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## Syndication and Robust Collusion in Financial Markets

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TEXTO PARA DISCUSSÃO

No. 522

Syndication and Robust Collusion in  
Financial Markets

Vinicius Carrasco  
Gustavo Manso



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# Syndication and Robust Collusion in Financial Markets

Vinicius Carrasco and Gustavo Manso\*

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## Abstract

This paper investigates the extent to which syndication in financial markets is related to collusive behavior. A group of financiers who have private information regarding their capability of monitoring an entrepreneur must decide whether to provide a loan individually in a competitive fashion, or provide it collectively. When deciding whether to provide the loan collectively, the lenders bargain over their participation, on who will be monitoring the lender (the leader), and on pricing. It is shown that if the bargaining stage is robust to timing of communication of their private information (Ex-Post Incentive Compatibility), and if the lenders believe it is better to agree on a collective deal than competing, positive participation in the loan is given to all lenders even when side payments are allowed. Hence, we show that syndication is the optimal response of colluding lenders to the communication costs resulting from the negotiations between them for a given loan. Syndication improves on pricing but introduces a distortion by leaving the most effective monitor with less than full participation in the loan. Necessary conditions for syndication prevailing over competition are provided.

## 1 Introduction

Syndicated loans have become a significant source of financing over the past few years. The numbers speak for themselves: syndicated loans worth over a trillion dollar are signed annually

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these days, and they already represent 51 percent of total corporate financing in the U.S. (The American banker). Some enthusiasts even attribute the shortening and shallowness of recent recessions to both the rapid growth of syndicated financing, as well as the development of a secondary market associated with it.<sup>1</sup> Syndication is also extremely recurrent in the Venture Capital Industry; in fact, a huge fraction of venture capital deals tend to have more than one investor involved with it.

Syndication occurs whenever two or more financial institutions provide jointly (and under common terms) funding to a borrower/entrepreneur. An interesting question to be asked is what are the economic forces driving such kind of financial arrangements. The prevalent answers have had as focus issues related to risk sharing, restrictions on the lending capacities of intermediaries, and technological complementarities. The first and second points involve the assumption that intermediaries are somewhat small to provide a loan. In the first explanation, the size of the loan under consideration, if sufficiently large, may represent a big risk for the institutions under analysis. By pooling resources together, the intermediaries can share the risks associated to a potential default. The second point is self-explanatory: an intermediary may not have enough funds to provide the amount of financing needed by an entrepreneur, or, analogously, the latter may be able to resort on more funds if backed up by more than one intermediary. The third explanation in turn assumes that different intermediaries may have complementary expertise that may be useful towards, say, a better monitoring of how an entrepreneur is using the borrowed resources, or, as it often argued with respect to venture capitalists, the performance of an active managerial role in the enterprises they finance.

One possibility that has been ignored by the literature is the potential advantage that lenders may enjoy in terms of being able to extract more rents from the entrepreneur if they are to provide the loan collectively. If that is the case, however, one could ask whether syndication is the best mechanism through which they can improve their bargaining power *vis a vis* the entrepreneur's. In fact, if monitoring the entrepreneur is of relevance, a joint loan, by diluting the lenders' participation in the returns, would dampen their incentives to monitor. Hence, the pooling of resources to get better deals would come at a cost. Instead of distorting the

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<sup>1</sup> “[W]ithout the type of credit crunch that accompanied earlier business cycles, our recovery has been swifter thanks to loan syndication and trading”, says the Director of Capital Studies of Milken Institute, an Economic Think Tank (see <http://www.milkeninstitute.org/newsroom>).

monitor's incentives by making use of participation in a loan, one could imagine a situation in which, either contractually or in a self-enforced fashion, the lenders agree to not compete for the loan, and split the potential gains of a monopolistic interaction with the entrepreneur through side payments.

The above story ignores that the use of side payments in a collusive scheme may, in addition to the possible need of self-enforceability, be subject to other "transaction costs". In this paper, we show that *communication* costs that are intrinsic to bargaining contexts force lenders to resort (at least partly) to loan participation instead of side payments in an optimally designed collusive scheme which is robust to the way lenders communicate relevant information for the loan. Therefore, syndication is the optimal response of colluding lenders to the communication costs resulting from the negotiations between them.

In our model, after funds are provided, lenders have to exert some (non-contractible) costly effort to monitor the entrepreneur. The lenders are assumed to have private information regarding their costs of monitoring the borrower. There are many ways to motivate this. In the venture capital industry, for instance, lenders play an active managerial role in the projects they finance. Presumably, how much value they can add to a particular project (i.e., their productivity) depends on the expertise they have in different areas. This, in turn, is, to some extent, the venture capitalist's private information as their areas of expertise depend on features such as their staff, and previously financed projects (which may not be fully observable by the other lenders). Additionally, lenders may differ in the amount, quality, and the processing of information about different projects and such differences are likely to induce some privacy of information regarding monitoring capabilities. The same reasoning applies to other types of loans. Banks, for example, differ in their assessment of projects. This, in turn, may be founded on differences regarding some acquired information about specific projects. Banks that are better informed (or equivalently, that process the available information in a better way) should be more productive in monitoring the borrowers.

As a result of the privacy of information regarding their costs of monitoring the borrower, when negotiating a joint offer, the lenders bargain among themselves under information asymmetry. We require this negotiation stage to be robust to the way they communicate such information. More specifically, we impose ex-post incentive compatibility on the mechanism that determines participations, pricing, monitors and side payments in the joint offer. Such im-

position guarantees that the negotiation among lenders will not be affected by specific details of the communication protocol – e.g., whether the private information is announced simultaneously or sequentially. Ex-Post IC seems to be the most appropriate criterion to impose on the mechanism designed by the lenders for a couple of reasons. First, in our model, the communication of costs is intrinsically related to the negotiations of the joint offer. Every lender has an incentive to be the last to speak as the more is known about the other lenders, the better he is in terms of his bargain position in the negotiation. Moreover, it seems unlikely that the lenders could either commit to announcing simultaneously their costs, or rely on a third party to enforce such simultaneity. The latter by the very purpose (collusion) they are designing the mechanism. The criterion we use guarantees that the negotiation/communication stage does not breakdown when all these considerations are taken into account.

The models also explicitly considers the fact that lenders face the option of providing the loan individually rather than as a group. If no agreement on a joint offer is reached, we assume that lenders compete in an open auction. Along the requirement that the joint offer has to be ex-post IC, we also impose ex-post participation constraints on the mechanism. The requirement of Ex-Post Individual Rationality Constraints assures that none of the lenders would have an incentive to cheat the others and make a deal on the side with the entrepreneur after learning their peers' costs. In practice, upon the failure of a joint negotiation and in possession of a handful of information regarding his competitor, a financier can approach the borrower and make an individual offer. The imposition of Ex-Post IR captures this possibility in our model.

Under this framework, we are able to show that lenders must decide between only two Ex-Post Incentive Compatible Mechanisms: competition or syndication. On the one hand, competition induces truthful revelation by making the price at which the winner provides the loan (i) not dependent on his announcement and (ii) so that he makes non-negative profits only in the states in which he is the most efficient monitor. In particular, when the two most efficient lenders have almost the same cost, the winner will have a payoff close to zero, which is the payoff of the lenders who lose under competition.

The Mechanism that generates syndication, on the other hand, induces truthful revelation by granting the same payoff to *all* lenders so that their announcements can only affect the size of the surplus to be (equally) divided: a false announcement by a lender can only reduce total surplus and, as a consequence, his payoff. Such scheme guarantees that *all* lenders' payoffs are

bounded away from zero irrespective of costs realizations. Therefore, whenever the most efficient lender's Participation Constraint is satisfied, syndication prevails over competition.

The forces driving the equality of payoffs whenever non-monitors have positive profits are simple to describe. Since monitoring levels are *substitutes* in the model, at most one lender will monitor the entrepreneur.<sup>2</sup> The only way to guarantee that non-monitor lenders report their costs truthfully is to make their payoff do not dependent on their announcements; otherwise, a non-monitor lender could, given the announcements made in equilibrium by the other lenders, report his costs in a way that the choice of the monitor is not changed and his payoff is increased. Intuitively, as non-monitors do not incur in any monitoring cost, their cost announcements are cheap talk. More interestingly, through the payoff interdependence generated by the monitoring stage (i.e., the monitoring determines the outcome of the project the entrepreneur has access to and, consequently, the payoffs of all lenders), in any mechanism that does grant non-zero payoff for non-monitors, the monitor's payoff can only depend on his costs. Therefore, if the monitor's payoff were to be higher than the non-monitors', for realizations so that the second most efficient lender's cost is close enough to the monitor's, the former would have an incentive to under report its cost so to capture the gains associated with the monitoring task.

Equality of payoffs in a setting in which Incentive Compatibility is required rules out of a fully efficient outcome for the lenders.<sup>3</sup> If efficiency – i.e., the monitor being granted full participation in the loan, which in turn would be priced so to extract all the surplus of the entrepreneur – were to be attained, the only component of the monitor's allocation that would be sensitive to his announcement (and that could be used to guarantee the required equality in the payoffs) would be the side payments. In such case, any two “types” of monitor would want to announce the cost that minimizes the amount of side payments. In other words, the collusive scheme must punish monitors that announce high costs and the punishment can only be effective if either participation or pricing (or both) are distorted. Syndication follows because, by distorting the monitor's participation, the group of lenders minimizes the required punishment.

Using this result as formal theoretical justification, we move on to characterize the optimal

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<sup>2</sup>Substitutability in the monitoring technology is the right assumption to be made if one wants to tell the complementarities from the collusive motives for syndication.

<sup>3</sup>That is, the outcome in which loan is provided fully by the most efficient lender who is also made the full residual claimant of the project.

mechanism without side payments. The lender in charge of monitoring is shown to be the one with the lowest (realized) cost. Moreover, whenever syndication prevails over competition, he is given a higher participation in the deal and a more favorable price than the other syndicate members, as this provides him with stronger monitoring incentives. We also find that syndication is more likely to take place for all possible cost realizations when the project under consideration is good, or when the lenders' are reasonably homogeneous regarding their costs. Last, we show that the joint profits of the lenders are higher when the project is financed through equity than when the project is financed through debt. This suggests that borrowers prefer to issue debt over equity when there is the possibility of collusion.

