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Article

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A neo-Kaleckian model of capital accumulation, income distribution and financial fragility^{☆,☆☆}

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Abstract

This paper develops a neo-Kaleckian dynamical model that investigates how an increased financial instability affects the investment rate and the wage share of income in the long run. It is shown that a rising benchmark interest rate affects negatively the capital accumulation and the wage share of income. The main argument is developed in two-steps. First, it is build a two-dimensional model to analyse the stability conditions of the dynamical interaction between wage share and capital accumulation, given a constant debt-capital ratio. Second, by allowing endogenous variations of the debt of firms as a proportion of their capital stock, the extended model explores the stability conditions of the steady-state equilibrium solution in a three-dimensional dynamic system. In doing so, this paper contributes to the literature by setting the conditions in which the debt-capital ratio, the income distribution and the process of capital accumulation can be simultaneously stable in the long run.

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JEL classification: C62; D33; E12; E22; E44

Keywords: Capital accumulation; Increasing risk; Income distribution; Financial instability

Resumo

Este trabalho desenvolve um modelo dinâmico neo-kaleckiana que investiga como um aumento da instabilidade financeira afeta a taxa de investimento e da participação dos salários de renda no longo prazo. Mostra-se que uma elevação na taxa básica de juros afeta negativamente a acumulação de capital e a participação dos salários de renda. O principal argumento é desenvolvido em duas etapas. Primeiro, desenvolvemos um modelo bidimensional para analisar as condições de estabilidade da interação dinâmica entre a participação dos salários e acumulação de capital, dada a razão dívida-capital constante. Em seguida, ao permitir variações endógenas da dívida das empresas como proporção do seu estoque de capital, o modelo estendido explora as condições de estabilidade da solução de equilíbrio em um sistema dinâmico tridimensional. Ao fazê-lo, este trabalho contribui para a literatura, definindo as

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condições em que a razão dívida-capital, a distribuição de renda e o processo de acumulação de capital possam ser simultaneamente estáveis no longo prazo.

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Palavras chave: Acumulação de capital; Risco crescente; Distribuição de renda; Instabilidade financeira

1. Introduction

The issue of long-run stability of capitalist economies has always been in the centre of the debate in the economic growth literature. Roy F. Harrod in 1939 and Evsey Domar in 1946 pioneered this discussion by developing a model where deviations from the warrant growth rate, which is the growth rate compatible with the equilibrium between saving and planned investments, make the actual growth rate to continuously veer off from that growth rate. Neoclassical economists, on the other hand, depart from the Harrod-Domar model by claiming shortcomings with respect to the instability of its solution. In this vein, Robert M. Solow in 1956 sets forth a formal model in which full employment prevails and the long-run economic growth is a stable process wherein the actual growth rate equals the natural growth rate of the economy determined by the growth of the labour force plus the growth of labour productivity. Through formal dynamic models of growth and distribution, early post-Keynesian economists like Nicholas Kaldor and Joan Robinson in the 1950s and 1960s proposed theoretical frameworks in which capacity utilisation is constant in the long run and economic growth is stable and inversely related to wages. Conversely, a newer post-Keynesian growth literature along Kaleckian lines advanced mainly by Robert Rowthorn and Amitava K. Dutt in the 1980s considers the capacity utilisation adjusts in the long run to restore the equilibrium in the goods market, which enables a positive relationship between growth and real wages.

More recently, Dutt (1994) reexamines the question of long-run stability of modern economies through a dynamic model of capital accumulation and income distribution. This model gives two long-run equilibrium solutions. The first equilibrium, which is a saddlepoint, takes place when the economy is operating with excess capacity; given the instability of this equilibrium solution, it is possible that the economy will move over time with increasing capital accumulation and wage share until it reaches full capacity utilisation. The second long-run equilibrium solution, which is usually stable, is obtained when the economy is operating at full capacity utilisation. By incorporating technological change into the model through the ‘learning-by-doing’ hypothesis, his model allows for oscillations around the long-run equilibrium, with the economy alternating between periods of full and excess capacity. Lima (2004) extends Dutt’s (1994) model by defining technological innovation as a non-linear function of distributive shares (wages and profits), with the latter determining both the incentive to innovate and the availability of funding needed to undertake it. Such an extension allows for the existence of a stable solution in an economy with excess capacity.

However, the theoretical frameworks developed by Dutt (1994) and Lima (2004) do not take into account the perceptually salient aspect of modern economies which is the impact of financial dynamics on accumulation and distribution. The diagrams below illustrate how an increasing firms’ indebtedness might affect negatively both accumulation and wage share. Fig. 1 shows the total debt-equity ratio of non-financial corporations and the annual rate of growth of gross fixed capital formation of a selected set of countries for 2012.¹ It is observed that, by and large, the higher the level of indebtedness of non-financial corporations, the lower the growth rate of investment. In other words, the data suggests that greater financial weakness of non-financial companies, as measured by its level of indebtedness, is accompanied by a lower growth rate of capital accumulation, particularly when the debt level exceeds 100%.

Simultaneously, as shown in Fig. 2, in countries where the debt-capital ratio of non-financial corporations is higher, the wage share of income is relatively low. In this context, it is suggested that greater financial fragility reduces the power of workers in the collective bargaining process and, consequently, the labour share of GDP.

