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Working Paper

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Discussion Paper, No. 2018/8

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Suggested Citation: Schreiber, Sven (2018) : Are bootstrapped cointegration test findings unreliable?, Discussion Paper, No. 2018/8, Freie Universität Berlin, School of Business & Economics, Berlin

This Version is available at:

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Are bootstrapped cointegration test findings unreliable?

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Discussion Paper

Economics

2018/8

Are bootstrapped cointegration test findings unreliable?*

by Sven Schreiber[†]

May 2018

Abstract

Applied time series research often faces the challenge that (a) potentially relevant variables are unobservable, (b) it is fundamentally uncertain which covariates are relevant. Thus cointegration is often analyzed in partial systems, ignoring potential (stationary) covariates. By simulating hypothesized larger systems Benati (2015) found that a nominally significant cointegration outcome using a bootstrapped rank test (Cavaliere, Rahbek, and Taylor, 2012) in the bivariate sub-system might be due to test size distortions. In this note we review this issue systematically.

Apart from revisiting the partial-system results we also investigate alternative bootstrap test approaches in the larger system. Throughout we follow the given application of a long-run Phillips curve (euro-area inflation and unemployment). The methods that include the covariates do not reject the null of no cointegration, but by simulation we find that they display very low power, such that the (bivariate) partial-system approach is still preferred. The size distortions of all approaches are only mild when a standard HP-filtered output gap measure is used among the covariates. The bivariate trace test p-value of 0.027 (heteroskedasticity-consistent wild bootstrap) therefore still suggests rejection of non-cointegration at the 5% but not at the 1% significance level. The earlier findings of considerable test size distortions can be replicated when instead an output gap measure with different longer-run developments is used. This detrimental effect of large borderline-stationary roots reflects an earlier insight from the literature (Cavaliere, Rahbek, and Taylor, 2015).

JEL codes: C32 (multiple time series), C15 (statistical simulation methods), E31 (inflation)

Keywords: bootstrap, cointegration rank test, empirical size

*I am grateful for very valuable feedback and advice from Robert Taylor, Anders Rahbek and Giuseppe Cavaliere. They are not responsible for any errors that may remain.

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1 Introduction

The cointegration rank test conducted in a multivariate system (“Johansen procedure”) is a widespread and popular tool for applied time series analysis. It has long been known that asymptotic inference with that test suffers from substantial size distortions in small samples typical of macroeconomic datasets. Johansen himself developed a finite-sample Bartlett correction for the trace test statistic (Johansen, 2002), and as PCs became faster some bootstrap techniques were also proposed (Cavaliere, Rahbek, and Taylor, 2012, 2015). This could be considered as the state of the art.

Recently, however, by conducting an extensive array of simulations Benati (2015) arrived at the interesting result that even the bootstrapped version of the rank test could still be subject to considerable size distortions. Benati’s paper was not meant as an econometrics methods study but investigated the existence of long-run Phillips curve relationships in various economies (synthetical euro area, UK, USA, Canada, and Australia). In one of the many simulations in his paper he essentially analyzed the performance of the bootstrapped rank test in a partial system, i.e. in a situation where the VAR used for the test is lower-dimensional than the DGP, even when only stationary covariates are omitted, not variables in the cointegration relationships themselves: $x_t = (\pi_t, u_t)'$ with $N = 2$ versus $x_t^* = (\pi_t, u_t, l_t - s_t, \Delta s_t, y_t)'$ with $N = 5$.¹

We focus here on the results for the synthetical euro area and follow the choice of Benati’s sample that actually predates the introduction of the euro –quarterly data 1970-1998– due to the apparently different properties of the series afterwards. For the bivariate system he reports in his Table 2 a p-value of 0.049 for the bootstrapped test of a cointegrating rank $r = 0$ versus $r = 1$. This finding would usually suggest to reject non-cointegration of euro-area inflation and unemployment at the 5% level of significance. He then found a considerable size distortion of the bootstrapped test based on x_t ($N = 2$) when the DGP was assumed to contain x_t^* ($N = 5$) and dismissed the nominal findings of cointegration as a “statistical fluke”.

