

Tamayo, Cesar E.

**Working Paper**

## Bankruptcy choice with endogenous financial constraints

IDB Working Paper Series, No. IDB-WP-824

**Provided in Cooperation with:**

Inter-American Development Bank (IDB), Washington, DC

*Suggested Citation:* Tamayo, Cesar E. (2017) : Bankruptcy choice with endogenous financial constraints, IDB Working Paper Series, No. IDB-WP-824, Inter-American Development Bank (IDB), Washington, DC,  
<https://doi.org/10.18235/0000728>

This Version is available at:

<https://hdl.handle.net/10419/173879>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*



<http://creativecommons.org/licenses/by-nc-nd/3.0/igo/legalcode>

IDB WORKING PAPER SERIES N° IDB-WP-824

# **Bankruptcy Choice with Endogenous Financial Constraints**

Cesar E. Tamayo

**Inter-American Development Bank  
Institutions for Development Sector**

**July 2017**

# **Bankruptcy Choice with Endogenous Financial Constraints**

Cesar E. Tamayo

Cataloging-in-Publication data provided by the  
Inter-American Development Bank  
Felipe Herrera Library

Tamayo, César E.

Bankruptcy choice with endogenous financial constraints / César E. Tamayo.  
p. cm. — (IDB Working Paper Series ; 824)

Includes bibliographic references.

1. Bankruptcy. 2. Corporate reorganizations. I. Inter-American Development Bank. Connectivity, Markets and Finance Division. II. Title. III. Series.  
IDB-WP-824

<http://www.iadb.org>

Copyright © 2017 Inter-American Development Bank. This work is licensed under a Creative Commons IGO 3.0 Attribution-NonCommercial-NoDerivatives (CC-IGO BY-NC-ND 3.0 IGO) license (<http://creativecommons.org/licenses/by-nc-nd/3.0/igo/legalcode>) and may be reproduced with attribution to the IDB and for any non-commercial purpose, as provided below. No derivative work is allowed.

Any dispute related to the use of the works of the IDB that cannot be settled amicably shall be submitted to arbitration pursuant to the UNCITRAL rules. The use of the IDB's name for any purpose other than for attribution, and the use of IDB's logo shall be subject to a separate written license agreement between the IDB and the user and is not authorized as part of this CC-IGO license.

Following a peer review process, and with previous written consent by the Inter-American Development Bank (IDB), a revised version of this work may also be reproduced in any academic journal, including those indexed by the American Economic Association's EconLit, provided that the IDB is credited and that the author(s) receive no income from the publication. Therefore, the restriction to receive income from such publication shall only extend to the publication's author(s). With regard to such restriction, in case of any inconsistency between the Creative Commons IGO 3.0 Attribution-NonCommercial-NoDerivatives license and these statements, the latter shall prevail.

Note that link provided above includes additional terms and conditions of the license.

The opinions expressed in this publication are those of the authors and do not necessarily reflect the views of the Inter-American Development Bank, its Board of Directors, or the countries they represent.



Contact: Cesar Tamayo, [ctamayo@iadb.org](mailto:ctamayo@iadb.org).

# Bankruptcy Choice with Endogenous Financial Constraints\*

Cesar E. Tamayo<sup>†</sup>

Inter-American Development Bank

## Abstract

In this paper we study firm dynamics and industry equilibrium when firms under financial distress face a non-trivial choice between alternative bankruptcy procedures. Given limited commitment and asymmetric information, financial contracts specify default, renegotiation and reorganization policies. Renegotiation entails a redistribution of social surplus, while reorganization takes the form of enhanced creditor monitoring. Firms with better contract histories are less likely to default, but, contingent on default, firms with better outside options successfully renegotiate, in line with the empirical evidence. Unless monitoring is too costly, renegotiation leads to reorganization, which resembles actual bankruptcy practice. We calibrate the model to match certain aspects of the data on bankruptcy and firm dynamics in the U.S.. Our counterfactual experiments suggest that poorly designed bankruptcy arrangements can increase substantially the fraction of firms facing financial constraints, with sizable negative implications for aggregate output and TFP.

Keywords: Corporate bankruptcy, default, renegotiation, reorganization, financial constraints, firm dynamics.

JEL Classification Numbers: D21, E22, D82, G32, G33, L25

---

\*My special thanks to Roberto Chang, Todd Keister and Cristina Fuentes-Albero for their support and advice. I have benefited from detailed suggestions by Gian Luca Clementi, Pablo D'Erasmus, Matthias Doepke, Nicholas Trachter and three anonymous referees. For useful comments I also thank Aloisio Araujo, Niccolo Battistini, Andres Fernandez, Ricardo Reis, Felipe Schwartzman and seminar participants at Rutgers, IDB-RES, Richmond Fed, Midwest Macro Meetings 2014, LACEA-LAMES 2014 and the 2015 Econometrics Society World Congress (Montreal). Part of this work was completed at the research departments of the Federal Reserve Bank of Richmond and of the IDB; their hospitality and financial support is gratefully acknowledged. A previous version of this paper circulated under the title "Financial Constraints, Bankruptcy Regimes and Firm Dynamics". The views expressed in this paper are my own and not necessarily those of the IDB, its Board of Directors, or the countries it represents. All remaining errors are my sole responsibility.

<sup>†</sup>1300 New York Avenue, NW, Washington, D.C. 20577, email: ctamayo@iadb.org

# 1 Introduction

Models of financial frictions and firm dynamics typically ignore the possibility that troubled firms are rehabilitated.<sup>1</sup> However, recent data from both developed and developing countries strongly suggest that alternatives to liquidation are important resolution mechanisms for financially distressed firms. In the U.S., over a third of bankruptcy filings in any given quarter are for Chapters 11 and 13, while two thirds of overall business failures (including informal bankruptcy and private workouts) are resolved under some reorganization procedure.<sup>2</sup> Moreover, during the last decade, over 21 economies have introduced or improved (in- and/or out-of-court) reorganization or debt restructuring proceedings.<sup>3</sup>

The main contribution of this paper is to propose and calibrate a model of bankruptcy and firm finance, and use it to quantitatively assess the implications of alternative bankruptcy arrangements. A key innovation with respect to the few available models of reorganization and firm dynamics is that, in line with the available empirical evidence, bankruptcy procedures can help alleviate moral hazard in addition to governing renegotiations. Our quantitative analysis shows that this feature of bankruptcy law can have important consequences for industry dynamics and aggregate outcomes.

In the model, entrepreneurs can operate a long-lived project but, due to insufficient wealth, must enter a long-term contractual relationship with a bank to finance set-up costs and working capital in each period. Once started, the firm can be liquidated at any time and the bank can recover a positive scrap value.

Two frictions prevent entrepreneurs from running firms at their optimal scale. First, entrepreneurs have access to a random outside option that is unobservable by the bank, and cannot commit to fulfilling the long-term contract. A novel feature of our model is that the parties can "renegotiate" the terms of the contract (at some cost) when an entrepreneur is tempted to leave the relationship. In the long-term contract, such renegotiation takes the form of a contingency that allows for the entrepreneur's value to increase when she receives a

---

<sup>1</sup>Recent exceptions are [Corbae and D'Erasmus \(2016\)](#) and [Senkal \(2014\)](#).

<sup>2</sup>Chapter 7 of the U.S. Bankruptcy Code governs the process of liquidation in which the assets of a corporation are sold either piecemeal or as a going concern. Alternatives to liquidation are chapters 11 and 13 of the Code, under which an exchange of securities is formally proposed in a reorganization plan.

<sup>3</sup>Some examples of these reform episodes include Spain in 2009, Austria, Colombia and Italy in 2010, Denmark and South Africa in 2011, and more recently, Chile and Germany.

high enough outside option.<sup>4</sup> Secondly, project returns are random and private information to the entrepreneur, which introduces the need for truth-telling incentives. Again, a key innovation of our framework is that, in each period, the parties may decide to reduce this agency problem by paying a cost.

In the optimal contract, only firms with poor contract histories are tempted to leave the relationship, a situation we label as default since it leads to either renegotiation or liquidation. Conditional on default, however, firms receiving better outside options are more likely to renegotiate. Reducing agency is optimal only when the entrepreneur’s continuation value is sufficiently low, a situation we label as reorganization since it allows creditors to exert tighter control over the firm’s revenues. In fact, an instance of default and renegotiation in the current period will always lead the parties to choose a reduced agency path in the following period. Thus, equilibrium in our contracting problem shares some features with modern corporate bankruptcy practice. For instance, liquidation in our model corresponds to Chapter 7 in the U.S. bankruptcy code. Likewise, renegotiation entails a redistribution of surplus and leads to a enhanced creditor oversight, both salient features of most reorganization plans under chapters 11 and 13 of the U.S. bankruptcy code.

We extend the contracting problem to allow for ex-ante heterogeneity in project types, and show that the family of dynamic contracts to which our model belongs introduces financial selection. That is, firms that are ex-ante more productive face a lower unconditional probability of liquidation. We then embed the contracting problem into a standard industry equilibrium framework.

Using data on firm dynamics and on the incidence of bankruptcy from the U.S., we calibrate the model and quantitatively evaluate its implications. The calibrated model can account well for some observed patterns of firm dynamics such as age-specific exit rates and employment shares. Moreover, the model is also successful in replicating certain bankruptcy and capital structure patterns such as the bankruptcy (default) rate, the relative frequency of Chapter 11 versus Chapter 7, the size of firms in Chapter 11 relative to those in Chapter 7, the various recovery rates, and the leverage of new, bankrupt and undistressed firms.

We use the calibrated model to quantify the implications of operating under alternative bankruptcy regimes. Compared with an economy in which entrepreneurs can commit not

---

<sup>4</sup> This redistribution of surplus bears some resemblance with other renegotiation models such as that found in [Yue \(2010\)](#) for the case of sovereign debt.

to breach financial contracts, the main consequence of introducing a default option is to reduce firm exit. However, the effect on aggregate variables such as output and total factor productivity (TFP) depends upon the costs and benefits associated with renegotiation and reorganization. Our counterfactual exercises suggest that a poorly designed bankruptcy code -where renegotiation and reorganization are costly and unattractive- can reduce aggregate output and TFP by as much as 72% and 5%, respectively.

The paper is related to the vast literature on financial markets and firm dynamics pioneered by [Cooley and Quadrini \(2001\)](#), where information and enforcement frictions in financial contracting have implications for the entry, growth and exit of firms. In particular, we borrow heavily from recent developments in the theory of dynamic financial contracting.<sup>5</sup> Studying dynamic contracts is important because there is growing evidence that long-term credit relationships are particularly attractive for small and opaque firms ([Bharath et al. \(2007, 2011\)](#)). Moreover, renegotiations and state contingency -salient features of dynamic contracts- appear to be norm in actual financial contracting ([Roberts and Sufi \(2009\)](#)).

Unlike most of the literature, however, the model we study introduces both information and enforcement frictions simultaneously.<sup>6</sup> As in [Clementi and Hopenhayn \(2006\)](#) and [Quadrini \(2004\)](#), in our model there is asymmetric information in production, and dynamic incentives for truth-telling are required. Additionally, entrepreneurs have access to outside opportunities and cannot commit not to leave the financial contract as in [Albuquerque and Hopenhayn \(2004\)](#) and [Hopenhayn and Werning \(2007\)](#).

A central element in our theory is that institutional arrangements -which we broadly define as bankruptcy law- can help agents cope with financial frictions. Thus, our paper is also related to the long-standing literature on financial distress and corporate bankruptcy.<sup>7</sup> In the theoretical strand of this literature, [White \(1994\)](#) first introduced the notion of corporate bankruptcy as a filtering device: bankruptcy law should be designed so as to force inefficient firms into liquidation and allow efficient ones to be rehabilitated.

Rehabilitated firms must negotiate with their creditors to reduce or postpone interest

---

<sup>5</sup>Recent models of financial frictions and firm dynamics that instead take an incomplete contracts approach and rely on short-term debt include [Arellano et al. \(2012\)](#) and [Midrigan and Xu \(2014\)](#).

<sup>6</sup>A recent exception is [Verani \(2014\)](#) who also studies asymmetric information and limited commitment together in a model of firm dynamics under aggregate uncertainty.

<sup>7</sup>There is also a rich literature that studies consumer bankruptcy. Representative papers in this literature include [Chatterjee et al. \(2007\)](#) and [Li and Sarte \(2006\)](#).



and principal payments. However, as pointed out by [Aghion and Bolton \(1992\)](#), creditors are willing to renegotiate or write-off a fraction of their claims only if they can be credibly protected against borrowers' future opportunistic behavior. The process by which this happens is typically governed by provisions such as chapters 11 and 13 of the U.S. bankruptcy code. In fact, detailed studies of U.S. reorganization cases suggest that creditors often condition renegotiations on the replacement of incumbent management and board of directors ([Gilson \(1990\)](#); [Jostarndt and Sautner \(2008\)](#); [Ayotte and Morrison \(2009\)](#); [Baird and Rasmussen \(2003\)](#)). Moreover, creditors of reorganized firms usually seek to exert greater control over the firms' policies by introducing more stringent debt covenants ([Nini et al. \(2012\)](#); [Bharath et al. \(2013\)](#)).

