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## Public versus private health care in a national health service

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# PUBLIC VERSUS PRIVATE HEALTH CARE IN A NATIONAL HEALTH SERVICE

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# PUBLIC VERSUS PRIVATE HEALTH CARE IN A NATIONAL HEALTH SERVICE

## Abstract

This paper studies the interaction between public and private health care provision in a National Health Service (NHS), with free public care and costly private care. The health authority decides whether or not to allow private provision and sets the public sector remuneration. The physicians allocate their time (effort) in the public and (if allowed) in the private sector based on the public wage income and the private sector profits. We show that allowing physician dual practice "crowds out" public provision, and results in lower overall health care provision. While the health authority can mitigate this effect by offering a higher wage, we find that a ban on dual practice is more efficient if private sector competition is weak and public and private care are sufficiently close substitutes. On the other hand, if private sector competition is sufficiently hard, a mixed system, with physician dual practice, is always preferable to a pure NHS system.

JEL Code: I11, I18, J42, L33.

Keywords: health care, mixed oligopoly, physician dual practice.

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# 1 Introduction

Most health care systems involve a mixture of public and private provision. However, as pointed out by Besley and Gouveia [1], in National Health Service (NHS) systems the role for private health care is different and more limited compared with private and mixed health care systems. In a NHS, health care is mainly provided publicly and financed by general taxation. Still, in most countries with a NHS system there exist a parallel (and growing) private sector alongside the public one.<sup>1</sup>

In a NHS, the physicians therefore have the opportunity to work in the private sector – a phenomenon typically referred to as *physician dual practice*. Interestingly, we observe that a substantial share of the physicians spend time in both sectors. For instance, in the UK most private medical services are provided by physicians whose main commitment is to the NHS. The UK Monopolies and Merger Commission [3] estimated that about 61% of the NHS consultants had significant private work. According to Johnson [4], physician dual practice is common in many countries like, for instance, France, Spain, Portugal, and the Scandinavian countries.

In this paper, we analyse the interaction between public and private health care provision in a NHS system, where publicly employed physicians may work in the private sector. Notably, physician dual practice introduces close links between the public and the private sector on both the demand and the supply side. In particular, physicians may shift patients seeking public care to their private practice, and they can allocate their labour supply according to which sector that provides the higher benefit.<sup>2</sup> While there exists some literature on the interaction between public and private health care provision, the issue of physician dual practice has received surprisingly little attention, despite being a common phenomenon. The purpose of this paper is to help fill this gap in the literature.

When analysing public and private health care provision in the presence of physician dual practice, we address the following questions: How does the private option for NHS physicians affect their public sector labour supply and the public provision of health care? What role does competition among physicians play for public and private health care provision? Finally, is a mixed health care system always desirable, or should the health authority enforce a pure NHS

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<sup>1</sup>In the UK, for instance, Propper [2] reports that private health care expenditures have increased from 9% of total health care expenditures in 1979 to 15% in 1995.

<sup>2</sup>This activity is sometimes labeled moonlighting. A recent newsletter from the Health Economics & Financing Programme at the London School of Hygiene & Tropical Medicine reported on the importance of "moonlighting physicians". See *Exchange* Summer 2003.

system by not allowing physician dual practice?

To analyse these questions, we consider a health care market characterised as a NHS, where public (in-plan) health care is free of any charges at the point of consumption, while patients seeking private (out-of-plan) care are charged a payment. In this market there is a health authority (e.g., the Ministry of Health) responsible for providing health care to individuals in need for medical treatment. The health authority decides whether or not to allow private (out-of-plan) provision of health care alongside the NHS, and determines the public sector remuneration (wage level).<sup>3</sup>

The provision of health care is very labour-intensive, implying that the physicians' labour supply in the two sectors are important for the amount of public and private health care that will be provided. In the public sector, the physicians' are on salary, while in the private sector they earn profits. Thus, the physicians' allocation of time between the two sectors will depend on the public sector wage income and the private sector profits. On the demand side, we assume that public and private care are (horizontally) differentiated products, reflecting different service mixes, specialisations, treatment methods, etc.<sup>4</sup> The degree of differentiation can also be interpreted in geographical terms, reflecting physical distance between the hospitals and the patients. The fact that public health care is free, while private health care is charged a price, implies that most patients prefer to be taken care of in the public sector. We show that public rationing always takes place in our model.<sup>5</sup>

Allowing physicians to offer (substitutable) private services outside the NHS system may have several potential effects on the provision of public health care. In this paper, we show that the private option imposes a *crowding-out* effect – not only on the public provision – but in fact on overall health care provision. Since the price of private care is decreasing in the public sector capacity, as well as the private sector supply, the physicians have an incentive to restrict their labour supply in both sectors. This is a standard *market power* incentive due to imperfect competition in the private sector. The strength of this incentive, obviously, depends on the number of physicians in the market and the degree of substitutability between public and

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<sup>3</sup>We also observe that Health Authorities sometimes impose restrictions on the private earnings of the publicly employed physicians. In the UK, for instance, full-time NHS consultants are not allowed to earn more than 10% of their NHS salary on their private practice. This issue is analysed by González [5].

<sup>4</sup>The empirical study by McAviney and Yannopoulos [6] shows that public and private health care in the UK NHS system are (imperfect) substitutes.

<sup>5</sup>We do not explicitly model waiting time in the NHS system. There are some papers that considers the effect of private health care provision on the waiting list in the public sector [7 – 9]. However, neither of these papers deals with physician dual practice and the incentive to shift patients from the public to the private sector.

private health care.

The health authority can mitigate the crowding out effect by offering a higher public sector wage. However, since this increases the public expenditures, a ban on physician dual practice may be a more efficient policy. We show that this is the case if the number of physicians in the market is low and public and private health care are sufficiently close substitutes. In this case private sector competition is weak and the crowding out effect strong, resulting in a substantial efficiency loss in the private sector and a concern for undersupply of public health care. On the other hand, we show that a mixed public-private health care system is always beneficial if the number of physicians is sufficiently high. However, the scope for a private (out-of-plan) option alongside the NHS is decreasing in the weight the health authority attaches to the patients' health benefit relative to physician profits. In the extreme case, where the health authority is concerned about maximising patients' health benefit net of public and private medical expenditures only, there is a strong bias towards a large public sector.

This paper relates to the literature on public and private health care markets in general.<sup>6</sup> A common assumption in these studies is that public and private providers are separate entities. Our paper departs from this literature in that we allow physicians to work in both sectors. Obviously, this substantially changes the physicians' incentives to provide public and private care, as there are now close links between the two sectors on both the demand and the supply side. As a consequence, welfare and policy implications are qualitatively different.

The literature on physician dual practice in mixed health care markets is, as mentioned above, rather limited but growing. A related paper is Rickman and McGuire [13]. Building on the model by Ellis and McGuire [14], they study the optimal public reimbursement cost sharing rule when a physician can provide both public and private services (which may be either substitutes or complements). Their focus and modelling approach differ from ours in several aspects. In Rickman and McGuire, physicians do not receive wage income in the public sector, but instead a share of the public hospital's profits (revenues), which establishes the link to the reimbursement system. Thus, public sector remuneration is not a part of their analysis. Moreover, they assume private sector monopoly, implying that the degree of competition plays no role in their analysis. Finally, they do not address the issue of whether or not physician dual

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<sup>6</sup>Some relevant papers are Barros and Martinez-Giralt [10] who analyse the effect of different reimbursement rules on quality and cost efficiency; Jofre-Bonet [11] who analyses the interaction between public and private providers when consumers differ in income, and Marchand and Schroyen [12] who analyse the desirability of mixed health care systems when distributional aspects matters.

practice should be banned.

In addition, there are four other papers applying a principal-agent framework to analyse effects of physician dual practice [5, 15 – 17]. These papers focus on potential moral hazard problems in the public provision that may arise due to the physicians’ private activities. Gonzalez [5] presents a model where a physician has an incentive to provide excessive quality in the public sector to raise her prestige in the private sector, and focuses on the policy issue of whether or not physician dual practice should be restricted. Barros and Olivella [15] analyse a model with waiting list in the public sector and study a physician’s decision to cream-skim. A similar approach is taken by Gonzalez [16], who focuses on different systems for remunerating the physician in the two sectors. Finally, Biglaiser and Ma [17] consider quality incentives of physician dual practice in a model with no explicit incentives in the public sector and two types of physicians; dedicated doctors and moonlighters, where the latter type shifts patients to their private practice if this is beneficial. As none of these studies are concerned with physicians’ labour supply nor the role of private sector competition, they differ substantially from ours.

The paper is organised as follows. In Section 2 we present the model. In Section 3 we derive and characterise the physicians’ labour supply in the public and the private for a given public sector remuneration (wage). In Section 4 we analyse the optimal wage setting by the health authority. In section 5 we analyse the health authority’s decision of whether or not to allow physician dual practice. Finally, Section 6 concludes the paper.

## 2 Model

We consider a health care market characterised as a National Health Service (NHS) with health care being provided by public hospitals free of any charges at the point of consumption. In this market there are three different agents: First, there is a health authority, which decides whether or not to allow private (out-of-plan) provision of health care alongside the NHS, and the public sector remuneration (wage level). Second, there is a set of physicians determining their labour supply in the public sector and, if allowed, in the private sector. Finally, there is a set of individuals demanding medical treatment from the NHS and private (out-of-plan) providers, if this is an option.