The paper is organized as follows. The next section describes the set-up of the model and the timing of events. In section 3, we derive the lenders' (endogenous) outside options. These are given by the payoffs resulting from the competition stage which follows a non-agreement for a joint offer. Section 4 describes formally the robustness criterion we impose to the negotiation stage. and derives the result that syndication is a necessary implication of a robust collusive scheme. Section 5 characterizes the optimal mechanism for the case in which the lenders are constrained to not use side payments altogether. It also derives a necessary condition for syndication prevailing over competition for all possible cost realizations and describes a possible way in which the lenders can implement the syndication. In the same section, we compare the lenders profits when the project is financed by equity and debt. Section 6 discusses the related literature. The concluding remarks are drawn in section 7. All proofs are relegated to the Appendix.

## 2 Setup

We consider a setting in which a risk neutral entrepreneur has access to an indivisible project but lacks resources to fund it. Financing can be provided by  $N$  risk neutral lenders, who approach the entrepreneur with proposals of funding. The project requires  $I$  units of the consumption good to be undertaken, and is financed through the issuance of equity.<sup>4</sup> Each of the lenders is fully endowed with resources to provide the loan individually, but they can potentially agree on a joint offer. The entrepreneur solely objective is to obtain the financing under the most

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<sup>4</sup>One can allow financing by debt without significant changes in the results. For the sake of exposition, we focus on equity throughout the text.



favorable financial terms for her (the smallest price).<sup>5</sup>

The project's return,  $y$ , depends stochastically on a non-contractible measure of effort exerted by the lenders,  $z$ . We take for granted that such an effort from the lenders' part is needed and leave unmodelled the specific reasons why this is so. One could think of situations in this lending context that such effort is exerted for monitoring purposes as if, for instance, the entrepreneur is able to divert the loan to other ends. One may also think of  $z$  as a contribution to the productive process embedded in the project. Indeed, it is often argued that venture capitalists, for example, have an active managerial role in the enterprises they finance (e.g., Sorensen (2005), Gorman and Sahlman (1989)). Keeping in mind the possibility of a myriad of interpretations for  $z$ , the general term "monitoring intensity" will be used throughout.

The cost of exerting monitoring activities is assumed to be lender's private information. More specifically, the cost for lender  $i$  of exerting monitoring intensity  $z_i$  is given by  $c_i h(z_i)$ , where  $h(\cdot)$  is common across lenders, while the parameter  $c_i$  is lender  $i$ 's private information. The  $c_i$ 's are distributed in an i.i.d. fashion with atomless density function  $f(c_i)$  over  $[\underline{c}, \bar{c}]$ , where  $0 < \underline{c} < \bar{c}$ .

We assume throughout the paper that the monitoring technology presents an extreme form of substitutability so that, whenever more than one lender is monitoring, the overall monitoring intensity is given by the highest individual amount among those exerting it:  $z(z_1, \dots, z_N) = \max_i \{z_i\}$ . This assumption, while extreme, captures the fact that at least some of the monitoring activities exerted by the financial intermediaries are redundant. Also, that seems to be the right assumption to make on technology if one wants to isolate the collusive motive for syndication from a technological one. Any technology that specifies some complementarity in the monitoring intensities will trivially induce positive participation from more than one lender. Last, it has also a great component of convenience. More general technologies would introduce non-trivial strategic interdependence between the stage in which the lenders communicate their costs and the one in which they monitor. These interactions would complicate tremendously the optimal assignment of participations and securities. Since we are mainly

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<sup>5</sup>For some interpretations of what monitoring is, this could not be a good description of what the entrepreneur is after. As an example, if the entrepreneur can run away with the money, he could possibly prefer a lender demanding a higher price but that would monitor him less. As Proposition 2 shows, under some conditions, the best mechanism for the entrepreneur will be equivalent to the one in which he cares only about pricing irrespective of how monitoring is interpreted.

interested in the bargaining stage, assuming away this interaction isn't problematic.

## 2.1 Timing of the Events and the Contract Space

The timing of events is depicted in figure 1. In the first period, each lender privately learns how costly it is the provision of monitoring,  $c_i$ . In the second period, the  $N$  lenders get together and each announces a cost realization  $\hat{c}_i$ . After all announcements are made, the decision of whether to provide jointly the loan is taken. Such a decision is required to be unanimous.<sup>6</sup> Two possibilities then arise.

First, if an agreement on a joint offer is reached, lenders bargain over the quadruple  $\{\{\chi_i(\hat{c})\}\{\alpha_i(\hat{c})\}, \{d_i(\hat{c})\}, t_i(\hat{c})\}_{i=1}^N$ , where  $\chi_i(\hat{c}) \in \{0, 1\}$  is a monitoring rule that specifies whether lender  $i$  is assigned to perform some monitoring,  $\alpha_i(\hat{c}) \in [0, 1]$  is lender  $i$ 's participations in the loan,  $d_i(\hat{c}) \in [0, 1]$  indexes the ex-post payment  $d_i(\hat{c})y$  the lender will have participation  $\alpha_i(\hat{c})$  of<sup>7</sup>, while  $t_i(\hat{c})$  are the side payments he potentially makes (or receives) to (from) the other lenders. The side payments must be so that

$$\sum_i t_i(\hat{c}) \leq 0 \text{ for all } \hat{c}.$$

This condition just says that, while the lenders cannot resort on outside sources of money, they can, if needed, burn money.

If no agreement is reached, lenders compete among themselves. Competition is modelled as an open auction in which lenders alternate individual offers to the entrepreneur, starting from  $\bar{d} = 1$ . As in a button auction, on his turn to make the offer, lender  $i$  decides on whether to lower his offer by a very small exogenous decrement. The competition stage ends when no lender decides by lowering his offer.

Figure 1 Here

Note that we take the assignment of monitors as a contractible decision. This is in accordance with what happens in practice. In fact, it is often the case that one or more "leaders" are chosen among those performing a joint loan. The leaders are contractually responsible of tasks such as monitoring annual and interim company accounts and project evaluations (McDonald, 1982).

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<sup>6</sup>As it will be shortly seen, this will be without loss of generality.

<sup>7</sup>In case the project is financed by debt, the relevant index would be  $D_i(c) \in [0, \bar{y}]$  and the agent would the have participation  $\alpha_i(c)$  over the ex-post payment given by  $\min\{y, D_i(c)\}$ .

In Period 3, in case some offer of funding is accepted by the entrepreneur, the loan is made in accordance to the previous periods, and the project is undertaken. In Period 4, the monitoring activities are performed. If no agreement on a joint offer was reached, the monitoring is made by the lender winning the competition stage. Otherwise, it is performed by those assigned to monitoring activities in the negotiation stage. The monitoring intensity is a function of the individual monitoring activities performed by the lenders,  $z_i$ . Each  $z(z_1, \dots, z_N) \in [0, 1]$  induces a distribution  $G(.|z)$ , with correspondent density function  $g(.|z)$ , over  $[0, \bar{y}]$ ,  $\bar{y} < \infty$ , for the project's return. Finally, in the last period, the project's  $y$  realize and payments are made in accordance.

We assume that there is no discounting. Also, we adopt the following more stringent assumptions

(A1)  $g(y|z)$  is atomless and continuously differentiable in  $z$ ,

(A2)  $\frac{g_z(y|z)}{g(y|z)}$  is strictly increasing in  $y$ ,

(A3)  $E(y|z)$  is concave in  $z$ ,

(A4)  $h(0) = 0$ ,  $h(1)$  and  $h'(1)$  are finite, large numbers,  $h'(\cdot)$ ,  $h''(\cdot) \geq 0$ , with strict inequality for  $z > 0$ ,

(A5) For all  $c \in [c, \bar{c}]$ , there exists a  $d \in [0, 1]$  so that  $\max_{z \in [0, 1]} \frac{1}{N} [dE(y|z)] - I - ch(z) > 0$ .

(A1) is made mainly for convenience. (A2) implies that higher monitoring induces distributions for the returns that strictly first order stochastically dominates the ones induced by lower monitoring. Assumptions (A3) and (A4) guarantee a unique (and interior) optimal level of monitoring intensity. Uniqueness is important for the derivation of one characterization result (Lemma 1). (A5) implies that the potential team problem generated by a joint offer is not severe enough to preclude the possibility of a joint offer.

### 3 The Competitive Stage

We proceed by solving the model backwards, starting from a subgame in which lenders could not agree on a joint offer, and moved to competition. In the competitive stage, they sequentially – say, starting from lender 1 until lender  $N$  and then, if needed, restarting the process – alternate individual offers to the entrepreneur starting from  $\bar{d} = 1$ . The game ends when neither of the lenders lower their bids.

On his turn to make the offer, lender  $i$  has to decide whether or not to lower the lowest outstanding offer by an arbitrarily small exogenous decrement. In principle, this decision could depend on what he believes the other lenders will do in subsequent turns. However, at each of those turns, the lender has a (weakly) dominant action to take: to lower the lowest outstanding offer whenever the former is not his and positive expected profits can be attained.

More formally, take a subgame in which the announcement profile was  $\widehat{c}$ , there was no agreement on a joint offer, and lender  $i$ , with cost  $c_i$ , is the one performing the loan. If he was granted the right to a security with index  $d$ , at the monitoring stage, he will choose monitoring intensity to solve

$$\max_{z \in [0,1]} d(E(y)|z) - I - c_i h(z).$$

Let  $z(d, c_i)$  be the solution to this problem and define  $d^*(c_i)$  implicitly by<sup>8</sup>

$$d^*(c_i)E(y|z(d^*(c_i), c_i)) - I - c_i h(z(d^*(c_i), c_i)) = 0 \quad (1)$$

Clearly,  $d^*(c)$  is the lowest security index at which a lender with cost  $c$  is willing to perform the loan at the competitive stage. Any security index smaller than  $d^*(c)$  yields negative expected profits to a lender with cost parameter  $c$ . More importantly, it completely pins down the profile of equilibrium strategies of the lenders in such a subgame, and defines their perceived outside option at the stage in which they negotiate a joint offer.

