

5 A SAM and Multiplier Analysis: Economic Linkages and Multiplier Effects

5.1 What are the Economic Linkages and Multiplier Effects?

The strongest argument in favour of the Walras equilibrium—as opposed to the Marshall *ceteris per ibis* approach—will find its momentum once industry linkages and multiplier effects are envisaged. This is so because in these circumstances thinking about partial equilibrium becomes less sustainable. In fact, the effect of a shock from one industry may have direct and indirect impact on the whole system defined by different industries. Let us analyse below a shock generated by the demand side. When we talk of “*exogenous demand-side shocks*” to an economy, we refer to changes to final control demand aggregates, i.e., export demand, government spending, or net investment demand of stocks. The effects of these shocks are both direct and indirect.

The direct effects are to those sectors that affront the shock. For example, an exogenous increase in demand for Polish manufactured exports has a direct impact on the manufacturing industry, which results in increased inputs, production, sales, and value-added. However, the positive consequences of such a shock go beyond the manufacturing industry. It may also have indirect effects stemming from manufactures’ linkages to other industries inside the economy. These indirect linkages can be classified into supply-side and demand-side. When we add up all direct and indirect linkages, we get a measure of the shock’s multiplier effect, or how much an initial effect is amplified or multiplied by indirect linkage effects. Supply-side linkages are determined by industry production technologies, which can be depicted from an input-output table. Next, they are differentiated into backward and forward linkages. Backward production linkages are the demand for additional inputs used by producers to supply additional goods or services. For instance, when production (of manufacturers) expands, it requires additional intermediate goods or services like raw material, machinery, and transport services. This demand then stimulates production of other industries that supply these intermediate goods. Technical coefficients supply information on the input intensity of the production technology used. The more an industry’s production technology is input intensive, the stronger its backward linkages.

Forward linkages allude to supply inputs to upstream industries. For instance, increased manufacturer production should lead to increased supply of goods to the construction industry, which, in turn, stimulates, among others, service industries. As in the case of backward linkages, the more important an industry is regarding upstream industries, the stronger its forward linkages will be and multipliers will definitely become larger.

The conceptual structure of the input-output matrix only allows for deriving multipliers that measure the effects of supply linkages. Since the input-output table does not show secondary income distribution, it is not possible to consider consumption linkages, which arise when an expansion of production generates additional incomes for factors and households, which are then used to purchase goods and services. Continuing the same example as above, when manufacturing production expands, it raises households' incomes, which are used to buy consumer goods. Depending on the share of domestically produced, tradable, and imported goods in households' consumption baskets, domestic producers benefit from greater demand for their products. The size of consumption linkages depends on various factors, including the share of net factor income distributed to households; for an open economy, the level of gross domestic product per inhabitant, which exercises an influence on the composition of the consumption basket; and the relative price between locally produced and imported goods which determines in Armington fashion the share of domestically supplied goods in consumer demand.

Consequently, SAM multipliers tend to be larger than input-output multipliers because they capture both production and consumption/income linkages.

Following Breisinger et al. (2009), *“while economic linkages are determined by the structural characteristics of an economy (evidenced through technical coefficients and/or the composition of households' consumption baskets) and remain thus static, multiplier effects capture the combined dynamic effects of economic linkages over a period of time through different auto-generated rounds”*.

Three types of multipliers are generally reported in empirical research. First, an output multiplier combines all direct and indirect (consumption and production) effects across multiple rounds and reports the final increase in gross output of all production activities. Second, a GDP multiplier measures the total change value-added or factor incomes caused by direct and indirect effects. Finally, the income multiplier measures the total change in household incomes.

The dampening path of multipliers is consecutive to the level of leakage inside economic circular flows. Ultimately, higher leakages stemming from income allocated to imported goods or from government taxes make the round-by-round effects slow down more quickly and reduce the total multiplier effect.

In empirical research, one must often deal with two kinds of economic hypotheses. First, still in the context of the above example, one can suppose that demand shock will encounter no constrained response from the supply side. The second case is the one where demand shock is constrained. This can happen when supply is not able to completely satisfy increased demand. In this hypothesis, multipliers will follow a modified dynamic path towards slowing down. Let us still follow Breisinger et al. (2009) and then succinctly analyse both cases.

5.1.1 A SAM Unconstrained Multiplier

Let us present below a simplified SAM where presented accounts are just those required to derive a multiplier matrix.

