

3 Optimal Behaviour And The General Equilibrium Model

3.1 Introduction

The fact that the model is “computable” means that a numerical solution exists (e.g., Arrow-Debreu, 1954; Mc Kenzie, 1959; Ginsburgh and Keyzer, 1997), and “general equilibrium” refers to simultaneously matching demand and supply on all markets.

In the example below, note the difference between a partial and a general equilibrium in the traditional way of analysing a market handed down by the Marshall and Walras schools. Let us suppose a Cobb Douglas two-sector economy with two commodities X_i two sector inputs L_i, K_i (labour and capital sectors) and two sector income Y_i , with $i = 1, 2$. Then, the partial equilibrium model is defined by the next optimal program:

Objective:

$$\text{Max}_{x_1, x_2} \longrightarrow U = X_1^\gamma X_2^{1-\gamma}$$

Market clearance:

$$Y_i = X_i \quad i = 1, 2$$

Production:

$$Y_i = A_i L_i^{\alpha_i} K_i^{1-\alpha_i}$$

Resource constraints:

$$L_1 + L_2 = \bar{L}$$

$$K_1 + K_2 = \bar{K}$$

In the case of a general equilibrium, we need to add an income balance restriction to ensure that all inflows and outflows are balanced.

Income balance:

$$P_1 X_1 + P_2 X_2 \leq w\bar{L} + r\bar{K}$$

with p_i ($i = 1, 2$), w , r representing the prices of the two commodities, the sectors labour and capital respectively.

Let us now generalize the above formulation and consider a simple economy with m finite number of producers, n finite number of consumers, r commodities, and let us suppose that the Walras hypotheses are fulfilled. Thus, under these conditions, let us present, below, the behavioural functions of economic representative agents and conditions of market equilibrium.

Producer behaviour. Each producer, j ($j = 1..m$), is confronted with a set of possibilities of production v_j , the general element v_j of which is a program of production with dimension r , where outputs have a positive sign and inputs a negative sign. The objective of each producer is to select, for a given price p ($p = 1..r$), an optimal program of profits pv_j .

Consumer behaviour. Each consumer i ($i = 1..n$) is supposed to have an initial endowment of goods w_i (outputs or inputs) that the consumer is ready to exchange against remuneration by the producer.

Thus, the consumer is confronted with X_i possibilities of consumption of which the general element is x_i with dimension r . The consumer is never saturated in consuming X_i and his endowment w_i allows him to survive. For a given price p of dimension r , consumer i has the objective of maximizing total utility $U_i(x_i)$ under his given budgetary constraints:

$$pw_i + \sum_j \theta_{ij} pv_j = px_i \text{ with } x_i \in X_i$$

where θ_{ij} ($i = 1, 2, \dots, n$; $j = 1, 2, \dots, m$) is a fraction of profits realized by the producer j and transferred to consumer i .

Producer function. m producers maximize individual total profits:

$$\text{Max } pv_j = \tilde{p}\tilde{v}_j \quad (5.1)$$

subject to:

$$v_j \in V_j$$

Consumer function. n consumers maximize individual total utility:

$$\text{Max } U_i(x_i) = U_i(\tilde{x}_i) \quad (5.2)$$

subject to:

$$\tilde{p}x_i = \tilde{p}w_i + \sum_j \theta_{ij} \tilde{p}\tilde{v}_j \quad (5.3)$$

$$x_i \in X_i$$

This is definitely a general equilibrium solution $(\tilde{p}, \tilde{v}_j, \tilde{x}_i)$ from a decentralized system.

Market clearance. Excess demand for r goods is not positive:

$$\sum_i \tilde{x}_i - \sum_j \tilde{v}_j - \sum_i w_i \leq 0 \quad (5.4)$$

Commodities with supply excess, i.e., free commodities, have price zero while other commodities have a positive price:

$$\tilde{p}(\sum_i \tilde{x}_i - \sum_j \tilde{v}_j - \sum_i w_i) = 0 \quad (5.5)$$

where

$$pv_j = \tilde{p}\tilde{v}_j$$

A general equilibrium solution. This is definitely a general equilibrium solution $(\tilde{p}, \tilde{v}_j, \tilde{x}_i)$ guarantying that each of the markets will have realizable equilibrium. This, too, is an equilibrium for a decentralized economy since it guarantees compatibility of consumer and producer behaviours (Equations 5.1 and 5.2). This is a competitive equilibrium. The price from Equation (5.3) is imposed on all actors of the market.

