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The Social Opportunity Cost of Consumption for Canada, 1965 to 1986

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THE SOCIAL OPPORTUNITY COST OF CONSUMPTION FOR CANADA, 1965 TO 1986

Ian M. McDonald University of Melbourne and Queen's University

> Luca Tacconi University of Melbourne DISCUSSION FAPER NO. 776

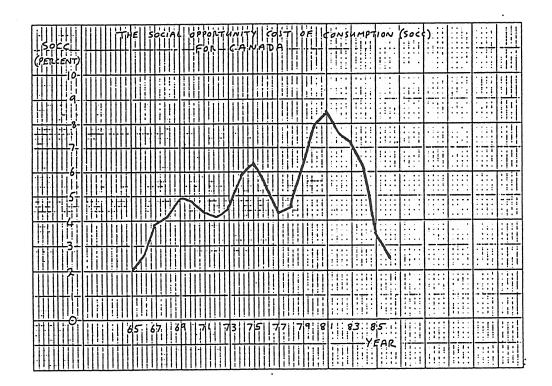
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ABSTRACT

The Social Opportunity Cost of Consumption for Canada, 1965 to 1986

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The Social Opportunity Cost of Consumption for Canada, 1965 to 1986¹

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The Social Opportunity Cost of Consumption (SOCC) for an economy is the rate by which a resident can trade current consumption for consumption in the future. In this paper series for the period 1965 to 1986 for Canada are calculated. As the Fisherian analysis suggests (see e.g. McDonald (1985)) for a small open economy such as Canada the SOCC is determined by foreign interest rates. In this paper foreign interest rates are the basis from which the SOCC for Canada is calculated.

The SOCC is an important determinant of the optimal level of the current account surplus or deficit and the optimal level of investment. For example if the SOCC is high, a resident's optimal decision will be to invest little in the domestic capital stock and, instead, to lend to foreign borrowers. For the economy as a whole this pattern will generate a low aggregate level of investment and a high aggregate level of lending to foreigners. The high level of lending to foreigners will require a surplus on the current account of the balance of payments.

In the public debates over economic management a lot of attention is paid to the aggregate level of investment and the current account outcome of the balance of payments. Commentators frequently argue that these aggregates are at undesirable levels. However much of this comment appears to ignore

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the influence that the SOCC should exert on the desirable levels. This ignorance may be due to a lack of understanding of the role of the SOCC for optimal decision making. It may also be due to a lack of knowledge of what the SOCC is. By attempting to measure the SOCC for Canada, this paper seeks to provide a data series which will be an input into an improved understanding of the optimal levels of the aggregate level of investment and the current account outcome for the Canadian economy.

In measuring the SOCC for Canada, the calculations in this paper are based on the assumption of perfect foresight. In particular it is assumed that the future values of the exchange rate and the consumer price index are known. Perfect foresight is assumed for the following reason. One factor which may cause the actual outcomes of investment and the current account surplus to deviate from their optimal levels is incorrect prediction of the future. Given that forecasts are public goods, it is likely that an inefficiently small amount of resources are allocated by private agents to forecasting. Under-provision of resources for public goods is the usual implication from economic analysis. This under-provision is a type of market failure. This market failure can be expected to lead to sub-optimal decision-making. If, instead, the perfect foresight estimates of the SOCC calculated here are used to construct optimal levels of investment and the current account surplus then the sub-optimality associated with incorrect forecasts will be avoided.

One weakness of the perfect foresight assumption should be borne in mind. It is probably the case that perfect foresight is not socially optimal because it is costly to improve the accuracy of forecasts. There must come a point when further improvement in the accuracy of prediction is not worth the cost. However one would, given the present state of knowledge in this area,

have little idea on the socially optimal degree of predictive accuracy. Given the impossibility at present of defining a socially optimal prediction, the best alternative is to assume perfect foresight.

A Single Asset Model

To initiate the development of the concept of the SOCC, assume there is only one overseas asset available for purchase. An individual, taken to be a Canadian resident, considers the rate of return from refraining from consumption now (at time t), in order to increase consumption in the future (at time t+n). Assume current consumption is reduced by \mathbf{X}_{t} units of consumption. This makes the amount of $P_t^{X_t}$ of domestic currency available for lending where $P_{\underline{t}}$ is the price of a unit of consumption at time t. At an exchange rate of F_{t} units of foreign currency for one unit of domestic currency, foreign assets to the value of $F_tP_tX_t$ can be purchased. If these assets are expected to earn a nominal rate of return of i per period then, after n periods, the loan will be worth $(1+i)^n F_t P_t X_t$. Deflating this amount by F_{t+n} , the value of the exchange rate expected at time t+n, gives the expected value of the loan in units of domestic currency at time t+n and deflating further by the price of a unit of consumption at time t+n, P_{t+n} , gives the expected value in terms of units of consumption of the loan at t+n. This is labelled X_{t+n} . Thus

$$X_{t+n} = \frac{(1+i)^{n} F_{t} P_{t} X_{t}}{F_{t+n} P_{t+n}}$$
 (1)

The SOCC, r, is then defined by

$$(1+r)^n = \frac{x_{t+n}}{x_t} \tag{2}$$

Substitution of (1) into (2) gives the familiar formula

$$1+r = \frac{1+i}{1+p} \tag{3}$$

where

$$1+p = \left(\frac{F_{t+n}^{P_{t+n}}}{F_{t}^{P_{t}}}\right)^{\frac{1}{n}}$$
 (4)

The formula is familiar in that one plus the real rate of return is equal to one plus the nominal rate of return divided by one plus the expected rate of inflation. However the definition of the expected rate of inflation is not so familiar. It depends on both the expected change in the exchange rate and the expected change in the domestic price of a unit of consumption.

