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## Political Competition (A theory with applications to the distribution of income)

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*Political Competition*

*A theory with applications to the distribution of income*

by

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## Chapter 0: INTRODUCTION

The formal model of political competition almost ubiquitously employed by students of political economy is one in which political parties play no role. That model, introduced by Anthony Downs (1957) over forty years ago, portrays a competition between candidates, whose sole motivation for engaging in politics is to enjoy the power and perquisites of office holding. Although voters care about policies, the candidates do not; for them, a policy is simply an instrument to be used, opportunistically, as an entry ticket to a prosperous career. Political parties, however, have, throughout the history of democracy, cared about policies, perhaps because they are formed by interest groups of citizens. Therefore the Downsian model cannot be viewed as an historically accurate model of party competition.

Democratic history is one of competition between parties that represent, perhaps imperfectly, contesting interest groups among the polity. Contesting interest groups can be represented, abstractly, as possessing different preferences over policies that are to be implemented by the government. An historically accurate model should therefore represent political competition as one between parties, each of which has preferences over policies, and each of which seeks, in the game of political competition, to propose the policy that maximizes its preference order, or utility.

Such a model was indeed introduced by Donald Wittman (1973), but it was not carefully developed until recently, and has only been used in applications by a small number of researchers. The Wittman model is less user-friendly than the Downs model, in two ways: first, more data are required to specify the political environment with Wittman politics than with Downs politics (e.g., one must specify the preferences of the active political parties), and second, the computation of equilibrium, at least in the interesting case of the presence of uncertainty, is more complex in the Wittman model. But as Einstein said, good science consists in constructing models that are as simple as possible, but no simpler, and in this case, I contend that the Downs model is too simple - the price of its simplicity is the elimination of politics from political competition.

In this book, I attempt to develop, in a systematic and rigorous fashion, the theory of competition between political parties in a democracy. Although the Downs model is not the one of choice, I develop the Downs theory as well, for it has obviously played an important role in the formal political theory. It is, moreover, important to understand when the theory of competition between *parties* produces political equilibria that differ from the Downs equilibrium, for it does not always do so. Thus, it is the case, in certain situations, that although competition occurs between partisan parties, the result is no different than it would have been, had the competition been between policy-disinterested candidates. I shall, however, argue, that the two models only predict the same

equilibrium policies in cases that are historically unrealistic, ones in which there is no uncertainty surrounding elections.

We begin with a model of a polity composed of citizens who possess preferences over a policy space. I assume a continuum of citizens; this is the model of choice when we are concerned to understand political competition with large polities. There are two political parties, whose preferences ( or payoff functions) are first specified sufficiently abstractly that both Downs politics and Wittman politics are special cases. We model political equilibrium as the Nash equilibrium of the game in which each party maximizes its payoff function over a strategy space which is the policy space. A party's payoff, under both the Wittman and Downs specifications, depends, *inter alia*, upon its probability of victory, given the policies played by it and its competitor.

The book consists in further articulation of this general model. In Chapter 1-7, we study the properties of the Nash equilibria of the political game, under eight different specifications of the model. These specifications are the eight possibilities in the cross-product of models:

$$\{\text{Downs, Wittman}\} \times \{\text{certainty, uncertainty}\} \times \{\text{unidimensional, multidimensional}\}.$$

Downs/Wittman refers to the motivation of the political actors -- whether they seek to maximize the probability of victory (Downs), or to maximize expected utility associated

with a preference order over the policy space (Wittman). Certainty/uncertainty refers to whether parties know for certain the distribution of voter types, or have only a probability distribution over the possible distributions of voter types. Parties are certain about how the vote will be distributed between the parties, once policies have been announced, if and only if they are certain about the distribution of voter types.

Unidimensional/multidimensional refers to the dimension of the policy space.

The classical 'median voter theorem' is a description of the Nash equilibrium in political competition in the case {Downs, certainty, unidimensional}, and probably 95% of the formal literature in political economy since Downs has employed this particular specification. In my view, this model is ahistorical in all three ways: democratic politics are never Downsian, parties are never certain about the mapping from policy pairs (proposed by the two parties) to the vote distribution, and, I contend, (national) politics are never unidimensional.