Although these data do not prove the existence of a direct causal relationship between debt-equity ratio, growth of investment, and the wage share of GDP, it can be said that both diagrams above show some stylised facts of modern economies in a recent period consistent with the results of the model proposed in this work. In this context,

¹ The sample of countries and the period was defined by the availability of data.

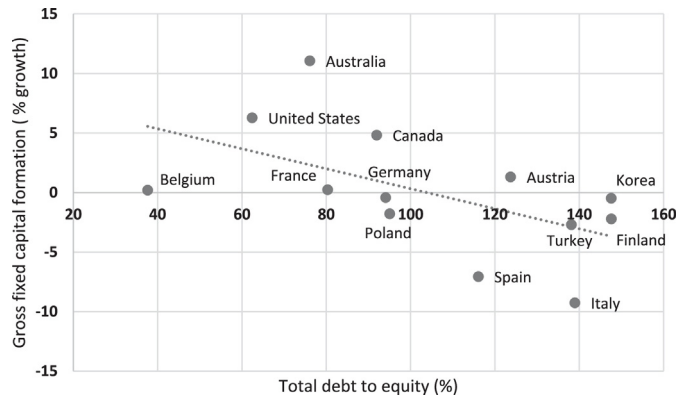


Fig. 1. Non-financial corporation debt-to-equity ratio and average annual growth of gross fixed capital formation by selected country (2012). Source: IMF and World Bank. Authors' own elaboration.

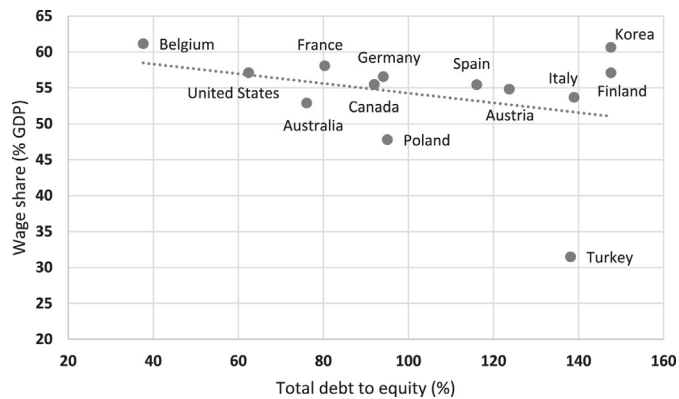


Fig. 2. Wage share and non-financial corporation debt-to-equity ratio by selected country (2012). Source: Ameco and IMF. Authors' own elaboration.

these phenomena indicate the relevance of taking into account the financial structure of companies in the dynamical interaction between accumulation and income distribution.

This paper contributes to the literature by developing a neo-Kaleckian dynamical model of capital accumulation, functional income distribution (henceforth simply income distribution) and financial fragility for a closed economy with no government activity. There are two classes in the economy, workers and capitalists. The later, in turn, is constituted by entrepreneurs and rentiers. Like in Lima (2004), production is carried out by oligopolistic firms that operate with excess capacity, which means that capacity utilisation adjusts to maintain the equilibrium in the goods market. Investment decisions depend on the expected profit rate, and the propensity of firms to save. It is also assumed that capital goods can be financed either by retained profits or external funds from commercial banks. Commercial banks calculate the nominal market interest rate by setting a mark-up over the base nominal interest rate, which is determined by the monetary authority. It is assumed that Central Bank's supply of money accommodates any increase in firms' demand for loans. From the commercial bank perspective, the higher the firms debt as a proportion of their stock of capital, the higher the default risk, which leads to a higher risk and liquidity premia, and a higher market interest rate, thus discouraging firms to financing new capital goods.

The model examines the impact of changes in the base interest rate on the investment rate and income distribution. Changes in the benchmark interest rate affect negatively the expected profitability of current investments and the actual value of firms' collateral, thus reducing the investment rate. A rising debt service also reduces the wage share of income due to its negative effect on the employment rate and firms' retained profits, thus undermining the bargaining power of workers in the wage decision-making process. That said, we investigate the impact of a financially unstable economy on capital accumulation and income distribution in the long run. Provided that, we explore the stability conditions of the steady-state equilibrium solution of the three-dimensional dynamical system formed by the capital-effective labour

supply ratio, the wage share, and the debt-capital ratio. This paper contributes to the literature by setting the conditions in which the debt-capital ratio, the income distribution and the process of capital accumulation can be simultaneously stable.

The remainder of this paper is organised in the following manner. Section 2 lays out the foundations of the model. In Section 3 we present the behaviour of the model in the short run, whereas in Section 4 we explore the dynamical relationship between capital accumulation, income inequality and financial instability in the long run. The paper ends with a summary of the main findings of the model.

2. The structure of the model

We assume a closed economy that produces only one good for both consumption and investment. We also assume a fixed-coefficient production function where homogeneous labour (L) and capital (K) are used as factors of production. Since the capital-potential output ratio is assumed to be constant and normalised to unity, the capacity utilisation (u) is given by the output-capital ratio. Given that domestic firms are assumed to be operating below full capacity, the rate of capacity utilisation must adjust to accommodate excess demand or supply.