We use the representative consumer approach to characterise the patients’ utility and choice of health care.<sup>7</sup> The utility of the representative consumer corresponds as usually to the ag-

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<sup>7</sup>The representative consumer approach is extensively used in economics. In the health economic literature,

gregate utility of individuals. For a specific individual, at a given point of time, the choice of public and private hospital care is typically a discrete choice (as most other choices). The representative consumer approach is a simplified way of aggregating potentially heterogeneous individuals' preferences and choices.<sup>8</sup>

In the case of a private (out-of-plan) option, the representative patient's utility from medical treatment is given by the following standard quasi-linear utility function:

$$U(X, Y, Z) = X + Y - \frac{1}{2}(X^2 + Y^2 + 2bXY) + Z, \quad (1)$$

where  $X$  and  $Y$  are the amounts of public and private health care, respectively, and  $Z$  is the amount of a numeraire good. The parameter  $b \in (0, 1)$  is a measure of the substitutability between public and private care. A high  $b$  reflects that public and private health care are close substitutes, while the opposite is true for a low  $b$ . We interpret  $b$  as a measure of the differences in service mix or speciality of the public and the private hospital, with a low  $b$  reflecting a high degree of differentiation or specialisation in services and treatments offered. Alternatively, we may think of  $b$  as a measure of the geographical distance between the public and the private hospital, with a high  $b$  reflecting close hospital locations (e.g., in the same city).

The optimal consumption choice of the representative consumer is restricted by a budget constraint. We assume that public (in-plan) health care is free of any charges, while private (out-of-plan) health care is subject to a charge  $p > 0$ . Normalising, as usual, the price of the numeraire good to one, we can write the budget constraint as follows:

$$m - pY - Z \geq 0, \quad (2)$$

where  $m$  is the representative consumer's disposable income. The restriction in (2) says that consumption expenditures cannot exceed disposable income. Assuming a binding budget constraint, we can insert (2) into (1), and rewrite the utility function of the representative patient as follows:

$$U(X, Y) = X + Y - \frac{1}{2}(X^2 + Y^2 + 2bXY) - pY + m. \quad (3)$$

Notably, the utility function is separable in income. Besides income the representative patient's

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there are also several studies employing this approach; see e.g., Rickman and McGuire [13], Ellis and McGuire [14], Brekke [18], Calem et al. [19], Kesteloot and Voet [20], and Siciliani [21].

<sup>8</sup>A further discussion on this issue can be found in, for instance, Anderson et al. [22], which also provide a formal correspondence between the heterogenous consumer approach and the representative consumer approach.



utility now consists of the gross utility from receiving public and private health care net of the expenditures on private health care.

Optimal consumption and demand for public and private health care can now be derived. Since private (out-of-plan) health care is costly for the patients, while public health care is free of any charges, it is very likely that public rationing will take place. In deriving the demand, we therefore assume that the demand for public health care always exceed the public sector capacity.<sup>9</sup> This is well in line with empirical observations from NHS systems.<sup>10</sup>

In addition, we assume ‘efficient rationing’, which means that the patients with the higher willingness to pay for health care is taken care of in the public sector.<sup>11</sup> Since the utility function, given by (3), is separable income, the patients’ willingness to pay can be directly interpreted as reflecting the patients’ severity level. Thus, ‘efficient rationing’ by the public sector corresponds to a situation where the NHS is rationing patients according to severity of illness, leaving the easier (milder) cases for the private sector.<sup>12</sup>

Under the assumption of (efficient) public rationing, the private sector demand is obtained by maximising (3) with respect to  $Y$ , yielding the following inverse demand function for private health care:

$$p = 1 - Y - bX. \quad (4)$$

We see that the (market) price for private health care is decreasing in the overall provision of health care. This is a standard demand effect simply reflecting decreasing marginal utility (or willingness-to-pay) of consumption. The magnitude of the negative effect of public provision on the private sector price depends on the degree of substitutability between the two sectors. If  $b$  is small – for instance, due to distant hospital locations or different treatment methods – then a substantial increase in the public provision has only a small negative impact on the price of private care, and vice versa.

In the market there is a finite number of physicians, given by  $N \geq 1$ , qualified to perform

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<sup>9</sup>In Appendix B, we derive the demand system when public health care is not rationed, and show that non-rationed demand for public health care always exceed the public sector provision in equilibrium. Thus, demand for public health care is always rationed in our model.

<sup>10</sup>According to the extensive literature survey by Cullis et al. [23] rationing by waiting lists for public (in-plan) care is a common feature of NHS systems.

<sup>11</sup>An alternative rationing rule is ‘random rationing’, where all consumers have the same probability of being rationed. For analytical purposes, we have decided to use the efficient rationing rule only, which is also undoubtedly the most common one in the literature. See Tirole [24] (pp. 212-214) for discussion on various rationing rules.

<sup>12</sup>This may follow from a strict priority plan enforced by the health authority. Alternatively, it may be the outcome from the physicians’ (cream-skimming) decisions, see e.g., Gonzalez [16], Olivella [9] and Barros and Olivella [15].

hospital treatments. The physicians allocate their time in the public and, if allowed, in the private sector. We assume, as Ellis and McGuire [14] and several others, that the physicians earn profits in the private sector, while they are on salary in the public sector, earning the wage  $w$  per unit of labour.<sup>13</sup> Normalising, for simplicity, input and output so that one unit of a physician's labour supply equals one unit of hospital care, we can write physician  $i$ 's utility as follows:

$$V_i(X, Y) = wx_i + py_i - (x_i + y_i)^2, \quad i = 1, \dots, N, \quad (5)$$

where  $x_i$  and  $y_i$  denote the amount of labour supplied by physician  $i$  in the public and the private sector, respectively. While the two first terms are public sector income and private sector profits, respectively, the last term represents the time costs associated with the disutility of working (measured in monetary terms). We see that the time cost is increasing and convex in the total amount of labour supply. This can be justified by a simple opportunity cost argument. The more time spent at the work place, the higher value of leisure. Moreover, there are physical limitations to how much one can work each day, implying an individual capacity constraint. This is captured by the convexity.

Hospital production is labour-intensive, and the vast amount of treatments are carried out by physicians. Thus, the provision of hospital care is highly sensitive to physicians' labour supply. The assumption that one unit of labour equals one unit of hospital care, implies that the total public and private provision of health care simply are the sums of individual labour supply in the two sectors, i.e.,

$$X = \sum_{i=1}^N x_i \quad \text{and} \quad Y = \sum_{i=1}^N y_i. \quad (6)$$

While this clearly is a simplification, the results do not rely on this, but only require that there is a positive relationship between the physicians' labour supply and the amount of health care provided.

The interaction between public and private health care provision within a NHS system is analysed by considering a four-stage game with the following sequence of events:

- Stage 1: The health authority decides whether or not to allow physicians to work in the

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<sup>13</sup>Alternatively, we can, as Rickman and McGuire [13], think of  $w$  as a prospective reimbursement, e.g., DRG-price or fee-for-service. They argue that even though NHS hospitals are prohibited from making profits and physicians receive a fixed salary, we may link the physicians' utility to the reimbursement mechanism as long as the physicians become residual claimant to hospital surplus – through the acquisition of better facilities, equipment, staff, etc.

private sector, i.e., dual practice.

- Stage 2: The health authority determines the public sector remuneration (wage).
- Stage 3: The physicians allocate their time in public sector, and if allowed, in the private sector.
- Stage 4: Patients receive medical treatment.

The model is, as usual, solved by backward induction.<sup>14</sup>

### 3 Physicians' labour supply

In this section we derive the equilibrium allocation of time by physicians. First, we consider the benchmark case where physicians are not allowed to work in the private sector. We label this as the pure public health care system (*PS*). Second, we derive and characterise the physicians' labour supply in the public and the private sector when dual practice is allowed. We label this case as the mixed public-private health care system (*MS*).

#### Pure public health care system

Let us start by analysing the physicians' allocation of time in the public sector when dual practice is not allowed, and we have a pure public health care system with no private sector.<sup>15</sup> Maximising (5) with respect to  $x_i$ , under the assumption that  $y_i := 0$ , we obtain the following first-order condition

$$\frac{\partial V_i(x_i, 0)}{\partial x_i} := w - 2x_i = 0 \quad (7)$$

Imposing symmetry, the equilibrium labour supply for each physician in a pure public system is given by:

$$x^{PS}(w) = \frac{w}{2}. \quad (8)$$

Aggregating each physician's labour supply, we find the following total public provision of hospital care in equilibrium:

$$X^{PS}(w) = N \left( \frac{w}{2} \right). \quad (9)$$

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<sup>14</sup>Note that we keep  $N$  fixed throughout the analysis. This can be justified by the fact that entry into the physician market is highly restricted not only at medical schools but also at the hospital level in terms of specialisation. Moreover, our interest in  $N$  is mainly as a measure of competition in the private sector.

<sup>15</sup>A ban on dual practice does not necessarily imply a pure public system. Instead this can induce the physicians to make a discrete choice between working in the public or the private sector. In our model, this will never be the case.

Thus, for any wage above the reservation wage (normalised to zero), the physicians will provide some public health care. A higher wage increases individual labour supply, resulting in more patients receiving hospital care in the public sector. Finally, we see that the more physicians in the public sector, the more patients receive public health care.

Inserting (8) and (9) into (5), we obtain each physician's equilibrium profits:

$$V^{PS}(w) = \left(\frac{w}{2}\right)^2, \quad (10)$$

which is increasing (and convex) in the public sector wage, and independent of the number of physicians in the sector. Note also that for any  $w > 0$ , every physician earns positive profits, and thus will accept to work in the public sector.