Finally, the paper is closely related to the few studies that quantify the costs of bankruptcy. For the U.S., early attempts to do so include [Warner \(1977\)](#) and [Altman \(1984\)](#), while the most recent estimates of such costs can be found in [Bris et al. \(2006\)](#). The latter paper shows that, taken together, direct and indirect bankruptcy costs result in recovery rates under liquidation that are on, average close to 50%, much lower than under reorganization where this average is closer to 80%. These results coincide remarkably well with the cross-country survey data presented in [Djankov et al. \(2008\)](#) for high income countries.

The remainder of the paper is organized as follows. In section 2 we present the financial contracting environment and the industry equilibrium framework. In Section 3 we discuss the the baseline calibration of the model along with the main counterfactual exercises. Section 4 concludes. Proofs and derivations not provided in the main text can be found in the Appendix.

## 2 The model economy

We now present a theory of firm finance in which contracting parties are presented with different alternatives to deal with the possibility of financial distress. After describing the environment and contracting problem, we introduce ex-ante project heterogeneity and then embed the contract into a standard industry equilibrium framework.

## 2.1 Preferences and technology

There are two types of agents: entrepreneurs and banks. The entrepreneur has access to a project characterized by a production technology  $F : \{0, 1\} \times R_+ \rightarrow R_+$  which combines working capital,  $k_t \in R_+$ , with project-level productivity  $z_t$ . More specifically,  $F(k_t, z_t) = z_t f(k_t)$ , with  $f(0) = 0, f' > 0, f'' < 0$  and  $\lim_{k \rightarrow 0} f'(k) = \infty, \lim_{k \rightarrow \infty} f'(k) = 0$ . In turn,  $z_t$  is the realization of an i.i.d. random variable  $Z$  with support  $\{z_L, z_H\} = \{0, 1\}$  and  $\Pr(Z = z_H = 1) = \pi$ . The project requires, in addition to  $k_{t \geq 0}$ , an initial set-up cost  $I_0$ . The entrepreneur has wealth  $M < I_0$  so, to operate the technology, she must enter a financial contract with a bank with deep pockets. We refer to projects that are successfully initiated as "firms". In each period in which the firm operates and returns  $z_t$  are realized, the bank expects a repayment from the entrepreneur,  $\tau_t$ , per period.

Both the entrepreneur ( $e$ ) and the bank ( $b$ ) are risk-neutral, discount cash flows at a common rate  $\beta \in (0, 1)$ , and seek to maximize the expected present value of dividends:  $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t d_t^j$ , for  $j = e, b$ . The following assumption is introduced to guarantee the existence of a firm:<sup>8</sup>

**Assumption 1.**  $\{\pi f[(f^{(-1)})'(1/\pi)] - [f^{(-1)}]'(1/\pi)\} (1 - \beta)^{-1} > I_0$

## 2.2 First-best

Under symmetric information and perfect enforcement, this problem is trivial enough: the properties of  $f(\cdot)$  imply that there exists a unique:

$$\tilde{k} = \arg \max_k [f(k) \mathbb{E}Z - k] = \arg \max_k [\pi f(k) - k],$$

which is referred to as the first-best level of working capital for a firm. A planner facing no information or commitment constraints, and concerned with maximizing social surplus only, will choose  $k_t = \tilde{k} \forall t \geq 0$ . Thus, the first-best value of the firm is given by  $\tilde{W} = [\pi f(\tilde{k}) - \tilde{k}] / (1 - \beta)$ , with  $\tilde{V} = \pi f(\tilde{k}) / (1 - \beta)$  being the lifetime expected value accruing to the entrepreneur. The solution to the first-best problem, therefore, implies that all entrepreneurs are able to borrow the first-best level of working capital  $\tilde{k}$  and, once started, firms will never grow, shrink or exit.

---

<sup>8</sup>In what follows, the terms "entrepreneur" and "firm" will be used interchangeably.

## 2.3 Contracts under private information and limited commitment

The problem becomes interesting when private information and limited commitment are introduced as follows. At the beginning of each period, the entrepreneur can leave the project and take an outside option  $s$  which is itself the realization of an i.i.d. random variable with support  $\mathcal{S} = [\underline{s}, \bar{s}]$  and differentiable cdf  $G(s)$ . The entrepreneur will therefore take her outside option whenever  $s$  is higher than the value that she can expect from continuing with the project, given the current terms of the contract. However, as in [Aguiar et al. \(2016\)](#),  $s$  is private information to the entrepreneur so that contract terms may not depend upon  $s$  directly as it does in other models of limited commitment ([Thomas and Worrall \(1990\)](#), [Albuquerque and Hopenhayn \(2004\)](#)). Instead, the contract may be contingent upon reports about  $s$  elicited by the entrepreneur, which we label  $\hat{s}$ .

It is assumed throughout that the outside opportunity is never more valuable than the total value of the project, when the latter were operated at full efficiency, i.e.,  $\bar{s} < \tilde{W}$ . This implies that for some values of  $s$ , the entrepreneur will not want to leave the contract for the outside option.

In every period, for those values of  $s$  for which the entrepreneur may want to leave the contract, reports  $\hat{s}$  may be elicited, at which point the firm may be liquidated or the contract "renegotiated".<sup>9</sup> Although involuntary separations will not happen in equilibrium, we label this situation as "default" since it results in either liquidation or renegotiation. Define  $x_t$  as taking the value of 1 if the entrepreneur defaulted in the period  $t$  and zero otherwise. If the firm is liquidated, the bank appropriates the scrap value of the project,  $\Delta$ , and the entrepreneur receives her outside option. Alternatively, the parties can pay proportional costs,  $\theta$ , to renegotiate the original terms of the contract. Let  $\ell_t = 1$  if the firm was liquidated upon default and  $\ell_t = 0$  if the parties renegotiated. As a timing convention, we assume that liquidation payoffs are received and renegotiation costs are paid at the beginning of the following period.

Next, for those values of  $s$  for which the entrepreneur does not want to leave the contract, the outside option becomes irrelevant and the parties move on to a production stage, where they face an investment decision under asymmetric information. Specifically, as in [Clementi](#)

---

<sup>9</sup>We call this situation "renegotiation" because it entails a redistribution of surplus, although formally this is merely a contingency within the dynamic contract.

and Hopenhayn (2006) (henceforth CH) project returns  $z_t$  are not observed by the bank, so that he must rely on reports by the entrepreneur,  $\hat{z}_t$ .<sup>10</sup> The latter can misrepresents actual returns and divert cash flows at some cost  $\rho^i$ , which depends on the monitoring technology used, itself a decision variable at the beginning of the period (after observing the default decision). In particular,  $\rho^r > 0$  if the parties decide to use a high-quality monitoring technology which costs a fraction  $\mu_r$  of the firm's returns and  $\rho^u = 0$  if they use a low-quality monitoring technology which is costless. Let  $\mathbf{u}_t = 0$  if the parties decide to use the high-quality monitoring technology in  $t$  and  $\mathbf{u}_t = 1$  otherwise. In what follows -and for reasons that will become apparent shortly- we refer to the case in which  $\mathbf{u}_t = 0$  as "reorganization" and label the case of  $\mathbf{u}_t = 1$  as "undistressed".<sup>11</sup> It is assumed that when indifferent the agent will not divert cash flows and that costs  $\theta, \mu_r$  are borne by the bank.

To complete the formal statement of the problem, let  $\tilde{h}^{t-1} = (h^{t-1}, x_t, \mathbf{u}_t)$  denote the interim public history after the default and monitoring quality decisions have been made. For notational convenience, we label choices made after observing  $\tilde{h}^{t-1}$  with a superscript  $i_t \in \{u, r\}$ , where  $i_t = r$  ("reorganization") if  $\tilde{h}^{t-1} = (h^{t-1}, 0, 0)$  so the firm is financed under the high-quality monitoring technology and  $i_t = u$  ("undistressed") if  $\tilde{h}^{t-1} = (h^{t-1}, 0, 1)$  so the firm operates under the low-quality monitoring technology. That is, in what follows, we write, e.g.,  $k_t(h^{t-1}, 0, \mathbf{u}_t)$  as  $k_t^{i_t}(h^{t-1})$ . Furthermore, we adopt the convention  $\hat{z}_t^i = k_t^i = \tau_t^i = \mathbf{u}_t = \emptyset$  if  $x_t = 1$ , and  $\ell_t = \emptyset$  if  $x_t = 0$ . End-of-period public histories are defined recursively as  $h^t = \{h^{t-1}, h_t\} \in \mathcal{H}^t$  where  $h_t = \{x_t, \mathbf{u}_t, \ell_t, \hat{s}_t, [k_t^i, \hat{z}_t^i, \tau_t^i]_{i=u,r}\}$ . The timing of events is depicted in Figure 1.

A contract,  $\sigma$ , is a collection of functions specifying a default decision, a reorganization policy,  $\mathbf{u}_t$ , a liquidation decision,  $\ell_t$ , as well as capital advancements,  $k_t^i$ , and repayments to the bank,  $\tau_t^i : \sigma = \{x_t(h^{t-1}), \mathbf{u}_t(h^{t-1}, x_t), \ell_t(h^{t-1}, x_t, \mathbf{s}_t), [k_t^i(h^{t-1}), \tau_t^i(h^{t-1})]_{i=u,r}\}_{t=0}^\infty$ . This contract implicitly defines discounted sums of future dividends for the bank,  $B_t$ , and for the firm,  $V_t$ . In what follows, we refer to  $V_t$  as the firm's equity.

---

<sup>10</sup>While the agent's reporting strategy may be arbitrarily complicated, the Revelation Principle can be invoked to identify the support of  $Z$  as the set of admissible reports.

<sup>11</sup>As mentioned in the introduction, this costly high-quality monitoring technology shares some features with formal or informal reorganization procedures. First, the use of this alternative is costly as are most (all) cases of reorganization, where dismissing management entails learning costs and payments to trustees, accountants or courts are made. Second, it allows creditors to exert tighter control over the firm's revenues which is one of the purposes of most reorganization cases. Finally, in equilibrium, this reduced agency path will only be taken when the firm is under financial distress -after having experienced a long enough sequence of bad revenue shocks- but before deciding on liquidation.

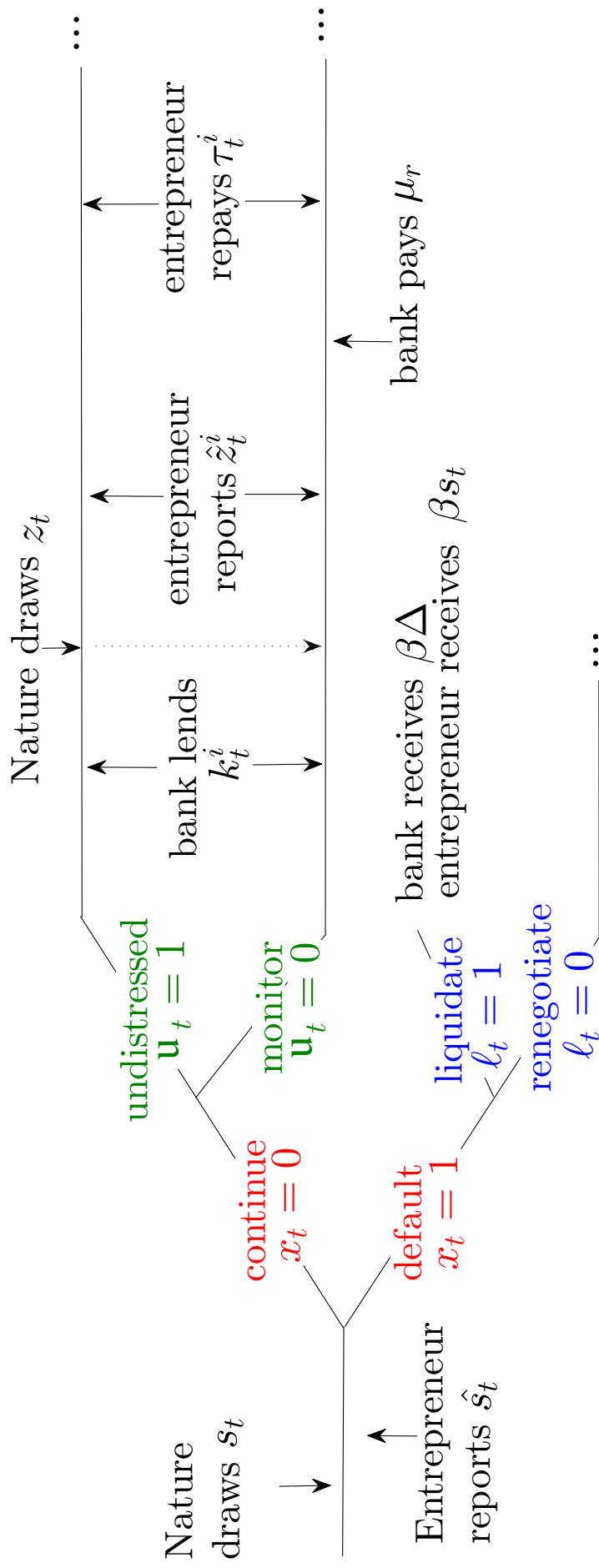


Figure 1: Timing of events within a period

The total asset value after history  $h^t$  is defined by  $W_t = V_t + B_t$ . As in [Spear and Srivastava \(1987\)](#),  $V_t$  effectively summarizes all information provided by history up to  $t - 1$ , and can be used as state variable in a recursive formulation of the contracting problem. The points  $(B(V), V)$  trace the Pareto frontier and  $W(V) = B(V) + V$  is usually referred to as the "value of the firm".

