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The Evaluation of Optimal Product Positions

Klaus Brockhoff

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Klaus Brockhoff*

Abstract: Marketing managers who are confronted with solutions from optimal product positioning models may well doubt whether it is possible to transfer results as given into the market place. Some reasons for that are shortly presented. It is then suggested to calculate the deviations from the optimal solution by one-dimensional, two-dimensional or multi-dimensional analyses. These can be used to evaluate the stability of product positions, especially the stability of optimal product positions. A simple measure of solution stability is introduced along with suggestions for its use in decision making.

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Introduction

The art of designing new products or of redesigning already existing products is supported by models of optimal product positioning. These approaches promise to determine that level of product characteristics that would result in maximum sales from potential customers.

The optimal product positioning models that have been suggested so far differ from eachother primarily with respect to the underlying assumptions on buying behavior. Shocker and Srinivasan [7] suggest to maximize a likelihood of buying. It is determined analogously to the logit model [6] from the distances of the new product and all perceptions on existing products to an ideal product perception. Distances are measured in a subjective attribute space and are weighted according to the importance (salience) assigned to them by each individual.

In another approach it is assumed that the likelihood of buying takes on values of one for the product that is perceived closest to the ideal product perception and values of zero for all other products [4]. This is called a single-choice model. Alternative algorithms have been suggested to solve the single-choice model [1,10], and have been compared for their performance under simulated conditions [2].

Optimal product positioning as a result of either one of these models has been criticized mainly for not considering dynamic aspects of preference changes, and for presenting only one solution point that could be missed easily by a new product development [8]. The first argument can be dealt with at least on the theoretical level [3], however, it has to await additional empirical research to add to its operationalization. Therefore, only the second argument is considered in the following. In the following

paragraph we offer some reasons for the difficulty to translate the optimal algorithmic solution into a marketable product. The next paragraph deals with possibilities to measure the instability of the solution. Finally, a suggestion to evaluate alternative solutions is made.

Operationalization of Product Positions

Let us assume that an optimal product position is suggested by the algorithmic solution of either one of the models mentioned above. The marketing manager would then have to translate the optimal levels of each dimension of the characteristics space into a new product. Shocker and Srinivasan suggest that this may be possible if the individual attributes are "actionable" [7]. However, as the attribute space and the product positions within it are derived from individual and subjective product evaluations, in general, actionability may not be guaranteed. Some of the attributes could have been aggregated from a number of "objective" product characteristics. I.e., roominess of a car as interpreted by some potential buyers may be derived from impressions on leg space, head space, length of the car or even trunk volume. For the producer it is not immediately obvious which of these would have to be changed to initiate a change in the roominess perception, provided such change is not to be achieved by some kind of advertising.
Another problem may arise from some fuzzyness in the relationship between objective product characteristics and their subjective perception. This may be true if the problem mentioned above does not apply, such that a one-to-one relationship between objective criteria and subjective attributes could be assumed. In that case one would like to have some idea on the membership function

that maps, i.e., mpg into the idea of the economy of a car. If fuzzyness as well as multiple objective criteria per one subjective attribute should arise at the same time, the membership function would have to be built up from membership functions for each objective criterion. These are connected by operators that indicate whether aggregation is multiplicative ("and") or additive ("or"). As has been shown, the behavioral interpretation of such operators may not coincide with laws of Boolean algebra, such that more flexible approaches have to be used [9] to encompass actual behavior. Measurement errors are another source of unreliable representations of market data collection, i.e. sampling errors, as well as from data processing and interpretation, i.e. choice of the number of characteristics to be considered.

Even if more arguments could be presented it is obvious that it is not at all easy to translate an optimal product position from an attribute space into a real product. Furthermore, the real and ideal product positions in "reality" may not be fully identical with the one presented in the model. This would have consequences for the determination of an optimal product position.

To avoid such difficulties it is suggested to develop a few new product alternatives, and to have them evaluated by potential customers [5]. This procedure may help to discover product designs that promise certain sales and may be preferred to other designs by a producer and a certain number of sellers alike. However, it is limited to very few design alternatives, and its test results cannot indicate whether optimal designs have been hit or missed.

For these reasons management should be interested in the consequences of deviations from the optimal product positions in the attribute space in terms of the objective function of the model. That means: Is the optimal solution rather stable or not? Are optimal solutions more or less flat or rather steep? In which direction from the optimal solution would one have to expect more or less deviations from the optimal value of the objective function, i.e. maximum sales?

An answer to the latter question could also be used to evaluate cost of design changes in one specific direction as compared with expected changes in sales.

If product characteristics can be altered in finite steps, such changes in positions would immediately become obvious. We assume that attributes can take on any value on the axes of the attribute space.

Representations of Solution Stability in One or Two Dimensions

Consider an attribute space with some distribution of ideal and real product perceptions and an optimal product position. Let us assume further that we deal with optimization models of the single choice type.

The first step in the evaluation of solution stability would be to consider a one-dimensional problem. By this we want to answer the question, what changes in the value of the objective function of the product positioning model can be expected along a given ray through the optimal solution and specifically within its neighborhood. Such a ray may be of interest because of an evaluation of changes in only one product characteristic or a prespecified combination of such attributes or because the sales on a trace between the position of a competing product and the new product should be demonstrated etc.

