

## 2 Extending to an Environmental Model

Let us start by giving hypotheses for the model. The first hypothesis states that pollution is treated as a collective good associated with private production. This means it enters inside the utility function of all agents. The second hypothesis has pollution being treated in the model as a factor of production since it can be substituted for classical inputs through pollution abatement activities as alternative uses. Thus abatement activities might be regarded as an opportunity cost for productive inputs in the context of general equilibrium. The third and last hypothesis is that the model is of general equilibrium.

**The model relations.** The objective is to maximize utility of representative consumer 1, that is:

$$U_1(X_{i1}, E)$$

under constraints:

- a) the utility of each consumer  $j$  other than 1 is predetermined and is at least equal to ; that is:

$$U_j(X_{ij}, E) \geq U_{jA}$$

$i=1\dots n$   
 $J=2\dots m.$

- b) production functions of all enterprises  $k$  are already predefined, i.e.,

$$Y_k = F_k(L_{ik}, E_k) \geq U_{jA}$$

$K=1\dots h,$   
 $i=1\dots n.$

- c) the general equilibrium constraint is:

$$\sum_1^m X_{ij} - \sum_k^h L_{ik} \leq R_i$$

where:

$L_{ik}$ : inputs involved in production by enterprises,

$X_{ij}$ : consumed goods,

$R_i$ : available resources inside the economy,

$E_k$ : externalities or pollution as an input.

To solve for the above model, we need to formulate its Lagrange as follows:

$$V = U_1(X_{i1}, E) + \sum_{j=2}^m \lambda_j [U_j(X_{ij}, E) - U_{jA}] + \sum_{k=1}^h \mu_k [Y_k - F_k(L_{ik}, E_k)] +$$

$$+ \sum_{i=1}^n \omega_i \left( R_i - \sum_{j=1}^m X_{ij} + \sum_{k=1}^h L_{ik} \right)$$

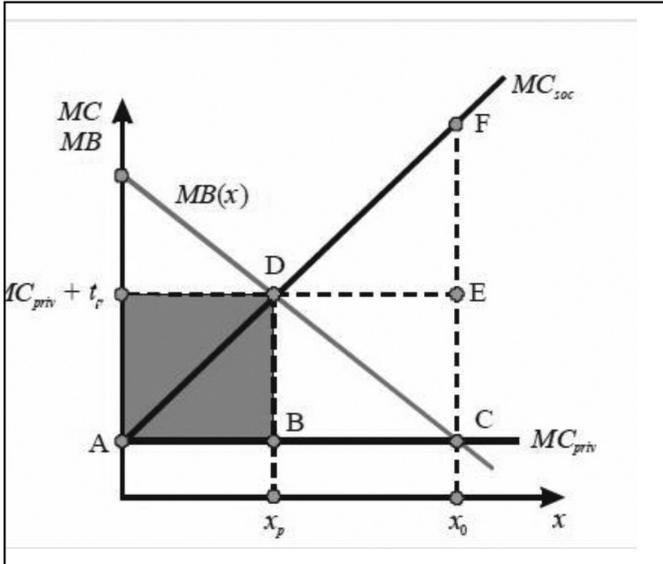


Figure 10: Optimal Pigovian tax

where  $\alpha_j$ ,  $\mu_k$  and  $\omega_i$  are Lagrange multipliers.

Deriving the model for the first order conditions with respect to independent variables  $L_{ik}$ ,  $X_{ij}$  and  $E_k$ , and checking for the second order, we finally obtain the following two optimum conditions:

1. 
$$\mu_k \cdot \frac{\partial F_k}{\partial E_k} = \frac{\partial U_1}{\partial E} + \sum_{j=2}^m \lambda_j \cdot \frac{\partial U_j}{\partial E}$$
2. 
$$\lambda_j \cdot \frac{\partial U_j}{\partial X_{ij}} = \mu_k \cdot \frac{\partial F_k}{\partial L_{ik}}$$

The first condition says that firms will carry out production activity up to the level where marginal productivity of pollutant emission (lhs of cond.1) equals marginal cost of that emission (rhs of cond 1), i.e., the social loss of utilities associated with production.

