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Article

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Suggested Citation: de Carvalho Griebeler, Marcelo; Hillbrecht, Ronald Otto (2015) : Producers, parasites and poverty traps, EconomiA, ISSN 1517-7580, Elsevier, Amsterdam, Vol. 16, Iss. 3, pp. 310-320,
<https://doi.org/10.1016/j.econ.2015.07.002>

This Version is available at:

<https://hdl.handle.net/10419/179602>

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Producers, parasites and poverty traps[☆]

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Received 4 March 2015; received in revised form 27 July 2015; accepted 31 July 2015

Available online 27 October 2015

Abstract

We studied the population dynamics of producers and parasites in a developing economy through the Lotka–Volterra model. Our baseline model found a cyclical equilibrium between these two groups of agents. When the equilibrium output is low, we propose that economy is in a poverty trap, such that the only way to achieve superior equilibrium is through improvements in the institutional parameters (e.g. property rights). By adding expectations and time delays, the cyclical result may no longer hold. Nevertheless, institutional improvements promote development through an increase in the level of producers and a decrease of parasites. Still, such improvements positively affect the stability of the modified model.

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JEL classification: E58; E52

Keywords: Lotka–Volterra model; Institutions; Economic development; Parasites

Resumo

Estudamos a dinâmica populacional de produtores e parasitas em uma economia em desenvolvimento através do modelo de Lotka–Volterra. Nosso modelo base encontra um equilíbrio cíclico entre esses dois grupos de agentes. Quando o produto de equilíbrio é baixo, estabelecemos que a economia está em uma armadilha de pobreza, tal que o único meio de alcançar um equilíbrio superior é através de melhora nos parâmetros institucionais (p.e. direitos de propriedade). Ao adicionar expectativas e defasagem temporal, o resultado cíclico pode deixar de existir. De todo modo, melhoras institucionais promovem desenvolvimento através de um aumento no nível de produtores e um decréscimo no de parasitas. Além disso, tais melhoras afetam positivamente a estabilidade do equilíbrio do modelo modificado.

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Palavras-chave: modelo de Lotka–Volterra; instituições; desenvolvimento econômico; parasitas

[☆] Peer review under responsibility of National Association of Postgraduate Centers in Economics, ANPEC.

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1. Introduction

Developing economies have in common the presence of a large number of unproductive businesses, which are financed by and survive at the expense of productive activities. These type of parasitic activities assume several forms, such as violent criminal organizations and gangs affiliated with corrupt politicians. Although parasites do not produce, they have the same objective as a regular firm, namely profit. Economic consequences of parasitic activities may be very serious, such that it is possible that it could result in an economy falling into a poverty trap (Mehlum et al., 2003a).

The following parasitic behavior is quite typical: on the one hand, they provide protection and enforce contracts of small businesses, but on the other hand the “price” of these services is charged through extortion. This is the well known *modus operandi* of mafia: problem solving, which should be a state responsibility, is handled by violent groups that extort the agents.¹ Although it is possible to find examples of such groups in developed countries (e.g. Sicilian mafia), the presence of parasites is more common in developing nations (see Campos, 2000 for eastern European economies in transition in general, Volkov, 2002 for Russia and Naím, 2006 for Latin American countries, for example).²

The aim of this paper is to model the population dynamics of producers and parasites in a developing economy. In order to do so, we use the Lotka–Volterra model, which describes the evolution over time of populations of predator and prey. The idea is to treat producers as prey and parasites as predators. Our baseline model finds cyclical equilibrium between these two groups of agents. When the equilibrium output is low, we claim that the economy is in a poverty trap, such that the only way to achieve a superior equilibrium is through improvements in the institutional parameters (e.g. property rights). By adding expectations and time delays, the cyclical result no longer holds. Nevertheless, institutional improvements promote development through an increase in the level of producers and a decrease of parasites. Still, such improvements positively affects the stability of the modified model.

Our paper contributes to the literature on economic development that considers the institutional environment and its influence on the economic growth (e.g. Besley and Ghatak, 2010; Mehlum et al., 2003a,b; Grossman, 1998) by stressing the importance of parameters like property rights and law enforcement, for example. Concerning to the use of Lotka–Volterra model in Economics, we follow the tradition of seminal paper by Goodwin (1967). Among its many extensions (e.g. Desai, 1973; van der Ploeg, 1988; Sportelli, 1995), one of the most innovative is Vadasz (2007), which incorporates time lag (and expectations) in the economy’s dynamics in order to consider informational failure in the model. Because of this desirable feature, we adopt in our modified model the same framework as proposed by the latter author.

This paper is divided into two sections, besides this introduction. Section 2 presents the baseline model of producer and parasite dynamics as well as the modified one, which includes a time lag, and studies the stability of the found equilibria. Section 3 concludes by presenting the possibility of identifying a poverty trap and how it is possible to overcome it. Our findings also provide some policy implications, discussed in this section. Appendix A shows the proofs omitted from the text.