**Proposition 1** *The only (sequential) equilibrium of a subgame induced by an announcement profile  $\widehat{c}$  and no agreement on a joint offer is characterized by:*

(i) *At any of his turns to make the offer, lender  $i$  will lower his offer if, and only if, the outstanding winning offer is larger than  $d^*(c_i)$  and is not his.*

(ii) *On the equilibrium path, the lender with the smallest realized cost will provide the loan and will receive a security indexed by  $d^*(c_{(2)})$  (where  $c_{(2)}$  is the second - bottom to top - order statistic).*

(iii) *The equilibrium payoff of lender  $i$  is given by  $\int \mathbf{1}_{\{c_i \leq c_j, \forall j\}} h(z(d^*(\min_{j \neq i} c_j), \tau)) d\tau$ .*

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<sup>8</sup>Such a  $d^*$  exists, since  $\underline{dy} = 0$  for all  $y$ , (A5) implies that for all  $c$ , there is a  $d$  such that  $\max_{z \in [0,1]} dE(y|z) - I - ch(z) > 0$ , and by Berge's Theorem of the Maximum  $\max_{z \in [0,1]} dE(y|z) - I - ch(z)$  is continuous in  $d$ .

From Proposition 1, it is seen that only the most efficient lender has positive outside option at the stage in which a joint offer is negotiated. Therefore, on the path of the play (i.e., when announcements are truthful), the decision of whether to make a such an offer will be ultimately taken by such lender. This may suggest that, if syndication occurs, he will enjoy substantial rents relative to the other lenders. This conjecture fails to take into account that, at the negotiation stage, the cost parameter is lender's private information. Intuitively, if in a joint offer the benefits assigned to the lender who claims to have the best outside option are very large, all lenders will have incentives to misreport their costs downward. The question then is how much the privacy of cost's information will bound the profits the most efficient lender can collect under a robust negotiation.

## 4 The Architecture of a Robust Syndicate

We model the stage in which the lenders negotiate the possibility of a joint offer as a mechanism to which lenders report costs  $\hat{c} = \{\hat{c}_i\}_{i=1}^N$  and are then assigned, respectively (i) participation in the loan, (ii) individual securities to which the participations apply, (iii) whether they will be perform monitoring activities, and (iv) possibly side payments,  $\{\{\alpha_i(\hat{c})\}, \{d_i(\hat{c})\}, \{\chi_i(\hat{c})\}, t_i(\hat{c})\}_{i=1}^N$ .

A profile of announcements  $\hat{c}$  induce a game of incomplete information among the lenders in the monitoring stage. The lender's expected profits upon participating of a joint offer depends then on the equilibrium outcome of the monitoring stage<sup>9</sup>. Let  $z_{-i}^*$  be the anticipated (equilibrium) vector of monitoring intensities exerted by lenders other than  $i$  when lender  $i$  is assigned to monitor, and  $z_j^*$  be the anticipated equilibrium profile of monitoring intensities when lender  $i$  is not assigned to monitor. Both these objects depend on the announcement of costs made by the lenders. The ex-post – i.e., after costs realize and the side payments are made – payoff of lender  $i$ , with cost  $c_i$ , upon announcing  $\hat{c}_i$  given that the other lenders are (truthfully) announcing  $c_{-i}$ , is given by

$$\begin{aligned} \Pi_i(\hat{c}_i, c_{-i}|c_i) = & \\ & \max_{z_i \in [0,1]} \alpha_i(\hat{c}_i, c_{-i}) E(S(y, d_i(\hat{c}_i, c_{-i})) | z(z_i, z_{-i}^*) - I - c_i h(z)) \text{ if } \chi_i(\hat{c}_i, c_{-i}) = 1 \\ & [\alpha_i(\hat{c}_i, c_{-i}) E(S(y, d_i(\hat{c}_i, c_{-i})) | z(z_j^*) - I), \text{ otherwise} \end{aligned}$$

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<sup>9</sup>There is no *a priori* guarantee that an equilibrium exists for all possible announcements  $\hat{c}$ . However, if one assumes that off-equilibrium beliefs have full support, the payoff structure of the model allow us to evoke Theorem 2 in Athey (2002) to assure that an equilibrium always exists.

It is customary to impose Interim Incentive Compatibility on the mechanism, i.e., that truthtelling is a Bayesian Nash Equilibrium of the game induced by  $\{\{\alpha_i(\hat{c})\}, \{d_i(\hat{c})\}, \chi_i(\hat{c}), t_i(\hat{c})\}_{i=1}^N$  (letting, with slight abuse of notation,  $\Pi_i(c_i, c_{-i}) \equiv \Pi_i(c_i, c_{-i}|c_i)$ ):

$$E_{c_{-i}}[\Pi_i(c_i, c_{-i}) - t_i(c_i, c_{-i})] \geq E_{c_{-i}}[\Pi_i(\hat{c}_i, c_{-i}|c_i) - t_i(\hat{c}_i, c_{-i})], \forall i, c_i, \hat{c}_i$$

We believe, however, that since in our setting the mechanism is being designed by the lenders themselves and not by a third party, it should satisfy a stronger requirement. More specifically, we impose that the mechanism has to satisfy Ex-Post Incentive Compatibility (henceforth, IC ex-post). IC ex-post requires that truthful announcement is a Nash Equilibrium of the game for *all* possible cost realizations<sup>10</sup>:

$$\Pi_i(c_i, c_{-i}) - t_i(c_i, c_{-i}) \geq \Pi_i(\hat{c}_i, c_{-i}|c_i) - t_i(\hat{c}_i, c_{-i}), \forall i, c_i, \hat{c}_i \quad (2)$$

A mechanism that satisfies (2) is "regret free" in the sense that agents, even if allowed to do so, would not want to change their own announcement after observing the (truthful) announcement of the others. This seems to be the right criterion to impose on the mechanism when agents agree on the contract after observing the state of the world, or when communication is not simultaneous and there are advantages of being the last to speak (Miller (2003)). The latter is the case in our model as the communication of costs is intrinsically related to the negotiations of the joint offer. Every lender has an incentive to be the last to speak as the more is known about the other lenders, the better he is in terms of his bargain position in the negotiation. Moreover, it seems unlikely that the lenders could either commit to announcing simultaneously their costs, or rely on a third party to enforce such simultaneity. The latter by the very purpose (collusion) they are designing the mechanism. (2) guarantees that the negotiation/communication stage does not breakdown when all these considerations are taken into account.

Along with the requirement of IC, we will impose ex-post Participation Constraint on the mechanism, so that, using Proposition (1), the following must hold if a joint offer is to be made

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<sup>10</sup>It is easily seen that a mechanism that satisfies this condition will satisfy its Bayesian counterpart for *all* prior beliefs regarding costs. This is an additional source of robustness implied by this criterion (Chung and Ely, 2002)

$$\Pi_i(c_i, c_{-i}) - t_i(c_i, c_{-i}) \geq \int \mathbf{1}_{\{c_i \leq c_j, \forall j\}} h(z(d^*(\min_{j \neq i} c_j), \tau)) d\tau, \text{ for all } i, c \quad (3)$$

## 5 Syndication as an Implication of Collusion

Assumption (A3) implies that, on the path of the play (i.e., when announcements are truthful), at most one lender will be exerting positive amount of monitoring. Thus we can without loss of generality impose the condition  $\sum_{i=1}^N \chi_i(c) = 1$  on the mechanism.<sup>11</sup>

Its is well known from the Incentive Theory literature that the constraints imposed by (2) can be equivalently re-stated in terms of a "first order condition" for an optimal truthful announcement and a monotonicity condition, which guarantees that if a local deviation from truthtelling is not optimal, the same will be true for a global deviation. In our setting an additional condition must be added. The lender whose cost realization hits the exact threshold that determines who monitors must be indifferent between monitoring or not. The formal statement of these conditions are presented next.

**Lemma 2** *A mechanism  $\{\alpha_i(\cdot), d_i(\cdot), \chi_i(\cdot), t_i(\cdot)\}_{i=1}^N$  satisfying  $\sum_{i=1}^N \chi_i(c) = 1$  is IC ex-post if and only if for all  $i$  and  $c = \{c_i\}_i$*

(i) *There are functions  $c_i^*(c_{-i}) \in [\underline{c}, \bar{c}]$  and  $K_i(c_{-i})$  such that  $\chi_i(c) = 0$  and  $\Pi_i(c_i, c_{-i}) = K_i(c_{-i})$ , whenever  $c_i > c_i^*(c_{-i})$*

(ii)  $K_i(c_{-i}) = \max_{z_i \in [0,1]} \alpha_i(c_i^*(c_{-i}), c_{-i}) (d_i(c_i^*(c_{-i}), c_{-i}) E(y|z) - I) - c_i^*(c_{-i}) h(z) - t_i(c_i^*(c_{-i}), c_{-i})$

(iii)  $\Pi_i(c) - t_i(c) = K_i(c_{-i}) + \int_{c_i}^{\bar{c}} \chi_i(\tau, c_{-i}) h(z(\tau, c_{-i}; \tau)) d\tau,$

where  $z(\tau, c_{-i}; \tau) = \arg \max_{z_i \in [0,1]} \alpha_i(\tau, c_{-i}) (E(S(y, d_i(\tau, c_{-i}))|z) - 1) - \tau h(z)$

(iv)  $z(\tau, c_{-i}; c_i)$  is (weakly) decreasing in  $\tau$ .

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<sup>11</sup>Note that restricting monitoring activities to one lender could be optimal for more general technologies. Assume, for instance, that, if one lender is already monitoring, any additional monitors is just slightly productive. By imposing  $\sum_i \chi_i(c) = 1$ , the productive loss associated with not having this additional lenders may be more than off-set by the gains that one has in relaxing the Incentive Constraints associated to their monitoring activities. We thank Ilya Segal for pointing this out.