The structure of the book is given in the flow chart below. Chapter 1 characterizes political equilibrium in the cases {Downs, certainty, unidimensional} and {Wittman, certainty, unidimensional}. In both cases, there is a 'median voter theorem,' but it is a different theorem in the two cases -- in particular, additional premises are needed to prove the theorem in the Wittman context. The conclusion of both theorems is that both parties announce the same policy at equilibrium, which is the ideal policy of the voter whose ideal point is median in the distribution of voter ideal points. If we believe

that parties never propose the same policies in actual democracies, then these models must be inaccurate.

It may, however, be a good approximation to say that, in some elections, both parties proposed the same policy. Chapter 1 tells us, importantly, that such an observation is not indirect evidence for the validity of the Downs characterization of politics, for Wittman politics will produce the same result. In a world of certainty, Downs and Wittman politics are observationally equivalent (unless we can somehow observe party motivations).

Chapter 2 introduces party uncertainty about voter behavior. There are, indeed, a variety of methods for modeling this kind of uncertainty, and three are explicated in this chapter. All three approaches to uncertainty deliver *aggregate uncertainty*, despite the fact that there is a continuum of voters. Thus, it will not do to assume that each voter behaves stochastically, and that the random variables that describe the behavior of individual voters are independently distributed, for in that case, uncertainty at the aggregate level would disappear. Even though we have a continuum of voters, parties do not know for sure the mapping from pairs of policies to the vote distribution.

Let  $(t^1, t^2)$  be the probability that Party One defeats Party Two at the policy pair  $(t^1, t^2)$ . The three approaches to uncertainty offered here each provide microfoundations (to varying extents) which allow us to *compute* the function  $\cdot$ . That function is not a datum of our problem, but is deduced from more primitive assumptions. This

philosophically desirable approach renders our analysis somewhat difficult at times, because it is often the case that the computed function is rather badly behaved, even when the underlying primitives are unexotic. For instance, is typically not everywhere continuous, nor everywhere differentiable where it is continuous ; nor does it standardly possess the kind of convexity properties which are useful in equilibrium theory.

Armed with these several approaches to uncertainty, Chapter 3 studies the cases {Downs, uncertainty, unidimensional} and {Wittman, uncertainty, unidimensional}. Now the Downs and Wittman formulations *do* generate equilibria which are different: while the Downs model continues to predict that both parties propose the same policy in equilibrium, the Wittman model predicts that equilibrium policies will be differentiated.

Thus, of the four model types we have studied in the unidimensional case -- {Downs, certainty}, {Wittman, certainty}, {Downs, uncertainty}, {Wittman, uncertainty} -- only the last one generates the realistic outcome that parties offer different policies in equilibrium. We can conclude that both the Downs assumption and the certainty assumption are poor ones, if we believe that in reality policies are differentiated in elections.

Although a theorem proving the existence of political equilibrium in the {Wittman, uncertainty, unidimensional} case is presented in Chapter 3, it is not fully

satisfactory or general, for its premises are complex in the sense of not being stated in terms of the primitive data of the model. Here, the unpleasant behavior of the function comes home to roost. It is nevertheless the case that in many applications of the model, these equilibria exist and can be calculated, even if we do not have a fully general theorem asserting the existence of equilibrium. Chapter 4 offers four applications of the {Wittman, uncertainty, unidimensional} model, and computes equilibria. These applications are offered for two reasons: first, to show that the model is indeed tractable, and can be used as a tool in political economy, and second, for the particular substantive results deduced.

Until now, we have taken the preferences of parties as given, much as the Arrow-Debreu model takes the technologies of firms as given. In Chapter 5\*, we relax this assumption, and propose two models in which the parties' preferences, in the Wittman model, are endogenously derived from the preferences of the citizens. Our conception is one of 'perfectly representative democracy,' an ideal type that is not realistic when parties are financed by private contributions, as they are in the United States ; nevertheless, there is value in comparing what these models predict to what we observe.

Thus, we propose notions of 'equilibrium in the process of party formation;' one of these we dub 'Condorcet-Nash equilibrium.' With these models, we have a relatively complete theory of political economy, one which derives political equilibrium beginning only with knowledge of the distribution of voter preferences. (Of course, the classical

Downs model does that as well, but we are dissatisfied with it as a model of politics.) As an application, we show how at least one important claim about the relationship between taxation and income distribution, which is true when politics are Downsian, is not true in this setting of endogenous Wittmanesque parties. This provides another reason to reject the Downs model of politics. We also calculate the Condorcet-Nash equilibrium for a Euclidean model, and demonstrate an interesting relationship between the preferences of parties and the voters who support them.