2. The dynamics of producers and parasites

2.1. The baseline model

Let us consider a developing economy where two types of agents, producers, and parasites exist. Producers include individuals and firms that engage in legal productive activities, which make the economy’s output grow. These agents are the only ones that are productive. We make the simplifying assumption that all producers are identical and each one produces only one unit of output, such that the economy’s aggregate output is equal to producers’ number. We also assume that there is no barrier to entry and exit, and the market for factors is able to meet producers’ demand without affecting their cost.

¹ It is important to stress that, based on the above definitions, the appropriation of income made by this group is different from regular rent-seeking. Such agents capture income in a direct manner from an active – and usually large – state through regulation of private business, for example. On the other hand, parasitic action takes advantage of gaps due to an absence or weakness of the state.

² In order to have an idea of economic consequences of parasitic action, let us analyse the case of piracy, one of the most widespread activities of this sector. According Naím (2006), U.S. companies, for instance, estimate a loss of revenues due to falsification between 200 and 250 billion dollars. Furthermore, the European Union has reported that the cost due to illegal copies, in terms of lost jobs, may achieve 100 thousand.

On the other hand, parasites are those agents defined in the introduction, which exist due only to poorly-defined or non-existent property rights. Such agents appropriate part of the producers output and “feed” their business through extortion and other illegal means. As cited above, parasites do not perform productive activities; therefore, their output is null. We assume that their cost is null and there is no barrier to entry and exit. Thus, the dynamics of these populations in the economy can be described by the system

$$\dot{x} = \alpha_1 x - \gamma_1 xy \quad (2.1)$$

$$\dot{y} = \theta_1 xy - \beta_1 y, \quad (2.2)$$

where x is the density of producers in the economy, y is the density of parasites, α_1 is the producers’ profit rate, β_1 is the rate of parasite extinction (due to state’s actions and its police power), γ_1 is the rate of producer extinction due to parasite extortion (a parameter that measures how much productive agents are sensitive to illegal activity from parasites) and θ_1 is the rate of conversion of extortion into more parasites (how much extortion is used to increase unproductive industry; the profit rate of parasite activity is a good proxy). All parameters are positive and constant.³

Eq. (2.1) illustrates that the number of producers in the economy increases as their profit rate (α_1) increases, because attracts more firms and individuals to productive activities. Rather, the number of producers decreases as sensitivity to parasite extortion (γ_1) is higher, given that it stimulates agents to leave their businesses. Similarly, Eq. (2.2) indicates that the parasite population increases as its profit rate (θ_1) increases by attracting new agents to this group. Furthermore, an improvement in state’s police power (an increase in β_1) decreases the number of parasites.

Let us now investigate the role of two of the parameters in the model’s dynamics. It is easy to verify in the system (2.1) and (2.2) that, in absence of parasites, producer populations grows exponentially at constant rate $\alpha_1 > 0$; and, in the absence of producers, parasite populations decrease exponentially until extinction at a constant rate $\beta_1 > 0$. This result is intuitive to the extent that we know the dynamics between the two sectors: in absence of a group that extorts part of its income, the number of producers will increase without limits; and in absence of producers, parasites do not find a means to “feed” their activities and survive, such that they vanish from the economy.

2.2. Equilibrium and stability of the system

Proposition below shows that two equilibria exist in (2.1) and (2.2).⁴

Proposition 2.1. *Consider the system (2.1) and (2.2). Then, we have the following equilibria, $E_1 = \{x=0, y=0\}$ and $E_2 = \{x=\beta_1/\theta_1, y=\alpha_1/\gamma_1\}$.*

The first equilibrium represents an extinction of both groups. With an absence of producers and parasites, the economy will stay indefinitely so. The second equilibrium represents a point at which both producers and parasites maintain the size of their populations – in a positive number – and, in the absence of shocks, it will stay indefinitely so. As shown in the dynamic system (2.1) and (2.2), the level of each population depends exclusively on exogenous parameters.

After identifying the equilibria, we must see how deviations from these points caused by exogenous shocks affect the model’s dynamics. Proposition 2.2 deals with the stability of the origin.

Proposition 2.2. *The equilibrium $E_1 = \{x=0, y=0\}$ of the system (2.1) and (2.2) is a saddle point.*

Although in equilibrium this point is of little interest, because both groups are absent, it is important to study its stability. Note that if such equilibrium was stable, populations in positive numbers might be attracted to it, such that the dynamics of the model would cause extinction of both producers and parasites for many initial levels of a population. However, given that it is a saddle point, extinction is an uncommon result in our model. In fact, the only possibility for

³ Observe that the economically relevant state space of our model is \mathbb{R}_+^2 , different from one studied by Goodwin (1967), which is $\{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 1, 0 \leq y \leq 1\}$.

⁴ Because of the economic interpretation, we chose to call a solution of the system (2.1) and (2.2) when $\dot{x} = \dot{y} = 0$ an equilibrium, instead of a fixed point.

extinction would be if producers were artificially eradicated, such that parasites would not have anyone to extort and so would disappear as well.