It was explained in the introduction that the aim of the paper is to measure what the SOCC would have been if economic agents had had perfect foresight. For the expected rate of inflation (p) the assumption of perfect foresight suggests the use of actual values for inflation. That is the approach taken in this paper. So to define p it is assumed that the predictions agents make of the future values of the price level and of the exchange rate are the actual future values. For example, to calculate the SOCC over a 10 year period starting in 1965 it is assumed that in 1965 economic agents predicted for 1975 the values of P and F that actually occurred in 1975. (For more recent years, where the end of the forecast period has not actually occurred, predicted values are manufactured using an extrapolative technique.)

Adjusting the Nominal Interest Rates for Imperfect Foresight

The aim of the paper is to construct a series of the SOCC for Canada assuming that agents had perfect foresight. However the market nominal rates

of interest observed are affected by actual predictions of inflation. The Fisher effect says that the market nominal rate of interest moves one for one with the expected rate of inflation. So if economic agents made incorrect predictions about the future rate of inflation then the market nominal rate of interest will differ from the perfect foresight nominal rate of interest. For example, if, as seems likely, in the late 1960's people underpredicted the future rate of inflation then that error would have reduced the market nominal rate of interest. To attempt to create perfect foresight would require for the late 1960's an upward adjustment to the market nominal rate of interest. That adjustment is explained in this section.

Consider Figure 1. In the figure IS and LM curves for the world economy are shown. The curves IS (actual) and LM (actual) are drawn assuming that the expected rate of inflation held by agents is less than the perfect foresight rate of inflation, as seems reasonable for the late 1960's. The intersection of IS (actual) and LM (actual) determines the market nominal interest rate, i_a, and the level of real output, Y₁. If, instead, economic agents had had perfect foresight they would have predicted a higher rate of future inflation. This would have raised the expected nominal rate of return to investment and shifted up the investment function. That would have shifted the IS curve upwards. A shift is shown to IS (perfect foresight). Assuming governments had aimed at the same level of aggregate demand, monetary policy would have been tighter yielding an LM curve at LM (perfect foresight). So with perfect foresight the market nominal rate of interest would have been i_p.

To calculate the adjustment factor for converting the market nominal interest rate that is observed into the perfect foresight nominal interest rate, we assume that the implicit change in the expected rate of inflation

does not affect the real rate of interest. In practice there are likely to be distortions in the economic system through which changes in the expected rate of inflation may be non-neutral and affect the real rate of interest. For example, a tax system which is non-neutral with respect to inflation. But it is not easy to predict the net effect of the many possible distortions. Some distortions would cause the nominal interest rate to under-adjust and others to over-adjust to changes in the expected rate of inflation (see Jha, Sahu and Meyer (1989) for a formal review of the various distortions). We assume here that these distortions are offsetting and so that the real rate of interest is independent of the expected rate of inflation.

Defining $\mathbf{p}_{\mathbf{a}}$ as the expected rate of inflation which people actually held and $\mathbf{i}_{\mathbf{a}}$ as the actual nominal rate of interest, the real rate of interest is given by

$$1+r = \frac{1+i_{a}}{1+p_{a}}$$
 (5)

To define the adjustment factor for the nominal rate of interest which takes account of using the expected rate of inflation with perfect foresight, define the perfect foresight nominal rate of interest, i_p , by

$$1+r = \frac{1+i_{p}}{1+p_{p}}$$
 (6)

where p_f is the expected rate of inflation with perfect foresight. Combining (5) and (6) gives the perfect foresight nominal rate of interest as

$$1+i_{p} = \left(\frac{1+p_{a}}{1+p_{p}}\right)(1+i_{a})$$
 (7)

The adjustment factor, $(1+p_a)/(1+p_p)$ is labelled A.

In the empirical work in this paper nominal interest rates from several countries are used. For each country adjustment factors using equation (7) are calculated. To make these calculations a series for the expected rate of inflation (p_a) is constructed from an autoregression on the GDP deflators for the country concerned. For example, to adjust the West German nominal interest rate a series which forecasts the West German GDP deflator is constructed. The construction of these forecasts is explained in greater detail in the empirical section below.

The adjustment factor has been explained in terms of a scenario where interest rates bear all the brunt of the adjustment for the divergence between perfect foresight and actual foresight. It is possible to conceive of an alternative scenario where exchange rates bear some of the adjustment. This alternative scenario would yield a similar adjustment factor and so the adjustment procedures undertaken here also apply to cases where exchange rates adjust. There is no need to make an additional adjustment for hypothetical exchange rate movements.

Allowing for Several Assets

There is not a single foreign asset. Instead a Canadian resident wishing to shift consumption forward in time can choose to hold wealth in a number of foreign assets. Assets can be categorized by length to maturity, risk type and country. In this paper two SOCC's are calculated for Canada, one based on a portfolio of assets which have five years to maturity and the other on assets with ten years to maturity. The assets in the portfolio

have a similar risk type (government bonds) but are from different countries. For the 5-year SOCC the portfolio consists of 5-year government bonds from four countries (U.S., U.K., W. Germany and Switzerland). For the 10-year SOCC the portfolio consists of 10-year government bonds from three countries (U.S., U.K., W.Germany). To combine these assets the share of each asset in the portfolio has to be calculated. The basis for calculating the asset shares is set out in this section.