## 5.1 A Benchmark

We aim to show that syndication may arise solely as an implication of a collusive motive from the lenders' perspective. To make this point the clearest, we first argue that the entrepreneur, if designing a mechanism to screen lenders to finance his project, would not, under some parametric conditions, select a pool of lenders to provide the loan. The argument is straightforward. The entrepreneur would want to maximize the value of the project net of the surplus awarded to the lenders, and subject to their Incentive Compatibility and the Participation Constraints. Using Lemma 1, his Program would read<sup>12</sup>

$$\max_{\{\alpha_i(\cdot)\}, \{d_i(\cdot)\}, \{\chi_i(\cdot)\}, \{K_i(c_{-i})\}_{ii}} E_c \left[ E(y|z(c)) - I - \left[ \sum_i (K_i(c_{-i}) + \chi_i(c)(c_i + \frac{F(c_i)}{f(c_i)})h(z(c))) \right] \right],$$

where  $z(c)$  is as defined in Lemma 1. It is clear that it is always optimal to set  $K_i(c_{-i}) = 0$  for all  $i$ . For any given realization of costs, by taking the derivative of the objective function with respect to the monitoring intensity, letting  $j$  be the lender so that  $\chi_j(c) = 1$ , one has

$$\frac{dE(y|z(c))}{dz} - \left( c_j + \frac{F(c_j)}{f(c_j)} \right) h'(z(c)).$$

By noting that  $\frac{dE(y|z(c))}{dz} = \frac{c_j h'(z(c))}{\alpha_j(c) d_j(c)}$ , the above expression can be re-written as

$$h'(z(c)) \left[ \frac{c_j}{\alpha_j(c) d_j(c)} - c_j - \frac{F(c_j)}{f(c_j)} \right].$$

Assuming that for all  $c$ ,

$$\frac{c}{d^*(c)} - c - \frac{F(c)}{f(c)} \geq 0 \tag{4}$$

one has that for all possible assignments of participation and pricing to the monitor that leaves the non-monitors with zero payoff, the objective function is increasing in  $z$ .<sup>13</sup> Moreover, in order to maximize monitoring intensity, it can be shown that full participation in the loan, and equity participation  $d^*(c_{(2)})$  must be given to the lender: in other words, competition among the lenders is what is best for the entrepreneur. We then have

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<sup>12</sup>Note, in addition, that the borrower could only make matters better for him if he was to forbid the lenders to make side payments among themselves. The argument uses this fact.

<sup>13</sup>This condition is satisfied if competition is fierce, so that  $d^*(c)$  is low, and/or the informational rents to be given to lenders for truthful revelation, captured by the term  $\frac{F(c)}{f(c)}$ , are not too high.



**Proposition 3** *Under (4), an entrepreneur would never want to have a pool of lenders providing the loan. More specifically, competition among the lenders yields his most preferred outcome.*

## 5.2 The Lenders' Optimal Scheme

The previous section showed that, under (4), if the entrepreneur could design his most desirable mechanism to screen the lenders and collect funds for his project, he would not want to induce syndication of the loan. This section, in turn, analyzes the lenders' optimal scheme.

By integrating (ii) from Lemma 1 by parts, it follows that, if the lenders want to maximize their ex-ante, i.e. before costs realize joint profits the optimal mechanism chooses  $\{\alpha_i(c), \{d_i(c)\}, K_i(c_{-i}), c_i^*(c_{-i}), t_{-i}(c)\}_{i=1}^N$  to maximize

$$\sum_{i=1}^N (E_{c_{-i}}(K_i(c_{-i})) + E_c[\mathbf{1}_{\{c_i \leq c_i^*(c_{-i})\}} h(z(c; c_i) \frac{F(c_i)}{F(c_i)})]) \quad (5)$$

subject to (ii), (iii), (iv),  $\sum_i t_i(c) \leq 0$ , and  $\sum_{i \in \tilde{N}} \chi_i(c) = 1$ .

It is worth noting that due to the ex-ante symmetry of the lenders, in search for the optimal collusive scheme, we can focus on symmetric mechanisms. More importantly in characterizing the scheme, an optimal mechanism never relies on unbalanced mechanisms. The reason is quite simple. Despite the fact that there are  $N$  lenders, at most one lender will have an incentive problem. In fact, among all incentive constraints, either one of two will bind: either the assigned monitor will have an incentive to report a lower cost to save some monitoring costs, or the second most efficient lender may want to report a cost that makes him the assigned monitor and reap some benefits. The only role that money burning could possibly play the role would be to guarantee that those two constraints are satisfied. Intuitively, however, since  $N - 1$  lenders have no incentive problems any money burned can be passed to them without affecting incentives and improving the overall payoff of the coalition.

**Proposition 4** *An optimal collusive scheme must be budget balanced; that is*

$$\sum_i t_i(c) = 0 \text{ for all } c.$$

The set of mechanisms satisfying budget balance, and all constraints implied by Incentive Compatibility is not empty. In fact, there exists a mechanism that fully replicates the outside

option of the lenders. This can be seen by setting  $c_i^*(c_{-i}) = \min_{j \neq i} \{c_j\}$ ,  $\chi_i(c) = \alpha_i(c) = 1$  if  $c_i \leq c_i^*(c_{-i})$ ,  $t_i(c) = 0$  for all  $i$  and  $c$ , and  $d_i(c) = d_j(c) = d^*(c_{(2)})$ , which obviously imply  $K_i(c_{-i}) = 0$ . In words, as in the competitive bidding process, each lender has, at each stage, a (weakly) dominant strategy, competitive bidding is trivially Ex-Post IC. Moreover, the participation constraints are also trivially satisfied. Therefore, competition is one of the (Ex-Post IC and IR) mechanisms available to the lenders.

A particular feature of the competitive mechanism is that it grants zero profits for non-monitors. There may be other mechanisms, differing from the latter one by the assignment of positive participation to all lenders, with this feature. In all such mechanisms, the joint profits of the lenders depend solely on how much the assignment of participations and securities can induce of monitoring. Therefore, the potential benefit of such mechanisms would come through the possibility of assigning to the monitor a security  $d > d^*(c_{(2)})$  so to increase the incentives to monitor. On the other hand, the obvious cost comes from assigning a smaller participation to the monitor (with a corresponding countervailing effect on his monitoring). The question then is how responsive is the leader's monitoring intensity to a (incentive compatible) substitution of participation by a higher security that keeps non-monitors with zero profits. The answer turns out to be clear-cut: the substitution of participation for "price" always reduces the monitor's effort.

**Proposition 5** *The best (i.e., ex-ante optimal for the lenders) mechanism among those in which non-monitors have zero profits is the one that replicates the lenders' outside options.*

Proposition 3 shows that lenders cannot do better than the competitive outcome if non-monitors are left with their outside option payoffs. Therefore, if it is to be the case that a robust collusive scheme improves the lenders' prospects when compared to pure competition, one needs to focus on mechanisms in which all lenders have positive payoffs. The question then is how restrictive this is. It turns out that the assignment of positive profits to all lenders, along with the consequent interdependence of values implied by a joint offer, restricts enormously the way payoffs have to be assigned. More specifically,

**Proposition 6** *Any mechanism for which non-monitors have positive profits must guarantee that all lenders – monitor and non-monitors – have the same expected profits.*

The above is key for the syndication result to come so a few words on the reason why expected payoffs have to be the same when all lenders have positive payoffs are called for. Lemma 1 states that a non-monitor payoff cannot depend on his own cost. Moreover, it must also be the case that the monitor's payoff cannot depend on the non-monitor's announcements. Otherwise, either the payments the monitor performs to non-monitor's or the participation and pricing he gets from the loan would depend on the non-monitors' announcement. In either case – the latter through the fact that participation and pricing affect monitoring and, therefore, the payoff of the other lenders –, the non-monitors' payoff would depend on their announcement which would violate incentive compatibility. Given that, if payoffs were different, say, if the monitor were to have a higher payoff, for cost realizations of the second most efficient lender close enough to the monitor, he would have an incentive to understate his costs. The only possible way to avoid such misrepresentation is if all payoffs are the same.

Using Proposition 3, one can argue that, even if side payments are available to sustain collusion, some inefficiency (from the lender's perspective) will necessarily arise in a robust collusive scheme. To see that, assume that full efficiency could be attained. One would then have the most efficient lender, say lender  $j$ , providing fully the loan,  $\alpha_j = 1$ , and being assigned as the sole claimant of the returns,  $d_j = 1$ . The total size of the surplus generated would then be given by

$$E(y|z_j^*(c_j)) - I - c_j h(z_j^*(c_j)).$$

By Proposition 4, it must be the case that all lenders' payoffs are equal to  $\frac{1}{N}[E(y|z_j^*(c_j)) - I - c_j h(z_j^*(c_j))]$ , where Therefore, lender  $j$ 's side payment must equal to  $(N - 1)$  times this expression. Such payment, however, depends only on his announcement – while his participation in the loan and the assignment of the returns do not –, so the lender will clearly over report his costs rendering the scheme incompatible. One then concludes the some sort of inefficiencies must be present. They could come either through the assignment of a less favorable security  $d_j < 1$  to the leader, or through a smaller participation in the loan  $\alpha_j < 1$ . The inefficiencies are minimized by distorting the monitor's participation so we have

**Proposition 7** *If the optimal mechanism is not the one resulting in the competitive outcome, all lenders are assigned positive participation in the loan.*

It is interesting to note that syndication follows from the fact that an Incentive Compatible

scheme must somehow “punish” the monitor if he announces a high cost and that side payments alone cannot implement such punishments. In other words, the scheme must prevent that a “low cost” monitor imitates a high cost monitor and the side payments cannot accomplish such task.

A successful collusive scheme that satisfies Ex-Post Incentive Compatibility necessarily induces syndication. The exact characterization of the optimal mechanism is rather complicated so we will from now on restrict attention to the case in which side payments are not used altogether. Proposition 5 provides a (partial) formal theoretical justification for this: the use of side payments does not fully substitute the need of providing participation in the loan for all potential lenders. On top of that, this also seems to be a reasonable assumption for two related reasons. First, antitrust law in the U.S. forbids such kind of arrangement (2 Clayton Act, 15 U.S.C.<sup>14</sup>) implying that court enforced payments should not be feasible. Second, even if, say, repetition could make such payments self enforced as in the Relational Contract literature (e.g., Levin (2003) and Rayo (2005)), lenders may *decide* not to use them in designing their optimal collusive scheme as its use would magnify the chances of them being caught by the antitrust authority (Athey and Bagwell, (2001)).