Because the reliability of the cointegration test is crucial for many applied research areas, to investigate the relevant statistical issues we use similar euro area data and focus on the issue of bivariate cointegration between the GDP deflator growth (inflation) and the unemployment rate.² Simulations using the actual data are also supplemented with some simulations of artificial data. One of our findings is that the inflated size stems from a large root in the five-dimensional null model. This insight

¹The variables are inflation π_t and unemployment u_t , the short- and long-term interest rates s_t and l_t are transformed a priori to the stationary term spread $(l - s)_t$ and the differenced short rate Δs_t , and the output gap y_t . See section 4.2 for a plot of the output gap and the data appendix for further plots.

²Benati also considered cointegration ranks $r > 1$ including interest rate levels, and checked CPI inflation as a variant. The datasets are not strictly identical, but with our proxy from the ECB’s area-wide model (AWM) we obtain qualitatively the same results, see section 4.2. For the bootstrap procedures we use the `johansensmall.gfn` function package (version ≥ 2.6) by Sven Schreiber and Andreas Noack Jensen, for the open-source `gretl` program and freely available online from within `gretl`. Similar code for Matlab is for example available on De Angelis’ homepage <https://sites.google.com/view/luca-de-angelis/research>.

is similar to a simulation result in Cavaliere, Rahbek, and Taylor (2015), with the difference that the large root is introduced by extra variables in a larger background system.

Furthermore it appears that the choice of the output gap measure introduced this large root, and that for example with a standard HP-filter gap the distortions disappear. This suggests that the evidence for bivariate cointegration is stronger than concluded by Benati. Overall we conclude that the problems of the bootstrapped rank test are less than previously suggested and that it is still to be recommended for applied research.

2 Test specifications

Throughout this note we focus on the popular case of an unrestricted constant, which was formally justified in Cavaliere, Rahbek, and Taylor (2015).³ For lag length selection in the test VARs we deliberately choose not to use information criteria. The reason is that the non-autocorrelation of residuals is essential for the validity of the bootstrap, and some of the lag order suggestions by information criteria led to substantial remaining residual autocorrelation. Thus we specify lag orders based on passing a diagnostic autocorrelation test instead.

Benati considered two structural-VAR cases: one where only a single structural shock has a permanent impact on inflation, and another one where up to four shocks may have such an impact. We focus on the case with four permanent inflation shocks because it leaves the reduced-form coefficients of the VAR unchanged. Only the remaining fifth shock might then be restricted in the five-dimensional system, and this single restriction would not affect the likelihood function of the model. In contrast, the case where four of the five structural shocks are restricted implies that estimation cannot be done by standard ML anymore. Here we wish to focus on the properties of the cointegration rank test in otherwise unrestricted systems which is the most common case in practice.

The original simulation study used a five-dimensional DGP including inflation and unemployment that imposed absence of cointegration, and then applied the bootstrapped rank test of the null hypothesis $r = 0$ vs. $r \geq 1$ to the bivariate sub-system of simulated inflation and unemployment (in levels) in each simulation draw. Benati's result in his Table 3 was that the bootstrap procedure rejected the null hypothesis of no cointegration at a nominal 5% significance in 18.3% of the simulation draws. Thus he concluded that the bootstrap test grossly exceeded its nominal significance level, and that therefore the original test rejection with a p-value of just under 5% might be "a fluke".

Benati's simulation design is reasonable in principle. If we test two variables for

³While Benati is not explicit about the deterministic specification used in his setup, this seems to be his choice there as well.

cointegration, we do not typically care about other aspects of the DGP, and we require (at least ideally) that the test outcome should only depend on whether or not cointegration is indeed present. However, this test approach is not the only possible one, at least two different test variants come to mind when further variables are suspected to be relevant for the system dynamics. We enumerate the following three possibilities of cointegration testing with stationary co-variates in small samples:

1. (Bivariate, Benati's method) The null model is given by an unrestricted autoregression for the vector $x'_{0,t} = (\Delta u_t, \Delta \pi_t, y_t, \Delta s_t, l_t - s_t)$, where y_t is the output gap, and $l_t - s_t$ is the term spread between longer-term and short-term interest rates. To ensure a common lag length in levels, the K -th lag coefficients for the differences of unemployment and inflation are set to zero for the simulation DGP:

$$x_{0,t} = c + \sum_{i=1}^{K-1} A_i x_{0,t-i} + (0_{5,2} | \tilde{A}_K) x_{0,t-K} + \varepsilon_t,$$

where \tilde{A}_K is an unrestricted 5×3 matrix for the K -th coefficients of the three stationary co-variates. Use this model to generate pseudo data, then run the Cavaliere, Rahbek, and Taylor (2015) bootstrapped cointegration test with an unrestricted constant on each simulated draw of the bivariate data $x'_{2,t} = (u_t^*, \pi_t^*)$ with a lag order K .⁴

2. (Swensen, unmodelled covariates method) Finally, another bootstrap possibility in the presence of stationary covariates is given by Swensen (2011). The null model is again set up and simulated as in 1, and the bootstrap test is also applied to the bivariate vector $x'_{2,t} = (u_t^*, \pi_t^*)$. However, the test system is augmented with lags of the co-variates $x'_{3,t} = (y_t^*, \Delta s_t^*, (l_t - s_t)^*)$, i.e. $x_{3,t-1}^* \dots x_{3,t-K}^*$ are added as unrestricted regressors.⁵
3. (Full system method) If the researcher suspects that there are some important covariates which are known to be $I(0)$, it seems natural to simply include them in the test system. Thus the null model and the bootstrap framework is again given as in method 1, but here the vector to be tested is $x'_{5,t} = (u_t^*, \pi_t^*, y_t^*, \Delta s_t^*, (l_t - s_t)^*)$, and since the co-variates add three stationary directions to the system already under the null, the relevant hypothesis to test cointegration between unemployment and inflation is $r = 3$ vs. $r = 4$ (again with K lags).

⁴It is not obvious from Benati's description how exactly he handles the lag structure in his simulation, i.e. whether or not he chooses a different lag length for the bivariate subsystem. We determine the lag length in each rank test based on autocorrelation diagnostics.

⁵We do not include contemporaneous values of the covariates as this would obviously violate the necessary assumption of uncorrelatedness. These pseudo covariates are re-generated in each simulation run, but are then held fixed for the inner bootstrap. This corresponds to the test variant described in remark 6 in Swensen (2011). His remark 3 also applies in our implementation, as we use the restricted non-cointegrated model in the bootstrap algorithm.

| (simulated rejection frequencies under H0) | as-if-iid | wild |
|---|-----------|-------|
| Bivariate, $r_0 = 0$ | 0.069 | 0.083 |
| Swensen 2 + 3 covariates, $r_0 = 0$ | 0.079 | 0.077 |
| Full 5-dim, $r_0 = 3$ | 0.033 | 0.040 |

Notes: Simulation of the size of the bootstrapped rank test. Nominal 5%; 2000 simulation replications; the bootstrap test in each simulation draw uses 1000 replications. The time series length is $T = 109$.

3 Simulation results

As explained before, by not explicitly modelling the structural shocks we implicitly allow (but do not force) many shocks to have permanent effects. The underlying system is a 5-dimensional VAR as introduced above, using the cycle component of a standard Hodrick-Prescott (HP) filter applied to real GDP as the relevant measure of the output gap.

3.1 Simulated empirical size

First of all we simulate the effective size (rejection probability under the null) of the cointegration test in the three different test strategies. Following Benati’s approach we take the parameters of a non-cointegrated 5-dimensional VAR as the DGP; the two $I(1)$ variables are differenced and the stationary variables are left as is. We use 4 lags to obtain the parameters under the null, as this satisfies both the AC and ARCH residual tests.⁶ For fitting the model to the simulated data in each draw we do not impose the original lag length but the algorithm chooses the lag order endogenously based on diagnostic residual testing.

Table 1 reports the size simulation results. For the rightmost column “wild”, the rank test is based on a wild bootstrap scheme from the cited literature to account for potential heteroskedasticity. The takeaway from that simulation is that there are only mild size distortions, and that the empirical sizes of the bivariate partial-system test and of Swensen’s approach are roughly equal. The full-system approach is mildly conservative which implies that its size is only about half of the sizes of the other approaches (for a nominal 0.05 level).