Table 16: A simplified SAM for multiplier analysis

	Activities		Commodities		Factors	Households	Exogenous	Total
	A1	A2	C1	C2	F	H	E	demand
A1			X_1					X_1
A2				X_2				X_2
C1	Z_{11}	Z_{12}				C_1	E_1	Z_1
C2	Z_{22}	Z_{22}				C_2	E_2	Z_2
F	V_1	V_2						V
H					$V_1 + V_2$			Y
E			L_1	L_2		S		E
Total	X_1	X_2	Z_1	Z_2	V	Y	E	

Source: own elaboration, based on Breisinger, Thomas, and Thurlow (2009).

We divide columns by their total to derive the coefficient matrix (M-matrix) excluding the exogenous components of demand.

Table 17: Transformed Table 16

	Activity		Commodities		Factors	Households	Exogenous
	A1	A2	C1	C2	F	H	demand
A1			$b_1 = X_1 / Z_1$				
A2			$b_2 = X_2 / Z_2$				
C1	$a_{11} = Z_{11} / X_1$	$a_{12} = Z_{12} / X_2$				$c_1 = C_1 / Y$	E_1
C2	$a_{21} = Z_{21} / X_1$	$a_{22} = Z_{22} / X_2$				$c_2 = C_2 / Y$	E_2
F	$v_1 = V_1 / X_1$	$v_2 = V_2 / X_2$					
H					$1 = (V_1 + V_2) / V$		
E			$l_1 = L_1 / Z_1$	$l_2 = L_2 / Z_2$		$s = S / Y$	
Total	1	1	1	1	1	1	E

Source: own elaboration, based on Breisinger, Thomas, and Thurlow (2009).

Symbols:

a) Values:

X: Gross output of each activity (i.e., X_1 and X_2)

Z: Total demand for each commodity (i.e., Z_1 and Z_2)

V: Total factor income

Y: Total household income

E: Exogenous components of demand shares

b) Share:

a: Technical coefficients

b: Share of domestic output in total demand

v: Share of value-added or factor income in gross output

l: Share of the value of total demand from imports or commodity taxes

c: Household consumption expenditure shares

s: Household savings rate

To derive equations representing the relationships in the above SAM, we start by setting up simple demand equations:

$$Z_1 = a_{11}X_1 + a_{12}X_2 + c_1Y + E_1$$

$$Z_2 = a_{21}X_1 + a_{22}X_2 + c_2Y + E_2 \quad (4.23)$$

Total demand = intermediate demand + household demand + exogenous demand.

The next relationships tell us that domestic production X is only part of total demand Z.

$$X_1 = b_1Z_1$$

$$X_2 = b_2Z_2$$

Since household income Y depends on the share each factor earns in each sector, then:

$$Y = v_1X_1 + v_2X_2$$

or,

$$Y = v_1b_1Z_1 + v_2b_2Z_2$$

Now replacing all X and Y in Equation (4.23), moving everything except for E onto the left-hand side, and grouping Z together, we finally obtain:

$$(I - M)Z = E, \quad (4.24)$$

where

$$(I - M) = \begin{pmatrix} (1 - a_{11}b_1 - c_1v_1b_1)(-a_{12}b_2 - c_1v_2b_2) \\ (-a_{21}b_1 - c_2v_1b_1)(1 - a_{22}b_2 - c_2v_2b_2) \end{pmatrix}.$$

We note that M is a square matrix, the elements (share values) of which are not negative. Each column sum (see Table 16) is less or equal to unity. Thus, an inverse matrix of $(I - M)$ exists and should display non-negative values, suggesting the non-negativity of the multiplier matrix.

Formally, from (4.24) we directly get the final multiplier equation of the form:

$$Z = (I - M)^{-1}E \quad (4.25)$$

Total demand = multiplier matrix \times exogenous demand

The above formulation tells us that when exogenous demand E increases, one will end up with a final increase in total demand equal to Z , owing to all the direct and indirect multiplier effects $(I - M)^{-1}$.

5.1.2 Equation System for Constrained SAM Multiplier

Often when factor allocation is not optimal, exogenous demand shocks may encounter limited response from producing sectors. Let us analyse below how much a multiplier will change if some producing sectors are unable to correctly respond. The expected issue is that if we fix one of two sectors Z , for instance Z_2 . In that case, imports should substitute for domestic supply, thus eliminating any growth linkages from this sector.