3.2 Economic Efficiency Prerequisites for a Pareto Optimum

The purpose of this section is to clarify the connection between the general equilibrium model and the optimum Pareto state. This will allow us in the next chapter to go beyond such an equilibrium and to analyse impact on social welfare. We must then check whether or not the three conditions below are fulfilled.

a) **Equality of marginal rates of technical substitution for different producers.**

Let us limit our generalization to an economy with two goods q_1 and q_2 , and two limited inputs x_1 and x_2 , for two respective producers.

$$q_1 = f_1(x_{11}, x_{12}), \text{ for producer 1,}$$

$$q_2 = f_2(x_{21}, x_{22}), \text{ for producer 2,}$$

This means that $\bar{x}_1 = x_{11} + x_{21}$ and $\bar{x}_2 = x_{12} + x_{22}$

Let us maximize the quantity produced of q_1 under restriction of known quantity \bar{q}_2 .

Using the Lagrange multiplier, we have:

$$L = f_1(x_{11}, x_{12}) + \lambda[f_2(\bar{x}_1 - x_{11}, \bar{x}_2 - x_{12}) - \bar{q}_2]$$

Finally, we get:

$$\frac{\frac{\partial f_1}{\partial x_{11}}}{\frac{\partial f_1}{\partial x_{12}}} = \frac{\frac{\partial f_2}{\partial x_{21}}}{\frac{\partial f_2}{\partial x_{22}}} = TmST_1 = TmST_2$$

The Pareto criterion having been satisfied, it becomes impossible to increase q_1 without decreasing q_2 and vice versa.

- b) **Marginal rate of substitution of products for different consumers.** Let $U = f(q_1, q_2)$ be the total utility of any consumer and let q_1 and q_2 be the quantities consumed of two products. Assuming a constant level of total utility, the next relations follow:

$$dU = \frac{\partial U}{\partial q_1} dq_1 + \frac{\partial U}{\partial q_2} dq_2$$

$$-\frac{dq_2}{dq_1} = TmSP = \frac{\frac{\partial U}{\partial q_1}}{\frac{\partial U}{\partial q_2}}$$

where

$$\frac{\partial U}{\partial q_1}$$

and

$$\frac{\partial U}{\partial q_2}$$

are marginal utilities of the two goods.

As for the first condition, limiting our generalization to two consumers and two products which supply them, then one can pose:

$$U_1 = f_1(q_{11}, q_{12})$$

and

$$U_2 = f_2(q_{21}, q_{22})$$

U_1 and U_2 represent levels of utilities for the two consumers. The quantities q_1 , q_2 are, respectively, consumed by consumer one and two. Thus, maximizing the utility of consumer 1 under the restriction of a given quantity of consumer 2 and using the Lagrange multiplier, we obtain:

$$L = f_1(q_{11}, q_{12}) + \lambda[f_2(\bar{q}_1 - q_{11}, \bar{q}_2 - q_{12}) - \bar{U}_2]$$

and finally:

$$\frac{\frac{\partial f_1}{\partial q_{11}}}{\frac{\partial f_1}{\partial q_{12}}} = \frac{\frac{\partial f_2}{\partial q_{21}}}{\frac{\partial f_2}{\partial q_{22}}} = TmSP_1 = TmSP_2$$

Thus, the Pareto criterion is satisfied: it is impossible to increase U_1 without decreasing U_2 and vice versa.

- c) **About the marginal rate of transformation.** The marginal rate of transformation of products is a measure in a global economy (and in absolute value) of how much supply of one product will increase as a consequence of an infinitesimal decrease in the supply of a second product.

We have:

$$-\frac{dq_2}{dq_1} = \frac{\frac{\partial q_2}{\partial x_{21}}}{\frac{\partial q_1}{\partial x_{11}}} = TmTP$$

Here the numerator and denominator of the second equality explain marginal physical productivities of inputs. To summarize conditions of attainment of the Pareto optimum or economic efficiency, we must have simultaneously fulfilled the three following prerequisites:

1. $TmST_1 = TmST_2 = \frac{r_1}{r_2}$ (r_i is the price of the input i)
2. $TmSP_1 = TmSP_2 = \frac{p_1}{p_2}$ (p_i is the price of the product i)
3. $TmTP = \frac{cm_1}{cm_2} = \frac{p_1}{p_2}$ (cm_i is the marginal cost of the product i)

In the competitive market case, these three conditions (1, 2, 3 above) are simultaneously fulfilled, and we have:

4. $TmTP = TmST_1 = TmST_2 = \frac{p_1}{p_2}$

At the same time, this is a socially optimum Pareto. Resource allocation is optimal and leads to equality between marginal rate of substitution of products TmSP of consumer and the marginal rate of transformation between products TmTP inside the economy. Out of this optimal point, *better* for an individual would mean *worse* for another. In this context, a competitive market not only guaranties economic efficiencies but also social equity. We shall come back later to this aspect when we present some particular assumptions, shifting the economy from competitive market conditions towards a disequilibrium.