To construct the SOCC it is assumed that the shares of assets are chosen to minimize the variance of the return of the portfolio. Identifying variance with risk, the shares give the minimum-risk portfolio. To calculate the variance-minimizing shares define the return on a portfolio of n assets (R) as

$$R = \sum_{i=1}^{n} a_i r_i, \text{ with } \sum_{i=1}^{n} a_i = 1,$$

$$i = 1$$
(8)

where a_i is the share of asset i and r_i is one plus the rate of return on asset i. The variance of the return on the portfolio (V) is

$$V = E[R^2] - (E[R])^2$$
 (9)

where E[] is the expectations operator. The shares a_i are chosen to minimize V. So the n-1 first order conditions are

$$\frac{\partial V}{\partial a_i} = \frac{\partial E(R^2)}{\partial a_i} - \frac{\partial (E(R))^2}{\partial a_i} = 0 \quad \text{for i=1...n-1}$$
 (10)

The nth equation needed to determine the n shares is the constraint

 $\Sigma a_i = 1$. In the empirical work for Canada four assets are used for the i=1

5-year real interest rate and three assets are used for the 10-year real interest rate. The specific form of the first order conditions (10) for the

four and three asset cases are derived in the appendix.

A basic assumption underlying the method of calculation is that the borrowing and lending rates of interest are equal. Since government bond rates are used in the empirical section this is a reasonable assumption. The Canadian government can be reasonably considered as risky as (and no more risky than) the governments whose bonds are used (U.S., U.K., West Germany and Switzerland). For the calculated rate of interest to be a borrowing rate one can imagine the Canadian government issuing bonds denominated in foreign currency (U.S. dollars, pound sterling, German marks or Swiss francs). Furthermore the optimal shares are not constrained to be positive. A negative value for an a implies the coexistence of borrowing and lending. Calculations

The construction of the series for the SOCC for Canada can be divided into three stages.

(1) <u>Calculating a series of adjustment coefficients for the interest rates</u> <u>of each country</u>

For each country (U.S., U.K., W. Germany and Switzerland) a series for the expected rate of inflation was calculated by estimating an autoregressive equation on the country's GDP deflator. These autoregressive equations were used to forecast future values of the GDP deflator. From these forecasts the expected rate of inflation was calculated.

Before running the autoregression the GDP deflator series was tested in its levels, its first difference and its second difference to find a stationary form. The stationary form of the series was used in the autoregression. Having determined the stationary form, the predictions for the GDP deflator in 5 years and 10 years time were calculated as follows. For a particular year (say 1965) the series for the GDP deflator up to that

year was used to estimate an autoregression. The current value was regressed on lagged values plus a constant. The number of lags and the inclusion of the constant term was decided on the basis of significance of the coefficients, determined by inspection of t-values. Then the regression equation is projected 5 years and 10 years ahead to generate the 5 and 10 year forecasts of the GDP deflator. These predicted values of the GDP deflator are used to generate the expected rate of inflation. Then the expected rate of inflation is used to calculate the adjustment factor, as defined in equation (7). This procedure is repeated for each year.

The actual GDP deflator, the forecast GDP deflator and the adjustment coefficients are shown in tables 1 to 7. Tables 1 to 4 cover 5 year rates and tables 5 to 7 cover 10 year rates. These tables should be read as follows. Take the first row in table 1. This refers to the year 1965 for the U.S. In 1965 the 5-year bond rate (the nominal interest rate) was 4.69 percent. The U.S. GDP deflator five years hence (i.e., in 1970) was 43.20. The forecast in 1965 of the U.S. GDP deflator for five years ahead, based on the estimated forecasting equation, was 38.86. This forecast implies that inflation was underpredicted for the 1965 to 1970 period in the U.S. Based on this underprediction the adjustment factor for the nominal interest rate was 1.0214 which is an adjustment of the order of about two percentage points.

(2) Calculating country real rates of interest for Canada

The sixth column in table 1 shows the actual inflation rate for the next five years of the price series F_tP_t . This is the Canadian consumer price index converted into foreign currency. Thus in the first row of table 1 the entry 1.0482 means that the Canadian consumer price index converted into U.S. dollars grew by 4.821 percent per annum over the five year period 1965 to

1970. Using this inflation rate to deflate the 5-year U.S. bond rate from column 2 yields a U.S. real rate of interest for Canada of -0.13 percent for 1965 (see column 8). However if the U.S. 5-year bond rate is adjusted by the adjustment factor in column 5 and then deflated by the inflation rate of column 6, an adjusted figure for the U.S. real rate of interest for Canada of 2.01 is obtained (see column 7).

(3) Calculating the SOCC for Canada

The uncertainty facing Canadian lenders is the uncertainty about the future values of P_tF_t , the Canadian consumer price index expressed in foreign currency. Nominal interest rates from holding bonds to maturity are, of course, known with certainty. Variances and covariances of the price series P_tF_t are used to calculate the shares of each country's asset in the portfolio of assets. For the portfolio of 5-year assets and for 10-year assets the shares are given in Table 8. These shares are then used to sum the country real rates of interest for Canada to yield the SOCC for Canada. The SOCC's are shown in Table 9. Four rates, unadjusted and adjusted five and ten year rates, are shown in Table 9.

Discussion of the Calculated Series for SOCC

In chart 1 both the series for the SOCC for Canada based on 5 year bond rates and the country specific 5 year real rates of interest are drawn. As can be seen, for the country specific series there are large divergences between the U.S., on the one hand, and the U.K., W. Germany and Switzerland on the other. Furthermore the interest rate series derived from the European countries show a greater volatility than does the interest rate series derived from the U.S. The main reason for the differences in behaviour between the U.S. interest rate and the European interest rates lies in the currency movements over the period. The Canadian dollar has moved more

closely with the U.S. dollar than with the European currencies. For example, the relatively low levels of the U.K., W. Germany and Swiss real rates of interest for Canada around 1978 are due to the increase in the value of the Canadian and U.S. dollars in the European currencies in the 1981 to 1983 period.