The next equation is related to the non-constrained case and expresses total demand as the sum of its parts.

$$(1 - a_{11}b_1 - c_1v_1b_1)Z_1 + (-a_{12}b_2 - c_1v_2b_2)Z_2 = E_1$$

$$(-a_{21}b_1 - c_2v_1b_1)Z_1 + (1 - a_{22}b_2 - c_2v_2b_2)Z_2 = E_2$$

Grouping exogenous terms on the right-hand side (i.e., E_1 and Z_2) and rearranging⁴², we finally obtain:

$$\begin{pmatrix} Z_1 \\ E_2 \end{pmatrix} = (I - M^*)^{-1}B \begin{pmatrix} E_1 \\ Z_2 \end{pmatrix} \quad (4.26)$$

where

$$(I - M^*) = \begin{pmatrix} (1 - a_{11}b_1 - c_1v_1b_1, 0) \\ (-a_{21}b_1 - c_2v_1b_1, -1) \end{pmatrix}$$

⁴² For derivation details, see Breisinger, Thomas, and Thurlow (2009).

and

$$B = \begin{pmatrix} 1, a_{12}b_2 + c_1v_2b_2 \\ 0, -1 + a_{22}b_2 + c_2v_2b_2 \end{pmatrix}$$

Interpretation of the above equation is the following: an exogenous increase in demand for the unconstrained sectors [E_1] leads to final increase in total demand for these sectors [Z_1], including all of the forward and backward linkages $(I - M^*)^{-1}$. For the sectors with constrained supply (in our case sector Z_2), it is net exports that decline. This means that the current trade balance must worsen if we have to amortize demand shock in the case of constrained supply. If exports remained unchanged, then the alternative of reducing exports would be increasing imports so as to meet additional exogenous demand in the context of this constrained supply.

5.1.3 On Modelling Multiplier Impact for an Ill-Behaved SAM

Let us now return back to the central problem of this presentation and suppose that the matrix is unbalanced, which implies that multiplier values are not reliable. The way to avoid this should consist of only estimating parameters of the model without taking into account the obligation that the whole SAM be internally consistent. Thus, we should maximize (or minimize) entropy for probabilities related to the multiplier matrix under traditional restrictions, plus an additional constraint declaring values of an already balanced SAM to be taken as a prior.

Remembering about the interpretation of estimated parameters through the maximum entropy principle, it would be easy to make a link between the multiplier effect and maximum entropy modelling. In fact, in a linear model, parameters estimated by entropy formalism are interpreted as the long-run (equilibrium) impact of one unit change of regressor x on regresand y . Thus, long-run impact means that direct and indirect effects of the multiplier are accounted for with respect to the shock.

Annex C. Proof of Economy Power Law Properties

1. Definition of Power Law Distribution

Since we already know existing relationships between power law function and non-extensive entropy from Part II of this work, let us now present the main properties of the former in the context of a SAM.

Using a simplified formulation, a power law is the relation of the form $f(x) = Kx^\alpha$ where $x > 0$ and K and α are constants. While power laws can appear in many dif-

ferent contexts, the most common are those where $f(x)$ describes a distribution of random variables or the autocorrelation function of a random process.

The formulation above has the advantage of being intuitive. However, it does not show the real attributes of that distribution, which displays asymptotical characteristics.

Thus, the notion of a power law as it is used in extreme value theory is an asymptotic scaling relation. Let us first explain what we understand by equivalent scaling. Two functions f and g have equivalent scaling, $f(x) \sim g(x)$ in the limit $x \rightarrow \infty$ ⁴³ if:

$$\lim_{x \rightarrow \infty} \frac{L(x)f(x)}{g(x)} = 1 \quad (4.27)$$

with $L(x)$ is a slowly varying function, thus satisfying the relation:

$$\lim_{x \rightarrow \infty} \frac{L(tx)}{L(x)} = 1,$$

for any finite constant $t > 0$. Slowly varying functions are, for example, $L(x) = C$ and $L(x) = \ln(x)$, that is, a constant and a logarithmic function, respectively.

A power law is defined as any function satisfying $f(x) \sim x^\alpha$. This definition then implies that a power law is not a single function but an asymptotical composite function. The slowly varying function $L(x)$ can be thought of as the deviation from a pure power law for finite x .