### Mixed public-private health care system

Consider now the situation with a private (out-of-plan) option alongside the NHS. In this case the physicians allocate their work time between the two sectors in order to maximise their payoff. Physician  $i$  maximises (5) with respect to  $x_i$  and  $y_i$ , yielding the following first-order conditions:

$$\frac{\partial V_i}{\partial x_i} := w - by_i - 2x_i - 2y_i = 0, \quad (11)$$

$$\frac{\partial V_i}{\partial y_i} := 1 - 4y_i - Y_{-i} - bx_i - bX_{-i} - 2x_i = 0, \quad (12)$$

where  $X_{-i}$  and  $Y_{-i}$  denote the aggregate labour supply in the public and the private sector, respectively, by all other physicians than physician  $i$ .

Comparing (11) with (7) we see that optimal public sector labour supply now depends, not only on the wage and the marginal costs of time spent in the public sector, but also on the private sector labour supply. The private option affects public sector labour supply in two ways: First, since total time spent at providing medical care matters, the allocation in the private sector must take into account the initial labour supply in the public sector, and vice versa. Second, the marginal benefit from the public sector not only depends on the wage but also on the private sector earnings. More precisely, the more patients physician  $i$  treats in the public sector, all else equal, the lower will the price in the private sector be.

Imposing symmetry, we obtain from (11)-(12) the following individual equilibrium labour supply in the public and the private sector:<sup>16</sup>

$$x^{MS}(w) = \frac{w(3+N) - 2 - b}{2(1+N)(1-b) - b^2N}, \quad (13)$$

$$y^{MS}(w) = \frac{2 - w(2 + bN)}{2(1+N)(1-b) - b^2N}, \quad (14)$$

respectively. Aggregating individual labour supply, we find that total equilibrium public and private health care provision are given by:

$$X^{MS}(w) = Nx^{MS}(w) \quad \text{and} \quad Y^{MS}(w) = Ny^{MS}(w), \quad (15)$$

respectively. Inserting (13)-(15) into (4) and (5), we obtain the following private sector price and individual physician profits in equilibrium:

$$p^{MS}(w) = \frac{2(1-b) + wN(2-3b)}{2(1+N)(1-b) - b^2N}, \quad (16)$$

$$V^{MS}(w) = \frac{[4(1-b) - b^2][1 - w(2 + Nb)] + w^2\Omega}{[2(1+N)(1-b) - b^2N]^2}. \quad (17)$$

where

$$\Omega := 5 - 6b + N(2 - 3b^2) + N^2(1 - b)^2.$$

Let us start by establishing the conditions for a mixed health care system to occur as an equilibrium. Observe that the public provision is increasing in  $w$ , while the private provision is decreasing in  $w$ . Thus, we can set (13) and (14) equal to zero, and solve with respect to  $w$ , to obtain the lower and upper bound on  $w$ , i.e.,

$$\underline{w} = \frac{2+b}{3+N} \quad \text{and} \quad \bar{w} = \frac{2}{2+bN}. \quad (18)$$

We can now establish the following result:

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<sup>16</sup>The equilibrium in (13)-(15) is unique and stable if the determinant of the Jacobian is positive, i.e.,  $2(1+N)(1-b) - b^2N > 0$ , which is true if

$$b < \hat{b} := \frac{1}{N} \left( \sqrt{4N + 3N^2 + 1} - N - 1 \right) \in (0.73, 0.83).$$

**Proposition 1** *There exists an equilibrium with a mixed health care system, where the physicians spend some time in both the public and the private sector, if and only if  $w \in (\underline{w}, \bar{w})$ . Otherwise, if  $w \geq (\leq) \bar{w}$ , the equilibrium is a pure public (private) health care system, where the physicians work exclusively in the public (private) sector.*

A proof is provided in Appendix A.

Thus, we have mixed public-private health care system if the public sector wage is not too high nor too low. If the health authority offers a too low wage the physicians will find it profitable to spend their time exclusively in the private sector. On the other hand, if the public sector wage becomes too high, private provision becomes unprofitable, and the physicians work exclusively in the public sector. Recall that the health authority not only sets the wage, but also determines whether or not dual practice is allowed. This means that the health authority can always eliminate the private option for the physicians if this is desirable from a welfare point-of-view. As will be shown later, it is never optimal for the health authority to close down the NHS, implying that the pure private health care system never occurs as an equilibrium outcome of the whole game. Since our focus is on the frequently observed physician dual practice in NHS systems, we therefore ignore this outcome in the following of this section.

### **Effects of the public sector wage**

Restricting attention to the equilibrium where physicians spend some time in both sectors, i.e., the mixed health care system, we first consider the effect of a higher wage on the physicians' labour supply, and in turn the provision of health care. By analysing (13)-(15), we obtain the following result:

**Proposition 2** *In a mixed health care system, a higher public sector wage induces (i) a shift in the labour supply from the private to the public sector, with a corresponding increase in the public provision relative to the private provision; and (ii) an increase in the total provision of health care.*

A proof is provided in Appendix A.

The first part of the result is quite intuitive. A higher wage induces the physicians to work more in the public sector, but at the same time reduce their private sector labour supply. The reason for the reduction in the private labour supply is two-fold. First, there is a cost effect. Since the physicians' disutility of working is increasing in the total time spent at providing care

– irrespectively of whether this is in the public or private sector – it is optimal for the physicians to reduce their private sector labour supply in response to the increase in the time spent in the public sector. Second, there is a revenue effect. The increase in the public provision triggered by the higher wage will, all else equal, result in a lower price for private treatments, since public and private care are substitutes. A lower price implies a lower (marginal) revenue from private provision, which makes it profitable for the physicians to reduce the time spent in the private sector.

The second part of Proposition 2 is less clear. Since the public provision increases, while the private provision decreases, in the wage level, the net effect on total health care provision is in general ambiguous. However, the proposition demonstrates that the positive effect on public provision of a wage increase always exceed the negative effect on private provision, i.e.,  $\partial X^{MS}/\partial w > |\partial Y^{MS}/\partial w|$ . As a consequence, overall health care provision becomes higher following a public sector wage increase.

Turning to the wage effects on private sector price and physician profits, we obtain from (16) and (17) the following result:

**Proposition 3** *In a mixed health care system, a higher public sector wage increases both the price of private care and the physicians' profits if  $b < 2/3$ . Otherwise, if  $b > 2/3$ , the price of private care is decreasing in the wage, and the physicians' profits are decreasing (increasing) at low (high) wage levels.*

A proof is provided in Appendix A.

These results are rather counter-intuitive and need some explanation. Consider first the public sector wage effect on the *private sector price*. To identify the countervailing effects, we can differentiate the equilibrium price (on general form) with respect to  $w$ :

$$\frac{dp^{MS}}{dw} = -\frac{\partial Y^{MS}}{\partial w} - b\frac{\partial X^{MS}}{\partial w}.$$

We know from Proposition 2 that a higher wage induces an increase in the public provision that exceeds the decrease in the private provision – i.e.,  $\partial X^{MS}/\partial w > |\partial Y^{MS}/\partial w|$  – resulting in a higher overall health care provision. This suggests that the price of private care should fall as a response to a higher wage. However, this is not true. The reason is that the impact of public provision on the price of private care is determined by the degree of substitutability between public and private care, measured by  $b$ . The more differentiated public and private care are (low

*b*) – e.g., due to long distance or different service mixes between public and private hospitals – the less impact has changes in the public provision on the price of private care. In fact, for any  $b < 2/3$ , the positive effect of a lower private provision more than offsets the negative effect of a higher public provision, resulting in a higher price of private care following a wage increase.

While the wage effect on *physician profits* is closely connected to the above discussion, additional effects also enter. To understand the net effect of a wage change on profits, it is useful to differentiate the profit (on general form) with respect to  $w$ :

$$\begin{aligned} \frac{dV^{MS}}{dw} &= \underbrace{w \frac{\partial x^{MS}}{\partial w} + x^{MS}}_{\text{Public income effect}} + \underbrace{\frac{\partial p^{MS}}{\partial w} y^{MS} + p^{MS} \frac{\partial y^{MS}}{\partial w}}_{\text{Private profit effect}} \\ &\quad - \underbrace{2(x^{MS} + y^{MS}) \left( \frac{\partial x^{MS}}{\partial w} + \frac{\partial y^{MS}}{\partial w} \right)}_{\text{Disutility / time cost effect}} \end{aligned}$$

Thus, we see that the wage effect on profits can be decomposed into three effects. First, there is a *public income effect*. This is always positive partially because of the higher wage itself, and partially because of the larger public provision induced by the wage increase. Second, there is a *private profit effect*. If  $b > 2/3$ , then a higher public sector wage reduces not only the private sector labour supply, but also the private sector price, implying a negative private profit effect following a wage increase. However, if  $b < 2/3$ , then the price of private care is increasing in the wage, making the wage effect on private profit ambiguous. Finally, there is a *time cost effect* of a higher wage associated with the disutility of working. Since the increase in public provision always exceed the decrease in private provision – i.e.,  $\partial x^{MS}/\partial w > |\partial y^{MS}/\partial w|$  – overall working time increases. As a consequence, the time cost effect is unambiguously negative.

Obviously, the net effect of a wage increase on physician profits is determined by the relative sizes of the three above mentioned effects. As Proposition 3 shows, physicians' profits will increase as a response to a higher wage, if public and private health care are sufficiently differentiated, i.e.,  $b < 2/3$ . In this case, the time cost effect is more than offset by the public and private revenue effects. On the other hand, if  $b > 2/3$ , the price of private care decreases as a response to a wage increase, and the private profit effect becomes unambiguously negative. In this case, the net effect on profit crucially relies on the strength of the public income effect, which again is depending on the absolute wage level. This explains the last part of the Proposition 3.