The second condition means that weighted marginal utilities of consumers  $j$  must equalize weighted marginal productivity of firms  $k$ , for any consumed good  $X_i$  or used input  $L_i$ .

**Criterion of externality internalization.** The optimum presented above represents the social and economic optimum as defined by equalization of marginal profit of producers (mpp) with social marginal cost (smc) of pollution owing to production activity. The principle polluter-payer results when  $smc \geq mpp$ . The next figure displays the essence of the above formula.

Without environmental regulations, the market equilibrium is  $x_0$ . As shown in Figure 10, the welfare loss in such equilibrium is represented by the box CDF, where the marginal social costs (MCsoc) exceed the marginal benefits of consumption (MB(x)) for all units consumed in excess of  $x_p$ . Tx is a tax which is equal to the marginal environmental damage (MED). It would make recovery of the social damages CDF possible. It is represented by a grey area.

Before ending this section, it is worthwhile to add that, apart from taxes, there are many other environmental policy instruments such as efficiency targets, quotas, and tradable permits.

## 2.1 The Model of Carbon Tax

The following models include carbon tax policy within sector activity and institutions and show the uniqueness of the solution in the context of the much revisited CGE model.

### Production of goods:

Production function adopted here is the one taken from Cobb-Douglas technology for the production of two goods (good 1 and good 2) with capital, labour, and the environmental pollution input poll.

$$x_i = A_i n_i^{\alpha_i} k_i^{1-\alpha_i} e^{\gamma \cdot POLLtax}$$

### Tax on Pollution $POLLtax$

Let us define pollution from the production in the following way:

$$POLLtax_i = E_{tax} \cdot x_i$$

$E_{tax}$ : tax on pollution per one unit, one unit of goods used or consumed.

### Demand for production factors:

The cost function with tax can be presented as follows:

$$\phi_i(w, r, x_i) = wn_i + rk_i + TAX_{POLL} \cdot P_i \cdot x_i$$

$w$  = return to labour,  $r$  = return to capital and  $n_i$  and  $k_i$  the amount of labour and capital, respectively. TAXPOLL is the tax rate per unit of pollution. Given this level of return, optimal inputs that minimize the above function for the production of  $x_i$  become:

$$n_i = A_i^{-1} \left( \frac{\alpha_i r}{(1-\alpha_i)w} \right)^{1-\alpha_i} \cdot x_i$$

$$k_i = A_i^{-1} \left( \frac{\alpha_i r}{(1 - \alpha_i) w} \right)^{-\alpha_i} \cdot x_i$$

**Supply of goods:**

With perfect competition, the price of a good equals the marginal cost of production:

$$p_i = \frac{\partial \phi_i(w, r, x_i)}{\partial x_i} = A_i^{-1} \alpha_i^{-\alpha_i} (1 - \alpha_i)^{(\alpha_i - 1)} w^{\alpha_i} r^{(1 - \alpha_i)} + TAX_{POLL} \cdot P_i$$

**Households:**

The income of households is defined as expected:

$$m = wn + rk + \text{taxtransfer}$$

and the budget restriction is:

$$m = p_1 c_1 + p_2 c_2$$

**Utility function:**

Let us assume the household/households to have the classical Cobb-Douglas utility function but where pollution is added as a separate term; that is:

$$U(H) = c_1^\beta \cdot c_2^{1-\beta} \cdot e^{\gamma \cdot POLL}$$

$\beta$  states how the households value their consumption of good 1 and good 2.

$\gamma$  is the parameter that states how the households value pollution. If  $\gamma$  is negative the households are assumed to dislike pollution.

**Demand:**

When we combine the budget restriction and the utility function of the household, we can derive the demand function for the different goods:

$$c_1(p_1, p_2, m) = \frac{\beta m}{p_1}$$

and,

$$c_2(p_1, p_2, m) = \frac{(1 - \beta)m}{p_2}$$

**Equilibrium conditions:**

We have two equilibriums to be formulated below: one concerns the goods market and prices and the other the factor market.

