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Tax Holidays in a Business Climate

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November 1992

TAX HOLIDAYS IN A BUSINESS CLIMATE

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Abstract

This paper provides a new explanation for "tax holidays," as well as their subsequent removal in a tax reform stage. In a two-period model, I assume that perfectly competitive foreign investors are uncertain about the host country government's propensity for public spending, and that infinitely divisible capital is subject to strictly convex adjustment costs. The host country government's current period tradeoff between public spending and the associated deadweight loss from distortionary taxation may signal the host's type and spare the investors from an unanticipated future tax hike. A separating equilibrium requires a deep tax concession early on, which corresponds to a tax holiday. When there are overlapping generations of foreign investors the tax profile flattens out over time as the information from tax holidays is exhausted; this is the tax reform phase.

JEL: 026, 321, 441
key words: foreign investment, signalling, tax holiday, time inconsistency

Preliminary

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TAX HOLIDAYS IN A BUSINESS CLIMATE

> *The willingness of potential investors to invest in a country depends not only upon the list of advantages and disadvantages in the tax laws and the rules of the game as they are today but, perhaps to a greater extent, upon the degree of confidence of the investor in the permanence of the arrangement. No investor can afford to ignore the possibility of expropriation.*
>
> —DAN USHER (1977)

1. Introduction

A tax holiday may signal a country's genuine commitment to long term private sector activity. Potential foreign investors are concerned with both the current tax rate on foreign capital income and the risk of an adverse future change in government tax policy. In this paper, I show that a tax holiday may provide foreign investors with the required confidence in the good faith of the governments of countries in which they invest. The results may be particularly relevant to a developing country that is in transition between a regime of substantial government spending and a regime that favours the free market.

A "tax holiday" refers to a statutory tax rate on capital income that begins at a low rate, but rises over time. One standard explanation for tax holidays is that foreign investors incur sunk costs that give the host country government *ex post* bargaining power over the foreign owned firm. Once the firm has committed its capital, the government maximizes tax revenue by raising the future tax rate. The time consistent solution is for foreign investors to require "up front" compensation in the form of a low initial tax rate, and for the government, indeed, to raise its tax rate in the future. This line of reasoning has been explored in several papers

beginning with Doyle and van Wijnbergen (1984). Bond and Samuelson (1985) demonstrate that the tax holiday profile emerges as the signalling solution to a bilateral bargaining game between a single firm with sunk fixed costs and a tax revenue maximizing government that has private information about the productivity of the potential investment on its territory. The private information may be due to the government's superior knowledge of local infrastructure, for instance. More recently, King and Welling (1989), and also King, McAfee and Welling (1990) examine the tax holiday profile in a model with a single firm, but with many governments competing for its location. They model the competition as a two-period second-price auction in which governments bid for the firm subject to non-negative discounted tax revenue for the game as a whole. The solution has the government with the highest productivity potential offering the firm a lump sum subsidy in the first period and a tax in the second.

This paper examines tax holidays and foreign investment in a somewhat different light. Unlike the papers cited above, the capital market is assumed to be perfectly competitive with infinitely divisible capital, and the government maximizes a social welfare function that depends on both public and private spending. In a two-period version of the model, sunk costs are captured with a continuous and convex capital adjustment cost equation, rather than fixed costs. The use of these neoclassical assumptions result in three distinct motives for why a host country government may wish to tax foreign capital income earned on its territory. The first is that if the host country is large in the world capital market, then its treatment of foreign investment can alter the world rate of interest. Kemp (1976) showed that the host can exploit this monopoly power by setting a positive tax rate on foreign investment earned in its borders. Second, if the marginal social cost of

public funds from domestic taxation is high, then as a second best solution the host will want to resort to some taxation of foreign investment income as a means to pay for public expenditure. This motive may be particularly important in a developing country, where the government is typically in the difficult position of facing a high domestic demand for public spending on social services and infrastructure, while being unable to collect significant tax revenue from residents because of poor tax collection technology. Third, in the absence of commitment the presence of capital adjustment costs results in a time inconsistency problem: the host will want to raise taxes on foreign capital income after the capital has been committed to its territory. The explanation for tax holidays in this paper makes use of the second and third motives.

The host country government is assumed to have private information about a parameter $\theta \in \{\theta^L, \theta^H\}$ of its social welfare function that summarizes its relative preference for public versus private expenditure. A host characterized by θ^H has a high preference for public spending, while θ^L means that it is more concerned about the private consumption of its citizens. The marginal cost of obtaining public funds from domestic taxation is assumed to be prohibitive, so that the government's only source of revenue is to tax foreign capital income. [1]

Foreign investors must infer from the currently observed tax rate on foreign investment, what rate they are likely to face in the future. On that basis, they undertake a certain amount of foreign investment, knowing that they will bear adjustment costs if they change their allocation in the future. Hence, from the point of view of investors, a "high" type of host ($\theta = \theta^H$) is bad news. There are

[1] This assumption is stronger than necessary, but is made to focus attention on the other aspects of the model. The results of the paper only require that there cannot be lump sum domestic taxation.

two periods in the model; the first period is the observable present, and the second period is the future.

Foreign investment is valuable to a country not only because it generates tax revenue, but also because it increases private domestic consumption by raising the productivity of land and labour. A government that values both private and public spending faces a tradeoff, because with infinitely divisible capital the deadweight loss from capital income taxation increases exponentially with the tax level. Only a government that gets extra satisfaction from public spending would tolerate much deadweight loss; conversely, a low spender cares more about efficiency. When the host country government's preferred tradeoff is unknown to potential foreign investors the hidden information poses an adverse selection problem: a high spender might tax foreign capital moderately in the early goings to disguise itself as a low spender, but will greatly raise the tax rate later when capital adjustment costs become significant. However, early tax concessions to potential foreign investors can sometimes reveal private information about the value of θ. I construct a model in which signalling with a tax holiday allows a host country government to overcome its adverse selection problem in a perfectly competitive international capital market. The analysis has implications for the time series behavior of taxes and capital levels. In particular, in a separating equilibrium, not only is the initial tax rate of a low spender below the tax rate of a pooling equilibrium; the future tax rate may well be higher. This seems paradoxical, because the purpose of the signal is precisely to reveal that the government does not have a high propensity for spending and taxing. However, a tax holiday attracts capital not only because it signals that the government is a low spender, but also simply because of its financial value to investors in the first period. Because of strictly convex adjsutment costs, the tax

holiday's two-fold boost to investment makes each investor more vulnerable to a future tax hike. Even a low spender will wish to take advantage of this.

I also extend the two-period model to one of overlapping generations of foreign investors and a long-lived government. The extension allows for a "tax reform" stage in which tax holidays are phased out, once their information content has been successfully communicated.

The model in this paper is an extension of Kemp's (1976) static model of foreign investment. Therefore, I begin the analysis with a review of Kemp's model to motivate the two-period model that follows. The two-period model is analyzed for two cases: first, investors know the value of θ; second, investors are uncertain about θ. The first case shows that the time consistent solution to the two-period model when there is complete information requires a rising tax profile, regardless of the value of θ. However, for my purposes, a tax holiday is something more drastic. It is the additional gap between the first and second period tax rates needed to signal the host's type, when there is incomplete information. I provide a diagrammatic interpretation of the results and a simulation of the model with quadratic functions. The extension to overlapping generations of foreign investors is then discussed, followed by my conclusions.

2. The Static Model

There are two countries. A capital-rich country contains a continuum of identical investors each endowed with an equal share of one aggregate unit of divisible capital (or savings), which can be invested at "home" or in the capital-poor "host" country. For simplicity, assume the capital-poor country is endowed with zero capital initially. Both countries use concave production functions to produce the same good. Capital

owners receive a competitive return on their capital, while rents accrue to the fixed factors in each country. The home country government is a passive player in not behaving strategically; the tax rate in the home country is set to zero. In the host country an *ad valorem* tax rate applies to capital income earned by foreigners within its boundaries. A social welfare function represents the host government's taste public and private consumption.

Let X represent the aggregate amount of capital invested in the host country, so that 1-X remains at home. Let the production functions in the home and host countries be g(1-X) and let y(X), respectively. The tax rate on foreign capital income is t and the social welfare function $W(\mathrm{C}, G)$ is assumed to be linear; C and G are private and public consumption. The host's problem is:

$$\max_t W(\mathrm{C}, G) = \mathrm{C} + \theta G \qquad \textit{subject to}: \tag{1}$$

$$g'(1-X) = (1-t)y'(X) \qquad \textit{arbitrage constraint} \tag{2}$$

$$\mathrm{C} = y(X) - Xy'(X) \qquad \textit{household budget} \tag{3}$$

$$G = tXy'(X) \qquad \textit{government budget.} \tag{4}$$

The general solution to the host's problem is $t^* = \tilde{t}(W, y, g)$. With the linear specification of the welfare function the solution to the host's problem is:

$$t^* = -\frac{\{\theta\mu_f\epsilon_f + (\theta-1)\mu_d\epsilon_d\}}{\theta(1-\mu_f\epsilon_f)}, \tag{5}$$

where $\mu_f = X/(1-X)$ is the ratio of the lending (i.e., home or "foreign") country's foreign investment to its domestic investments, and $\epsilon_f = (1-X)\,g''(1-X)/g'(1-X)$ is the reciprocal of the elasticity of demand for capital in that country; ϵ_d is the corresponding inverse elasticity for the host (i.e., "domestic") country; and $\mu_d = X/X$ is the share of investment in the host country that comes from foreign investment. [2] The optimal tax depends on a weighted average of the values of $\epsilon \cdot \mu$

[2] $\mu_d = 1$ only because of the assumption of zero endowment in the host country.

of the home and host countries. Note that t^* is non-decreasing in θ. Kemp asked what tax rate the host country should set in order to maximize national income. When $\theta = 1$, the social welfare function (1) is equivalent to the definition of national income. In this case, it is clear from (5) that t^* is non-negative and depends entirely on the elasticity of demand for capital in the lending country. Intuitively, the host country tolerates some deadweight loss of rents on fixed factors in order to take a share of capital income that would otherwise be fully repatriated to the home country. I assume a lower bound of $\theta = 1$ on the government's taste for public spending; such a government sets the same tax rate as a government that seeks to maximize national income.