The following procedure serves to solve the problem.
Let

K: a set of customers,

I: a set of existing brands,

J: a set of mutually independent attributes which are considered as potentially relevant to constitute brand preferences;
the attribute space is $R^{\bar{j}}$, where $\bar{j}$ gives the number of dimensions (cardinality) in J,

H: subset of J, such that $H \subseteq J$, and with cardinality two,

e_{kij}: an estimate of the kth ($k \varepsilon K$) customer's perception of the existing brands $i \varepsilon I$, expressed by the coordinate value of the jth ($j \varepsilon J$) coordinate,

c_{kj}: an estimate of the kth ($k \varepsilon K$) customer's perception of an ideal product as expressed by the coordinate value of the jth ($j \varepsilon J$) coordinate,

s_{kj}: a salience, measuring the relative importance of each attribute $j \varepsilon J$ to the customer $k \varepsilon K$,

y_j: the coordinate value for the location of the new product ($j \varepsilon J$),

Δ: distance from y_j to define a neighborhood, measured with some metric parameter m and with $s_{kj} = 1 (k \varepsilon K, j \varepsilon J)$,

m: metric parameter.

For the solution of the single choice model it has been determined

$$(1) \quad d_{ik} = \left| \left[\sum_{j \varepsilon J} s_{kj} \mid c_{kj} - e_{kij} \mid^{m} \right]^{1/m} \right|, \quad i \varepsilon I, \quad k \varepsilon K$$

and

$$(2) \quad d_k = \min \{ d_{ik} \mid i \varepsilon I \}, \quad k \varepsilon K.$$

We would now like to calculate intersections $z_{jk}(j\varepsilon H, k\varepsilon K)$ of

$$(3) \quad \left| \left[\sum_{j\varepsilon H} s_{kj} \mid c_{kj} - z_{jk} \mid^m \right]^{1/m} \right| - d_k = 0 \quad , \; k\varepsilon K,$$

with a ray through y_j $(j\varepsilon H)$

$$(4) \quad \sum_{j\varepsilon H} a_j z_{jk} + b = 0, \quad k\varepsilon K.$$

where $a_j(j\varepsilon H)$ and b are parameters. If the direction of the ray is chosen in advance, then $a_j(j\varepsilon H)$ are known. The other parameter is determined from the condition that the ray passes through $y_j(j\varepsilon J)$ by $b = - \sum_{j\varepsilon H} a_j y_j$.

From equating (3) with (4) it is possible to determine whether z_{jk} exist[1)], and, if so, whether at least one of these for any $k\varepsilon K$ meets

$$(5) \quad \left| \left[\sum_{j\varepsilon H} |y_j - z_{jk}|^m \right]^{1/m} \right| \leq \Delta, \; k\varepsilon K.$$

Let us call these z_{jk_l}, where $l\varepsilon\{1,2\}$ according to whether the ray intersects with (3) or is tangent to it or, in the case of an intersection, whether one of the solutions is excluded by (5). These can be brought into an increasing order along any one of the dimensions $j\varepsilon H$. Lower values (l=1) and upper values (l=2) for any $k\varepsilon K$ define segments on the ray (4) where $k\varepsilon K$ would be a customer of a product positioned between z_{jk_1} and z_{jk_2} on the ray. As a number of such segments intersect, it is easy to look up consecutively the customer indices of all intersecting segments at any point z_{jk_1} and their respective sales, r_k. From this we calculate the possible sales at z_{jk_1} that are

1) See the Appendix for the calculation

compared with the value of the objective function at $y_j, j \in J$. This procedure is sketched schematically in Fig. 1.

> Fig. 1 about here <

Two simulated examples may serve to demonstrate the results obtained from such calculations. In Fig. 2 we show a highly segmented market, in Fig. 3 customers are rather homogeneously distributed over the two-dimensional attribute space. The figures show the individually perceived ideal product positions and the ellipsoids that are defined by the closest distances from all individually perceived real products.

For simplicity, we have assumed $r_k = 1$, $k \in K$. Thus, the number of customers attracted by a new product position could be an optimization criterion.

The development of that number along the rays shown in Fig. 2 and Fig. 3 is depicted in Fig. 4 and Fig. 5, respectively.

> Fig. 2 through Fig. 5 about here <

The optimum product position is located at the coordinate value .0 on the rays that are identical to the abscissa in these figures. In Fig. 4 it is seen clearly that along rays passing through the optimum with 20°- and 50°- angles the expected sales is almost identical in its immediate neighborhood. While the 20°-ray passes through one of the clusters it shows relative maxima at a distance of roughly 1.1 from the optimum. The 50°-ray only touches on this cluster such that it does not reach as high a relative optimum. What seems to be important is that the sales (or number of customers according to our assumptions) is almost identically distributed on the rays that pass

through the optimum and in its immediate neighborhood.

