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## Interest Rates Modeling and Forecasting: Do Macroeconomic Factors Matter?

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# Interest Rates Modeling and Forecasting: Do Macroeconomic Factors Matter?

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## **Abstract:**

Recent studies documented a sufficient forecasting performance of shadow-rate models in the low yields environment. Moreover, it has been shown that including the macro-variables into the shadow-rate models further improves the results. We build on these findings and evaluate for the U.S. Treasury yields, whether the lower bound proximity was truly the only issue to reflect in the interest rate modeling since the Great Recession. Surprisingly, we discover that the relative importance of yield curve factors has changed as well. More specifically, instead of macroeconomic factors, financial market sentiment factors became dominant since the recent financial turmoil. Based on such finding, we show, that extending the macro-finance interest rate models by financial market sentiment proxies further improves the forecasting performance.

**Keywords:** Interest Rate, Yield Curve, Macro-Finance Model, Affine Model, Nelson-Siegel

**JEL:** C38, C51, C58, E43, E47

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# 1 Introduction

The low yields environment present in the U.S. since the Great Recession opened a discussion about the validity of canonical affine Gaussian dynamic term structure models (DTSMs). In an important paper, Christensen and Rudebusch (2016) document a significant drop of the forecasting ability of a standard DTSM in the lower bound proximity. Furthermore, they show an improvement of the forecasting accuracy after using a shadow-rate DTSM. An identical finding is obtained by Bauer and Rudebusch (2016), who additionally utilize a set of macro-factors to build a macro-finance shadow-rate DTSM. As they show, the macro-finance version of the shadow-rate DTSM performs the best, compared to both yields-only and non-shadow-rate (Gaussian) DTSMs. Consequently, the latter study has shown, that the macro-finance DTSM framework, popular prior to the Great Recession, remains best-performing also in the new environment, after reflecting the lower bound proximity.

In these studies, a *technical* feature of the new environment (an asymmetry of the yields due to the lower bound) has been well reflected. Contrary, a discussion of *economic* features of post-crisis yield dynamics have been largely neglected. In practice, the macro-variables introduced by Bauer and Rudebusch (2016) into the macro-finance shadow-rate DTSM represent real activity and price dynamics. Such approach is close to the macro-finance DTSMs prior to the Great Recession, when real activity, price dynamics and often also a monetary policy variable were usually introduced into the macro-finance models.

We fill this gap in the present macro-finance literature by evaluating, whether variables related to real activity, price dynamics and the monetary policy are still the optimal external factors to be included into macro-finance models. To do so, in the first part of the analysis, we start by estimating a canonical DTSM (Duffie and Kan 1996). Using the estimated model, we decompose yields into risk-neutral yields and term premia. Reason for performing the decomposition is a possibility of a different response of each of the components to various macroeconomic and financial shocks. As a next step, we estimate a relation of the components and a set of macroeconomic and financial variables within a vector auto-regression (VAR) model. Using a historical decomposition of the shocks from each variable, we evaluate the importance of each macroeconomic or financial variable for explaining the evolution of the components, and consequently also the yields. The analysis is conducted on a rolling-window, covering periods both prior to and since the Great Recession.

Our findings are encouraging. As we show, the traditional macro-variables were particularly important in the period prior to and during the recent financial turmoil. Contrary, after the crisis, the importance of factors reflecting a financial market

sentiment and portfolio allocation shifts increased<sup>1</sup>. More precisely, the financial market sentiment dynamics is tightly related to the term premium, which has played a dominant role in explaining the volatility of U.S. yields over last years, whereas the risk-neutral yields reflecting macroeconomic conditions became less volatile. Our findings are in line with the empirical evidence: as an example, in July 2016, the U.S. yields reached the lowest-ever level despite favorable economic conditions and positive expectations about a future monetary policy rates evolution. Contrary, a market reaction to the result of the Brexit referendum related to a high level of global markets uncertainty and a sizable portfolio reallocation was the most important factor. Since U.S. bonds are considered a safe-haven asset, the demand for them raised in relation to this event, which reduced the yields. Furthermore, as a second related finding, we additionally confirm that the two components of yields (the risk-neutral yields and the term premium) truly often respond differently to shocks, both in the extent and the sign.

In the light of our findings, the performance of macro-finance models under post-crisis conditions could be further improved, beyond the improvement offered by introducing shadow rates into the models (Christensen and Rudebusch 2016). To evaluate such possibility quantitatively, we advance to the second part of the analysis and propose adjustments of the canonical interest rate models. These adjustments follow our previous findings: we add proxies for the financial market sentiment into the set of factors entering the models, and also replace yield data entering the models by the separated yield components.

Afterwards, we compare the forecasting performance of a wide set of interest rate models. Apart from the DTSM used for the yields decomposition, we utilize also the Dynamic Nelson-Siegel (DNS) framework introduced by Diebold and Li (2006) and a simple time series analysis. The utilization of multiple modeling frameworks supports the robustness of our results: due to the economic background of our findings (a change of yield factors), any interest rate model should benefit from the proposed adjustments. All modeling frameworks are used in both the traditional representations and in the versions including our newly proposed adjustments. As we show, the proposed adjustments result in an improvement of the forecasting accuracy. More precisely, the separate inclusion of yield components into the models is crucial for achieving the best forecasting performance for yields of the shortest maturities. On the other hand, for longer maturities, introducing the market sentiment variables is necessary to achieve an improved forecasting accuracy.

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<sup>1</sup>Macroeconomic news are a frequent source of changes of the financial market sentiment. However, in the recent history, market uncertainty and sentiment shifts were often triggered by forces, which were not directly related to the U.S. macroeconomic conditions, for example August 2015 and January 2016 market crashes or the weeks following UK EU-leave referendum. Consequently, in this paper, shocks to the market sentiment are considered as autonomous, not being triggered by macro-news. In the vector autoregression model specification described in the section 5, we obtain such shock identification by ordering macro-variables first.

Consequences of the lower bound proximity are handled carefully in our methodology. An asymmetry of yields near the lower bound needs to be reflected in all steps of our analysis: the estimation the DTSM and yield components, the VAR analysis as well as the forecasting exercise. To keep the interpretation of our results straightforward, we do not use the shadow-rate macro-finance DTSM as of Bauer and Rudebusch (2015). Instead, we propose a simplified approach: First, we convert observed yields into shadow yields (rates), using Krippner (2013) and Christensen and Rudebusch (2014) methodology. Afterwards, all steps described above (the decomposition of yields, the VAR analysis and the forecasting performance evaluation) are done using these shadow rates. Consequently, our results are related to the shadow rates rather than the observed yields. However, it can be argued that our conclusions are valid generally, since a conversion of the shadow rates to the observed yields is straightforward.<sup>2</sup>

The rest of the paper is structured as follows: the next section covers the related literature, which the paper builds on. The third section presents modeling frameworks used for obtaining the shadow rates, for their decomposition as well as for the forecasting. Data are described in the fourth section. The fifth section presents obtained components and results of the VAR analysis linking together yields and their factors. The sixth section compares forecasting performance of a wide set of model specifications, both gathered from the literature and newly proposed, motivated by the previous section results. We particularly focus on the forecasting accuracy improvement following the introduction of the market sentiment proxies into the models and the benefits of modeling the yield components separately. Finally, the last section concludes.

## 2 Related Literature

A theoretical basis for the DTSM framework builds on short rate models of Vasicek (1977) and Cox *et al.* (1985). In their setup, the evolution of the whole yield curve is governed by dynamics of a short rate. Under a risk-neutral measure, the expectation hypothesis is used to derive longer yields as an average of expected future short rates. A transfer to the real-world (data-generating) measure is then done by introducing a term (risk) premium. The approach became especially popular after an introduction of an affine class of short-rate models by Duffie and Kan (1996). The affine representation became the canonical dynamic term structure modeling approach thanks to its ability to derive a closed-form formula for the yields dynamics, which allows for an efficient estimation of the model. Duffee (2002) uses the affine model (originally designed for fitting the term structure) also for capturing real-world dynamics and particularly for forecasting. In one of the most important macro-finance papers, Ang

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<sup>2</sup>The methodology is outlined in the section 3 of this paper.

and Piazzesi (2003) extend the framework by macroeconomic factors, i.e. form the affine Gaussian macro-finance term structure model.

The second framework used in the paper builds on a functional representation of the term structure of interest rates as proposed by Nelson and Siegel (1987). Diebold and Li (2006) view the parameters of the function (so-called level, slope and curvature of the yield curve) as dynamic factors, which form the dynamic Nelson-Siegel (DNS) model. Thanks to its parsimony, the approach became popular both in the academia and in the modeling practice. The macro-financial extension of this framework was subsequently introduced by Diebold *et al.* (2006). Later, Christensen *et al.* (2011) specified an affine no-arbitrage version of the DNS model, which linked together DNS and DTSM frameworks<sup>3</sup>.

Macroeconomic factors included into the macro-finance models (both DTSM and DNS) usually represent business activity, price dynamics and a monetary tool or a monetary aggregate. Diebold *et al.* (2006) include manufacturing capacity utilization, the Federal funds rate and an annual price inflation. Ludvigson and Ng (2009) and De Pooter *et al.* (2010) utilize a large dataset including macroeconomic and financial variables and use the most important principal components representing the common factors. Based on their relation to the original series, De Pooter *et al.* (2010) show, that the first principal component is related to real activity, the second component to price dynamics and the third component to monetary variables. A similar set of variables is utilized also in later studies, building mostly on the DTSM framework: Kim and Wright (2005) studied a specific role of the inflation for explaining yield dynamics, Dai and Philippon (2005) focused on an importance of fiscal policy and Bikbov and Chernov (2010) offered an enhanced approach to measure the association among financial and macroeconomic factors. Ludvigson and Ng (2009) and Wright (2011) aim at explaining the macro-drivers of only one component of yields: the term-premium. Recently, Joslin *et al.* (2014) raised questions about the validity of affine macro-finance DTSMs. Authors show, that a large proportion of the macroeconomic variation is not spanned by the yield curve, which contradicts canonical DTSM implications. Authors build a new “unspanned” affine DTSM model solving this caveat, and call for a replacement of the canonical macro-finance DTSM by the new framework. However, Bauer and Rudebusch (2015) show a statistic insignificance of the added value of the unspanning restrictions, and reject the necessity to replace the prevailing macro-finance DTSM framework.

Apart from these advances, an important part of the interest rate modeling discussion is concerned with the implications of a presence of a lower bound<sup>4</sup>. Several

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<sup>3</sup>In practice, the affine no-arbitrage DNS model is considered as a member of the DTSM family of models. To avoid confusion, we refer to this model as to an affine DTSM model, since the no-arbitrage property is the crucial feature of the model.

<sup>4</sup>The existence of the lower bound is explained by the availability of physical currency. Consequently, investors can escape negative bond yields by converting their funds to physical currency.



studies evaluated consequences of the post-crisis low yield environment for the canonical models' forecasting performance. Trück and Wellmann (2015) detect a drop in the DNS model (without macro factors). Similarly, Chung and Iiboshi (2015) compare the forecasting accuracy of affine and quadratic macro-finance DTSMs, and show a reduced performance of the affine DTSM near the lower bound. An extensive overview of negative consequences of using affine DTSM in the lower bound proximity offers Krippner (2015). Author presents a range of shadow-rate models, which build on the methodology of Black (1995) and view the observed bond price (i.e. also the observed yield) as a difference between a shadow bond price (a price of a bond in a hypothetical world without physical currency) and a value of a option covering the possibility, that yields would be negative<sup>5</sup>. The shadow rates are used also by Christensen and Rudebusch (2014), who incorporate them into an affine DTSM. Christensen and Rudebusch (2016) use this model to show an improved forecasting performance of the shadow-rate affine DTSM compared to the canonical affine DTSM. Bauer and Rudebusch (2016) use the shadow-rate affine DTSM extended by a set of macro-factors and document, that the macro-finance affine shadow-rate DTSM outperforms the "yields-only" models.