When $\theta > 1$ there is an increased emphasis on government spending in the welfare function. However, the exponential increase in deadweight loss from distortionary taxation limits the amount of taxation the government is willing to undertake, even if the linear social welfare places a relatively heavy weight on public spending.

Kemp also considers what the optimal tax rate is on repatriated earnings for a *lending* country that seeks to maximize its national income. In calculating this rate he assumes the host country's tax on foreign capital is zero. Hamada (1966) and more recently Mintz and Tulkens (1990) examine the Nash equilibria arising from simultaneous and therefore strategic tax setting behavior by the borrowing and lending countries. In order to stress the main points of my paper, I will assume that the lending (i.e., home) country behaves passively; its tax rate on repatriated earnings is exogenous. To economize on notation the tax is set to zero.

3. Two-Period Model with Complete Information

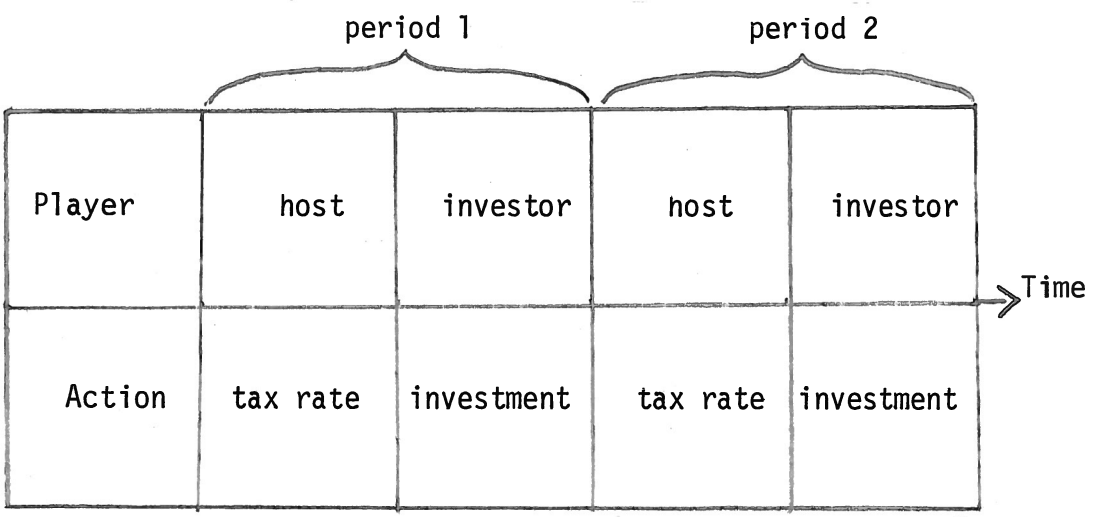
In this section, I assume that investors know the value of $\theta \in \{\theta^L, \theta^H\}$. There are two periods. Initially there is a unit of savings from the capital-rich country that must be allocated between the home and capital-poor foreign country, as in the previous section. Savings are liquid, so there are no transactions costs associated with the initial allocation. However, once the investments have been undertaken, there are costs to adjusting capital subsequently. This type of formulation of the lifetime of a pool of savings is essentially the same as in van Wijnbergen (1985), where he examines how the credibility of trade reform can affect the division of savings between physical capital and liquidity.

The government of the host country imposes a tax on foreign capital income invested on its territory in each period in order to maximize the welfare of its citizens. In performing this maximization, the host country government takes for granted that the economy is in equilibrium and that arbitrage profits are nil on the international capital market. After observing the tax rate in a given period, each foreign investor makes a savings (capital) allocation to maximize his return, subject to his costs of adjusting capital in the second period; as one member of a continuum of investors, he takes the rates of return in each country and each period as given. It will be necessary to distinguish between the individual investor's capital allocation, denoted with a lower case letter x, and the aggregate allocation, denoted by a capital letter X. It is the aggregate level X that determines the rates of return, but the individual investor's level of x that determines his cost of adjustment. Since all investors are identical, the market's investment behavior is characterized by a representative investor. Of course, in equilibrium, the representative investor's investment x must coincide with the aggregate value of X.

An investor's initial investment decision has a bearing on his second period payoff, because there are costs to adjusting the level of capital away from the initial allocation. The costs of capital adjustment may be thought of as lost output, or simply as transactions costs paid to a third party, not explicitly modelled. The cost of adjusting capital is given by a strictly convex function $C(|x_1 - x_2|)$, with $C(0) = 0$, $C'(0) = 0$, $C'(\delta) > 0$ for any $\delta \neq 0$, and $C''(0) > 0$, where the subscript denotes the period. Each investor has perfect foresight of the second period equilibrium tax rate, although he perceives no link between his first period investment and the future tax rate; this just means that the individual investor is too small a player to influence policy, but the market as a whole predicts the future correctly. Loosely speaking, if it is known that the host is a high type of spender, then the market arbitrage condition would be based on an appropriately higher expected future tax rate, other things being equal. The discount factor is assumed to equal one; this does not matter for the derivation of the results. Figure 1 summarizes the timing of events.

FIGURE 1

Sequence of Events



The Host's Payoff

Substituting the private and public budget constraints, (2) and (3), into the social welfare function (1) in each period yields W_1, the host's objective function or payoff for the two-period game. In symbols:

$$W_1 = y(X_1) - X_1 y'(X_1) + \theta t_1 X_1 y'(X_1) + W_2 \tag{6}$$

$$W_2 = y(X_2) - X_2 y'(X_2) + \theta t_2 X_2 y'(X_2). \tag{7}$$

The Representative Investor's Payoff

The representative investor takes the rates of return on capital as given by the market and he simply seeks the highest expected return on his portfolio net of taxes and net of his second period capital adjustment costs, which depend in part on his initial allocation x_1. The second period net capital income is denoted by U_2 and the payoff for the entire game is U_1:

$$U_1 = (1 - x_1) g'(1 - X_1) + (1 - t_1) x_1 y'(X_1) + U_2(x_2, t_2, x_1) \tag{8}$$

$$U_2 = (1 - x_2) g'(1 - X_2) + (1 - t_2) x_2 y'(X_2) - C(|x_1 - x_2|). \tag{9}$$

Equilibrium Concept for the Complete Information Game

The appropriate equilibrium concept for the complete information game is that the solution must be subgame perfect. Subgame perfection requires that the solution induced by the strategies of each player for the game as a whole form a Nash equilibrium starting at each subgame. In the context of the game between the host

and the investors, an interior solution is obtained by recursively solving the first-order conditions of each player's objective function given the history of the game. Hence, a subgame perfect solution, expressed in the form of closed loop strategies, is any correspondence

$$\{\tilde{t}, \tilde{x}\} \equiv \{\tilde{t}_1(\theta), \tilde{x}_1(t_1, \theta), \tilde{t}_2(\theta, X_1), \tilde{x}_2(t_2, x_1)\}$$

such that the following conditions hold:

(A) $\tilde{x}_2 = \{x_2 : \ argmax \ U_2(\tilde{t}_2, x_1, x_2)\}$,

(B) $\forall \ \theta, \ \tilde{t}_2 = \{t_2 : \ argmax \ W_2(t_2, \tilde{X}_2, \theta)\}$,

(C) $\tilde{x}_1 = \{x_1 : \ argmax \ U_1\big(\tilde{t}_1, x_1, U_2(\tilde{t}_2, x_1, \tilde{x}_2)\big)\}$,

(D) $\forall \ \theta, \ \tilde{t}_1 = \{t_1 : \ argmax \ W_1\big(t_1, \tilde{X}_1, \theta, W_2(\tilde{t}_2, \tilde{X}_2, \theta)\big)\}$,

(E) $\forall \ x_1 \in \tilde{x}_1, \ X_1 = x_1$ (i.e., $\tilde{X}_1$) and $\forall \ x_2 \in \tilde{x}_2, \ X_2 = x_2$ (i.e., $\tilde{X}_2$).

Note that at the start of the second period the first period tax rate is inconsequential; what matters to the host and investors is the level of capital carried into period two. Of course, recursively substituting the optimal capital and tax allocations would yield an equivalent open loop expression for the equilibrium, where the best reply mappings would depend on θ alone. Denote this equivalent solution as:

$$\{t^*, x^*\} = \{t_1^*(\theta), x_1^*(\theta), t_2^*(\theta), x_2^*(\theta)\}.$$

The conditions A and B define the Nash equilibrium in the second period subgame, while conditions C and D define the Nash equilibrium for the game as a whole.