2.1. Classes

There are two classes in the economy, workers and capitalists. Workers earn only wages (W) and consume all their income. Capitalists are divided into entrepreneurs and rentiers. Entrepreneurs earn profits of their firms and save a constant fraction of their income. Rentiers earn income from the stock of credit granted to firms at a given market interest rate (i) and also save a constant fraction of their income. Total income is given by

$$Y = \left(\frac{W}{P}\right)L + \Pi \quad (1)$$

$$u = \sigma u + r \quad (2)$$

$$\sigma = \left(\frac{W}{Pa}\right) \quad (3)$$

$$r = \frac{\Pi}{K} = (1 - \sigma)u \quad (4)$$

where P is the price level, σ is the wage share of income, Π is total profit, r is the profit rate and $a = Y/L$ is the labour productivity. Following Hein (2008), we split total profits (Π) into profits of firms (Π_c) and income of rentiers (Π_f)

$$\Pi = \Pi_c + \Pi_f = \Pi_c + iB \quad (5)$$

where B is the stock of credit granted to firms.

The debt-capital ratio is

$$\lambda = \frac{B}{K} \quad (6)$$

2.2. Interest rate and the endogenous risk premium

It is assumed that commercial banks calculate the nominal market interest rate (i) by marking up the base nominal interest rate set by the Central Bank (i_{CB}). Here it is assumed that any increase in the demand for loans and, consequently, deposits with commercial banks is fully accommodated by Central Bank's supply of monetary reserves. Ergo, i_{CB} is assumed to be exogenous.

$$i = hi_{CB} \quad (7)$$

where $h > 1$ is the mark-up set by commercial banks. This mark-up is a risk and liquidity premia set by commercial banks and depends on the debt-capital ratio, as follows

$$h = \mu\lambda \quad 0 < \lambda < \lambda_{\max} \quad (8)$$

where $\mu > 0$ is a constant of proportionality and λ_{max} is the maximum amount of credit as a proportion of the capital stock that lenders are willing to grant to firms. For simplicity, let us set μ equal to unity henceforth ($\mu = 1$). From commercial banks' standpoint, the higher the firms debt as a proportion of their stock of capital, the higher the default risk perceived by the banks, and hence the higher the risk and liquidity premia lenders will require in order to financing new investments. However, one should expect that commercial banks will not be willing to always supply credit regardless of the riskiness of the borrower's project. Thus, it is assumed that when the debt-capital ratio attains its maximum value λ_{max} the supply of credit is drastically interrupted, as lenders consider that firms will certainly fail to fulfill their obligation for any value of λ greater than λ_{max} .

Therefore, from the lenders' perspective, the increasing risk following a raising debt-capital ratio can hinder the supply of loans granted to firms and hence hamper the process of capital accumulation. This mechanism is inspired by [Kalecki's \(1937\)](#) principle of increasing risk. According to the author, as investment grows, this happens because: (i) the greater is the investment the more is the wealth position of the entrepreneur endangered in the event of unsuccessful business; ii) the greater is the investment the more is the danger of illiquidity. Even though, in this case, [Kalecki's \(1937\)](#) principle refers to the risk perception from the firms' standpoint, we suggest herein that it also can be employed to examine how lenders perceive risk. As Kalecki himself pointed out, “[i]f, however, the entrepreneur is not cautious in his investment activity it is the creditor who imposes on his calculation the burden of increasing risk charging the successive portions of credits above a certain amount with rising rate of interest” ([Kalecki, 1937](#), p. 442). Moreover, it is worth mentioning that in this work the principle of increasing risk is not used only with respect to the capital stock, as in [Kalecki \(1937\)](#). Instead, this principle is associated herein with the share of capital goods financed by loans in the total capital stock. Thus, a more accurate relationship can be established between capital accumulation and borrowers' solvency, as perceived by lenders. Therefore, by assuming that the risk and liquidity premia is positively related to changes in the amount of credit granted to firms, we fall in line with the Post-Keynesian structuralist endogenous money theory ([Minsky, 1975](#); [Palley, 1996](#)).

2.3. Capital accumulation

Now we assume that the rate of capital accumulation desired by firms depends on the expected rate of retained profit on investment. In this sense, following [Kalecki \(1971\)](#) and [Dutt \(1984\)](#), we argue that the higher the expected profit rate, the higher the propensity of firms to invest. If the present is usually assumed to be a “serviceable guide to the future” ([Keynes, 1937](#), p. 214), for convenience, we can state that profits expected by firms are equal to current profits. Thus, the linear function of the capital accumulation plans is given by

$$g^d = \frac{\Delta K}{K} = \frac{I}{K} = g_0 + \alpha_1 u + \alpha_2 s_c (r - i\lambda) \quad (9)$$

where g_0 is the autonomous investment rate, $\alpha_1 > 0$ is the sensitiveness of desired investment to capacity utilisation, $\alpha_2 > 0$ accounts for the responsiveness of the desired investment rate to retained profits of firms, and s_c is entrepreneurs' propensity to save out of retained profits.

Assuming a bank-based financial system, there are two main forms in which capital goods can be financed: internal funds consisting of retained profits and external funds/debts.

2.4. Aggregate saving

If domestic firms operate below full capacity the equality between investments and savings is brought about by the adjustment of the capacity utilisation. In this scenario, the aggregate saving normalised by the stock of capital is given by

$$g = \frac{S}{K} = \frac{S_c + S_f}{K} \quad (10)$$

where S is total savings, S_c is entrepreneurs' saving out of retained profits, and S_f is the financial capitalists' saving out of the income received from the stock of credit granted to firms at a given market interest rate.