Goods market:

The supply (i.e., the production) should equal the demand (i.e., the consumption of households):

$$x_1 = \frac{\beta m}{p_1}$$

$$x_2 = \frac{(1 - \beta)m}{p_2}$$

*Ceteris paribus*, the price equals marginal cost of production:

$$p_1 = A_1^{-1} \alpha_1^{-\alpha_1} (1 - \alpha_1)^{(\alpha_1 - 1)} w^{\alpha_1} r^{(1 - \alpha_1)} + TAX_{POLL} \cdot P_1$$

$$p_2 = A_2^{-1} \alpha_2^{-\alpha_2} (1 - \alpha_2)^{(\alpha_2 - 1)} w^{\alpha_2} r^{(1 - \alpha_2)} + TAX_{POLL} \cdot P_2$$

### Factor market:

The supply (i.e., the households' total resources) should equal the demand for the different factors, and then we have an optimal factor market:

$$n = A_1^{-1} \left( \frac{\alpha_1 r}{(1 - \alpha_1) w} \right)^{1 - \alpha_1} x_1 + A_2^{-1} \left( \frac{\alpha_2 r}{(1 - \alpha_2) w} \right)^{1 - \alpha_2} x_2$$

$$k = A_1^{-1} \left( \frac{\alpha_1 r}{(1 - \alpha_1) w} \right)^{\alpha_1} x_1 + A_2^{-1} \left( \frac{\alpha_2 r}{(1 - \alpha_2) w} \right)^{\alpha_2} x_2$$

We then have six equations and six endogenously determined variables  $x_1$ ,  $x_2$ ,  $p_1$ ,  $p_2$ ,  $w$  and  $r$ . This means that such an extended CGE model has a solution.

## 2.2 Carbon Tax Model And Double-Dividend Hypothesis

Pigou (Pigou, 1932) suggested imposing a tax per unit of pollution at a rate  $t_\alpha$ , equal to the marginal external damages per unit of pollution. Carbon tax means that not only the level of taxes depends upon the consumed quantity of carbon but also on its consumed quality. The targeted objective being the reduction of more polluting energy (e.g., that which results in  $CO_2$ ) in favour of cleaner energy technologies. At the same time, such a strategy allows for an interiorizing of negative externalities through introduction of an ad hoc tax that is socially equitable. Accordingly, such a tax policy should help in recovering lost macroeconomic equilibrium and social Pareto optimum welfare under the free market hypothesis. However, like any tax, a carbon tax should negatively impact economic growth.

Thus, this leads to a fundamental question as to whether or not the above mentioned positive effects are sufficient to explain its introduction. Tullock (Tullock, 1967)

raised the possibility that government revenue would be “free,” while Terkla (Terkla, 1984) estimated the amount of revenue and the efficiency gains from using it. For Lee and Misiolek (Lee & Misiolek, 1986), the whole benefit of imposing a pollution tax depends on whether it raises revenue. We now show how this emphasis on revenue continues in the double-dividend literature.

David Pearce (1991) is probably the first writer to use the term *double-dividend*:

Governments may then adopt a fiscally neutral stance on the carbon tax, using revenues to finance reductions in incentive-distorting taxes such as income tax or corporation tax. This “double-dividend” feature of a pollution tax is of critical importance in the political debate about the means of securing a “carbon convention.”

Thus, the idea is that budgetary income from a carbon tax will be used to reduce other taxes so that one will get positive effects not only with respect to environmental targets but also economic targets.

### **Validity of the hypothesis (strong form)**

The validity of the double-dividend hypothesis cannot logically be settled as a general matter. For instance, the environmental tax may have its own distorting effects on labour supply and, therefore, can have the same excess burden as a tax on labour income. Each proposal must be evaluated individually. This evaluation must fully specify the policies already in place as well as the reform under consideration.