Proposition 2.3. *The equilibrium $E_2 = \{x = \beta_1/\theta_1, y = \alpha_1/\gamma_1\}$ of the system (2.1) and (2.2) is not hyperbolic, that is, it is neither stable nor unstable.*

Given that the eigenvalues of the Jacobian evaluated in E_2 are purely imaginary (see Appendix A), one may not conclude anything through linear analysis. However, the proposition below provides an auxiliary result in understanding the dynamics in the equilibrium's neighborhood.

Proposition 2.4. *The system (2.1) and (2.2) moves in closed orbits. In addition, it admits a constant of motion $C = x^{\beta_1} y^{\alpha_1} e^{-\theta_1 x - \gamma_1 y}$.*

Proposition 2.4 states that the levels of producer and parasite populations oscillate cyclically around equilibrium $E_2 = (\beta_1/\theta_1, \alpha_1/\gamma_1)$. Intuitively, what the result states is that the cycle's length depends on the initial number of each group as well as the values of exogenous parameters.

Therefore, the cyclical dynamics of our model works in the following way: the number of parasites increases whenever there is a large number of producers to be extorted, but by extorting they encourage productive agents to quit their business and so the possibilities of extortion decrease, making their own population declines. Thus, parasite populations decrease, making the number of producers increase again. This dynamics results in a cycle of growth and decline of both groups.

2.3. The modified model: adding expectations and bounding the number of producers

The baseline model assumes that, in the absence of parasites, the producer population increases exponentially toward infinity at a constant rate. This means that output would grow infinitely. Furthermore, such an assumption implies that the entry rate of new producers in the economy is not affected by the number of agents already established. In other words, in this case it is necessary a demand which is increasing at the same rate, such that every new producer may sell his production. Thus, the profitability of productive sector is independent of the number of productive agents.

In order to overcome the above difficulty and add more reality to our model, we assume that producer populations grow according to a logistic function (saturation). Formally, we have now⁵

$$\dot{x} = \alpha_2 \left(1 - \frac{x}{K}\right) x - \gamma_2 xy.$$

It well known that the solution of the above equation, in the absence of parasites, is given by

$$x = \frac{Kx_0 e^{\alpha_2 t}}{K + x_0(e^{\alpha_2 t} - 1)},$$

where K is a positive constant and x_0 is the initial producer population.

We can easily verify that, in the absence of parasites, x is increasing (decreasing) over time if the initial producer population is lower (greater) than K . Furthermore, in both cases, the solutions tend to K as time goes to infinity. Observe that now the number of producers already established affects the attractiveness of the sector. If, for instance, in an economy without parasites, excessive initial number of producers exist ($K < x_0$), the variation in their population will be negative, indicating that there is excess of supply in the economy, what decreases sector's attractiveness.

The second modification adds lag in the relation between producers and parasites. An increase in the number of parasites does not have an instantaneous effect on producers' number and vice-versa. Indeed, there exists a lag between the increase in one population and the perception and response of another. For example, potential parasites recognize the increase in the number of producers – and thus a greater availability of targets for extortion – after some time, and only after that, they decide whether to entry into the sector. This lag may be caused by the fact that some productive

⁵ Definitions of parameters α_2 and γ_2 are the same as those of α_1 and γ_1 in the baseline model, respectively. We chose to change the subscripts in order to highlight the different population dynamics in the two models.

activities are not instantly measurable,⁶ for example. Specifically, we assume that the parasite sector has less available information – maybe due to its level of organization, given they are composed of illegal activities, in general – than productive one, such that the response of its population to changes in producer numbers is not instantaneous.

The lag above described may be modeled by adding in Eq. (2.2) a weight function that takes into account past movements in producer populations. Formally, we replace x in Eq. (2.2) by

$$z = \int_0^t x(\tau)G(t-\tau)d\tau \quad (2.3)$$

where $G(\cdot)$ is the weight function, which is non negative and integrable, with the property of $\int_{-\infty}^t G(t-\tau)d\tau = \int_0^\infty G(s)ds = 1$.

An alternative suggested by the literature (e.g. Farkas, 1984; Vadasz, 2007) is to study the system with the following weight function $G(s) = G_1(s) = \phi_2 e^{-\phi_2 s}$, $\phi_2 > 0$. This function assumes a discount $1/\phi_2$ in the parasites' response to changes in the number of producers. In addition, function G has the desirable feature of giving less weight to levels of producers' population which occurred in the distant past (large s), such that they have a lower effect on the parasites' population than recent ones. In the limit, as s goes to infinity, the effect is null.