⁶Having approximately white noise innovations is preferable because we use resampling for the simulation. If we drew the simulation innovations from a parametric model instead the lag length would of course be less important.

Table 2: Bootstrapped cointegration rank tests (inflation / unemployment)

| (bootstrapped p-values) | iid | wild |
|---------------------------------|-------|-------|
| Bivariate | 0.011 | 0.027 |
| Swensen 2 + 3 covar., $r_0 = 0$ | 0.182 | 0.213 |
| Full 5-dim, $r_0 = 3$ | 0.159 | 0.185 |

Notes: 4999 replications; lags are chosen based on diagnostic tests: bivariate – 7 lags, Swensen’s approach – 7 lags, full system – 4 lags. The respective sample size T is 113 minus the lag order.

3.2 Test results

Given that we use similar but not identical data as Benati did, it is interesting to compare the test results on the actual data. (See below for further analysis of the issue of a different output gap measure.)

While the residuals are free from autocorrelation in the bivariate specification with seven lags, there are always remaining ARCH effects, so the wild bootstrap variant (right column) may be preferred for the bivariate case. We obtain similar results to Benati in this bivariate setup, also rejecting the null of no cointegration at the 5% level (p-value with the wild bootstrap 0.027).

Swensen’s approach, where the bivariate system is augmented with the stationary covariates, is also subject to ARCH-type residuals, again suggesting the use of the wild bootstrap. Here the bootstrapped p-value is far above conventional critical levels (0.213), suggesting non-rejection of no cointegration.

Finally, the full-system setup with four lags is well behaved, so the iid bootstrap is the method of choice, but it shares with Swensen’s setup the non-rejection result (p-value 0.159).

3.3 Power assessment

The test results in Table 2 represent a dilemma. Given that in Table 1 we found that the size distortions of the bootstrapped rank test variants are not dramatic, we do not prefer one approach in Table 2 over any other based on the size assessment (at least if we share the prior belief that the chosen covariates are actually part of the DGP). But obviously the test outcomes are very different, so a test decision is difficult.

Therefore we turn to an assessment of the empirical power of the three test approaches. The question is how large are the rejection probabilities under the alternative hypothesis of cointegration (between inflation and unemployment)? To this end we run a similar simulation as before in Section 3.1, but as the DGP we use a cointegrated system instead: the parameters are taken from the estimated error correction system (VECM) of the actual data under an assumed rank of 4, including the cointe-

Table 3: Test power simulations

| (simulated rejection frequencies under H0) | iid | wild |
|---|-------|-------|
| Bivariate, $r_0 = 0$ | 0.810 | 0.798 |
| Swensen 2 + 3 covariates, $r_0 = 0$ | 0.139 | 0.128 |
| Full 5-dim, $r_0 = 3$ | 0.224 | 0.235 |

Notes: Simulation of the power of the bootstrapped rank test for the fixed alternative given by the cointegrated system (cointegration between unemployment and inflation plus the three stationary covariates) estimated from actual data. Nominal 5%; 2000 simulation replications; the bootstrap test in each simulation draw uses 1000 replications. The time series length is $T = 109$.

gration coefficients β .⁷ Then we simulate artificial data many times with resampled innovation processes, and each time we run the bootstrapped cointegration rank test on the artificial data.

The results of that simulation exercise are reported in Table 3. There is a surprisingly large gap between the power of around 80% in the bivariate case and the power below 25% or even 15% in the full-system and Swensen approaches. This means that the latter two approaches would quite rarely result in rejection of the null hypothesis even if it were false. Against this background it appears that the bivariate setup is the most reliable, combining only mild size distortions with large power advantages. Overall the most natural test conclusion would therefore be that euro area unemployment and inflation are cointegrated, based on a conventional significance level of 5% (but not at the 1% level).

4 Revisiting earlier results of size distortions

Especially the simulated size results in Section 3.1 are different from the analogous results in Benati, and we now turn to further analysis of the underlying causes of this discrepancy.

