phi}^*) \frac{1-\beta}{\beta} \right) + (\nu - 1) \tilde{\phi}^* \left( 4 \left( \frac{1-\beta}{\beta} \right)^2 + (\alpha - \delta)^2 \right) \right]}{1 + \tilde{\phi}^*} \end{aligned} \quad (68)$$

with  $\det D \geq 0$  for  $\nu \geq 1$  and  $\text{tr} D > 0$  for  $\nu > 1$ . As  $\text{tr} = EV_1 + EV_2$  and  $\det = EV_1 \cdot EV_2$  (with  $EV_1$  and  $EV_2$  being the eigenvalues of the system), one eigenvalue is negative for  $\det < 0$  while for  $\det > 0$  and  $\text{tr} > 0$  both eigenvalues are positive. Since our system contains one jump variable and one predetermined variable, the economy is saddle-path stable for  $\nu < 1$ .

## B. Generalization of results

Let us shortly consider the case of additional heterogeneity with respect to the gains of specialization. Recall that when sectors only differ with respect to research intensities, (30) describes how BGP research is affected by sectoral heterogeneity. This equation modifies in the presence of heterogeneous gains of specialization to

$$\frac{1 - \beta_z}{\beta_z} g_m^* - \frac{1 - \beta_x}{\beta_x} g_n^* = (\alpha - \delta) \rho \quad (69)$$

with  $\beta_x$  and  $\beta_z$  representing the gains of specialization in sector  $x$  and  $z$  respectively. Higher gains of specialization in the sector  $x$  (i.e.  $\beta_z > \beta_x$ ) would therefore imply an even stronger bias towards  $z$ -research.

Note that research productivities do not enter (30) and (69). Sectoral differences in  $a$  affect the labor input in each research sector as well as the allocation of labor between research and intermediates (and thereby the levels of  $g_n$  and  $g_m$ ). They do, however, not affect the functional relationship between  $g_n$  and  $g_m$ .

## C. Policy analysis 1

### C1. Derivation of dynamic system

The policy maker can employ three types of instruments: resource taxes, research subsidies and labor subsidies. The governmental budget constraint reads

$$\tau p_R R = s_m w L_m + s_n w L_n + s_w w + T, \quad s_n, s_m, s_w < 1 \quad (70)$$

where  $\tau$ ,  $s_n$ ,  $s_m$  denote the resource tax rate and the subsidy rates on  $x$ - and  $z$ -sector research respectively.  $s_w$  is the subsidy rate on labor.  $T$  denotes lump-sum taxation or subsidization of households that balance the government's budget at every point in time.

The profit function of the individual intermediate producer in the  $x$ -sector reads after taxation and subsidization

$$\Pi_{x_i} = p_{x_i} x_i - \bar{\tau} p_R R_{x_i} - \bar{s}_w w L_{x_i} \quad (71)$$

and equivalent for producers in sector  $z$ . Please note that for notational convenience we denote  $\bar{\tau} = 1 + \tau$  and  $\bar{s}_w = 1 - s_w$ . It is assumed that individual producers do not take account of the effect of their production on public good provision, such that the modified first-order conditions for labor and resource input are given by

$$L_{x_i} = \alpha \beta \frac{\tilde{\phi}}{1 + \tilde{\phi}} \frac{C}{\bar{s}_w w} \quad \text{and} \quad R_{x_i} = \alpha \beta \frac{\tilde{\phi}}{1 + \tilde{\phi}} \frac{C}{\bar{\tau} p_R} \quad (72)$$

and firms' equilibrium profits are still equal to (7).