## 6 Optimal Mechanism Without Side Payments

The objective of this section is twofold. First, ignoring the monitor’s outside option, we characterize the optimal mechanism among those in which the lenders are constrained to not use side payments. Second, by explicitly taking into account the participation constraint for the most efficient lenders, we provide necessary conditions for syndication – rather than competition – taking place.

Toward the first objective, one must first realize that the result in Proposition 3 goes through if the lenders are constrained to not make side payments: it must still be the case that an optimal mechanism without side payments (and that differs from the competitive outcome) leaves all lenders with the same payoff. It remains to identify the lender’s participation in the loan,  $\alpha_i$ ’s, and the indexes  $d_i$ ’s that specify the ex-post payments the lenders will have participation  $\alpha_i$  of. One thing to notice is that, given the equality of payoffs required in a IC-ex-post mechanism, if

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<sup>14</sup>See the U.S. Department of Justice Antitrust Manual at <http://www.usdoj.gov/atr/foia/divisionmanual/ch2.htm>

a mechanism is such that the monitor is assigned both participation in the loan and an index that do not depend on his announcement, he has no incentives to misreport his costs: a false announcement may induce the assignment of a less efficient lender to the monitoring activities which, in turn, would reduce his payoff. Under such a mechanism, the non-monitors would have to be assigned indexes that guarantee that their payoffs are equal to the monitor's. In the same fashion as in the proof of Proposition 4, one can see that it is never optimal to distort the monitor's index so that he is assigned  $d = 1$ . In such case, incentive compatibility calls for his participation in the loan not to depend on his cost announcement. The best mechanism then assigns the highest possible participation that is compatible with there being securities that guarantee that the equality of payoffs is attainable

**Proposition 8** *Letting  $i^*$  be the most efficient lender, the best mechanism when side payments are not available takes the following form:*

$$\begin{aligned}
(a) & \chi_{i^*}(c) = 1, \alpha_{i^*}(c) = \bar{\alpha}, d_{i^*}(c) = 1 \\
(b) & \alpha_j(c) = \frac{1-\bar{\alpha}}{N-1}, d_j(c) \equiv \tilde{d}(c_{i^*}, \bar{\alpha}), \text{ for } j \neq i^* \\
& \text{where } \bar{\alpha} = \max_{c_{i^*}} \bar{A} = \bigcap_{c_{i^*}} \{\alpha' : \exists \tilde{d}(c_{i^*}, \alpha') \in [0, 1] \text{ with } (\alpha' E(y|z(\alpha', \bar{d}, c_{i^*})) - I) - c_{i^*} h(z(\alpha', \bar{d}, c_{i^*})) = \\
& \frac{1-\alpha'}{N-1} (\tilde{d}(c_{i^*}, \alpha') E(y|z(\alpha', \bar{d}, c_{i^*})) - I).
\end{aligned}$$

With such characterization in hands, a natural question to be made – and that leads to the second goal of this section – is whether an offer satisfying the requirement of Proposition 3 exists. Assumption (A6) guarantees that  $\bar{A}$  is not empty ( $\frac{1}{N}$  is in  $\bar{A}$ ) so that syndication is always feasible. Also one may ask whether for a given profile  $c$  a joint offer satisfy the monitor's I.R., a constraint that has been ignored up to now. Such an offer will be made if (and only if) the answer to this question is positive. On one hand, syndication brings gains in terms of pricing, but requires the monitor to relinquish a big deal of participation. On the other hand, the monitor's outside option looks better the more efficient he is compared to the other lenders and the larger the number of potential lenders. If the competition is very fierce, the number of lenders is small and the project prospects are very good, the I.R. most likely will slack. However, there are situations in which the outside option looks very tempting to him. The next result provides a necessary condition for syndication occurring irrespective of cost realizations in case the costs of monitoring are potentially small. Let  $D^*(c)$  be implicitly defined by

$$E(\min\{y, D^*(c)\}|z(c, D^*(c))) - I - ch(z(c, D^*(c))) - 0$$

In words,  $D^*(c)$  is the equivalent of  $d^*(c)$  for the case in which the project is financed by debt. We have

**Proposition 9** *Assume there exists  $z < 1$  so that  $\frac{1}{N}(E(y|z) - I) \leq E(\text{Min}\{y, D^*(\bar{c})\}|z)$ . Then, if  $\underline{c}$  is positive but close to zero, syndication occurs for all cost realizations only if  $\frac{1}{N}(\bar{y} - I) \geq D^*(\bar{c}) - I$ .*

By assuming such a  $z$  exists, the above result provides some hints on under which circumstances a purely collusion driven syndicate cannot prevail. The larger the number of lenders, the less promising the project – measured by a small  $\bar{y} - I$  –, the more efficient the most efficient lender is and the less efficient the second better lender, the more likely is the prevalence of competition. The latter suggests that a syndicate outcome driven by collusion is more likely to occur among equals and if the costs of monitoring are significant. Heterogeneity, measured by  $\bar{c} - \underline{c}$ , plays against collusion in our setting.

The above result makes use of the fact that we have assumed that the formation of a syndicate should be consensual among all lenders. One could also think of a situation in which, for example, upon a lender or more refusing to participate in the syndicate and deciding to bid individually for the loan, the remaining lenders could still possibly agree on a joint offer. In such case, the first thing to notice is that at most one lender would want to move to competition. This is so because at most one of such deviating lenders can have positive payoffs if they do not join the syndicate. Clearly, the lender who is most eager to move to competition is the one whose realized cost is the smallest. Therefore, the relevant Participation Constraint for the case in which a subset of the lenders can form a pool while the remaining bid for the loan individually is still the most efficient lender's. Noting, moreover, that, in competing with the most efficient lender, the best the syndicate formed by the remaining lenders can do is to have the second most efficient lender bidding alone for the loan, the relevant Participation Constraint is exactly the same as for the case in which the lenders must consensually agree on a joint offer.

### 6.0.1 Syndication in a Numerical Example

As an example of a case in which a joint offer is always made, we consider the following. The project needs  $\frac{1}{8}$  units of the consumption good to be undertaken and can yield 1 with probability  $z$ , and 0 with complementary probability. There are two lenders with costs uniformly distributed

over  $[\frac{1}{2}, 1]$ . Their costs of exerting effort level  $z$  are given by  $c_i \frac{z^2}{2}$ ,  $i = 1, 2$ .

Simple calculations show that  $d^*(c_i) = \sqrt{\frac{c_i}{4}} \leq \frac{1}{2}$  for all  $c_i$ . Therefore, the most equity participation a lender can have under competition is  $\frac{1}{2}$ . We claim that for such an example a joint offer will always be optimal. A mechanism that dominates competition is, for instance, one in which both lenders contribute with half of the resources needed to undertake the project, the most efficient lender gets an equity participation of  $\frac{1}{2}$  and the least efficient one is granted equity participation of  $\frac{1}{2}(\frac{1}{4c_{i^*}})$ , where  $c_{i^*}$  is the cost parameter of the most efficient lender. The entrepreneur keeps the remaining participation (which is always positive). This assignment of equity participations guarantee that both lenders have the same expected profits as required by Proposition 3.

### 6.0.2 Implementing The Optimal Mechanism

Whenever the optimal mechanism prescribes a joint offer, monitor and non-monitors will be assigned participations over securities with differing indexes. This kind of arrangement is not common in practice. However, the scheme in Proposition can be implemented as follows for the cases in which the financing is made through either equity or debt.

In case of equity financing, the most efficient lender,  $i^*$ , provides  $\bar{\alpha}I$  to the entrepreneur and is granted equity participation of  $\bar{\alpha}$ . Each of the other lenders participate with  $(\frac{1-\bar{\alpha}}{N-1})I$  of the loan and are granted equity participation of  $(\frac{1-\bar{\alpha}}{N-1})\tilde{d}(c_{i^*})$ .

In case the project is financed through debt, each of non-monitors would have a participation  $(\frac{1-\bar{\alpha}}{N-1})$  over a debt contract indexed by  $\tilde{D}(c_{i^*})$ . Lender  $i^*$  would have participation  $\bar{\alpha}$  over the same debt contract and, additionally, would receive a fee of  $\bar{\alpha}(y - \tilde{D}(c_{i^*}))$  from the entrepreneur whenever returns are in excess of  $\tilde{D}(c_{i^*})$ . In practice, it is often the case that syndication managers (monitors in our setting) receive fees from borrowers for services related to the loan. Indeed, some of the anecdotal evidence of collusion in private loan markets comes from statements about high fees charged by managers in syndicated deals. In accordance with such evidence, the implementation of the optimal mechanism indicates that the collusive gains derived by the monitor may come from high fees, not from pricing itself.

## 6.1 Debt vs. Equity: How Should the Project be Financed?

The analysis so far has assumed that the project was financed by equity. As claimed throughout, one could re-write all the results in terms of debt financing. Up to now, the security financing the project was held fixed in the paper. We could move one step back and ask what type of security would be chosen if a syndication was to occur. The answer will of course depend on the lender's relative bargain power *vis a vis* the entrepreneur's. We could think of a case in which the lenders have all bargain power as one in which, in addition to agreeing on participations, prices and the monitor assignment, they can demand the type of security they want to hold. In a situation in which the entrepreneur, for this matter at least, has relative more bargain power, the type of security will be chosen before the offers are made by the lenders. The following ranks the joint profits of lenders under syndication when the securities can either be debt or equity.

**Proposition 10** *Under syndication, the joint profits of the lenders are strictly higher if the project is to be financed through equity.*

Proposition 5 helps to answer the security design question we just raised. It suggests that lenders have strict preferences for equity over debt. On her turn, an entrepreneur solely concerned with the terms upon which she can finance her project should issue debt. This provides an additional rationale for debt financing. In a seminal paper, Myers and Majluf (1984) establish the superiority of debt over equity in a setting in which a manager cares about incumbent shareholders and has private information about the prospects of the firm. Townsend (1979) and Gale and Helwig (1984) show that debt is the optimal way to finance projects if verification of states is costly. Aghion and Bolton (1992) and others have pointed out the optimality of debt as a mechanism to allocate contingent control in a world of incomplete contracts. In the setting of our paper, debt would be chosen by an entrepreneur concerned with the terms of the financing as it minimizes the joint profits of lenders colluding through a joint offer.