Condition E requires the solution to be consistent with capital market clearing. For ease of notation the investor's payoff is written without the aggregate level of capital X in its arguments; this should not lead to confusion, as long as it is understood that the equality between x and X is only imposed after the representative investor has optimized. To simplify the presentation of the main ideas of the paper, I will make assumptions to ensure that the first order approach yields a unique optimum, when there is an interior solution. While the investor's problem is always concave, it is well known (see, for example, Wildasin (1989)) that social welfare functions are generically nonlinear in tax rates, which may lead to nonconcavities in the host's payoff. Moreover, it is unclear how players in the real world focus on a particular equilibrium when there are multiple solutions arising from nonconcavities in the host's payoff functions. In Appendix B, I provide more general conditions that are sufficient for uniqueness. The remainder of the paper uses the first-order approach. The correspondences $\{\tilde{t}_1(\theta), \tilde{x}_1(t_1), \tilde{t}_2(\theta, X_1), \tilde{x}_2(t_2, x_1)\}$ may therefore be interpreted as functions, and $\{t_1^*, x_1^*, t_2^*, x_2^*\}$ is the reduced form of the equilibrium for a given value of θ. It will be shown in this section that $t_2^* > t_1^*$ and $x_2^* < x_1^*$.

Assumptions

Assumption 1. The production functions of the host and home countries are concave and the cost function is strictly convex. The third and higher order derivatives of these functions are equal to zero.

Assumption 2. At least one of the production functions is strictly concave.

Assumption 3. In each period, $y'(X) + Xy''(X) \geq 0$.

Without assumption 2 the solution is trivial: the tax rate would equal zero for all θ. The non-negativity assumption on $y' + Xy''$, which ensures that the host's indifference curves slope downward, is equivalent to assuming that the demand for capital in the host country is relatively inelastic. Heller and Kauffman (1963) have argued that this is a reasonable assumption for developing countries. [3] The rest of the paper maintains the assumptions 1 to 3. In order to derive the subgame perfect solution the model must be solved recursively. I therefore begin the analysis with the investor's second period capital allocation decision x_2, which is conditional on his first period choice x_1 and the tax rate t_2.

The Investors' Second Period Reaction Function

At the beginning of the second period, the host reveals the second period tax rate t_2 on foreign capital income. Each investor then decides on the proportion of capital x_2 that he will maintain in the host country, in order to maximize his capital income given by (9). The first order condition for (9) defines the investor's optimal strategy $\tilde{x}_2$:

$$g'(1 - X_2) + \{sgn(x_1 - x_2)\}C'(x_1 - \tilde{x}_2) + (1 - t_2)\, y' X_2 = 0. \qquad (10)$$

Substituting the equilibrium conditions $X_1 = x_1$ and $X_2 = x_2$ into (10) leads to (11), which is the second period capital market arbitrage constraint. [4]

$$g'(1 - X_2) - C'(X_1 - X_2) = (1 - t_2)y'(X_2) \qquad \textit{arbitrage constraint.} \qquad (11)$$

[3] "...it would seem unlikely that investor responses would be highly sensitive to small increases in return rates, and an elasticity as great as unity would be an optimistic assumption...."(p.160) The authors attribute the low elasticity to risk factors and inadequate financial resources in less developed countries.

[4] In equilibrium it must be the case that $X_1 \geq X_2$, as shown later in proposition 2. Hence, I omit the 'sgn' term for clarity.

This means that the net rates of return are the same in both countries; (11) is a generalization of equation (4). Equation (11) implicitly defines a function that can be interpreted as the supply of capital to the host country in the second period for any choice of t_2 given X_1:

$$X_2 = \tilde{X}_2(t_2, X_1). \tag{12}$$

Note from differentiating (11) that the supply is decreasing in t_2:

$$\frac{dX_2}{dt_2}_{|X_1} = \frac{y'(X_2)}{g''(1-X_2) - C''(X_1 - X_2) + (1-t_2)y''(X_2)} < 0. \tag{13}$$

It will prove useful later to note the curvature of (12).

Lemma 1. The supply of capital (12) is strictly concave in the tax rate t_2.

Proof. See the appendix.

The Host's Second Period Optimal Tax Rate

At the beginning of the second period, the host anticipates the market's supply of capital response (12), when she chooses t_2 to maximize $W_2(t_2, \tilde{X}_2(t_2, X_1), \theta)$. Although the host's indifference curves between X_2 and t_2 slope downward, their convexity property is ambiguous over the tax range $t_2 < 1/2\theta$. To simplify the analysis, I add the following assumption, noting that it is satisfied by the simulation model presented later.

Assumption 4. W_2 is strictly quasi-concave.

The implication of this assumption is that the host's indifference curves over X_2, t_2 are strictly convex. If an interior solution exists, it is given uniquely by the tangency

between the host's indifference curve map and the supply of capital curve (12). [5] The first order condition for the host's second period optimization problem is:

$$\frac{dW_2}{dt_2}_{|x_1} = [\tilde{X}_2 y''(\tilde{X}_2) + \theta t_2 y'(\tilde{X}_2) + \theta t_2 \tilde{X}_2 y''(\tilde{X}_2)] \frac{dX_2}{dt_2}_{|x_1} + \theta \tilde{X}_2 y'(\tilde{X}_2) = 0 \tag{14}$$

Substituting (13) into (14), if an interior solution exists, it is implicitly given by:

$$t_2^* = \frac{\theta \tilde{X}_2 [C'' - g''] + (1-\theta)\tilde{X}_2 y''}{\theta y'}. \tag{15}$$

Note that for large values of X_1 and θ the solution provided by the first order condition may lie above the admissible set in which case the optimal tax rate is $t_2^* = 1$.

Using the second period arbitrage condition (11), equation (15) can be re-arranged into an expression that is analogous to the optimal tax expression discussed in the section on the static model with one exception. The reciprocal of the elasticity of demand for capital in the home country (i.e., "foreign") must now be modified to include the effects of the adjustment costs. Letting $\hat{\epsilon}_f \equiv \frac{(1-\tilde{X}_2)(g''(1-\tilde{X}_2)-C''(X_1-\tilde{X}_2))}{(g'(1-\tilde{X}_2)-C'(X_1-\tilde{X}_2))}$,

$$t_2^* = -\frac{\{\theta \mu_f \hat{\epsilon}_f + (\theta - 1)\mu_d \epsilon_d\}}{\theta(1 - \mu_f \hat{\epsilon}_f)}. \tag{16}$$

Note that t_2 is strictly positive. The unique best reply t_2^* that solves (16) for any given X_1 and θ is denoted by the function:

$$t_2^* = \tilde{t}_2(X_1, \theta). \tag{17}$$

The relationship between adjustment costs and t_2^* is the following.

[5] What I call the supply curve is also the capital market's offer curve, reaction function for given tax rate; that is why it is combined with an indifference curve map.

Proposition 1. For any value of θ the higher are C' and C'', the higher is the optimal second period tax rate (within admissible values).

Proof. The proof follows from inspection of (16). ■

Second Period Nash Equilibrium

If an interior solution for the level of the capital stock X_2 exists it can be found by substituting $\tilde{t}_2$ into $\tilde{X}_2(\tilde{t}_2, X_1)$. The Nash equilibrium for the second period subgame, given X_1 and θ, is then:

$$\{\tilde{t}_2(X_1,\theta), \tilde{X}_2(\tilde{t}_2, X_1)\}, \quad \text{or equivalently} \quad \{t_2^*(X_1,\theta), X_2^*(X_1,\theta)\}.$$

Define the host's and representative investor's optimal value functions for the second period continuation game, $W_2^*(X_1,\theta)$ and $U_2^*(x_1,\theta)$, by substituting the equilibrium values into the respective payoff functions and noting that $x_1 = X_1$ and $x_2 = X_2$. The comparative statics of the second period equilibrium are given in the next lemma.

Lemma 2. For any θ we have the following:

$$(i)\ 0 < dt_2^*/dX_1\,;\ (ii)\ 0 < dX_2^*/dX_1 < 1\,;$$

$$(iii)\ d^2t_2^*/dX_1^2 < 0\,;\ (iv)\ dX_2^{2*}/dX_1^2 = 0\,;\ (v)\ dW_2^*/dX_1 > 0\,.$$

Proof. See the appendix.