The divergences between the country specific real rates of interest are large enough to cause one to question their consistency. Would international arbitrage have prevented the occurrence of divergences of the size shown in Chart 1?² The aim of the paper is to provide a measure of what SOCC would have been if economic agents had had perfect foresight. Perhaps with perfect foresight international flows of capital would have narrowed the gap between the country real rates and so changed the country real rates. This narrowing would have lead to a different measure of SOCC. It would also have changed the shares in the risk-minimizing portfolio since these are affected by the pattern of exchange rates which would have been sensitive to the international capital flows. To adjust the measurement of SOCC to allow for the possible effects of international arbitrage would require a model of the international capital market that is beyond this paper. So instead the measures of SOCC are not adjusted for this effect. This might not be an important omission. As the shares in table 8 show, the portfolio of assets

²To avoid misunderstanding, it should be realised that the large size of the divergences between the country-specific real rates of interest calculated in this paper does not, in itself, prove that international arbitrage is weak. The country-specific real rates of interest that wealth-holders would have anticipated and acted on at the time would not have been the same as the country-specific rates calculated here. In this paper we have tried to calculate perfect foresight rates. In practice wealth-holders may not have had perfect foresight about future movements in prices and exchange rates. Indeed in this paper we have argued that expectations about prices were sufficiently in error to have required an adjustment to the country nominal rates of interest.

which minimizes risk is dominated by U.S. bonds. In Chart 1 this dominance is reflected in the closeness of the SOCC for Canada with the U.S. real rate of interest. Now if, in reducing the interest rate gaps, international arbitrage had had more effect on the European rates and less effect on the U.S. rates then the bias in our measure of SOCC may be rather small.

From table 9 it can be seen that the adjustment made to nominal rates of interest in the four countries to allow for errors in predicting inflation transformed negative values for the SOCC for Canada into positive values for each year in the period 1965 to 1971. Later on, after 1981, the adjustment tends to reduce the SOCC, reflecting the tendency to overpredict inflation during the period of slowdown of inflation. As 1986 approaches the adjustment goes to zero by definition because the forecasting equation is used to predict the future values (i.e., post-1986) of the GDP deflators.

The fluctuations in the series for the SOCC for Canada are quite large. For the series based on adjusted nominal rates of interest the peak value, which is for the year 1980, is 9.39 percent. This peak is 8.46 times the lowest rate of 1.11 percent which occurred in 1965. The general trend of the series is increasing to 1980 and then decreasing to 1986.

For the 10 year rates, Chart 2 shows significant divergences between the country rates, although these divergences are not as great as for the 5 year rates. The difference is due to the smoothing effect on exchange rate fluctuations of the 10 year time horizon. As for the 5 year rate, the U.S. rate dominates the SOCC for Canada. The high weight of the share of U.S. assets in the risk-minimizing portfolio shown in table 8 yields a series of the SOCC for Canada which follows very closely the U.S. real rate of interest for Canada. The SOCC for Canada based on 10 year bond rates shows a significant amount of fluctuation. The highest value in 1980 of 9.05 percent

is 4.62 times the lowest value of 1.96 percent for 1986. However this is less than the fluctuation of the SOCC based on 5 year bond rates. The pattern of the 10 year series is similar to the 5 year series. The SOCC based on 10 year bond rates increases from 1965 to a peak in 1980 and then declines to 1986. There is also, as with the 5 year series, a small local peak in 1975.

As noted above the divergences between the country 10 year real rates are smaller than the divergences between the country 5 year real rates. For this reason the 10 year real rates are not such an affront to the laws of international arbitrage. Because of this the SOCC based on 10 year rates may be judged a more reliable measure than the SOCC based on 5 year rates.

Conclusion

An important part of macroeconomic performance is the provision made for future consumption purposes. A determinant of the socially optimal level of this provision is the social opportunity cost of consumption (SOCC). This paper has calculated the SOCC for Canada for the period 1965 to 1985.

The series calculated for SOCC show considerable variation. The intertemporal tradeoff faced by Canada between present and future consumption is not constant. Generally the SOCC for Canada rises from 1965 to a peak around 1980-1 and then declines to 1986. The peak value is at least 4 times the size of the lowest value recorded in 1965.

The large fluctuations in the SOCC are derived from an estimating procedure in which nominal interest rates were adjusted to remove the influence of incorrect inflationary expectations. In the late 1960's and early 1970's the estimates reported here suggest that people underpredicted future the rate of inflation. Because of this underprediction of inflation, in this

paper the nominal interest rates were adjusted upwards for these years. The aim of this procedure was to generate a series for the SOCC based on the assumption of perfect foresight. If this adjustment had not been made the SOCC would have shown even greater fluctuations. As it is, the fluctuations in the series reported here cannot be blamed on incorrect foresight. That is to say the low rates for the SOCC in the late 1960's and early 1970's were not due to an underprediction of inflation. And similarly the high rates in the 1980's were not due to an overprediction of inflation. This makes more intriguing the question of why the SOCC has fluctuated by such a large amount. Does the fluctuation only reflect "real" factors such as changes in technology, tastes and the demographic composition of these economies? These questions deserve study.

The variation in the SOCC for Canada during the period 1965 to 1986 is so large that it is inconceivable that the socially optimal levels of investment and the current account surplus have been constant over this period. This implies that the economic debate over the performance of these aggregates is ignoring an important influence. Hopefully this omission will be rectified by further work on the SOCC and its implications for the socially optimal levels of investment and the current account surplus.