We will characterize contracts recursively by specifying value functions at the different decision stages within a period. Working backwards, consider first the problem of a firm which has not defaulted in the current period ( $x = 0$ ) and is being financed under monitoring quality  $i$ . This problem can be written recursively as:

$$\widehat{W}_i(V_c^i) = \max_{k^i, \tau^i, V_H^i, V_L^i} \pi(1 - \mu_i) f(k^i) - k^i + \beta [\pi W(V_H^i) + (1 - \pi) W(V_L^i)] \quad (\widehat{P}_i)$$

$$s.t. : f(k^i) - \tau + \beta V_H^i \geq (1 - \rho^i) f(k^i) + \beta V_L^i,$$

$$V_c^i = \pi (f(k^i) - \tau^i) + \beta [\pi V_H^i + (1 - \pi) V_L^i],$$

$$f(k^i) \geq \tau^i \quad \text{and} \quad V_H^i, V_L^i \geq 0$$

In  $(\widehat{P}_i)$ ,  $\mu_u = 0$  and  $V_z^i$ ,  $z = H, L$ , is the firm's value of equity beginning the following period after a revenue shock  $z$  has been reported. Moreover, this formulation of the problem already uses the fact that from limited liability  $z_L^i = 0 \Rightarrow \tau_L^i = 0$  and hence  $\tau_H^i$  can be written as  $\tau^i$ . The second constraint in  $(\widehat{P}_i)$  imposes individual rationality (the so-called promise-keeping constraint), while constraints  $f(k^i) - \tau^i, V_H^i, V_L^i \geq 0$  capture the entrepreneur's limited liability. The first constraint in  $(\widehat{P}_i)$  requires that contracts are incentive compatible. Since a realization of  $z_L^i = 0$  will never result in the agent reporting  $z_H^i$ , only one incentive constraint is required. The left hand side of this constraint shows what the entrepreneur receives if she reports truthfully, while the right hand side is what she gets by concealing project returns. Notice that for  $i = u$ ,  $\rho^u = 0$  and we are back to the incentive constraint found CH, while for  $i = r$ ,  $\rho^r > 0$  and the constraint is looser. Loosening the incentive constraint is the sense in which reorganization reduces agency problems.

Next, given  $x = 0$  the problem of choosing monitoring quality is given by:<sup>12</sup>

$$W^c(V_c) = \max_{V_c^u, V_c^r, u} u \widehat{W}_u(V_c^u) + (1 - u) \widehat{W}_r(V_c^r) \quad (P^c)$$

$$s.t. : V_c = u V_c^u + (1 - u) V_c^r \text{ and } V_c^u, V_c^r \geq 0$$

where  $\widehat{W}_i(V_c^i)$ ,  $i = u, r$  satisfy  $(\widehat{P}_i)$ . Now consider the problem of the match when  $x = 1$  and the entrepreneur has reported  $\hat{s}$ . At this point we assume that the entrepreneur will prefer to renegotiate and keep the firm if she gets at least her (reported) outside option in the renegotiated contract. That is, we assume that the bank has all the bargaining power. This implies that for  $x = 1$  and  $\ell \in \{0, 1\}$ , the entrepreneur's payoff is  $\beta \hat{s}$ . Hence, truthful reporting of the outside option by the entrepreneur requires:<sup>13</sup>

$$x(s)\beta s + [1 - x(s)] V_c \geq x(\hat{s})\beta \hat{s} + [1 - x(\hat{s})] V_c \quad \forall s, \hat{s} \in \mathcal{S} \quad (IC)$$

As long as constraints (IC) are satisfied, we can let the terms of the contract depend directly upon  $s$ . Fortunately, the following Lemma allows us to replace these constraints with a simple reservation policy:<sup>14</sup>

**Lemma 1.** *Suppose that for all  $\hat{s} \in \mathcal{S}$ ,  $x(\hat{s}) = 1$  if  $\beta \hat{s} \geq V_c$  and  $x(\hat{s}) = 0$  otherwise. Then constraints (IC) are satisfied.*

**Proof.** See appendix A. ■

In other words, by imposing a simple constraint on the function  $x(\cdot)$  we are able to write our contracting problem in terms of the actual value of the outside option as in most of the limited commitment literature. Now, the decision of whether to liquidate the firm or to continue with the relationship by renegotiating the original contract, for a given realization  $s$ , solves:

$$W_d(s) = \max_{\ell(s) \in \{0, 1\}} \ell(s) W_\ell(s) + [1 - \ell(s)] (1 - \theta) \beta W(s), \quad (P_d)$$

---

<sup>12</sup>Hereafter, the dependence of the policy functions on equity is suppressed and we write e.g.,  $x(s, V)$  as  $x(s)$ . Occasionally we revert to e.g.,  $x(s, V)$  when characterizing these policies as functions of  $V$ .

<sup>13</sup>Again, the Revelation Principle can be used to identify  $\mathcal{S}$  with the set of admissible reports

<sup>14</sup>A static version of this cut-off rule is frequently found in models of financial contracting that feature costly state verification (see, e.g., [Tamayo \(2015\)](#)).

where  $W_\ell(s) = \beta(\Delta + s)$  and we have used the fact that, upon default, the entrepreneur is indifferent between renegotiation and liquidation, so that  $s$  is the appropriate argument for  $W(\cdot)$ . Then the optimal default policy can thus be found as the solution to:

$$W(V) = \max_{x(s) \in \{0,1\}} W^c(V_c) \int_S [1 - x(s)] dG(s) + \int_S x(s) W_d(s) dG(s) \quad (P)$$

$$s.t. \quad V = V_c \int_S [1 - x(s)] dG(s) + \beta \int_S x(s) s dG(s)$$

$$x(s) = \begin{cases} 1, & \text{if } \beta s > V_c \\ 0, & \text{otherwise,} \end{cases}$$

where  $W^c(V_c)$  and  $W_d(s)$  in the objective function of (P) satisfy, respectively, (P<sup>c</sup>) and (P<sub>d</sub>) and the promise-keeping constraint already uses the fact that upon default the entrepreneur receives exactly the value of her outside option.

## 2.4 Optimal contracts

We begin our characterization of optimal contracts by studying the solution to problems ( $\widehat{P}_i$ ) and (P<sup>c</sup>). That is, we first consider optimal policies when  $x = 0$ . Notice that for each  $i = u, r$ , the problem in ( $\widehat{P}_i$ ) is virtually identical to that of CH and hence their main results apply. In particular, capital advancement policy satisfies  $k^i(V^i) < \tilde{k}^i = \arg \max_k [\pi(1 - \mu_i) f(k^i) - k^i]$  as long as  $V^i \leq \tilde{V}^i = \pi(1 - \rho_i) f(\tilde{k}^i) / (1 - \beta)$ ; that is, the firm is borrowing constrained. Along with risk neutrality, this implies that allowing the equity value  $V^i$  to reach the threshold  $\tilde{V}^i$  in the shortest possible time is optimal, i.e.,  $V^i \leq \tilde{V}^i$  implies  $f(k^i) = \tau^i$ . This allows capital to increase with equity values so that the endogenous financing constraints tend to relax as the firm's equity grows. Finally, for  $V^i \leq \tilde{V}^i$  future equity values satisfy  $V_L^i(V^i) < V^i < V_H^i(V^i)$ , implying that the firm's equity value increases with a good shock and decreases with a bad shock ("cash-flow sensitivity"). These results allow us to reduce ( $\widehat{P}_i$ ) to unidimensional maximization problems:



$$\widehat{W}_i(V_c^i) = \max_{k^i} \pi(1 - \mu_i) f(k^i) - k^i + \beta \left\{ \pi W \left[ \frac{V_c^i + (1 - \pi)(1 - \rho^i) f(k^i)}{\beta} \right] + (1 - \pi) W \left[ \frac{V_c^i - \pi(1 - \rho^i) f(k^i)}{\beta} \right] \right\} \quad (P_i)$$

Notice that for  $i = u$ , and using the notation and definitions of the previous paragraph, one has that  $\tilde{k} = \tilde{k}^u$ ,  $\tilde{W} = \tilde{W}^u$  and  $\tilde{V} = \tilde{V}^u$ . For the reminder of the paper, and given that  $\mu_u = \rho^u = 0$ , we write  $\mu = \mu_r$  and  $\rho = \rho^r$ . Denote by  $\tilde{W}^r$  the value of a firm that is currently under "reorganization" and operated at scale  $\tilde{k}^r$ . Then:

**Lemma 2.**  $\mu > 0 \Rightarrow \tilde{W}^r < \tilde{W}$ .

**Proof.** See appendix A. ■

Lemma 2 establishes that for large enough values of equity, leaving the firm undistressed is optimal. Finding conditions under which  $u = 0$  is optimal requires some more work:

**Proposition 1.** *There exist  $\Delta, \mu, \rho \in (0, 1)$  with  $\mu_r < \rho^r$  such that for some  $0 < V_{\mathcal{R}} < \tilde{V}^r$ ,  $V_c < V_{\mathcal{R}} \Rightarrow W^c(V_c) = \widehat{W}_r(V_c^r)$  and  $V_c \geq V_{\mathcal{R}} \Rightarrow W^c(V_c) = \widehat{W}_u(V_c^u)$*

**Proof.** See Appendix A. ■

Heuristically, Proposition 1 says that reorganization is optimal for intermediate values of equity and the firm is left undistressed if equity is large enough. The content and intuition for Proposition 1 can be seen graphically in the left panel of Figure 2, where the function  $W^c(V_c)$  is shown as the upper envelope of the functions  $\widehat{W}_u(V_c^u)$  and  $\widehat{W}_r(V_c^r)$ . An immediate consequence of the proposition is that for some combinations of parameters, the value of the firm is higher when reorganization is an option than when only liquidation is available as in CH (see right panel of Figure 2).

Figure 2 traces the value of the firm as a function of continuation (i.e., no-liquidation) equity  $V_c$ . In the region to the right of  $V_{\mathcal{R}}$  the firm is undistressed but may be financially constrained. In turn, the equity region in which the firm is financially distressed (left of  $V_{\mathcal{R}}$ ) can be divided into liquidation and reorganization. The right panel of Figure 2 compares

the value of the firm under a contract which allows for costly high quality monitoring, with a contract in which only the low quality monitoring technology is available (the CH contract).

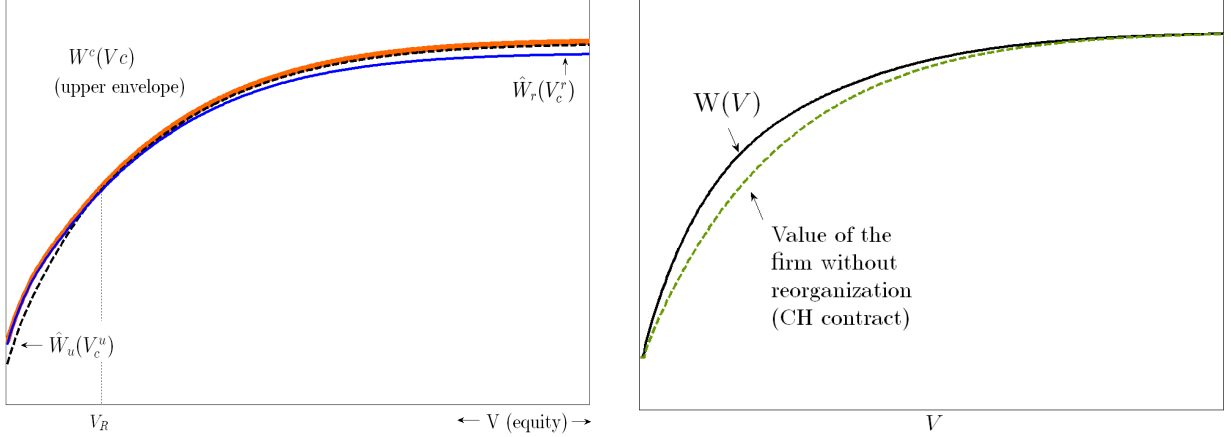


Figure 2: **The reorganization option and the value of the firm.** The left panel depicts the value function as the upper envelope of the values of reorganization and undistressed. The right panel compares the value of the firm with reorganization to the CH contract.