By replacing G into (2.3), we have a new system:

$$\dot{x} = \alpha_2 \left(1 - \frac{x}{K}\right) x - \gamma_2 xy$$

$$\dot{y} = \theta_2 y \int_0^t x(\tau) \phi_2 e^{-\phi_2(t-\tau)} d\tau - \beta_2 y,$$

which is equivalent to (see Farkas, 1984)

$$\dot{x} = \alpha_2 \left(1 - \frac{x}{K}\right) x - \gamma_2 xy \quad (2.4)$$

$$\dot{y} = \theta_2 yz - \beta_2 y \quad (2.5)$$

$$\dot{z} = \phi_2(x - z). \quad (2.6)$$

The economic interpretation of our new system is simple and more realistic than the baseline model: parasites no longer react to current levels of producer populations; now they form expectations about future levels based on the past before acting. Still, their expectations changes and are continually updated, as expressed by (2.6) in the above system.

2.4. Stability and equilibrium of the modified system

Our modified model presents one additional equilibrium.

Proposition 2.5. *Consider the system (2.4)–(2.6). Then, we have the following equilibria, $E_3 = \{x=0, y=0, z=0\}$, $E_4 = \{x=K, y=0, z=K\}$ and $E_5 = \{x=\beta_2/\theta_2, y=\alpha_2/\gamma_2(1-(\beta_2/\theta_2 K)), z=\beta_2/\theta_2\}$.*

Regarding the economic interpretation, we have the trivial equilibrium $E_3 = (0, 0, 0)$, with absence of both producers and parasites; $E_4 = (K, 0, K)$, in which the economy achieves the maximal number of producers, K , and minimal (absence) of parasites; and $E_5 = [\beta_2/\theta_2, \alpha_2/\gamma_2(1-(\beta_2/\theta_2 K)), \beta_2/\theta_2]$, with both populations in a positive number, since that $\beta_2 < \theta_2 K$. Recall that variable z only has an auxiliary role, being strictly technical. Moreover, observe that values of equilibrium are independent of the intertemporal discount rate ϕ_2 .

The equilibrium corresponding to the origin continues to be unstable (see Appendix A), which ensures that, when one starts in an environment with both populations in positive numbers, there is no possibility of extinction.

Proposition 2.6. *The equilibrium $E_3 = \{x=0, y=0, z=0\}$ of the system (2.4)–(2.6) is a saddle point.*

⁶ Another possible cause is informational failures; something common in developing economies.

Now we can see that the other equilibria do not show cyclical behavior.

Proposition 2.7. *Consider the equilibrium $E_4 = \{x=K, y=0, z=K\}$ of the system (2.4)–(2.6). With respect to its stability we have: (i) if $\beta_2 > \theta_2 K$, then E_4 is asymptotically stable; and (ii) if $\beta_2 < \theta_2 K$, then E_4 is unstable.*

What is relevant to observe in Proposition 2.7 is that $\beta_2 > \theta_2 K$ is a sufficient condition to stability.⁷ In other words, if the state's power to combat parasitic activity is high enough relative to profit rate of the illegal sector, weighted by the number of producers, we will have stability of E_4 . This fact is explained by E_4 being an equilibrium in which producers are in maximal number and parasites do not exist, such that police power must be very strong – or the parasite's profit rate must be very low – in order for this situation to remain stable.

Equilibrium E_5 is of interest to us by presenting positive numbers for both populations.

Proposition 2.8. *Consider the equilibrium $E_5 = \{x = \beta_2/\theta_2, y = \alpha_2/\gamma_2(1 - (\beta_2/\theta_2 K)), z = \beta_2/\theta_2\}$ of the system (2.4)–(2.6). With respect to its stability we have: (i) if $\beta_2 < \theta_2 K$, then E_5 is asymptotically stable; and (ii) if $\beta_2 > \theta_2 K$, then E_5 is unstable.*

Now we have a positive number for both populations, such that enforcement power cannot be too strong, otherwise parasites would be extinct ($\beta_2 = \theta_2 K$, for example). In other words, the parasite's profit rate, weighted by the number of potential prey, must be high relative to enforcement power in order for parasites to remain in a positive number.

3. Policy implications and poverty traps

A direct conclusion from Section 2.2 (from Proposition 2.3, in particular) is that a developing economy, once in equilibrium, is not able to grow continuously. Such a situation may be considered a poverty trap.

Definition 3.1. We say that an equilibrium E_i is a poverty trap whenever it is characterized by low output and the economy by itself (endogenously) is not able to overcome it. Moreover, deviations from E_i cause a cyclical behavior around it.

Observe that we define a poverty trap slightly different than the literature (e.g. Azariadis and Stachurski, 2005): there is an upper bound that the economy's output must not surpass; and the output presents cyclical behavior. Therefore, the equilibrium $E_2 = (\beta_1/\theta_1, \alpha_1/\gamma_1)$ may be considered a poverty trap, depending on its parameter values.

It is well documented that developing countries present values for $\alpha_1, \beta_1, \gamma_1$ and θ_1 which often lead to a poverty trap. For example, they present weak property rights, reflected in high values of γ_1 and θ_1 . However, the main parameter to be affected by their bad institutions is β_1 , the rate of parasite extinction, because states' actions are ineffective and their police power is weak. Thus, β_1/θ_1 is low in these countries, such that E_2 may be characterized as a poverty trap.