As can be seen from Fig. 5(that refers to the spherical design of a market structure)this is not always the case. First, the shape of the sales distributions along the 0^{o}-and the 90^{o}-rays through the optimum is not symmetrical. In the .1-neighborhood of the optimum the possible sales are more stable along the 90^{o}-ray than along the 0^{o}-ray. More observations can be made if directions of change from the optimum along any one ray are additionally considered.

The analysis of solution stability along one ray that passes through the optimal solution y_j, $j \varepsilon J$, may prove to be rather limited. A first extension could be to change the direction of that ray systematically and to compare the results from such changes. As this may become unintelligible, another way to present the results in two dimensions is suggested. Isoquants of sales or customers can be calculated from the previous results and can be shown in the Δ-neighborhood of the optimal solution. This is demonstrated in Fig. 6. It gives the marketing manager a good impression of the stability of the optimal product position, albeit in only two dimensions.

Underlying Figure 6 is the spherical market structure. The optimum sales level is depicted by the cross-hatched area in Fig. 6. The asymmetrical sales levels along rays passing throug the optimum is again clearly visible as in Fig. 5. The 0^{o}- and 90^{o}-rays of Figure 5 could also be depicted in Fig. 6 (which shows a much closer neighborhood of the optimum solution than Fig. 4; this is obvious from a comparison of the scales used in both figures). However, the picture presents much more information than the individual rays that were considered before.

> Fig. 6 about here <

To surmount the restriction to two dimensions a more general procedure can be considered.

A Concept of Solution Stability

Let Δ be used to define a $\bar{j}$-dimensional ball around $y_j, j\varepsilon J$. Furthermore, let $x_k \varepsilon\{0,1\}$, $k\varepsilon K$, be a variable that indicates, whether the k-th customer buys a new product or not. If x_k^* is the optimal solution to a product positioning problem, we would expect maximum sales

$\sum_{k\varepsilon K} r_k x_k^*$. (A similar argument would hold if x_k were likelihoods of buying as in [7]). It may now be possible to calculate minimum sales of the new product at some position within the ball with radius Δ around $y_j, j\varepsilon J$. Let us call this number S_Δ, where Δ is added as an index to indicate that the value may depend on the size of the ball. To facilitate comparisons with many problems a standardization of Δ would seem appropriate. It should be dependent on the length of the characteristics axes, i.e. 1/20 of that lenth. However, this figure may depend on the importance that the manager assigns to changes along the axes. Thus, the problem is similar to the one of determining levels of significance in statistical tests.

The stability of the optimal solution can be evaluated in a worst case analysis by

$$(6) \qquad ST_\Delta = \frac{\sum_{k\varepsilon K} r_k x_k^* - S_\Delta}{\sum_{k\varepsilon K} r_k x_k^*} .$$

Our next problem would be to calculate S_Δ-values (the Δ-index is not carried along in the following, assuming some standardization).

The following model would solve our problem analytically:

$$(7) \quad S = \min \sum_{k\varepsilon K} r_k x_k$$

such that

$$(8) \quad \left| \left[\sum_{j\varepsilon J} |y_j - z_j|^m \right]^{1/m} \right| \leq \Delta$$

$$(9) \quad \left| \left[\sum_{j\varepsilon J} |c_{kj} - z_j|^m \right]^{1/m} \right| - d_k \geq - x_k M, \quad k\varepsilon K,$$

$$(10) \quad x_k \varepsilon \{0,1\}, \quad k\varepsilon K,$$

where M is some sufficiently great number, and $z_j, j\varepsilon J$, is an optimal solution in terms of the coordinates.
The constraint (8) is equivalent to (5), except for the number of attributes or characteristics considered in the summation. By (9) it is determined whether some customer with ideal product perception c_{kj}, $k\varepsilon K$, $j\varepsilon J$, would become a buyer of a new product located at z_j, $j\varepsilon J$, that is $x_k=1$ under the single choice assumption or whether he would rather not buy ($x_k=0$). The solution values for z_j give the coordinates of the point at which S can be achieved.

While (7) and (9) through (10) are similar to the product positioning problem [1,2,4], the addition of (8) poses additional problems. Furthermore, a multitude of solutions z_j, $j\varepsilon J$, may exist all of which lead to identical values S. Considering that Δ will be chosen small as compared with the total characteristics space, a heuristic will be more

appropriate to solve the problem. With Δ relatively small, grid searches around y_j, $j \varepsilon J$, can be very efficient.

Comparison of multiple solution points

The considerations presented above do apply in principle, even if y_j, $j \varepsilon J$, is not selected as a starting point. Any other suggested product position may be chosen instead. If so, one may want to compare among various positions. This may be hampered by the fact that the probability $p(\Delta)$ of realizing an actual product position within the Δ-neighborhood of some preestablished position could be different in various areas of the characteristics space.

The minimum expected sales due to a miss of the optimum (or some other preestablished) product position could then be determined from

$$(12) \qquad \sum_{k \varepsilon K} r_k x_k^* - p(\Delta) \left[\sum_{k \varepsilon K} r_k x_k^* - S_\Delta \right]$$

Using (6), it may also be possible to introduce a normation by finding a relative equivalent to (12), namely

$$(13) \qquad 0 \leq 1 - p(\Delta) \cdot ST_\Delta \leq 1$$

(where the bounds are rather unlikely to be observed in practice). We interpret (13) as the cost factor for not being able to translate a model solution into reality. Having come so far, one may even speculate about contrasting (13) with empirical results on actual product positions to analyze reasons for divergencies etc.

