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## The rise of the service economy and the real return on capital

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School of Economics Discussion Papers

# **The rise of the service economy and the real return on capital**

Miguel León-Ledesma and Alessio Moro

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# The rise of the service economy and the real return on capital\*

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May 6, 2016

## Abstract

We use a two-sector model of structural transformation and balanced growth to show that the real interest rate, measured as the return on capital in units of GDP or in units of aggregate consumption, declines as income grows. This is due to the differential TFP growth in the goods producing sector relative to the services sector. This differential drives a relative price change that triggers a steady decline in the rate of return on capital along the growth path. We calibrate the model to U.S. data to reproduce the behavior of GDP, the share of services in consumption, the relative price goods/services and the investment/output ratio in the period 1950-2015. We find that the calibrated model displays a decline of the real interest rate of 36% in terms of units of GDP and of 43% in terms of units of aggregate consumption during the period considered.

**JEL Classification:** E22; E24; E31; O41.

**Keywords:** Structural transformation, productivity of capital, two-sector model.

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## Non-technical summary

The possible tendency for the returns to investment in real capital to fall as economies develop has been a long-standing problem in Economics that already preoccupied classical economists. In standard models of economic growth, the concept of balanced growth path (BGP) implies that the real return on capital, the rate of growth of GDP, the capital-to-output, and investment-to-output ratios are all constant. In the data, this is consistent with the so-called “Kaldor stylized facts”. Models of growth that achieve BGP, thus, imply that the rate of return on capital is constant in the long run.

However, several observations are at odds with this. First, when measured in *real* terms, i.e. deflated by their respective prices, the capital-to-output and the investment-to-output ratios in the US economy both have a positive trend. Intuitively, this implies a lower average product of capital and, under certain conditions, a lower marginal product. Second, recent evidence for the US shows that the so-called “natural” or equilibrium real interest rate has declined quite substantially during the past 50 years. This has occurred at a time when there has been a massive transformation in the structure of the economy from manufacturing to services. Consumption of services was a mere 40% of total consumption expenditure in 1950, and is now close to 70%. This was accompanied by a very marked decline in the prices of goods relative to services.

In this paper, we propose an explanation for all these facts. We use a model of structural change from goods producing to services producing industries driven by productivity growth differences between the two sectors. Productivity grows slower in services. This differential explains the drop in the relative price of goods. The model generates BGP, with a constant real return to capital. However, this is only the case if this return is measured in terms of a chosen price, the price of goods (including investment goods). If we take the model-generated real return and GDP growth, and we measure them as it is done in national accounts, we actually would observe a secular decline. This is because services are now a larger share of consumption and GDP and their prices grow faster, hence reducing the actual number of units of consumption or output that one can buy after an investment project in real capital (i.e., *the* real return).

We estimate that, for the 1950-2015 period, the fall in the real rate of return due to this mechanism is of the order of 40%. The rate of growth of GDP falls by a less substantial 16% for the 65 years of the sample. These are both very significant shifts with important consequences for macroeconomic policy design, pension plans projections, and financial management.

# 1 Introduction

The stability of the real return to capital is one of the key features of neoclassical growth models. In the standard one-sector growth model, balanced growth implies that the marginal product of capital is constant as it is pinned down by agents' inter-temporal preferences and the rate of growth of TFP. This is roughly consistent with the observation of relatively constant trend growth rates of GDP and great ratios such as the nominal capital to output ratio. However, when measured in real terms (i.e. deflated by the relative price), both the capital to GDP and the investment to GDP ratios in U.S. data have displayed a positive trend since 1950 (see [Fernald \(2012\)](#) and [Gourio and Klier \(2015\)](#)). This appears to be a feature of non-balanced growth, which is inconsistent with the predictions of the one-sector growth model. The non-constancy of the investment-output and capital-output ratios might imply other important changes of the economy over time, such as a change in the rate of growth of GDP and the real return on capital, with important consequences for business investment, macroeconomic policy, and financial management. Casual observation suggests that the growth rate of GDP in the U.S. has been roughly stable in the post-war period, although there is some econometric evidence pointing to a slowdown. The real return on capital, instead, cannot be directly measured in the data, unless some theoretical assumptions are made.

In this paper we use a two-sector growth model of structural transformation from manufacturing to services to rationalize the increase in the investment/GDP ratio and measure its implications for the real return to capital and the growth rate of the economy. The model displays balanced growth with a constant real return to capital when measured in terms of an appropriately chosen numeraire. However, when measured in terms of units of GDP, or in terms of aggregate consumption, the marginal product of capital, and hence the real return, displays a trend because of a change in the relative price of services to goods in the economy. The model is then calibrated to replicate certain features of the U.S. economy in the past 65 years. Quantitatively, it can reproduce very closely the rate of growth of GDP, the observed change in the share of services in consumption, and the increase in the real investment to GDP ratio. The model generates a fall in the real return on capital of 36% in units of GDP and of 43% in units of aggregate consumption between 1950 and 2015. Also, in the model, GDP growth falls from 2.29% per year to 1.93% per year from the beginning to the end of the sample (a 16% decline). This is an economically significant reduction that is difficult to statistically detect in the data, given the observed magnitude of business cycle fluctuations in the U.S.