### 3 Models

In order to capture macro-financial linkages, the paper utilizes an extensive set of yield curve modeling frameworks. These are used in three separate steps. First, the shadow-rate affine DTSM introduced by Christensen and Rudebusch (2014) and Krippner (2013) is used to transform observed U.S. yields into shadow rates<sup>6</sup>, i.e. hypothetical yields "as if" no physical currency was present in the U.S. economy. By doing this, the paper ensures that the subsequent modeling steps are not influenced by consequences of the lower bound proximity (an asymmetry of the yields, in particular). Consequently, all further calculations utilize only these shadow rates<sup>7</sup>. As a second step, a canonical DTSM model is used to extract two components of the (shadow) yields. These components then enter a VAR model explaining their response (and the response of the yields) to macroeconomic and financial shocks. Using the model, we quantitatively evaluate the importance of various shocks for the yield dynamics. Third, and finally, the DTSM together with a DNS model are used in many sub-versions including both "yields-only" and macro-financial specifications

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Nevertheless, due to additional costs related to holding physical currency, the lower bound does not need to be exactly at the zero level. We use the term "lower bound" without assuming any exact level of the bound, however expecting it to be reasonably close to zero.

<sup>5</sup>The shadow rate is defined as a yield of a shadow bond.

<sup>6</sup>The term "shadow yields" would be more precise. However, the literature usually focuses only on short shadow yields, for which the term "rates" is meaningful. Consequently, we use both "shadow rates" and "shadow yields" interchangeably.

<sup>7</sup>For simplicity, we keep using the term "yields" in the following text, which however always refers to the shadow rates (yields).

to compare the forecasting performance of particular models.

In order to present the methodology in this section, it is useful to start with outlining the canonical frameworks (DTSMs and DNS models) used for the decomposition and the forecasting. The advanced methodology related to shadow rates is an extension of the DTSM and therefore is explained at the end of this section.

The paper utilizes two frameworks: a DNS approach as introduced by Diebold and Li (2006) and macro-extended by Diebold *et al.* (2006), and a macro-finance affine DTSM similar to the model used by Ang and Piazzesi (2003). In the section comparing the forecasting performance of the models, for each of these frameworks, we estimate both a “yields-only” version and a macro-finance version including a set of additional macroeconomic and/or financial variables. Additionally, a random walk process serves as a baseline model. Apart from the canonical models described in this section, several newly introduced model specifications are utilized for the forecasting as well – their description is postponed to the section 5.

Both DTSM and DNS frameworks result in a model, which can be defined in a state-space form. In this representation, yields are a linear function of factors, which follow a VAR(1) process:

$$y_t(\tau) = f(\mathbf{L}_t, \mathbf{M}_t, \tau) + \epsilon_{y,t}(\tau) \quad (1)$$

$$\begin{bmatrix} \mathbf{L}_t \\ \mathbf{M}_t \end{bmatrix} = \boldsymbol{\alpha} + \boldsymbol{\Gamma} \begin{bmatrix} \mathbf{L}_{t-1} \\ \mathbf{M}_{t-1} \end{bmatrix} + \boldsymbol{\epsilon}_{LM,t} \quad (2)$$

The vector  $\mathbf{L}_t$  includes latent (unobservable) factors, whereas the vector  $\mathbf{M}_t$  includes observable macroeconomic and financial variables. Depending on the particular model version,  $\mathbf{M}_t$  is either empty (i.e. “yields-only” model), includes the traditional macro-variables (representing real activity and price dynamics), or includes also a set of newly proposed variables (further described in the section 5). The function  $f(\bullet)$  is given by the Nelson and Siegel (1987) function in the DNS model; in case of the affine DTSM, the function results from a no-arbitrage assumption under the equivalent martingale (risk-neutral) measure (see Appendix I. for details on the methodology). Additionally,  $\boldsymbol{\alpha}$  and  $\boldsymbol{\Gamma}$  are a vector and a matrix of parameters, and  $\tau$  represents the maturity. Measurement error terms  $\epsilon_{y,t}(\tau)$  are forming a  $N \times 1$  vector error term  $\epsilon_{y,t} \sim \mathcal{N}(\mathbf{0}, \mathbf{R})$ , where  $N$  is the total number of maturities included into the sample. The vector  $\boldsymbol{\epsilon}_{LM,t} \sim \mathcal{N}(\mathbf{0}, \mathbf{Q})$  represents the random disturbances in the factors process. As usual for the basic state space representation,  $\epsilon_{y,t}$  and  $\boldsymbol{\epsilon}_{LM,t}$  are assumed to be mutually uncorrelated.

Apart from the multiple model specifications, also the estimation methods allow for multiple approaches. Both frameworks can utilize the Kalman filter procedure and the maximum likelihood estimation as a single-step estimation method. However, the function  $f(\bullet)$  can be non-linear in parameters, which makes the estimation non-

trivial. Consequently, two-step estimation approaches were developed to allow for a straightforward estimation, at costs of imposing additional restrictions.

In case of the affine DTSM, the two-step estimation is based on a transformation of the latent factors  $\mathbf{L}_t$  to observable factors. Such approach was utilized by Ang *et al.* (2006) and further promoted by Joslin *et al.* (2011). In practice, we follow this by using the first three principal components of yields as proxies for the factors. The transition equation (Equation 2) can be then estimated using least squares, either including only the principal components or adding the macro-financial factors as well. Afterwards, the parameters of the measurement equation (Equation 1) can be obtained using the least squares as well.

The various specifications of the affine DTSM used in the analysis are summarized in the Table 1. For the Kalman filter approach, the macro-finance specification is not included because of the size of the resulting model, which would made the estimation procedure slow and path-dependent.

Table 1: Canonical Affine Models

model name	estimation method	type
AFpcsPF	observable factors & LS	yields-only
AFpcsMB	observable factors & LS	macro-finance
AFkalPF	Kalman filter & MLE	yields-only

The symbol “AF” represents the affine DTSM framework; “pcs” and “kal” is related to the estimation approach (principal components and LS fit vs. Kalman filter with MLE); “PF” and “MB” represent pure-financial (i.e. yields-only) and macro-basic (i.e. the traditional macro-finance) specification. AFkalPF is also used to decompose yields to risk-neutral yields and term premia. LS means least-squares estimation; MLE means the maximum likelihood estimation.

In case of the two-step DNS estimation, the first step involves fitting the Nelson and Siegel (1987) function (i.e. estimating the measurement equation) in each period independently, using the least squares method. Afterwards, the obtained parameters of the function are considered as extracted latent factors. The transition equation can be then estimated directly. Furthermore, the DNS framework includes an exponential decay parameter, which can be also set in multiple ways. In a single-step estimation approach, the parameter can be estimated within the Kalman filter procedure using the maximum likelihood estimation. In the two-step approach, the parameter needs to be fixed before the estimation of the model. In this case, its value can be set either empirically, providing a desired shape of the yield curve, or at the level ensuring an optimal fit (described into detail in Appendix I.). Again, both these approaches are tested in order to obtain robust results. The set of utilized model versions of the DNS family is summarized in the Table 2 (again without the newly proposed specifications, which are introduced in the section 5).

After the models are estimated, the forecasts are obtained by iterating the vector of factors forward through the transition equation, utilizing the fact that  $E[\boldsymbol{\epsilon}_{LM,t}] = \mathbf{0}$ . The forecasts of the yields are then calculated directly from the mea-

Table 2: Canonical Dynamic Nelson-Siegel Models

model name	estimation method	type
NSfixPF	two-step, $\lambda$ set by curvature location	yields-only
NSfixMB	two-step, $\lambda$ set by curvature location	macro-finance
NSoptPF	two-step, $\lambda$ set by optimal fit	yields-only
NSoptMB	two-step, $\lambda$ set by optimal fit	macro-finance
NSkalPF	Kalman filter & MLE, $\lambda$ estimated directly	yields-only
NSkalMB	Kalman filter & MLE, $\lambda$ estimated directly	macro-finance

The symbol “NS” represents the DNS framework; “fix”, “opt” and “kal” is related to the estimation approach; “PF” and “MB” represent pure-financial (i.e. yields-only) and macro-basic (i.e. the traditional macro-finance) specification. MLE means maximum likelihood estimation.

surement equation.

Apart from the forecasts, the affine DTSM has the ability to decompose yields into two components: a risk-neutral yield and a term premium. We utilize this feature, as each of the components can react differently to macroeconomic and financial shocks, so the decomposition can offer a detailed insight into the actual dynamic properties of yields. To obtain the components, the estimated AFkalPF model<sup>8</sup> is used to generate forecasts of the short rate, which can be considered as estimates of expectations about the future risk-free rate development. For each maturity, an average of the short rate predicted on a horizon equal to the particular maturity represents the risk-neutral yield. The difference between the observed (shadow) yield and the risk-neutral yield forms the term premium.

Returning to the very first step of the analysis, a shadow-rate DTSM is used to convert observed yield data into the shadow yields (rates). The model can be specified identically to the DTSM as captured by the equations 1 and 2, only with the shadow rates replacing the observed yields  $y_t(\tau)$ . The link between the shadow rates and the observed yields lies in the possibility for the investors to hold the physical currency with a zero return in case the actual return from their financial asset (the bond) would be negative. As proposed by Black (1995), a value of such possibility can be calculated as a value of a put option. Christensen and Rudebusch (2014) and Krippner (2013) further elaborate on this approach and obtain an affine shadow-rate DTSM with a closed-form solution for yields.<sup>9</sup> Because of a relative robustness of this model, we utilize it in a specification including three latent factors.<sup>10</sup>

<sup>8</sup> As discussed by Bauer *et al.* (2014), least squares methods can lead to biased results of the decomposition. A small sample bias adjustment method is utilized by authors to obtain unbiased results. In order to avoid the estimation bias, we take a different approach: we use the one-step approach and estimate the model (without macro-financial factors) using the Kalman filter and the maximum likelihood estimation. The results of our approach presented in the next section are very close to results of the decomposition performed by Bauer *et al.* (2014) using the bias-adjustment method.

<sup>9</sup>As the specification of the model is extensive and is not directly related to the topic of this paper, we omit it here. The full description of the model can be found in Christensen and Rudebusch (2014) and Krippner (2013).