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Appendix

The calculation of z_{jk_1} is as follows, if $j\varepsilon\{1,2\}= H$, $a_2=1$ (which can be assumed without loss of generality) and m=2 (Euclidean distance):

$$s_{k1}(c_{k1} - z_{1k})^2 + s_{k2}(c_{k2} - b - a_1 z_{1k})^2 = d_k^2 , \quad k\varepsilon K$$

Let

$$p = (s_{k1}c_{k1}^2 + s_{k2}c_{k2}^2 + s_{k2}b^2 - 2s_{k2}c_{k2} - 2s_{k2}b - d_k^2)/$$

$$/(s_{k1}+s_{k2}a_1^2)$$

and

$$q = (s_{k1}c_{k1} + a_1 b + a_1 c_{k2})/(s_{k1} + s_{k2}a_1^2).$$

From this we get

$$z_{1k}^2 - 2q\, z_{1k} + p = 0$$

and

$$z_{1k_1} = -q \pm \sqrt{q^2 - p} \quad .$$

If $q^2 - p$ $\begin{cases} < 0, & \text{we have no intersection between the ray (4) and the k-th ellipsoid defined by (3), above,} \\ = 0, & \text{the ray (4) is tangent to (3), and we have only } z_{1k_1} = z_{1k_2} = -q. \\ > 0, & \text{the ray (4) intersects twice with (3), and we find } z_{1k_1} = -q + \sqrt{q^2-p} \text{ and } z_{1k_2} = -q - \sqrt{q^2-p}. \end{cases}$

From the solutions for z_{1k_1} (if they exist) we can calculate z_{2k_1} by inserting into (4).

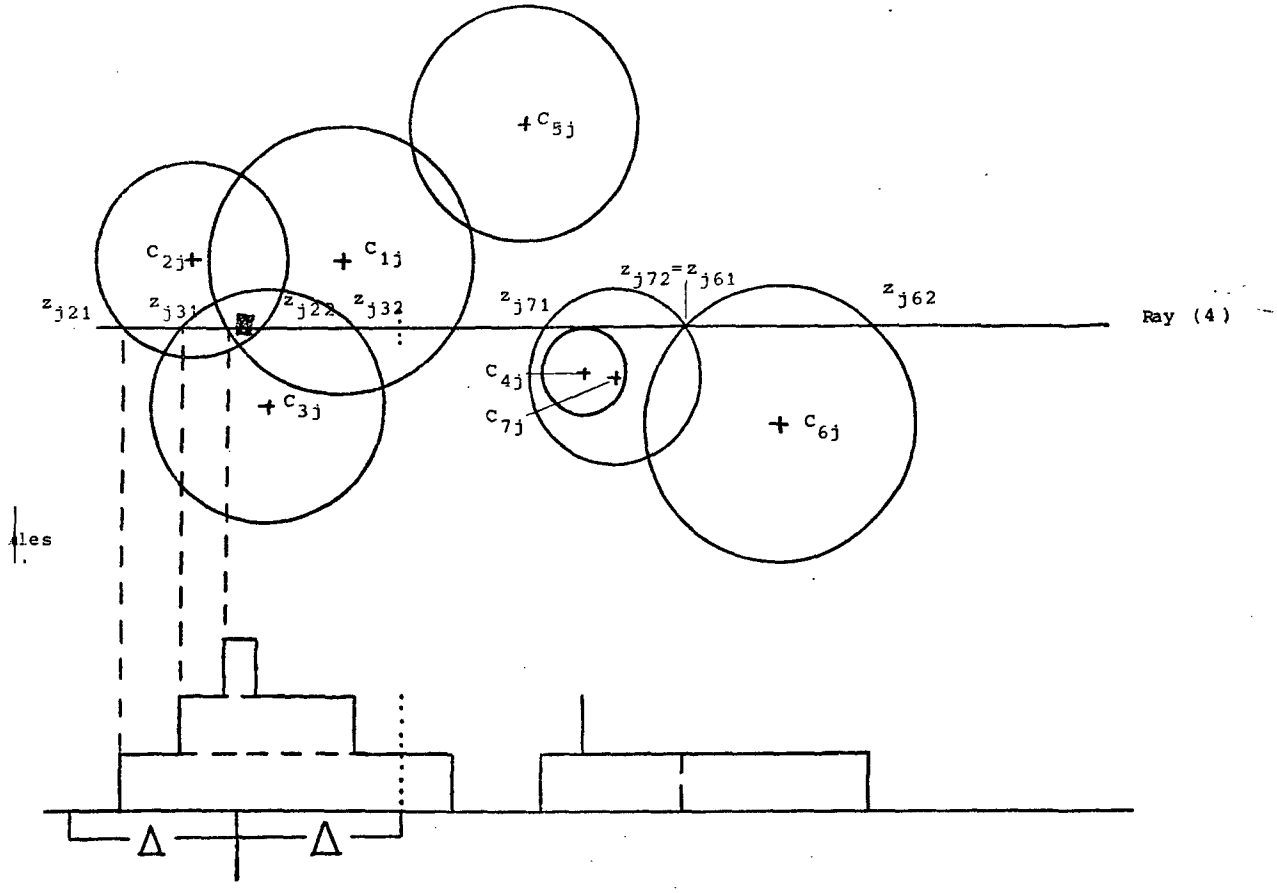


Fig. 1: Concept of deriving a distribution of sales along a ray through an optimum product position.