The model is a simplified version of [Boppart \(2014\)](#), where differential productivity

growth in the two sectors generates relative price changes and non-Gorman preferences allow for an aggregate balanced growth path measured in terms of the numeraire, which in this case is the price of goods. Along the balanced growth path, the real return on capital in terms of the numeraire is constant. The mechanism generating the fall in the marginal product of capital in our model is as follows. We measure capital returns in terms of the number of units of GDP (or in units of aggregate consumption) that can be bought after investing one unit of capital, obtaining the marginal product, and selling the non-depreciated capital at its price. That is, from the model, we generate a measure of the real return to capital consistent with the way in which GDP and aggregate consumption are constructed in national accounts. The price of GDP is a Fisher index of the prices of goods and services. When measured in terms of this deflator, the marginal product of capital falls. Put differently, growth is “unbalanced” in terms of GDP units in the tradition of [Baumol \(1967\)](#). As a result, the model generates a fall in the rate of growth of GDP and the return on capital. Quantitatively, the fall in GDP growth in our model is of the order of only 0.36 percentage points throughout the 65 year sample. Given that the standard deviation of annual per capita GDP growth in the U.S. between 1950 and 2015 is approximately 2.30%, statistically, it would be difficult to separate the trend fall from business cycles in the data.<sup>1</sup>

The problem we tackle has in fact a long tradition in economics. The secular decline of the profit rate was an issue that preoccupied already classical economists such as Smith and Marx. For Smith, profits would steadily decline due to increased market competition with the development of capitalism. There has also been a long lasting controversy about the “tendency of the rate of profit to fall” since Marx’s *Das Capital*. The Marxist argument was that capitalism leads to the accumulation of capital and the introduction of labor saving technologies that reduce the labor input and generates lower value according to the labor theory of value. The value-to-capital ratio would fall leading to a fall in the profit rate. This theory is marred by theoretical criticisms and problems related to the empirical counterpart to the “profit rate”. In our approach, we avoid these controversies and focus on the real marginal product of capital. We assess the quantitative performance of a two-sector model of structural change from manufacturing to services, which is an uncontroversial observation in the development process, and focus on the role of price changes and how empirical measures of the real return to capital are predicted to evolve according to our equilibrium model. The relationship between the growth rate of output and the return on capital also plays an important role in [Piketty \(2014\)](#), where a secular increase in the wealth to output ratio is

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<sup>1</sup>A simple trend fitted to GDP per capita growth suggests an annual average fall in GDP per capita of 0.03% per year. However, this trend heavily depends on the starting and ending year of the sample and is only marginally significant. Using more sophisticated econometric techniques, [Bai, Lumsdaine, and Stock \(1998\)](#) and [Eo and Morley \(2015\)](#), suggest there is fall in the growth rate of real GDP in the US.

attributed to returns to capital exceeding the rate of growth of output. In our model, even if the return to capital exceeds the rate of growth of GDP, our quantitative analysis shows that, with structural change into services, the decline in the return to capital exceeds the decline in the rate of growth of GDP.

Our work is related to several streams of the literature. Here we discuss those more closely related to our paper. The literature on structural transformation struggled for long to provide a model consistent with both structural change and aggregate balanced growth. [Kongsamut, Rebelo, and Xie \(2001\)](#) and [Ngai and Pissarides \(2007\)](#) represent the first models that succeeded in providing such a coincidence. However, there are features of the data that these models cannot account for. For instance in [Kongsamut, Rebelo, and Xie \(2001\)](#) the nominal and the real shares coincide, while in [Ngai and Pissarides \(2007\)](#) the real share of services declines. Both these predictions are at odds with the data. More recently, [Boppart \(2014\)](#) provides a model with non-Gorman preferences that displays balanced growth and is consistent both with an increasing relative price of services and an increasing real share of services. In this paper, we use a version of this model.

An important note is due here, which is also a key point of this paper. The definition of balanced growth in a model of structural transformation relies on expressing all variables, including aggregate output, in terms of a numeraire. This is usually the price of capital. Thus, the models in [Ngai and Pissarides \(2007\)](#) and [Boppart \(2014\)](#) find that there exists a dynamic equilibrium in which the real interest rate is constant and aggregate output and capital grow at the same constant rate. However, these models are silent about the properties of the economy if real variables are expressed in terms of units of another good. We show here that the concept of balanced growth strictly depends on the units variables are expressed in. This is relevant when bringing the model to the data, because GDP in the data differs from nominal aggregate output divided by the price of one good. Instead, real GDP in the data is constructed using a chain-weighted Fisher index. Roughly speaking, the Fisher index weights the growth rate of individual components of GDP by their shares in GDP. This implies that, even if variables grow at a constant rate, if these rates are different and there is structural transformation, the growth rate of GDP is non-constant over time. This point is also made in [Moro \(2015\)](#) in a model without capital. Here we extend that argument by showing that, when measuring GDP as in the data, even a model which displays balanced growth in terms of a numeraire shows a reduction in the growth rate of GDP and the return on capital when transitioning from manufacturing to services. This, in turn, implies a reduction in the real interest rate along the growth path.

Our paper is also related, albeit indirectly, to the recent literature on the “Secular Stagnation”. This originates from the fast decline in real interest rates occurred in the last

15 years which are accompanied by low growth prospects of the World economy after the Great Recession (Summers (2014)).<sup>2</sup> For instance, Laubach and Williams (2003), updated in Laubach and Williams (2016), report evidence suggesting that the real natural rate of interest in the U.S. drops from around 5% in 1960 to slightly negative in 2015. Several hypotheses have been advocated to explain such a fast decline in the natural rates of interest. Common explanations are: i) countries such as China and the former Soviet Union participating in a growing transfer of savings from the emerging economies to the advanced economies; ii) monetary policy; iii) changes in the distribution of income which raised the propensity to save. While all these explanations certainly play a role, the evidence in Laubach and Williams (2003) suggests that the decline in real rates is not an exclusive phenomenon of the last few years, but starts earlier in time. In our paper we do not focus on the observed decline in the past 15 years or the possible post- Great Recession stagnation. However, our results are relevant to this literature insofar as it can explain a long-run decline in real returns.