From Eqs. (4)–(6), we obtain

$$\frac{S_c}{K} = s_c \frac{\Pi_c}{K} = s_c \frac{\Pi - \Pi_f}{K} = s_c [r - i\lambda] \quad (11)$$

Rentiers also save a constant fraction, s_f , out of the income they receive from the loans

$$\frac{S_f}{K} = s_f \frac{\Pi_f}{K} = s_f i\lambda \quad (12)$$

Substituting from (12) and (11) into (10), we have

$$g = s_c [r - i\lambda] + s_f i\lambda = s_c r + (s_f - s_c) i\lambda \quad (13)$$

Assuming, for simplicity, $s_f = s_c = s$, we obtain

$$g = sr \quad (14)$$

Eq. (14) is the so-called Cambridge equation.

3. The model in the short run

In the short run the stock of capital, K , the wage share, σ , the debt-capital ratio, λ , and the market interest rate, i , are constant. When there is excess capacity, investment and saving are equalised by changes in the capacity utilisation. Thus, in the goods market equilibrium we have $g^d = g$. Substituting from (4), (7) and (8) into (9), also from (4) into (14), and then the resulting expressions into the equilibrium condition $g^d = g$, and solving for u , we have the equilibrium capacity utilisation

$$u^* = \frac{g_0 - \alpha_2 s i_{CB} \lambda^2}{s(1 - \alpha_2)(1 - \sigma) - \alpha_1} \quad (15)$$

Since $\sigma \in (0, 1)$, the stability condition of the equilibrium capacity utilisation is $s(1 - \alpha_2)(1 - \sigma) - \alpha_1 > 0$. Given that $u \in (0, 1)$, we must also assume $g_0 > \alpha_2 s i_{CB} \lambda^2$.

Other things held constant, from Eq. (15), the partial effect of a change in the base interest rate, in the debt-capital ratio and in the wage share on the equilibrium capacity utilisation is

$$u_{i_{CB}}^* = \frac{\partial u^*}{\partial i_{CB}} = -\frac{\alpha_2 s \lambda^2}{s(1 - \alpha_2)(1 - \sigma) - \alpha_1} < 0 \quad (16a)$$

$$u_{\lambda}^* = \frac{\partial u^*}{\partial \lambda} = -\frac{2\alpha_2 s i_{CB} \lambda}{s(1 - \alpha_2)(1 - \sigma) - \alpha_1} < 0 \quad \forall \lambda > 0 \quad (16b)$$

$$u_{\sigma}^* = \frac{\partial u^*}{\partial \sigma} = \frac{s(1 - \alpha_2)(g_0 - \alpha_2 s i_{CB} \lambda^2)}{[s(1 - \alpha_2)(1 - \sigma) - \alpha_1]^2} > 0 \quad (16c)$$

The impact of a raising base interest rate or debt-capital ratio on the equilibrium capacity utilisation, Eqs. (16a) and (16b) respectively, is negative and unambiguously signed. It means that an increase in the debt-service from firms to rentiers, due to either raising interest or the stock of debt relative to the total physical capital, reduces capacity utilisation and consequently the retained profits of firms. The partial differential (16c), in turn, shows that an increased wage share affects positively the equilibrium capacity utilisation u^* .

It follows from Eqs. (4) and (16a)–(16c) that the partial effects of a rising base interest rate, debt-capital ratio or wage share in income on the equilibrium profit rate is $r_{i_{CB}}^* < 0$, $r_{\lambda}^* < 0$ and $r_{\sigma}^* = s[(1 - \sigma)u_{\sigma}^* - u]$. In the neo-Kaleckian model it is assumed $r_{\sigma}^* > 0$, that is, an increasing wage share raises the equilibrium profit rate.

The short-run equilibrium rate of capital accumulation, g^* , can be obtained by substituting Eqs. (15) and (4) into either (9) or (14)

$$g^* = s(1 - \sigma)u^* = \frac{s(1 - \sigma)(g_0 - \alpha_2 s i_{CB} \lambda^2)}{s(1 - \alpha_2)(1 - \sigma) - \alpha_1} \quad (17)$$

From Eq. (17) we also obtain $g_{i_{CB}}^* < 0$, $g_{\lambda}^* < 0$ and $g_{\sigma}^* > 0$.

4. The long-run analysis

In this section we explore the interrelatedness of the debt-capital ratio trajectory, capital accumulation, and income distribution in a process of dynamical feedback. It is shown that an increased financial instability in the economy can lead to severe consequences in terms of capital accumulation and income distribution.

In this regard, first we model the dynamical interaction between capital accumulation and income distribution in order to assess the impact of a rising interest rate on the stability condition of the system when the debt-capital ratio is assumed to be constant. Then, we relax this hypothesis and allow for debt-capital ratio variation following changes in the interest rate in order to demonstrate how the inclusion of the financial dimension into the model increases the potential instability of the economy.

Further, in the long run it is assumed that the short-run equilibrium values of the variables always hold. In Section 4.1 the long-run dynamics of the economy will be examined through the behaviour of two short-run variables over time, namely, the wage share in income σ and the capital-effective labour supply ratio, i.e. $k = K/(Na)$, where N is the labour supply of the economy. In subsection 4.2 the financial dimension will be added to the analysis.

4.1. The two-dimensional system

In this subsection we extend the models developed by Dutt (1994) and Lima (2004) by allowing for the impact of debt service on retained profits and wage share. First, we analyse the stability conditions of the dynamical interaction between wage share and capital accumulation, given a constant debt-capital ratio λ . In the next subsection we relax this hypothesis.