Δ defines a neighborhood,
■ optimal product position y_j,
the circles are defined by (2).

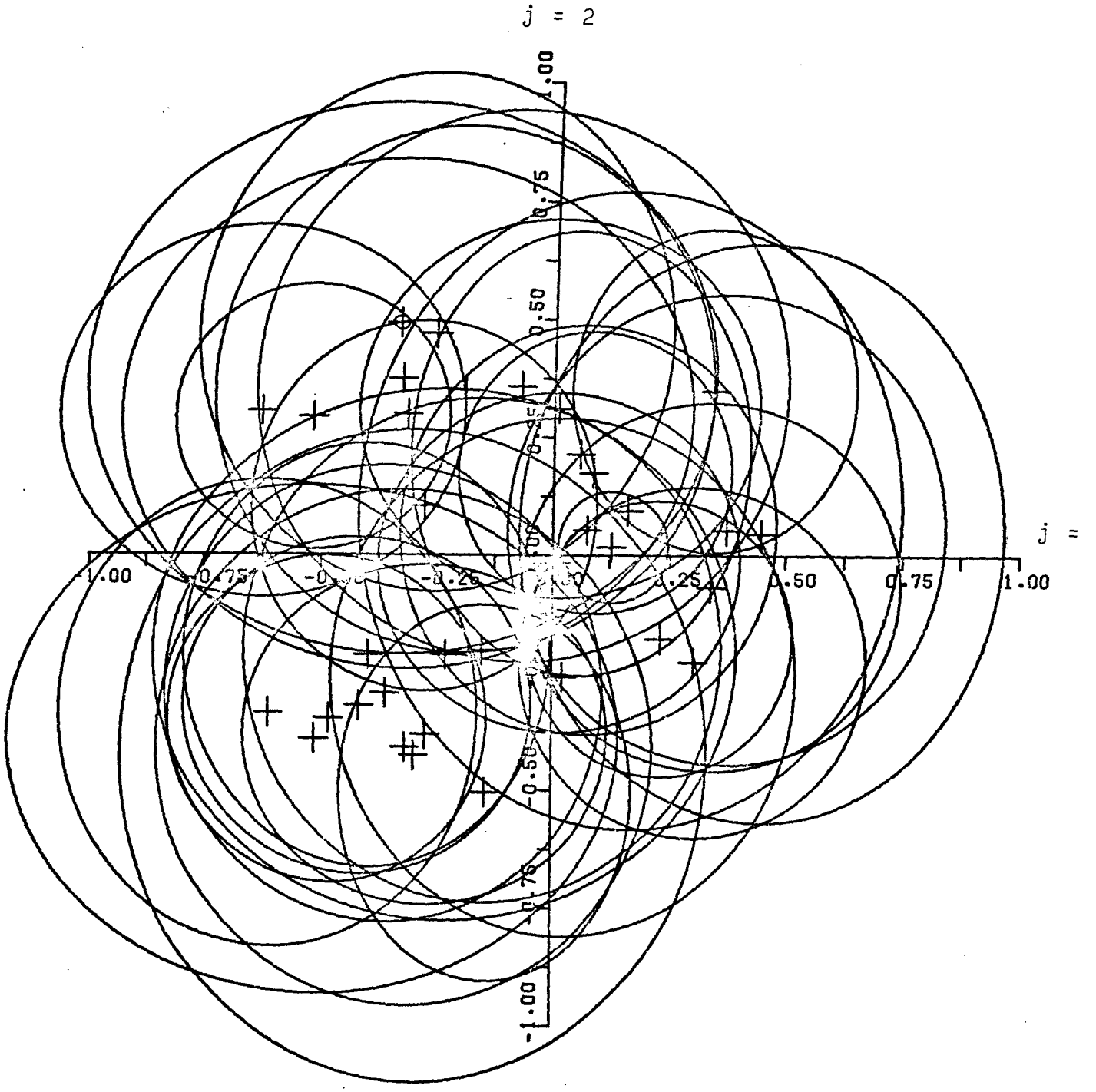


Fig. 3: Two-dimensional Spherical Market Structure

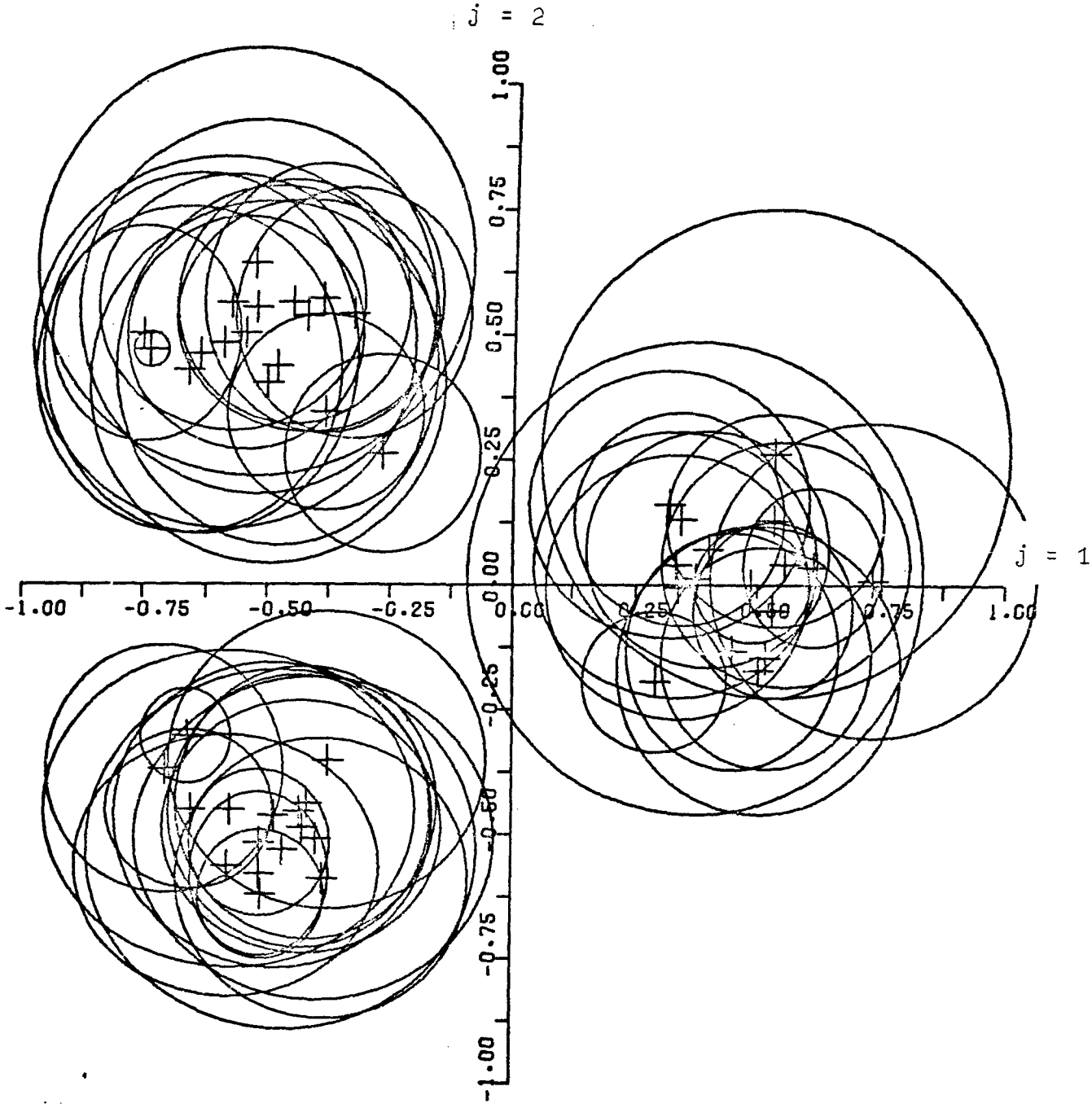