Chapter 5\* concludes the unidimensional analysis. In Chapter 5, we study the models {Downs, certainty, multi-dimensional} and {Wittman, certainty, multi-dimensional}. The story in these cases is that, except under singular specifications of the data, political equilibria do not exist. In the Downs case, this fact is well-known, and is essentially equivalent to the non-existence of Condorcet winners (when the policy space is multi-dimensional). Altering the specification of party payoff functions from Downs to Wittman does not enable us to escape the failure of the model in offering a prediction of political equilibrium in the multi-dimensional context.

Chapter 6 studies the {Downs, uncertainty, multi-dimensional} model. Several authors (Coughlin, Linbeck and Weibull) have shown that political equilibria exist in models of this type. We present their results. Their models assume, however, a finite polity, and the uncertainty that exists in the environments they postulate disappears when the polity approaches the continuum. Nevertheless, we are able to use the finite-type

model (introduced in Chapter 2) to show that equilibria continue to exist in a model with a continuum of voters. As before, in Downs equilibrium, both parties offer the same policy.

While we have shown that equilibrium can exist in the {Downs, uncertainty, multi-dimensional} case, it is a fragile existence: equilibrium does not exist in this case if we model uncertainty in the other two ways described in Chapter 2.

The fact that equilibrium does not exist in the models of Chapter 5, and exists only fragilely in the environment of Chapter 6, has induced many researchers to depart from equilibrium analysis and study 'cycling.' Cycling behavior is what presumably occurs in a real-world game with no Nash equilibrium: Party 1 plays  $t$ , Party 2 plays a best response to  $t$ , Party 1 plays a best response to what Party 2 played, and so on. If Nash equilibrium does not exist, this process will never converge.

We take, however, a different point of view -- that when a model has no equilibrium, it is probably a misspecification of the real-world phenomenon it is meant to portray. In Chapter 7, which is, in a dramatic sense, the center of the book, we offer a way of thinking of multi-dimensional political competition under uncertainty in which Nash equilibria do exist.

That conception marries the Downs and Wittman approaches. It conceives of a party as consisting of three factions -- militants, opportunists, and reformists. The opportunists are the *dramatis personae* of the Downs model -- they desire only to

maximize the probability of the party's victory. The reformists are the actors in the Wittman model -- they desire to maximize the expected utility associated with the party's preference order over policies. The militants are new characters: they desire to propose a policy as close as possible to the party's ideal point, and have no interest in winning the election at hand. I argue that political histories are replete with descriptions of these three kinds of party activist -- for instance, Schorske (1955) calls them the party bureaucrats, the trade union leadership, and the radicals. To summarize, in a word, the opportunists, reformists and militants are interested, respectively, in *winning*, *policy*, and *publicity*.

Each party is now postulated to be a coalition of these three factions. We propose that the *party's* preference order over policy pairs is the intersection of the preference orders of its three factions. Political equilibrium -- christened Party Unanimity Nash Equilibrium (PUNE) -- is now defined as Nash equilibrium where each party is equipped with the preferences just described. In English, this means that, at a policy pair  $(t^1, t^2)$ , Party 1 will deviate from  $t^1$  to a policy  $t^*$  only if all three of its factions (weakly) prefer  $(t^*, t^2)$  to  $(t^1, t^2)$ . We show that we should expect a (two-dimensional) continuum of such equilibria to exist in multi-dimensional party competition.

Once armed with the PUNE concept, we are also able to provide a conceptually simple proof of the fact that, except in singular cases, equilibria fail to exist in the {Wittman, uncertainty, multi-dimensional} model.

The existence of a continuum of equilibria in the {multi-faction, uncertainty, multi-dimensional} model means that the model is underdetermined -- if we believe that real political equilibria are locally unique. I argue, in Chapter 7, that the multiplicity of equilibria is the price we pay for not specifying a particular bargaining game among the factions of a party. There are, however, reasons not to specify such a game. Perhaps the main one is that bargaining among factions requires compromises, and compromises only make sense in a multi-period game -- for example, the militants will compromise today in return for a promise that the reformists will compromise in the next election. Thus, bargaining among factions is only sensible in a more complex model, one played over many periods.

It turns out, nevertheless, that in many applications, the cost we pay for not refining the equilibrium set is small, either because the PUNEs turn out to be quite locally concentrated in the policy space, and so the model gives us quite good predictions of the characteristics of equilibrium, or because we are able to establish characterizations of the entire equilibrium set. Chapters 8 through 11 present four applications of the {multi-faction, uncertainty, multi-dimensional } model.