The remainder of the paper is organized as follows. Section 2 presents the data facts that motivate our work; in section 3 we present the model and in section 4 we calibrate the model to U.S. data and use it as a measurement tool to assess the decline in the real interest rate induced by structural transformation. In section 5 we conclude.

## 2 Stylized facts

We present a set of facts that motivate our analysis and serve as quantitative targets for our model. Because of the need to match theory and data, we pay special attention to the measurement of variables in a way that is consistent with the two-sector model presented below. The key variables are the relative price of goods over services, the investment to GDP ratio measured in real terms, the capital-GDP ratio measured in real terms, and the nominal share of services consumption in total personal consumption expenditure. In the two-sector model below we assume that the manufacturing sector produces a good that can be used both for investment and for consumption of manufacturing. Thus, in the data we construct a *price of goods* which is a Fisher chain-weighted price index of consumption goods and gross domestic investment (GDI).<sup>3</sup>

The relative price goods/services is obtained from NIPA tables<sup>4</sup> as the *price of goods* (constructed as described above) relative to the price of services. The real GDI to GDP ratio

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<sup>2</sup>See the collection of essays in Teulings and Baldwin (2014) for an overview.

<sup>3</sup> In the appendix we present a three-sector model and present data for the relative prices goods/services and investment/services. As we show there the main message in the data and in the model is confirmed.

<sup>4</sup>NIPA Table 1.1.4 at [http://www.bea.gov/iTable/index\\_nipa.cfm](http://www.bea.gov/iTable/index_nipa.cfm).

is calculated as the ratio of real investment to real GDP. We deflate nominal GDI<sup>5</sup> using the same *price of goods* used to construct the goods/services price ratio. Note that when using the investment deflator from NIPA tables to deflate investment, the trend observed in the investment/GDP ratio is similar and statistically significant, but less pronounced.<sup>6</sup> This will be discussed further below because replicating a measure of the investment-output ratio deflated by the investment price requires a three-sector model.<sup>7</sup> Finally, real GDP is given by nominal GDP deflated by the GDP deflator.

Additionally, we present evidence on the evolution of the capital-GDP ratio. We are interested in the ratio between the real capital stock and real GDP, i.e. where each nominal measure has been deflated by its own price. Note that this will generally differ from the ratio of the two nominal measures as long as the relative price deflators for capital and GDP differ.

The measurement of capital is more controversial than that of investment. For this reason the estimates should be taken with caution. We use the measure coming from the BLS Multi Factor Productivity (MFP) project which calculates total capital services for the private business sector. The measure uses a Jorgensonian perpetual-inventory method aggregating from different types of capital. As discussed in [Gourio and Klier \(2015\)](#), BLS estimates are more reliable than BEA fixed assets accounts, as they use weights based on real user costs to aggregate capital stocks.<sup>8</sup> For comparison, we also show the measure of [Fernald \(2012\)](#), which accounts for the total business sector and is adjusted for capital utilization. In practice, the trends displayed by these two measures are very similar, as they mostly differ only in terms of business cycle volatility.

Finally, the share of services in total consumption expenditure is calculated as the nominal share of of personal consumption expenditure on services over total personal consumption expenditures (i.e. on services and goods). The data also come from NIPA tables (Table 1.1.5).

Figure 1 presents the data in logs (except for the consumption share of services) and a fitted trend line. The figure also contains the investment (GDI) to output (GDP) ratio in nominal terms from NIPA accounts for comparison. The price of consumption goods relative to services displays a very well defined negative trend implying a yearly growth of -1.57%.

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<sup>5</sup>NIPA Table 1.1.5.

<sup>6</sup>See Appendix A.

<sup>7</sup>Similarly, the price of total goods including investment and consumption relative to services, displays a very similar trend to that of the price of consumption goods relative to services. The former falls at a rate of 1.57% per year between 1950 and 2015, and the later at a rate of 1.61% per year.

<sup>8</sup>We also compared the estimates of [Fernald \(2012\)](#) with the original data from BLS's MFP. The observed trends in the data are very similar and they only appear to differ in terms of business cycle characteristics, something expected given the capital utilization adjustment in [Fernald \(2012\)](#).