## 7 Related Literature

Our paper relates to two distinct literatures. The first one stresses the reasons why financing may be provided by groups of investors. The second one characterizes optimal collusive schemes



as the solution of a Mechanism Design problem. Starting with the first strain, up to our knowledge, there are no papers relating syndication to collusion. Brander, Amit and Antweiler (2002) and Pichler and Wilhelm (2001), for instance, focus on the technological complementarity function of the syndicate in a financial context. The first paper, in a model a la Sah and Stiglitz (1985), focus on the advantage of a second opinion in the decision of which project to finance. In particular, as opposed to what is found in this paper, they find that projects of average quality are more prone to be syndicated as a second opinion is not so valuable if the signal received by the first lender is either too bad (and the project is not financed at all) or too good (so that the project is totally financed by the lender who receives the signal first). The second paper focus on the implications for the form of a syndicate (e.g., which lender will be chosen to be a leader) of the need of more than one lender to be a monitor. Wilson (1969) develops a general Theory of Syndicates by putting emphasis in its risk-sharing function.

Regarding the second strain of the literature, McAfee and McMillan (1992) model collusion in an auction setting as a mechanism. Athey, Bagwell and Sanchirico (2003) consider an infinitely repeated Bertrand Game in which producers costs are private information and analyse this game in its mechanism form. Both papers find that positive participation to all players is optimal under a specific condition (log-concavity) on the type's distribution. Compared to the settings to which these papers apply, collusion in financial markets tend to be facilitated by the fact that it is easy to write a court enforced contract defining the participations each lender will have in a deal. Such contractibilities are not present in those settings, so issues of self enforceability – which are not discussed here by obvious reasons – are of concern for them<sup>15</sup>. Additionally, our model differs from theirs in two main respects. First, in addition to a communication stage, we have a stage in which players have to exert some effort. This, on top of playing against giving positive participation to all lenders, generates some value interdependence in the lenders' payoff whenever the mechanism assigns positive payoff to all lenders. Secondly, we impose Incentive Compatibility Ex-Post on the collusive mechanism. In our setting, positive participation is – through the implied communication costs that such requirement impose on the mechanism – a consequence of the latter and not of any feature of the type distribution.

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<sup>15</sup>Such issues are explicitly dealt with in Athey et al (2003), but are not considered in McAfee and McMillan (1992).

## 8 Conclusion

This paper provided conditions under which the syndication of a loan is a necessary implication of collusion among lenders. More specifically, we showed that if lenders negotiate under some information asymmetry regarding their capability of monitoring the entrepreneur and if the way they communicate their private information is robust to the communication protocol – e.g., whether the private information is announced simultaneously or sequentially – positive participation in the loan must be granted to all lenders. This was shown to hold true even if side payments are allowed and if one “controls” for other forces that could also lead to syndication such as limited lending capacity, risk sharing, and complementarities.

Using our main result as a formal justification, and the fact that the antitrust law in the U.S. would forbid the use of side payments among the lenders as practical one, we also characterized the optimal mechanism without side payments. It was shown that the lender in charge of monitoring is be the one with the lowest (realized) cost of monitoring. Additionally, whenever syndication prevails over competition, he is given a higher participation in the deal than the other syndicate members, as this provides him with stronger monitoring incentives. We also found that syndication is more likely to take place when the project under consideration is good, or when the lenders’ are reasonably homogeneous regarding their costs of monitoring. Last, we showed that the joint profits of the lenders are higher when the project is financed through equity than when the project is financed through debt. This could suggest that borrowers prefer to issue debt over equity when there is the possibility of collusion among lenders.

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## 9 Appendix: Proofs

**Proof of Proposition 1** The first part holds because the competitive stage is an Independent Private Values English Procurement Auction for which lender  $i$  has valuation  $d^*(c_i)$ . For the second part, it suffices to show that  $d^*(\cdot)$  is strictly increasing. Suppose, toward a contradiction, there were  $c' > c''$  with  $d^*(c') \leq d^*(c'')$ . We would have

$$0 = d^*(c'')E(y, |z(d^*(c''), c'') - I - c''h(z(d^*(c''), c'')) \geq$$

$$d^*(c'')E(y|z(d^*(c'), c') - I - c''h(z(d^*(c'), c')) >$$

$$d^*(c'')E(y|z(d^*(c'), c') - I - c'h(z(d^*(c'), c')) \geq$$

$$d^*(c')E(y|z(d^*(c'), c')) - I - c'h(z(d^*(c'), c')) = 0$$

which cannot hold (the first equality follows from the definition of  $d^*(\cdot)$ , the first inequality from revealed preferences, the second from  $c' > c''$  and the third inequality from  $d^*(c') \leq d^*(c'')$ . The last equality follows again from the definition of  $d^*(\cdot)$ ) The third part follows from lemma 1 by noting that the competition stage has a direct revelation counterpart with (letting  $i^* = \arg \min_j \{c_j\}$ ):

$$\chi_{i^*}(c) = 1 = \alpha_{i^*}(c), d_{i^*}(c) = d_j(c) = d^*(\min_{j \neq i^*}(c_j)) \text{ and } K_i(c_{-i}) = 0. \blacksquare$$

**Proof of Lemma 1 Sufficiency:** Take a lender with cost  $c_i \leq c_i^*(c_{-i})$ . If he was to announce  $\hat{c}_i > c_i^*(c_{-i})$ , his payoff would be  $K_i(c_{-i})$ , which is smaller than  $\Pi_i(c_i, c_{-i}) - t_i(c_i, c_{-i})$ . Using (ii) and (iii), we have, for  $\hat{c}_i < c_i$ ,

$$\begin{aligned} \Pi_i(\hat{c}_i, c_{-i}) - t_i(\hat{c}_i, c_{-i}) - [\Pi_i(c_i, c_{-i}) - t_i(c_i, c_{-i})] &= \int_{\hat{c}_i}^{c_i} h(z(\tau, c_{-i}; \tau)) d\tau \leq \int_{\hat{c}_i}^{c_i} h(z(\hat{c}_i, c_{-i}; \tau)) d\tau = \\ &= \int_{\hat{c}_i}^{c_i} - \left( \frac{d \max_{z_i \in [0,1]} \alpha_i(\hat{c}_i, c_{-i})(d_i(\hat{c}_i, c_{-i})E(y|z) - I) - \tau h(z) - t_i(\hat{c}_i, c_{-i})}{d\tau} \right) d\tau \\ &= \Pi_i(\hat{c}_i, c_{-i}) - t_i(\hat{c}_i, c_{-i}) - [\Pi_i(\hat{c}_i, c_{-i}|c_i) - t_i(\hat{c}_i, c_{-i})], \end{aligned}$$

so that  $\Pi_i(c_i, c_{-i}) - t_i(c_i, c_{-i}) \geq \Pi_i(\hat{c}_i, c_{-i}|c_i) - t_i(\hat{c}_i, c_{-i})$  for all  $\hat{c}_i < c_i$  (the second equality in the chain follows from Corollary 4 in Milgrom and Segal (2002), as we have  $z \in [0, 1]$ ,  $\alpha_i(\hat{c}_i, c_{-i})(d_i(\hat{c}_i, c_{-i})E(y|z) - I) - \tau h(z)$  continuous in  $z$ , and with a continuous partial derivative with respect to  $\tau$ . The required uniqueness of the optimizer is implied by (A4) and (A5)). An analogous argument shows that  $\Pi_i(c_i, c_{-i}) - t_i(c_i, c_{-i}) \geq \Pi_i(\hat{c}_i, c_{-i}|c_i) - t_i(\hat{c}_i, c_{-i})$  for all  $\hat{c}_i \in (c_i, c_i^*(c_{-i})]$ . (i) implies that a lender with cost  $c_i > c_i^*(c_{-i})$  is indifferent between announcing his true cost or any other  $\hat{c}_i > c_i^*(c_{-i})$ . Moreover, condition (ii) and the monotonicity of  $\max_{z_i \in [0,1]} \alpha_i(\hat{c}_i, c_{-i})(d_i(\hat{c}_i, c_{-i})E(y|z) - I) - \tau h(z)$  in  $\tau$  assures that a lender who is not assigned to monitor cannot benefit from pretending being a monitor.

**Necessity:**  $\Pi_i(c_i, c_{-i}) - t_i(c_i, c_{-i}) = \max_{\hat{c}_i} \Pi_i(\hat{c}_i, c_{-i}|c_i) - t_i(\hat{c}_i, c_{-i})$ , assumptions (A4) and (A5) guarantee that  $\Pi_i(\hat{c}_i, c_{-i}|c_i)$  is everywhere differentiable with respect to  $c_i$ . The derivative being 0 if  $\chi_i(\hat{c}_i, c_{-i}) = 0$ , or  $-h(z(\hat{c}_i, c_i))$  otherwise (the latter by Corollary 4 in Milgrom and Segal (2002)). This implies that the partial derivative's modulus is bounded by  $h(1)$ . By Theorem 2 in Milgrom and Segal (2002),  $\Pi_i(c_i, c_{-i}) - t_i(c_i, c_{-i})$  is absolutely continuous and, therefore, can be written as

$$\Pi_i(c_i, c_{-i}) - t_i(c_i, c_{-i}) = \Pi_i(\bar{c}, c_{-i}) - t_i(\bar{c}, c_{-i}) + \int_{c_i}^{\bar{c}} \chi_i(\tau, c_{-i}) h(z(\tau, c_{-i}; \tau)) d\tau.$$

A second order necessary condition for truthtelling is that, whenever it exists,  $-\frac{d^2[\Pi_i(\hat{c}_i, c_{-i}|c_i) - t_i(\hat{c}_i, c_{-i}) - t_i(\hat{c}_i, c_{-i})]}{d\hat{c}_i^2}$   $\frac{d^2[\Pi_i(\hat{c}_i, c_{-i}|c_i) - t_i(\hat{c}_i, c_{-i})]}{dc_i d\hat{c}_i} |_{\hat{c}_i=c_i} = \frac{d-\chi_i(\tau, c_{-i})h(z(\tau, c_{-i}; c_i))}{d\tau} \geq 0$ . This monotonicity condition along with  $\chi_i(c) \in \{0, 1\}$ , and  $\frac{d\Pi_i(c_i, c_{-i}) - t_i(c_i, c_{-i})}{dc_i} = 0$  a.e. when  $\chi_i(c) = 0$  implies that there must exist  $c_i^*(c_{-i})$  and  $K_i(c_{-i})$  satisfying (i). (i) and the monotonicity condition implies (iv). Along with the integral representation of  $\Pi_i(c_i, c_{-i})$ , (i) implies (iii). (ii) is obviously necessary to guarantee continuity of  $\Pi_i(c_i, c_{-i})$  in  $c_i$ . ■

**Proof of Proposition 2** Follows from the discussion in the text plus the argument in the Proof of Proposition 3 (see below) that states that full participation to the most efficient lender with security  $d^*(c_{(2)})$  induces the largest amount of monitoring among mechanisms that leave non-monitors with zero payoff. ■

To prove Proposition 3, we will use the following result.