Chapter 8 considers the problem of multi-dimensional taxation. The question why income taxation is progressive in almost all democracies has never been given a satisfactory answer in the political economy literature, because of not having a specification of a multi-dimensional political contest in which equilibria exist. We

assume that the set of admissible income tax policies consists of quadratic functions of income, with certain properties (monotonicity of after-tax income in pre-tax income, etc.). The policy space is two-dimensional, after taking account of a budget-balancing constraint. There are two political parties, one representing relatively rich citizens, the other relatively poor citizens. The polity is one in which median income is less than mean income (as in all actual democracies). We show that, in all PUNEs of the game where parties compete by announcing tax policies, a *progressive* tax policy wins with probability one. This application demonstrates that, even though we have a continuum of equilibria, we can say something interesting about all of them.

Although the PUNE tool allows us to advance our understanding of the ubiquity of progressive taxation, it does not settle the question entirely. In particular, extending the argument to a policy domain of even cubic tax functions has not been done.

Chapter 9 attacks a venerable question in political economy, one of concern since the foundation of democracy. In a democracy, why do the poor not expropriate the rich through the tax system? Various answers have been offered; this chapter proposes a new formulation of an old answer that depends upon the multi-dimensionality of the policy space. That old answer is that the voters -- in particular, the poor -- care about other things as well as income, for example, religion. If the conservative party is religious and the labor or socialist party is not, a section of the poor will vote for the former, despite its

conservative policy on income redistribution<sup>1</sup>. This chapter proposes a polity in which each citizen type has a preference order over income and the religious position of the government; the polity is characterized as a distribution of these types. We ask: Is there a condition on the distribution of types under which it will be the case that, if religion is sufficiently salient, *all* PUNEs will involve *both left and right* political parties' proposing low tax rates? We indeed discover such a condition. Here is, again, an application in which the infinite multiplicity of PUNE turns out not to sabotage the possibility of analysis. We go on to show that, if the 'religious' issue is read as the race issue in the United States, then the condition in question holds for the US polity. Thus, to the extent that race is a salient issue for US voters, we should expect *neither* political party to propose highly redistributive policies.

This result shows that multi-dimensional politics can be, at first glance, counter-intuitive, or as some would say, paradoxical. We show, in Chapter 9, that if religion is sufficiently salient for voters, both parties will propose a *zero* tax rate, in all PUNEs, even though a majority of the population have an ideal tax rate of unity! The source of the paradox is that policies are not voted upon independently, but as a package, under party competition. (This is in contrast to the referendum process, where, presumably, each dimension of policy can be voted on independently.)

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<sup>1</sup> This is not quite the same thing as Marx's view that 'religion is the opiate of the masses.' For Marx, religion kept the masses from rebelling; here religion, may deter many workers from voting for the party

In Chapter 10, we apply the PUNE concept to another historical question : why did Socialists win in some countries, and Fascists in others, in interwar Europe? The chapter is inspired by the rich analysis of this question by Luebbert (1991). While Luebbert's analysis is complex, a rather simple story is its dominating feature. That story is one of a multi-class, competitive politics. Luebbert models countries as consisting of four classes: the urban workers (W), the urban middle class (M), the rural landed peasantry (P), and the agricultural laborers (A). (Of course, urban and rural upper classes existed too, but were too few in number to matter in terms of voting.) The key to political victory (whether by election or some other form of popular support), was to forge an alliance between either the workers and landed peasants, or between the middle class and the landed peasants. Presumably, the Left would win if the former alliance were cemented, and the Right would win if the latter alliance were cemented. Luebbert goes on to argue that the Left succeeded in forging the worker-peasant alliance only in countries in which peasant-agricultural worker class struggle was quiescent. The three European countries where that struggle was active were Germany, Italy, and Spain, which all became Fascist.

We design a model to test Luebbert's conjecture, which becomes a formulation about the probability of Left victory in an electoral competition between Left and Right, where rural class struggle is either quiescent or active. The 'Luebbert conjecture' is that

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that champions their economic interests.

that probability should be significantly larger when rural class struggle is quiescent. The policy space in this model is four-dimensional. We deduce strong but not ironclad support of the Luebbert conjecture.