For $f(x) = L(x)x^\alpha$, taking logarithms of both sides and dividing by $\log(x)$ gives

$$\log f(x)/\log(x) = -\alpha + \log L(x)/\log(x) \quad (4.28)$$

Remembering that $L(x)$ is a slowly varying function, in the limit, the second term on the right vanishes to zero as $x \rightarrow \infty$, and thus we have:

$$\log f(x)/\log(x) = -\alpha,$$

or equivalently,

$$f(x) = x^{-\alpha}, \text{ for } x \rightarrow \infty.$$

This means that the empirical form of the function becomes:

$$f(x) \sim x^{-\alpha} \quad (4.29)$$

and in terms of probabilities, a the cumulative function $P(S > x) = kx^{-\alpha}$ corresponds to a probability density function: $f(x) = k\alpha x^{-(\alpha+1)}$.

43 Note that this limit is not the one possible, but remains a realistic device, e.g., in finance.

2. Main Properties

We list below only properties directly related to two theorems proposed in this annex.

- a) The property that most interests us and that generally makes power laws special is that they describe scale free phenomena. A variable undergoes a scale transformation of the form $x \rightarrow Cx$. If x is transformed, we then obtain:

$$f(x) = kC^\alpha x^\alpha = C^\alpha f(x) \tag{4.30}$$

provided that given initial power law function is $f(x) = kx^\alpha$. Changing the scale of the independent variable thus preserves the functional form of the solution but with a change in its scale. This is an important property in our case. Scale-free behaviour strongly suggests that the same mechanism is at work across different sectors of the economy, the industrial structure of which remains constant over a relatively long period of time, measured with any time measurement (i.e., seconds, minutes, hours, days, years). A useful example that should be appealing for economists is price. We say that price is a homogenous function of degree zero with respect to income.

- b) A power law is just a linear relationship between logarithms (Breisinger et al., 2009) of the form:

$$\log f(x) = -\alpha \log (x) + \log k. \tag{4.31}$$

- c) Power law also has excellent aggregation properties⁴⁴. The property of being distributed according to a power law is conserved under addition, multiplication, polynomial transformation, min, and max. The general rule is that when combining two power law variables, the fattest power law (i.e., the one with the smallest exponent) dominates. This property could be helpful for empiricist researchers using this form of function.

Let X_1, \dots, X_n be independent random variables, and k , a positive constant. Let α_x be also the power law exponent of variable X . Following Gabaix (2008), Jessen and Mikosch (2006), we have the so-called inheritance mechanism for power law:

$$\alpha_{X_1+X_2+\dots+X_n} = \min(\alpha_{X_1}, \alpha_{X_2}, \dots, \alpha_{X_n}) \tag{4.32}$$

$$\alpha_{X_1 * X_2 * \dots * X_n} = \min(\alpha_{X_1}, \alpha_{X_2}, \dots, \alpha_{X_n}) \tag{4.33}$$

$$\alpha_{\max(X_1, X_2, \dots, X_n)} = \min(\alpha_{X_1}, \alpha_{X_2}, \dots, \alpha_{X_n}) \tag{4.34}$$

44 The interested reader is recommended to see the works of Jessen & Mikosch (Jessen & Mikosch, 2006) or Gabaix (Gabaix, 2008). As an example of relative facilities of proofs, if $P(X > x) = kx^{-\alpha}$, then

$$P(X^k > x) = kx^{-\alpha} = P(X > x^{\frac{1}{k}}) = kx^{-\frac{\alpha}{k}}, \text{ so } \alpha_{X^k} = \frac{\alpha}{k}.$$

$$\alpha_{\min(X_1, X_2, \dots, X_n)} = (\alpha_{X_1} + \alpha_{X_2} + \dots + \alpha_{X_n}) \quad (4.35)$$

$$\alpha_{kx} = \alpha_x \quad (4.36)$$

$$\alpha_{X^k} = \frac{\alpha_X}{k} \quad (4.37)$$

Thus, if we have two variables X and Y with different exponents, this property holds when Y is normal, lognormal, or exponential, in which case $\alpha_Y = \infty$. Hence, multiplying by normal variables, adding non-fat tail noise, or summing over independent and identically distributed (i.i.d.) variables preserves the exponent.