Next, we concentrate on the optimal default and liquidation policies. The first thing to notice is that the availability of an outside option truncates the equity domain from below (i.e., the strategy set shrinks). The lower bound of equity  $\underline{V} > 0$  depends upon the specification of  $s$  and will be derived in Proposition 2. Also, it is easy to see that if  $V > \beta \bar{s}$  the entrepreneur will not default since her maximum default payoff is precisely  $\beta \bar{s}$ . That is,  $V > \beta \bar{s}$  implies  $V_c > \beta \bar{s}$  which in turn implies  $x(s, V) = 0 \forall s$  and  $W(V) = W^c(V_c)$ . In order to provide a sharper characterization of the default and liquidation policies we add the following assumption:

**Assumption 2.** Suppose that  $s \sim U[0, \bar{s}]$  with  $\bar{s} < \tilde{W}^u$

Confronted with any contract, the entrepreneur will employ a reservation policy, taking any outside opportunities above some threshold  $s_d$  and rejecting the rest. That is, for each  $V$ ,  $s \geq s_d \Rightarrow x(s, V) = 1$  and  $s < s_d \Rightarrow x(s, V) = 0$ . The entrepreneur's lifetime utility evolves according to:

$$V = \int_0^{\bar{s}} \max\{\beta s, V_c\} dG(s) = V_c \int_0^{s_d} dG(s) + \beta \int_{s_d}^{\bar{s}} s dG(s)$$

We have seen that if  $V \geq \beta \bar{s}$  the entrepreneur will not default. For  $V < \beta \bar{s}$  the following proposition characterizes the default policy in the optimal contract:

**Proposition 2.** *Suppose that Assumption 2 is satisfied. Then in the optimal contract, for  $V < \beta \bar{s}$ , the default threshold,  $V \mapsto s_d(V)$ , is strictly increasing and  $s_d(V) > 0 \forall V$ .*

**Proof.** See Appendix A. ■

The intuition of Proposition 2 is straightforward: as the value delivered by the original contract increases, the entrepreneur requires a higher realization of the outside option to be tempted to default. The fact that  $s_d(V) > 0 \forall V$  follows from the truncation of the equity domain introduced by the risk of default. Notice also that  $V \mapsto s_d(V)$  strictly increasing implies that  $s_d^{-1}(\cdot)$  is well-defined and unique. Hence, for each  $s$ , we can find an equity value,  $V_D(s)$ , at which the entrepreneur is indifferent between defaulting or continuing.

We now turn to the formal characterization of the liquidation threshold. Notice that if the contract is renegotiated, the bank receives (after proportional renegotiation costs are paid)  $\beta(1 - \theta)W(s) - \beta s$ . Instead, if the firm is liquidated, the bank receives  $\beta\Delta$ . Thus, the liquidation threshold solves:

$$W(s_\ell) = \frac{\Delta + s_\ell}{1 - \theta} \quad (L)$$

Naturally, if  $\theta$  is too large, equation (L) will not have a solution which, once again, points to the role of bankruptcy costs in shaping renegotiation/liquidation decisions. Unfortunately, low renegotiation costs are not enough to find  $s_\ell$  as equation (L) may not have a unique solution. The following assumption introduces a sufficient condition for  $s_\ell$  to be unique and allows us to provide a sharper characterization of the liquidation decision:

**Assumption 3.**  $W(\bar{s}) > \frac{\Delta + \bar{s}}{1 - \theta}$

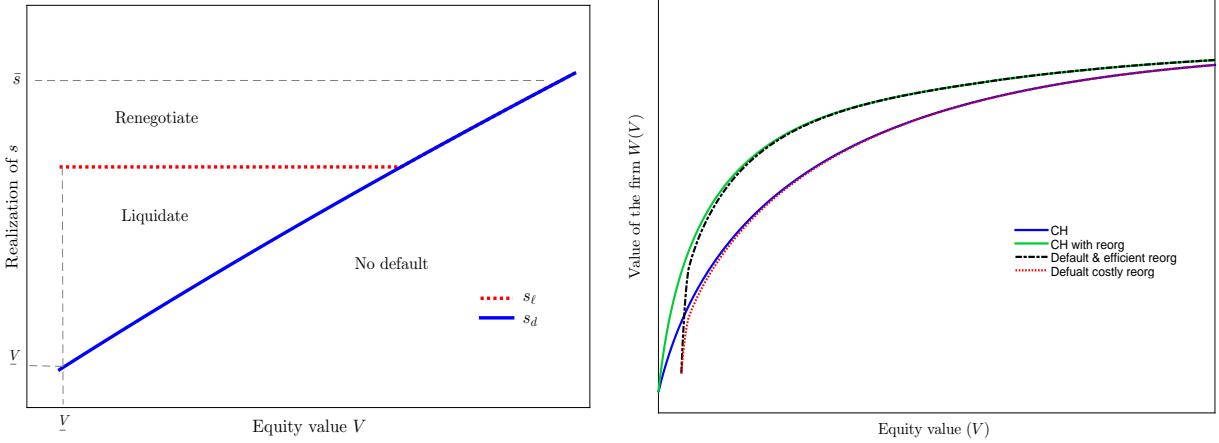
When assumption (3) is satisfied, the bank will find it optimal to renegotiate if  $s$  is sufficiently large and liquidate otherwise. This result is in line with the evidence discussed in the introduction according to which firms with better outside options in the form of alternative financing are more likely to successfully renegotiate their contracts with creditors. We summarize our previous discussion in the following proposition:

**Proposition 3.** *Suppose that assumptions 2 and 3 are satisfied. Then, for each  $V$ ,  $s_\ell$  is unique and satisfies  $s_\ell \geq s_d(V) \forall V$*

With the results from propositions (2)-(3) at hand, the problem in (P) can be conveniently reformulated, for  $V \in [\underline{V}, \bar{s}]$ , as:

$$\begin{aligned} W(V) &= W^c(V_c) \int_0^{s_d} dG(s) + \beta \int_{s_d}^{s_\ell} [\Delta + s] dG(s) + \beta(1-\theta) \int_{s_\ell}^{\bar{s}} W(s) dG(s) \\ &= \frac{1}{\bar{s}} \left\{ s_d W^c(V_c) + \beta \Delta (s_\ell - s_d) + \frac{\beta}{2} (s_\ell^2 - s_d^2) + \beta(1-\theta) \int_{s_\ell}^{\bar{s}} W(s) ds \right\} \end{aligned}$$

where,  $W^c(V_c)$  solves (P<sub>i</sub>),  $s_\ell$  solves (L) and  $s_d = \max\{\underline{V}, \sqrt{2\bar{s}V\beta^{-1} - \bar{s}^2}\}$ . The results from Propositions (2)-(3) can be seen graphically in the left panel of Figure 3 where we have depicted the optimal default, liquidation and renegotiation policies for a given parametrization, as well as the value of the firm when with and without renegotiation and reorganization.



**Figure 3: Liquidation-renegotiation policies and the value of alternative contracts.** The left panel depicts default, renegotiation and liquidation areas as a function of equity-outside option combinations. The right panel compares the value of the firm with and without renegotiation under different costs of reorganization

The right panel of Figure 3 illustrates how the default risk affects the value of the firm. Importantly for our purposes the figure shows that, while the contract with the reorganization option strictly dominates the CH contract, this may or may not be true for the contract with a default option. In particular, the CH contract may dominate the contract with default if the ratio  $\rho/\mu$  is too low; i.e., if the benefit of reorganization is low relative to its cost.

## 2.5 Outside opportunities and renegotiation: Discussion

A brief discussion about the liquidation and renegotiation decisions is in order. First, the fact that in some cases the bank may be willing to increase the firm's equity up to the point where it matches the outside option means that the value of debt,  $B(\cdot)$ , may be increasing in a subset of the equity domain (i.e., that the slope of  $W(\cdot)$  is greater than 1). Whether this actually happens will depend on the value of  $\Delta$  relative to the maximum value of debt,  $\bar{B}$ . If  $\bar{B} < \Delta$  then whenever the entrepreneur defaults the bank will find it more profitable to liquidate the firm regardless of the realization of  $s$ ; in other words,  $s_\ell = \bar{s}$  and there would be no renegotiation.

When  $\bar{B} \geq \Delta$  (so that  $B(\cdot)$  has an increasing segment) the question arises, couldn't the bank always provide the entrepreneur with the equity level that guarantees him  $\bar{B}$ ? A contract with this feature would eliminate any ex-post inefficiency, but the higher lower bound for equity would also mean that the strategy set shrinks further, limiting the incentives for truth-telling and lowering total welfare.<sup>15</sup> In other words, such a contract would be dominated by the contract with renegotiation, which in turn is -assuming no reorganization- dominated by the CH contract where there is two-sided commitment

The previous discussion makes it clear that the outside option does not need to bring any additional value to the firm. It simply makes the threat of separation credible and introduces a new contingency in the contract that increases equity for certain combinations of equity-outside option pairs. Thus, outside opportunities can, in the corporate finance context for instance, take the form of improved financial conditions as in [Roberts and Sufi \(2009\)](#). That paper documents that renegotiations are more likely to be favorable to the borrower when she has access to relatively inexpensive alternative sources of financing. In general equilibrium without aggregate risk, the outside option could correspond to the value of searching for a new project as in [Cooley et al. \(2004\)](#) or [Verani \(2014\)](#), once a stochastic penalty for breach of contract is introduced. Likewise, in an occupational choice framework, the outside option could be identified with an offer in the labor market as in [Buera et al. \(2011\)](#) or [Antunes et al. \(2008\)](#) with an idiosyncratic and stochastic labor ability component.

---

<sup>15</sup>Notice that in the contract with renegotiation some ex-post inefficiencies persist since the renegotiated contract does not result in the bank receiving  $\bar{B}$ . A thorough discussion of this ex-post inefficiency and ex-ante optimality trade-off can be found in [Dovis \(2016\)](#) for the case of sovereign default.

## 2.6 Heterogeneous projects

Suppose now that in every period a continuum of entrepreneurs are born, each of which has access to exactly one project of average productivity  $\pi$  which is drawn from a time-invariant distribution  $\Gamma(\cdot)$  with support  $\Pi = [\underline{\pi}, \bar{\pi}]$ . After project types are drawn, they become public information so that banks offer each entrepreneur a contract indexed by her type  $\pi$ . Accordingly, the value and policy functions are now written e.g.,  $W(V; \pi)$ .

As a first approximation to the effect of project heterogeneity, consider the simple case in which there is neither default nor reorganization (i.e.,  $\bar{s} = \rho = 0$ ), but the firm can be liquidated at the beginning of each period (i.e., the CH contract). A stochastic liquidation rule will specify a cutoff value of equity  $V_{p(\ell)}(\pi)$  such that for  $V \leq V_{p(\ell)}(\pi)$  the firm faces a strictly positive liquidation probability. We will next show that even if there is neither renegotiation nor reorganization, a financial contract that relies on intertemporal incentives for truthful reporting induces selection along the productivity dimension:

**Proposition 4.** *Suppose that  $\pi' > \pi$ . Then  $V_{p(\ell)}(\pi') < V_{p(\ell)}(\pi)$ .*

**Proof.** See Appendix. ■

The content of Proposition 4 is simple: A project with higher average productivity will face a smaller set containing positive liquidation probabilities. Since a firm reaches this set only after experiencing enough bad realizations of the revenue shock, and such low realizations are more likely with lower  $\pi$ , firms with higher average productivity face a lower unconditional probability of exit. In other words, the contract features financial selection.

This intuition can be carried over to the contracting problem of section 2.3. The reorganization option increases the slope of the value function at the origin. With heterogeneous projects, this effect is compounded so that projects with higher average productivity disproportionately benefit from the reorganization option and financial selection is enhanced. This issue will be pursued further in our quantitative analysis.

## 2.7 Industry equilibrium

In order to conduct a meaningful quantitative exercise, the contracting problems studied above are embedded into a standard industry equilibrium framework. The details of the industry follow closely those found in Li (2010).