4.1 Size simulations with artificial data

An insight that was already revealed in Cavaliere, Rahbek, and Taylor (2015) was that large stationary roots in the system affect the empirical size of the bootstrapped rank test. Here we briefly address a closely related issue in a partial-system setup where the DGP also contains stationary covariates. The artificial three-dimensional

⁷Three of the four columns of β are trivial unit vectors picking the stationary covariates, which technically increases the cointegration rank. The only “actual” cointegration relationship is still the one between unemployment and inflation.

Table 4: Test size simulation, artificial DGP

| (simulated rejection frequencies under H0) | resampling as-if-iid |
|--|----------------------|
| Bivariate, $r_0 = 0$ | 0.164 |
| Swensen 2 + 1 covar., $r_0 = 0$ | 0.128 |
| Full 3-dim, $r_0 = 1$ | 0.112 |

Notes: (nominal 0.05; 5000 replications); Sample size $T = 100$.

DGP is given as follows.

Consider the vector $v = (x, y, z)'$, where the first two components (x_t, y_t) are $I(1)$, while the last one (z_t) is a stationary co-variate. Due to the presence of z_t the formal cointegration rank (dimension of the stationary directions) of the system is one, even though the $I(1)$ variables are not cointegrated. The VECM representation is given by $\Delta v_t = \alpha\beta'v_{t-1} + \Gamma_1\Delta v_{t-1} + u_t$ with a diagonal covariance matrix and the trivial cointegration vector $\beta = (0, 0, 1)'$. The loading coefficients are $\alpha = (0.1, a_y, -0.2)'$, and the short-run dynamics are set to

$$\Gamma_1 = \begin{bmatrix} 0.4 & 0.3 & 0.1 \\ 0 & 0.5 & 0.1 \\ 0 & 0.2 & -0.3 \end{bmatrix}.$$

As usual, the corresponding levels form VAR with two unit roots is $v_t = B_1v_{t-1} + B_2v_{t-2} + u_t$, where $B_1 = \alpha\beta' + I_3 + \Gamma_1$ and $B_2 = -\Gamma_1$. With $a_y = 0.3$ for example, the roots of the system are all real: 1, 1, 0.948, 0.450, 0.400, -0.398 . Obviously the largest stationary root is quite close to the unit circle and implies considerable persistence.

Running the analogous test size simulation as in Section 3.1 using this DGP (with $a_y = 0.3$), we obtain the results in table 4. In contrast to the earlier results here we observe the same phenomenon as Benati did, namely a considerable size distortion. For a nominal significance level of 5% all test approaches display an effective size of over 10%, the bivariate approach even over 15%. This suggests that the effect that Benati observed was the same as in Cavaliere, Rahbek, and Taylor (2015) when (under the null) a large stationary root is present.

4.2 Results with the AWM gap

In the earlier tests and simulations we used a standard HP-filtered cycle component of real GDP as the output gap measure. This is not what appeared in Benati's system for the euro area, however, which was based on a certain vintage "from the ECB's database" (quote from the online appendix to Benati, 2015). The precise calculation method of that series is unknown.