The research firms' profit functions in case of labor *and* research subsidies read

$$\Pi_l = p_{V_l} \dot{l} - \bar{s}_l \bar{s}_w w L_l, \quad l = n, m \quad (73)$$

where  $\bar{s}_l = 1 - s_l$  and it is assumed that the research subsidy is paid on the basis of the wage bill after labor subsidization. In equilibrium the value of a patent has again to be equalized to marginal costs, such that

$$V_l = \frac{a w \bar{s}_l \bar{s}_w}{l}. \quad (74)$$

Proceeding as in Appendix A1 we get a modified system of differential equations that describe the system's dynamics:

$$\dot{L}_Z = \left[ \frac{1}{2a\delta} \left( \delta + \tilde{\phi}\alpha + \frac{1-\beta}{\beta} \left( \frac{1}{\bar{s}_m} + \frac{\tilde{\phi}}{\bar{s}_n} \right) \right) L_Z - \rho - \frac{1}{2a} + \frac{1}{2}(g_{\bar{s}_m} + g_{\bar{s}_n}) - \frac{\tilde{\phi}}{1+\tilde{\phi}} g_{\tilde{\phi}} \right] L_Z \quad (75)$$

$$\begin{aligned} \dot{\tilde{\phi}} = & \left[ \left( \frac{1}{2a} \frac{\alpha - \delta}{\delta} \left( \delta + \tilde{\phi}\alpha + \frac{1-\beta}{\beta} \left( \frac{1}{\bar{s}_m} + \frac{\tilde{\phi}}{\bar{s}_n} \right) \right) - \frac{1}{\delta a} \left( \frac{1-\beta}{\beta} \right)^2 \left( \frac{1}{\bar{s}_m} - \frac{\tilde{\phi}}{\bar{s}_n} \right) \right) L_Z \right. \\ & \left. - \frac{1}{2a}(\alpha - \delta) - \frac{1-\beta}{\beta}(g_{\bar{s}_m} - g_{\bar{s}_n}) + \frac{1}{2}(\alpha - \delta)(g_{\bar{s}_m} + g_{\bar{s}_n} + 2g_{\tilde{\tau}}) \right] (\nu - 1)\tilde{\phi}. \end{aligned} \quad (76)$$

The BGP values of  $\tilde{\phi}$  and  $L_Z$ , (35) and (36), follow from (75) and (76) by considering that along the balanced path  $g_{\tilde{\phi}} = g_{L_Z} = 0$ . The system is again saddle-path stable for  $\nu < 1$ .

## C2. Comparative statics

Using the BGP values of  $\tilde{\phi}$ ,  $g_R$  and  $g_C$  we can derive the comparative statics results for the three policy instruments where we denote<sup>20</sup>

$$\begin{aligned} G_1 &= aD_1 - A < 0 \\ G_2 &= \frac{\bar{s}_n}{\bar{s}_m} \frac{a(1-\beta)^2(2(1-\beta) + \beta(\alpha\bar{s}_n + \delta\bar{s}_m))}{G_1^2} > 0. \end{aligned}$$

As  $\bar{s}_i = 1 - s_i$ ,  $i = n, m$ , and  $\bar{\tau} = 1 + \tau$  we get  $g_{\bar{s}_i} = -\frac{s_i}{1-s_i}g_{s_i}$  and  $g_{\bar{\tau}} = \frac{\tau}{1+\tau}g_{\tau}$  such that  $\frac{d\bar{s}_i}{ds_i} = -1$  and  $\frac{d\bar{\tau}}{d\tau} = 1$  as well as  $\frac{dg_{\bar{s}_i}}{dg_{s_i}} < 0$  and  $\frac{dg_{\bar{\tau}}}{dg_{\tau}} > 0$ . The comparative statics results are given by