**Claim** Take two differentiable functions  $f, g : [0, 1] \rightarrow \mathfrak{R}$ . If  $f'(x) > g'(x)$  for all  $x$  in  $[0, 1]$ , then the solution to (i)  $\max_x f(x) - \tau h(x)$  is larger than the solution to (ii)  $\max_x g(x) - \tau h(x)$ , for all  $\tau$ .

**Proof:** Consider the parameterized maximization problem

$$\max_{x \in [0, 1]} \theta(f(x) - \tau h(x)) + (1 - \theta)(g(x) - \tau h(x)),$$

where  $\theta \in \{0, 1\}$ . It is easy to see that the objective function has increasing differences in  $(x, \theta)$ . By Topkis (1978), the solution of this parametrized maximization problem is then increasing in  $\theta$ . Noting that, when  $\theta = 1$ , the problem is exactly (i) and when  $\theta = 0$  the problem is exactly (ii) the result follows. ■

**Proof of Proposition 3** By Lemma 1, the expected joint profits of a mechanism in which non-monitors have zero profits is given by (using symmetry,  $K_i(c_{-i}) = 0$  for all  $i$ , and integrating (iii) by parts)  $E(\mathbf{1}_{\{c_i \leq c_j, \forall j\}} h(z(c, c_i) \frac{F(c_i)}{f(c_i)}))$ . Thus it suffices to show that the monitoring intensity induced by the outside option is pointwise higher than any other in which the monitor has less than full participation. Using the above claim, one only needs to argue that any pair  $(\alpha, \tilde{d}(c, \alpha))$

such that  $\max_{z_i \in [0,1]} \alpha(\tilde{d}(c, \alpha)E(y|z) - I) - ch(z) = 0$  must be such that  $\alpha\tilde{d}(c, \alpha) < d^*(c)$ .<sup>16</sup> If we had  $\alpha\tilde{d}(c, \alpha) \geq d^*(c)$ , the following would be true:

$$0 = \alpha(\tilde{d}(c, \alpha)E(y|z(\alpha, \tilde{d}(c, \alpha), c)) - I) - ch(z(\alpha, \tilde{d}(c, \alpha), c)) \geq$$

$$\alpha(\tilde{d}(c, \alpha)E(y|z(d^*(c), c)) - I) - ch(z(d^*(c), c)) >$$

$$d^*(c)E(y|z(d^*(c), c)) - I - ch(z(d^*(c), c)) = 0$$

which is a contradiction (the first equality comes from the definition of  $(\alpha, \tilde{d}(c, \alpha))$ , the first inequality from revealed preferences, the second from  $\alpha < 1$  and  $\alpha\tilde{d}(c, \alpha) \geq d^*(c)$  and the last equality from the definition of  $d^*(c)$ ). ■

**Proof of Proposition 4:** Condition (i) in Lemma 1, along with  $\sum_i \chi_i(c) = 1$ , implies that  $i^*$  will be assigned to monitor. For all other lenders  $j$ ,  $\chi_j(c) = 0$  which implies, again by condition (i) in Lemma 1, that their profits can only depend on lender  $i^*$ 's cost. Additionally, lender  $i^*$ 's profits can only depend on his own costs. Otherwise, either his participation, the security assigned to him or the side payments he makes will depend on the other lenders cost. If this is the case, his optimal monitoring intensity or the payment he makes to the others will depend on the other lenders' costs. On its turn, this implies that the non-monitors payoff will depend on their costs, which is a contradiction with (i) in Lemma 1. Thus both monitors and non-monitor's profits will only depend on  $c_{i^*}$ . If  $\Pi_{i^*}(c_{i^*}) > \Pi_j(c_{i^*}) = \Pi_k(c_{i^*}) > 0, \forall j, k \neq i^*$ , continuity of the profit function in costs is violated, as if the second most efficient lender "ties" with the most efficient one, his payoff will be, by symmetry,  $\Pi_{i^*}(c_{i^*})$ . For a cost realization of  $c_{i^*} - \delta$ , his payoff is  $\Pi_j(c_{i^*}) < \Pi_{i^*}(c_{i^*})$  no matter how close to zero  $\delta$  is. ■

**Proof of Proposition 5:** Noting that, for a fixed payment made to the other lenders, (i) participation and price are fully interchangeable in terms of the amount of induced monitoring, while a reduced participation dilutes the financing cost the most efficient lender incurs, and (ii) that all lenders must have the same profits, it is always optimal to grant some participation to all lenders. ■

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<sup>16</sup>Note that, in case non-monitors receive zero payoff, side payments among lenders are of no use.

**Proof of Proposition 6:** Since for all cost realizations the lenders will have the same expected profits, the ex-post sum of profits will be  $N$  times the payoff of the most efficient lender. Moreover, as argued above, the monitor's participation and security can only depend on his costs. Therefore, the ex-ante sum of profits will be given by (noting that  $K_{i^*}(c_{-i^*})$  is equal to what would be the profits of the second most efficient lender if he was the monitor),

$$N(E_c([\max_{z \in [0,1]} \alpha(c_{(2)})[d(c_{(2)})E(y|z) - I] - c_{(2)}h(z)] + \int_{c_{(1)}}^{c_{(2)}} h(z(c_{(1)}; c_{(1)}) \frac{F(c_{(1)})}{f(c_{(1)})})$$

where  $c_{(1)}$  and  $c_{(2)}$  are, respectively, the first and second order statistics. Thus any allocation that induces a monitoring intensity  $z(\tau; \tau)$  that is decreasing in the first argument can be replaced by one that induces  $z(\underline{c}; \tau) \leq z(\tau; \tau), \forall \tau$ . This substitution increases both terms of the joint profits: the second trivially, the first because to induce more monitoring more advantageous participations and/or securities must be assigned to the monitor. Therefore, without loss of optimality, we can restrict attention to monitor's participations and securities that do not depend on his announcement:  $\alpha_{i^*}(c) = \alpha, d_{i^*}(c) = d$ . By symmetry, as  $\sum_i \alpha_i(c) = 1$  the participations of non monitors will be given by  $\frac{1-\alpha}{N-1}$ . Clearly, their securities have to be chosen so to guarantee the equality of payoffs. Hence, we just need to argue that it is always optimal to set  $d = \bar{d}$ . We show that whenever  $d < \bar{d}$ , there is an alternative scheme that improves the monitor's profits. Take any scheme with  $\alpha_{i^*}(c) = \alpha, d_{i^*}(c) = d < \bar{d}$ . There must exist a  $\tilde{d}(c_{i^*}, \alpha)$  such that  $\alpha(dE(y|z(\alpha, d, c_{i^*})) - I) - c_{i^*}h(z(\alpha, d, c_{i^*})) = \frac{1-\alpha}{N-1}(E(S(y, \tilde{d}(c_{i^*}, \alpha)|z(\alpha, d, c_{i^*})) - I)$ . Replacing  $d$  by  $d' = d + \delta$  for a sufficiently small  $\delta$ , one increases the monitor's profits. The question is whether the equality of profits can be re-attained. Either one of two possibilities may arise

$$\alpha(d'E(y|z(\alpha, d', c_{i^*})) - I) - c_{i^*}h(z(\alpha, d', c_{i^*})) \leq \frac{1-\alpha}{N-1}(\tilde{d}(c_{i^*}, \alpha)E(y|z(\alpha, d', c_{i^*})) - I),$$

in such a case, by continuity, one can obviously find an alternative  $\tilde{d}(c_{i^*}, \alpha)$  that re-establishes the equality of profits (as  $\underline{dy} = 0$ ), or

$$\alpha(d'E(y|z(\alpha, d', c_{i^*})) - I) - c_{i^*}h(z(\alpha, d', c_{i^*})) > \frac{1-\alpha}{N-1}(\tilde{d}(c_{i^*}, \alpha)E(y|z(\alpha, d', c_{i^*})) - I).$$



In this case, again a  $\tilde{d}(c_{i^*}, \alpha)$  can be found. By choosing  $\epsilon$  properly, we will have

$$\frac{1-\alpha}{N-1}(E(y|z(\alpha, d', c_{i^*}) - I) > \alpha E(y|z(\alpha, d', c_{i^*}) - I) - c_{i^*}h(z(\alpha, d', c_{i^*}))$$

as

$$\frac{1-\alpha}{N-1}(E(y|z(\alpha, d, c_{i^*}) - I) >$$

$$\frac{1-\alpha}{N-1}(\tilde{d}(c_{i^*}, \alpha)E(y, |z(\alpha, d, c_{i^*}) - I) =$$

$$\alpha(dE(y|z(\alpha, d, c_{i^*}) - I) - c_{i^*}h(z(\alpha, d, c_{i^*}))$$

and the first and third term are continuous in  $d$ . Finally, given  $d_{i^*}(c) = \bar{d}$ , it is optimal to give as much participation to the monitor as feasible so that  $\bar{\alpha} = \max \bar{A}$  is the optimal assignment of participation<sup>17</sup>. ■

To prove Proposition 7, we will use the following result.