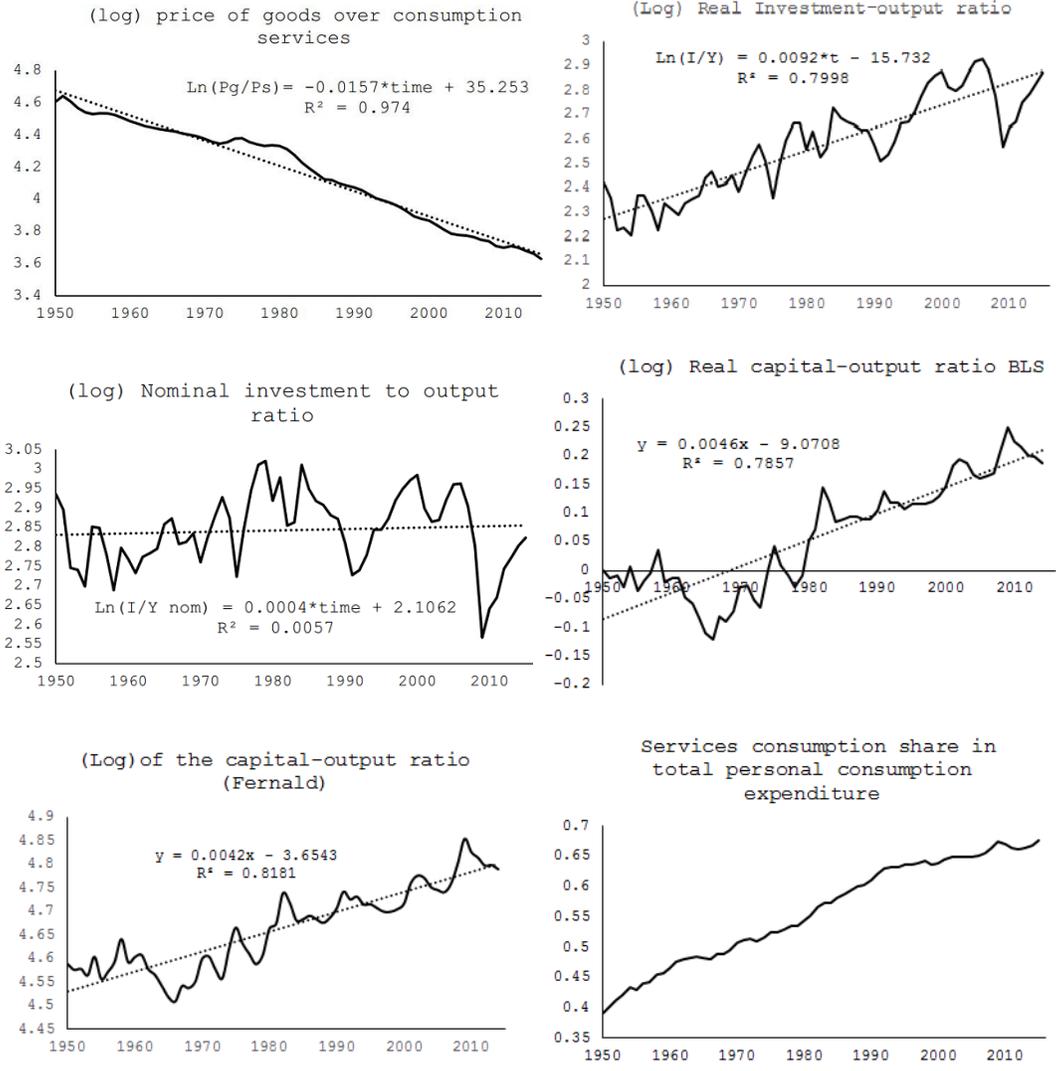


Figure 1: Price of goods relative to services consumption, real investment-output ratio, nominal investment-output ratio, BLS capital-output ratio, Fernald capital-output ratio, and share of services consumption in total consumption expenditure. All variables in logs and with a fitted linear trend except for consumption shares.

This is accompanied by an increase in the share of services in total private consumption expenditure from 40% in 1950 to 68.5% in 2015. This increase appears to be leveling off slightly during the last 15-20 years. The increase in the share of services in consumption and GDP is a well known fact in literature on the process of structural transformation (see [Herrendorf, Rogerson, and Valentinyi \(2014\)](#)). The data in figure 1 suggest that this process has been accompanied by a steady increase in the real measures of the investment to GDP ratio and capital to GDP ratio. The former increases at a rate of 0.92% per year and the latter at a rate of 0.46% per year (0.42% if using the measure by [Fernald \(2012\)](#) adjusted for capacity utilization). In contrast, the nominal investment-output ratio does not display any significant trend.

### 3 Model

This section presents a two-sector model of structural change with balanced growth. The model is a simplified version of [Boppart \(2014\)](#), where we abstract from household heterogeneity and focus on features related to structural transformation between goods producing and services producing sectors.

#### 3.1 Households

Time  $t$  is discrete. There are two types of goods in the economy: two consumption goods (manufacturing and services) and one investment good. The representative household in this economy has preferences given by

$$U = \sum_{t=0}^{\infty} \beta^t V(p_{st}, p_{gt}, E_t), \quad (1)$$

where  $\beta$  is the subjective discount factor,  $V(p_{st}, p_{gt}, E_t)$  is an instantaneous indirect utility function of the household,  $p_{st}$  is the prices of services,  $p_{gt}$  the price of manufacturing, and  $E_t$  is total consumption expenditure. The explicit functional form for  $V$  is

$$V(p_s, p_g, E) = \frac{1}{\epsilon} \left[ \frac{E}{p_s} \right]^\epsilon - \frac{\nu}{\gamma} \left( \frac{p_s}{p_g} \right)^{-\gamma} - \frac{1}{\epsilon} + \frac{\nu}{\gamma}, \quad (2)$$

where  $0 \leq \epsilon \leq \gamma \leq 1$  and  $\nu > 0$ . From now on we consider  $p_g$  as the numeraire of the economy and set it to one. These non-homothetic and non-Gorman type of preferences are the key to obtaining balanced growth in the original model by [Boppart \(2014\)](#).

The household owns the capital stock of the economy and rents it out to firms in the

market. It also inelastically supplies a unit of labor to firms each period in exchange for a wage. The budget constraint is

$$E_t + K_{t+1} = w_t + K_t(1 + r_t - \delta), \quad (3)$$

where  $w_t$  is the wage rate,  $K_t$  is the amount of capital owned by the household,  $r_t$  is the return on capital and  $\delta$  is the depreciation rate. Thus, the problem of the household is to maximize (1) subject to (2) and (3).