To begin with, we built upon Dutt (1994) and assume that the time derivative of the wage share of income is a positive function of the gap between the wage share desired by workers (σ^d) and the current wage share. Thus, the adjustment process of the wage share is explained by the dynamical equation below

$$\dot{\sigma} = \beta(\sigma^d - \sigma) \quad (18)$$

where $\beta > 0$ is the adjustment parameter. The wage share desired by workers depends on the bargaining power of workers which, in turn, is positively related to the rate of employment. That is, a tighter labour market increases the bargaining power of workers. Conversely, an increase in the debt service $i\lambda$, that is, the portion of financial capitalists in profit income, imposes a greater restriction on the demand of workers for higher wages in the collective bargaining process. Thus, the wages share desired by workers is given by

$$\sigma^d = \omega_0 + \omega_1 e - \omega_2 i\lambda \quad (19)$$

where $\omega_0, \omega_1, \omega_2 > 0$ are constants, and $e = L/N$ is the employment rate. The employment rate can be rewritten as follows

$$e = uk \quad (20)$$

Next, we define the ratio of capital stock to labour supply in productivity units k in time-rates of change as follows

$$\dot{k} = k(g - n - \hat{a}) \quad (21)$$

where n is the exogenous growth rate of labour supply, and \hat{a} is the rate of change of labour productivity a .

It is assumed here that technological innovation depends on the current income distribution and the debt service in the following way

$$\hat{a} = \xi_0 - \xi_1 \sigma - \xi_2 i\lambda \quad (22)$$

Lima (2004) defines innovation as a concave-down quadratic function of prevailing wage share. The author claims that at high levels of profit share the availability of funding for innovation is high but the incentives to innovate are low, whereas at low levels of profit share firms are encouraged to innovate but the availability of funding is low. In our model, for simplicity, we assume that the wage share is sufficiently high, so that increases in the wage share reduce retained profit of firms and hence hampers their ability to innovate. In other words, our model assumes a linear function in which the wage share is inversely related to technical progress. We also assume that increased debt services curb innovation by reducing entrepreneurial profit.

Plugging Eqs. (7), (8) and (20) into (19), and then the resulting equation into (18) we obtain the state transition function of σ ; also substituting Eqs. (14) and (22) into (21), we obtain the two-dimensional system formed by the dynamical interaction between σ and k , as follows

$$\dot{\sigma} = \beta(\omega_0 + \omega_1 uk - \omega_2 i_{CB} \lambda^2 - \sigma) \quad (23a)$$

$$\dot{k} = k(sr - n - \xi_0 + \xi_1 \sigma + \xi_2 i_{CB} \lambda^2) \quad (23b)$$

Assuming for convenience that the system has only one non-homogeneous solution in the relevant domain and taking the linear approximation around the equilibrium point, we find the Jacobian matrix, J , of the dynamical system constituted by Eqs. (21) and (22)

$$J = \begin{bmatrix} -\beta(1 - \omega_1 k u_\sigma) & \beta \omega_1 u \\ k(\xi_1 + sr_\sigma) & sr - n + \xi_0 - \xi_1 \sigma - \xi_2 i_{CB} \lambda^2 \end{bmatrix} \quad (24)$$

The Jacobian matrix in (24) shows that the impact of a change in the wage share on its own growth is negative, $\partial \dot{\sigma} / \partial \sigma = -\beta(1 - \omega_1 k u_\sigma) < 0$ and that the impact of changes in the capital-effective labour ratio on the wage share time variation is unambiguously signed, $\partial \dot{\sigma} / \partial k = \beta \omega_1 u > 0$. The matrix (24) also shows that the partial effect of changes in the wage share on the time variation of the capital-effective labour is ambiguous, $\partial \dot{k} / \partial \sigma = k(sr_\sigma + \xi_1) \geq 0$ and that the partial effect of changes in the capital-effective labour on its own time variation also can go either way $\partial \dot{k} / \partial k = sr - n - \xi_0 + \xi_1 \sigma + \xi_2 i_{CB} \lambda^2 \geq 0$.

Assuming, for convenience, that in equilibrium, i.e. $\dot{\sigma} = 0$ and $\dot{k} = 0$, the system has only one non-homogeneous solution in the relevant domain, the Jacobian matrix (23) can be rewritten as

$$J = \begin{bmatrix} -\beta(1 - \omega_1 k^* u_\sigma^*) & \beta \omega_1 u^* \\ k^*(\xi_1 + sr_\sigma^*) & 0 \end{bmatrix} \quad (25)$$

That said, the stability conditions of this dynamical system is given by $-\beta(1 - \omega_1 k^* u_\sigma^*) < 0$ and $k^*(\xi_1 + sr_\sigma^*) \beta \omega_1 u^* > 0$. Since the first inequality is negative, given that $\omega_1 k^* u_\sigma^* < 1$, the stability of the system depends exclusively on the sign of the second inequality. If $\xi_1 + sr_\sigma^* > 0$, then the system is stable. That is, if technological progress is more responsive than capital accumulation to wage share variations, then the system is stable.