**Lemma 11 (2)** *If two continuously differentiable functions  $f, g : [0, b] \rightarrow \mathbb{R}$  are such that (i)  $f(0) = g(0)$ , and (ii)  $f(x) = g(x) \Rightarrow f'(x) > g'(x)$ , then  $f(x) > g(x)$  for all  $x$  in  $(0, b]$*

**Proof** Take the set  $A_0 = \{x : x \in (0, b] \text{ and } f(x) = g(x)\}$ . Assuming it is not empty (if it is, there is nothing to be proved: by continuity, necessarily  $f(x) - g(x)$  cannot change of sign over its domain and, as  $f'(0) - g'(0) > 0$  and the first derivatives are continuous, the result has to hold), name its minimum element  $\underline{x}$ . (This is a well defined object as  $A_0$  is compact). For all  $x$  in  $(0, \underline{x})$ , we must have  $f(x) > g(x)$ , because, due to continuity and the definition of

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<sup>17</sup>This object is well defined for  $\bar{A}$  is compact. Boundness is obvious. As for closedness, take a sequence  $\{\alpha^n\} \subset A_{\alpha'}(c_{i^*}) \equiv \{\alpha' : \exists \tilde{d}(c_{i^*}, \alpha') \in D \text{ with } (\alpha' E(y|z(\alpha', \tilde{d}, c_{i^*}) - I) - c_{i^*}h(z(\alpha', \tilde{d}, c_{i^*})) = \frac{1-\alpha'}{N-1}(E(S(y, \tilde{d}(c_{i^*}, \alpha')|z(\alpha', \tilde{d}, c_{i^*}) - I)$ , with  $\alpha^n \rightarrow \alpha$ . By definition of  $A_{\alpha'}(c_{i^*})$ , there must then exist a sequence  $\{\tilde{d}^n\} \subset [\underline{d}, \bar{d}]$  that guarantes the equality of payoff among lenders. As  $[\underline{d}, \bar{d}]$  is compact there is a subsequence

$\{\tilde{d}^{n_k}\}$  of  $\{\tilde{d}^n\}$  with a limit in it. As for all  $k$ ,  $\alpha^{n_k} E(y|z(\alpha^{n_k}, \tilde{d}^{n_k}, c_{i^*}) - I) - c_{i^*}h(z(\alpha^{n_k}, \tilde{d}^{n_k}, c_{i^*})) =$ , taking the limit as  $k$  goes

to infinity, and noting that both sides are continuous in  $\alpha$ , and the right hand side is continuous in the security index, it follows that  $\alpha \in A_{\alpha'}(c_{i^*})$ . As  $\bar{A}$  is the intersection of closed sets, it is also closed.

$\underline{x}$ ,  $f(x) - g(x)$  cannot change of sign over  $(0, \underline{x})$ , and as  $f'(0) - g'(0) > 0$ , the claim must follow. Proceeding inductively (starting from replacing 0 by  $\underline{x}$  in the definition of the above set), we have  $f(x) \geq g(x)$  for all  $x$ . If equality was to hold for some  $x$ , (ii) and continuity of the derivatives would imply  $f'(y) > g'(y)$  for all  $y \in (x - \epsilon, x]$ , for some  $\epsilon > 0$  sufficiently small. Integrating both sides of the inequality over this set and using  $f(x) = g(x)$ , we would have  $f(x - \epsilon) < g(x - \epsilon)$ , which is a contradiction. ■

**Proof of Proposition 7** If the condition does not hold, there must exist  $\hat{y}$  such that  $\frac{1}{N}(y - I) > \min\{y, D^*(\bar{c})\} - I$  for all  $y < \hat{y}$ , and the reverse strict inequality for  $y > \hat{y}$ . By Lemma 5 in DeMarzo et al (2002), this along with (A2) implies that, whenever

$$E(\min\{y, D^*(\bar{c})\}|z) - I = \frac{1}{N}(E(y|z) - I),$$

one has that

$$\frac{dE(\min\{y, D^*(\bar{c})\}|z) - I}{dz} > \frac{d\frac{1}{N}(E(y|z) - I)}{dz}$$

We can split the analysis in two cases.

(i) If there exists  $\underline{z} < 1$  so that  $E(\min\{y, D^*(\bar{c})\}|\underline{z}) - I = \frac{1}{N}(E(y|\underline{z}) - I)$ , using Lemma 10,  $E(\min\{y, D^*(\bar{c})\}|z) - I > \frac{1}{N}(E(y|z) - I)$  for all  $z > \underline{z}$ . Also, by the definition of  $D^*(\bar{c})$  and  $d^*(\bar{c})$  there exists (a) a  $\tilde{y}$  such that  $\min\{y, D^*(\bar{c})\} - I > d^*(\bar{c})y - I$  for all  $y < \tilde{y}$ , and the reverse inequality for  $y > \tilde{y}$ , and (b) a  $\underline{\underline{z}} < 1$  so that  $E(\min\{y, D^*(\bar{c})\}|\underline{\underline{z}}) - I = d^*(\bar{c})E(y|\underline{\underline{z}}) - I$ .<sup>18</sup> Thus, by the same reasons as above, we must have

$$d^*(\bar{c})E(y|z) - I > (E(\min\{y, D^*(\bar{c})\}|z) - I) \text{ for all } z > \underline{\underline{z}}.$$

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<sup>18</sup>If there was not such a  $z$ , by continuity of such expected values in monitoring, strict inequality would have to prevail and if, say,

$$E(\min\{y, D^*(\bar{c})\}|z) < d^*(\bar{c})E(y|z)$$

for all  $z$ , one would have

$$0 = E(\min\{y, D^*(\bar{c})\}|z(D^*(\bar{c}), \bar{c})) - I - \bar{c}h(z(D^*(\bar{c}), \bar{c})) <$$

$$d^*(\bar{c})(E(y|z(D^*(\bar{c}), \bar{c})) - I - \bar{c}h(z(D^*(\bar{c}), \bar{c})) <$$

$$d^*(\bar{c})(E(y|z(d^*(\bar{c}), \bar{c})) - I - \bar{c}h(z(d^*(\bar{c}), \bar{c})))$$

which would contradict the definition of  $d^*(\cdot)$ .

Therefore, letting  $\bar{z} = \max\{\underline{z}, \underline{z}\}$ , the following holds:

$$\max_{z \in (\bar{z}, 1]} d^*(\bar{c})E(y|z) - I - \tau h(z) >$$

$$\max_{z \in (\bar{z}, 1]} E(\min\{y, D^*(\bar{c})\}|z) - I - \tau h(z) >$$

$$\max_{z \in (\bar{z}, 1]} \frac{1}{N}(E(y|z) - I) - \tau h(z).$$

If  $\tau = 0$ , the constraints in these maximization problems do not bind. Moreover,  $\frac{1}{N} = \bar{\alpha} = \max A_{\alpha'}(0)$ . As a consequence, for  $\underline{c}$  positive but close enough to zero,

$$\max_{z \in [0, 1]} d^*(\bar{c})E(y|z) - I - \underline{c}h(z) >$$

$$\max_{z \in [0, 1]} E(\min\{y, D^*(\bar{c})\}|z) - I - \underline{c}h(z) >$$

$$\max_{z \in [0, 1]} \bar{\alpha}(E(y|z) - I) - \underline{c}h(z) >$$

so that syndication will not occur for a positive measure of cost realizations. (ii) If there is no  $\underline{z} < 1$  so that  $E(\min\{y, D^*(\bar{c})\}|\underline{z}) - I = \frac{1}{N}(E(y|\underline{z}) - I)$ , the assumption in the proposition and continuity of the expected values in  $z$  implies that  $\frac{1}{N}(E(y|z) - I) < E(\min\{y, D^*(\bar{c})\}|z) - I$  for all  $z$ . Thus, in such a case again syndication does not occur for all cost realizations. ■

**Proof of Proposition 8** It suffices to show that for each cost profile  $c$  and  $\alpha' \in \bar{A}$  under debt financing, we can find a scheme using equity that is IC-Ex-Post and yields higher profits to the monitor. Under debt financing, if  $\alpha' \in \bar{A}$  there exists  $\tilde{D}(\alpha', c_{i^*})$  so that

$$\alpha' E(y|z(\alpha', \bar{D}, c_{i^*})) - I - c_{i^*} h(z(\alpha', \bar{D}, c_{i^*})) =$$

$$\frac{1 - \alpha'}{N - 1} (E(\min\{y, \tilde{D}(\alpha', c_{i^*})\}|z(\alpha', \bar{D}, c_{i^*})) - I)$$

$$< \frac{1 - \alpha'}{N - 1} (E(y|z(\alpha', \bar{D}, c_{i^*})) - I)$$

the first equality because of the definition of  $\bar{A}$ , and the inequality from the fact that  $\tilde{D}(\alpha', c_{i^*}) < \bar{D}$ . By continuity of the first and third terms in  $\alpha$ , there exists  $\epsilon > 0$  sufficiently small so that

$$\alpha'' E(y|z(\alpha'', \bar{D}, c_{i^*})) - I) - c_{i^*} h(z(\alpha'', \bar{D}, c_{i^*})) < \frac{1 - \alpha''}{N - 1} (E(y|z(\alpha'', \bar{D}, c_{i^*})) - I),$$

where  $\alpha'' = \alpha' + \epsilon$ . It is clear that there exists  $\tilde{d}(\alpha'', c_{i^*}) \in (0, 1)$  such that

$$\alpha'' E(y|z(\alpha'', \bar{D}, c_{i^*})) - I) - c_{i^*} h(z(\alpha'', \bar{D}, c_{i^*})) = \frac{1 - \alpha''}{N - 1} (\tilde{d}(\alpha'', c_{i^*}) E(y|z(\alpha'', \bar{D}, c_{i^*})) - I),$$

implying  $\alpha'' \in A_\alpha$  under equity financing. As the monitor's payoff is increasing in his participation, the result follows. ■

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