Table 1

U.S. Real Interest Rate for Canada - 5 Year Rate

al ate for (%) unadiusted	rx100		-0.13	-0.13	18	91.0-	1 24	-1.87	-0.39	1.44	2.24	2.21	1.04	-0.15	1.10	4.32	8.29	08.6	8.64	7.91	78.9	• •	• •
U.S. real interest rate Canada (%)	r×100		Z.01	7.52	60.0	3,90	3.07	2.66	4.30	5.82	6.31	6.84	5.90	3.91	4.03	6.01	8.51	9.21	8.20	7.43	5.87	2.86	•
Inflation rate for Canadian CPI converted to U.S.	$ \frac{\mathbf{F_tP_t}}{\mathbf{F_t+5^Pt+5}} $ $ = 1+p $	1 0/02	1.040Z	1.0557	1.0641	1.0718	1,0791	1,0783	1.0680	1,0584	1,0543	1.0515	1.0600	1.0781	1.0774	1.0593	1.0422	1.0349	1.0301		1.0455	1.0549	•
Adjustment factor for U.S. 5 year bond rate	A	1 021%	1.0214 1.0266	1.0287	1,0317	1,0397	1.0436	1.0462	1.0470	1.0432	1.0398	1.0452	1.0481	1.0406	1.0290	1.0163	1.0020	0.9946	0.9960	0.9956	0.9956	0.9981	0.9999
5 year ahead forecast of U.S. GDP deflator	l I	α	• «	41.67	7		9	2	5.	i.	7	72.03	ė.	3		•	•			•		134.56	
U.S. GDP deflator 5 years ahead	7. 	02.57	, L	48.00								89.85											138.34
U.S. 5 year bond rate (%)	ix100	,	, , ,	5.33	.,	Τ.		ω	۲,	er.	· .	٧.	٠.	Ψ.	0, 1	٠.	12.88	9	σ.	ς.	;		9
Description	Notation	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	19/6	1977	19/8	6/6T	1980	1981	1982	1983	1984	1985	1986

U.K. Real Interest Rate for Canada - 5 Year Rate

escription	U.K. 5 year bond rate (%)	U.K. GDP deflator years ahead	5 year ahead forecast of U.K. GDP deflator	Adjustment factor for U.K. 5 year	Inflation rate for Canadian CPI converted to U.K.	U.K. real interest rate Canada (%)	e for) unadinsted
otation	1×100	1	1		$ \left(\frac{F_{t}F_{t}}{F_{t+5}F_{t+5}}\right)^{0.2} $ = 1+p		rx100
1965 -966 -967 -968 -970 -971 -975 -976 -978 -980 -980 -981	6.70 6.80 7.24 8.31 8.35 7.20 7.36 9.40 11.85 10.81 10.52 12.17 13.10 14.39 13.67 11.60	28.35 30.85 33.20 35.85 44.80 53.90 61.60 69.25 78.20 91.75 115.75 123.15 129.00 135.55 141.95 141.95 163.29	8.7.20.00.00.00.00.00.00.00.00.00.00.00.00.	1.0190 1.0286 1.0286 1.0358 1.0771 1.0771 1.0002 0.9947 1.0198 1.0250 1.0047 0.9408 0.9666 0.9676	1.0788 1.0767 1.0626 1.0640 1.0822 1.1223 1.1519 1.0599 1.0338 1.0483 1.1762 1.1762 1.1762 1.1762 1.1762	0.78 2.03 4.56 6.63 7.75 2.88 7.72 7.79 7.79 2.84 -8.69 -5.71 9.07	-1.09 -0.81 0.94 1.79 0.11 -4.49 -6.80 5.86 3.21 5.71 0.14 -2.90 -3.84 0.23 5.71 3.52
985 986	0.4 9.5		192.25 196.94	0.9912	1.0187 1.0419	7.48 5.00	11.13 8.41 5.10

West German Real Interest Rate for Canada - 5 Year Rate

n real ate for (%) unadjusted	rx100	4.67 5.53 7.25 8.60 8.58 7.42 6.89 8.37 10.10 10.13 6.61 1.00 -4.53 -7.14 -6.54 0.40 8.73 11.98 7.73
West German real interest rate fo Canada (%) adjusted unad	rx100	6.01 7.81 10.19 12.27 11.52 7.98 7.98 8.97 9.07 6.37 -7.20 -7.25 -7.20 7.33 10.22 11.23 7.13
Inflation rate for Canadian CPI converted to West German currency	$\left(\frac{\mathbf{F}_{\mathbf{t}}\mathbf{P}_{\mathbf{t}}}{\left(\mathbf{F}_{\mathbf{t}+5}\mathbf{P}_{\mathbf{t}+5}\right)}\right)^{0.2}$ = 1+p	1.0241 1.0149 0.9919 0.9795 0.9882 1.0044 1.0117 1.0061 0.9882 1.0526 1.1531 1.1607 1.0901 1.0901 1.0903 0.9530 0.9530
Adjustment factor for West German 5 year bond rate	A	1.0128 1.0216 1.0274 1.0271 1.0052 1.0052 0.9941 0.9977 1.0021 0.9988 0.9988 0.9897 0.9897 0.9897 0.9897
5 year ahead forecast of West German GDP deflator	1	58.20 59.41 61.11 63.23 69.56 81.16 85.66 87.42 96.37 102.51 103.17 109.19 113.91 119.67 129.79 137.27 141.38
West German GDP deflator years ahead	1	62.00 66.10 69.95 74.65 79.50 83.30 86.40 89.85 97.70 106.30 110.35 113.20 115.55 118.60 124.61 128.36 132.69 137.48
West German 5 year bond rate (%)	1x100	7.20 6.38 6.38 7.30 7.89 8.14 9.04 7.68 6.32 6.32 7.92 7.92 7.92 6.00
escription	otation	965 966 967 969 970 971 975 975 978 978 980 981