### **Effects of private sector competition (the number of physicians)**



Let us briefly examine how the number of physicians affects the equilibrium outcomes reported above. Analysing (13)-(16), we obtain the following result:

**Proposition 4** *In a mixed health care system, a larger number of physicians triggers (i) a lower price of private care, (ii) a shift in the individual labour supply from the private to the public sector; (iii) a larger public provision; and (iv) a smaller (larger) private provision when the number of physicians is high (low).*

A proof is provided in Appendix A.

The first three effects are quite straightforward. A larger number of physicians, triggers competition in the private sector, inducing a shift in individual labour supply from the private sector to the public sector due to lower profitability from private care. Total public health care provision increases, not only because of the shift in individual labour supply, but also because of the new physicians spend some time in the public sector.

The fourth effect needs a closer explanation. If we differentiate the effect of a higher number of physicians on the private provision on general form, we obtain the following expression

$$\frac{dY^{MS}}{dN} = \underbrace{y^{MS}}_{\text{Direct provision effect}} + \underbrace{N \frac{\partial y^{MS}}{\partial N}}_{\text{Competition effect}}.$$

Thus, we can decompose the net effect on the private provision into two effects. First, a higher number of physician increases private provision directly, because every physician works in both sectors under a mixed health care system. Second, there is a negative competition effect. A higher number of physicians triggers private sector competition, inducing a shift in the labour supply from the private sector to the public sector, i.e.,  $\partial y^{MS}/\partial N < 0$ . We see that the competition effect is stronger the more physicians there are in the market. In fact, when  $N$  becomes sufficiently high, the competition effect more than offsets the positive direct provision effect, resulting in a reduction in private provision, as reported in the Proposition.

### **The scope for a mixed public-private health care system**

Let us briefly examine the scope for a mixed health care system. A mixed system requires that the physicians are active in both sectors, implying that the scope for a mixed system can be defined by the difference between the upper and the lower bound on the wage, i.e.,

$$\bar{w} - \underline{w} = \frac{2(1+N)(1-b) - b^2N}{(2+bN)(3+N)}. \quad (19)$$

Thus, the scope for a mixed system is determined by the degree of substitutability between public and private health care ( $b$ ) and the number of physicians in the market ( $N$ ). From (19), we obtain the following result:

**Proposition 5** *The scope for a mixed health care system is (i) decreasing in the substitutability between public and private health care, (ii) and increasing (decreasing) in the number of physicians if public and private care are sufficiently differentiated (substitutable).*

A proof is provided in Appendix A.

The first part of the proposition is not very surprising. The closer substitutes public and private health care are, the less is the scope for a mixed system. The reason is that as  $b$  becomes higher, the more sensitive the price of private care becomes to changes in the public provision. When  $b$  is high, a small increase in the public provision induces a large drop in the price of private care, which in turn results in a large reduction in the private sector labour supply. In fact, the physicians' allocation of time between the two sectors becomes a discrete choice when  $b$  is sufficiently high. In this case, if the private sector profits exceed the public sector income, the physician will spend all their time in the private sector, and vice versa.

The second part is less intuitive, and needs some closer explanation. *Prima facie* one should expect that a higher number of physicians in the market reduced the scope for a private sector provision, simply because of stronger price competition and thus less private sector profits. This is what happens if  $b$  is sufficiently high, as reported in the proposition. However, if  $b$  is small, then the shift-effect in labour supply from private to public provision is weak, implying a larger scope for a mixed health care system. Technically, it is easy to verify that  $|\partial\bar{w}/\partial N| < (>)$   $|\partial\underline{w}/\partial N|$  if  $b$  is sufficiently low (high).

### **Comparison of the pure public system and the mixed public-private system**

Before we start analysing optimal policy by the health authority, it is useful to briefly contrast the pure NHS system with the mixed health care system. A first observation is that the availability of a private option increases the physicians' reservation wage for working in the public sector. This follows straightforwardly by the fact that  $\underline{w} > 0$ . By comparing (8)-(9) with (13)-(15), we obtain the following result:

**Proposition 6** *Allowing the physicians to provide (substitutable) private health care, reduces the public health care provision and also the overall health care provision in the market for any  $w \in (\underline{w}, \bar{w})$ .*

A proof is provided in Appendix A.

Thus, the proposition shows that allowing publicly employed physicians to offer substitutable private (out-of-plan) services has a strong *crowding out effect* – not only on public health care provision – but in fact also on overall health care provision. Obviously, this means that the increase in private health care provision – resulting from the private sector option – is smaller than the corresponding reduction in public health care provision. The intuition is connected to two countervailing effects. First, there is *market power effect*. Since the price of private care not only depends on the private provision, but also on the public provision, the physicians have an incentive to restrict their labour supply in both sectors in order to obtain higher prices and profits in the private sector. Second, there is a negative *public sector income effect* associated with the reduction in the public sector labour supply, countervailing the former effect. The proposition demonstrates that the market power effect always dominates the public sector income effect, implying that overall health care provision becomes lower.

## 4 The health authority's wage setting

Let us now turn to stage 2 of the game. At this stage the health authority sets the public sector wage, anticipating the labour supply responses from the physicians, as described in the previous section. We assume that the health authority is concerned about social welfare, defined as usual as the sum of patients' surplus and providers' profits net of public transfers (wage expenditures). The welfare function can be specified as follows:

$$W = U + \gamma \left[ wX + pY - N(x + y)^2 \right] - wX, \quad (20)$$

where  $\gamma$  is the weight the health authority attaches to physicians' profits. We find it reasonable that the health authority is more concerned with the patients' benefit from a medical treatment than with the physicians' profits, so we assume that  $\gamma < 1$ .

### Pure public health care system

First, we derive the optimal public sector wage in a pure NHS system. In this case  $Y := 0$ , and the welfare function can be written as follows:

$$W^{PS}(w) = X^{PS}(w) - \frac{1}{2} [X^{PS}(w)]^2 - (1 - \gamma) wX^{PS}(w) - \gamma N [x^{PS}(w)]^2 \quad (21)$$

Inserting (8) and (9) into (21), and maximising (21) with respect to  $w$ , we obtain the following optimal wage:

$$w^{PS} = \frac{2}{4 + N - 2\gamma}. \quad (22)$$

We see that the optimal wage is decreasing in  $N$  but increasing in  $\gamma$ . The latter effect is due to the fact that as  $\gamma$  increases, the wage becomes a welfare neutral transfer between the government and the physicians, eliminating the incentive of keeping public expenditures down.<sup>17</sup> The former effect is due the positive relationship between  $N$  and overall public provision. Thus, when  $N$  is high, there is less need to use the wage to induce an optimal level of individual labour supply.

Inserting (22) into (8) and (9), we obtain the following equilibrium labour supply and provision of public health care:

$$x^{PS} = \frac{1}{4 + N - 2\gamma}, \quad (23)$$

$$X^{PS} = \frac{N}{4 + N - 2\gamma}. \quad (24)$$

As expected, we see that both the individual labour supply and the total public provision are increasing in  $\gamma$ . This is simply due to the fact a higher  $\gamma$  is equivalent to less concern about public expenditures, resulting in a higher wage, as explained above. A higher  $N$  decreases individual labour supply, because the optimal wage is decreasing in  $N$ . This makes the effect of  $N$  on total public provision ambiguous in principle. Taking the partial derivative of (24) with respect to  $N$ , we can show that:

$$\frac{\partial X^{PS}}{\partial N} = \frac{2(2 - \gamma)}{(4 + N - 2\gamma)^2} > 0.$$

Thus, the direct effect of an extra physician in the market always exceed the indirect negative effect on individual labour supply.

### Mixed public and private health care system

Turning to the mixed public-private health care system, welfare can now be written as follows:

$$W^{MS}(w) = \tilde{U} - (1 - \gamma) [p^{MS}(w) Y^{MS}(w) + w X^{MS}(w)] - \gamma N [x^{MS}(w) + y^{MS}(w)]^2, \quad (25)$$

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<sup>17</sup>We could of course attach a shadow cost of public funds to the wage expenditures, which would have provided a separate incentive to keep wage expenditures down, and thus limited the scope for a public sector. However, since the effect is straightforward and not very interesting, we have decided to leave this out.

where

$$\tilde{U} := X^{MS}(w) + Y^{MS}(w) - \frac{1}{2} \left\{ [X^{MS}(w)]^2 + [Y^{MS}(w)]^2 + 2bX^{MS}(w)Y^{MS}(w) \right\}$$

is the patients' gross utility from medical treatment. We see that a high weight on physicians' profits,  $\gamma \rightarrow 1$ , implies – on the one hand, a less concern for public *and private* medical expenditures – but, on the other hand, a larger concern for the physicians' costs of providing the treatments. In any case, the patients' gross utility from public and private health care are left unaffected.

To understand the mechanisms governing the health authority's wage setting, it is useful to differentiate (25) with respect to  $w$  on general form:

$$\begin{aligned} \frac{dW^{MS}}{dw} = & \underbrace{[1 - X^{MS} - bY^{MS} - w(1 - \gamma)] \frac{\partial X^{MS}}{\partial w}}_{\text{Social gain from public provision}} \\ & - \underbrace{(1 - \gamma) \left[ p^{MS} \frac{\partial Y^{MS}}{\partial w} + \frac{\partial p^{MS}}{\partial w} Y^{MS} \right]}_{\text{Social gain / loss from private provision}} - \underbrace{2\gamma (x^{MS} + y^{MS}) \left[ \frac{\partial x^{MS}}{\partial w} + \frac{\partial y^{MS}}{\partial w} \right]}_{\text{Social cost of public and private provision}} \end{aligned}$$

We see that the optimal wage setting can be decomposed into three elements: First, there is a direct social gain associated with public provision. This gain is decreasing in the size of the public and the private sector, simply reflecting decreasing marginal benefit from medical treatments. The gain is also decreasing in the wage itself, which can be interpreted as the marginal cost of public provision.