A direct implication of Definition 3.1 is that the state may promote growth by improving its police power, β_1 , and through other actions that decrease the profit rate of parasite activity, θ_1 .

Corollary 3.2. *Consider a economy represented by the system (2.1) and (2.2). Assume that its equilibrium output is $(\beta_1/\theta_1) < M$, where M is the output level which defines a poverty trap for this economy. Furthermore, assume that the producers' profit rate and the rate of conversion of extortion into more parasites are fixed. Then, the only way to overcome the poverty trap is by institutional improvement, by increasing the protection of property rights and police power.*

Although in marginally different versions, Corollary 3.2 is well established in the literature. In fact, Acemoglu and Robinson (2010) and Azariadis and Stachurski (2005), among others, identify different types of traps.⁸ The above statement is rather strong by putting in the hands of the state all the responsibility for the development. However, our

⁷ We excluded from the analysis of Propositions 2.7 and 2.8 the case $\beta_2 = \theta_2 K$. One can use expressions (A.7) and (A.9) to conclude that, in this case, in both equilibria we would have eigenvalues with null real part, such that the Hartman–Grobman theorem does not apply.

⁸ Acemoglu and Robinson (2010) uses the term “pitfall”.

basic model captures a crucial aspect of growth, relegated in many development theories: the creation – or improvement – of incentives for truly productive activities and the effort to inhibit those activities which do not aggregate value – and, in our model, even disaggregate. The main contribution of the Lotka–Volterra model is to provide the perception that whenever an incentive is created, it may affect both producers and parasites.

Good institutions may have another important impact on the overcoming of poverty traps: they may make the economy move away the cyclical equilibria by destroying the closed orbits. Given that one of the equilibria of system (2.1) and (2.2) is not hyperbolic, the associated model presents structural instability. This instability means that very small perturbations in the vector field may lead to qualitatively different behaviors. Thus, since institutional improvement may cause such perturbations in the vector field of our system, one possibility is that, for γ_1 high enough, such an improvement causes parasite extinction.

In the modified model (2.4)–(2.6) there may not be poverty trap, because all equilibria are hyperbolic. In fact, Definition 3.1 does not exclude the possibility of existence of periodic orbits around equilibria of the modified system. For example, if we are able to find a compact region of the $x - y - z$ space, and a trajectory of the dynamical system such that it remains in this region for all $t > 0$, then Poincaré–Bendixson theorem applies,⁹ and this trajectory possibly is a closed orbit or approaches a closed orbit. Thus, for K and β_2/θ_2 low enough, equilibria E_4 and E_5 , respectively, may satisfy Definition 3.1 and be considered poverty traps. However, such a possibility depends on the parameters' values, and it may not be straightforward to find the aforementioned compact region.

Despite that difference, improvements in the institutional variables in the system (2.4)–(2.6) have a effect on the economic growth similar to one of the baseline model. Equilibrium $E_5 = (\beta_2/\theta_2, \alpha_2/\gamma_2(1 - (\beta_2/\theta_2)K), \beta_2/\theta_2)$, for example, presents higher output (number of productive agents) and lower number of parasites as β_2 increases. Still, an improvement in the police power (higher γ_2) decreases the number of parasites. However, there is a bound for the institutional improvement, given that $\beta_2 > \theta_2 K$ implies in negative number of parasites. Even if we considered this situation possible, the equilibrium would become unstable. In fact, as Propositions 2.7 and 2.8 show, these parameters affect stability of equilibria. For instance, although $E_4 = (K, 0, K)$ is not directly affected, an institutional improvement may make it stable for higher values of K .

We conclude with two observations. First, our results do not change when the profit of both activities are endogenous. In order to see this, assume that $\alpha(x)$ and $\theta(y)$ are functions of their respective populations. We can assume that profits decrease as activities become more competitive, such that. Then, a positive shock in x implies a decrease of $\alpha(x)$. It turns out that this causes two effects: a direct one, because the decrease in the profit rate makes some producers leave the sector; and an indirect effect, because with a larger number of potential targets, the number of parasites increases. This latter effect makes $\theta(y)$ decrease and the cyclical dynamics restarts.

Second, we implicitly assume that the state can choose the values of β and γ without any budget constraint. Obviously, improvements in police power cost money, such that we can conjecture how our findings would change with the inclusion of such constraints. In this case, the state would have two roles: on the one hand it would extract income (a share of the output) of the productive sector in the form of taxes; on the other hand, it would restrain parasitic activity by means of the police. Thus, the state would act both as “producer”, by promoting growth, and as a parasite, by taxing people in order to finance its actions. The dominant effect probably would depend on the specific economy in study.