The indirect utility function  $V(p_{st}, p_{gt}, E_t)$  encompasses the static problem in which the household decides, given the level of consumption expenditure  $E_t$ , how much to spend in goods and services such that instantaneous utility is maximized and

$$E_t = p_{st}C_{st} + C_{gt},$$

holds, where  $C_{st}$  and  $C_{gt}$  are the optimal consumption levels of services and manufacturing.

### 3.2 Firms and Market Clearing

There are two representative firms in the economy operating in perfect competition. The first firm produces the manufacturing good with technology

$$y_{gt} = k_{gt}^\alpha (n_{gt} A_{gt})^{1-\alpha}, \quad (4)$$

where  $k_{gt}$ ,  $n_{gt}$  and  $A_{gt}^{1-\alpha}$  are capital, labor and total factor productivity (TFP) of the goods producing firm. This output can be used to build the capital stock or as consumption of manufacturing.<sup>9</sup> The second firm produces services with technology

$$y_{st} = k_{st}^\alpha (n_{st} A_{st})^{1-\alpha}, \quad (5)$$

with  $k_{st}$ ,  $n_{st}$  and  $A_{st}^{1-\alpha}$  being capital, labor and TFP of the service producing firm. The output of this firm is used as services consumption.

TFP in the two sectors evolves according to

$$\frac{A_{st+1}}{A_{st}} = 1 + \gamma_s, \quad (6)$$

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<sup>9</sup>In Appendix A we consider the case in which consumption of manufacturing and investment are produced in two different sectors.

$$\frac{A_{gt+1}}{A_{gt}} = 1 + \gamma_g, \quad (7)$$

where  $\gamma_s$  and  $\gamma_g$  are exogenous constant growth rates, and we assume that  $\gamma_s < \gamma_g$ .

In equilibrium all markets clear and the following must hold

$$y_{gt} = C_{gt} + K_{t+1} - (1 - \delta)K_t,$$

$$y_{st} = C_{st},$$

$$k_{gt} + k_{st} = K_t,$$

and

$$n_{gt} + n_{st} = 1.$$

### 3.3 The real interest rate in the model

As proved in Boppart (2014), the model described above displays a balanced growth path in which capital, wages, consumption expenditure and output in terms of the numeraire grow at the same rate. Along this path, the interest rate in investment units is constant and equal to

$$r_t = \frac{1}{\beta} (1 + \gamma_g)^{1-\alpha\epsilon} (1 + \gamma_s)^{(\alpha-1)\epsilon} - 1 + \delta. \quad (8)$$

The concept of balanced growth, however, is tightly linked to the units in which variables are measured. To see this, consider for instance expressing the real interest rate in units of services. From the technologies in the two sectors and the assumption of perfect competition, and assuming  $A_{g1} = A_{s1} = 1$ , the relative price of services to manufacturing is

$$p_{st} = \frac{(1 + \gamma_g)^{(t-1)(1-\alpha)}}{(1 + \gamma_s)^{(t-1)(1-\alpha)}},$$

and the interest rate in services units becomes

$$\frac{r_t}{p_{st}} = \frac{1}{\beta} (1 + \gamma_g)^{1-\alpha\epsilon-(t-1)(1-\alpha)} (1 + \gamma_s)^{(\alpha-1)\epsilon+(t-1)(1-\alpha)} - 1 + \delta, \quad (9)$$

which is a function of time  $t$  as long as  $\gamma_g \neq \gamma_s$ . Thus, the real interest rate in services units is non-constant in this model, and one of the requirements for balanced growth does not hold anymore.

### 3.4 The real interest rate in the data

The question that equation (9) naturally raises is which is the appropriate deflator in multi-sector models when confronting the model with the data. In one-sector models, this issue does not arise as all goods are produced with the same technology and output, investment, and consumption share the same price, commonly assumed to be the numeraire. In multi-sector models instead, the common practice is to express aggregate variables such as total output (GDP) and aggregate consumption in terms of the numeraire of the economy, usually the investment good. However, this is in contrast with standard aggregate measures in national accounts, that are used to contrast the model with the data.

For instance, in the U.S. the NIPA constructs real GDP using a chain-weighted Fisher index of sectoral value added. This is similar to a Divisia index, in which the growth of the various components of GDP is weighted by their shares in nominal GDP. As the shares change over time, the weight of the various components changes. Thus, if GDP is constructed in the model as it is in the data, even if all its individual components (consumption of manufacturing and services and investment in the model in this paper) grow at constant rates over time, structural transformation implies a non-constant growth of GDP over time. Instead, if GDP were measured in units of one of the goods of the economy, no change in the growth rate of GDP would be observed. This point is made in [Moro \(2015\)](#), who shows that structural transformation from manufacturing to services implies a decline in the growth rate of GDP as measured with a Fisher index. Equally, to construct measures of the real return to capital one needs to decide in terms of which units this is expressed. Standard measures of, for instance, real rates of interest, normally deflate nominal rates using GDP or aggregate consumption deflators, implicitly measuring the number of units of GDP or aggregate consumption that can be bought after an investment project.