<sup>10</sup> It must be admitted, that a three-factor model is shown for example in Christensen and Rude-

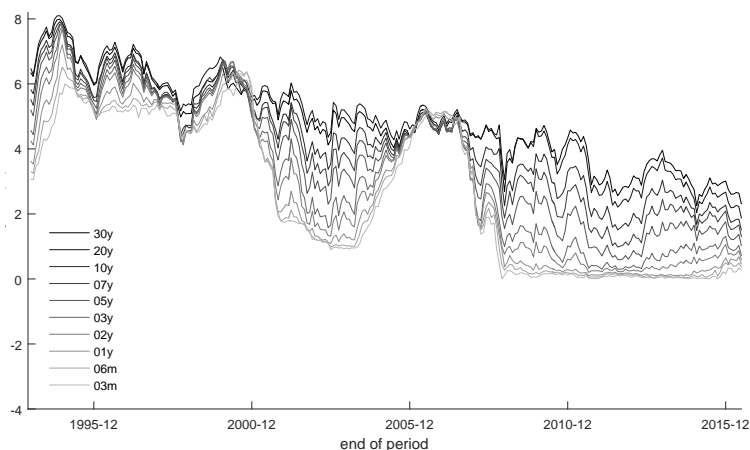
## 4 Data

### Yield Data and Shadow Rates

To estimate the models and evaluate their performance, a set of U.S. Government zero-coupon bond yields is utilized. More specifically, ten maturities are included, ranging from Bills of maturities 3, 6 and 12 months, over US Notes (2, 3, 5, 7 and 10 years) up to 20 and 30 year Bonds. We use constant maturity rates available in FRED (2016) database. These rates are calculated by the U.S. Treasury by interpolating the yield curve of non-indexed Treasury securities.<sup>11</sup>

Monthly frequency of yields was chosen using the end-of-month data. Such frequency provides a sufficient number of observations. At the same time, the monthly frequency allows us to match the observations of yields and the macroeconomic series, for which the monthly frequency is usually the highest available. The data cover a period since December 1993 until June 2016. The beginning of the period roughly coincides with an introduction of a new paradigm into the monetary policy, emphasizing the role of the interest rate as a monetary policy tool (described for example in Mankiw 2001). We consequently avoid a monetary policy structural break related to the change in monetary policy instruments (shifting from monetary aggregates to interest rates).

Figure 1: Observed Yields



Source: FRED (2016)

The historical evolution of the U.S. yields is included in the Figure 1. As apparent from the figure, the lower bound close to a zero level significantly influenced

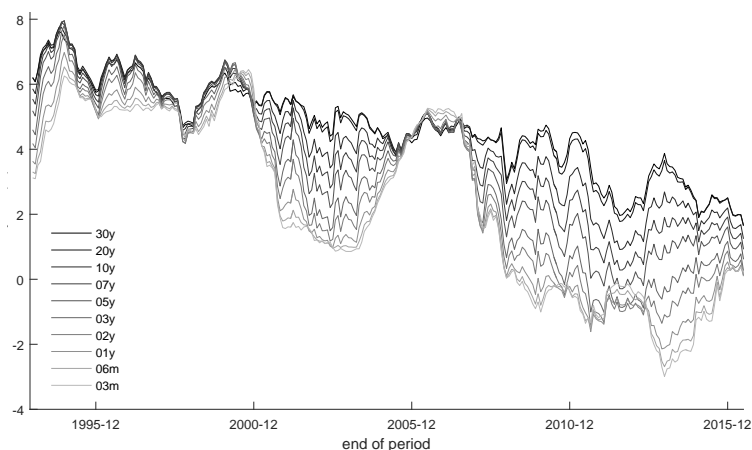
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busch (2014) to provide less robust results than a two-factor model. However, as all the models in the paper are set as three-factor, we need at least three factors to be driving the shadow rates. Otherwise, a problem of multi-collinearity could arise in the subsequent steps of the analysis.

<sup>11</sup>See FED (2016) and U.S. Treasury (2016) for detailed description of the calculation method. 30-years maturity yields data are missing in the period between February 2002 and February 2006. However, an adjustment term is offered by the U.S. Treasury, which allows us to approximate the missing observations.

yields since the end of 2008. As discussed by Krippner (2015), the effective lower bound implies biased results of canonical models, including their forecasts and the decomposition of yields. Consequently, as described the previous section, we utilize the methodology of Christensen and Rudebusch (2014) and Krippner (2013) and estimate shadow rates (yields), which are not influenced by the lower bound proximity. The extracted shadow rates are illustrated in the Figure 2.

Figure 2: Shadow Yields



Source: author's computations

When compared to the observed yields, the shadow rates are significantly lower for the yields that were originally close to zero, but only slightly different from the yields distinctively far from zero. This is in line with the shadow rates methodology, where the difference between the yields (more precisely, the difference between related bond prices) is viewed as a value of a call option on the shadow bond with a strike price at par (Christensen and Rudebusch 2014). As already noted in the previous section, only these shadow rates are utilized in the subsequent analyses, since the observed rates are biased by the lower bound proximity. For simplicity and clarity of the further text, the term "yields" will be kept, however meaning always the shadow rates.

## Macroeconomic and Financial Market Sentiment Data

The usual set of macro-variables introduced into macro-finance models represents real business activity, price dynamics and monetary policy development.<sup>12</sup> We follow this approach and use the industrial production index (IPI) as a proxy for real activity, the CPI index to represent price dynamics and the adjusted monetary base (MBA) to represent monetary policy steps. IPI is preferred to a GDP growth because of its monthly frequency. In the models, we also tried to replace MBA by the Fed funds rate, but the results were not significantly altered by this change. However, as

<sup>12</sup>The reference is included in the section 2.

the unconventional monetary policy is not directly reflected by the Fed funds rate changes, we believe that MBA is a better proxy for the monetary policy steps in the present environment.

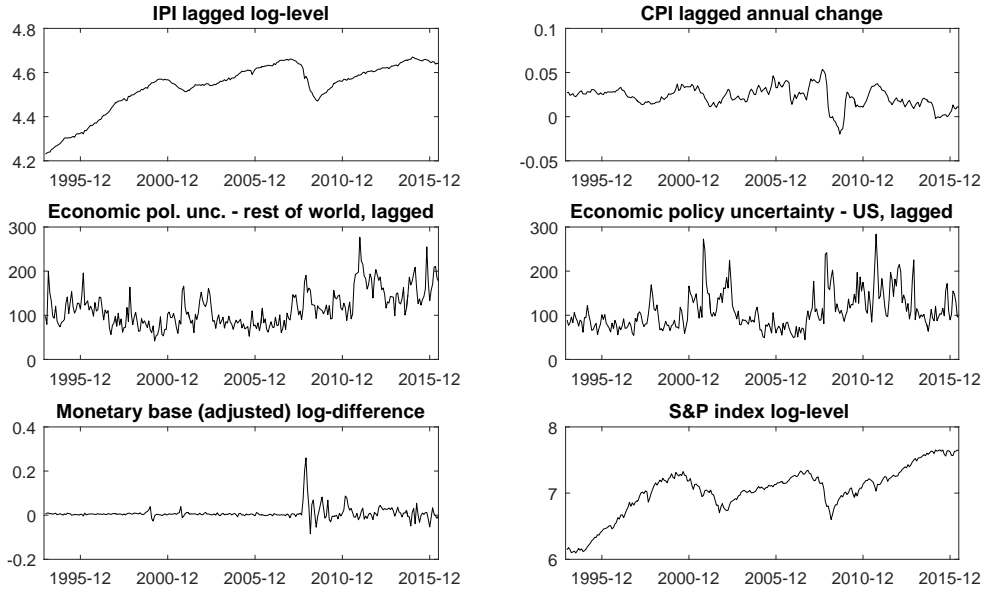
This set of variables is supposed to reflect most of the economic conditions development. However, recent events have shown, that yields can move significantly also in a response to financial market sentiment changes unrelated to new economic information. For example, during the stock market drop in January 2016 or after the UK Brexit referendum, the financial market uncertainty motivated investors to reallocate their portfolios into U.S. bonds, which represent a safe haven asset. This resulted in a significant decrease of yields of the 10 year U.S. Treasuries unrelated to a development in U.S. business activity, price dynamic or the monetary policy stance.

Following this motivation, in order to allow the models to capture the relation of the U.S. yields to financial market sentiment shocks explicitly, we augment the models by a set of three additional variables. First, we include the S&P 500 index. By doing so, we are able to capture changes in interest rates triggered by stock market crashes and the subsequent flight to safety – reallocation from U.S. stocks to U.S. bonds. Afterwards, we utilize a dataset of Baker *et al.* (2015), which includes news-based measures of an economic policy uncertainty in selected countries. Using data for EU, China, Japan and Russia, we extract the first principal component, which we call “Economic policy uncertainty - the rest of the world” (EPU-ROW). We consider changes in this variable to be a proxy to international shocks motivating capital flows to or from U.S. Treasuries, again because of their position as the safe haven financial instrument. Finally, we use the same data to get also the U.S. economic policy uncertainty (EPU-US). Reason for using EPU-US alongside with the S&P 500 index is a possible U.S. economic policy shocks, which may not be fully reflected by the stock market. Moreover, as the economic policy uncertainty measure in the U.S. and the rest of the world can be tightly related because of their news-based nature, EPU-US serves as a control variable allowing us to consider EPU-ROW changes as purely foreign shocks.

To link a response of yields to the macroeconomic and financial impulses correctly, IPI, CPI and EPU variables are one month lagged. The lag also ensures that when producing the forecasts, only the data available at the moment of the forecast are used. Except the EPU, the variables are taken as logarithms. For purposes of the section 5 (in-sample relations), all the variables including yields are transformed to first differences. For the forecasting purposes in the section 6, we stick to the literature and use the yields in levels despite their non-stationarity (evaluated using the Augmented Dickey-Fuller test – results are included in the Appendix II). In order to avoid a loss of information, also the other macro-financial variables are used in levels for the forecasting purposes, except for CPI and MBA, which follow a strong

trend and their inclusion in levels would result in estimates producing diverging forecasts. The historical evolution of macroeconomic and financial market sentiment variables is included in the Figure 3.

Figure 3: Macroeconomic and Financial Market Sentiment Variables



Source: FRED (2016), Baker *et al.* (2015), author's computations.

A VAR model is utilized in the next section. Within its specification, ordering of the variables is an important issue. As the shocks to macro-variables can be considered to have a contemporary impact on the financial market sentiment variables, they are ordered first. Such specification strengthens the interpretation of the shocks to financial sentiment variables as being unrelated to changes in macroeconomic conditions.

## 5 In-sample dynamic relations

### Procedure

In this section, we aim at evaluating a relative importance of factors determining changes of yields and their components: the term premium and the risk-neutral yield. As the Great Recession could result into structural changes in the Treasuries market, it is preferred to observe the results dynamically, i.e. capture their development over time. In order to obtain such outcome, the estimation procedure described below is repeated over a rolling window. The sample window size is set at 10 years in order to include enough data for the estimation while being able to capture changes in the results over time.

As a starting point, the sub-sample 1993/12–2003/11 is used. Based on this sub-sample, first, the yields are used to estimate the AFkalPF model. The estimated



model is then used to obtain the two components. For each maturity in the sample, these components together with macroeconomic and financial market sentiment variables enter a first order VAR model. The macroeconomic variables are ordered first in the model (in the order IPI, CPI, MBA), followed by the financial market sentiment variables (EPU-ROW, EPU-US, S&P 500) and the two yield components. From the estimated VAR model, impulse-response functions (IRFs) are derived. Furthermore, residuals (considered to be estimates of shocks) are used to obtain a historical decomposition of the yield changes: after pre-multiplying the residuals by IRFs, the changes of the two components in the last period of the particular sub-sample are expressed as a sum of responses to the contemporary and past shocks from the macroeconomic and financial variables. Finally, shares of the particular responses serve as a measure of the relative importance of each variable for explaining the changes of yields with respect to the last period of the given sub-sample.

Afterwards, the sample window is moved one month forward, keeping its length fixed. By this procedure, 152 sub-samples are evaluated, the last being from 2006/07 to 2016/06. Moreover, to increase the robustness, the two components ordered at the end of the VAR vector are ordered in both ways (the term premium the first, the risk-neutral yield the second and vice versa) and the results are compared. Consequently, the results of the analysis are 3-dimensional: 152 sub-samples, 2 ways of ordering and 10 various maturities.