Appendix A. Omitted proofs

Proof of Proposition 2.1. In equilibrium there is no change in both populations, such that $\dot{x} = \dot{y} = 0$. Then, the system (2.1) and (2.2) becomes

$$0 = x(\alpha_1 - \gamma_1 y)$$

$$0 = y(\theta_1 x - \beta_1),$$

which has solutions $\{x=0, y=0\}$ and $\{x=\beta_1/\theta_1, y=\alpha_1/\gamma_1\}$. \square

⁹ Although Poincaré–Bendixson theorem applies only to continuous dynamical systems on the plane, we can invoke it because our three dimensional system (2.4)–(2.6) is equivalent to one of two variables (Farkas, 1984).

Proof of Proposition 2.2. The Jacobian of the system (2.1) and (2.2) is

$$J_1(x, y) = \begin{bmatrix} \alpha_1 - \gamma_1 y & -\gamma_1 x \\ \theta_1 y & \theta_1 x - \beta_1 \end{bmatrix}.$$

Thus, we have

$$J_1(0, 0) = \begin{bmatrix} \alpha_1 & 0 \\ 0 & -\beta_1 \end{bmatrix},$$

which has eigenvalues $\lambda_1 = \alpha_1$ and $\lambda_2 = -\beta_1$. Given that $\alpha_1, \beta_1 > 0$, we have $(0, 0)$ is a saddle point. \square

Proof of Proposition 2.3. By evaluating J_1 in $(\beta_1/\theta_1, \alpha_1/\gamma_1)$ we have

$$J_1\left(\frac{\beta_1}{\theta_1}, \frac{\alpha_1}{\gamma_1}\right) = \begin{bmatrix} 0 & -\frac{\gamma_1 \beta_1}{\theta_1} \\ \frac{\theta_1 \alpha_1}{\gamma_1} & 0 \end{bmatrix},$$

whose eigenvalues are $\lambda = \pm i\sqrt{\alpha_1 \beta_1}$. Purely imaginary eigenvalues imply that the point is not hyperbolic. \square

Proof of Proposition 2.4. In order to show that the system (2.1) and (2.2) has closed orbits, we must find a function which is constant in any path and show that it is strictly monotone over the ray starting in an equilibrium point and increasing in the northeast direction of the plane producer-parasite. To understand the above reasoning, suppose we have a such function f and consider a path starting in a point \mathbf{a} of a ray, making a full turn around the equilibrium and achieving another point \mathbf{b} of the ray. Given that f is constant in the path, $f(\mathbf{a}) = f(\mathbf{b})$. However, f is monotone in the ray, such that $\mathbf{a} = \mathbf{b}$, which characterizes a closed orbit.

First, we divide both sides of (2.1) and (2.2) by x and y , respectively. Thus, we obtain

$$\frac{\dot{x}}{x} = \alpha_1 - \gamma_1 y \tag{A.1}$$

$$\frac{\dot{y}}{y} = \theta_1 x - \beta_1. \tag{A.2}$$

By dividing (A.1) by (A.2) we have

$$\frac{y}{x} \frac{\dot{x}}{\dot{y}} = \frac{\alpha_1 - \gamma_1 y}{\theta_1 x - \beta_1}.$$

After some algebraic operations, the resulting expression is

$$\frac{(\theta_1 x - \beta_1)}{x} \dot{x} = \frac{(\alpha_1 - \gamma_1 y)}{y} \dot{y}. \tag{A.3}$$

It is possible to note that (A.3) is equivalent to

$$\frac{d}{dt}(\theta_1 x - \beta_1 \ln x) = \frac{d}{dt}(\alpha_1 \ln y - \gamma_1 y). \tag{A.4}$$

Now, by integrating both sides of (A.4),

$$\begin{aligned} \int \frac{d}{dt}(\theta_1 x - \beta_1 \ln x) dt &= \int \frac{d}{dt}(\alpha_1 \ln y - \gamma_1 y) dt \\ A + \theta_1 x - \beta_1 \ln x &= \alpha_1 \ln y - \gamma_1 y, \end{aligned}$$

where A is the constant of integration.

Finally, the constant of motion can be obtained by taking the exponential of both sides and rearranging the terms

$$C = x^{\beta_1} y^{\alpha_1} e^{-(\theta_1 x + \gamma_1 y)}, \tag{A.5}$$

where $C = e^A$.

For the second part of the proof, consider a ray (x, y) starting in the equilibrium $(\beta_1/\theta_1, \alpha_1/\gamma_1)$ and moving in the northeast direction. We can write $x = (\beta_1/\theta_1)s$, $y = (\alpha_1/\gamma_1)s$, where s is a parameter which measures the distance from the equilibrium. Thus, if $s = 1$, $(x, y) = (\beta_1/\theta_1, \alpha_1/\gamma_1)$. By substituting it into (A.5) we have

$$\left(\frac{\beta_1}{\theta_1}\right)^{\beta_1} \left(\frac{\alpha_1}{\gamma_1}\right)^{\alpha_1} s^{\alpha_1 + \beta_1} e^{-s(\alpha_1 + \beta_1)} = C.$$

Now, by rearranging the terms and taking the $(\alpha_1 + \beta_1)$ th root of both sides,

$$se^{-s} = D, \quad (\text{A.6})$$

where $D = C^{1/(\alpha_1 + \beta_1)} (\theta_1/\beta_1)^{(\beta_1/\alpha_1 + \beta_1)} (\gamma_1/\alpha_1)^{(\alpha_1/\alpha_1 + \beta_1)}$.