Thus, to construct the real return on capital in the model, one has to measure what is the return in terms of output (GDP) tomorrow, of a unit of foregone output (GDP) today.<sup>10</sup> This requires to construct GDP from the model's equilibrium path as it is constructed in the data. Hence, to construct the real rate of return that results from the model we take the following steps:

1. We find the solution of the model;
2. We use the solution of the model to construct real GDP through a Fisher index;
3. By using this measure of real GDP and GDP in terms of the numeraire in the model we construct a measure of the GDP deflator divided by the numeraire;

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<sup>10</sup>Or, equivalently, the return in terms of aggregate consumption tomorrow of a unit of foregone aggregate consumption today.

4. We divide the real interest rate in terms of the numeraire in equation (8) by the GDP deflator divided by the numeraire obtained at point 3. This will give a measure of the real interest rate in units of GDP.
5. We repeat steps 2, 3 and 4 by substituting GDP with aggregate consumption to obtain a measure of the real interest rate in consumption units.

## 4 Quantitative analysis

We now calibrate the model to some aggregate targets of the U.S. economy to measure the decline in the real rate of return predicted by the model. We set some parameters to standard values in the literature. Thus we have  $\beta= 0.95$ , consistent with a yearly interest rate of 5%,  $\alpha= 0.34$ , and  $\delta= 0.06$  as in [Caselli and Feyrer \(2007\)](#).

By normalizing TFP levels in the two sectors in the first period to 1, we then need to calibrate three preference parameters  $\epsilon$ ,  $\gamma$  and  $\nu$ , and two growth rates of TFP,  $\gamma_g$  and  $\gamma_s$ . To calibrate these we choose the following targets in the data: 1) the growth rate of GDP per capita over the period considered (1950-2015); 2) the share of services in the initial period (1950); 3) the share of services in the final period (2015); 4) the growth rate of the real investment to output ratio during the period considered; and 5) the average growth in the relative price goods/services. In the model, we assume that the manufacturing sector produces both investment and consumption goods. Thus, to construct our target 5), as explained in section 2, we compute a Fisher index from the price of investment and the price of goods in the data, and take the ratio of this index and the price of services. Table 1 reports all parameter values while table 2 shows the fit of the calibrated model.

Figure 2 reports the simulation of the model and the data. The model does a good job at replicating the long run evolution of GDP and the services share. The evolution of the investment-output ratio is also reproduced fairly well, although this series in the data displays a high volatility. As we don't have a data counterpart for the real return on capital, we normalize this to one in 1950 for ease of computing the total decline over the period. Figure 2 shows that the real return in the model declines by roughly 36% over the period considered (0.6448 in 2015 in the model) in units of GDP and by 0.43% in units of aggregate consumption (0.5720 in 2015). Thus, if a unit of GDP invested in capital in 1950 was providing a gross return of a unit of GDP at  $t+1$ , in 2015 this return is only 0.64. The difference between 1950 and 2015 in terms of units of consumption is even more striking.

In the calibration, we target an average growth of real GDP per capita of 2.12% per year, the average rate measured in the U.S. in the 1950-2015 period. Note however that the

Table 1: Parameter Values

$\beta$	$\alpha$	$\delta$	$\epsilon$	$\gamma$	$\nu$	$A_{g1}$	$A_{s1}$	$\gamma_g$	$\gamma_s$
0.95	0.34	0.06	0.20	0.50	0.63	1	1	2.78%	0.40%

Table 2: Data targets

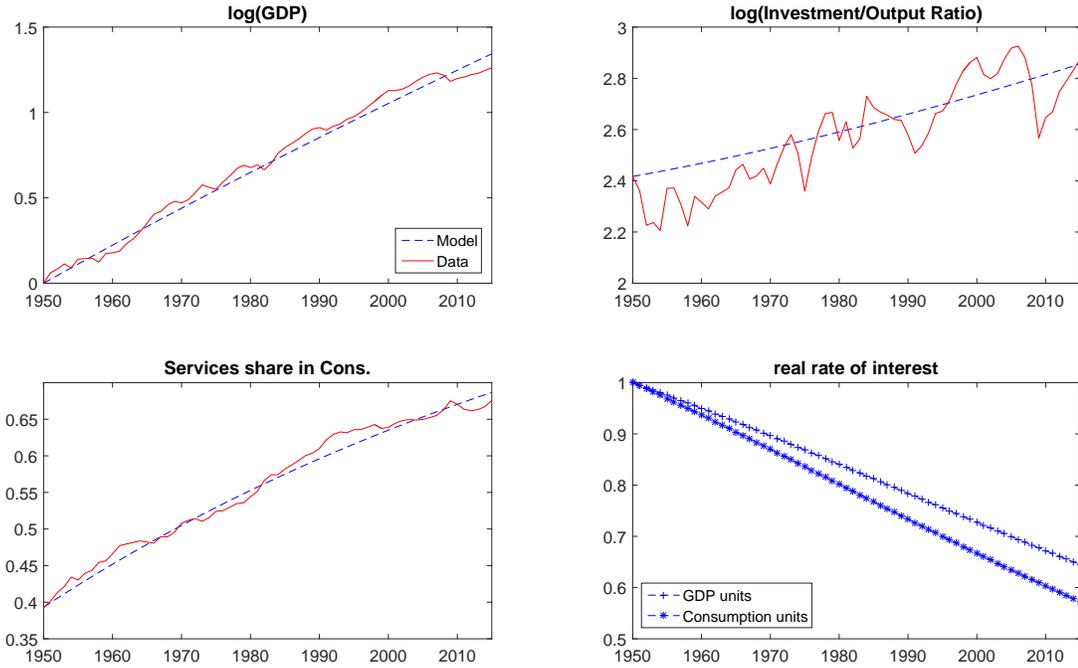
Target	GDPpc Growth	Initial share of services	Final share of services	Real I/Y growth	Growth of $p_g/p_s$
Data	2.12%	0.393	0.685	0.92%	-1.57%
Model	2.09%	0.392	0.687	0.68%	-1.57%

model predicts a declining growth rate of GDP, due to structural transformation between manufacturing and services. The growth rate of GDP in the model goes from 2.29% in the first period to 1.93% in the last period of the simulation. This is a decline of 16% in the rate of growth.