Second, there are two potentially countervailing private sector effects. On the one hand, a higher wage reduces private provision, which increases patients' surplus simply because a larger fraction of patients receives free public care. On the other hand, a higher wage may increase the price of private care. In Proposition 3, we showed that this was the case if public and private care are sufficiently differentiated, i.e.,  $b < 2/3$ . In this case, the net welfare effect via the private sector is ambiguous. However, if public and private care are sufficiently close substitutes, i.e.,  $b > 2/3$ , a higher wage not only shifts patients to free public health care, but also reduces the price for those seeking private care, resulting in an unambiguously positive welfare effect.

Finally, there is a cost effect associated with the provision of health care. Since overall provision becomes higher following a wage increase – i.e.,  $\partial x^{MS}/\partial w > |\partial y^{MS}/\partial w|$  – the cost effect is unambiguously negative. We see that if the health authority is not concerned with the

physicians' profits, i.e.,  $\gamma \rightarrow 0$ , the cost effect does not play a role for the optimal wage setting.

The optimal wage is derived by inserting (13)-(16) into (25), and maximising (25) with respect to  $w$ , yielding:<sup>18</sup>

$$w^{MS} = \frac{1}{\Delta} \left[ N(1-b)(14+7b-4b\gamma) + N^2(4+b)(1-b) \right. \\ \left. + 2(5-4\gamma) - b(1-\gamma)(2b+Nb^2+8) \right], \quad (26)$$

where

$$\Delta := 2(6-6b+6b\gamma-5\gamma) + N(21-16b-6b^2+6b^2\gamma-4\gamma) \\ + 2N^2(1-b)(5+b+b\gamma-\gamma) + N^3(1-b^2)$$

Inserting (26) into (13)-(15), we obtain the following equilibrium public and private labour supply and provision:

$$x^{MS} = \frac{1}{\Delta} \left[ (3+N^2)(1-b) + N(2-b) + \gamma(2-3b)(N-1) \right], \quad (27)$$

$$y^{MS} = \frac{1}{\Delta} \left[ (1-b)(N^2+5N-2N\gamma+2-2\gamma) - Nb^2(1-\gamma) \right], \quad (28)$$

with

$$X^{MS} = Nx^{MS} \quad \text{and} \quad Y^{MS} = Ny^{MS}. \quad (29)$$

Inserting (26) into (16), we obtain the following equilibrium price of private care:

$$p^{MS} = \frac{1}{\Delta} \left[ (12+5N^2)(1-b) + N(19-17b-2\gamma-3b^2(1-\gamma)) - 2\gamma(5-6b) \right] \quad (30)$$

From these expressions, the following result can be derived:

**Proposition 7** *The health authority offers a strictly positive wage. The optimal wage results in a mixed health care system, where the physicians work in both sectors.*

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<sup>18</sup>The second order condition is given by:

$$\frac{\partial^2 W^{MS}}{\partial w^2} = -N \frac{\Delta}{[2(1+N)(1-b)-b^2N]^2} < 0,$$

which implies that:  $\Delta := 2(6-6b+6b\gamma-5\gamma) + N(21-16b-6b^2+6b^2\gamma-4\gamma) + 2N^2(1-b)(5+b+b\gamma-\gamma) + N^3(1-b^2) > 0$ . It can easily be verified that this condition is always less strict than the equilibrium condition reported in Footnote 16.

A proof is provided in Appendix A.

The result may not seem very surprising, but recall that the health authority could induce a pure private system by setting the public sector wage sufficiently low, and, vice versa, a pure public system by setting the wage sufficiently high. The proposition shows that it is never optimal for the health authority to eliminate the NHS. If a pure private system is not desirable at a positive wage, i.e.,  $w^{MS} > 0$ , it is surely not so at any lower wage, e.g.,  $w = 0$ . Thus, a pure private system is never an equilibrium in this model.

The above proposition also shows that it is not optimal to implement a pure NHS system by offering so high wages that the physicians will want to exclusively work in the public sector. However, we cannot rule out that it may be welfare improving to eliminate the physicians' private option by banning dual practice. This case will be analysed in Section 5. Before we do this, we will briefly characterise the equilibrium outcomes under the mixed system. For expositional purposes, we focus on two special cases: first, we assume that  $\gamma = 0$ , and analyse the impact of the degree of competition (measured by the number of physicians) on the equilibrium outcomes. Second, we assume  $b = 0.5$ , and focus on the role of providers' surplus (physicians' profits) in the objective function (measured by  $\gamma$ ) on the wage setting and corresponding equilibrium outcomes.

### **The role of competition among physicians**

Starting with the case where the health authority is not concerned about physician profits', we see from (25) that the health authority's objective now is to maximise the patients' gross health benefit net of private and public health care expenditures. Analysing (26)-(30), under the assumption of  $\gamma = 0$ , we obtain the following result:

**Proposition 8** *A higher number of physicians induces (i) a lower public sector wage, (ii) a lower individual labour supply in both sectors, (iii) a higher provision of health care in both sectors, and (iv) a lower price of private care, given that  $\gamma = 0$ .*

A proof is provided in Appendix A.

Recall from Proposition 4 that – for a given wage  $w \in (\underline{w}, \bar{w})$  – a higher number of physicians induced a shift in labour supply from the private to the public sector, resulting in increased public provision. While effect on private provision was ambiguous, the price of private care was reduced by the number of physicians in the market. Endogenising the wage setting, the

above proposition shows that the optimal response for the health authority is to reduce the wage level as the number of physicians increase. The reason is three-fold. First, since a higher  $N$  increases public provision, there is less need to use the wage to stimulate individual public sector labour supply to induce optimal levels public care. Second, a higher public provision increases public expenditures, all else equal, which in turn provides a separate incentive to lower the wage. Finally, a higher number of physicians triggers private sector competition resulting in a lower price of private care. Since private medical expenditures also matters for the health authority (as long as  $\gamma < 1$ ), the incentive to use the wage to shift patients from the private to the public sector in order to enhance patients' surplus is weaker.

### **The role of physicians' surplus (profits)**

Turning to the issue of how the weight on physicians' profits may affect the optimal wage setting by the health authority, we assume, for simplicity, that  $b = 0.5$ .<sup>19</sup> We see from (25) that when  $\gamma > 0$ , the physicians' costs of providing health care now matters for the wage setting. Analysing (26)-(30), we obtain the following result.

**Proposition 9** *A higher weight on the physicians' profits relative to patients' surplus in the health authority's objective function leads to (i) a higher public sector wage, (ii) a shift from private to public health care provision, and (iii) a higher price of private care, given that  $b = 0.5$ .*

A proof is provided in Appendix A.

At first glance, this result may seem counterintuitive. However, recall that a higher  $\gamma$  implies that the health authority is less concerned with both public and private medical expenditures. In fact, in the extreme case of  $\gamma \rightarrow 1$ , we see from (25) that the wage expenditures, as well as the patients' expenditures on private health care, do not influence the health authority's wage setting. This explains why the public sector wage is increasing in  $\gamma$ . The two last parts of the proposition follows then straightforwardly. A higher wage induces a shift in individual labour supply from the private to the public sector, and a corresponding decrease in private provision and increase in public provision. The price effect follows from the fact that  $b = 0.5 > 2/3$ , see Proposition 3.

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<sup>19</sup>It can be shown that the result in Proposition 9 is valid for any  $b$ . However, the expressions are very long and the proof is quite tedious, so we have decided to leave it out. Interested readers may contact the authors for the proof.



## 5 The decision on physician dual practice

At stage 1, the health authority decides whether or not to allow for private (out-of-plan) provision of health care alongside the NHS. In our setting a pure NHS system is equivalent to not allowing publicly employed physicians to work in the private sector. To make this decision the health authority needs to compare the welfare level for the two different regimes. Inserting (22)-(24) into (21), we obtain the welfare level under a pure NHS system:

$$W^{PS} = \frac{N}{2(4 + N - 2\gamma)}. \quad (31)$$

In a similar way, we obtain the welfare level under a mixed public-private system by inserting (26)-(30) into (25):

$$W^{MS} = \frac{N}{2\Delta} [(1 - b)(1 - b + 2N + 2N^2 + 2\gamma(3N + 2 + b - 2\gamma)) + b^2\gamma^2] \quad (32)$$

By comparing (31) and (32), we obtain the following result:

**Proposition 10** *A mixed health care system (with physician dual practice) is welfare improving compared with a pure public health care system if the number of physicians is sufficiently high and/or the health authority is sufficiently concerned about physicians' profits. Otherwise, a pure health care system is welfare improving if public and private health care are sufficiently close substitutes.*

A proof is provided in Appendix A.

Thus, if the number of physicians is sufficiently high, the health authority should allow physician dual practice as this would always improve welfare compared with a pure public system. The reason is that a high number of physicians triggers competition in the private sector, resulting in low prices of private care. Thus, the welfare loss associated with market power in the private sector is reduced. In addition, a higher number of physicians makes the crowding-out effect of dual practice – as shown in Proposition 6 – less severe. Thus, under-supply of public health care is less a concern.