$$\begin{aligned} \frac{d\tilde{\phi}^{p1*}}{dg_{\tau}} &= -\beta \frac{\alpha - \delta}{\delta} L_Z^{p1*} \left( \delta + \tilde{\phi}\alpha + \frac{1-\beta}{\beta} \left( \frac{1}{s_{\bar{s}_m}} + \frac{\tilde{\phi}^{p1*}}{s_{\bar{s}_n}} \right) \right) G_2 \frac{dg_{\bar{\tau}}}{dg_{\tau}} < 0 \\ \frac{dg_R^{p1*}}{dg_{\tau}} &= -\frac{dg_{\bar{\tau}}}{dg_{\tau}} < 0 \\ \frac{dg_C^{p1*}}{dg_{\tau}} &= \left( \frac{\left( \frac{1-\beta}{\beta} + \alpha s_n \right)(\alpha - \delta)}{2\frac{1-\beta}{\beta} + (\alpha s_n + \delta s_m)} - (1 - \delta) \right) \frac{dg_{\bar{\tau}}}{dg_{\tau}} < \left( \frac{\frac{1-\beta}{\beta} + \alpha s_n}{2\frac{1-\beta}{\beta} + (\alpha s_n + \delta s_m)} - 1 \right) (1 - \delta) \frac{dg_{\bar{\tau}}}{dg_{\tau}} < 0 \end{aligned}$$

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<sup>20</sup> $G_1 = aD_1 - A < 0$  holds as it can be shown that  $L_Z^{p1*} = \frac{\beta\delta s_m}{(1-\beta)^2(2(1-\beta) + \beta(\alpha\bar{s}_n + \delta\bar{s}_m))}(-G_1)$  which is positive for an interior equilibrium, such that  $G_1 < 0$ .

$$\begin{aligned}
\frac{d\tilde{\phi}^{p1*}}{dg_{s_m}} &= ((1-\beta)(1+2a\rho-2ag_{\bar{s}_n}) - a\beta(\alpha-\delta)(\rho+g_{\bar{\tau}})) G_2 \frac{dg_{\bar{s}_m}}{dg_{s_m}} < 0 \\
\frac{d\tilde{\phi}^{p1*}}{dg_{s_m}} &= \frac{(1-\beta)\delta\bar{s}_m}{2(1-\beta) + \beta(\alpha\bar{s}_n + \delta\bar{s}_m)} \frac{dg_{\bar{s}_m}}{dg_{s_m}} < 0 \\
\frac{d\tilde{\phi}^{p1*}}{ds_m} &= (1-\beta) \frac{\bar{s}_n}{\bar{s}_m^2 G_1 G_2} \frac{d\bar{s}_m}{ds_m} > 0 \\
\frac{d\tilde{\phi}^{p1*}}{ds_m} &= \frac{\delta G_1}{a(2(1-\beta) + \beta(\alpha\bar{s}_n + \delta\bar{s}_m))} \frac{d\bar{s}_m}{ds_m} > 0 \\
\frac{d\tilde{\phi}^{p1*}}{dg_{s_n}} &= (-(1-\beta)(1+2a\rho-2ag_{\bar{s}_n}) - a\beta(\alpha-\delta)(\rho+g_{\bar{\tau}})) G_2 \frac{dg_{\bar{s}_n}}{dg_{s_n}} > 0 \\
\frac{d\tilde{\phi}^{p1*}}{dg_{s_n}} &= \frac{(1-\beta)\alpha s_n}{2(1-\beta) + \beta(\alpha\bar{s}_n + \delta\bar{s}_m)} \frac{dg_{\bar{s}_n}}{dg_{s_n}} < 0 \\
\frac{d\tilde{\phi}^{p1*}}{ds_n} &= ((1-\beta) \frac{1}{\bar{s}_n G_1 G_2} \frac{d\tilde{\phi}^{p1*}}{dg_{\bar{s}_n}} \frac{d\bar{s}_n}{ds_n} < 0 \\
\frac{d\tilde{\phi}^{p1*}}{ds_n} &= \frac{s_m}{s_n} \frac{\alpha\phi^{p1*} G_1}{a(2(1-\beta) + \beta(\alpha\bar{s}_n + \delta\bar{s}_m))} \frac{d\bar{s}_n}{ds_n} > 0.
\end{aligned}$$