It means that, in an unstable economy, that is $\xi_1 + sr_\sigma^* < 0$, the monetary authority can help to stabilise the system by raising the base interest rate i_{CB} until the absolute value of r_σ^* is sufficiently reduced. However, by claiming that a sufficiently high base interest rate stabilises the dynamical feedback between capital accumulation and income distribution does not sit comfortably with some of the most salient features of modern economies. In fact, growing financial weakness usually leads to ever-decreasing investments and greater income inequality.

By assuming that the amount of credit granted to firms relative to the capital stock λ remains constant, the two-dimensional models neglects the fact that changes in the price of loans (the interest rate) affect the quantity of loans. In the next subsection the hypothesis of a constant λ is relaxed, thus implying that a rising interest payments changes the debt structure of firms and hence affects the dynamic behaviour of capital accumulation and income distribution over time.

4.2. The three-dimensional system

In order to model the dynamics of the debt-capital ratio over time, we take the differential of λ with respect to time. From Eq. (6), we have

$$\dot{\lambda} = \lambda \left(\frac{\dot{B}}{B} - g \right) \quad (26)$$

In the long run, the additional amount of credit granted to firms is equal to rentier's saving (Hein, 2008). Thus, we have

$$\dot{B} = S_f = s_f \Pi_f = s_f i B \Rightarrow \frac{\dot{B}}{B} = s_f i = si \quad (27)$$

However, it is noteworthy that Eq. (27) does not imply that rentiers’ saving causes the expansion of credit and hence investment. Total saving of the economy, which, in our model, is formed by rentiers’ saving and firms’ retained profits, is created as a result of the process of capital accumulation being initially financed through short-run credit granted to firms by commercial banks.

By substituting from (7), (8), (14) and (27) into (26), we have

$$\dot{\lambda} = \lambda(si_{CB}\lambda - sr) \tag{28}$$

That said, the dynamical system formed by the differential Eqs. (23a), (23b) and (28) describing the endogenous adjustment of the investment rate, wage share, and debt-capital ratio respectively is given by

$$\dot{\sigma} = \beta(\omega_0 + \omega_1uk - \omega_2i_{CB}\lambda^2 - \sigma) \tag{23a}$$

$$\dot{k} = k(sr - n - \xi_0 + \xi_1\sigma + \xi_2i_{CB}\lambda^2) \tag{23b}$$

$$\dot{\lambda} = \lambda(si_{CB}\lambda - sr) \tag{28}$$

Assuming for convenience that the system has only one non-homogeneous solution and taking the linear approximation around the equilibrium point, i.e. $\dot{\sigma} = 0$ and $\dot{k} = 0$, we obtain the Jacobian matrix, J , of the three-dimensional system

$$J = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} \\ \Omega_{21} & \Omega_{22} & \Omega_{23} \\ \Omega_{31} & \Omega_{32} & \Omega_{33} \end{bmatrix} \tag{29}$$

where $\Omega_{11} = \partial\dot{\sigma}/\partial\sigma = -\beta(1 - \omega_1k^*u_\sigma^*) < 0$

$$\Omega_{12} = \partial\dot{\sigma}/\partial k = \beta\omega_1u^* > 0$$

$$\Omega_{13} = \partial\dot{\sigma}/\partial\lambda = \beta(\omega_1k^*u_\lambda^* - 2\beta\omega_2i_{CB}\lambda^*) < 0$$

$$\Omega_{21} = \partial\dot{k}/\partial\sigma = k^*(\xi_1 + sr_\sigma^*) > 0$$

$$\Omega_{22} = \partial\dot{k}/\partial k = 0$$

$$\Omega_{23} = \partial\dot{k}/\partial\lambda = k^*(sr_\lambda^* + 2\xi_2i_{CB}\lambda^*) \geq 0$$

$$\Omega_{31} = \partial\dot{\lambda}/\partial\sigma = -\lambda sr_\sigma^* < 0$$

$$\Omega_{32} = \partial\dot{\lambda}/\partial k = 0$$

$$\Omega_{33} = \partial\dot{\lambda}/\partial\lambda = s(2i_{CB}\lambda^* - r_\lambda^*\lambda^* - r^*) \geq 0$$

In order to keep the model tractable, we must impose some constraints on the parameter values. We assume that the impact of a change in the wage share on its own growth is negative, $\Omega_{11} < 0$; an increase in the capital-effective labour supply ratio invariably raises the growth of the wage share $\Omega_{12} > 0$; given that $u_\lambda^* < 0$, and a rising debt-capital ratio unambiguously reduces the growth of the wage share $\Omega_{13} < 0$. In the matrix (29) we also assume the stability condition of the two-dimensional system, i.e. $\xi_1 > |sr_\sigma^*|$, thus implying that $\Omega_{21} > 0$; in equilibrium, since $\dot{k} = 0$, we must have $\Omega_{22} = 0$; additionally, since $r_\lambda^* < 0$, the partial effect of a rising debt-capital ratio on the capital-effective labour productivity is ambiguous $\Omega_{23} \geq 0$. Given that $r_\sigma^* > 0$, we observe that changes in the wage share are inversely related to changes in the growth of the debt-capital ratio $\Omega_{31} < 0$; capital-effective labour supply ratio variations do not affect changes in the debt-capital ratio over time $\Omega_{32} = 0$; lastly, the impact of a change in the debt-capital ratio on its own growth rate is ambiguous, that is $\Omega_{33} \geq 0$.