On the other hand, if the number of physicians in the market is low, the desirability of a mixed health care system depends on relative sizes of the degree of substitutability between public and private health care ( $b$ ) and the weight the health authority attaches to physicians' profits ( $\gamma$ ). Figure 1 below illustrates the relationship for  $N = 2$ . We see that the scope

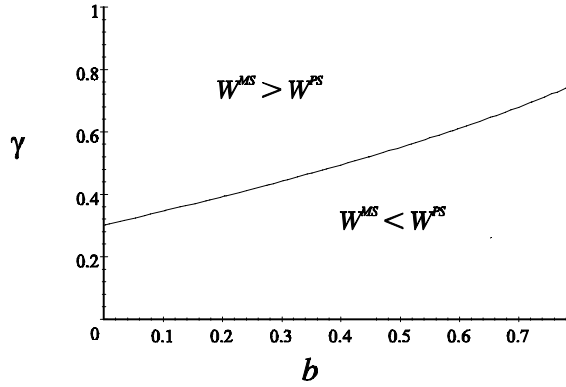


Figure 1: Welfare effect of physician dual practice;  $N = 2$

for a mixed health care system is decreasing in  $b$  and increasing in  $\gamma$ . The closer substitutes public and private health care are, the more likely it is that a pure public health care system is desirable from a welfare point-of-view. The reason is that when the number of physicians is low and private health care is a close substitute for public health care, then the crowding out effect very strong. A low number of physicians imply weak competition among the physicians, and thus high private sector price and profit. In addition, a high  $b$  makes the price of private care very responsive to small changes in the public sector provision. This combination implies a substantial reduction in public provision following from the possibility of physician dual practice.

Finally, Proposition 10 shows that the desirability of mixed health care system is increasing in the weight the health authority attaches to physician profits. In the extreme case of  $\gamma \rightarrow 1$ , where the health authority puts an equal weight on patients' surplus and physician profits, the mixed health care system is always preferable to the pure public system. The reason is that in this case the level of neither public nor private medical expenditures has no influence on the health authority's choice of health care system. On the other hand, if the health authority puts a strong emphasis on the public and private medical expenditures, then the scope for a mixed health care system is reduced.

## 6 Concluding remarks

In this paper we have analysed the interaction between public and private health care in a NHS system where physicians may work in both sectors. We have characterised the physicians' allocation of effort (number of treatments) between the two sectors, and analysed how this can

be affected by monetary incentives, as the public sector wage and the private sector price and profits. Moreover, we have derive the health authority's optimal public sector wage and analysed the decision of whether or not to allow private (out-of-plan) provision of health care.

We would like to highlight the following three main findings of the paper. First, we show that allowing publicly employed physicians to offer substitutable private (out-of-plan) services has a strong *crowding out effect* – not only on public health care provision – but in fact also on overall health care provision. The reason is that the that physicians can increase the private sector profits by restricting their labour supply in both sectors. Second, the health authority can mitigate the crowding out effect by increasing the public sector wage, and will do so if the number of physicians is small and they put a larger weight on patients' surplus than physicians' profits. Finally, we show that a private (out-of-plan) option alongside the NHS is desirable only if the number of physicians and/or the weight on physicians' profits is sufficiently high. A high number of physicians imply strong private sector competition and makes the crowding-out effect on public sector less severe. A high weight on physicians' profits implies less concern for public and private medical expenditures, which are the main arguments in favour of a pure NHS system. A pure public system is preferable if these conditions are not fulfilled, and, in addition, the private sector offers sufficiently differentiated services to the public sector – interpreted either in geographical terms or in product space.

Finally, we would like to stress some aspects of our model. First, we have assumed that the number of physicians is fixed, which can be justified by the severe restrictions on entry into the physician market. Free entry is an implausible assumption. On the other hand, we could of course let the health authority set the number of physicians. In practice, however, entry is determined by negotiations between the medical association and the health authority, with obvious conflicting interests. Thus, we decided to leave this out of the analysis. Second, the health authority could in principle regulate the price in the private sector. In this case, the market power incentives of raising the private sector price and profits would have be eliminated, which would in turn dampen the crowding out effect of the private option. The effects are, however, very straightforward. Finally, the production technology is very simple, and some of the results may rely on this. On the other hand, the crowding out effect and the basic trade-offs concerning the private sector option are quite general.

## 7 Appendix A: Proofs of Propositions

**Proof of Proposition 1:** By inspection of (13)-(15), the following is true:

$$x^{MS}(w) \quad \text{and} \quad X^{MS}(w) > (=) 0 \quad \text{iff} \quad w > (<=) \underline{w},$$

$$y^{MS}(w) \quad \text{and} \quad Y^{MS}(w) > (=) 0 \quad \text{iff} \quad w < (>=) \bar{w}.$$

It is easily verified that:

$$\bar{w} > \underline{w} \quad \Leftrightarrow \quad 2(1+N)(1-b) - b^2N > 0,$$

which is always true from the condition of equilibrium uniqueness and stability, see Footnote 16.

To prove the existence of a mixed public-private system, we need to verify that the physicians' profits are non-negative for any  $w \in (\underline{w}, \bar{w})$ . Evaluating (17) at the upper and lower wage bounds, we obtain:

$$V^{MS}(\underline{w}) = \frac{2}{(3+N)^2} > 0 \quad \text{and} \quad V^{MS}(\bar{w}) = \frac{1}{(2+bN)^2} > 0. \quad (\text{A1})$$

Checking the second-order condition, we get:

$$\frac{\partial^2 V^{MS}}{\partial w^2} = \frac{2\Omega}{[2(1+N)(1-b) - b^2N]^2} > 0, \quad (\text{A2})$$

which is true since  $\Omega > 0$  for any valid parameter values. Thus,  $V^{MS}(w)$  is convex function with a minimum in  $w$ . We find the extreme value by minimising (17), yielding

$$V(w^{\min}) = \frac{4(1-b) - b^2}{4\Omega} > 0, \quad (\text{A3})$$

where

$$w^{\min} = \frac{[4(1-b) - b^2](2+Nb)}{2\Omega}. \quad (\text{A4})$$

This completes the proof. *QED.*

**Proof of Proposition 2:** The first part can be shown by taking the partial derivatives of (13) and (14) with respect to  $w$ , yielding the following

$$\frac{\partial x^{MS}}{\partial w} = \frac{3+N}{2(1+N)(1-b) - b^2N} > 0, \quad (\text{A5})$$

$$\frac{\partial y^{MS}}{\partial w} = -\frac{2 + bN}{2(1 + N)(1 - b) - b^2N} < 0. \quad (\text{A6})$$

Since  $X^{MS}(w) = Nx^{MS}(w)$  and  $Y^{MS}(w) = Ny^{MS}(w)$ , the result in part (i) is established.

Part (ii) is obtained by comparing (A5)-(A6). We need to establish that:

$$\frac{\partial x^{MS}}{\partial w} - \left| \frac{\partial y^{MS}}{\partial w} \right| > 0 \quad \Leftrightarrow \quad N(1 - b) + 1 > 0$$

which is always true. Thus, a higher wage results in a increase in overall provision of health care, i.e.,  $\partial(X^{MS} + Y^{MS})/\partial w > 0$ . *QED*.

**Proof of Proposition 3:** Part (i) is established by taking the partial derivative of (16) with respect to  $w$ , yielding

$$\frac{\partial p^{MS}}{\partial w} = \frac{N(2 - 3b)}{2(1 + N)(1 - b) - b^2N}, \quad (\text{A7})$$

which is positive only if  $b < 2/3$ .

Part (ii) can be established by first taking the partial derivative of (17) with respect to  $w$ , yielding

$$\frac{\partial V^{MS}}{\partial w} = \frac{-[4(1 - b) - b^2](2 + Nb) + 2w\Omega}{(2(1 + N)(1 - b) - b^2N)^2}. \quad (\text{A8})$$

Taking the second partial derivative of (17) with respect to  $w$ , we obtain

$$\frac{\partial^2 V^{MS}}{\partial w^2} = \frac{2\Omega}{(2(1 + N)(1 - b) - b^2N)^2} > 0. \quad (\text{A9})$$

Thus, we know that  $V^{MS}(w)$  is a convex function with a unique minimum in  $w$ . It then suffice to evaluate (A8) at the lower wage bound, i.e.,

$$\left. \frac{\partial V^{MS}}{\partial w} \right|_{w=\underline{w}=\frac{2+b}{3+N}} = \frac{(N - 1)(2 - 3b)}{(N + 3)(2(1 + N)(1 - b) - b^2N)},$$

which is strictly positive (negative) for any  $b < (>) 2/3$ . This completes the proof. *QED*.

**Proof of Proposition 4:** The comparative statics results are obtained by taking the partial derivative of (13)-(16) with respect to  $N$ , yielding the following expressions:

$$\frac{\partial x^{MS}}{\partial N} = \frac{(2 + b)\Gamma}{[2(1 + N)(1 - b) - b^2N]^2} > 0 \quad (\text{A10})$$

$$\frac{\partial y^{MS}}{\partial N} = -\frac{2\Gamma}{[2(1+N)(1-b) - b^2N]^2} < 0 \quad (\text{A11})$$

$$\frac{\partial p^{MS}}{\partial N} = -\frac{2(1-b)\Gamma}{[2(1+N)(1-b) - b^2N]^2} < 0, \quad (\text{A12})$$

where

$$\Gamma := 2(1-b) - b^2 - w(2-3b). \quad (\text{A13})$$

To verify part (i) and (ii), it suffice to prove that  $\Gamma > 0$ . First, we see that the wage has no effect on  $\Gamma$  if  $b = 2/3$ . In this case we can show that  $\Gamma(b = \frac{2}{3}) \simeq 0.22$ . Assume then that  $b < 2/3$ , so that  $\Gamma$  is monotonically decreasing in  $w$ . In this case, it suffice to evaluate (A13) at the upper wage bound, i.e.,

$$\Gamma(\bar{w}) = b \frac{2(1+N)(1-b) - b^2N}{2 + bN} > 0.$$

Finally, for  $b > 2/3$ , then  $\Gamma$  is increasing in the wage. In this case, it suffice to evaluate (A13) at the lower wage bound, i.e.,

$$\Gamma(\underline{w}) = \frac{2(1+N)(1-b) - b^2N}{3 + N} > 0.$$

Part (iii) follows from the fact that  $X^{MS} = Nx^{MS}$ . Differentiating  $X^{MS}$  with respect to  $N$  on general form, yields:

$$\frac{dX^{MS}}{dN} = N \frac{\partial x^{MS}}{\partial N} + x^{MS} > 0,$$

which is always true since  $\partial x^{MS}/\partial N > 0$ .