Note that  $M = [(1-\beta)(1+2a\rho-2ag_{\bar{s}_n}) - a\beta(\alpha-\delta)(\rho+g_{\bar{\tau}})] > 0$ , as claimed for  $\frac{d\tilde{\phi}^{p1*}}{dg_{s_m}} < 0$ , can be proofed as follows: It was shown that  $G_1 = aD_1 - A < 0$  for  $L_Z^{p1*} > 0$  (see Footnote 6). From  $\tilde{\phi}^{p1*} = \frac{\bar{s}_n}{s_m} \frac{A-aD_1}{B-aD_2} > 0$ , this implies that also  $B - aD_2 > 0$ . Now it can be shown that

$$(A - aC) - (1-\beta)M = -a\beta\delta\bar{s}_m K \quad (77)$$

$$(B - aD) - (1-\beta)M = a(2(1-\beta) + \alpha\beta\bar{s}_n)K \quad (78)$$

with  $K = ((1-\beta)(g_{\bar{s}_n} - g_{\bar{s}_m}) + (\alpha-\delta)(g_{\bar{\tau}} + \rho))$ . As  $A - aD_1 > 0$  and  $B - aD_2 > 0$ , it follows from (77) and (78) that  $M < 0$  is not feasible, as in this case RHSs of the above two equations would have to be simultaneously positive.

## D. Policy analysis 2

### D1. Derivation of dynamic system

To endogenize  $C$  in (46), express (2) in terms of  $\tilde{X}$ , resp.  $\tilde{Z}$ , only. Recall that  $\tilde{Z} = \tilde{\phi}^{\frac{\nu}{1-\nu}} \tilde{X}$  follows from (3), such that (2) reads

$$C = \left( \frac{1+\tilde{\phi}}{\tilde{\phi}} \right)^{\frac{\nu}{\nu-1}} \tilde{X} = \left( 1+\tilde{\phi} \right)^{\frac{\nu}{\nu-1}} \tilde{Z}. \quad (79)$$

Inserting (79) into (46) and rearranging gives

$$\tilde{X} = n^{\frac{1-\beta}{\beta} \frac{1}{\alpha}} L_X R_X^{\frac{1-\alpha}{\alpha}} \mu_x^{\frac{1-\alpha}{\alpha}} \left( \frac{1+\tilde{\phi}}{\tilde{\phi}} \right)^{\frac{\nu}{\nu-1} \frac{1-\alpha}{\alpha}}. \quad (80)$$

$$\tilde{Z} = m^{\frac{1-\beta}{\beta} \frac{1}{\delta}} L_Z R_Z^{\frac{1-\delta}{\delta}} \mu_z^{\frac{1-\delta}{\delta}} (1+\tilde{\phi})^{\frac{\nu}{\nu-1} \frac{1-\delta}{\delta}}. \quad (81)$$

Again proceeding as in the no-policy section we derive the modified dynamic system of this economy:

$$\dot{L}_Z = \left[ \frac{1}{2a\delta} \left( \delta + \tilde{\phi}\alpha + \frac{1-\beta}{\beta} (1+\tilde{\phi}) \right) L_Z - \rho - \frac{1}{2a} - \frac{\tilde{\phi}}{1+\tilde{\phi}} g_{\tilde{\phi}} \right] L_Z \quad (82)$$

$$\begin{aligned} \dot{\tilde{\phi}} = & \frac{(\nu-1)(1+\tilde{\phi})}{\alpha(1-2\nu)(1+\tilde{\phi}) + \alpha - \delta} \left[ (\alpha - \delta) \left( \frac{1}{2a} \frac{1-\beta}{\beta} - \rho \right) + (\alpha(1-\delta)g_{\mu_z} - \delta(1-\alpha)g_{\mu_x}) \right. \\ & \left. - \frac{1}{2a\delta} \frac{1-\beta}{\beta} \left( \left( \frac{1-\beta}{\beta} (\alpha + \delta) - \alpha\delta \right) (1-\tilde{\phi}) - (\alpha^2\tilde{\phi} - \delta^2) \right) L_Z \right] \tilde{\phi}. \end{aligned} \quad (83)$$

The BGP values of  $\tilde{\phi}$  and  $L_Z$ , (47) and (48), follow from (82) and (83) by considering that along the balanced path  $g_{\tilde{\phi}} = g_{L_Z} = 0$ . As in the policy scenario 1 and the no-policy case, the system is saddle-path stable for  $\nu < 1$ .

