

Summary

This book provides a new and robust power-law (PL)-based, non-extensive entropy econometrics approach to the economic modelling of ill-behaved inverse problems. Particular attention is paid to national account-based general equilibrium models known for their relative complexity.

In theoretical terms, the approach generalizes Gibbs-Shannon-Golan entropy models, which are useful for describing ergodic phenomena. In essence, this entropy econometrics approach constitutes a junction of two distinct concepts: Jayne's maximum entropy principle and the Bayesian generalized method of moments. Rival econometric techniques are not conceptually adapted to solving complex inverse problems or are seriously limited when it comes to practical implementation.

In recent years, PL-based Tsallis entropy has been applied in many fields. Its popularity can be attributed to its ability to more accurately describe heavy tail, non-ergodic phenomena. However, the link between PL and economic phenomena has been neglected—probably because the Gaussian family of laws, which are globally sufficient for time (or space) aggregated data and easy to use and interpret. Recent literature shows that the amplitude and frequency of macroeconomic fluctuations do not substantially diverge from many extreme events, natural or human-related, once explained at the same time or space-scale by PL. In particular, in the real world, socioeconomic rare events may, through long-range correlation processes, have higher impact than more frequent events could. Because of this and based on existing literature, this monograph proposes an econometric extension called *Non-extensive Entropy Econometrics* or, using a less technical expression, *Superstar-Generalised Econometrics*.

Recent developments in information-theoretic built upon Tsallis non-additive statistics are powerful enough to put established econometric theory in question and suggest new approaches. As will be discussed throughout this book, long-range correlation and observed time invariant scale structure of high frequency series may still be conserved—in some classes of non-linear models—through a process of time (or space) aggregation of statistical data. In such a case, the non-extensive entropy econometrics approach generally provides higher parameter estimator efficiency over existing competitive econometrics procedures. Next, when aggregated data converge to the Gaussian attractor, as generally happens, outputs from Gibbs-Shannon entropy coincide with those derived through Tsallis entropy. In general, when the model involved displays less complexity (with a well-behaved data matrix) and remains closer to Gaussian law, computed outputs by both entropy econometrics approaches should coincide or approximate those derived through most classical econometric approaches. Thus, the proposed non-ergodic approach could at least be as good as the existing estimation techniques. On empirical grounds, it helps in ensuring stability of the estimated parameters and in solving some classes of, up to now, intractable non-linear PL-related models. Furthermore, the approach remains one of the most appro-

priate for solving all classes of inverse problems, whether deterministic or dynamic. It is a more general approach. Finally, this approach helps us better assess—thanks to the Tsallis- q parameter—the interconnection level (complexity) between economic systems described by the model.

Consequently, this book aims at providing a new paradigm for econometric modelling through non-extensive (cross) entropy information-theoretic. Reaching this goal requires some intermediary results obtained through a synthesis of the existing, sometimes sparse literature. There are, then, methodological issues to address. Among these is the application of non-extensive entropy to low frequency time series. This constitutes a new challenge and must be clarified. Next, generalizing Gibbs-Kullback-Leibler information divergence to the Tsallis non-ergodic econometric model with different constraining moment formulations in both classes of entropy model will require special attention since we are not aware of any publications on the subject. Another important intermediary result of this work will be the proposition of a new theorem linking PL and macroeconomics on both the supply and demand sides. Its demonstration will provide new keys for carrying out further Tsallis entropy econometric modelling. Finally, we will provide an *ad hoc* statistical inference corresponding to the new modelling approach presented here.

The first part of the monograph presents basic targets and principal hypotheses.

In the second part, we present definitions and quantitative properties of statistical theory of information. Progressively, a link between the statistical theory of information and the generalized ill-posed inverse problem is established. After having shown the properties of the Shannon-Jaynes maximum entropy principle in detail, techniques for solving ill-behaved problems, from the Moore-Penrose generalized inverse problem to non-extensive entropy, are compared. Intrinsic relationships between both forms of Shannon-Jaynes¹ and Tsallis entropies are also shown. After having presented Kullback-Leibler information divergence, a generalization of this concept to non-extensive entropy is developed. A general linear non-extensive entropy econometric model is then introduced. It will play an important role for models to be developed in subsequent chapters. Next, an inferential formalism for parameter confidence interval area is proposed. This part is concluded with an applications example: the estimation of a Tsallis entropy econometrics model using the case of labour demand anticipation with a time series, error-correction model. Its outputs are compared with those of other approaches through Monte-Carlo simulations.

The third and fourth parts of the book—and, to a certain extent, the fifth part—are closely related to each other since a social accounting matrix can be seen as a kind of input-output transaction matrix generalization. The separation of these two

¹ Here we prefer to shorten the name of this form of entropy. Scientists who have contributed to this form of entropy are many and cannot all be mentioned. It could be ‘succinctly’ named “Gibbs-Shannon-Jaynes-Kullback-Leibler Golan entropy econometrics.”

parts has avoided highly horizontal and vertical subdivisions in the book, thereby preserving the clarity of the study. These two parts provide economic applications of statistical theory presented through Part II. Part III focuses on updating and forecasting national accounts tables. In particular, a new efficient approach to forecast input-output tables—or their extended forms—is set forth. The RAS approach is presented as a competing technique with empirical application and comments. To show one of the possible fields of entropy model implementation, we provide an ecological model to be solved as an inverse problem.

After having proposed a theorem linking PL distribution and the macroeconomic aggregative structure of national accounts, the problem of balancing a social accounting matrix (SAM) in the context of non-ergodicity is posed and solved in Part IV. The example presented deals with the actual problems of updating a SAM in real-world conditions.

In Part V, a computable general equilibrium (CGE) model is presented as a national account-related model. Two important concepts are discussed in the context of optimum property that both of them convey: the maximum entropy principle and the Pareto-optimum. Next, we open a short, epistemological discussion on two competitive and frequently confused estimation approaches, the Bayesian approach and the maximum entropy principal. An approach using non-extensive relative entropy for parameter estimation in the case of a constant elasticity of substitution (CES) function is proposed through the presentation of the CGE model.

To show the extensions of the standard national accounts table and to go beyond the general equilibrium framework, an environmentally extended social accounting matrix and a subsequent theoretical model displaying externalities are presented in Part VI. Finally, a carbon tax and double dividend theory model is presented and its social welfare impact is derived as well.

The last part of the book concludes with the principal findings and proposes areas for further investigation and research.

Two examples are provided in Appendix C and D. The first concerns the use of GAMS as a platform for economic programming. The second presents some hints for solving inverse problems in the context of the proposed model.

To enable readers to better understand the results in the different chapters, they are accompanied by detailed examples or case studies and summarizing comments. As such, this book can be an ideal reference for students and researchers in many disciplines (infometrics, econometrics, statistics, national accounting, optimal control, etc.) interested in becoming familiar with approaches that reflect the most recent developments in statistical theory of information and their application for stochastic inverse problem modelling. Last but not least, the discussion in this book is limited to technical issues; it does not cover the philosophical implications of non-extensive entropy, whether general or within the discipline of economics.