Incumbent firms behave competitively, taking prices in output ( $p$ ) market as given. Aggregate demand for the product is given by the inverse demand function,  $p = D(Q)$ , where the function  $D$  is continuous, strictly decreasing, and satisfies  $\lim_{Q \rightarrow \infty} D(Q) = 0$  and  $0 < \lim_{Q \rightarrow 0} D(Q) < \infty$ . Notice that the output price is now a state variable and therefore value functions and policy functions depend upon it, for instance,  $W(V, \pi; p)$ .

We assume that all entrepreneurs have the same initial wealth,  $M$ , and that they draw a project from the fixed initial distribution  $\Gamma(\cdot)$  after signing a contract with the lender. This implies that all entrepreneurs receive identical value entitlement upon entry,  $V_0(p)$ , and will sign a contract only if  $V_0(p) \geq M$ . However, the parties know that once the contract is signed, the project type becomes common knowledge. Therefore, the lender will sign the contract only if his expected initial value (i.e., his share of the "average contract") is greater than the financing he provides:

$$\int_{\Pi} W(V_0(p), \pi; p) d\Gamma(\pi) - V_0(p) \geq I_0 - M \quad (P0)$$

To generate an invariant distribution of firms, we allow for some exogenous exit. More specifically, we assume that, in every period, entrepreneurs face a time-invariant exogenous probability  $1 - \eta$  of surviving into following period. For simplicity, we assume that if the entrepreneur dies exogenously, the firm becomes obsolete and the bank's payoff is zero.<sup>16</sup>

The state of the industry can be described by the distribution of firms over equity-type pairs  $(V, \pi)$ . Let  $\psi_t$  denote the measure of incumbent firms after (endogenous and exogenous) liquidation has taken place, and let  $E_{t+1}$  stand for the mass of new entrants at the beginning of  $t + 1$ . The law of motion for  $\psi_t$  satisfies:

$$\psi_{t+1} = \mathcal{T}^*(\psi_t, E_{t+1}; p) \quad (ST)$$

The expression for the mapping  $\mathcal{T}^*(\cdot)$  is derived in Appendix A for the economy with default and renegotiation. This is the more general economy and definitions are easily obtained for the other cases by applying suitable changes. We are now in a position to define an equilibrium:

---

<sup>16</sup>This way, exogenous exit merely implies a lower discount rate  $\hat{\beta} = \beta(1 - \eta)$  and does not require modifying the contracting problems of the previous sections.

**Definition 1.** A stationary competitive equilibrium for the industry consists of output  $Q^*$  and price  $p^* \geq 0$ ; policy functions  $[\ell(s, V, \pi; p^*), x(s, V, \pi; p^*)]_{s \in \mathcal{S}}, \mathbf{u}(V, \pi; p^*), V_c(V, \pi; p^*), V_\ell(V, \pi; p^*),$  and  $[V_c^i(V, \pi; p^*), k^i(V_c^i, \pi; p^*), \tau^i(V_c^i, \pi; p^*), V_H^i(V_c^i, \pi; p^*), V_L^i(V_c^i, \pi; p^*)]_{i=r,u},$  as well as value functions  $W(V, \pi; p^*), W_d(s, V_\ell, \pi; p^*), W^c(V_c, \pi; p^*), [\widehat{W}_i(V_c^i, \pi; p^*)]_{i=r,u};$  a measure of incumbent firms  $\psi^*$  and a mass of entrants  $E^*$  such that:

- (i). The value and policy functions solve  $(\widehat{P}_i), (P^c), (P_d),$  and  $(P)^{17}$
- (ii).  $p^* = D(Q^*)$  and  $Q^* = \int \pi f(k(V, \pi; p^*)) \psi(dV, d\pi; p^*)$
- (iii). if  $E^* > 0$ ,  $V_0(p^*) = M$  solves  $(P0)$  with equality, and
- (iv).  $\psi_t = \psi^*$  and  $E_t = E^*$  for all  $t$  solve  $(ST)$

Condition (i) states that all players must optimize while condition (ii) requires goods market clearing. Condition (iii) is the free entry condition for firms; when  $E^* > 0$  then  $V_0(p^*) = M$  and  $\int W(V_0(p^*), \pi; p^*) d\Gamma(\pi) = I_0$ , which pins down  $p^*$  in a stationary equilibrium with positive entry.

### 3 Quantitative analysis

We now calibrate the model presented above to match some salient features of the U.S. economy. With a calibrated model at hand, we then conduct counterfactual experiments aimed at assessing the quantitative importance of alternative bankruptcy regimes.

#### 3.1 Calibration

Our parameter values are listed in Table 1. The model period is a quarter and the price of output is normalized to unity. The discount factor is set at  $\beta = 0.978$  so as to match the average annual real return of the S&P500 over the 1980-2014 period which is 9.2%. As much of the finance literature, we view this as a better measure of the bank's opportunity cost than the risk-free interest rate often used in the RBC literature. We parametrize the

---

<sup>17</sup>With suitable changes in notation to include  $\pi, p$ .



production function as  $f(k) = k^\alpha$  and set  $\alpha = 0.88$ , which is consistent with the values found in the empirical literature that estimates returns to scale.<sup>18</sup>

Next, we choose  $\underline{\pi}$  so that  $s_\ell(\underline{\pi})$  is arbitrarily close to but greater than  $\bar{s}$ , ensuring that even the least productive firms (lowest quality projects) have a non-zero probability of renegotiating their debt and undergo reorganization. We parametrize the distribution of project types as a Pareto distribution:  $\Gamma(\pi) = 1 - [1 + R(\pi)]^{-\zeta}$  where  $R(\pi) \geq 1$  denotes the rank of  $\pi$  in the distribution.<sup>19</sup>

Table 1: Baseline calibration

parameter	description	value
$\beta$	Discount factor	0.978
$\alpha$	Returns to scale	0.88
$\Delta$	Liquidation value (bank)	3.142
$\bar{s}$	Outside option upper bound	4.239
$\theta$	Renegotiation costs	0.452
$\bar{\pi}$	Highest quality project	0.729
$\zeta$	Tail parameter for $\Gamma(\cdot)$	0.315
$V_0$	Initial equity of new firms	3.639
$\eta$	Exogenous exit	0.025
$\mu$	Cost of reducing agency	0.05
$\rho$	Cost of diverting cash flows	0.414

The remaining nine parameters  $\{\eta, \zeta, \bar{\pi}, V_0, \Delta, \bar{s}, \theta, \mu, \rho\}$  are chosen so as to approximately match the eight relevant moments of the U.S. data shown in the upper panel of Table 2. In the model, all these parameters affect all the target moments in a non-linear fashion. However, we can point to moments that are more informative to pin down a given parameter than others. The establishment exit rate –which in the data is 10.6%– is key to identify  $\eta$ , the exogenous exit probability in the model. The tail parameter in the distribution of projects,  $\zeta$ , and the highest quality project,  $\bar{\pi}$ , are chosen to approximate, respectively, the

<sup>18</sup>Using industry-level data Basu and Fernald (1997) Using plant-level data, Lee (2005) finds that returns to scale in manufacturing vary from 0.83 to 0.92.

<sup>19</sup>Having chosen  $\underline{\pi}$ , for any given  $\bar{\pi}$ , we discretize the set  $[\underline{\pi}, \bar{\pi}]$  so that it contains 10 equally spaced gridpoints. This is done for computational reasons; notice that for each simulation of the model one must solve the contracting problem for each  $\pi_j$ ,  $j = 1, \dots, 10$ .

employment share and the average size (relative to all) of the largest 20% of establishments (in the data 78% and 3.92 respectively). The initial equity value,  $V_0$ , is mainly pinned down by the share of employment of establishments from new or entrant firms (ages 0-2) which stands at 5.8%. These four statistics are computed as averages for the 1980-2014 period from the U.S. Census Bureau’s Business Dynamic Statistics (BDS).<sup>20</sup>

Table 2: Moments: data and model

<b>Targeted moments</b>	<b>data</b>	<b>model</b>
Exit rate	0.106	0.106
Default rate	0.0082	0.0084
Relative frequency of reorganization	0.343	0.394
Employment share of largest 20%	0.782	0.741
Avg relative size among largest 20%	3.92	3.63
Recovery rate liquidation	0.514	0.536
Employment share of entrants	0.058	0.028
Relative leverage in reorganization	1.50	1.34
<b>Additional moments</b>	<b>data</b>	<b>model</b>
Exit by liquidation (endog. liq. rate)	0.0047	0.0044
Employment share of ages 0-10yrs	0.246	0.268
Exit rate ages 0-2yrs	0.214	0.233
Exit rate ages 3-10yrs	0.122	0.108
Ratio of assets in reorg. to liquidation	3.18	2.60
Reorganization recovery rate (pre-fees)	1.07	1.098
Leverage of undistressed firms	0.357	0.598
Leverage of entrants	0.721	0.709

Next, the upper bound of the outside option,  $\bar{s}$ , mainly impacts the default rate. This figure is computed as the time series average of the ratio of total business bankruptcies

<sup>20</sup>The BDS dataset includes measures of establishment and firm births and deaths, job creation and destruction by firm size, firm age, and several other statistics on business dynamics for all non-farm sectors of the U.S. economy. For a thorough description of this dataset and its underlying survey –the Longitudinal Business Database– see [Haltiwanger et al. \(2013\)](#).

reported by the American Bankruptcy Institute to total active firms reported by the U.S. Census Bureau, yielding a 0.8% rate. The frequency of renegotiation/reorganization relative to liquidation, which can be found as 34% in Table F-2 of the U.S. Bankruptcy Courts, is used to obtain a value for renegotiation costs,  $\theta$ . The liquidation value that the bank receives,  $\Delta$ , mainly affects the recovery rate under Chapter 7, which is taken from [Bris et al. \(2006\)](#) as 51.4%. Finally, the parameters  $\mu, \rho$  are not easily identified separately. However, from the model it is clear that their ratio determines the slope of  $\widehat{W}_r(V_c^r)$ , which in turn determines the additional leverage that firms are able to obtain when under reorganization. Accordingly, we choose  $\rho/\mu$  so as to approximately replicate the leverage of firms under reorganization relative to the leverage of non-bankrupt firms reported by [Corbae and D’Erasmus \(2016\)](#).<sup>21</sup>

The model is reasonably successful in matching the target moments in the data. In terms of firm dynamics, the model performance is best in hitting the establishment exit rate and worst in matching the employment share of entrants. However, the model does replicate very closely the employment share of establishments from young firms (ages 0-10), and the leverage of new firms –defined as one minus the ratio of internal equity to total financing– as reported in the first wave of the Kauffman Survey ([Sanyal and Mann \(2010\)](#)).

The model results are also consistent with some empirical regularities in terms of firm growth and survival. As in the data, the model exit rate of new establishments is much higher than that of young ones, although this pattern appears to be smoother in the data. Moreover, the left panel of Figure 4 shows that the model is able to generate the negative correlations observed in the data between age and growth, and age and growth dispersion. Notice that, in contrast with CH, [Quadrini \(2004\)](#) and [Verani \(2014\)](#), the model produces significant growth dispersion in the stationary distribution. This follows from the heterogeneity in project types, a feature that is absent in these papers.

With respect to the incidence of bankruptcy procedures, the model matches very closely the bankruptcy rate and the exit by liquidation which in the data stands at 0.47%.<sup>22</sup> In the model, recovery rates for liquidation are calculated as  $\beta\Delta/B_{t-1}$ , where  $B_{t-1} = W_{t-1} - V_{t-1}$

---

<sup>21</sup>Specifically, we fix  $\mu$  at 0.051 so as to approximate median fees as a fraction of assets (1.9%) reported in [Bris et al. \(2006\)](#), and then choose  $\rho$  to match the leverage ratio. From the model we compute the ratio of median leverage of reorganization firms relative to the median leverage of non-bankrupt firms. In [Corbae and D’Erasmus \(2016\)](#), the ratio of the means (1.45) is not too different from the ratio of the medians (1.50); see Table 1 in that paper.

<sup>22</sup>This is very close to the 0.42% firm exit by liquidation reported in [Corbae and D’Erasmus \(2016\)](#).

is the lender's claim prior to default. Likewise, recovery rates for reorganization are defined as  $\beta B(s_t)/B_{t-1}$  where  $B(s_t) = W(s_t) - s_t$  is the lender's claim after renegotiation. The model approximates both recovery rates fairly well.<sup>23</sup> Finally, the fit is less tight in terms of the leverage ratio, mainly due to the higher leverage of undistressed firms implied by the model.