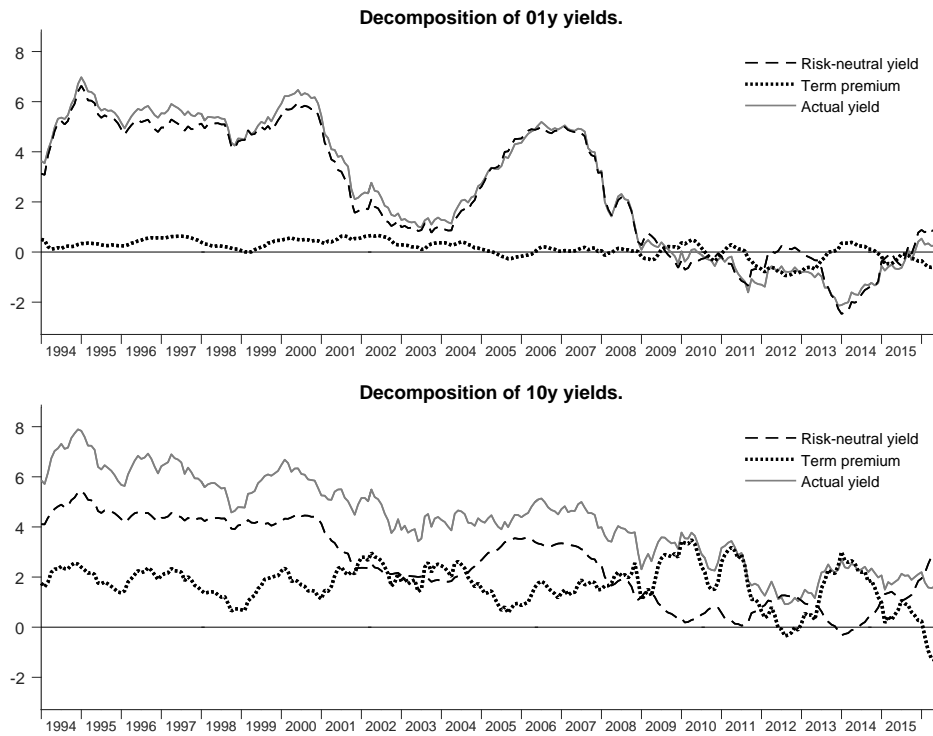
## Results

For each of 152 sub-samples, the components are estimated for each period of the sub-sample. For the particular sub-sample, these components then enter the VAR model described above and are used in the forecasting exercise as well. However, from the full sample perspective, the sub-samples are overlapping, so multiple estimates of the components are available for most periods. These estimates are generally different, since various sub-samples result in different estimations. Consequently, in order to illustrate the historical evolution of the components graphically, an average of all available estimates was used for each period. Such representation is offered in the Figure 4.

As obvious from the figure, the shorter rate is governed mostly by the risk-neutral yield, whereas for the longer yield, the term premium is important as well. Results confirm so-called Greenspan's conundrum<sup>13</sup> in 2005 as being explained by a drop of the term premium. Moreover, the 10-year yields term premia are rather counter-cyclical, which is in line with the prevailing consensus in the literature (see Bauer *et al.* 2014 for discussion). Since 2014, the risk-neutral yield is rising, which is in line with improving U.S. economic conditions. The most recent drops of yields in

<sup>13</sup>Greenspan's conundrum is related to the inability of the U.S. Federal Reserve Board to influence yields of longer maturities by increasing the short rates.

Figure 4: Yield Components



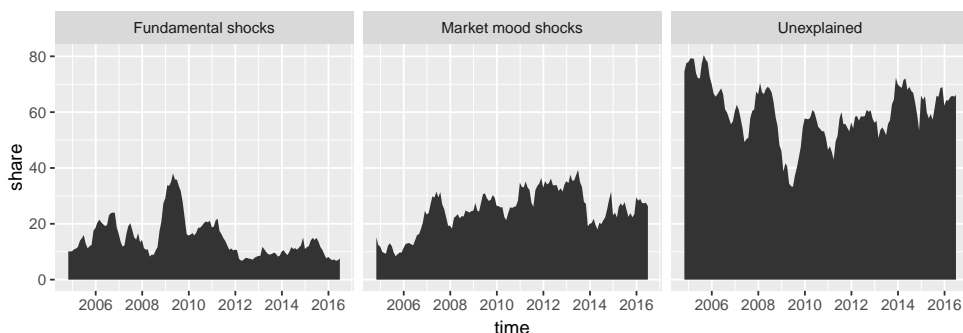
The components for each period were obtained by calculating an average value of all estimates for the particular period. Actual yields display the shadow rates.

January 2016 and June 2016 are explained by a drop of the term premium, which is in line with the above-expressed intuition about the flight-to-quality behavior in case of adverse financial market conditions. All these stylized facts confirm the validity of extracted components as being in line with the economic and financial conditions.

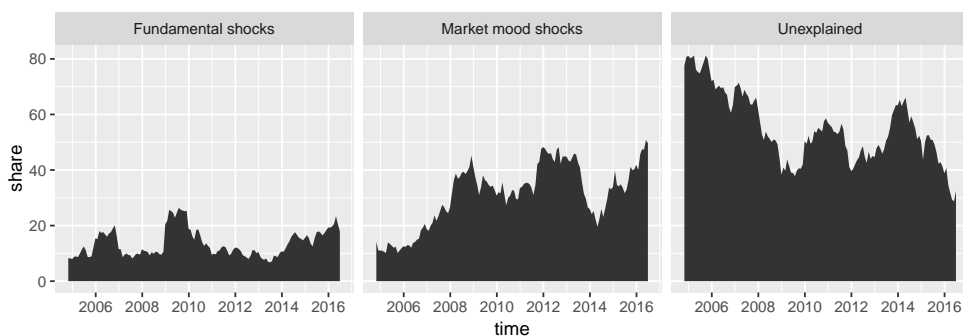
The calculated shares of particular factors for explaining changes of (shadow) yield components are robust to the ordering of the two components, so only results for the term-premium ordered in front of the risk-neutral yield are displayed. As the shares are related to the last observation of each sub-sample, they are calculated for the periods 2003/11–2016/06. For purposes of a graphical representation of the shares, they are aggregated (i) by summing the shares of the three macroeconomic variables (to create a joint “fundamental” share) and of the three financial market sentiment variables (to create a joint “market mood” share), and (ii) by summing the response of the two components to calculate the final response of yields. Aggregated results for 1-year and 10-year yields are included in the Figure 5. Since the shares estimated from the single sub-samples are quite volatile, the figure displays a 12-month moving average of the estimated shares so that the long-term trends are apparent. Figures displaying a detailed (unaggregated) split of the shares of particular variables for explaining changes of the single components are included in the Appendix III.

Figure 5: Aggregated Shares of Factors

1 year maturity:



10 years maturity:



Fundamental shocks include an aggregated share of shocks from IPI, CPI and MBA. Market mood shocks include an aggregated share of shocks from EPU-ROW, EPU-US and S&P 500. The unexplained share includes shocks from the term premium and the risk-neutral yield themselves. The shares are smoothed by a 12-period moving average, results for the first 12 periods are not displayed. The year marks show a start of the year.

At the aggregate level, the figure demonstrates a growth of the share of yield moves explained by the market sentiment shocks. At the end of the sample, their (smoothed) share is close to 30 % for the 1-year maturity and more than 40 % for the 10-year maturity. In both cases, the share roughly doubled over the whole period and fluctuated between 20 % and 50 % over the last five years. On the other hand, the share explained by shocks to macroeconomic variables is usually below 20 %, maximum reaching in 2009 during the U.S. recession period.

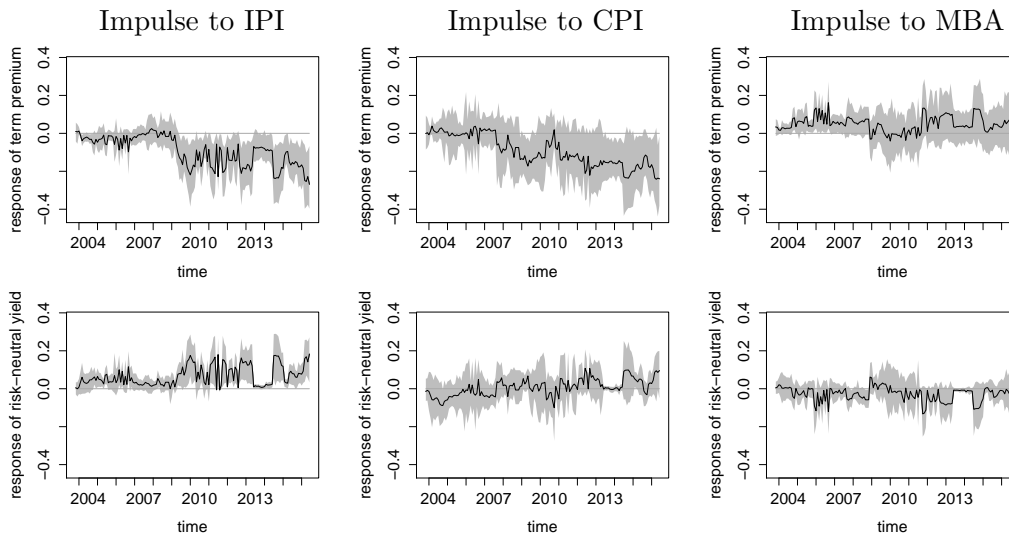
According to the detailed unaggregated results (Appendix III.), the market sentiment factors EPU-ROW and S&P 500 are the most important drivers of the term premium movements. EPU-US contributed to the variations significantly until 2014, its share is diminishing since then. S&P 500 is an important factor for explaining the risk-neutral yield changes as well, as this variable can be seen as implicitly including expectations about the health of the U.S. economy. Nevertheless, the shocks to IPI are the most important source of the risk-neutral yield variation. A temporary

growth of MBA share over 2009 can be explained by Federal Reserve Board policy steps during the economic slowdown.

The detailed results discovered, that the market sentiment shocks are primarily related to the term premium changes. At the same time, as obvious from the Figure 4, the term premium is (since the Great Recession) a dominant source of a variation in longer yields, compared to its lesser role in a variation of the short yields. This explains the higher importance of the market sentiment shocks for explaining changes of yields of longer maturities.

The presented shares of yield dynamics attributable to particular factors support the initial idea to extend the models by financial market sentiment variables. The results also highlight a different response of the two components on different shocks. To evaluate the link between the variables and the components explicitly, impulse-response functions are utilized. For each sub-sample, a response (at a 1-year horizon) of the components to the shocks to macroeconomic and financial sentiment variables are gathered. Results for yields of maturity 10 years are shown in the Figure 6 and the Figure 7, the presented findings are roughly valid also for the other maturities.