This is a reason for hope for empiricists, that power law exponents can be measured even if the data are noisy. Although noise affects statistics (moments), it will not affect the PL exponent. The problem of missing data may not affect information contained inside data, either.

3. Statistical Complexity of a SAM

A social accounting matrix represents an economic table aggregating information about complex interchanges within different sectors and/or institutions. These interchanges have been described in Figure 1 where a general scheme of income flow in the economy was described. In economics, the main purpose of human activity is increasing income, the principal source of well-being. Changes in that income are usually assessed through gross domestic product growth (GDPG). However, this GDPG is itself an aggregate accounting of income growth from different sectors and institutions. Since a SAM is built under the principle of double entry bookkeeping, income, and expense totals should balance. This aspect has been previously alluded to. Remaining within the Walrasian economy⁴⁵ which rules out that expenses growth are absolutely co-integrated, in the context of Granger time series analysis, with incomes or wealth, over any time period.

As earlier suggested, the economic system described is defined by different interactive subsystems, each represented by respective actors and characterized by optimizing behaviour. Households, which tend to maximize a certain utility function, remain the owner of factors of production and are the final consumer of produced commodities; firms maximize profits by optimal renting of these factors from households for the production of goods and services. In this model, government has the passive role of collecting and disbursing taxes. Furthermore, the economy analysed

⁴⁵ However, according to recent research, power law consistency with equilibrium theory has, so far, failed to address this. Nevertheless, such consistency is expected for both theories (Doyné, & Geanakoplos, 2009)

is small and open, and entirely prone to world fluctuations, owing to, among other things, the country's status of 'price taker' from the international market.

Furthermore, as already noted, due to different and sometimes contradictory sources of collected statistical information, SAM cannot be balanced. Such statistical data may display, as partially coming from statistical surveys, systematic and stochastic errors, thus missing some normal Gaussian properties.

A Proof of SAM Power Law Distribution Properties.

For the next step, we provide propositions evidenced by the above properties of power law functions and by other recent works in econophysics (e.g., Stanley et al., 1998).

Proposition 1. Under general Walrasian conditions,⁴⁶ the present level of sectorial (or institutional) income or expense total is a linear function of cumulated past and present sectorial or institutional wealth (income) or expenses growth rates of the global economy.

Proposition 2. Income (profit) growth rate follows exponential law within sectors or institutions with similar activity scale while this distribution becomes a power law among firms with different activity scales.

The first proposition simply explains a cumulative character of wealth from additional net incomes over time, in this case, sectorial industry or an institution. The property (4.32) guarantees plausibility of this proposition provided the independence of growth rates.

The second proposition, follows results found in the case of the U.S. economy (Stanley et al., 2001), where firms with the same level of activity display an exponential distribution of income (profit) growth rate while that distribution becomes a power law when we confront firms with different levels of activity. Furthermore, in this last case, the above authors have noted seven different fractals within that distribution.

Assumption 1 (structural stability): We will adopt an economy where factors of production are mobile among sectors of production and different scales of sectors are not affected by structural differences in factor productivities. To make this assumption more realistic, this means that the level of productivity and of factors within different sectors of the economy are identical enough so that there are no observed factor movements towards a given sector over a long period $T \rightarrow \infty$.

Assumption 2 (convergence). A cumulated economic growth rate trend is positive. This means that if we assume g_{jt} to be any growth rate in the economic or institutional sector j for period t , we have:

$$\sum_{t=1}^{\infty} g_{jt} \rightarrow \infty \quad (4.38)$$

⁴⁶ Here, we have particularly the principles of market *clearance* and of *income balance* in mind.

In other terms, positive growth rates always mark an advantage on negative ones and economic progress in the long run is guaranteed. This assumption is a stylized version of human economic development through history, owing to their natural capacity to innovate.

Theorem 1. For a given non-centralized economy, disaggregated subaccounts of a vector (matrix) additively defining micro-elements of an entire system account by row or by column (being a cumulative income growth over finite lengths, periods n) display by row (expenses) or column (incomes) a power law distribution.