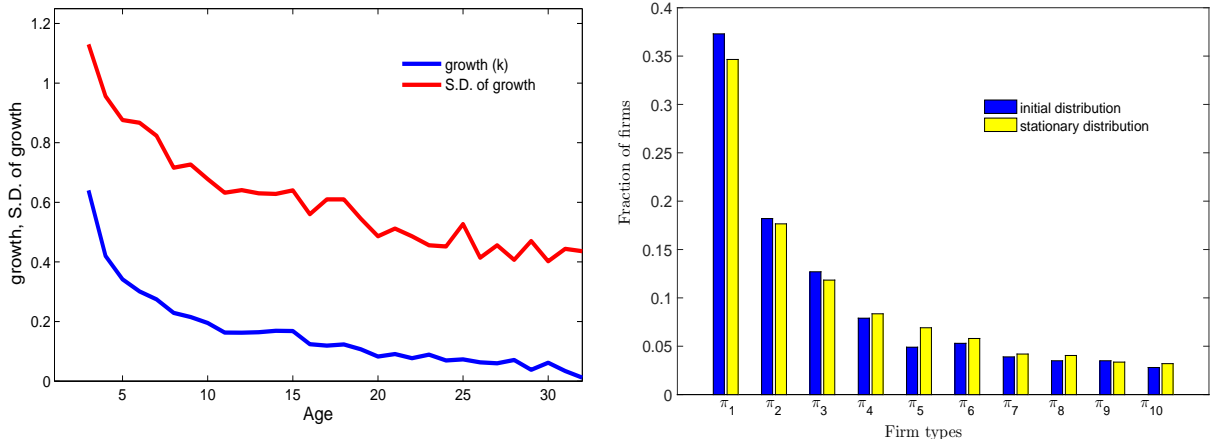


Figure 4: **Financial selection and firm dynamics** The left panel shows age-specific firm growth and standard deviation of firm growth computed as the long-run average from a large sample of firms simulated under the financial contract of section 2 and the baseline calibration. The right panel compares the distribution of project types from  $\Gamma(\cdot)$  with the long-run or stationary distribution of types under the baseline calibration.

The simulations from the model are also in line with the well established fact that firms filing for reorganization are generally larger than those that undergo Chapter 7 liquidation. In fact, the model is able to replicate well the ratio of (median) Chapter 11 assets to (median) Chapter 7 assets as reported in Corbae and D'Erasmus (2016).<sup>24</sup>

The calibrated version of the model generates some financial selection as defined in Section 2.6 above. The right panel of Figure 4 shows that the stationary distribution of project types differs from the initial distribution,  $\Gamma(\cdot)$ . In particular, the stationary distribution has lower frequencies for the three lowest project types and higher frequencies for every other project type.

<sup>23</sup>Our target for liquidation recovery rate corresponds to what Bris et al. (2006) label Chapter 7 "optimistic" secured recovery rate in their Table XIII. This is the most relevant measure since in our model long-term debt is collateralized by  $\Delta$ . The target for reorganization recovery rate can be found as the ratio of post-bankruptcy, pre-fees assets to pre-bankruptcy assets reported in Bris et al. (2006)'s Table III.

<sup>24</sup>Assets in Chapter 11 are defined in Corbae and D'Erasmus (2016) as assets in the initial period of a Chapter 11 bankruptcy. Accordingly, for the numerator of this ratio we compute the median of assets in the period of default but after renegotiation.

### 3.2 Counterfactuals

The counterfactual exercises that we conduct are aimed at quantifying the aggregate and firm dynamics implications of an economy operating under alternative bankruptcy regimes. In terms of the model presented in section 2, such regimes are characterized by different values of the parameters  $\rho, \mu, \theta$  and  $\bar{s}$ .

Table 3 presents some statistics that result from the simulation of economies under different bankruptcy arrangements. The moments from the baseline calibration (labeled "Baseline U.S.") can be found in the first column. When computing aggregate output,  $Y$ , working capital,  $K$ , and TFP, we have defined:

$$Y = \int \pi f(k(V, \pi; p^*)) \psi(dV, d\pi; p^*)$$

$$K = \int k(V, \pi; p^*) \psi(dV, d\pi; p^*)$$

$$TFP = \frac{Y}{K^\alpha}$$

In column two of Table 3, under the label "Liquidation only" we have simulated an economy that operates under the CH contract. That is, we assume that  $\bar{s} = \rho = \mu = 0$ . While  $\bar{s} = 0$  is certainly a strong assumption, it is interesting to consider how the equilibrium would change if agents were able to commit. When comparing these results with those from the baseline calibration, we can see that under the CH contract exit is almost 1.5 times higher, while aggregate output and TFP are 2.3% and 1.3% lower, respectively. The high exit rate in the CH contract also translates into an economy with much younger firms and a larger fraction of unconstrained firms (defined as firms operating at  $\tilde{k}(\pi)$ ).

Next we maintain the commitment assumption ( $\bar{s} = 0$ ) but allow agents to use a very efficient monitoring technology (i.e. a high  $\rho/\mu$  ratio). According to Figure 3, for each project type, this is the contract that provides the highest welfare. Column three in Table 3 (labeled "Liquidation & eff reorg") shows that this result can be extended to the overall economy: the contract with commitment and efficient reorganization maximizes aggregate output and TFP, which are, respectively, 44% and 2.9% higher than the under the baseline calibration. This economy also exhibits the largest share of unconstrained firms.

moment	Baseline "U.S."	Liquidation only	Liquidation & eff reorg	Default only	Default, renege & & eff reorg
Exit rate	0.106	0.149	0.122	0.111	0.109
Aggregate output	1.000	0.977	1.445	0.774	1.331
Aggregate TFP	1.000	0.987	1.0292	0.971	1.019
Average firm age	9.31	7.6	8.4	9.22	9.38
Max firm age	94	84.5	90.4	99	99.3
% of unconstrained	0.054	0.068	0.073	0.053	0.064

Table 3: **Aggregate and firm dynamics implications of alternative bankruptcy regimes** The first column presents the results from the baseline calibration. The second column uses a contract without renegotiation or reorganization. The third column uses a contract without renegotiation but with reorganization using a value of  $\mu$  that is 50% lower and a value of  $\rho$  that is 50% higher than the baseline. Column four uses a contract with  $\bar{s}$  as in the baseline and  $\theta = 1, \rho = \mu = 0$ . In column five,  $\bar{s}$  is as in the baseline but  $\rho$  and  $\mu$  are, respectively, 50% larger and 50% smaller than in the baseline.

In columns four and five of Table 3 we study the implications of limited commitment (i.e.,  $\bar{s} = 4.24$  as in the baseline) under different assumptions about renegotiation and reorganization. Column four, under the label "Default only" presents the results of the simulated economy where renegotiation is very costly and reorganization has little benefit ( $\theta = 1, \rho = \mu = 0$ ). Again, extending the the individual firm specific results of Figure 3, this economy displays the lowest levels of aggregate output and TFP, as well as the smallest fraction of unconstrained firms. Finally, we look at the equilibrium from an economy in which renegotiation is allowed ( $\theta = 0.451$  as in the baseline) and reorganization is relatively efficient (high  $\rho/\mu$ ). These moments, presented in column five under the label "Default, renege & eff reorg", come fairly close to those obtained from the commitment economy: aggregate output and TFP are, respectively, 33% and 1.8% higher than in the baseline calibration. Likewise, this economy exhibits the oldest firms and the largest fraction of unconstrained firms among those economies with default.

These counterfactual exercises tell us that the design of corporate bankruptcy arrangements can have quantitatively important consequences in terms of the aggregate economy. For instance, by comparing columns four and five in Table 3 we can conclude that a poorly designed bankruptcy law, where high costs and limited creditor oversight make renegotiation and reorganization unattractive, can reduce aggregate output and TFP by as much as 72% and 5%, respectively.

## 4 Concluding remarks

In this paper we have presented an industry equilibrium-dynamic contracts approach to corporate default, liquidation and reorganization. In the model, costly bankruptcy arrangements help agents cope with enforcement and information frictions. Importantly, the model allows for a financial selection mechanism whereby firms that are ex-ante less productive face a higher unconditional probability of exit. However, the degree of financial selection depends upon the costs associated with bankruptcy and the benefits from reorganization plans in terms of greater creditor oversight.

A calibrated version of the model is able to reproduce fairly well most firm dynamics and bankruptcy related statistics. Moreover, the counterfactual analysis shows that differences in bankruptcy law can have substantial implications for aggregate outcomes and firm dynamics. In particular, when reorganization is effective in increasing creditor control at a relatively low cost, financial selection can improve capital allocation and increase TFP. The effect on aggregate output is amplified because, as in other models of financial constraints and heterogeneous projects, more productive entrepreneurs tend to run larger firms.

A number of promising areas for further research open up from here. To begin with, the analysis could be extended so as to allow richer dynamics along the entry margin. Some recent work on bankruptcy procedures and firm dynamics suggests that this mechanism can be quantitatively important ([Rodriguez-Delgado \(2010\)](#)). Although tractability issues may arise, the model could also be extended to study the role of bankruptcy arrangements in amplifying or ameliorating the effects of aggregate shocks as in [Verani \(2014\)](#), and in shaping recoveries from pronounced recessions. Circumstantial evidence can be found in [Bergoeing et al. \(2007\)](#) suggesting that differences in bankruptcy law can help account for the asymmetric recoveries of Chile and Mexico from similar negative shocks experienced in the early 1980s.

## 5 Appendix

### 5.1 Proofs and derivations

**Proof of Lemma 1.** The hypothesis of the Lemma, along with the linearity of the outside option and the fact that  $V_c$  is independent of  $s$  implies that  $\exists s^*$  such that  $s \geq s^* \implies x(s) = 1$  and  $s < s^* \implies x(s) = 0$ . Choose any  $s' > s''$ . There are three cases to consider. First, if  $s^* \geq s' > s''$  then  $x(s') = x(s'') = 0$  and the constraint reduces to  $V_c \geq V_c$  which trivially holds. Next, if  $s^* \geq s' > s''$  then  $x(s') = x(s'') = 1$  and the constraint reduces to  $s' \geq s''$  which also holds. Finally, if  $s' \geq s^* > s''$  then  $x(s') = 1 - x(s'') = 1$  and the constraint reduces to  $\beta s' \geq V_c$  which holds by the hypothesis of the Lemma. ■

**Proof of Lemma 2.** A firm currently under reorganization, and operating at  $\tilde{k}^r$  can either become undistressed next period with probability  $\pi$ , or remain under reorganization and have the chance to become undistressed in the following period:

$$\begin{aligned} \tilde{W}_r &= \pi(1 - \mu)f(\tilde{k}^r) - \tilde{k}^r + \beta[\pi\tilde{W} + (1 - \pi)\tilde{W}_r] \\ &= \frac{\beta\pi\tilde{W} + \pi(1 - \mu)f(\tilde{k}^r) - \tilde{k}^r}{1 - \beta(1 - \pi)} \end{aligned} \quad (1)$$

since  $\frac{\beta\pi}{1 - \beta(1 - \pi)} < 1$ , it suffices to show that:

$$\frac{\pi(1 - \mu)f(\tilde{k}^r) - \tilde{k}^r}{1 - \beta(1 - \pi)} < \left[1 - \frac{\beta\pi}{1 - \beta(1 - \pi)}\right]\tilde{W}$$

some tedious algebra and  $\tilde{W} = \frac{\pi f(\tilde{k}) - \tilde{k}}{1 - \beta}$  reduces this to showing that:

$$\pi f(\tilde{k}) - \tilde{k} > \pi(1 - \mu)f(\tilde{k}^r) - \tilde{k}^r$$

which holds given the strict concavity of  $f(\cdot)$  and  $\mu > 0$ . ■

**Proof of Proposition 1.** First notice that  $\widehat{W}_i(V^i) > \widehat{W}_{j \neq i}(V^{j \neq i})$  implies  $V = V^i$  and  $W(V) = \widehat{W}_i(V^i) = \widehat{W}_i(V)$ . Now,  $\widehat{W}_u(V) > \widehat{W}_r(V)$  for  $V$  large enough follows from continuity and Lemma 2, while  $\widehat{W}_u(V) > W_\ell$  is true by Assumption 1 and continuity. Next, to show that  $\widehat{W}_r(V) > \widehat{W}_u(V)$  for some  $V$ , first define  $W_u^*(V)$  as the value of a firm that cannot be liquidated or reorganized and  $W_r^*(V)$  as the value of a firm that always operates under reorganization and



cannot be liquidated. Results analogous to those of the CH contract hold for each of these sub-problems, so one has that  $\tau_*^i = f(k_*^i) = \frac{\beta(V_{H*}^i - V_{L*}^i)}{1 - \rho^i}$ , and therefore  $V = \beta [\pi V_{H*}^i + (1 - \pi) V_{L*}^i]$  for  $i = u, r$ . Now  $W_u^*(V), W_r^*(V)$  can be written:

$$\begin{aligned} W_u^*(V) &= \max_{V_*^{1H}, V_*^{1L} \geq 0} V - \beta V_{L*}^u - f^{-1} \left( \frac{V - \beta V_{L*}^u}{\pi} \right) + \beta \mathbb{E} W_u^*(V) \\ \text{s.t.} \quad & V = \beta [\pi V_{H*}^i + (1 - \pi) V_{L*}^u] = \beta \mathbb{E} V_*^u \end{aligned} \quad (2)$$

and:

$$\begin{aligned} W_r^*(V) &= \max_{V_*^{0H}, V_*^{0L} \geq 0} \frac{(1 - \mu)(V - \beta V_{L*}^r)}{1 - \rho} - f^{-1} \left[ \frac{V - \beta V_{L*}^r}{(1 - \rho)\pi} \right] + \beta \mathbb{E} W_r^*(V) \\ \text{s.t.} \quad & V = \beta [\pi V_{H*}^r + (1 - \pi) V_{L*}^r] = \beta \mathbb{E} V_*^r \end{aligned} \quad (3)$$

clearly the functions  $W_u^*(V), W_r^*(V)$  are increasing, concave and differentiable so the Envelope Theorem applies and:

$$\frac{dW_u^*(V)}{dV} = 1 - \frac{1}{\pi} \left[ f^{-1} \left( \frac{V - \beta V_{L*}^{1L}}{\pi} \right) \right]'$$

where  $[f^{-1}(y)]' = \frac{df^{-1}(y)}{dy}$ , while:

$$\frac{dW_r^*(V)}{dV} = \frac{1 - \mu}{1 - \rho} - \frac{1}{(1 - \rho)\pi} \left[ f^{-1} \left( \frac{V - \beta V_{L*}^r}{(1 - \rho)\pi} \right) \right]'$$