Figure 6: Responses of Yield Components to Macroeconomic Shocks

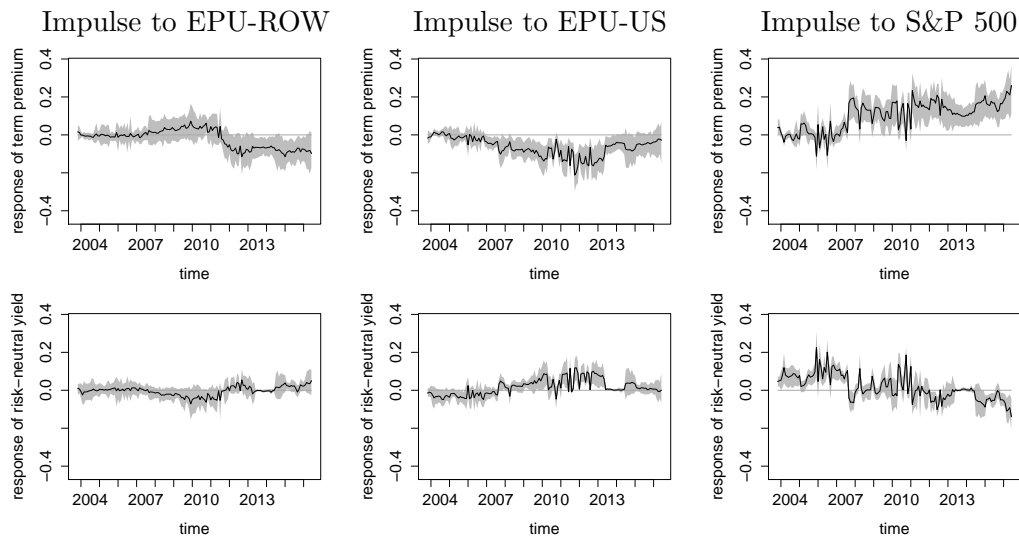


The plots capture dynamics of 12-month horizon responses. The gray area displays the 90% confidence interval obtained by bootstrapping.

Responses of the risk-neutral yield and the term premium to the shocks to macroeconomic variables are opposite in many cases. This is in line with the procyclicality of the risk-neutral yields and the counter-cyclicality of the risk premium at the same time. Regarding the responses to the financial market sentiment shocks, since 2011, the response of the term premium to a growth of EPU-ROW is significantly negative. Such development signals strengthening of the position of U.S. Treasuries as a safe haven asset. In case of an increased global uncertainty, the investors require a smaller premium to hold U.S. Treasuries. A similar pattern is obvious in the relation

of the term premium to the S&P 500 index. A positive shock in the equity market motivates investors to reallocate their portfolios towards equity (which leads to an increase of bond yields), and vice versa, a drop in the equity market causes the flight to safe U.S. Treasuries. Finally, also a growth of EPU-US results into the flight-to-quality behavior. However, in this case, the estimated significance of the channel is smaller, as part of the dynamics was already included in the S&P index change.

Figure 7: Responses of Yield Comp. to Market Sentiment Shocks



The plots capture dynamics of 12-month horizon responses. The gray area displays the 90% confidence interval obtained by bootstrapping.

## Newly proposed models

Following the described results, we propose two extensions of the models. First, we extend the set of three macroeconomic variables used in the macro-finance models NSfixMB, NSoptMB, NSkalMB and AFpcsMB by the financial market sentiment variables, as these were shown to explain an important share of yield movements. The extended models are called NSfixME, NSoptME, NSkalME and AFpcsME (“ME” meaning “macro-extended”).

Second, the results have also shown a significantly different response of the two components (the term premium and the risk-neutral yield) to macroeconomic and financial shocks. Consequently, when estimating a response of their sum (i.e. the yield) to the shocks, as the canonical models do, the responses of the components are partially mutually compensated. This leads to a loss of information about the underlying movements. To reflect this, we propose introducing both components separately into the models. In practice, we estimate the DNS model (NSfix.. and NSopt.. versions) for components instead of yields, using the two-step estimation approach. In the first step, for each component, we estimate the Nelson-Siegel function in each period and so obtain three latent factors per component. In the second steps, we

link the three factors from both components together within a VAR model (the transition equation in DNS). That means, that the resulting “yields-only” VAR model with separated components includes six variables. As in the other models, these factors can be accompanied by the macroeconomic and financial variables (“MB” and “ME” models). We name the models with separated components by adding “sep” to the end of the model code.

Moreover, we additionally propose a set of simplified models including single maturities. For each maturity, the risk-neutral yield and the term premium enter directly a VAR model, either by themselves (TSvarPFsep model) or together with macroeconomic (TSvarMBsep) and financial market sentiment (TSvarMEsep) variables.<sup>14</sup> The set of all newly proposed models is included in the Table 3.

Table 3: Newly Proposed Models

model name	based on	extension	type
NSfixME	NSfixMB	fin. market sentiment variables	macro-finance extended
NSoptME	NSoptMB	fin. market sentiment variables	macro-finance extended
NSkalME	NSkalMB	fin. market sentiment variables	macro-finance extended
AFpcsME	AFpcsMB	fin. market sentiment variables	macro-finance extended
NSfixPFsep	NSfixPF	separated components	yields-only
NSfixMBsep	NSfixMB	separated components	macro-finance basic
NSfixMEsep	NSfixME	separated components	macro-finance extended
NSoptPFsep	NSoptPF	separated components	yields-only
NSoptMBsep	NSoptMB	separated components	macro-finance basic
NSoptMEsep	NSoptME	separated components	macro-finance extended
TSvarPFsep	time series	separated components	yields-only
TSvarMBsep	time series	separated components	macro-finance basic
TSvarMEsep	time series	separated components	macro-finance extended

The symbols “NS” and “AF” represent the DNS and ATSM frameworks; “fix”, “opt”, “pcs”, “kal” and “var” are related to the estimation approach; “PF”, “MB” and “ME” represent pure-financial (i.e. yields-only), macro-basic (i.e. the traditional macro-financial) and macro-extended (including financial market sentiment variables) specifications; “sep” at the end represents models utilizing separated components of yields. “TS” symbol reflects the simple time-series VAR analysis.

## 6 Forecasts Comparison

To confirm and further extend the findings from the previous section, we present results of a yield forecasting exercise in this section. We generate forecasts of (shadow) yields on a rolling window, using the sub-samples identical to the in-sample analysis. For each sub-sample, the variables are gathered: yields, yield components (estimated for that particular sample) and macroeconomic and financial market sentiment variables. Afterwards, using these data and the methods described in the section 3, the

<sup>14</sup>Such approach was not included in the original set of canonical models, as usually considered as performing poorly for forecasting purposes – for example in De Pooter *et al.* (2010). However, by using the components instead of yields, we believe the performance may increase.

models are estimated and yields are predicted. All the models from the Tables 1–3 are used to generate the forecasts.

Yield forecasts are calculated up to 12 months ahead. Results for the longest horizon are displayed below; for shorter horizons, the results were generally similar, however slightly less conclusive. To compare the forecasting performance of the models prior to and after the recent crisis, the set of sub-samples is split into two sample groups. The first sample group starts with the first sub-sample (ending 2003/11, i.e. the first forecast is calculated for 2004/11) and ends with the sub-sample ending 2008/08 (one month before the Lehman Brothers collapse. The period of the crisis is not evaluated, since structural changes were present in that period. For our purposes, we arbitrarily set the end of the crisis period at the end of 2009 to obtain a sufficiently wide gap. Consequently, the second sample group consists of the sub-samples ending between 2010/01 and 2015/06 (included), i.e. the last forecast is generated for the most recent observation in the whole sample (2016/06). The first (second) sample group includes 58 (66) sub-samples.

For each sample group, model and maturity, an average of squared differences between the predicted values and the actual realizations (mean squared error, MSE) is calculated (the mean is taken over all sub-samples in the particular sample group). Moreover, to evaluate the robustness of our results, we use bootstrapping, i.e. we calculate multiple MSEs for sub-samples randomly drew from the given sample group. Afterwards, we take a difference of the MSE calculated for the random walk forecasts<sup>15</sup> (i.e. the forecasts set at the level of the last observation in each sub-sample) and the single MSEs from the bootstrapping. This difference (MSEdif) shows an average improvement of a model’s forecasting accuracy over the accuracy of the random walk.

The aim of the forecasting exercise is to evaluate the change of the forecasting accuracy after (i) introducing macroeconomic and/or financial market sentiment factors into the models, and (ii) replacing yield data by the separated components. To do so, we aggregate MSEdifs according to two criteria. First, focusing on factors entering the models, we group together MSEdifs of all “PF” (“MB”, “ME”) models (each group includes 5000 bootstrapped MSEdifs per each model included in that group). Second, using the components separation as the second criterion, we similarly split all MSEdifs into two groups: the first group includes the MSEdifs of all “sep” models (see Table 3), whereas the second group includes MSEdifs of all other models (we call this group as “joint” models).

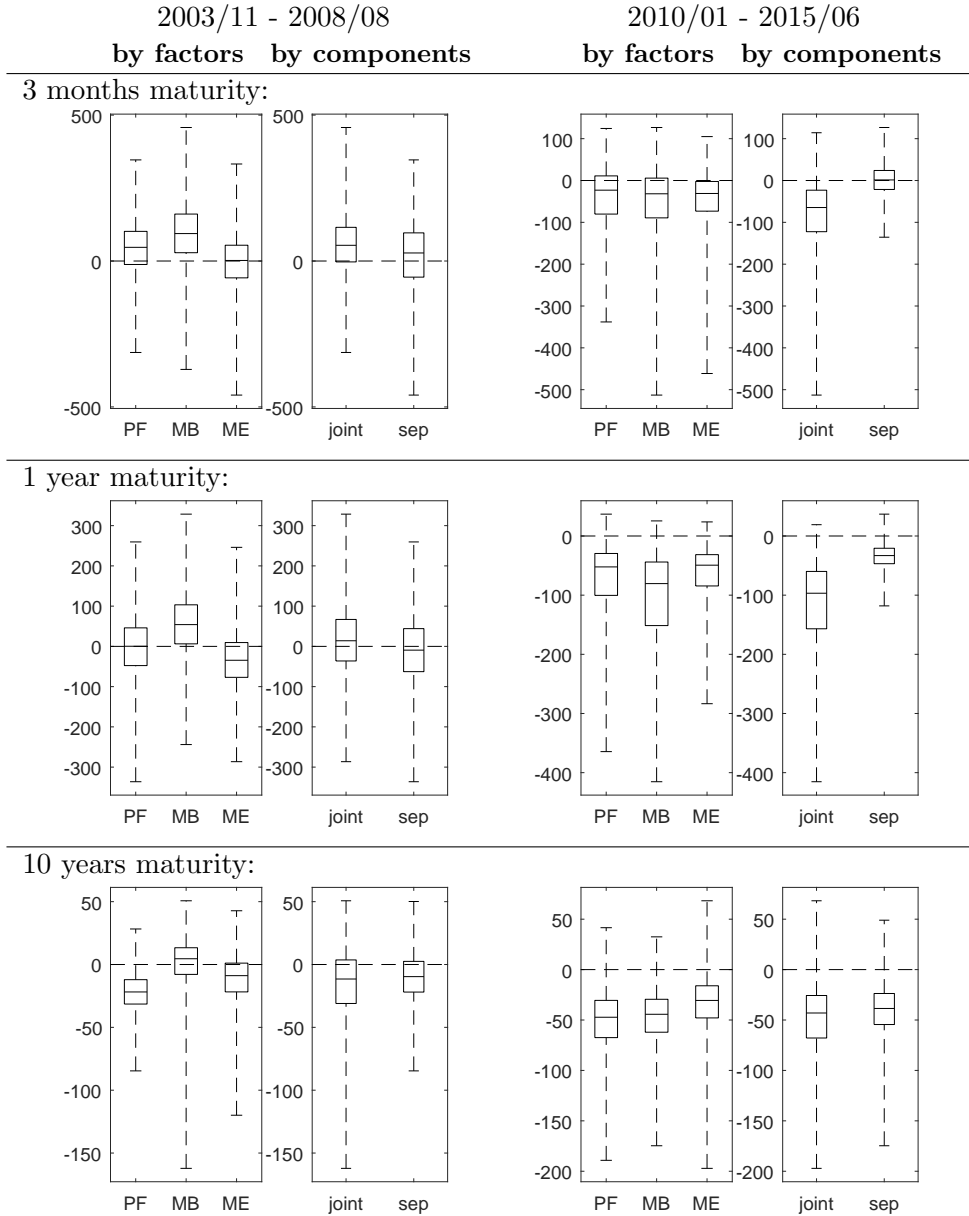
Below, we display forecasting results for three maturities: 3 months, 1 year and 10 years (Figure 8). For each group of models and each sample period, a box plot displays the median, the upper and the lower quartile and the extremes calculated

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<sup>15</sup>More specifically, we bootstrap over the random walk forecast as well. For each model, the set of 5000 MSEs generated by the bootstrapping is compared to the set of 5000 MSEs of the random walk. Such approach should provide quite robust results.

over all MSEdifs in the particular group. The number indicates by how much the MSE of the random walk is higher than of the particular model group (in basis points), i.e. a positive numbers indicates that a forecast of the particular model group is better than of the random walk. The detailed results for particular models (unaggregated) are included in the Appendix IV.