It is also useful to note that the the slope of the host's second period indifference curves are steeper the larger is θ; that is, a government that places a high valuation on public spending requires relatively more capital inflow as compensation for

reducing her tax rate. This so-called "single-crossing" property for W_2 is given by:

$$-\partial\left(\frac{\partial W_2/\partial t_2}{\partial W_2/\partial X_2}\right)\Big/\partial\theta \;=\; \frac{{X_2}^2\, y'\, y''}{(-X_2 y'' + \theta t_2(y' + X_2 y''))^2} \;<\; 0. \qquad (18)$$

Corner Solution

If the solution $t_2^* \notin [-1, 1]$ or $X_2^* \notin [0, 1]$, then the equilibrium is on the boundary of the action spaces. A corner solution may arise when the marginal cost of adjustment is so large that the first order revenue gains from further taxation, weighted by the relative value of public spending in the welfare function, exceed the lost rents from capital flight even at 100 percent taxation. Investors prefer to simply "abandon" some capital in the second period rather than bear the transactions costs from repatriating the whole stock of foreign capital. From equation (11), a 100 percent second period tax rate implies that

$$-g'(1 - X_2) + C'(X_1 - X_2) \geq 0\,,$$

and, therefore, that the residual amount of capital in left in the host country if capital is taxed at 100 percent is:

$$X_2 = Max\ \{0,\ X_1 - {C'}^{-1}(g'(1 - X_2))\}. \qquad (19)$$

The Investors' First Period Reaction Function

In the first period the representative investor observes t_1 and anticipates the second period Nash equilibrium, when choosing his capital allocation to maximize his capital income over both periods. The investor's first order condition for maximizing

(8) is:

$$\frac{dU_1}{dx_1} = -g'(1 - X_1) + (1 - t_1)y'(X_1) + \frac{dU_2^*(x_1, \theta)}{dx_2^*}\frac{dx_2^*}{dx_1} + \frac{dU_2^*(x_1, \theta)}{dx_1} = 0. \quad (20)$$

Applying the envelope theorem and the equilibrium conditions $X_1 = x_1$ and $X_2 = x_2$ we obtain the first period arbitrage condition:

$$g'(1 - X_1) = (1 - t_1)\, y'(X_1) - C'(X_1 - X_2^*(X_1, \theta)) \qquad \textit{arbitrage constraint.} \quad (21)$$

Implicit in (21) is the first period supply of capital to the host country as a function of both t_1 and the expected second period equilibrium, which depends on θ. Denote this as: [6]

$$X_1 = \tilde{X}_1(t_1, \theta). \quad (22)$$

Totally differentiating (21) with respect to t_1 shows that the supply is decreasing in the tax rate:

$$\frac{dX_1}{dt_1} = \frac{y'(X_1)}{g''(1 - X_1) + (1 - t_1)y''(X_1) - C''(X_1 - X_2^*)[1 - dX_2^*/dX_1]} < 0. \quad (23)$$

Lemma 3. The first period supply of capital to the host country $\tilde{X}_1(t_1, \theta)$ is stricly concave.

Proof. See the appendix.

In making their first period capital decision, investors realize that the second period equilibrium tax rate that they will face depends on the host's taste for public spending. This is reflected in the fact that the first period supply of capital to the host country decreases in the value of θ for a given t_1.

Lemma 4. $dX_1/d\theta_{|_{t_1}} < 0$.

[6] It is easy to show that (21) has a unique fixed point for X_1 given any t_1, θ.

Proof. See the appendix.

The Host's First Period Optimal Tax Rate

The host anticipates the investor's best-reply function before choosing her first period tax rate on foreign capital income. An optimal first period action for the host is given by the correspondence $\tilde{t}_1(\theta)$ defined by

$$\tilde{t}_1(\theta) = \{t_1 : argmax\ W_1(t_1, \tilde{X}_1(t_1), \theta), W_2^*(t_2^*(\tilde{X}_1(t_1), \theta), X_2^*(\tilde{X}_1(t_1), \theta))\}. \quad (24)$$

The host's first period indifference curve over the space of x_1, t_1 is obtained by totally differentiating W_1. The first order condition for the problem is:

$$dW_1 = [-X_1 y''(X_1) + \theta t_1 (y'(X_1) + X_1 y''(X_1))]dX_1 + \theta X_1 y'(X_1) dt_1 + dW_2^* = 0, \quad (25)$$

$$dW_2^* = (\partial W_2^*/\partial X_2)(dX_2/dX_1)dX_1 + (\partial W_2^*/\partial t_2)(dt_2/dX_1)dX_1. \quad (26)$$

Combining these expressions, the slope of the indifference curves are:

$$\frac{dX_1}{dt_1} = \frac{-\theta\, X_1\, y'(X_1)}{-X_1\, y''(X_1) + \theta\, t_1\, (y'(X_1) + X_1\, y''(X_1)) + dW_2^*/dX_1} < 0. \quad (27)$$

To establish that the host's payoff function is strictly quasi-concave the host's first period indifference curves must be strictly convex, which requires that the derivative of (27) be positive. The sign of one term in the derivative of (27), $\frac{d^2 W_2^*}{dX_1^2}$, is ambiguous; furthermore, the term increases in absolute value with increases in C'', until the second period tax rate reaches its upper bound. The following lemma provides information on the convexity property of the host's first period indifference curves.

Lemma 5. There exists a critical value $\bar{c}$, such that for $C''(1) < \bar{c}$ the host's first period indifference curves are strictly convex over the region $t_1 \geq 1/2\theta$.

Proof. See the appendix.

A similar restriction on C'' generally ensures that the first period indifference curves obey the single-crossing property; that is, they become steeper (downward) the higher is the value of θ:

$$\frac{\partial\{\frac{\partial W_1}{\partial t_1}/\frac{\partial W_1}{\partial X_1}\}}{\partial\theta} < 0. \tag{28}$$

Intuitively, a higher value for θ means that the government places a large weight on public spending and therefore requires more X_1 to compensate for a marginal decline in t_1.

Lemma 6. There exists a critical value $\bar{\bar{c}}$, such that for $C''(1) < \bar{\bar{c}}$ the host's indifference curves obey the single-crossing property (28).

Proof. See the appendix.

On the basis of lemmas 6 and 7, I impose the following assumption, noting that it is satisfied by the illustration model given in section 4.

Assumption 5. The host's first period indifference curves are strictly convex and obey the single-crossing property.

Using assumption 5 and substituting (23) into (27), the unique optimal first period tax rate is characterized the following expression, which bears comparison with (15):

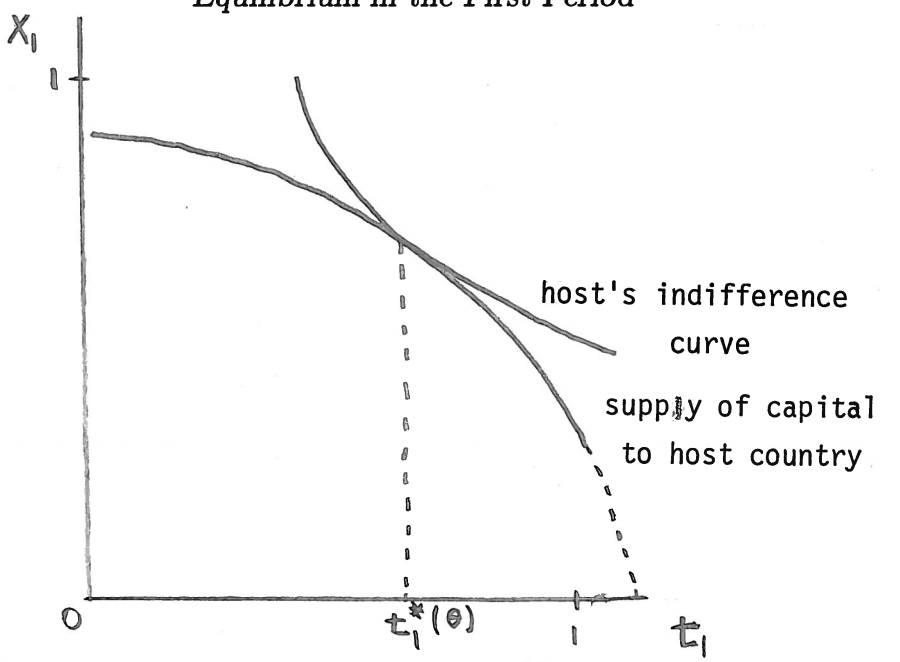
$$t_1^* = \frac{\{\theta X_1(C'' \cdot (1 - \frac{dX_2}{dX_1}) - g'') + (1-\theta)X_1 y'' - \frac{dW_2}{dX_1}\}}{\theta y'.} \tag{29}$$

Equilibrium in the Complete Information Game

The first period equilibrium occurs at the point of tangency between a host's indifference curve and the capital supply curve. This is depicted in Figure 2. It will prove useful later to denote the optimal first period tax rate for a low (high) type of host as t_1^L or t_1^H, as the case may be.

FIGURE 2

Equilibrium in the First Period



The main result of the complete information game is summarized as a proposition.

Proposition 2. A unique subgame perfect equilibrium to the complete information version of the game exists, and is characterized by a tax profile that increases over time and a capital allocation in the host country that is decreasing: $t_1^* < t_2^*$ and $X_2^* < X_1^*$.

Proof. See the appendix.

The intuition for the proposition is evident from a comparison of the market's response to a tax increase in each of the periods, as given by equations (23) and (13). Convert these differentials into first and second period market elasticities and evaluate them at the same point. Then it is clear that the presence of the term dX_2^*/dX_1 in the denominator of (23) makes the first period supply elasticity of capital in the host country more elastic than in the second period. The result $t_1^* < t_2^*$ accords with optimal taxation theory, which says that the least elastic goods should face the highest tax. The tax profile indicated in proposition 2 is similar to the sunk cost explanation for tax holidays found in the literature. For my purposes a tax holiday is something more drastic: it is a first period tax concession beyond the implication of proposition 2 to signal the host government's commitment to the private sector, when information about its social welfare function is incomplete. Before proceding to the case incomplete information version of the two-period model, I present a numerical illustration of the properties indicated in the propositions 1 and 2.