Now  $\rho > \mu$ ,  $V = 0 \Rightarrow V_{L*}^i = 0$  and  $[f^{(-1)}(0)]' = \frac{df^{-1}(0)}{dk} = 0$  together imply that  $\frac{dW_{L*}^r(0)}{dV} > \frac{dW_u^*(0)}{dV}$ . By continuity of the value functions,  $\exists! V^{++}$  such that  $\frac{dW_u^*(V)}{dV} < \frac{dW_r^*(V)}{dV} \forall V \in (0, V^{++})$ . Given  $W_u^*(0) = W_r^*(0) = 0$ , it can be concluded that  $W_u^*(V) < W_r^*(V) \forall V \in (0, V^{++})$ . Next, let  $W_u^\Delta(V)$  denote the value of a firm that is currently undistressed and can be liquidated with scrap value  $\Delta$ , but cannot be reorganized. Define  $W_r^\Delta(V)$  analogously for a firm currently under reorganization. Clearly,  $\lim_{\Delta \rightarrow 0} W_u^\Delta(V) = W_u^*(V)$  and  $\lim_{\Delta \rightarrow 0} W_r^\Delta(V) = W_r^*(V)$ . Then continuity ensures that  $\exists! \Delta^S$  and  $V^S$  such that  $\Delta \in (0, \Delta^S) \Rightarrow W_r^\Delta(V) > W_u^\Delta(V)$  for  $V \in (0, V^S)$ . It remains to show that  $W_r^\Delta(V) > W_u^\Delta(V)$  for some  $V$  is sufficient for  $\widehat{W}_r(V) > \widehat{W}_u(V)$  to hold for some  $V$ . To see this, suppose otherwise (and find a contradiction). That is, suppose  $W_r^\Delta(V) > W_u^\Delta(V)$  for some  $V$  but  $\widehat{W}_r(V) \leq \widehat{W}_u(V) \forall V$ . Then  $\widehat{W}_u(V) \geq \widehat{W}_r(V) \forall V \Rightarrow \widehat{W}_u(V) = W_u^\Delta(V)$ . On the other hand, it must be true that  $\widehat{W}_r(V) \geq W_r^\Delta(V)$  since a policy of never leaving the firm undistressed is clearly feasible and incentive compatible for the problem in  $\widehat{W}_r(V)$ . In other words,  $\widehat{W}_u(V) = W_u^\Delta(V) \geq \widehat{W}_r(V) \geq W_r^\Delta(V) \forall V$ , a contradiction since we

have already shown that  $W_r^\Delta(V) > W_u^\Delta(V)$  for some  $V$ . ■

**Proof of Proposition 2.** Notice that the entrepreneur will default if and only if  $\beta s > V_c$ , so the indifference point is  $\beta s_d = V_c$ . Using the expression for the entrepreneur's lifetime utility we find that  $s_d$  satisfies  $V = \beta s_d \int_0^{s_d} dG(s) + \beta \int_{s_d}^{\bar{s}} s dG(s)$ . Solving yields  $s_d(V) = \sqrt{2\bar{s}V\beta^{-1} - \bar{s}^2}$  so that  $V \mapsto s_d(V)$  is strictly increasing. It is left to show that  $s_d(V) > 0 \forall V$ . It suffices to show that  $V > \beta E(s)$  so we show that  $\underline{V} > \beta E(s)$ . Suppose that  $\underline{V}$  is promised for sure starting in the following period. Then the continuation equity in the current period is  $V_c^i = \beta[\pi \underline{V} + (1 - \pi) \underline{V}] = \beta \underline{V}$ . Hence,  $\underline{V}$  satisfies  $\underline{V} = \beta \underline{V} \int_0^{\underline{V}} dG(s) + \beta \int_{\underline{V}}^{\bar{s}} s dG(s)$ . That is,  $\underline{V}$  solves  $(\beta/2\bar{s}) \underline{V}^2 - \underline{V} + (\beta\bar{s}/2) = 0$ . We are interested in the solution to this quadratic equation that satisfies  $\underline{V} < \bar{s}$ . Thus,  $\underline{V} = -2(\bar{s}/\beta)((1/2)(-\beta^2 + 1)^{1/2} - (1/2)) > \beta(\bar{s}/2) = \beta E(s) > 0$ . ■

**Proof of Proposition 4.** First, we reproduce the statement of the problem as it is found in CH. The value of the firm that is not liquidated at the beginning of the period is given by:

$$\begin{aligned} \widehat{W}(V; \pi) &= \max_{k, V^H, V^L, \tau} \pi f(k) + \beta [\pi W(V_H; \pi) + (1 - \pi) W(V_L; \pi)] \\ s.t. : V_c &= \pi(f(k) - \tau) + \beta[\pi V_H + (1 - \pi) V_L], \\ \beta(V_H - V_L) &\geq \tau, \quad f(k) \geq \tau \quad \text{and} \quad V_H, V_L \geq 0 \end{aligned}$$

while the value of the firm prior to the liquidation decision is:

$$\begin{aligned} W(V; \pi) &= \max_{\ell \in [0, 1]} (1 - \ell) \widehat{W}(V; \pi) + \ell \Delta \\ s.t. : V &= \ell Q + (1 - \ell) V_c, \quad Q \geq 0 \quad \text{and} \quad V_c \geq 0. \end{aligned}$$

As shown in CH, the function  $W(\cdot; \cdot)$  is concave and therefore almost everywhere differentiable, as is  $\widehat{W}(\cdot; \cdot)$ . Moreover, using the result  $f(k) = \tau$  it follows that for  $V < \tilde{V}(\pi)$ :

$$\frac{\partial \widehat{W}(V; \pi)}{\partial \pi} = f(k) + \beta \pi [W(V_H; \pi) - W(V_L; \pi)] (V_H - V_L) \geq 0$$

where the inequality is ensured by  $W(V_H; \pi) \geq W(V_L; \pi)$  and  $V_H \geq V_L$ . Next, notice that  $\frac{\partial \widehat{W}(V; \pi)}{\partial V}$

is increasing in  $\pi$ . To see this, notice that:

$$\frac{\partial \widehat{W}(V; \pi)}{\partial V} = 1 - \frac{1}{\pi} \left[ f^{-1} \left( \frac{V - \beta V^L}{\pi} \right) \right]' = 1 - \frac{1}{\pi} \left\{ \frac{1}{f' \left[ f^{-1} \left( \frac{V - \beta V^L}{\pi} \right) \right]} \right\}$$

where  $[f^{-1}(y)]' = \frac{df^{-1}(y)}{dy}$  and the last equality is by the Inverse Function Theorem. Now,  $f$  increasing implies  $f^{-1}$  is increasing. Therefore,  $f^{-1} \left( \frac{V - \beta V^L}{\pi} \right)$  decreases with  $\pi$ . Moreover,  $f$  concave implies that  $f' \left[ f^{-1} \left( \frac{V - \beta V^L}{\pi} \right) \right]$  increases with  $\pi$ , which in turn means that the term in braces decreases with  $\pi$ . Summarizing, one has that  $\pi' > \pi$  implies  $\widehat{W}(V; \pi') > \widehat{W}(V; \pi)$  and  $\frac{\partial \widehat{W}(V; \pi')}{\partial V} > \frac{\partial \widehat{W}(V; \pi)}{\partial V}$ . Since  $\widehat{W}(0; \pi') > \widehat{W}(0; \pi) = \beta \Delta$ , this establishes that  $V_\ell(\pi') < V_\ell(\pi)$ . ■

### Invariant distribution of firms

Let  $\phi(V, \pi) = 1$  if  $V \in E$  and  $\pi \in Q$ , and zero otherwise. Then  $\psi_t$  satisfies the law of motion:

$$\begin{aligned} \psi_{t+1}(\mathcal{E}, \mathcal{Q}; p) &= (1 - \eta) \int \pi \sum_{i=u,r} \sum_{j=u,r} \left\{ \phi \left[ V_c^j \left( V_H^i(V, \pi; p), \pi; p \right), \pi \right] \times \right. \\ &\quad \left. \left[ 1 - s_\ell \left( V_H^i(V, \pi; p), \pi; p \right) \left( 1 - s_d \left( V_H^i(V, \pi; p); p \right) \right) / \bar{s} \right] \right\} \psi_t(dV, d\pi; p^*) \\ &+ (1 - \eta) \int (1 - \pi) \sum_{i=u,r} \sum_{j=u,r} \left\{ \phi \left[ V_c^j \left( V_L^i(V, \pi; p), \pi; p \right), \pi \right] \times \right. \\ &\quad \left. \left[ 1 - s_\ell \left( V_L^i(V, \pi; p), \pi; p \right) \left( 1 - s_d \left( V_L^i(V, \pi; p); p \right) \right) / \bar{s} \right] \right\} \psi_t(dV, d\pi; p^*) \\ &+ (1 - \eta) \int \sum_{j=u,r} \left\{ \phi \left[ V_c^j \left( V_\ell(V, \pi; p), \pi; p \right), \pi \right] \times \right. \\ &\quad \left. \left[ 1 - s_\ell \left( V_\ell(V, \pi; p), \pi; p \right) \left( 1 - s_d \left( V_\ell(V, \pi; p); p \right) \right) / \bar{s} \right] \right\} \psi_t(dV, d\pi; p^*) \\ &+ E_{t+1} \int \phi \left[ V_0(p), \pi \right] d\Gamma(\pi) \end{aligned}$$

The first four lines add up all the firms that have not defaulted in  $t$ , and who survive exogenous exit and endogenous liquidation in  $t + 1$ . The next two lines add up all the firms that defaulted (and renegotiated) in  $t$  and survive exogenous exit and endogenous liquidation in  $t + 1$ . The final line accounts for new entrants (whose type is drawn from  $\Gamma(\pi)$ ). Notice that  $V_{t+1}$  for the firms that did not default depends on the realization of  $z_t$  while for the defaulted firms does not (i.e.,  $V_{t+1} = V_\ell$ ). Notice also that we have used the fact that  $s_d(V, \pi; p) = s_d(V; p)$  which follows directly from Proposition 2.

## 5.2 Calibration algorithm

The calibration of the model presented in the quantitative analysis section is done using the following seven step iterative procedure:.

- (i). Choose values for each of the parameters in the vector  $\Theta = \{\eta, \zeta, \bar{\pi}, V_0, \Delta, \bar{s}, \theta, \mu, \rho\}$
- (ii). Find  $\underline{\pi}$  so that  $s_\ell(\underline{\pi}) = \bar{s} - \epsilon$  for arbitrarily small  $\epsilon$ .
- (iii). Discretize the set  $[\underline{\pi}, \bar{\pi}]$  into equally spaced values  $\underline{\pi}, \pi_2, \dots, \bar{\pi}$ .
- (iv). Solve the contracting problem  $P$  for each project type  $\pi_j$ ,  $j = 1, \dots$ .
- (v). Draw a large number of project-firms  $i = 1, \dots, N$  from the initial distribution  $\Gamma(\cdot)$  and initialize them all with the same equity  $V_0$ .
- (vi). Using the policy functions obtained in (iv), for each  $i = 1, \dots, N$ , simulate  $\{V_{it}, k_{it}, W_{it}\}_{t=1}^T$  for large  $T$ . Upon exit, a firm is replaced with a new one drawn from  $\Gamma(\cdot)$ , so that the total mass of firms is constant.
- (vii). Compute the moments of table 2 using the second half of the simulated sample so as to obtain the long-run or stationary distribution, and compare them to the data moments.