Figure 8: Aggregated Forecasting Results



The y-axis measures an average improvement (if positive) of the particular model group over the random-walk forecasts, measured in basis points. “PF” = yields-only models, “MB” = macro-finance models with a “traditional” set of macroeconomic variables, “ME” = models including financial market sentiment variables. “joint” models include yield data or their factors, whereas “sep” models include separated yield components (or their factors) instead. Whiskers show the maximum and the minimum and boxes display the interquartile range, both calculated using bootstrapping with 5000 iterations.



The canonical macro-financial (“MB”) models resulted in the most accurate forecasts in the period prior to the Great Recession. Such finding is in line with the literature, for example Diebold and Li (2006) and De Pooter *et al.* (2010). Adding the financial market sentiment variables leads to poorer results. Similarly, modeling the yield components separately doesn’t generally improve the results, although reduces the variance of the forecasting accuracy for long maturities.

The situation is significantly different in the period since the Great Recession. First of all, none of the models is able to systematically outperform the random walk forecasts in the low yield environment. Such finding is common in the recent literature (Christensen and Rudebusch 2016). Furthermore, the change of the relative importance of yield factors documented in the previous section influenced the forecasting results as well. The added value of the “MB” models over the yields-only (“PF”) models is kept only for long maturities. Contrary, the models with financial market sentiment factors (“ME”) perform better, as their median accuracy outperforms the “MB” model group median accuracy for all maturities. Similarly, replacing the yields in the models by their components proves as highly beneficial, since the forecasts generated by the “sep” models are generally more accurate and with a lower variance than the canonical “joint” models.

The results consequently show, that both presented adjustments of the models have a positive implications for the forecasting accuracy of the models. However, the impact is different for short and long maturities. For the short maturities, it is advisable to include the components (the term premium and the risk-neutral yields) separately into the models. This is in line with the findings of the previous section: the shocks have often different impact on both components. If the yield is included directly (i.e. as a sum of the two components), a significant amount of information is lost, which weakens the results. Contrary, for the long maturities, including the financial market sentiment variables is crucial for improving the forecasting results. The intuition is straightforward: for the longer maturities, the term premium explains a significant share of the yield movements. As it has been shown in the previous section, since the recent crisis, the term premium is explained by the financial market sentiment factors in a large extent. Consequently, these factors are increasingly crucial to capture the long yields’ movements. The separation of the components is less important for longer maturities, as the dominance of the term premium movements for explaining the long yields variation is relatively high (so the amount of information lost by not separating the components is quite small).

To conclude, the proposed extensions of the interest rate models resulted in a partially improved forecasting accuracy. This confirms that the economic consequences of the new environment since the Great Recession had an impact on the models’ performance and should be carefully reflected. On the other hand, our forecasting results are still relatively worse compared to the performance of interest rate models

prior to the recent crisis. It can be argued, that a part of the economic consequences of the new environment could represent a new feature of the yields dynamics that can not be addressed by the models directly. An example could be a possible lower persistence of the financial market shocks, compared to the macroeconomic shocks. The difference in the persistence of shocks is however not quantified in this paper and the issue is left opened for a further research.

## 7 Summary and Conclusions

The paper evaluates economic consequences of the environment following the Great Recession for the interest rate modeling and forecasting. By this, the paper goes beyond the recently popularized shadow-rates models, which focus primarily on the technical consequences of the lower bound proximity. As a result, the aim of the presented analysis is to contribute to the discussion about the causes of the post-crisis drop of the forecasting performance of canonical interest rate models.

Using shadow rates extracted from the U.S. Treasury yields of ten maturities, we evaluate dynamic relations of yields, yield components, macroeconomic and financial market sentiment variables. Our results clearly show, that shocks unrelated to macroeconomic news have become an increasingly important source of yields variation since the Great Recession. These shocks represent changes of the financial market sentiment triggered by political and purely financial events, for example Brexit or recent Chinese stock market drops. Furthermore, the two yield components often respond to shocks in an opposite way. Consequently, the final response of yields includes partially compensated responses of these two components and a significant amount of information about the underlying dynamics can be lost.

To reflect these findings, we propose two modifications of the canonical models. Namely, we extend them by the financial market sentiment proxies, and replace the (shadow) yield data by their separated components. Afterwards, we compare the forecasting performance of a wide set of the canonical models with these newly enhanced. As we find out, our extensions truly improve the forecasting accuracy, although the extent of the improvements depends on the yield maturity. More specifically, as we show, the forecasting accuracy of the short yields improves after using the separated yield components. On the other hand, for the longer yields, introducing the financial market sentiment variables explicitly into the models is crucial. Despite these advances, we still fail to obtain a forecasting accuracy comparable to the performance of models prior to the recent crisis. We argue that this can signal new features of the yields dynamics, for example a lower persistence of the shocks, which cannot be reflected by the present modeling techniques. However, this question remains opened and is left for a further research.

With the U.S. economy gradually leaving the lower bound proximity, the technical

problems related to the low yield environment could become less crucial. On the other hand, the new macro-financial factor dynamics and the way how it impacts the yield curve can remain. In that case, further advances in the analysis of the yield factors and the responses of the yield components would be necessary, which opens a wide area of the future research.

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## Appendices

### Appendix I: Models setup

#### Dynamic Nelson-Siegel Approach

The basic building block of the Dynamic Nelson-Siegel model is the Nelson and Siegel (1987) representation of the yield curve as a function of the maturity  $\tau_i$ :

$$y(\tau_i) = \beta_1 + \beta_2 \left( \frac{1 - e^{-\lambda\tau_i}}{\lambda\tau_i} \right) + \beta_3 \left( \frac{1 - e^{-\lambda\tau_i}}{\lambda\tau_i} - e^{-\lambda\tau_i} \right) \quad (3)$$

where  $\beta_1$ ,  $\beta_2$ ,  $\beta_3$  and  $\lambda$  are parameters. Later, the function was further extended by Svensson (1995) by including an extra term to enhance a flexibility of the function when fitting the term structure. However, the original form is often considered as being flexible enough, so the additional element is not utilized in order to maintain parsimony.

The three indexed beta parameters are of a special interest, as they can be considered as representatives of the main characteristics of the term structure of interest rates: the latent factors. For example Diebold and Li (2006) show, that these factors can be, following their factor loadings, viewed as a level, a slope and a curvature of the yield curve. Such specification is plausible, being in line with the nature of the first three principal components extracted from the yield curve data set, as shown by Litterman and Scheinkman (1991). Finally, when allowing for the three factors to be time-varying following Diebold and Li (2006), the static functional form changes to the Dynamic Nelson-Siegel model, which can be written in a form of a state space model:

$$y_t(\tau_i) = \beta_{1,t} + \beta_{2,t} \left( \frac{1 - e^{-\lambda\tau_i}}{\lambda\tau_i} \right) + \beta_{3,t} \left( \frac{1 - e^{-\lambda\tau_i}}{\lambda\tau_i} - e^{-\lambda\tau_i} \right) + \epsilon_{y,t}(\tau_i) \quad (4)$$

$$\beta_t = \alpha + \Gamma\beta_{t-1} + \epsilon_{\beta,t} \quad (5)$$

where  $\beta_t = (\beta_{1,t}, \beta_{2,t}, \beta_{3,t})'$ , whereas  $\alpha$  and  $\Gamma$  are a vector and a matrix of parameters. The measurement error terms  $\epsilon_{y,t}(\tau_i)$  are forming a  $N \times 1$  vector error term  $\epsilon_{y,t} \sim \mathcal{N}(\mathbf{0}, \mathbf{R})$ . A  $3 \times 1$  vector  $\epsilon_{\beta,t} \sim \mathcal{N}(\mathbf{0}, \mathbf{Q})$  represents random disturbances in the latent factors process. As usual for the basic state space representation,  $\epsilon_{y,t}$  and  $\epsilon_{\beta,t}$  are assumed to be mutually uncorrelated.

Following Diebold *et al.* (2006), the macro-financial extension of the model can be done directly. The measurement equation does not change, however the state vector is enriched by  $m$  observable macroeconomic and/or financial variables  $\theta_t$ , forming a new  $(3 + m)$ -dimensional state vector  $\eta_t = (\beta_t', \theta_t')'$ . The new transition equation is:

$$\eta_t = \mu + \Phi\eta_{t-1} + \epsilon_{\eta,t} \quad (6)$$

where  $\boldsymbol{\mu}$ ,  $\boldsymbol{\Phi}$  and  $\boldsymbol{\epsilon}_{\eta,t}$  are similar to  $\boldsymbol{\alpha}$ ,  $\boldsymbol{\Gamma}$  and  $\boldsymbol{\epsilon}_{\beta,t}$ , differing only in their dimensions. As the macro-variables  $\boldsymbol{\theta}_t$  do not enter the measurement equation, they do not influence the yields directly, but only through their joint dynamics with the original yield factors  $\boldsymbol{\beta}_t$ .

Two estimation methods can be used. A one-step estimation includes the usual methodology for estimating state space models: the maximum likelihood estimation within the Kalman filter procedure. Contrary, a two-step approach begins with fitting the yield curve by the Nelson-Siegel function (the Equation 3) in each period separately, using least squares. Obtained parameters  $\boldsymbol{\beta}_t$  then enter a VAR(1) model. An advantage of the one-step method, compared to the two-step approach, is an optimal estimation of  $\boldsymbol{\beta}_t$  factors with respect to the measurement and transition random errors. Contrary, in the two-step approach, the measurement errors are minimized first. The transition equation is then estimated keeping the extracted factors (i.e. also the measurement errors) fixed.

Within the estimation method, a determination of the  $\lambda$  parameter is crucial. In the two-step estimation method,  $\lambda$  must be determined before the estimation. Two possible approaches can be used. First, following Diebold and Li (2006),  $\lambda$  can be chosen arbitrarily to ensure that the yield curve will maintain a “usual” shape. Diebold and Li (2006) set  $\lambda$  equal to 0.0609, which maximizes the curvature factor loading at the maturity 30 months. The second approach is less arbitrary:  $\lambda$  minimizing the measurement error is employed. In this case, the first step of the two-step estimation method (fitting the yield curve in each period) is run repeatedly for various values of  $\lambda$ , until the sum of squared measurement errors over all maturities and all periods is minimized. In both cases, the final  $\lambda$  value is fixed for all periods. For the single-step approach, the two-step  $\lambda$  values can be used as well. As an alternative, the parameter can be also estimated directly using the maximum likelihood method within the Kalman filter procedure.