Swiss Real Interest Rate for Canada - 5 Year Rate

U.S. Real Interest Rate for Canada - 10 Year Rate

¥	1 ate for (%) unadjusted	rx100	-1.65 -1.44 -0.77 0.03 0.67 -0.67 -0.73 1.98 2.66 4.95 7.47 7.47 8.66 6.49
	U.S. real interest rate Canada (%) adjusted ur	rx100	1.73 2.32 3.23 4.21 5.18 6.00 6.00 7.49 7.09 8.26 7.09 8.26 7.09
	Inflation rate for Canadian CPI converted to U.S.	$ \frac{\begin{bmatrix} \mathbf{F_tP_t} \\ \mathbf{F_{t+10}}^T \\ \mathbf{F_{t+10}} \end{bmatrix}^{0.1} }{\begin{bmatrix} \mathbf{F_tP_t} \\ \mathbf{E_{t+10}} \end{bmatrix}^{0.1}} $	1.0636 1.0653 1.0618 1.0613 1.0630 1.0652 1.0690 1.0731 1.0468 1.0468 1.0468 1.0442 1.0442 1.0442 1.0477 1.0477
	Adjustment factor for U.S. 10 year bond rate	Ą	1.0344 1.0382 1.0404 1.0408 1.0478 1.0502 1.0594 1.0255 1.0196 1.0130 1.0196 0.9994 0.9977 0.9977 0.9990
	10 year ahead forecast of U.S. GDP deflator	1	43.65 44.82 46.96 50.09 52.98 56.36 59.47 63.60 70.55 78.81 84.34 90.00 98.48 108.76 120.55 146.97 152.42 166.46
	U.S. GDP deflator 10 years ahead	1	61.20 65.20 69.75 75.40 82.15 89.85 97.00 101.95 112.65 115.80 119.52 123.80 123.30 143.55 166.31
	U.S. 10 year bond rate (%)	1×100	4.60 5.36 6.16 7.01 6.76 6.19 7.20 7.92 8.93 112.69 113.46 11.82 11.82 11.57 8.03
)escription	lotation	965 966 967 968 970 971 975 975 978 979 981 982 983

U.K. Real Interest Rate for Canada - 10 Year Rate

	11 (%) unadiusted	rx100	-3.00 -2.33 0.30 1.55 0.31 0.37 0.37 4.57 4.57 7.59 8.23 8.23
	U.K. real interest rate Canada (%)	rx100	2.64 2.50 8.77 11.23 8.60 4.56 3.52 -0.60 0.06 4.69 7.04 7.04
	Inflation rate for Canadian CPI converted to U.K.	$ \frac{\begin{bmatrix} F_{\mathbf{t}} P_{\mathbf{t}} \\ F_{\mathbf{t}+1} O^{P}_{\mathbf{t}+1} 0 \end{bmatrix}^{0.1} }{\begin{bmatrix} F_{\mathbf{t}+1} O^{P}_{\mathbf{t}+1} O \end{bmatrix}^{0.1}} $	1.1003 1.1137 1.0982 1.0796 1.0710 1.1193 1.1165 1.0862 1.0653 1.0653 1.0733 1.0783 1.0783 1.0271 1.0271
	Adjustment factor for U.K. 10 year bond rate		1.0582 1.0683 1.0766 1.0766 1.0845 1.0953 1.0596 1.0556 0.9724 0.9863 0.9863 0.9447 0.9447 0.9892 0.9892 0.9892
	10 year ahead forecast of U.K. GDP deflator		30.63 31.83 33.10 34.76 36.93 49.23 64.88 71.69 94.72 179.40 222.71 180.20 158.40 158.40 158.40 249.72 254.72 249.12 271.68
	U.K. GDP deflator years ahead	1	53.90 61.60 69.25 78.20 91.75 105.75 115.75 129.00 135.55 141.95 141.95 141.95 173.01 183.92 173.01 183.92 223.65 223.65 223.28
	U.K. 10'year bond rate (%)	1×100	6.73 6.86 7.26 8.29 8.76 8.35 8.42 12.91 14.14 12.57 11.78 13.87 11.78 11.15 10.34
081)escription	Votation	965 966 967 969 970 971 975 975 978 978 980 981

Table 7

West German Real Interest Rate for Canada - $10\,\mathrm{Year}$ Rate

<pre>in real ate for (%)</pre>	unadjusted	r×100	5.72	5.73	6.52	7.79	8.61	7.19	4.87	3.31	2.42	1.93	2.95	4.10	2.88	2.34	3.18	5.57	6.92	7.29	7.72	7.01	4.73	2.12
West German real interest rate for Canada (%)	adjusted	r×100	7.69	7.89	9.02	10.39	10.35	7.01	4.65	3.62	2.42	1.46	2.28	3.66	2.24	1.57	2.21	4.50	5.83	6.26	6.95	6.52	4.41	1.98
Inflation rate for Canadian CPI converted to	West German currency	$ \left(\frac{\mathbf{F}_{\mathbf{t}}\mathbf{P}_{\mathbf{t}}}{\mathbf{F}_{\mathbf{t}+10}\mathbf{P}_{\mathbf{t}+10}}\right)^{0.1} $ = 1+p	1.0142	1.0133	0.9990	0.9872	0.9882	1.0071	1.0319	1.0561	1.0711	1.0709	1.0492	1.0261	1.0343	1.0483	1.0507	1.0356	1.0169	1.0066	1.0003	0.9994	1.0164	1.0419
Adjustment factor for West German	10 year bond rate	¥	1.0186	1.0204	1.0235	1.0241	1.0160	0.9983	0.9978	1.0030	0.9999	0.9954	0.9935	0.9957	0.9938	0.9925	9066.0	0.9898	0.9898	0.9904	0.9929	0.9954	0.9969	0.9987
0 0 1	GDP deflator	1	6	70.58	71.24	73.70	83,33	103.73	108.65	107.08	113.24	120.95	126.60	126.96	132.64	138.44	145.78	•	•	•	165.31	167.57	ij	175.37
West German GDP deflator 10 years	ahead	1	3.3	•	•	•	97.70			•	•		118.60	•	•	•	•	•	•	œ	3	160.02	6.3	173.02
West German 10 year bond rate	(%)	1x100	•	7.13	•	•	7.33	•	•	•	•	•	•	•	•	•	•	•	•	•	.7	6.	6.45	• 4
escription		otation	965	996	296	896	. 696	970	971	972	973	974	975	916	977	978	676	086	981	982	983	984	985	986