The process iterates until the distance between the simulated and data moments is small enough. In (iv), the contracting problem is solved using value function iteration on a non-uniform equity grid with a higher density of gridpoints for low values of equity.

## 5.3 Financial constraints and cash flow sensitivity

Many early studies of financial constraints associated the statistical significance of cash flow coefficients in investment equation regressions as evidence of the existence of financial constraints (Fazzari et al. (1988)). And while this practice has recently become controversial due to potentially omitted variables (Gomes (2001)), in our controlled context (where we know precisely what the data generating process is) it is perfectly correct to run such a regression with simulated data. More specifically, we follow Clementi and Hopenhayn (2006) and estimate the following model:

$$\Delta k_{it} = \gamma_0 + \gamma_1 y_{it-1} + \gamma_2 V_{it-1} + \varepsilon_{it}, \quad (4)$$

where  $y_{it-1} = z_{it-1} k_{it-1}^\alpha$ . Table 4 below presents the results from estimating equation (4) on a panel of firms simulated using the financial contract described in section 2. The results are in line with the quantitative analysis of the main text. That is, in an economy where renegotiation and reorganization become unavailable due to high costs and little benefits, firms face more stringent financial constraints: the cash flow coefficient is between 20% and 30% larger than in the economy with renegotiation and efficient reorganization.

Table 4: Investment equation regressions (simulated data)

	Renegotiation & eff reorg		Default only	
	FE	RE	FE	RE
Revenue (t-1)	0.0543*** (-0.00218)	0.047*** (-0.0021)	0.065*** (-0.0014)	0.061*** (-0.00266)
equity (t-1)	-0.0029*** (-0.0002)	-0.0028*** (-0.0001)	-0.004*** (-0.0001)	-0.004*** (-0.0002)
Constant	-0.131*** -0.0114	-0.069*** -0.0077	-0.0519*** -0.0081	-0.028*** -0.0058
Observations	389,122	389,122	388,960	388,960
R-squared (overall)	0.211	0.212	0.311	0.311
Number of firms	10,965	10,965	11,063	11,063

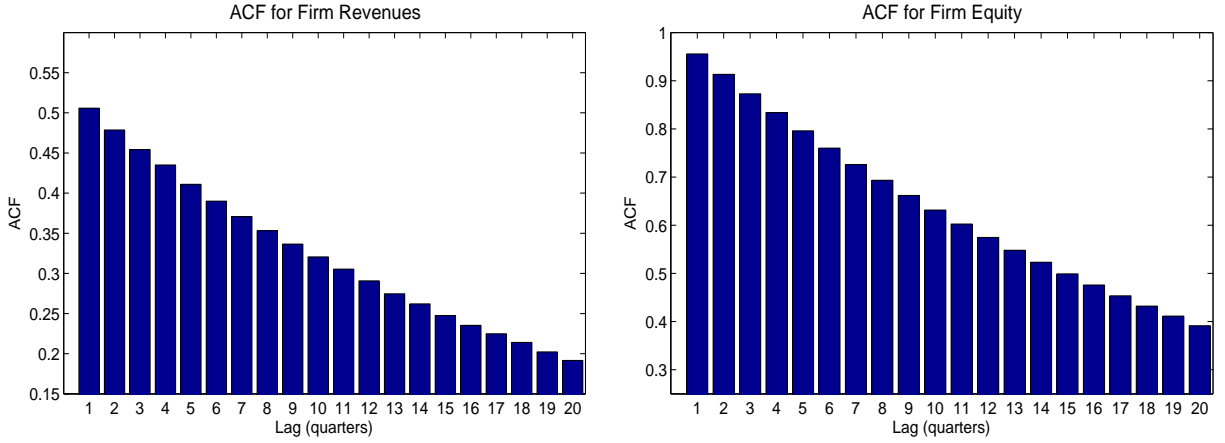
\*\*\* denotes statistical significance at the 1% level. FE columns correspond to fixed effects models while RE columns to random effects models. The Hausman test favored the use of FE models.

## 5.4 Persistence and history dependence

The model we have presented in this paper assumes that the two shocks that affect financial contracting are iid over time. It is therefore interesting to know what are the implications of this assumption for the behavior of the main endogenous variables. Figure 5 below presents the autocorrelation functions for  $k_t$  and  $V_t$  taken as the average across firms.

It is noteworthy that a model with such a simple stochastic structure in the firm-level productivity process is able to generate quite some persistence on firm-level revenue. On the other hand, the empirical literature on firm dynamics in the U.S. reports higher persistence

Figure 5: Persistence in firm performance



on this front: [Cooper and Haltiwanger \(2006\)](#) reports AR(1) cash flow coefficients that are close to 0.9. This could be achieved by allowing some persistence in the exogenous component of the production function. For instance, [Quadrini \(2004\)](#) shows that if the density function of the production shock depends on a persistent factor that is public information, the main results of a contracting problem such as that of section 2 can be easily extended once a new state variable is included (the previous realization of the persistent factor).

## References

- AGHION, P. AND P. BOLTON, “An Incomplete Contracts Approach to Financial Contracting,” *Review of Economic Studies* 59 (1992), 473–94.
- AGUIAR, M., M. AMADOR, H. HOPENHAYN AND I. WERNING, “Take the Short Route: Equilibrium Default and Debt Maturity,” NBER Working Papers 22847, National Bureau of Economic Research, Inc, November 2016.
- ALBUQUERQUE, R. AND H. A. HOPENHAYN, “Optimal Lending Contracts and Firm Dynamics,” *Review of Economic Studies* 71 (2004), 285–315.
- ALTMAN, E., “A further empirical investigation of the bankruptcy cost question,” *Journal of Finance* 39 (1984), 1067–1089.
- ANTUNES, A., T. CAVALCANTI AND A. VILLAMIL, “The effect of financial repression and en-

- forcement on entrepreneurship and development,” *Journal of Monetary Economics* 55 (2008), 278–297.
- ARELLANO, C., Y. BAI AND J. ZHANG, “Firm dynamics and financial development,” *Journal of Monetary Economics* 59 (2012), 533–549.
- AYOTTE, K. AND E. MORRISON, “Credit control and conflict in chapter 11,” *Journal of Legal Analysis* 1 (2009), 511–551.
- BAIRD, D. AND R. RASMUSSEN, “Chapter 11 at twilight,” *Stanford Law Review*, 673 56 (2003), 673–699.
- BASU, S. AND J. G. FERNALD, “Returns to Scale in U.S. Production: Estimates and Implications,” *Journal of Political Economy* 105 (April 1997), 249–283.
- BERGOEING, R., P. J. KEHOE, T. J. KEHOE AND R. SOTO, “A Decade Lost and Found: Mexico and Chile in the 1980s,” in T. J. Kehoe and E. C. Prescott, eds., *Great depressions of the twentieth century* (Federal Reserve Bank of Minneapolis, 2007).
- BHARATH, S., S. DAHIYA, A. SAUNDERS AND A. SRINIVASAN, “So what do I get? The bank’s view of lending relationships,” *Journal of Financial Economics* 85 (2007), 368–419.
- , “Lending relationships and loan contract terms,” *Review of Financial Studies* 24(4) (2011), 1141–1203.
- BHARATH, S., V. PANCHAPEGESAN AND I. WERNER, “The changing nature of Chapter 11,” *IIM Bangalore Research Paper No. 461* (2013).
- BRIS, A., I. WELCH AND N. ZHU, “The costs of bankruptcy: Chapter 7 liquidation versus Chapter 11 reorganization,” *Journal of Finance* 61(3) (2006), 1253–1303.
- BUERA, F., J. KABOSKI AND Y. SHIN, “Finance and development: a tale of two sectors,” *American Economic Review* 101 (2011), 1964–2002.
- CHATTERJEE, S., D. CORBAE, M. NAKAJIMA AND J.-V. RIOS-RULL, “A Quantitative Theory of Unsecured Consumer Credit with Risk of Default,” *Econometrica* 75 (2007), 1525–1589.
- CLEMENTI, G. AND H. HOPENHAYN, “A A theory of financing constraint and firm dynamics,” *Quarterly Journal of Economics* 121 (2006), 229–265.

- COOLEY, T., R. MARIMON AND V. QUADRINI, "Aggregate consequences of limited contract enforceability .," *Journal of Political Economy* 112(4) (2004), 817–847.
- COOLEY, T. AND V. QUADRINI, "Financial markets and firm dynamics," *American Economic Review* 91(5) (2001), 1286–1310.
- COOPER, R. W. AND J. C. HALTIWANGER, "On the Nature of Capital Adjustment Costs," *Review of Economic Studies* 73 (2006), 611–633.
- CORBAE, D. AND P. D'ERASMO, "Reorganization or Liquidation: Bankruptcy Choice and firm dynamics," *manuscript* (2016).
- DJANKOV, S., O. HART, C. MCLIESH AND A. SHLEIFER, "Debt enforcement around the world," *Journal of Political Economy* 116 (6) (2008), 1105–1149.
- DOVIS, A., "Efficient Sovereign Default," Technical Report, Working Paper, 2016.
- FAZZARI, S., G. HUBBARD AND B. PETERSEN, "Financing constraints and corporate investment," *Brooking Papers on Economic Activity* 1 (1988), 141–206.
- GILSON, S., "Bankruptcy, Boards, Banks, and Blockholders," *Journal of Financial Economics* 27 (1990), 355–387.
- GOMES, J., "Financing investment," *American Economic Review* 91 (2001), 1263–1285.
- HALTIWANGER, J., R. JARMIN AND J. MIRANDA, "Who Creates Jobs? Small versus Large versus Young," *The Review of Economics and Statistics* 95(2) (2013), 347–361.
- HOPENHAYN, H. AND I. WERNING, "Equilibrium Default with Limited Commitment," *Meeting Papers, Society for Economic Dynamics* (2007).
- JOSTARNDT, P. AND Z. SAUTNER, "Financial distress, corporate control, and management turnover," *Journal of Banking and Finance* 32(10) (2008), 2188–2204.
- LEE, Y., "The importance of reallocations in cyclical productivity and returns to scale: evidence from plant-level data," Working Paper 0509, Federal Reserve Bank of Cleveland, 2005.
- LI, S. M., "Employment Flows with Endogenous Financing Constraints," *The B.E. Journal of Macroeconomics* 10(1) (2010), 1–40.



- LI, W. AND P.-D. SARTE, "U.S. consumer bankruptcy choice: The importance of general equilibrium effects," *Journal of Monetary Economics* 53 (April 2006), 613–631.
- MIDRIGAN, V. AND D. XU, "Finance and misallocation: evidence from plant-level data," *American Economic Review* 104(2) (2014), 442–458.
- NINI, G., D. SMITH AND A. SUFI, "Creditor control rights, corporate governance, and firm value," *Review of Financial Studies* 26 (2012), 1713–1761.
- QUADRINI, V., "Investment and liquidation in renegotiation-proof contracts with moral hazard," *Journal of Monetary Economics* 51(4) (2004), 713–751.
- ROBERTS, M. AND A. SUFI, "Renegotiation of financial contracts: Evidence from private credit agreements," *Journal of Financial Economics* 93 (2009), 159–184.
- RODRIGUEZ-DELGADO, J., "Bankruptcy and Firm Dynamics: The Case of the Missing Firms," *IMF Working Paper WP/10/41* (2010).
- SANYAL, P. AND C. L. MANN, "The financial structure of startup firms: the role of assets, information, and entrepreneurs," WP 10-17, Federal Reserve Bank of Boston, 2010.
- SENKAL, A., "Misallocation and Recovery Rates," *Manuscript, University of Virginia* (2014).
- SPEAR, S. AND S. SRIVASTAVA, "On repeated moral hazard with discounting," *Review of Economic Studies* 54 (1987), 599–617.
- TAMAYO, C., "Investor protection and optimal contracts under risk aversion and costly state verification," *Economic Theory* 59 (August 2015), 547–577.
- THOMAS, J. AND T. WORRALL, "Income fluctuations and asymmetric information: An example of the repeated principal-agent problem," *Journal of Economic Theory* 51 (1990), 367–390.
- VERANI, S., "Aggregate Consequences of Dynamic Credit Relationships," *Manuscript* (2014).
- WARNER, J., "Bankruptcy cost: Some evidence," *Journal of Finance* 32 (1977), 337–347.
- WHITE, M., "Corporate bankruptcy as a filtering device: Chapter 11 reorganizations and out-of-court debt restructurings," *Journal of Law, Economics, and Organization* 10 (1994), 268–295.
- YUE, V. Z., "Sovereign default and debt renegotiation," *Journal of International Economics* 80 (March 2010), 176–187.