## Affine No-arbitrage Model

To specify an affine model, we build on the work of Duffie and Kan (1996) and a discrete-time specification presented by Ang and Piazzesi (2003). Under a risk-neutral measure ( $\mathcal{Q}$ ), a value of a zero-coupon bond  $P_t(\tau + 1)$  equals the expected value of the bond one period forward, discounted by an instantaneous (short) interest rate  $r_t$ :

$$P_t(\tau + 1) = E_{\mathcal{Q}} \left[ \frac{P_{t+1}(\tau)}{e^{r_t}} \right] \quad (7)$$

where  $E_{\mathcal{Q}}$  denotes expectations under the risk-neutral measure. Discounting by the short (risk-free) interest rate is perfectly consistent under the risk-neutral measure, as risk neutrality is equivalent to an absence of a term premium.

The model is affine, which means that the short rate can be expressed as an affine function of a  $l$ -dimensional vector of latent factors  $\mathbf{L}_t$ :

$$r_t = a_0 + \mathbf{a}_1^T \mathbf{L}_t \quad (8)$$

where  $a_0$  is a scalar parameter and  $\mathbf{a}_1$  is a  $l$ -dimensional vector of parameters. The latent factors follow a first-order VAR process:

$$\mathbf{L}_t = \boldsymbol{\gamma}_0 + \boldsymbol{\Gamma}_1 \mathbf{L}_{t-1} + \boldsymbol{\Sigma}_L \boldsymbol{\epsilon}_t \quad (9)$$

where  $\boldsymbol{\gamma}_0$  is a  $l \times 1$  vector and  $\boldsymbol{\Gamma}_1$  represents a  $l \times l$  matrix of parameters, whereas  $\boldsymbol{\epsilon}_t$  is a  $l$ -dimensional random term assumed to follow  $N(0, \mathbf{I})$ . Moreover,  $\boldsymbol{\Sigma}_L$  term is in this case a  $l \times l$  matrix of coefficients.

Under such specification, utilizing the fact that  $P_t(0) = 1$ , the model is fully defined under the  $\mathcal{Q}$ -measure. To shift between  $\mathcal{Q}$ -measure and the data-generating (real-world)  $\mathcal{P}$ -measure, a time-varying market price of risk  $\boldsymbol{\lambda}_t$  is employed.<sup>16</sup> The market price of risk is an affine function of the latent factors:

$$\boldsymbol{\lambda}_t = \boldsymbol{\lambda}_0 + \boldsymbol{\lambda}_1 \mathbf{L}_t \quad (10)$$

with  $\boldsymbol{\lambda}_0$  being a  $l$ -dimensional vector and  $\boldsymbol{\lambda}_1$  being a  $l \times l$  matrix of parameters. The market price of risk is used to convert the measures. Within the conversion, a Radon-Nikodym derivative is utilized and a stochastic discount factor (pricing kernel, a  $\mathcal{P}$ -measure analogy of the risk-free discount factor  $e^{-r_t}$ ) is derived. A full description of the conversion can be found for example in Ang and Piazzesi (2003).

The only variables determining (recursively) the bond prices under the  $\mathcal{P}$ -measure are  $r_t$  (from the Equation 7) and the measure-converting  $\lambda_t$ . Both  $r_t$  and  $\lambda_t$  are an affine function of the latent factors. As a result, bond prices as well as the yields can be expressed in terms of the latent factors, which are the only source of the bond price dynamics. An elegant advantage of the affine class of no-arbitrage models is that this relation has a convenient form (Duffie and Kan 1996):

$$P_t(\tau) = e^{\mathbf{B}_1(\tau) + \mathbf{B}_2^T(\tau) \mathbf{L}_t} \quad (11)$$

$$y_t(\tau) = -\frac{\ln P_t(\tau)}{\tau} = -\frac{\mathbf{B}_1(\tau)}{\tau} - \frac{\mathbf{B}_2^T(\tau) \mathbf{L}_t}{\tau} = \mathbf{C}_1(\tau) + \mathbf{C}_2^T(\tau) \mathbf{L}_t \quad (12)$$

Ang and Piazzesi (2003) show, that the functions  $\mathbf{B}_1(\tau)$  and  $\mathbf{B}_2(\tau)$  are defined

<sup>16</sup> The  $\boldsymbol{\lambda}_t$  symbol is used in order to follow the notation common in the literature, although having a different interpretation than the  $\lambda$  symbol in the Nelson-Siegel function.



recursively for increasing maturities:

$$\mathbf{B}_1(1) = -a_0, \quad \mathbf{B}_2(1) = -\mathbf{a}_1 \quad (13)$$

$$\mathbf{B}_1(\tau + 1) = \mathbf{B}_1(\tau) + \mathbf{B}_2^T(\tau) (\gamma_0 - \Sigma_L \boldsymbol{\lambda}_0) + \frac{1}{2} \mathbf{B}_2^T(\tau) \Sigma_L \Sigma_L^T \mathbf{B}_2(\tau) - a_0 \quad (14)$$

$$\mathbf{B}_2^T(\tau + 1) = \mathbf{B}_2^T(\tau) (\Gamma_1 - \Sigma_L \boldsymbol{\lambda}_1) - \mathbf{a}_1^T \quad (15)$$

Consequently, the whole model can be written in the state-space representation:

$$y_t(\tau) = \mathbf{C}_1(\tau) + \mathbf{C}_2^T(\tau) \mathbf{L}_t + \epsilon_{y,t}(\tau) \quad (16)$$

$$\mathbf{L}_t = \boldsymbol{\gamma}_0 + \Gamma_1 \mathbf{L}_{t-1} + \boldsymbol{\epsilon}_{L,t} \quad (17)$$

where the inclusion of  $\epsilon_{y,t}(\tau)$  results from the fact that the observed yields might not be fitted perfectly, i.e. some measurement error is present. Similarly to the Dynamic Nelson-Siegel model,  $\epsilon_{y,t}(\tau)$  are forming a  $N \times 1$  vector error term  $\boldsymbol{\epsilon}_{y,t} \sim \mathcal{N}(\mathbf{0}, \mathbf{R})$ . A  $l \times 1$  vector  $\boldsymbol{\epsilon}_{L,t} \sim \mathcal{N}(\mathbf{0}, \mathbf{Q})$  is a rewritten form of the  $\Sigma_L \boldsymbol{\epsilon}_t$  term from the Equation 9 ( $\mathbf{Q} = \Sigma_L \Sigma_L^T$ ) and represents random disturbances in the latent factors process.  $\boldsymbol{\epsilon}_{y,t}$  and  $\boldsymbol{\epsilon}_{L,t}$  are assumed to be mutually uncorrelated.

The presented state-space representation of the affine no-arbitrage model is very close to the Dynamic Nelson-Siegel form. Consequently, the macro-financial extension is done in a similar way, enriching the original  $\mathbf{L}_t$  vector by a  $\boldsymbol{\theta}_t$  vector of macroeconomic and/or financial variables to form an extended macro-financial state vector. However, in case of the affine model, the  $\boldsymbol{\theta}_t$  vector enters directly also the measurement equation (the Equation 16), so a change of the observed macro-financial factors has a direct impact on the yield curve.

Similarly to the Dynamic Nelson-Siegel model, either a one-step or a two-step estimation approach is possible. The one-step approach utilizes the Kalman filter and the maximum likelihood estimation. The two-step approach firstly finds a proxy for the latent factors, making them observable. For example, the common approach is to use the principal components of yields, representing yields of some bond portfolios, in line with Joslin *et al.* (2011). A VAR process of these observable factors  $\mathbf{L}_t$  is then estimated (the Equation 17). Afterwards, a proxy for the short rate is found (for example an over-night money market rate) a the Equation 8 is estimated. Finally, all remaining parameters of the Equation 16 are obtained by minimizing the measurement errors over the remaining parameters of the equations 13–15.

## Appendix II: Results of stationarity tests

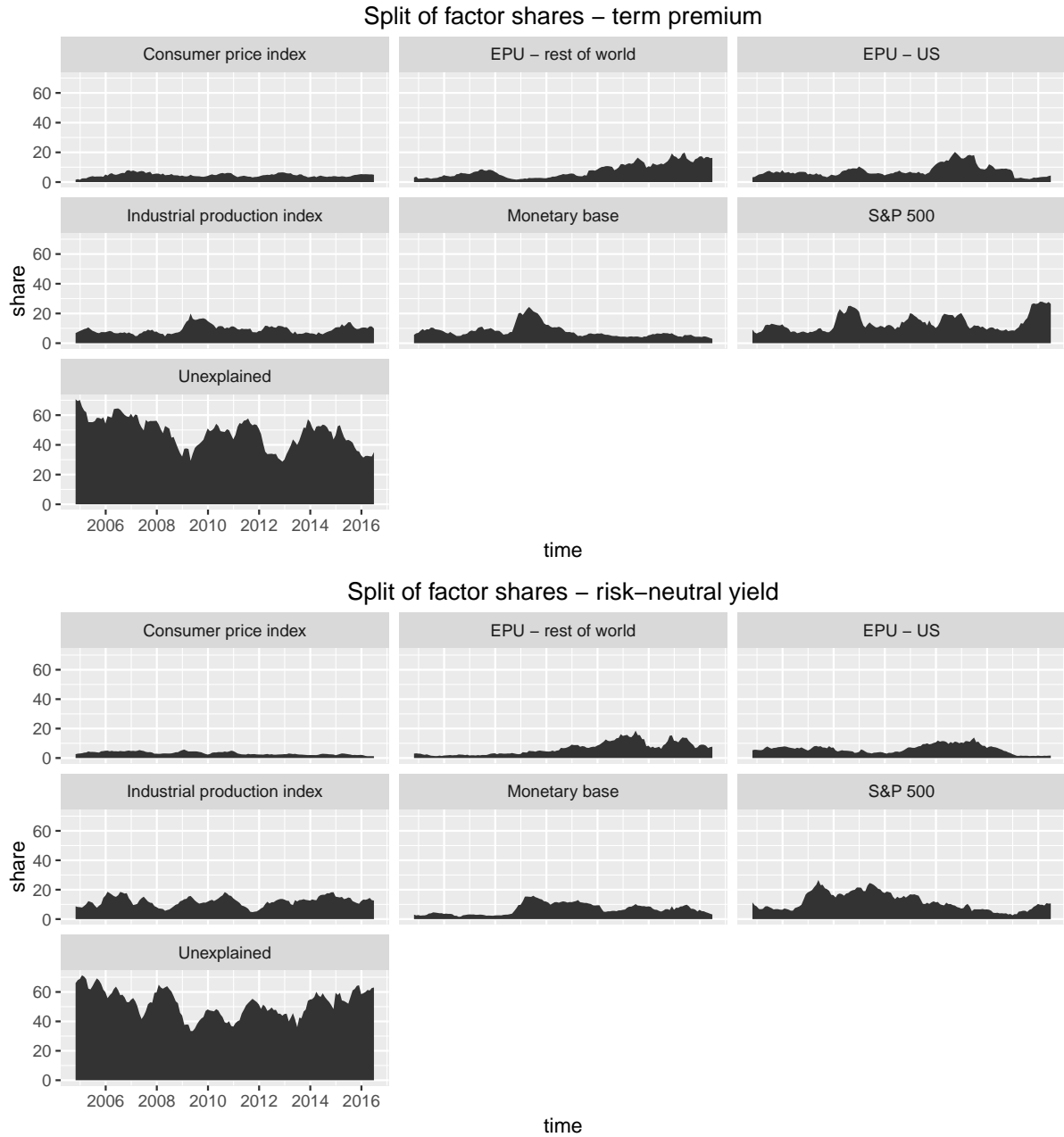
Table 4: Augmented Dickey–Fuller Test Results

variable	test statistic of a model:	
	in levels	in differences
3-month yield	-0.6524	-4.3921
6-month	-0.6561	-4.5422
1-year	-0.6630	-5.2732
2-year	-0.6768	-6.3237
3-year	-0.6777	-6.8038
5-year	-0.6568	-7.2470
7-year	-0.6326	-7.4999
10-year	-0.6080	-7.7752
20-year	-0.5802	-8.2854
30-year	-0.5743	-8.5016
monetary base	5.1970	-7.3099
CPI	6.8707	-7.4827
IPI	4.3673	-6.7734
S&P 500	1.4648	-4.4208
EPU U.S.	-1.1999	-8.9163
EPU rest of world	-0.9982	-10.2898
critical value (90%)		-2.58
critical value (95%)		-1.95
critical value (99%)		-1.62

Only the first estimation sub-sample is used for testing the stationarity. Shadow rates are not tested, they are very close to the observed yields in the first estimation sub-sample. For the model in differences, the first difference is used.