Proof⁴⁷: Let us first provide the demonstration on the income generating side of an accounts table and thus consider a non-centralized economy (system) made of M sectors and institutions (micro-elements) j ($j = 1..M$) generating each income w_{ijt} where i ($i = 1..M$) means one of the M sectors receiving incomes from one of the j sectors during the period t_n . Aggregative sectorial income is

$$\sum_{j=1..M} w_{.j} = W_{.}$$

Let us consider two free periods of time 0 and t . We have then w_{j0} and w_{jt} , two successive incomes during two periods, and we defined wealth growth $g_{.j}$ as $g_{.j} = w_{jt} / w_{j0}$, meaning a relative growth of wealth at period t . Equivalently, we have the growth rate $\ln(g_{.j})$. In probabilistic terms, we will assume there is some collection $g_{.j}$ of possible wealth growth in a fixed sector i by a finite number of sectors j that each can generate with associated probabilities $\{p_{.j}\}$. For a fixed sector i , we have

$$\sum_{j=1..M} p_{.j} = P_{.} = 1 \quad (4.39)$$

Let us now introduce maximum entropy formalism to the problem.

Let us consider a continuous case where we have wealth growth $g_{.j}$ and its density of probability to be found, $f(g_{.j})$. We maximize the entropy (Carter, 2011).

$$\text{Max}H(f) = -\int_1^{\infty} f(g_{.j}) \ln((g_{.j})) dg_{.j} \quad (4.40)$$

subject to

$$\frac{r^{q-1} - 1}{q-1} \geq 1 - \frac{1}{r} \quad (4.41)$$

$$\int_1^{\infty} f(g_{.j}) \ln((g_{.j})) dg_{.j} = k \ln(g_{.j}); \quad (4.42)$$

⁴⁷ It will suffice to demonstrate the case of, for example, wealth growth and to deduct the case of expenditure growth, thanks to Walrasian aspects of our economy or by referring to the co-integrating character of both variables on a longer interval of time.

where constant k is the average number of inter-sectorial transactions per time step. The next step consists of applying the calculus of variations to maximize over a class of functions. Thus, solving an external problem of the functional:

$$\int F[z, f(z), f'(z)] dz \tag{4.43}$$

and look then at solving:

$$\frac{\partial F}{\partial f(z)} - \frac{d}{dz} \left(\frac{\partial F}{\partial f'(z)} \right) = 0$$

We define the Lagrange of the form:

$$L \equiv - \int_1^{\infty} f(g_{\bullet j}) \ln((g_{\bullet j})) dg_{\bullet j} - \mu \left(\int_1^{\infty} f(g_{\bullet j}) dg_{\bullet j} - 1 \right) - \lambda \left(\int_1^{\infty} f(g_{\bullet j}) \ln((g_{\bullet j})) dg_{\bullet j} - k \ln(g_{\bullet j}) \right)$$

Finally, we get from conditions of first order:

$$f(g_{\bullet j}) = e^{-(\lambda_0 - \lambda \ln(g_{\bullet j}))} = g_{\bullet j}^{-\lambda} e^{-\lambda_0}$$

where $\lambda_0 = 1 + \mu$.

One can use the normalization (4.41) condition to solve for $e^{-\lambda_0}$,

$$e^{-\lambda_0} = \int_1^{\infty} g_{\bullet j}^{-\lambda} dg_{\bullet j} = \left[\frac{g_{\bullet j}^{1-\lambda}}{1-\lambda} \right]_1^{\infty} = \frac{1}{\lambda - 1}$$

after assuming $\lambda > 1$.

Rearranging the above terms, we get the density probability functional form:

$$f(g_{\bullet j}) = (\lambda - 1)(g_{\bullet j})^{-\lambda} \tag{4.44}$$

This is the sought *density probabilities* of wealth growth rate of economic sectors and institutions. It displays a power law distribution form.

Next, Theorem 1 above and properties (4.32– 4.36) guarantee that the cumulated by the past growth sectorial incomes should continue to display a power law distribution irrespective of which form of transition economy evolves from period t to the next period $t + 1$. In particular, we note that property (4.33) ensures that multiplicative transitory combinations of different growth rates continue to keep the power law property of economic sectorial movements unmodified. Assumptions 1 and 2 guarantee that cumulated income growth rates are an increasing function of time, guaranteeing increasing sectorial wealth over generations.

The demonstration is proven.

Theorem 2. *Economic growth rate movements of any open economy display power law function properties.*

Proof: Its proof results from the demonstration of Theorem 1 and the additive property (4.32) since the global economic growth rate is derived as a weighted linear combination of sectorial growth rates. This ends the demonstration.

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