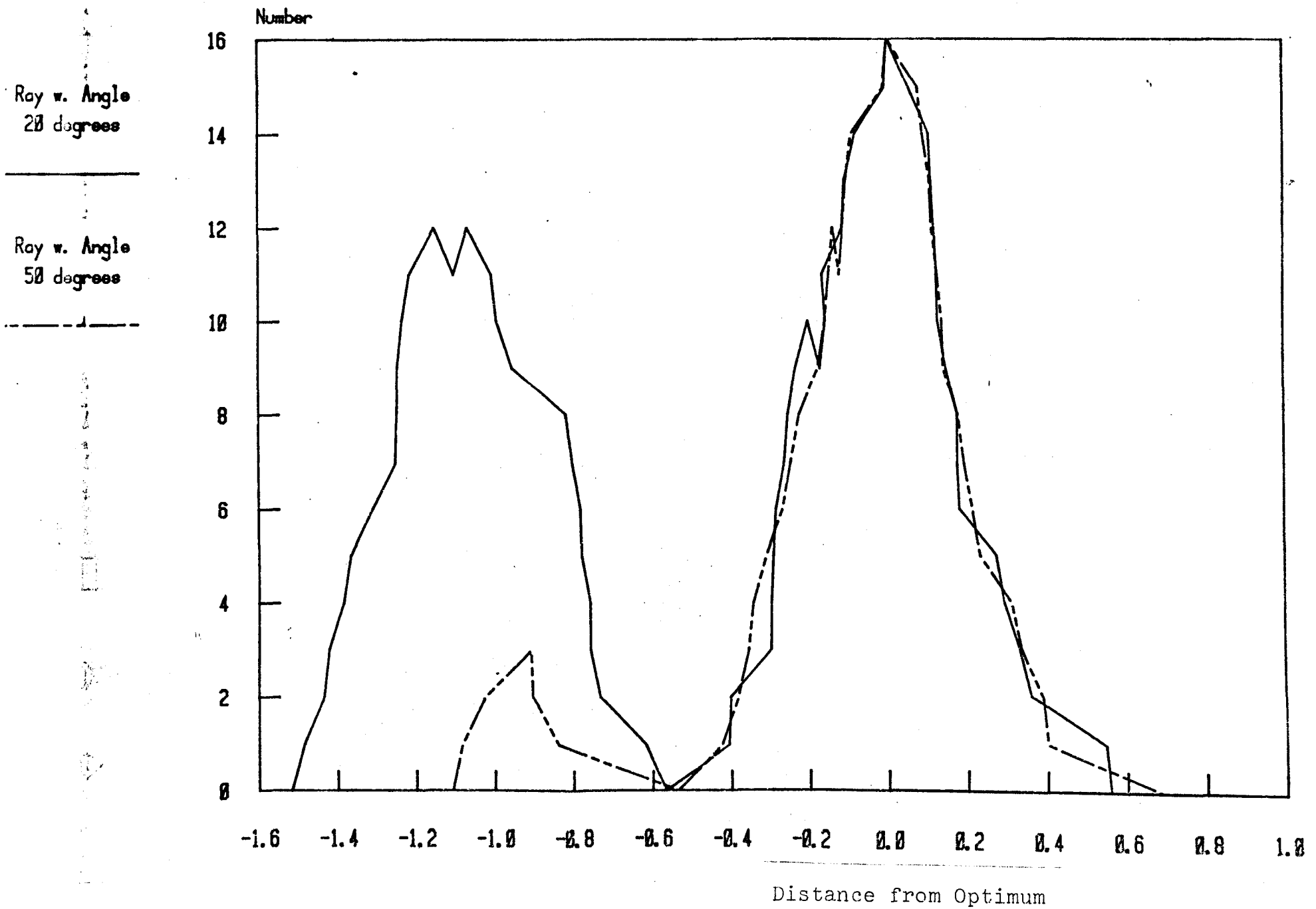
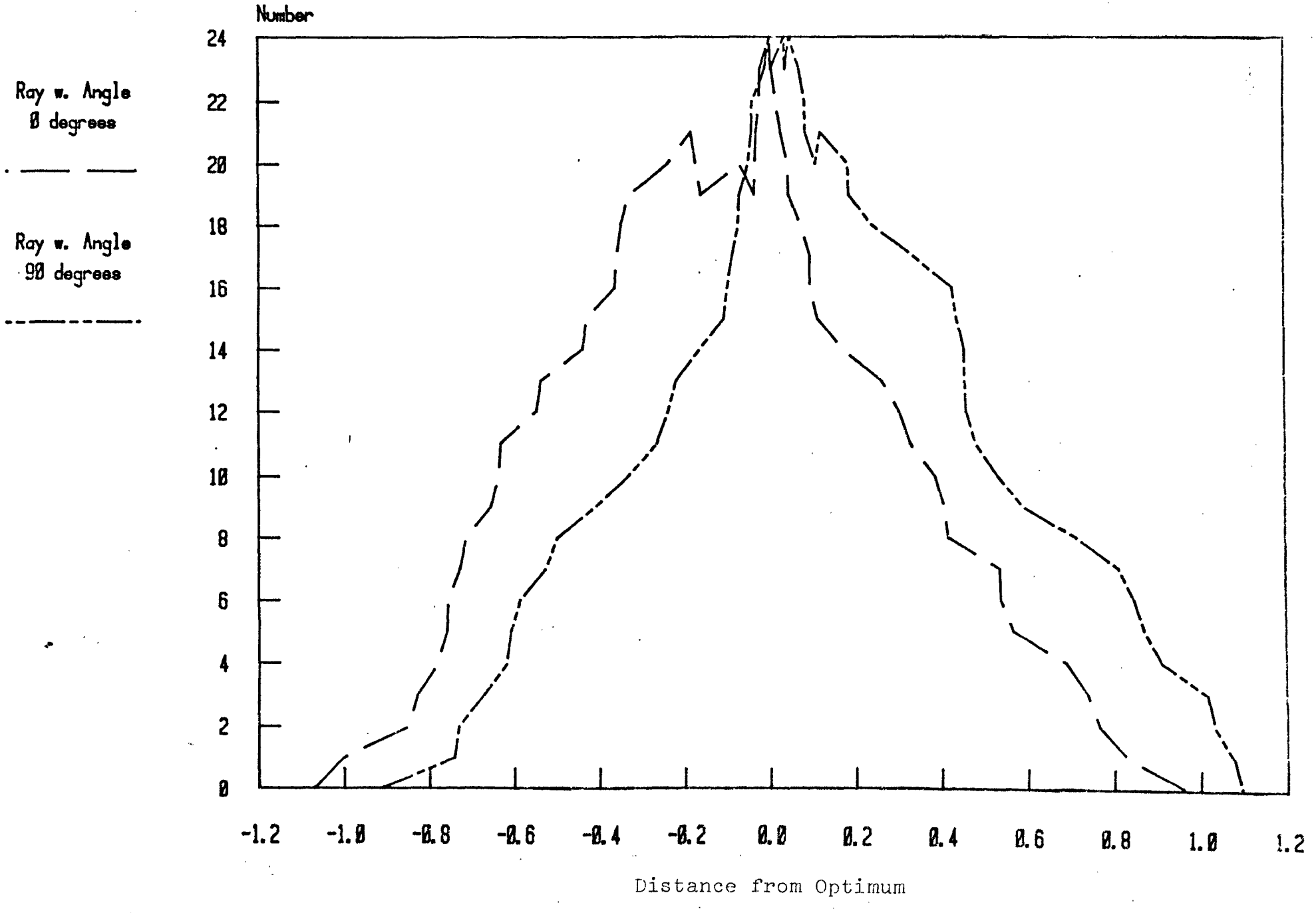