4. An Illustration

In this section I provide the results of a simulation of a version of the model in which the production and adjustment cost functions are quadratic; and the production functions are identical in each country. That is, for $\tau = 1, 2$:

$$y(X_\tau) = X_\tau - .5X_\tau^2$$

$$g(1 - X_\tau) = (1 - X_\tau) - .5(1 - X_\tau)^2.$$

The cost of capital adjustment is given by

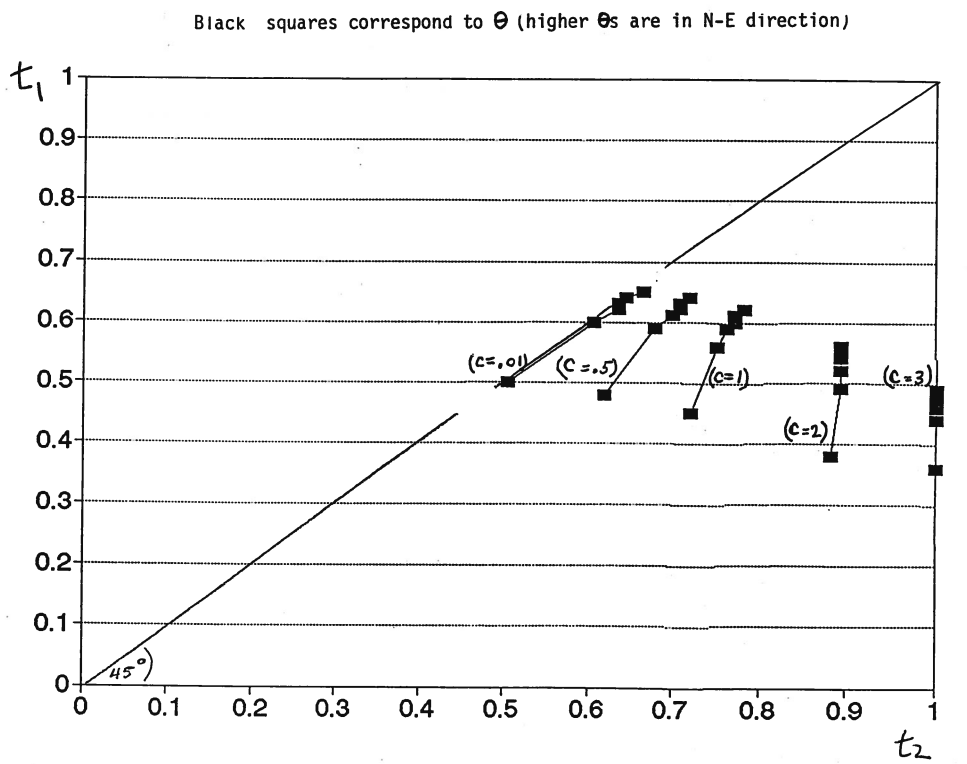
$$C = c(|x_1 - x_2|)^2,$$

where c is a positive constant. Note that with these assumptions on the production functions the expressions for the marginal product of capital in the home and host

countries are X_τ and $1 - X_\tau$, repsectively. Since $X_\tau \in [0, 1]$ the marginal products are also bounded between 0 and 1. The assumption that $y' + Xy'' \geq 0$, is satisfied in equilibrium, but is not globally necessary for uniqueness in this example. The next figure gives comparative static results for a range of values for θ from 1 to 9 and for c from .01 to 3. Each line segment in the figure corresponds to one value for c, given in brackets, while each black square in a line segment corresponds to a value of θ. The equilibrium pair $\{t_1^*, t_2^*\}$ always lies below the 45^o line, showing that $t_2^* > t_1^*$. In every case, movements in the north-east direction correspond to higher values of θ; the point $\{t_1^*, t_2^*\}$ rapidly converges to a limit, as θ increases. Note that for high values of c there is a corner solution for t_2^*.

FIGURE 3

Equilibrium Tax Rates for Different θ and c



5. The Two-Period Model with Incomplete Information

Suppose now that investors are unsure of which of the two potential types of host they face. When there is incomplete information about the host's propensity for public spending the investors must infer what tax rate to expect in the second period on the basis of prior information and the host's first period tax rate. The host's optimal strategy then depends on a comparison between selecting a first period tax rate that leads investors to infer her type, or choosing a tax rate that leads investors to believe she is the other type. Other things equal, all types of hosts prefer higher tax rates and higher capital levels. Therefore, the adverse selection of one type of host masquerading as the other type only works in one direction: both high and low types would like investors to believe they are low spenders. I have described the sequence of economic events as taking place in two periods, where the periods correspond to the life cycle of a pool of savings *cum* capital. A formal description of the incomplete information game requires a more precise characterization of periods into sub-periods, or "stages." Formally, this is a so-called "multi-stage" game with observed actions and incomplete information. The extensive form of the game is summarized below. [7]

The Game Structure

S.0 Nature chooses a value for $\theta \in \{\theta^L, \theta^H\}$.

The stage S.0 establishes the source of asymmetric information in the model.

S.1 The host observes θ and chooses a tax rate $t_1 \in [-1, 1]$.

S.2 Each investor revises his beliefs about the value of θ

[7] The game is analogous to a principal and agent problem, where the capital market plays the role of principal, and the host is the agent.

using Bayes' rule and chooses a level of investment $x_1 \in [0,1]$

as a function of his beliefs and t_1.

S.3 The host chooses a tax rate $t_2 \in [-1,1]$

as a function of θ and x_1.

S.4 Each investor chooses an allocation

$x_2 \in [0,1]$ as a function of t_2 and x_1.

End of the game.

If chronological time is represented by "periods," period one corresponds to stages S.1 and S.2, and the second period to stages S.3 and S.4. Note that there can be no gains for the host from concealing her type at S.3, since this is her last move and the investor's payoff does not depend directly on θ. For all intents and purposes, then, the second period subgame is one of complete information. Therefore, the investor's belief revision at S.4 is trivial and is omitted from the description of the game. The informed player moves before the uninformed, so this is a signalling game. A solution to this game is defined by a Perfect Bayesian Equilibrium (PBE). [8]

Definition of Perfect Bayesian Equilibrium

Roughly speaking, a perfect Bayesian equilibrium in pure strategies is a strategy profile for each player where no one can gain by a unilateral defection given their beliefs at each information set. A player's strategy profile specifies his action at each of his information sets as a function of the history of the game up to that point. Using a similar notation as in section 3, let $\tilde{t}_\tau$, $\tau =, 1, 2$ be a correspondence from the

[8] A PBE is equivalent to a sequential equilibrium for this simple game.

history of the game at each period (measured at the beginning of the stage where the host chooses her tax rate) to the interval of admissible tax rates. Similarly, let $\tilde{x}_\tau$, $\tau = 1,2$ map from the investor's beliefs and the history of the game at each period (measured at the beginning of the stage where the investors make their allocation decisions) to the interval [0,1]. Also, let ρ_0 denote the common knowledge prior probability that the representative investor assigns to a low spending type of host and $\rho_1(t_1)$ the updated beliefs (posterior distribution) that the host is a low type after investors observe t_1. Since there are only two types of host in this model, the posterior belief that the host is a high type is $1 - \rho_1(t_1)$. A PBE for this game consists of a strategy profile for the host

$$\tilde{t} \equiv \{\tilde{t}_1(\theta), \tilde{t}_2(\theta, X_1)\}$$

and a strategy profile for the representative investor

$$\tilde{x} \equiv \{\tilde{x}_1(t_1), \tilde{x}_2(t_2, \tilde{x}_1)\}$$

and posterior beliefs $\rho_1(t_1)$ such that these conditions hold:

(A) $\forall\, \theta,\ \tilde{t} = \{t:\ argmax\ W_1(t, \tilde{X}, \theta)\}$,

(B) $\tilde{x} = \{x:\ argmax\ E_{\theta|_{t_1}}\ U_1(\tilde{t}, x, \tilde{X})\}$, [9]

[9] Note that in comparison to the complete information game, the investor's expected payoff and period one best reply mapping are modified as follows:

$$\begin{aligned} E\,U_1 =& g'(1 - X_1)(1 - x_1) + (1 - t_1)\, y'(X_1)\, x_1 \\ &+ \rho_1(t_1)\, U_2^*(x_1, \theta^L) + (1 - \rho_1(t_1))\, U_2^*(x_1, \theta^H); \end{aligned}$$

$$\tilde{x}_1 = \{x_1 :\ argmax\, E\,U_1(t_1, x_1, X_1, \rho_1(\theta|t_1))\} \cap \{X_1 = x_1\}.$$

(C) $\forall\, x \in \tilde{x},\ X_1 = x_1$ and $X_2 = x_2$ (i.e., $\tilde{X}_1$ and $\tilde{X}_2$),

(D) $\rho_1(t_1) = \rho_0\, Prob(t_1 \in \tilde{t}_1(\theta^L))/\{\rho_0\, Prob(t_1 \in \tilde{t}_1(\theta^L))$
$+(1-\rho_0)\, Prob(t_1 \in \tilde{t}_1(\theta^H))\}$,
if the denominator is greater than zero; $\rho_1(t_1)$ is any probability distribution on $\{\theta^L, \theta^H\}$ if the denominator equals zero.

Conditions (A) and (B) are the conditions for perfectness. (A) says that the host maximizes her welfare given the representative investor's choice of capital allocations, which by condition (B) must be optimal given the market rates of return and posterior beliefs about θ. (C) Says that in equilibrium the aggregate behavior of the continnum of identical investors is consistent with that of the representative investor. Finally, (D) corresponds to the application of Bayes' rule. If t_1 is not part of the host's optimal strategy for some type, observing t_1 is a zero-probability event. Any posterior beliefs $\rho_1(t_1)$ are then admissible, so any action x_1 that is a best response for some beliefs can be played. Note that a characterization of the investor's best-reply mapping $\tilde{x}$ depends on his posterior beliefs, which cannot be determined independently from the host's strategy. That is, beliefs must be consistent with the Nash equilibrium for the game.