Two points deserve attention here. First, even if GDP appears to grow at a constant rate, the model suggests that the rate of growth declines over time. Given the size of the U.S. business cycle, which displays a standard deviation of GDP growth of 2.3% over the period considered, it would be difficult to detect such trend decline in the data. In fact, figure 3 shows that the trend in our model lies very close to the case in which GDP grows at a constant rate of 2.12% as in the data. However, the standard deviation of the log-deviation of the data from the linear trend is 15% larger than that from the model. Second, the magnitude of the decline in real GDP growth is less than half that in the real return on capital. Thus, decline in the return to capital in terms of GDP is not fully reflected in a decline in the rate of growth of the economy.

The quantitative results are able to explain a trend decline in the rate of return on capital when measured in a way that is consistent with standard data measurements. [Laubach and Williams \(2016\)](#), using state space methods and based on a Wicksellian definition of the “natural” real rate of interest, show that this has declined by around 5 percentage points since 1960. Although their definition and ours differ in terms of the underlying model, if we started from a 5% interest rate, our model could explain a full 2 percentage points of the observed decline in real rates. This fall in the model is also much more pronounced than the fall in GDP growth. In this sense, in the process of structural transformation, the decline in the returns to capital would outweigh the decline in GDP growth. This is important when considering the [Piketty \(2014\)](#) statement that the wealth to output ratio increases due to

Figure 2: Model versus Data

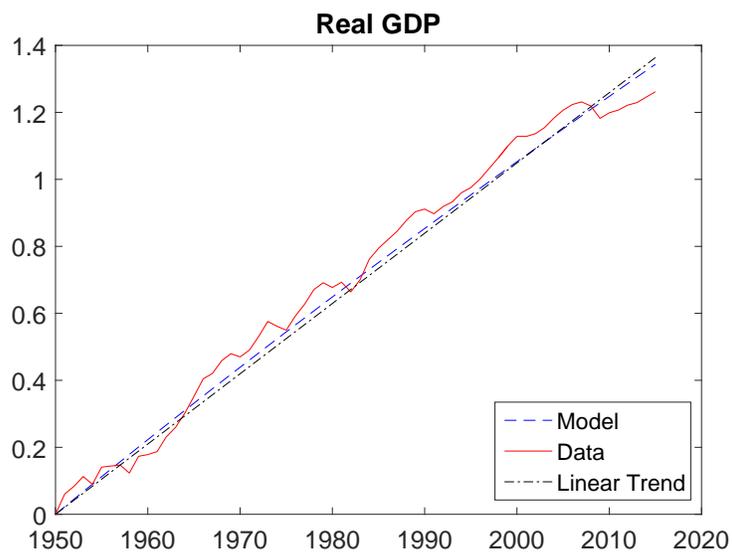


the excess capital returns over GDP growth. In a two-sector growth model, the dynamics of these two variables will be affected by relative price changes.

## 5 Conclusions

The decline in the return on capital is a fundamental problem in economics. In fact, it is probably the most important question facing macroeconomics in advanced economies after the financial crisis started in 2007. In this paper we present a two-sector growth model to show that the rise of services naturally implies a decline in the real return on capital. Our main working assumption is a constant growth rate of TFP in manufacturing and services which generates a declining trend in the relative price of goods over services. This decline, together with the increase in the share the services sector relative to manufacturing, implies that both the growth rate of GDP and the real rate of return decline along the growth path when measured in a way that is consistent with national accounts. In 65 years, while the rate of growth of GDP per capita declines by 16% in the model, the real return on capital declines by 36%. Our results are thus consistent with a declining growth rate of GDP that is not easily detectable in the data, due to the magnitude of business cycle fluctuations.

Figure 3: Model versus Data



Our analysis suggests the possibility that these underlying trends generated by structural change will exert a downward pressure on growth and capital returns in the future. This unpleasant possibility is also very relevant to understand the evolution of living standards in poor as well as in advanced economies. The decline in the real rate of return can also have important consequences for the design of pension plans, the management of public and private debt, and business investment projections.

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# Appendix

## A. The three-sector model

Here we present a version of the model in which the consumption good and the investment good are produced in two different sectors. The main implication is that the price of the two goods is now different. There are now three representative firms in the economy operating in perfect competition. The first firm produces the manufacturing consumption good with technology

$$y_{gt} = k_{gt}^\alpha (n_{gt} A_{gt})^{1-\alpha}, \quad (10)$$

where  $k_{gt}$ ,  $n_{gt}$  and  $A_{gt}^{1-\alpha}$  are capital, labor and total factor productivity (TFP) of the firm. The second firm produces services with technology

$$y_{st} = k_{st}^\alpha (n_{st} A_{st})^{1-\alpha}, \quad (11)$$

with  $k_{st}$ ,  $n_{st}$  and  $A_{st}^{1-\alpha}$  being capital, labor and TFP. The output of this firm is used as services consumption. Finally, the third firm produces the investment good with technology

$$y_{It} = k_{It}^\alpha (n_{It} A_{It})^{1-\alpha}, \quad (12)$$

with  $k_{It}$ ,  $n_{It}$  and  $A_{It}^{1-\alpha}$  being capital, labor and TFP.