If we show that the left-hand of (A.6) is strictly monotone for $s > 1$, then any $s > 1$ which satisfies (A.6) is unique. Thus, by taking the derivative of the left-hand we have

$$e^{-s} - se^{-s} = (1 - s)e^{-s},$$

which is strictly negative for $s > 1$. This completes the proof. \square

Proof of Proposition 2.5. Equilibrium requires $\dot{x} = \dot{y} = \dot{z} = 0$, such that the system (2.4)–(2.6) becomes

$$0 = x \left(\alpha_2 - \frac{\alpha_2 x}{K} - \gamma_2 y \right)$$

$$0 = y(\theta_2 z - \beta_2)$$

$$0 = \phi_1(x - z).$$

Now it is possible to see that the solutions are $\{x=0, y=0, z=0\}$, $\{x=K, y=0, z=K\}$ and $\{x=\beta_2/\theta_2, y=\alpha_2/\gamma_2(1 - (\beta_2/\theta_2 K)), z=\beta_2/\theta_2\}$. \square

Proof of Proposition 2.6. The Jacobian of the system (2.4)–(2.6) is given by

$$J_2(x, y, z) = \begin{bmatrix} \alpha_2 \left(1 - \frac{2x}{K}\right) - \gamma_2 y & -\gamma_2 x & 0 \\ 0 & \theta_2 z - \beta_2 & \theta_2 y \\ \phi_2 & 0 & -\phi_2 \end{bmatrix}.$$

By evaluating J_2 in $(0, 0, 0)$ we have

$$J_2(0, 0, 0) = \begin{bmatrix} \alpha_2 & 0 & 0 \\ 0 & -\beta_2 & 0 \\ \phi_2 & 0 & -\phi_2 \end{bmatrix},$$

which provides directly its eigenvalues: $\lambda_1 = \alpha_2$, $\lambda_2 = -\beta_2$ and $\lambda_3 = -\phi_2$. Because $\alpha_2, \beta_2, \phi_2 > 0$, $(0, 0, 0)$ is a saddle point. \square

Theorem A.1 (Descartes' rule of signs). *The number of positive roots in a polynomial equation $p(x)$ with real coefficients is never higher than the number of sign changes T in the sequence of its non-zero coefficients, and if it is lower, then always it is an even number.*

Proof of Proposition 2.7. The Jacobian of the system (2.4)–(2.6) evaluated in the equilibrium $(K, 0, K)$ is

$$J_2(K, 0, K) = \begin{bmatrix} -\alpha_2 & -\gamma_2 K & 0 \\ 0 & \theta_2 K - \beta_2 & 0 \\ \phi_2 & 0 & -\phi_2 \end{bmatrix}.$$

The characteristic polynomial of $J_2(K, 0, K)$ is then

$$-\lambda^3 - \lambda^2(\alpha_2 + \phi_2 + \beta_2 - \theta_2 K) - \lambda[\alpha_2 \phi_2 + (\alpha_2 + \phi_2)(\beta_2 - \theta_2 K)] + \alpha_2 \phi_2(\theta_2 K - \beta_2). \quad (\text{A.7})$$

For the item (i), we can apply [Theorem A.1](#) in order to show that $\beta_2 > \theta_2 K$ is a sufficient condition for (A.7) to have only negative roots. For, consider the following notation for (A.7): $a_0 \lambda^3 + a_1 \lambda^2 + a_2 \lambda + a_3$. Note that if $a_i < 0$ for all i , then (A.7) does not change sign – presents the following sequence $(-, -, -, -)$ –, and, by [Theorem A.1](#), it has no positive roots.

As $a_0 = -1 < 0$, we must show that $\beta_2 > \theta_2 K$ implies $a_1, a_2, a_3 < 0$. Given that $\alpha_2 + \phi_2 > 0$, for $a_1 = -(\alpha_2 + \phi_2 + \beta_2 - \theta_2 K) < 0$ it suffices that $\beta_2 > \theta_2 K$. For $a_2 = -[\alpha_2 \phi_2 + (\alpha_2 + \phi_2)(\beta_2 - \theta_2 K)] < 0$, we also have $\alpha_2 \phi_2 > 0$, such that $\beta_2 > \theta_2 K$ is sufficient. Finally, for $a_3 = \alpha_2 \phi_2 (\theta_2 K - \beta_2) < 0$, we once again require that $\theta_2 K < \beta_2$. Thus, $\beta_2 > \theta_2 K$ is a sufficient condition for the existence of only positive real roots in (A.7).