There are two types of equilibria. A *separating equilibrium*, if it exists, is a PBE in which the different types of hosts have strategies that lead them to undertake different actions in the first period. Investors can therefore infer the host's type from her choice of t_1. The continuation game beginning at S.2 (where each investor chooses his first period capital level x_1) is then one of complete information and

$$\rho_1(\theta = \theta^L | t_1) = 1 \text{ or } 0.$$

A *pooling equilibrium* is a PBE in which different types of hosts would select

the same action in the first period. Hence, no new information is revealed by the host's first period tax rate and the posterior probability that the host's type is low remains the same as the prior:

$$\rho_1 = \rho_0.$$

Separating equilibrium

An equilibrium is separating if the first period tax rate t_1 selected by the low type differs from that of the high type. A high type may wish to disguise herself as a low type in order to draw in more investment than would be the case if investors knew the truth. The disguise consists of mimicking the first period tax rate that a low type would choose. Given certain prior beliefs held by investors it might be in the interest of a low type to set an exceptionally low tax rate in the first period in order to make it clear that she is in fact low. This strategy succeeds if a high type cannot bring herself to forgo so much public expenditure in the initial period, even if mimicking a low type's behavior would result in a large capital inflow that is vulnerable to high taxation in the second period. On the other hand, separation may fail to be an equilibrium if the low type host is required to make too steep a concession in the first period: she may prefer to maintain a higher t_1 and put up with the losses in investment from the investors' uncertainty over her type. Separation may fail if the θ^H and θ^L are close together.

A separating equilibrium in pure strategies, if it exists, can be constructed in a two step procedure. First, propose a strategy combination for the investor and the host that generate posterior beliefs that are degenerate. Second, verify that each player's proposed strategy is in fact a best reponse to the other's strategy given

these beliefs. I carry out these steps below.

Investor's strategy at S.2 and host's strategy at S.1.

Some additional notation will facilitate the description of strategies. Let $x_1^L(t_1)$ be the first period best response of the representative investor to the tax rate t_1 given that he believes the host to be a low type, and assuming the capital market is in equilibrium. In a similar way define $x_1^H(t_1)$. Also, recall that $x_2^*(x_1, \theta) \equiv \tilde{x}_2(\tilde{t}_2, x_1)$. Finally, recall from the complete information equilibrium that t_1^H denotes the optimal first period tax rate for a high type if her type is common knowledge at the beginning of the game. Now consider the following pair of strategies: [10]

INVESTOR: "If $t_1' \leq t_1^c$, I play $\{x_1^L(t_1'), x_2^*(x_1^L, \theta^L)\}$;
otherwise, I play $\{x_1^H(t_1'' > t_1^c), x_2^*(x_1^H, \theta^H)\}$."

HOST: "If $\theta = \theta^L$, I play $t_1' \leq t_1^c$ and $t_2^*(X_1, \theta^L)$;
otherwise, I play $\{t_1^H, t_2^*(X_1, \theta^H)\}$."

As a consequence of the host's proposed strategy the posterior beliefs of investors are degenerate: either ρ_1 equals one or zero, depending on the observed t_1. Consequently, the first order condition for the investor's expected payoff can be written simply as:

$$\begin{aligned} \frac{dU_1}{dx_1}|_{t_1} = &- g'(1 - X_1) + (1 - t_1')\, y'(X_1) + \frac{dU_2^*}{dx_1}(x_1^L, \theta^L) = 0, \quad \text{for } t_1' \leq t_1^c \\ &- g'(1 - X_1) + (1 - t_1'')\, y'(X_1) + \frac{dU_2^*}{dx_1}(x_1^H, \theta^H) = 0, \quad \text{for } t_1'' > t_1^c. \end{aligned} \tag{30}$$

The first order conditions together with the equilibrium requirement that X_1 equals x_1^L or x_1^H, as the case may be, define the market's first period supply of capital to

[10] Only these strategies are considered; other possibilities seem implausible for this game.

the host country in the incomplete information game:

$$\tilde{X}_1(t_1) \equiv \{x_1^L(t_1'), x_1^H(t_1'')\}. \tag{31}$$

The host chooses between selecting a first period tax rate t_1' less than or equal to the critical value t_1^c and thereby securing a capital level $x_1^L(t_1')$, or a tax rate greater than the critical level and obtaining the lower capital level $x_1^H(t_1'')$. For the critical value t_1^c to be the cut off level for a potential separating equilibrium, two self-selection constraints have to hold: the high spending type of host must prefer the outcome with $\{t_1'' > t_1^c, X_1^H(t_1'')\}$, while the low type must prefer $\{t_1' \leq t_1^c, X_1^L(t_1')\}$. In symbols, the self-selection constraints for the low and high types are:

$$\begin{gathered}
\exists\ t_1' \leq t_1^c, \text{ such that } \forall t_1'' > t_1^c: \\
W_1\{t_1', X_1^L(t_1'), W_2^*\big(t_2^*(X_1^L(t_1'), \theta^L), X_2^*(X_1^L(t_1'), \theta), \theta^L\big), \theta^L\} > \\
W_1\{t_1'', X_1^H(t_1''), W_2^*\big(t_2^*(X_1^H(t_1''), \theta^L), X_2^*(X_1^H(t_1''), \theta^L), \theta^L\big), \theta^L\};
\end{gathered} \tag{32}$$

and

$$\begin{gathered}
\exists\ t_1'' > t_1^c, \text{ such that } \forall t_1' \leq t_1^c: \\
W_1\{t_1'', X_1^H(t_1''), W_2^*\big(t_2^*(X_1^H, \theta^H), X_2^*(X_1(t_1'', \theta^H), \theta^H\big), \theta^H\} > \\
W_1\{t_1', X_1^L(t_1'), W_2^*\big(t_2^*(X_1^L(t_1'), \theta^H), X_2^*(X_1^L(t_1'), \theta^H), \theta^H\big), \theta^H\}.
\end{gathered} \tag{33}$$

In order to characterize separating equilibria, it is useful to define sets of tax rates based on the incentive compatibility conditions. For a given value of t_1^c such that $t_1' \leq t_1^c$ and $t_1'' > t_1^c$, let $T_1'(t_1^c)$ be the set of t_1' such that a low type (weakly) prefers to choose t_1' and have investors believe she is a low type, rather than choose *any* $t_1'' > t_1^c$ and be believed to be a high. That is,

$$T_1'(t_1^c) = \{t_1' \leq t_1^c:\ W_1\big(t_1', X_1^L(t_1'), \theta^L\big) \geq W_1\big(t_1'', X_1^H(t_1''), \theta^L\big)\}. \tag{34}$$

Similarly, construct the set $T_1''(t_1^c)$ to be all $t_1'' > t_1^c$ such that a high type (weakly) prefers to choose t_1'' and have the market believe she is a high over *any* $t_1' \leq t_1^c$ and be mistaken as a low type. The next proposition gives a necessary condition for a separating equilibrium.

Proposition 3. A tax rate t_1^c supports a separating equilibrium only if the sets $T_1'(t_1^c)$ and $T_1''(t_1^c)$ are non-empty.

Proof. Using the strategy pair given above for a given t_1^c, each type of principal would prefer an element of T_1' or T_1'', as the case may be. Hence, the types of host are separated and the investor's beliefs are consistent with the outcome. ■

For a separating value t_1^c a low type picks her best element in T_1' and a high picks the best in T_1''. It is obvious that a t_1^c that supports a separating equilibrium must be a tax rate less than the full information optimal tax rate for a high type. Hence, the set $T_1''(t_1^c)$ must contain the first best full information choice for the high, t_1^H. This leads immediately to the following characterization of a separating equilibrium.

Proposition 4. In a separating equilibrium a high type chooses $t_1'' = t_1^H$. A low type chooses the element in $T_1'(t_1^c)$ that maximizes $W_1\left(t_1', X_1^L(t_1'), \theta^L\right)$.

Proof. The proof of the proposition follows from the preceding discussion.

A separating equilibrium does not exist if the tax rate required to identify a low spender falls below the set of admissible values, leaving T_1' empty for any t_1^c for which T_1'' is non-empty. On the other hand, there can be uncountably many separating equilibria. Before refining the potential equilibria by placing restrictions on out-of-equilibrium beliefs, let us look at a special case of a separating equilibrium.

Natural Separation

In a separating equilibrium for a given t_1^c the set $T_1'(t_1^c)$ does not generally contain the low's optimal tax rate for the full information game t_1^L. When it does contain t_1^L the separating equilibrium is called "natural." In such a case the low type host does not make a tax "concession." The adverse selection imposed on a low by a high type is of no consequence, because the low host can identify herself coincidentally by simply choosing her first best tax rate. A high type must place a rather heavy weight on public spending for a natural separating equilibrium to occur. Note that if the marginal costs of capital adjustment were zero separation would arise naturally.