TFP in the three sectors evolves according to

$$\frac{A_{st+1}}{A_{st}} = 1 + \gamma_s, \quad (13)$$

$$\frac{A_{gt+1}}{A_{gt}} = 1 + \gamma_g, \quad (14)$$

$$\frac{A_{It+1}}{A_{It}} = 1 + \gamma_I, \quad (15)$$

where  $\gamma_s$ ,  $\gamma_g$  and  $\gamma_I$  are exogenous constant growth rates.

In equilibrium all markets clear and the following must hold

$$y_{gt} = C_{gt},$$

$$y_{st} = C_{st},$$

$$y_{It} = K_{t+1} - (1 - \delta)K_t$$

Table 3: Parameter Values

$\beta$	$\alpha$	$\delta$	$\epsilon$	$\gamma$	$\nu$	$A_{g1}$	$A_{s1}$	$A_{I1}$	$\gamma_g$	$\gamma_s$	$\gamma_I$
0.95	0.34	0.06	0.17	0.50	0.63	1	1	1	3.05%	0.62%	2.60%

Table 4: Data targets

Target	(1)	(2)	(3)	(4)	(5)	(6)
Data	2.12%	0.393	0.685	0.67%	-1.61%	-1.31%
Model	2.13%	0.390	0.685	0.46%	-1.61%	-1.31%

$$k_{gt} + k_{st} + k_{It} = K_t,$$

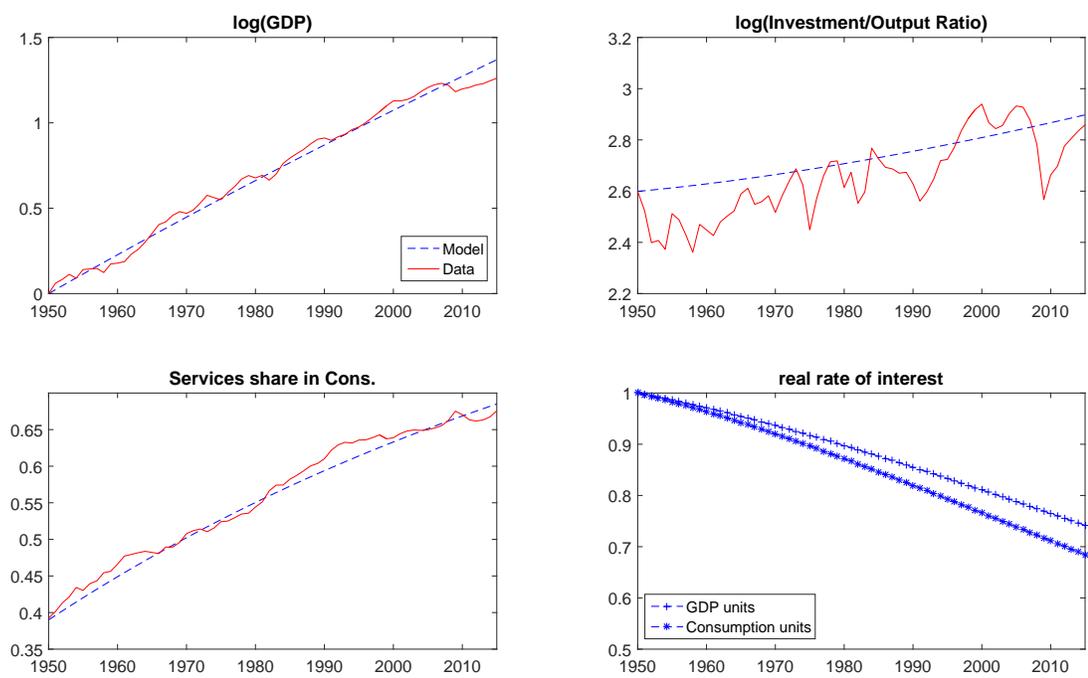
and

$$n_{gt} + n_{st} + n_{It} = 1.$$

By normalizing TFP levels in the three sectors in the first period to 1, we then need to calibrate three preference parameters  $\epsilon$ ,  $\gamma$  and  $\nu$ , and three growth rates of TFP,  $\gamma_s$ ,  $\gamma_g$  and  $\gamma_I$ . Thus we need an additional target with respect to the two-sector model. Also, in the two-sector model, target 5) uses a Fisher index of the price of consumption goods and investment, because we assume that manufacturing goods and investment are produced in the same sector. Instead, here we target 5) the average growth in the relative price goods/services (-1.61%, where now we use the price of consumption goods as the numerator); and 6) the average growth in the relative price investment/services (-1.31%). Table 3 reports all parameter values while table 4 shows the fit of the calibrated model.

In the two-sector model, we deflated the nominal investment-output ratio by the relative price manufacturing/GDP. To compare model and data, the price of manufacturing in that case is a Fisher index of the price of consumption goods and investment in the data. In the data in Figure 4 instead, the nominal investment-output is deflated by the price of investment over the price of GDP. In this case, the real investment-GDP ratio increases by 0.67% per year, compared to the 0.92% figure in section 3. Also in this case the model fits remarkably well GDP growth and the evolution of the share of services, but reproduces a growth of the investment-GDP ratio smaller than in the data (0.46% versus 0.67%). The decline in the real interest rate in this case is 25% in terms of GDP and 32% in terms of aggregate consumption.

Figure 4: Model versus Data: three-sector model



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