Now we use the Routh-Hurwitz Criteria in order to analyse the stability around the equilibrium point. This method sets the stability conditions of the 3×3 dynamical system (see the mathematical appendix). The stability conditions are

$$A. \underbrace{\Omega_{11}}_{<0} + \underbrace{\Omega_{33}}_{\leq 0} < 0$$

$$B. \left(\underbrace{\Omega_{11}}_{<0} \underbrace{\Omega_{33}}_{\geq 0} - \underbrace{\Omega_{31}\Omega_{13}}_{>0} \right) - \left(\underbrace{\Omega_{21}\Omega_{12}}_{<0} \right) > 0$$

$$C. \left(\underbrace{\Omega_{12}}_{>0} \underbrace{\Omega_{23}}_{\leq 0} \underbrace{\Omega_{31}}_{<0} \right) - \left(\underbrace{\Omega_{33}}_{\geq 0} \underbrace{\Omega_{21}\Omega_{12}}_{>0} \right) < 0$$

$$D. -AB + C > 0$$

Given the constraints imposed on the parameter values, the stability of the system depends largely on the sign and magnitude of Ω_{33} . Since Ω_{33} is ambiguously signed, we must examine two possible cases: $\Omega_{33} \geq 0$ and $\Omega_{33} < 0$. Given that $\Omega_{33} = s(2i_{CB}\lambda^* - r_\lambda^*\lambda^* - r^*)$, the higher the initial value of the base interest rate set by the central bank i_{CB} , more likely it is to obtain $\Omega_{33} > 0$. In other words, if i_{CB} is sufficiently high, then we have $\Omega_{33} > 0$. Next, we analyse the four stability conditions above for $\Omega_{33} \geq 0$ and $\Omega_{33} < 0$. It is noteworthy that a stable system must satisfy all the four conditions above.

The stability condition A depends exclusively on the sign of Ω_{33} . If Ω_{33} is negative, then the stability condition is fulfilled. However, if Ω_{33} is strictly positive, then the condition A is ambiguous. Hence, a sufficiently high interest rate i_{CB} yields a positive value of Ω_{33} and destabilises the economy.

The stability condition B is ambiguously signed regardless of the sign of Ω_{33} . However, it must be pointed out that a sufficiently high base interest rate i_{CB} increases the possibility of economic instability. If $\Omega_{33} < 0$, then we have $\Omega_{11}\Omega_{33} > 0$, which makes it easier to satisfy this condition. On the other hand, when $\Omega_{33} > 0$ the term $\Omega_{11}\Omega_{33}$ becomes negative and this makes it more difficult to obtain a positive value on the left-hand side of the inequality given by the stability condition B.

The stability condition C, on the other hand, becomes more easily satisfied after a raise in interest rate. The term $\Omega_{12}\Omega_{23}\Omega_{31}$ is ambiguously signed as $\Omega_{23} \geq 0$. However, a sufficiently high interest rate yields a positive value of Ω_{23} and hence the term $\Omega_{33}\Omega_{21}\Omega_{12}$ becomes negative. Since we have $\Omega_{21}\Omega_{12} > 0$, a sufficiently high interest rate yields positive values of Ω_{23} and Ω_{33} and hence fulfills the stability condition $\Omega_{12}\Omega_{23}\Omega_{31} - \Omega_{33}\Omega_{21}\Omega_{12} < 0$.

Lastly, the stability condition D. assuming that the first three stability conditions are satisfied, we have $A < 0$, $B > 0$ and $C < 0$, which implies that the condition D is ambiguously signed. Given that $\Omega_{33} < 0$ is a necessary stability condition for B (and consequently for the whole system), a higher base interest rate i_{CB} raises Ω_{33} , thus reducing $|A|$, as B and $|C|$ increases. If we assume, for simplicity that variations in $|A|$ and B cancel each other out, then the fall in C following a rising base interest rate i_{CB} makes the stability condition D less likely to be fulfilled.

In short, the stability condition B guarantees that if $\Omega_{33} \geq 0$, which means that the base interest rate is sufficiently high, then the dynamical system is unambiguously unstable; if $\Omega_{33} < 0$, then the stability will depend on all the four conditions.

The three-dimensional model analyses the role of the interest rate and the amount of debt granted to firms as a proportion of capital stock in the intra-capitalist conflict. Since rising interest payments redistributes profits in favour of rentiers, retained profit by firms is reduced, thus bringing down the investment rate. Increased interest payments also impose severe consequences over the labour market, as it reduces capital accumulation, thus lowering the employment rate and, consequently, the capacity of workers to bargain for higher wages. In the scenario of rising financial instability, an explosive debt dynamics leads to ever-decreasing investments and wage share.

To sum up, several differences between the two-dimensional and the three-dimensional systems can be pointed out. The stability of the two-dimensional model, in which the debt-capital ratio λ remains constant, is attained when the inequality $\xi_1 > |sr_\sigma^*|$ is satisfied. On the other hand, even if the stability condition of the two-dimensional system holds, i.e. $\Omega_{21} = k^*(\xi_1 + sr_\sigma^*) > 0$, the three-dimensional system can still be unstable if at least one of the four stability conditions above (A, B, C and D) is not fulfilled. By relaxing the hypothesis of a constant λ , the three-dimensional model shows formally how a changing debt-capital ratio increases the financial fragility and the potential instability of the economy. Further, the three-dimensional model also demonstrates how a sufficiently high base interest rate destabilises the system. In terms of policy implication, it is worth noting that the central banks of developing countries

with relatively high base interest rates might have limited space to raise interest rates in order to steer prices without destabilising the economy.