Table 8
Portfolio Shares

	5 Year Assets	10 Year Assets
United States	1.08192	1.13382
United Kingdom	-0.09375	-0.30472
West Germany	-0.19918	0.17090
Switzerland	0.21101	

Table 9

The Social Opportunity Cost of Consumption (SOCC)

Canada, 1965-86

		5 year	rate		10 year r	ate
	Portfolio s based on minimizat	risk	Equal Portfolio Shares	Portfolio s based on minimizat	risk	Equal Portfolio Shares
	Unadjusted	<u>Adjusted</u>	Adjusted	<u>Unadjusted</u>	Adjusted	Adjusted
1965 6 7 8 9	-1.01 -0.97 -1.09 -0.97 -0.25	1.11 1.58 1.74 2.20 3.78	2.44 3.81 5.59 7.87 8.85	0.02 0.58 0.95 1.27 1.76	2.47 3.22 3.63 3.88 4.23	3.98 4.19 5.74 7.71 8.83
1970 1 2 3 4 5 6 7 8	-0.67 -1.22 0.17 2.05 2.43 2.35 0.97 0.00 4.73 4.52	3.83 2.91 3.95 5.63 6.15 6.54 6.06 4.30 4.35 7.15	6.29 3.93 5.84 8.04 7.30 6.55 4.55 -0.02 -3.41 -4.37	1.29 0.47 0.30 0.74 1.90 2.46 1.88 1.71 2.67	4.26 4.29 4.14 4.49 6.54 7.18 5.41 3.95 4.50 7.10	6.80 4.46 3.73 3.46 2.06 2.75 4.25 3.74 3.06 2.21
1980 1 2 3 4 5 6	5.40 9.54 8.44 7.04 5.58 2.33 1.57	9.39 9.14 7.70 6.73 5.25 2.23 1.56	-0.02 5.72 7.84 9.23 9.11 5.85 2.41	7.51 8.69 7.39 7.13 6.00 2.98 1.96	9.05 8.91 7.81 7.08 5.93 2.99 1.96	4.06 6.23 6.44 6.96 6.65 4.65 3.11

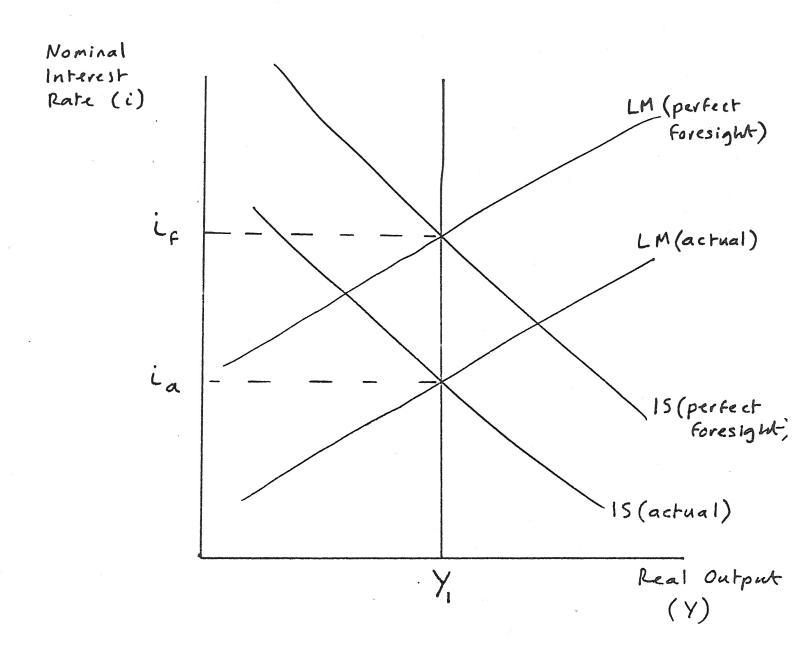


Figure 1

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APPENDIX

In this appendix expressions determining the asset shares that minimize portfolio risk are derived for 4 asset and 3 asset portfolios.

On a 4 asset portiolio the return R is

$$R = a_1 r_1 + a_2 r_2 + a_3 r_3 + a_4 r_4 \tag{A1}$$

Using the constraint $\sum_{i=1}^{4} a_i = 1$, (A1) can be written

$$R = a_1(r_1-r_4) + a_2(r_2-r_4) + a_3(r_3-r_4) + r_4$$
 (A2)

and

$$R^{2} = A_{1}a_{1}^{2} + A_{2}a_{2}^{2} + A_{3}a_{3}^{2} + r_{4}^{2} + 2A_{12}a_{1}^{2}$$

$$+ 2a_{1}a_{3}A_{13} + 2a_{1}B_{1} + 2a_{2}a_{3}A_{23}$$

$$+ 2a_{2}B_{2} + 2a_{3}B_{3}$$
(A3)

where
$$A_1 = (r_1 - r_4)^2$$

 $A_2 = (r_2 - r_4)^2$
 $A_3 = (r_3 - r_4)^2$
 $A_{12} = (r_1 - r_4)(r_2 - r_4)$
 $A_{13} = (r_1 - r_4)(r_3 - r_4)$
 $A_{14} = (r_2 - r_4)(r_3 - r_4)$
 $A_{15} = (r_2 - r_4)(r_3 - r_4)$
 $A_{16} = a_4(r_1 - r_4)$
 $A_{17} = a_4(r_2 - r_4)$
 $A_{18} = a_4(r_3 - r_4)$