Refining the Set of Separating Equilibria

When there are many potential separating equilibria, we can reduce the set to a unique equilibrium by placing a natural restriction on out-of-equilibrium beliefs. The restriction I impose derives from Cho and Kreps (1987) and is used by Vickers (1986) among others. The restriction begins by assuming, quite reasonably, that the host takes it for granted that the market response to any first period tax rate is sequentially rational. Suppose that there exists a t_1^c that supports a separating equilibrium; suppose, also, that another separating equilibrium exists based this time on $t_1^{c\prime}$, where $t_1^{c\prime} > t_1^c$. In the original equilibrium with t_1^c the representative investor's strategy calls for a response of $x_1^H(t_1'')$ to any $t_1'' > t_1'$, leaving the low type worse of than if she were to choose $t_1' \leq t_1^c$. Suppose, nonetheless, that investors observe a tax rate $\hat{t}'' \in T_1''(t_1^c)$, such that $\hat{t}'' = t_1' \in T_1'(t_1^{c\prime})$. By construction a high type could *never* do better by choosing $\hat{t}_1'' = t_1' \in T_1'(t_1^{c\prime})$ than by choosing $t_1^H \in T_1''(t_1^{c\prime})$ and be known to be high. Recall that proposition 4 ensures that

$t_1^H \in T''$ in any separating equilibrium. Therefore, it would not be sequentially rational for investors to carry out their original strategy after observing $\hat{t}_1'' > t_1^c$. In fact, the investor should attach zero probability to the event of a high type choosing a tax rate in $T_1'(t_1^c)$ for all t_1^c that support a separating equilibrium. Given these new beliefs the low type host can choose among any the sets $T_1'(t_1^c)$ that are part of a separating equilibrium. That is, the refinement eliminates all weakly dominated strategies in the set

$$\cup_{t_1^c} T_1'(t_1^c).$$

Note that a natural separation requires $t_1^L \in \cup T_1'$. The refinement allows a sharper characterization of a separating equilibrium.

Proposition 5. In a separating equilibrium with the refinement introduced above the optimal action for a low spending principal is $t_1^{s*}(\theta^L)$:

$$t_1^{s*}(\theta^L) = t_1^L \text{ if } t_1^L \in \cup T_1'; \text{ otherwise, } t_1^{s*}(\theta^L) = sup\{\cup_{t_1^c} T_1'(t_1^c)\}. \qquad (35)$$

Proof. The welfare function W_1 is quasi-concave in t_1 (over the range $t_1 \geq 1/(2\theta)$). Hence, at a tax rate less than the full information optimum for a low type t_1^L, the welfare function is increases monotonically provided beliefs remain fixed, which is the case for any $t_1 \leq t_1^c, \forall t_1^c$. Hence,

$$t_1' \leq t_1^L \Rightarrow W_1(t_1', \theta^L) \leq W_1(t_1^L, \theta^L). \quad \blacksquare$$

A method for solving for the unique separating tax rate $t_1^{s*}(\theta^L)$ is the following. Calculate the value of the social welfare function for a high host when it is common knowledge she is a high. Then find the tax rate when she is believed to be low that

leaves her indifferent between the resulting utility level and the level calculated previously. Verify that a low host's incentive compatibility constraint is not violated at the tax rate found in the previous step. Figure 4 illustrates a separating equilibrium in terms of indifference curves and the first period arbitrage constraint. Figure 5 illustrates the relationship between the different tax rates characterized above.

FIGURE 4

Separating Equilibrium

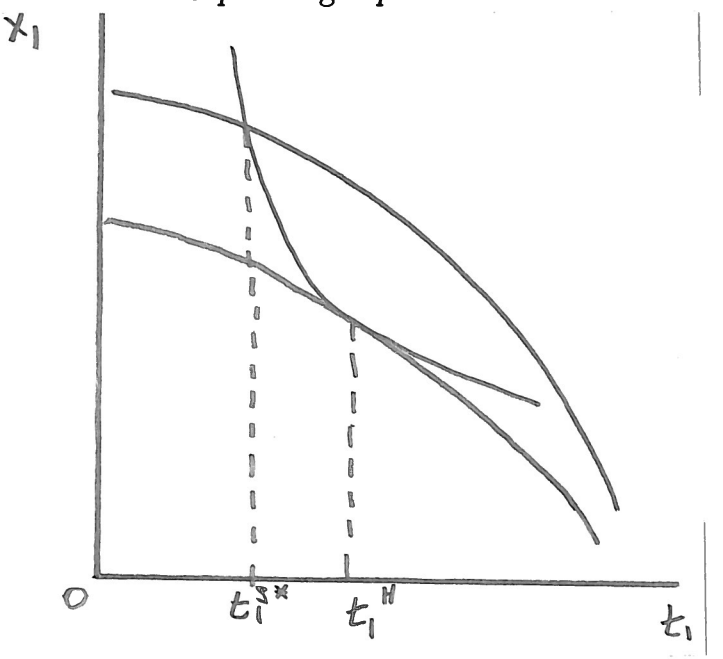
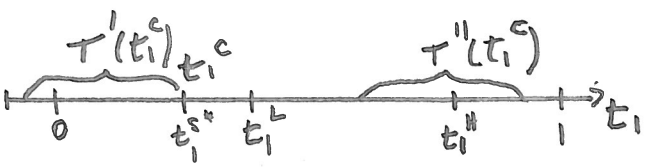


FIGURE 5

Relative Positioning of Tax Rates



Pooling Equilibrium

When the tax policy of a host country government does not signal its type (and in the absence of any further political information) it is perhaps natural for foreign investors to attach subjective probabilities over spending types that correspond to the proportions of high and low spenders in the general population of capital-poor countries; I construct pooling equilibria based on such "passive beliefs." A pooling equilibrium in the first period can only arise if both the high and low spending types select the same first period tax rate, which I denote t_1^p. A pooling equilibrium requires that $\rho_1(\theta|t_1^p) = \rho_0$; the capital market must be in equilibrium based on expectations of the second period tax rate; and out-of-equilibrium beliefs must be such that neither spending type has an incentive to deviate from the pooling tax rate. The investor's first period problem is to maximize U_1 with beliefs given by $\rho_1 = \rho_0$. This maximization generates an equilibrium foreign investment level of $X_1^p \equiv \tilde{X}_1(t_1^p)$ in the first period and an *ex ante* expected second period equilibrium:

$$\{\,[\rho_0 \cdot t_2^*(X_1^p, \theta^L) + (1-\rho_0) \cdot t_2^*(X_1^p, \theta^H)], [\rho_0 \cdot X_2^*(X_1^p, \theta^L) + (1-\rho_0) \cdot X_2^*(X_1^p, \theta^H)]\,\}. \quad (36)$$

For ease of notation I denote this *ex ante* second period equilibrium as

$$\{t_2^p(\theta), X_2^p(\theta)\},$$

for a given prior distribution of θ. Of course, when the second period actually occurs the host's type becomes known: $\theta = \theta^L$ or θ^H. Let us therefore also indicate the *ex post* realization of the second period equilibrium as:

$$\{t_2^*(X_1^p, \theta^i), X_2^*(X_1^p, \theta^i)\}, \text{ for i} = \text{L, H}.$$

To facilitate the characterization of pooling equilibria, I follow Vicker's (1986) analysis in defining the following sets. Let $\mathcal{T}_i^-$ (resp. $\mathcal{T}_i^+$) be the lowest (resp. highest) level of t_1 such that a type-i host is indifferent between

(a) choosing $t_1 = \mathcal{T}_i^-$ (resp. $\mathcal{T}_i^+$) and have investors base
their second period expectations on their prior that a low type
occurs with probability equal to ρ_0—that is they
expect $t_2 = t_2^p(\theta)$; and
(b) choosing the best tax rate given that investors believe that the host
is a high type, so that $\tilde{X}_1 = X_1^H(t_1)$, $\forall t_1$. In this
case the expected second period tax rate is $t_2^*(X_1^H(t_1), \theta^i)$.

Let $\mathcal{T} \equiv [\mathcal{T}_L^- \mathcal{T}_L^+] \cap [\mathcal{T}_H^-, \mathcal{T}_H^+]$. The next proposition characterizes pooling equilibria.

Proposition 6. If $\{t_1^p, X_1^p(t_1^p)\}$ is a pooling equilibrium, then $t_1^p \in \mathcal{T}$. [11]

Proof. See the appendix.

The resolution of the adverse selection problem by signalling has time series implications for the tax and investment profiles; this is given as a proposition.

Proposition 7. Compare the first and second period tax rates set by a low spender in a separating equilibrium with the corresponding rates in a pooling equilibrium, where in the second period it is revealed that $\theta = \theta^L$. Then, not only is the first period tax rate lower in a signalling equilibrium than in a pooling equilibrium; the second period tax rate will be *higher*. That is,

$$t_1^{s*} < t_1^p \text{ and } t_2^*(X_1^L(t_1^{s*}, \theta^L) > t_2^*(X_1^p, \theta^L). \tag{37}$$

Proof. In a separating equilibrium with $\theta = \theta^L$ the level of first period investment X_1 is higher than in a pooling equilibrium for two reasons. First, with separation

[11] Nothing rules out the possibility of uncountably many pooling equilibria, although a strong refinement on out-of-equilibrium beliefs may be used to break all pooling equilibria, following Kreps (1990).

investors know the host is a low type in the first period, which encourages investment because $t_2^*(\theta^L, X_1) \leq [\rho_0 \cdot t_2^*(X_1, \theta^L) + (1 - \rho_0) \cdot t_2^*(X_1, \theta^H)]$, $\forall X_1$. Second, the tax concession required for signalling is itself a financial inducement for investment. That is, $\tilde{X}_1(\theta^L, t_1^c) > \tilde{X}_1(\theta^L, t_1^p)$. Since it was established in lemma 2 that the second period tax rate is an increasing function of first period investment, the signalling equilibrium produces a higher second period tax rate than the pooling equilibrium for an *ex post* low type of host. ■

In fact, it may be the case that the separating equilibrium produces a higher second period tax rate than even a *high* type would set in a pooling equilibrium, as a resut of the tax holiday stimulus to investment. What this analysis suggests is that investors cannot simply infer from a tax hike subsequent to their initial investment that their view about the host government's spending type were incorrect; even a low type host government will want to raise capital income taxes to profit from an investment "boom" resulting from a tax holiday. If there were more than two periods, then after the first period investment boom and the resultant tax increase, the rate on foreign capital income can be expected to settle to a lower level— the level corresponding to a low type's optimal rate when its SWF is common knowledge. However, before considering the case of overlapping generations, I present the example of a simulation model that meets the assumptions of the paper.