## D2. Comparative statics

Taking the derivatives of  $\tilde{\phi}^{p2*}$ ,  $g_R^{p2*}$  and  $g_C^{p2*}$ , as given by (47) to (51), with respect to  $\mu_k$  and  $g_{\mu_k}$  gives:

$$\begin{aligned} \frac{d\tilde{\phi}^{p2*}}{dg_{\mu_x}} &= -(1-\alpha)\delta \frac{a^2\beta^2(1-\beta)(1+2a\rho)}{H_1H_2} < 0 \\ \frac{dg_C^{p2*}}{dg_{\mu_x}} &= \frac{1-\beta}{\beta} \frac{1-\alpha}{a} (1-\beta + \alpha\beta)H_2 > 0 \\ \frac{d\tilde{\phi}^{p2*}}{dg_{\mu_z}} &= (1-\delta)\alpha \frac{a^2\beta^2(1-\beta)(1+2a\rho)}{H_1H_2} > 0 \\ \frac{dg_C^{p2*}}{dg_{\mu_z}} &= \frac{1-\beta}{\beta} \frac{1-\delta}{a} (1-\beta + \delta\beta)H_2 > 0 \end{aligned}$$

with

$$\begin{aligned} H_1 &= (\delta B - aE_2)^2 > 0 \\ H_2 &= \frac{a\beta}{(1-\beta)((\alpha + \delta)(1-\beta) + \beta(\alpha^2 + \delta^2))} > 0. \end{aligned}$$

## E. Proof of Proposition 3

Due to the productivity effect of public goods, labor inputs in  $x$ - and  $z$ -sector research change as follows compared to the no-policy scenario (assuming that  $g_{\mu_k} = 0$ ,  $k = x, z$ ):

$$L_n^{p2*} - L_n^* = (1 - \beta + \delta\beta)\Omega \quad (84)$$

$$L_m^{p2*} - L_m^* = -(1 - \beta + \alpha\beta)\Omega \quad (85)$$

with  $\Omega = f(a, \alpha, \beta, \delta, \rho, g_{\mu_z}, g_{\mu_x})$ . From (84) and (85) it follows that

$$\text{sgn}(L_n^{p2*} - L_n^*) = -\text{sgn}(L_m^{p2*} - L_m^*), \quad (86)$$

i.e. a policy induced rise in  $L_n$  (resp.  $L_m$ ) has to be accompanied by a decline of  $L_m$  (resp.  $L_n$ ).

Furthermore, public good provision modifies the sectoral equilibrium growth rates (28) and (29) to

$$g_{\tilde{X}}^{p2*} = \frac{1}{\alpha} \left( \frac{1 - \beta}{\beta} \frac{L_n^{p2*}}{a} - (1 - \alpha)\rho \right) \quad (87)$$

$$g_{\tilde{Z}}^{p2*} = \frac{1}{\delta} \left( \frac{1 - \beta}{\beta} \frac{L_m^{p2*}}{a} - (1 - \delta)\rho \right). \quad (88)$$

If  $L_n^{p2*}$  and  $L_m^{p2*}$  were unchanged compared to the no-policy scenario, this would imply  $g_{\tilde{X}}^* < g_{\tilde{X}}^{p2} < g_{\tilde{Z}}^{p2}$  where the relation  $g_{\tilde{X}}^{p2} < g_{\tilde{Z}}^{p2}$  is not compatible with BGP growth (see (27)). Therefore (87) and (88) together with (86) imply that a post-policy BGP with  $g_{\tilde{X}} = g_{\tilde{Z}} = g_C$  can only be compatible with  $L_n^{p2*} - L_n^* > 0$  and  $L_m^{p2*} - L_m^* < 0$ . From (87) we see that this increase in  $L_n$  raises growth.

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