5. Concluding remarks

This paper develops a neo-Kaleckian dynamical model of capital accumulation, income distribution and financial fragility. This theoretical framework contributes to the literature by setting the conditions in which the debt-capital ratio, the income distribution and the process of capital accumulation can be simultaneously stable. Our model shows how a growing financial weakness leads to a poorer performance of investments and worst income distribution in the economy.

First we model the two-dimensional dynamic system between capital-effective labour supply ratio and wage share. Such a system meets the methodological purpose of serving as a baseline scenario where the debt-capital ratio remains constant. Based on this analysis, we conclude that the stability of the two-dimensional system is attained when technological progress is more responsive than capital accumulation to wage share variations. Further, it is shown that around the equilibrium solution the higher the base interest rate, the more stable the economic system. However, the idea that a higher base interest rate helps to stabilise the dynamical interaction between capital accumulation and income distribution does not sit comfortably with stylised facts of modern economies. In fact, the empirical evidence suggests that an increasing financial fragility usually leads to ever-decreasing investments and greater income inequality.

The three-dimensional model, on the other hand, relaxes the hypothesis of a constant λ , which implies that a rising debt service changes the debt structure of firms and hence affects the dynamic behaviour of capital accumulation and income distribution over time. The analysis of equilibrium solution of the 3×3 dynamical system through the Routh-Hurwitz Criteria shows how the economy becomes potentially more unstable when we allow for variations of the debt of firms as a proportion of their capital stock into the model. Unlike the 2×2 dynamical system, the three-dimensional model also formally demonstrates that a sufficiently high base interest rate increases the financial fragility of the economy, which leads to weaker investments and redistribution of income in favour of financial capitalists. Such a conclusion is particularly relevant to understand why central banks of developing countries with relatively high base interest rates might have limited space to conduct a contractionary monetary policy through open market operations, as any increase in the interest rate might be enough to drag the economy into a financially unstable scenario of rising indebtedness, decreasing investment, and growing income inequality.

Mathematical appendix. The Routh-Hurwitz criteria

At a given steady-state solution we have the following Jacobian matrix

$$J = \begin{pmatrix} a_{11} & \cdots & a_{1k} \\ \vdots & \ddots & \vdots \\ a_{k1} & \cdots & a_{kk} \end{pmatrix}$$

By solving $\text{Det}(J - \nu I)$ we obtain the general characteristic equation

$$\nu^k + a_1 \nu^{k-1} + a_2 \nu^{k-2} + \cdots + a_k = 0$$

The roots of the characteristic equation are the eigenvalues of the matrix J . The steady-state solution is stable if all the eigenvalues have negative real parts.

That said, let us illustrate the Routh-Hurwitz criteria for $k=2$. The Jacobian matrix is

$$J = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

The characteristic equation is

$$v^2 + A_1v + A_2 = 0$$

⋮

$$v^2 - \text{Tr}(J)v + \text{Det}(J) = 0$$

where $A_1 = -\text{Tr}(J)$ and $A_2 = \text{Det}(J)$. Therefore, if $\text{Tr}(J) < 0$ and $\text{Det}(J) > 0$ the solution is stable.

Now, let us examine the Routh-Hurwitz criteria for $k = 3$. The three-dimensional Jacobian matrix is

$$J = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$$

The characteristic equation is

$$v^3 + A_1v^2 + A_2v + A_3 = 0$$

⋮

$$v^3 - (a + e + i)v^2 + (ae + ie + ia - gc - hf - db)v - (aei + bfg + cdh - gce - hfa - dbi) = 0$$

⋮

$$v^3 - \text{Tr}(J)v^2 + [\text{Det}(J_1) + \text{Det}(J_2) + \text{Det}(J_3)]v - \text{Det}(J) = 0$$

where $J_1 = \begin{pmatrix} e & f \\ h & i \end{pmatrix}$, $J_2 = \begin{pmatrix} a & c \\ g & i \end{pmatrix}$, $J_3 = \begin{pmatrix} a & b \\ d & e \end{pmatrix}$, $A_1 = -\text{Tr}(J)$, $A_2 = [\text{Det}(J_1) + \text{Det}(J_2) + \text{Det}(J_3)]$, and $A_3 = -\text{Det}(J)$. Hence, $\text{Det}(J_1)$, $\text{Det}(J_2)$ and $\text{Det}(J_3)$ are the three principal minors of matrix J .

By this criteria, the stability condition holds if $A_1 > 0$, $A_2 > 0$, $A_3 > 0$, and $A_1A_2 - A_3 > 0$. Thus, we can obtain the stability conditions (25a)–(28a)

$$\text{Tr}(J) < 0$$

$$\text{Det}(J_1) + \text{Det}(J_2) + \text{Det}(J_3) > 0$$

$$\text{Det}(J) < 0$$

$$-\text{Tr}(J)[\text{Det}(J_1) + \text{Det}(J_2) + \text{Det}(J_3)] + \text{Det}(J) > 0$$

To sum up, the Routh-Hurwitz criteria to analyse the stability of the steady-state solution for $k = 2, 3$ are:

If $k = 2$, then $A_1 > 0$, $A_2 > 0$.

If $k = 3$, then $A_1 > 0$, $A_2 > 0$, $A_3 > 0$, and $A_1A_2 - A_3 > 0$.

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