From equation (10) in the text the risk minimizing shares are determined by

$$\frac{\partial V}{\partial a_i} = \frac{\partial E[R^2]}{\partial a_i} - \frac{\partial (E[R])^2}{\partial a_i} = 0 \text{ for } i = 1...3.$$
 (A4)

From differentiation of (A3)

$$\frac{\partial E(R^2)}{\partial a_1} = 2a_1 E[A_1] + 2a_2 E[A_{12}] + 2a_3 E[A_{13}] + 2E[B_1]$$
 (A5)

$$\frac{\partial E(R^2)}{\partial a_2} = 2a_2 E[A_2] + 2a_1 E[A_{12}] + 2a_3 E[A_{23}] + 2E[B_1]$$
 (A6)

$$\frac{\partial E[R^2]}{\partial a_3} = 2a_3 E[A_3] + 2a_1 E[A_{13}] + 2a_2 E[A_{23}] + 2E[B_3]$$
 (A7)

and $\frac{\partial (E[R])^{2}}{\partial a_{1}} = 2a_{1}(\hat{r}_{1} - \hat{r}_{4})^{2} + 2a_{2}(\hat{r}_{1} - \hat{r}_{4})(\hat{r}_{2} - \hat{r}_{4}) + 2a_{3}(\hat{r}_{1} - \hat{r}_{4})(\hat{r}_{3} - \hat{r}_{4}) + 2a_{3}(\hat{r}_{1} - \hat{r}_{4})(\hat{r}_{3} - \hat{r}_{4})$ $+ 2 \hat{r}_{4}(\hat{r}_{1} - \hat{r}_{4})$ (A8)

$$\frac{\partial (E[R])^{2}}{\partial a_{2}} = 2a_{2}(\hat{r}_{2}-\hat{r}_{4})^{2} + 2a_{1}(\hat{r}_{1}-\hat{r}_{4})(\hat{r}_{2}-\hat{r}_{4}) + 2a_{3}(\hat{r}_{2}-\hat{r}_{4})(\hat{r}_{3}-\hat{r}_{4}) + 2\hat{r}_{4}(\hat{r}_{1}-\hat{r}_{4})$$

$$+ 2\hat{r}_{4}(\hat{r}_{1}-\hat{r}_{4})$$
(A9)

$$\frac{\partial (E[R])^{2}}{\partial a_{3}} = 2a_{3}(\hat{r}_{1} - \hat{r}_{4})^{2} + 2a_{1}(\hat{r}_{1} - \hat{r}_{4})(\hat{r}_{3} - \hat{r}_{4}) + 2a_{2}(\hat{r}_{2} - \hat{r}_{4})(\hat{r}_{3} - \hat{r}_{4}) + 2\hat{r}_{4}(\hat{r}_{3} - \hat{r}_{4})$$

$$+ 2\hat{r}_{4}(\hat{r}_{3} - \hat{r}_{4})$$
(A10)

where \hat{r}_{i} is the expected value of r_{i} .

Substituting (A5) to (A10) into (A4) yields

$$\begin{bmatrix} \sigma_{1}^{2} + \sigma_{4}^{2} - 2\text{cov}_{14} & \sigma_{4}^{2} + \text{cov}_{12} - \text{cov}_{14} - \text{cov}_{24} & \sigma_{4}^{2} + \text{cov}_{13} - \text{cov}_{14} - \text{cov}_{34} \\ \sigma_{4}^{2} + \text{cov}_{12} - \text{cov}_{14} - \text{cov}_{24} & \sigma_{2}^{2} + \sigma_{4}^{2} - 2\text{cov}_{24} & \sigma_{4}^{2} + \text{cov}_{23} - \text{cov}_{24} - \text{cov}_{34} \\ \sigma_{4}^{2} + \text{cov}_{13} - \text{cov}_{14} - \text{cov}_{34} & \sigma_{4}^{2} + \text{cov}_{23} - \text{cov}_{24} - \text{cov}_{34} & \sigma_{3}^{2} + \sigma_{4}^{2} - 2\text{cov}_{34} \end{bmatrix} \begin{bmatrix} a_{1} \\ a_{2} \\ a_{3} \end{bmatrix}$$

$$= \begin{bmatrix} \sigma_4^2 - \cos_{14} \\ \sigma_4^2 - \cos_{24} \\ \sigma_4^2 - \cos_{34} \end{bmatrix}$$
 (A11)

where σ_i^2 is the variance of the return on asset i and cov_{ij} is the covariance between the returns on asset i and asset j. (All) determines the shares a_1 , a_2 and a_3 which minimize the variance of the 4 asset portfolio. The fourth share is given by $a_4 = 1 - a_1 - a_2 - a_3$.

For a 3 asset portfolio simply eliminate asset 1 from (All). So the shares that minimize the variance of a 3 asset portfolio are given by

$$\begin{bmatrix} \sigma_{2}^{2} + \sigma_{4}^{2} - 2\text{cov}_{24} & \sigma_{4}^{2} + \text{cov}_{23} - \text{cov}_{24} - \text{cov}_{34} \\ \sigma_{4}^{2} + \text{cov}_{23} - \text{cov}_{24} - \text{cov}_{34} & \sigma_{3}^{2} - \sigma_{4}^{2} - 2\text{cov}_{34} \end{bmatrix} \begin{bmatrix} a_{2} \\ a_{3} \end{bmatrix} = \begin{bmatrix} \sigma_{4}^{2} - \text{cov}_{24} \\ \sigma_{4}^{2} - \text{cov}_{24} \end{bmatrix}$$
(A12)

with the third share given by $a_4 = 1 - a_2 - a_3$.