Fig. 2: Two-dimensional Clustered Market Structure

Fig. 4: Results of a Sensitivity Analysis
(Clustered Structure)



5. Results of a Sensitivity Analysis
(Spherical Structure)



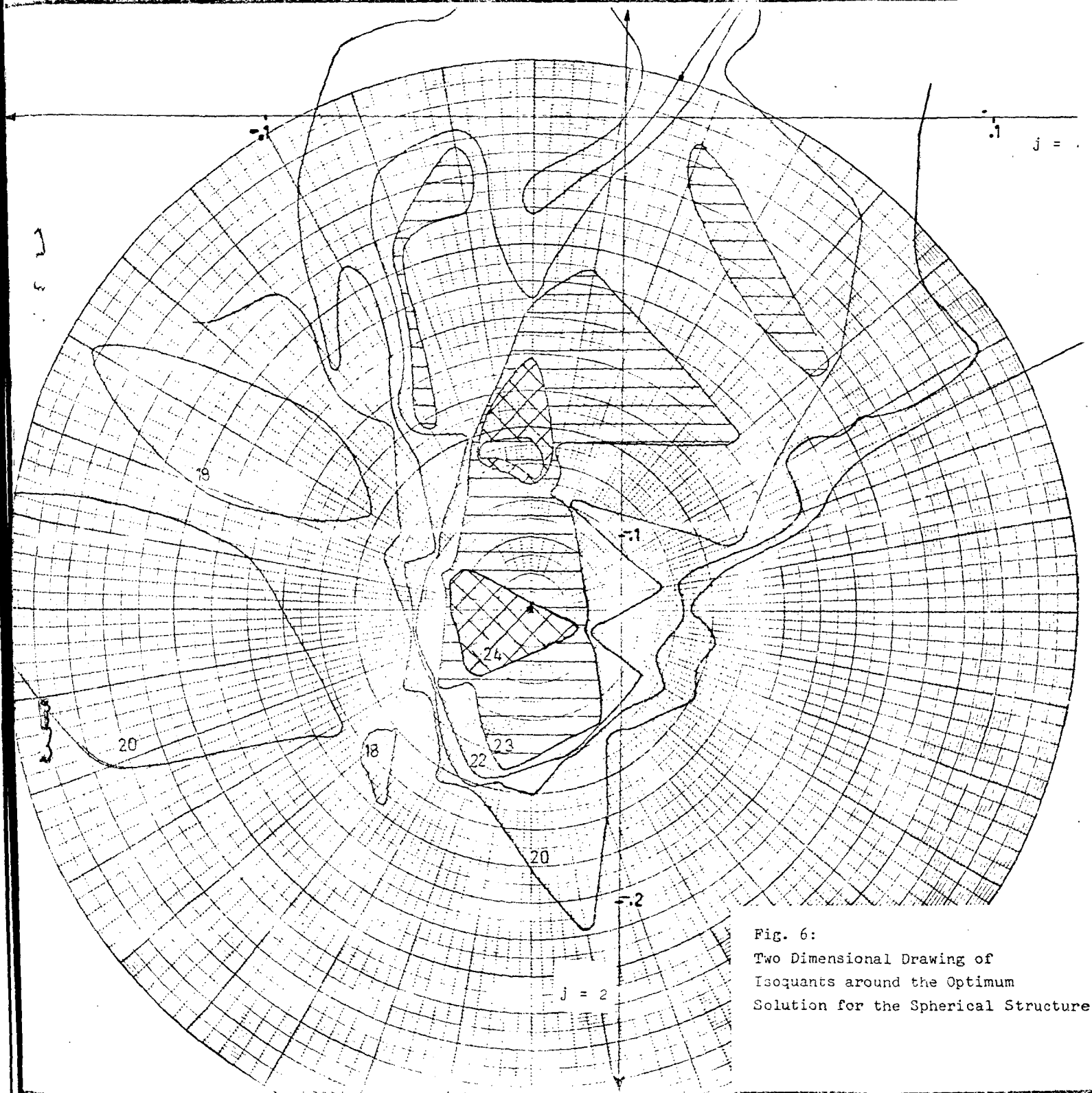


Fig. 6:
Two Dimensional Drawing of
Isoquants around the Optimum
Solution for the Spherical Structure