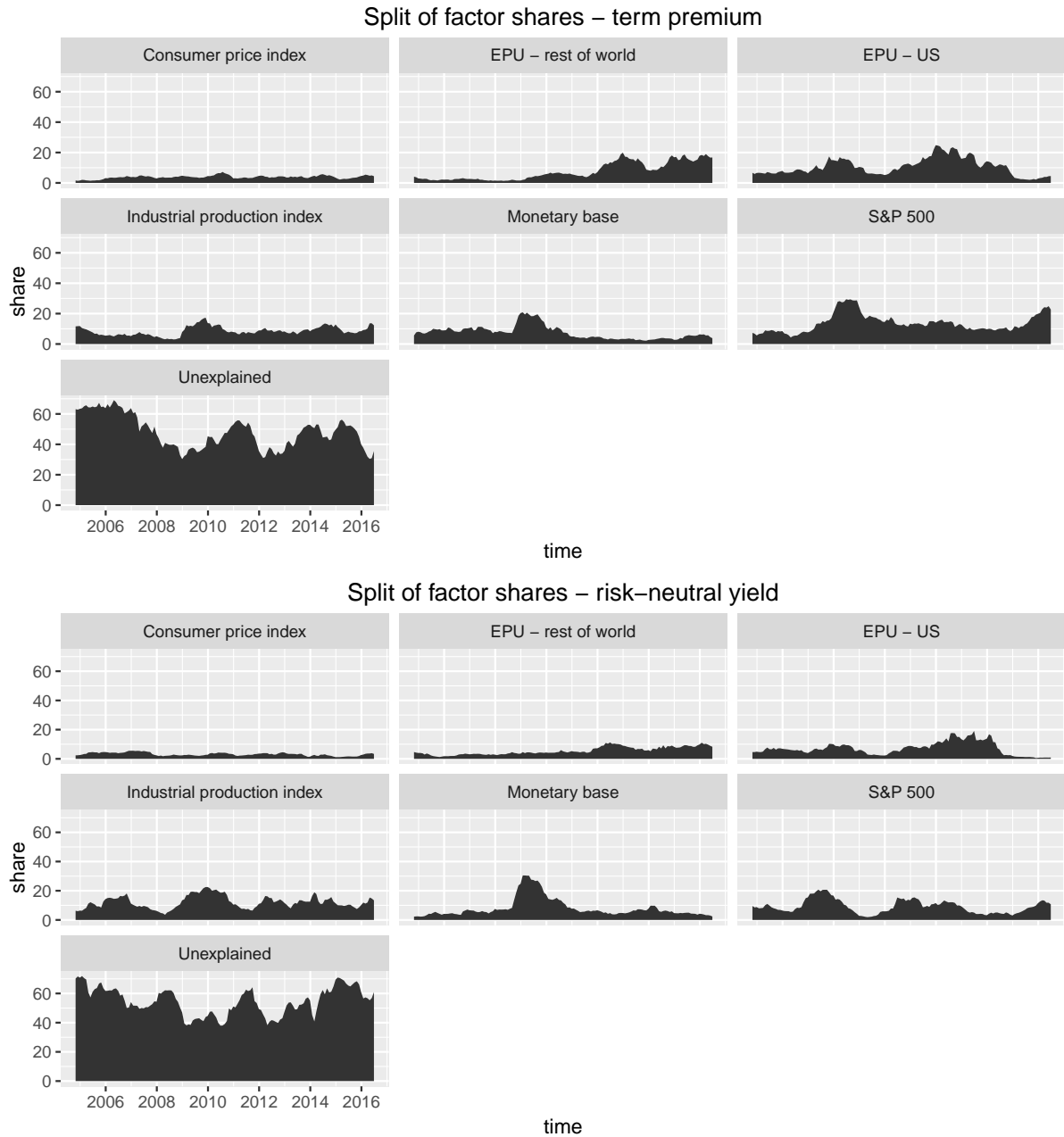
### Appendix III: In-sample results

Figure 9: Shares of Factors: 1-year Yield Components



The unexplained share includes shocks from the term premium and the risk-neutral yield themselves. The shares are smoothed by a 12-period moving average, results for the first 12 periods are not displayed. The year marks show a start of the year.

Figure 10: Shares of Factors: 10-year Yield Components

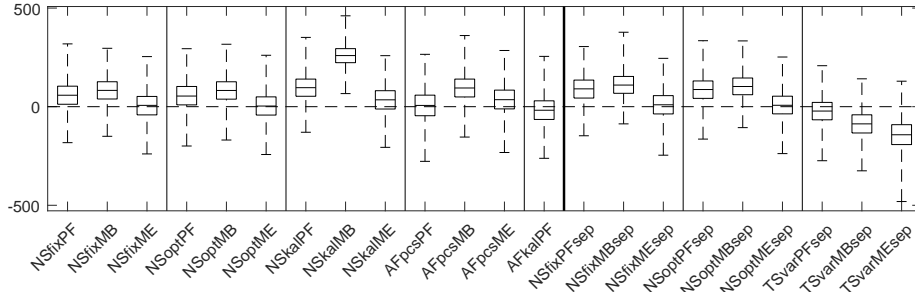


The unexplained share includes shocks from the term premium and the risk-neutral yield themselves. The shares are smoothed by a 12-period moving average, results for the first 12 periods are not displayed. The year marks show a start of the year.

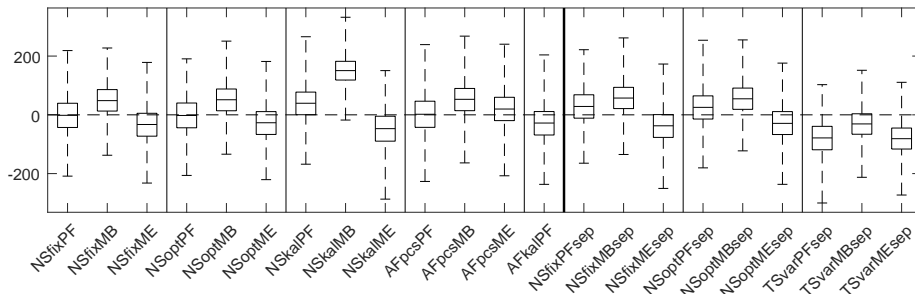
## Appendix IV: Forecasting results

Figure 11: Detailed Forecasting Results: 2003/11–2008/08

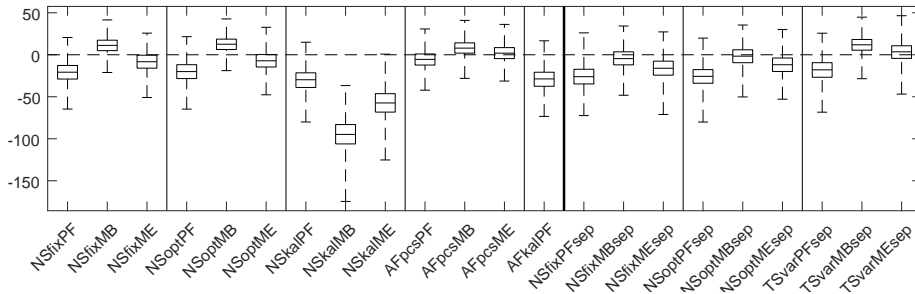
3 months maturity:



1 year maturity:



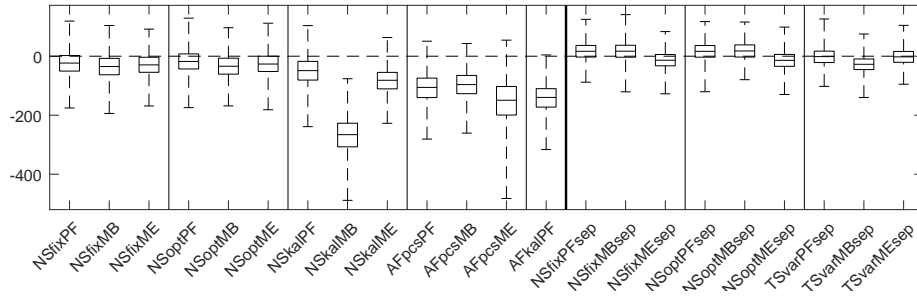
10 years maturity:



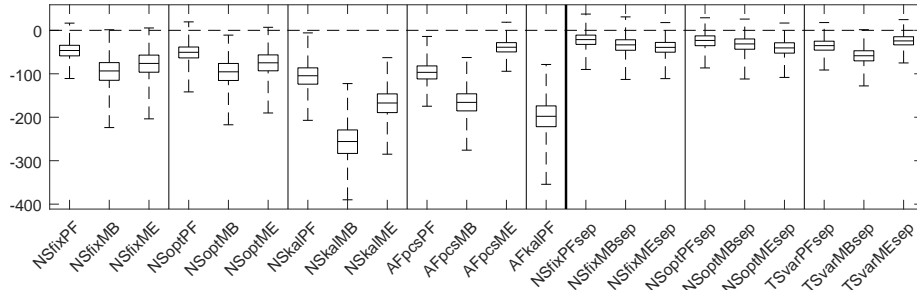
The y-axis measures an average improvement (if positive) of the particular model group over the random-walk forecasts, measured in basis points. The description of the particular models is included in the tables 1–3. Whiskers show the maximum and the minimum and boxes display the interquartile range, both calculated using bootstrapping with 5000 iterations.

Figure 12: Detailed Forecasting Results: 2010/01–2015/06

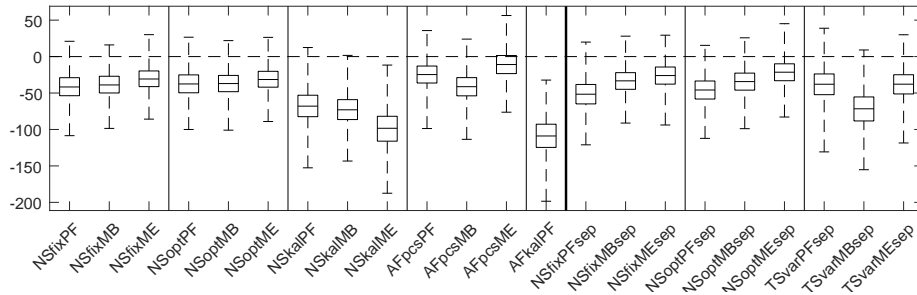
3 months maturity:



1 year maturity:



10 years maturity:



The y-axis measures an average improvement (if positive) of the particular model group over the random-walk forecasts, measured in basis points. The description of the particular models is included in the tables 1–3. Whiskers show the maximum and the minimum and boxes display the interquartile range, both calculated using bootstrapping with 5000 iterations.

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