Figure 1: Output gap comparison

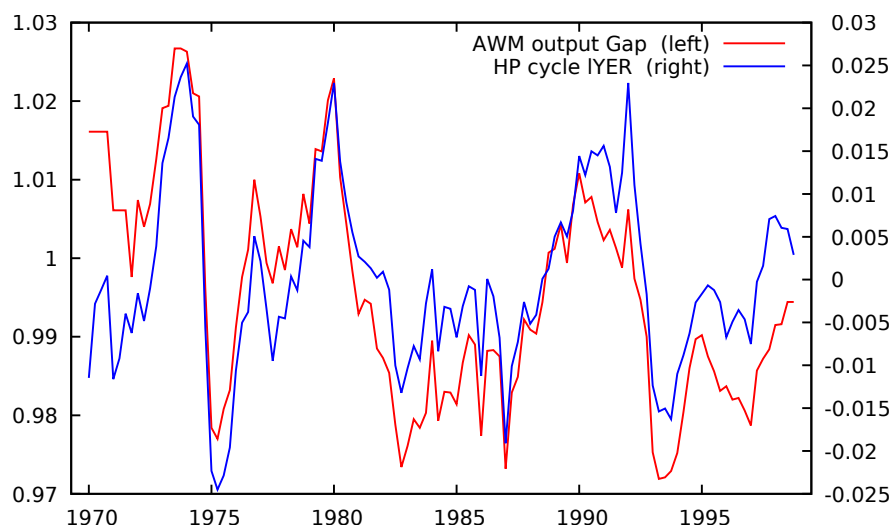


Table 5: Test size simulations under 5-dim DGP with YGA

| (simulated rejection frequencies under H0) | resampling as-if-iid | wild |
|--|----------------------|-------|
| Bivariate, $r_0 = 0$ | 0.349 | 0.327 |
| Swensen 2 + 3 covar., $r_0 = 0$ | 0.067 | 0.086 |
| Full 5-dim, $r_0 = 3$ | 0.023 | 0.023 |

Notes: nominal level 0.05; 2000 replications

As a proxy we use the output gap series from the ECB’s area-wide model (AWM) database. In Figure 1 the two variants are compared, where the AWM gap series is taken from the dataset shipped with the *gretl* program. At business-cycle frequencies the two series are highly correlated, as should be expected. However, while the HP cycle measure fluctuates around a constant mean (by construction), the AWM gap is more persistent in the longer run, starting with a sequence of higher-than-average values and finishing the sample with many lower-than-average values. Its AR(1) root is 0.90, opposed to the slightly lower root of the HP cycle of 0.85.

In the test size simulations that work as before in Section 3.1, but use the described AWM gap instead and by consequence require 7 lags under the null to obtain innovations close to white noise, we observe (in Table 5) that again the full-system approach is somewhat conservative, Swensen’s approach is mildly oversized, but that now the bivariate partial-system test approach is dramatically oversized with an empirical size over 30% for a nominal 5%. This appears even more drastic than Benati’s original finding (based on a different lag length and possibly slightly different data).

The bootstrapped actual test results are reported in Table 6. Of course the bivariate test by definition does not depend on the output gap variable and therefore is the

Table 6: Test results with actual data (AWM gap)

| (bootstrapped p-values) | iid | wild |
|---------------------------------|-------|-------|
| Swensen 2 + 3 covar., $r_0 = 0$ | 0.007 | 0.011 |
| Full 5-dim, $r_0 = 3$ | 0.366 | 0.335 |

Notes: 2000 replications; lag choices: Swensen – 5 lags, Full-system – 7 lags.

same as in Table 2 and is not reproduced again. For the Swensen approach there is always remaining ARCH effects, thus the wild bootstrap results may be preferred, with a p-value of 0.011 suggesting rejection of no cointegration at the 5% significance level. Given the only mild size distortions of the Swensen approach this appears to be a valid result. The full-system approach here implies well-behaved residuals, so the preferred variant is the iid bootstrap, yielding a p-value of 0.366, not providing evidence in favor of cointegration.

5 Conclusions

The issue of how cointegration rank tests behave when they are applied in partial systems is important, because applied research often faces the challenge that (a) either potentially relevant variables are unobservable, or (b) it is fundamentally uncertain which covariates might be relevant. As Benati (2015) showed, and as this note has partly confirmed, the worrying insight is that rejection results in partial systems may be misleading. A closer analysis revealed that this is the effect of the fact that the full non-cointegrated DGP in the background contains large (stationary) roots. It should be acknowledged, however, that a very similar result was already known from the original literature that proposed the bootstrapped rank test (e.g., Cavaliere, Rahbek, and Taylor, 2015).

At least for the given issue of a euro-area long-run Phillips curve we could show that it does not pay off to consider instead full-system methods, as they suffer from a severe lack of power. The claimed size distortions, however, turn out to hinge on a very specific choice of the output gap measure which in the given sample introduces a very persistent root into the posited DGP. With a standard HP filtered gap measure the distortions largely disappear, even though its univariate autoregressive root is also still around 0.85. Therefore, the econometric evidence for cointegration in this sample and between these variables remains intact, unless one is convinced that the true output gap is extremely persistent (e.g., closer to the AWM gap in Figure 1 than to the HP cycle).