We still have to obtain conditions for (A.7) to have only negative real roots. Observe that by replacing λ by $(-\lambda)$ in (A.7) we have

$$\lambda^3 - \lambda^2(\alpha_2 + \phi_2 + \beta_2 - \theta_2 K) + \lambda[\alpha_2 \phi_2 + (\alpha_2 + \phi_2)(\beta_2 - \theta_2 K)] + \alpha_2 \phi_2 (\theta_2 K - \beta_2). \quad (\text{A.8})$$

By assuming $\beta_2 > \theta_2 K$, we have that (A.8) changes signs three times – presents the sequence $(-, +, -, +)$. Therefore, [Theorem A.1](#) implies that (A.7) has three, two or no negative real root. As two or none are impossible options – complex roots occur in conjugate pairs –, we have that (A.7) only has negative real roots. Thus, we conclude that $\beta_2 > \theta_2 K$ is a sufficient condition for the stability of the system (2.4)–(2.6).

To prove (ii), notice that, if $\beta_2 < \theta_2 K$, then $a_3 = \alpha_2 \phi_2 (\theta_2 K - \beta_2) > 0$ in both (A.7) and (A.8). As $a_0 > 0$ in (A.8), that polynomial may change signs either twice [either $(+, +, -, +)$ or $(+, -, +, +)$] or not at all $[(+, +, +, +)]$. By applying [Theorem A.1](#), we have that, in the first possibility (A.7) has two or no negative real roots and; in the second possibility, no negative real root. In any case, it is impossible to have three negative real roots or one negative real and two complex with negative real part, cases that would ensure the stability. This proves (ii). \square

Proof of Proposition 2.8. The Jacobian of (2.4)–(2.6) evaluated in E_5 is

$$J_2 \left(\frac{\beta_2}{\theta_2}, \frac{\alpha_2}{\gamma_2} \left(1 - \frac{\beta_2}{\theta_2 K} \right), \frac{\beta_2}{\theta_2} \right) = \begin{bmatrix} -\frac{\alpha_2 \beta_2}{\theta_2 K} & -\frac{\gamma_2 \beta_2}{\theta_2} & 0 \\ 0 & 0 & \frac{\theta_2 \alpha_2}{\gamma_2} \left(1 - \frac{\beta_2}{\theta_2 K} \right) \\ \phi_2 & 0 & -\phi_2 \end{bmatrix}.$$

By calculating the determinant of $J_2(\beta_2/\theta_2, \alpha_2/\gamma_2(1 - (\beta_2/\theta_2 K)), \beta_2/\theta_2) - \lambda \mathbf{I}$ we have

$$-\lambda^3 - \lambda^2 \left(\phi_2 + \frac{\alpha_2 \beta_2}{\theta_2 K} \right) - \lambda \left(\frac{\alpha_2 \beta_2 \phi_2}{\theta_2 K} \right) + \alpha_2 \beta_2 \phi_2 \left(\frac{\beta_2}{\theta_2 K} - 1 \right). \quad (\text{A.9})$$

To prove (i) and (ii) we use the notation $a_0 \lambda^3 + a_1 \lambda^2 + a_2 \lambda + a_3$ for (A.9). Firstly, consider that $\beta_2 < \theta_2 K$. Then, $a_3 = \alpha_2 \beta_2 \phi_2 ((\beta_2/\theta_2 K) - 1) < 0$ and we have no sign change in (A.9), because $a_0 = -1 < 0$, $a_1 = -(\phi_2 + (\alpha_2 \beta_2/\theta_2 K)) < 0$ and $a_2 = -(\alpha_2 \beta_2 \phi_2/\theta_2 K) < 0$. By [Theorem A.1](#), therefore, (A.9) has no positive real root.

By substituting λ by $(-\lambda)$ into (A.9) we have

$$\lambda^3 - \lambda^2 \left(\phi_2 + \frac{\alpha_2 \beta_2}{\theta_2 K} \right) + \lambda \left(\frac{\alpha_2 \beta_2 \phi_2}{\theta_2 K} \right) + \alpha_2 \beta_2 \phi_2 \left(\frac{\beta_2}{\theta_2 K} - 1 \right), \quad (\text{A.10})$$

which presents three sign changes $[(+, -, +, -)]$, which means that (A.9) has three, two or no negative real root, where we use [Theorem A.1](#). Because complex roots occur in pairs, the only possibility is one of three negative real roots. This proves stability.

Now consider that $\beta_2 > \theta_2 K$. This makes $a_3 = \alpha_2 \beta_2 \phi_2 ((\beta_2/\theta_2 K) - 1) > 0$ in both (A.9) and (A.10). Observe that in (A.10) we have two sign changes $[(+, -, +, +)]$, which makes (A.9) have two or no negative real root. In both cases, one root is positive and real, such that the system (2.4)–(2.6) is unstable. This proves (ii). \square

Proof of Corollary 3.2. It follows directly from [Proposition 2.4](#) and [Definition 3.1](#). \square

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