To prove part (iv), we first differentiate  $Y^{MS}$  on general form with respect to  $N$ , yielding:

$$\frac{dY^{MS}}{dN} = N \frac{\partial y^{MS}}{\partial N} + y^{MS}.$$

Since  $\partial y^{MS}/\partial N < 0$ , we know that:

$$\frac{dY^{MS}}{dN} < (>) 0 \quad \text{iff} \quad N \left| \frac{\partial y^{MS}}{\partial N} \right| > (<) y^{MS}.$$

Inserting (A11) and (14), we obtain the following inequality

$$\frac{2N\Gamma}{[2(1+N)(1-b) - b^2N]^2} > \frac{2 - w(2 + bN)}{2(1+N)(1-b) - b^2N}. \quad (\text{A14})$$

First observe that for  $w = \bar{w}$ , the RHS of (A14) becomes zero – i.e.,  $y^{MS} = 0$  – while the LHS is positive, i.e.,

$$\frac{2bN}{(Nb+2)(2(1+N)(1-b)-b^2N)} > 0.$$

Thus, for  $w = \bar{w}$ , then  $dY^{MS}/dN$  is always negative. However, since  $y^{MS}$  is monotonically decreasing in  $w$ , the sign may turn positive. Evaluating the inequality in (A14) for  $w = \underline{w}$ , we obtain the following after some calculations:

$$N > \frac{2(1-b)}{b(2+b)}.$$

Since the RHS of (A14) approaches infinity if  $b \rightarrow 0$ , the inequality cannot hold, and the sign of  $\partial Y^{MS}/\partial N$  will turn positive. Generally, we have that, for any  $w \in (\underline{w}, \bar{w})$ ,

$$\frac{dY^{MS}}{dN} < (\geq) 0 \quad \Leftrightarrow$$

$$N > (\leq) \frac{2bw - 2b^2w - 2\sqrt{2bw - 2bw^2 - 4b^2w + b^3w + b^4w + 5b^2w^2 - 3b^3w^2}}{-2bw + 2b^2w + b^3w}.$$

*QED.*

**Proof of Proposition 5:** The first part of the result is obtained by taking the partial derivative of (19) with respect to  $b$ , i.e.,

$$\frac{\partial(\bar{w} - \underline{w})}{\partial b} = -\frac{6N + 4Nb + 2N^2 + N^2b^2 + 4}{(Nb+2)^2(N+3)} < 0. \quad (\text{A15})$$

To prove the second part, we first take the partial derivative of (19) with respect to  $N$ , i.e.,

$$\frac{\partial(\bar{w} - \underline{w})}{\partial N} = \frac{8 - 14b + N^2b^3 - 2bN(N+2)(1-b)}{(Nb+2)^2(N+3)^2}. \quad (\text{A16})$$

Setting (A16) equal to zero and solving for  $N$ , we can show that

$$\frac{\partial(\bar{w} - \underline{w})}{\partial N} < (\geq) 0 \text{ iff } N < (\geq) \hat{N} := \frac{4b - 4b^2 - 2\sqrt{18b^4 + 12b^3 - 40b^2 + 16b}}{2(b^3 - 2b + 2b^2)}.$$

It is straightforward to show that  $\hat{N}$  is decreasing in  $b$ , which verifies part (ii). *QED.*

**Proof of Proposition 6:** To verify the result, we must show that the following is true:  $X^{PS}(w) > X^{MS}(w) + Y^{MS}(w)$ . Inserting the expressions from (9) and (15), we can write the

inequality as follows:

$$\frac{2 - w(2 + Nb)}{2(1 + N)(1 - b) - b^2N} > 0$$

Since the denominator is always positive (see Footnote 16), it remains to check the numerator. It is straightforward to show that the numerator is zero for  $w = \bar{w} := 2/(2 + Nb)$ . Since the numerator is monotonically decreasing in  $w$ , the inequality must be true for any  $w \in (\underline{w}, \bar{w})$ . This completes the proof. *QED*.

**Proof of Proposition 7:** We know that  $\Delta > 0$  from the second-order condition (see Footnote 16), so it suffice to check the sign of the numerators of (26)-(28) to verify the proposition. Starting with  $w^{MS}$ , we can set the numerator of (26) equal to zero and solve for  $\gamma$ , yielding the following critical value

$$\hat{\gamma}_w = \frac{N(14 - 7b - 7b^2 - b^3) + (1 - b)(10 + 2b + 4N^2 + N^2b)}{(4 - 4b - b^2)(2 + Nb)}. \quad (\text{A17})$$

If  $\hat{\gamma}_w \notin [0, 1]$ , the sign of the numerator is unambiguously positive. We see by inspection that (A17) is always positive. Thus, it remains to check whether  $\hat{\gamma}_w < 1$ , which is equivalent to

$$\begin{aligned} N(14 - 7b - 7b^2 - b^3) + (1 - b)(10 + 2b + 4N^2 + N^2b) &< (4 - 4b - b^2)(2 + Nb). \\ &\Downarrow \\ N(1 - b)(4N + 3b + Nb + 14) + 2 &< 0, \end{aligned}$$

which is never true.

To check the sign of  $x^{MS}$ , observe first that the only potentially negative term in the numerator of (27) is the last one, which turns negative if  $b > 2/3$ . In this case, a higher  $\gamma$  will make it even more negative. Thus, it suffice to check the numerator of (27) at the following limit

$$\begin{aligned} \lim_{\gamma \rightarrow 1} [(3 + N^2)(1 - b) + N(2 - b) + \gamma(2 - 3b)(N - 1)] \\ = N(N + 4)(1 - b) + 1 > 0. \end{aligned}$$

Thus,  $x^{MS}$  is always positive for any valid parameter values.

Finally, we see from (28) that the numerator of  $y^{MS}$  is decreasing in  $b$ . Thus, it suffice to



check the sign at the upper limit of  $b$  (see Footnote 16), i.e.,

$$\begin{aligned} \lim_{b \rightarrow \hat{b} := \frac{1}{N}(\sqrt{4N+3N^2+1}-N-1)} & [(1-b)(N^2+5N-2N\gamma+2-2\gamma) - Nb^2(1-\gamma)] \\ & = (N+3) \left( 2N - \sqrt{4N+3N^2+1} + 1 \right) > 0, \end{aligned}$$

which is always true for any valid parameter value. *QED.*

**Proof of Proposition 8:** The result can be verified by taking the partial derivatives of (26)-(30) with respect to  $N$ . Before we evaluate the expressions recall that the equilibrium condition, reported in Footnote 16, requires that:

$$b \in (0, \hat{b}), \text{ where } \hat{b} := \frac{1}{N} \left( \sqrt{4N+3N^2+1} - N - 1 \right) \in (0.73, 0.83).$$

Part (i): The partial derivative of (26) with respect to  $N$  is given by:

$$\begin{aligned} \frac{\partial w^{MS}}{\partial N} &= - \left( \frac{1-b}{\Delta^2} \right) \left[ N^4 (1-b^2) (4+b) + 2N^3 (1+b) (14-7b-7b^2-b^3) \right. \\ &\quad \left. + N^2 (86+7b-74b^2-24b^3-2b^4) + 2(1-b) (21+4b+52N+28Nb+4Nb^2) \right]. \end{aligned} \quad (\text{A18})$$

We see that there are two potentially negative terms. Evaluating these, we can show that  $(86+7b-74b^2-24b^3-2b^4) > 0$  for  $b \lesssim 0.97$  and  $(14-7b-7b^2-b^3) > 0$  for  $b \lesssim 0.96$ . Since the upper bound  $\hat{b}$  always ensures that these are true, it follows that  $\partial w^{MS}/\partial N < 0$ .

Part (ii): The partial derivatives of (27) and (28) with respect to  $N$  is given by:

$$\begin{aligned} \frac{\partial x^{MS}}{\partial N} &= \left( \frac{1-b}{\Delta^2} \right) \left[ N^3 (1-b) (N+2b+Nb) + N^2 (8+4N-5b^2) \right. \\ &\quad \left. + 2N (18-12b+5Nb-6b^2) + 3 (13-12b-6b^2) \right], \end{aligned} \quad (\text{A19})$$

$$\begin{aligned} \frac{\partial y^{MS}}{\partial N} &= - \left( \frac{1-b}{\Delta^2} \right) \left[ N^3 (1+b) ((N+10)(1-b)-2b^2) + N^2 (35-24b-20b^2-2b^3) \right. \\ &\quad \left. + 8N (2+b)(1-b) + 28b - 18 \right], \end{aligned} \quad (\text{A20})$$

respectively. It is easily verified that  $\partial^2 x^{MS}/\partial N^2 < 0$  and  $\partial^2 y^{MS}/\partial N^2 < 0$ . Thus, it suffice to

evaluate (A19) and (A20) at the lower limits, i.e.,

$$\lim_{N \rightarrow 1} \frac{\partial x^{MS}}{\partial N} = 2(44 - 24b - 19b^2) > 0,$$

$$\lim_{N \rightarrow 1} \frac{\partial y^{MS}}{\partial N} = -(44 - 4b - 41b^2 - 4b^3) < 0 \text{ if } b \lesssim 0.95.$$

Thus,  $\partial x^{MS}/\partial N > 0$  and  $\partial y^{MS}/\partial N < 0$  are true for any valid parameter values.