Chapter 11 presents a rather schematic, three-class model of US politics, whose purpose is to study the question: why are the interests of large capital represented in both the Democratic and Republican Parties? (We presume this to be the case, without argument.) The economy consists of three types of individual: capitalists, who own a large firm, and hire labor; workers, who sell their labor-power to the large firm; and the petit bourgeoisie, who work in their own shops and hire no labor. It is supposed that the large firm uses government provided infrastructure as an input, besides capital and labor, while the petit bourgeois shops use only the labor of their owners. The political problem is to determine a uniform tax rate on the income of all three classes, and to divide tax revenues between a lumpsum transfer to all citizens and spending on infrastructure. This is a two-dimensional policy space.

We presume that the petit bourgeoisie and large capital are both represented in the Republican Party. ( This party turns out to have five factions.) We study two alternative membership scenarios for the Democratic Party: either it represents only workers (Scenario One), or it is a coalition of workers and large capital (Scenario Two). These alternatives are easily captured with the machinery of factions. We show that capitalists do better in the political equilibria (PUNE) of Scenario Two than in those of Scenario

One, which provides an explanation of the presence of both large capital and labor in the Democratic Party. Why does large capital not join the British Labor Party and the Scandinavian social democratic parties? Perhaps it has. If not, this analysis would suggest that entry of the capitalist class into those parties is forbidden.

Chapter 12 revisits the problem of (endogenous) party formation, but now in the multi-dimensional context. We begin with a distribution of types and assume that two parties will form. We compute what the preferences of those parties will be under the assumption of a perfectly representative democracy that was introduced in Chapter 5\*. We develop two applications. First, we estimate the distribution of US voter preferences on the two issues of taxation and race, using data from the National Election Survey, and then compute the endogenous parties that would form in a perfectly representative democracy. Second, we take the model of Chapter 8, in which the set of policies is quadratic income taxes, and compute what parties should arise, given the distribution of income in the United States, and assuming that all voters are interested in maximizing their after-tax income. We show, in both of these examples, that the parties are quite polarized in their preferences, and that policies in political equilibrium are quite far apart. We end the main body of this chapter with a short section that contemplates the nature of party formation in an imperfectly representative democracy, where the preferences of parties are determined by their financial contributors. A reprise takes up, once again, the

issue of why the poor don't expropriate the rich in democracies, summarizing what we have learned.

Chapter 13 begins an exploration of the theory of three-party competition. It is assumed that, if no party wins a majority of the vote, then a majority coalition, consisting of two parties, must form. We now introduce two kinds of voter -- sincere and strategic -- and model political equilibrium as a Nash equilibrium of a game that takes place at three dates: first, parties announce policies; second, citizens vote (perhaps after a series of opinion polls); and third, a coalition government forms and announces a policy. The main purpose of the chapter is to show that, under the supposition of uncertainty about voter behavior, the main concepts introduced in the two-party model generalize to the three party case -- that is, with a unidimensional policy space, a generalized Wittman equilibrium exists, and with a multi-dimensional policy space, a generalized PUNE exists. As in the two party case, the policies of parties are differentiated at these equilibria.

[extra line space here]

Throughout this book, there is an emphasis on the computation of equilibrium and there is an attempt to use examples that are realistic. We instruct the reader in some detail about the computational procedures, so that he or she could replicate the computations we have made. We have used *Mathematica* to compute equilibria but, of course, other programs can be used as well. The student who wishes to apply the theory

developed in this book should learn the computational aspect. Indeed, we strongly recommend that the student learn to compute. This is a book of tools that can be used to unearth concrete answers to problems involving political competition, but mastering computation is essential to that end.

The mathematics employed in this book is almost all covered in a course in linear algebra and an advanced calculus course. Concepts or theorems invoked that do not fit that characterization, such as Farkas' Lemma and other terminology of convex analysis, and the elements of probability theory, are briefly presented in the Mathematical Appendix. As is often said, a certain level of mathematical sophistication is desirable, for which an advanced calculus course is a proxy. Any student who has completed the first year Ph.D. course in microeconomic theory will be adequately prepared, but that preparation is probably not necessary. Some significant exposure to economic modeling is, however, surely necessary. The game theory used in the book is elementary: the only equilibrium concept used is Nash equilibrium. Nevertheless, the reader will develop a relatively sophisticated understanding of the equilibrium approach if she or he masters the text. A number of examples assume a familiarity with the concept of general economic equilibrium, but those examples are fairly simple.

The centerpiece of the book, as I wrote earlier, is the presentation of a tractable, and historically motivated concept of political equilibrium when politics are multi-

dimensional. This tool, and variations upon it, expand our ability to model political competition in its fuller complexity.