6. An Illustration of the Signalling and Pooling Equilibria

Tables 1 to 4 correspond to the separating and pooling equilibria of the incomplete information version of the illustration model introduced in section 4. In this set of simulations I assume that c=.75, $\theta^H = 4$ and $\theta^L = 1$. I set $\rho_0 = 0.5$ and I

assume that investors believe that the host is a high type if she deviates from the pooling level of the tax rate. In the second period of a pooling equilibrium the true value of θ is revealed, although investors were, of course, unaware of the value of θ at the time of their first period actions. I assume that in the pooling equilibrium $\theta = \theta^L$, *ex post*. Note how the second period tax rate in the signalling equilibrium exceeds the second period rate of the pooling model, when *ex post* the host's type is low. Furthermore, note that the equilibrium tax rates are in the region of strict quasi-concavity of the host's welfare function; that is, $t_\tau > 1/(2\theta)$.

TABLE 1 COMPLETE INFORMATION θ=4

period	*tax*	*foreign capital*
1	t_1=.5353	X_1=.2324
2	t_2=.7203	X_2=.2237

TABLE 2 COMPLETE INFORMATION θ=1

period	*tax*	*foreign capital*
1	t_1=.4078	X_1=.2847
2	t_2=.6462	X_2=.2697

TABLE 3 SIGNALLING

period	*tax*	*foreign capital*
1	t_1=.2422	X_1=.3320
2	t_2=.7804	X_2=.2776

TABLE 4 POOLING $\rho = .5$

period	*tax*	*foreign capital*
1	t_1=.4125	X_1=.2778
2	t_2=.6424	X_2=.2493

7. Finite Overlapping Generations

The analysis in the previous section has led to a characterization of the tax treatment of a representative foreign investor's savings or capital over the two period lifetime of the asset, when there is an adverse selection problem. Let us now suppose that there are overlapping generations of foreign investors, each with an identical pool of savings to be invested either at home or abroad, as in the previous sections. In this case, the tax holiday solution will change from one generation of investors to the next. In particular, after a tax holiday has been used to signal a host country government's type the need for tax concessions as a signal disappears and the tax profile faced by the next generation of investors is less steep. However, with no discounting, one may wonder whether it is in the interest of a low type host to make a large tax concession immediately in order to separate types in the first period, or to do so gradually with a moderate tax holiday for the first generation of investors and an even more moderate one for the next. To examine this question, suppose there are only two generations of foreign investors. Hence, in period two, at the same time that the first generation investors are adjusting their capital allocation subject to adjustment costs, the second generation investors make their initial decision about where to place their savings. In the third period, the second generation adjusts its allocation subject to adjustment costs. That is the final period of the game. Assume that the host first announces the second period tax rate applicable to the first generation, and then announces the first period tax rate applicable to the second generation. A solution requires a host to choose an optimal sequence of taxes $\{t_1^{\{1\}}(\theta), t_2^{\{1\}}(\theta), t_1^{\{2\}}(\theta), t_2^{\{2\}}(\theta)\}$, where the superscript denotes the generation of the investor to which the particular tax applies, and the subscript plays the same role as in section 4, while θ denotes the host's type. Note

that $t_2^{\{1\}}$ precedes $t_1^{\{2\}}$ chronologically. Suppose, for a moment, that a low type host were to use the same strategy against the first generation of investors as the strategy worked out in section 5 (where there was only one generation of investors). That is, $t_1^{\{1\}}(\theta^L) = t_1^{s^*}$. Then, a high type host who wishes to mimic a low type in order to induce more investment in the first period must also choose this tax rate. Furthermore, in the second period the high type must again mimic the low type even though she would prefer to choose $t_2^{\{1\}} = t_2^*(X_1, \theta^H)$; but, to do otherwise would reveal her type before any benefits from mimicking are realized. Now, suppose the initial tax rate set by a low type for the second generation of investors coincides with the optimal tax profile for the complete information model from section 3. That is, $t_1^{\{2\}}(\theta^L) = t_1^L$. Again, a high type must also choose this rate in order to continue mimicking the low; that is the only way to justify her earlier tax revenue loss relative to her preferred choice. Finally, in the last period of the game the high type can reveal herself and choose a high tax rate: $t_2^{\{2\}} = t_2^*(X_1^L, \theta^H)$.

The alternative to mimicking the low type's strategy would be for the high type host to choose her best tax rate profile, given that investors could infer her type. Therefore, compare the payoffs for a high type under the alternative strategies of mimicking, and revealing herself, where W^M denotes the payoff across both generations from mimicking, and W^R the payoff from revealing her type.

$$W_M = W_1\big(t_1^{s^*}, t_2^*(X_1(t_1^{s^*}, \theta^L), \theta^L), \theta^H\big) + W_1\big(t_1^L, t_2^*(X_1(t_1^L, \theta^L), \theta^H), \theta^H\big)$$

$$W_R = W_1\big(t_1^H, t_2^*(X_1(t_1^H, \theta^H), \theta^H), \theta^H\big) + W_1\big(t_1^H, t_2^*(X_1(t_1^H, \theta^H), \theta^H), \theta^H\big).$$

Notice that the first term in W_M is strictly lower than the first term in W_R, since this was precisely the choice of payoffs that led to separation in the one-generation version of the game in section 5. However, the second term in the expression for W_M exceeds the second term in W_R, unless there exists a natural separation for the one-generation game. That is, mimicking a low type host in dealing with the first

generation of investors may eventually pay off, if the second generation of investors is convinced by earlier play that they are dealing with a low type. If $W_M > W_R$, then separation fails unless a low type is prepared to either deepen the initial tax holiday by setting $t_1^{\{1\}} < t_1^{s^*}$, or to grant a tax holiday to the second generation of investors in addition to the first generation, so that $t_1^{\{2\}} < t_2^L$. However, note that if the latter strategy is pursued by the low type, the tax holiday need not be as extreme as it was for the first generation of investors, under any reasonable beliefs, because the first term of W_M is strictly less than the first term in W_R. The above discussion is summarized as a proposition.

Proposition 8. Any separating equilibrium for the two-generations version of the game has the following property:

$$t_1^{\{1\}}(\theta^L) \leq t_1^{\{2\}}(\theta^L)$$
$$t_2^{\{1\}}(\theta^L) \geq t_2^{\{2\}}(\theta^L).$$

The inequality is strict unless there is a natural separating equilibrium in the one-generation version of the game; i.e., if and only if $t_1^{s^*} = t_1^L$.

Proof. The proof follows from the discussion above.

Note that an application of the intuitive criterion would result in an equilibrium where a low type of host separates in the first period of the first generation, if possible, rather than doing so gradually over two generations. The reason for this is as the first period tax rate is reduced below t_1^L the high type's welfare decreases more rapidly than the low type's, as a result of the single-crossing property of the first period indifference curves. Therefore, a "very low" first period tax rate for the first generation is the least costly way for a low type to discourage a high type from mimicking her behavior. Hence, second generation investors should infer that

only a low type could gain by setting an exceptionally low first period tax rate on the first generation of investors. Such an equilibrium sharpens the conclusion that the tax profile becomes less steep across successive generations of investors. This provides an explanation for why many developing countries reform their tax system by eliminating investment incentives after a number of years of generous tax holidays. [12]

8. Conclusion

"Shy like a deer," is the piquant description of international capital recently given by the German economics minister regarding foreign investment in developing countries. [13] Today there are many developing countries that appear newly converted to free market economics; whereas in the recent past they may have engaged in heavy handed-government intervention, countries such as Vietnam and India, among many, now appeal for direct foreign investment and proclaim a political and economic climate that is favourable to business. Potential foreign investors undoubtedly look for evidence of "the permanence of the arrangement" before sinking their capital in a host country. In this paper, I constructed a model to show how a tax holiday may signal a host country government's commitment to the private economy relative to government spending. By setting a low tax rate on foreign capital income and thereby foregoing public expenditure a low-spending government may resolve a credibility problem when it announces its pro-market reforms. In a separating equilibrium the tax holiday stimulates foreign investment as a result of both the direct financial incentive of a low tax rate, and the indirect effect of

[12] In 1984, Indonesia eliminated the use of tax holidays. One stated view was that the tax holidays were of "dubious value," and were the mistake of previous policy-makers. However, an interpretation based on section 7 would say that the tax holidays were removed only after they had successfully signalled the government's pro-market orientation.

[13] *The Globe and Mail*, February 7, 1992.

establishing credibility about the government's low-spending type. Paradoxically, the increase in investment encourages a higher second period tax rate as even a relatively low-spending government attempts to exploit the convex costs of capital adjustment. However, when there is a second generation of investors, the tax rate eventually drifts back downward "justifying" the foreign investors' confidence in the permanence of the arrangement. The analysis provides a new interpretation of the tax holiday phenomenon in developing countries and suggests that the time series behavior of tax policy and foreign investment may not be straightforward.

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