Finally, it should be acknowledged that this note has addressed a very specific aspect of Benati (2015), which also includes an impressive amount of other empirical and theoretical work. It is not the purpose of this note to question the broad

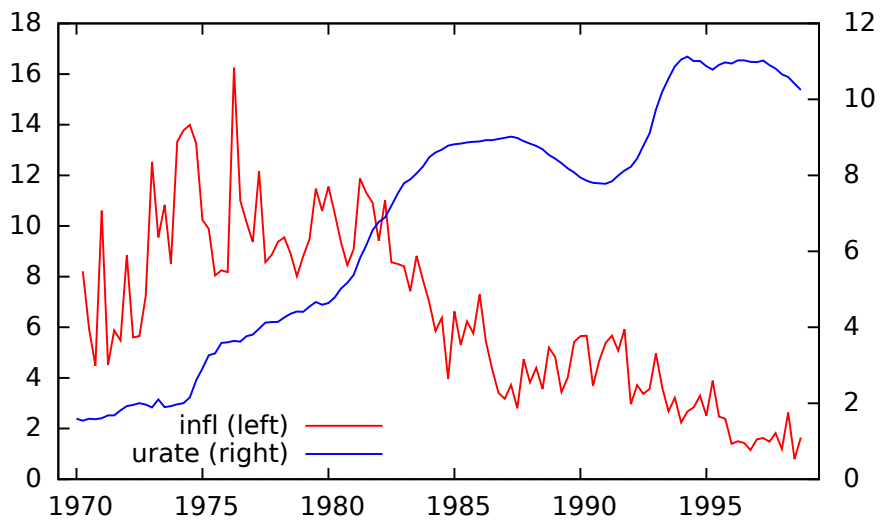
conclusions of his work, which he himself summarizes as “uncertainty ... is ... substantial” (p. 27). We fully agree. Nevertheless, some of his conclusions depend on the alleged weaker-than-expected evidence for cointegration, and we regard it as important to clarify for applied economists that conducting cointegration tests in small samples with a bootstrap remains a justified practice and that its results cannot be easily discarded as “statistical flukes”.

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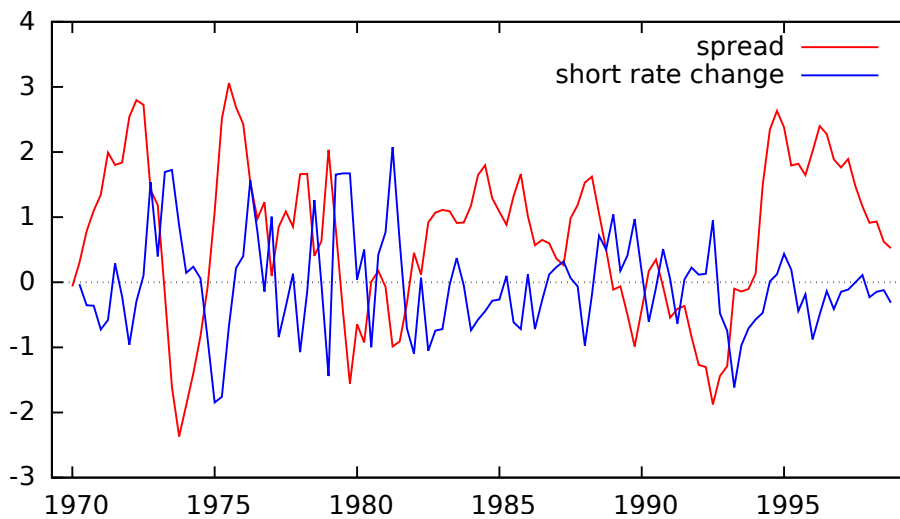
A Data appendix

Figure 2: Inflation and unemployment rates



Notes: Data from the ECB's AWM, $400 \times \Delta \log(YED)$ and $100 \times URX$.

Figure 3: Interest rates



Notes: Data from the ECB's AWM, $LTN - STN$ and ΔSTN .

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