Part (iii): Since  $X^{MS} = Nx^{MS}$ , it follows that  $\partial X^{MS}/\partial N > 0$ . The sign of  $\partial y^{MS}/\partial N$  is, however, not clear, so we need to take the partial derivative of (29) with respect to  $N$ :

$$\begin{aligned} \frac{\partial Y^{MS}}{\partial N} = \frac{1}{\Delta^2} [ & N^4(1-b)(5-8b+4b^2+b^3) + 2N^3(1-b)(19-16b-4b^2) \\ & + N^2(121-221b+53b^2+42b^3+6b^4) + 24N(1-b)(5-5b-b^2) + 24(1-b)^2 ] \end{aligned} \quad (\text{A21})$$

It is straightforward to verify that every term in the brackets are strictly positive for any  $b \in (0, \hat{b})$ . Thus, it follows that  $\partial Y^{MS}/\partial N > 0$  must be true.

Part (iv): Since both  $X^{MS}$  and  $Y^{MS}$  are increasing in  $N$ , then  $\partial p^{MS}/\partial N < 0$  must be true. *QED.*

**Proof of Proposition 9:** The result can be verified by taking the partial derivatives of (26)-(30) with respect to  $\gamma$ , given the assumption of  $b = 0.5$ . We obtain the following expressions:

$$\frac{\partial w^{MS}}{\partial \gamma} = \frac{(16N + 20N^2 + 5N^3 + 8)(3N + 4)}{2(16\gamma - 46N + 10N\gamma - 22N^2 - 3N^3 + 2N^2\gamma - 24)^2} > 0, \quad (\text{A22})$$

$$\frac{\partial x^{MS}}{\partial \gamma} = \frac{2(16N + 20N^2 + 5N^3 + 8)(N + 3)}{(16\gamma - 46N + 10N\gamma - 22N^2 - 3N^3 + 2N^2\gamma - 24)^2} > 0, \quad (\text{A23})$$

$$\frac{\partial y^{MS}}{\partial \gamma} = -\frac{(16N + 20N^2 + 5N^3 + 8)(N + 4)}{(16\gamma - 46N + 10N\gamma - 22N^2 - 3N^3 + 2N^2\gamma - 24)^2} < 0, \quad (\text{A24})$$

$$\frac{\partial p^{MS}}{\partial \gamma} = \frac{(16N + 20N^2 + 5N^3 + 8)N}{(16\gamma - 46N + 10N\gamma - 22N^2 - 3N^3 + 2N^2\gamma - 24)^2} > 0. \quad (\text{A25})$$

Since  $X^{MS} = Nx^{MS}$  and  $Y^{MS} = Ny^{MS}$ , it follows that

$$\frac{\partial X^{MS}}{\partial \gamma} = N \frac{\partial x^{MS}}{\partial \gamma} > 0 \quad \text{and} \quad \frac{\partial Y^{MS}}{\partial \gamma} = N \frac{\partial y^{MS}}{\partial \gamma} < 0,$$

which completes the proof. *QED.*

**Proof of Proposition 10:** Comparing (31)-(32), we can show that  $W^{MS} > W^{PS}$  is equivalent to:

$$\Delta W := N^3 (1 - b)^2 + 2N^2 (1 - b) (2\gamma - b\gamma - b) - N (1 - \gamma) \Psi - 2 (1 - \gamma)^2 \Phi > 0, \quad (\text{A26})$$

where

$$\Psi := 12 - 16\gamma (1 - b) - 7b^2 - 6b + b^2\gamma,$$

$$\Phi := 2 (1 - b) (2 - 2\gamma + b) + b^2\gamma.$$

From (A26) we see that the first term is unambiguously positive. The second term is positive if  $\gamma$  is sufficiently high relative to  $b$  (i.e.,  $\gamma > b / (2 - b)$ ), which may or may not be true. The two last terms are both negative since both  $\Psi > 0$  and  $\Phi > 0$ . Thus, the inequality surely hold for a sufficiently high  $N$ , while it may be negative for a low  $N$ . Checking the limits, we see that this is indeed true:

$$\lim_{N \rightarrow \infty} \Delta W = (1 - b)^2 > 0$$

$$\lim_{N \rightarrow 1} \Delta W = 2\gamma^3 (4 - 4b - b^2) - \gamma^2 (40 - 36b - 9b^2) + 4\gamma (14 - 11b - 4b^2) + 6b + 14b^2 - 19,$$

which may be positive or negative depending on the combination of parameter values of  $b$  and  $\gamma$ .

Moreover, we see from (A26) that  $\Delta W$  is increasing in  $\gamma$ . Checking the upper and lower limit, we can show that:

$$\lim_{\gamma \rightarrow 1} \Delta W = N^2 (1 - b)^2 (N + 4) > 0,$$

$$\lim_{\gamma \rightarrow 0} \Delta W = N^3 (1 - b)^2 - 2bN^2 (1 - b) - N (12 - 6b - 7b^2) - 4 (2 + b) (1 - b),$$

where the lower limit is ambiguous. The effect of  $b$  upon  $\Delta W$  is verified by inspection of (A26), noting that  $\Delta W < 0$  at low values of  $N$  and  $\gamma$ . *QED*.

## 8 Appendix B: Derivation of demand

Let us first derive optimal consumption and, thus, demand of hospital care, assuming no binding capacity constraint in the public sector, despite public health is free of any charges. Maximising

(3) with respect to  $X$  and  $Y$  yields the following first-order conditions:

$$\frac{\partial U}{\partial X} := 1 - X - bY = 0, \quad (\text{B1})$$

$$\frac{\partial U}{\partial Y} := 1 - Y - bX - p = 0. \quad (\text{B2})$$

By solving the set of first-order conditions, we derive optimal consumption, and thus the direct demand functions, of public and private health care:

$$X^* = \frac{1 - b + bp}{1 - b^2}, \quad (\text{B3})$$

$$Y^* = \frac{1 - b - p}{1 - b^2}, \quad (\text{B4})$$

respectively. We see that  $X^* > Y^*$  for any  $p > 0$ . Thus, most consumers prefer public health care simply because of the extra cost associated with the private (out-of-plan) treatment. In the extreme case of public and private health care being perfect substitutes, i.e.,  $b \rightarrow 1$ , then everybody prefer to be treated at a NHS-hospital.

In the paper, we assume public rationing, i.e.,  $X^{MS} < X^*$ . However, since we endogenise the capacity level of the public sector, we need to check whether this is actually taking place in equilibrium. Inserting (16) into (B3), we obtain the following condition to be satisfied:

$$X^{MS} < X^* \quad \Leftrightarrow \quad \frac{wN(3 - 2b + N - Nb^2) - (1 - b)(4N + Nb + 2)}{(1 - b^2)(2(1 + N)(1 - b) - b^2N)} < 0 \quad (\text{B5})$$

The denominator is strictly positive by the equilibrium condition reported in Footnote 16. Thus, it suffice to check the sign of the numerator. The last term of the numerator is negative. However, the first term is positive and increasing in  $w$ . Thus, it suffice to evaluate the numerator at the upper wage bound:

$$\begin{aligned} & \lim_{w \rightarrow \bar{w} := \frac{2}{2+bN}} [wN(3 - 2b + N - Nb^2) - (1 - b)(4N + Nb + 2)] \\ & = -\frac{(Nb - N + 2)[2(1 + N)(1 - b) - b^2N]}{(Nb + 2)} < 0. \end{aligned}$$

Thus, public rationing always takes place in equilibrium, i.e.,  $X^{MS} < X^*$  for any  $w \in (\underline{w}, \bar{w})$ .

## References

- [1] Besley T, Gouveia M. Alternative systems of health care provision. *Economic Policy* 1994; **19**: 200-258.
- [2] Propper C. The demand for private health care in the UK. *Journal of Health Economics* 2000; **19**: 855-876.
- [3] UK Monopolies and Mergers Commission. *Private Medical Services: A report on the agreements and practices relating to charges for the supply of private medical services by NHS consultants*. Cm 2452. HMSO: London, 1994.
- [4] Johnson N. *Private Markets in Health and Welfare: An International Perspective*. Berg Publishers Ltd.: Oxford, 1995.
- [5] Gonzalez P. Should physicians' dual practice be limited? An incentive approach. *Health Economics* 2004; **13**: 505-524.
- [6] McAviney ID, Yannopoulos A. A cost function approach to the choice between private and public acute health care. *Scottish Journal of Political Economy* 1994; **41**: 194-211.
- [7] Iversen T. The effect of a private sector on the waiting time in a national health service. *Journal of Health Economics* 1997; **16**: 381-396.
- [8] Hoel M, Sæther EM. Public health care with waiting time: the role of supplementary private health care. *Journal of Health Economics* 2003; **22**: 599-616.
- [9] Olivella P. Shifting public-health-sector waiting lists to the private sector. *European Journal of Political Economy* 2002; **19**: 103-132.
- [10] Barros PP, Martinez-Giralt X. Public and private provision of health care. *Journal of Economics & Management Strategy* 2002; **11**: 109-33.
- [11] Jofre-Bonet M. Health care: private and/or public provision. *European Journal of Political Economy* 2000; **16**: 469-489.
- [12] Marchand M, Schroyen F. Can mixed health care systems be desirable on equity grounds? *Scandinavian Journal of Economics* 2005; **107**: 1-23.

- [13] Rickman N, McGuire A. Regulating providers' reimbursement in a mixed market for health care. *Scottish Journal of Political Economy* 1999; **46**: 53-71.
- [14] Ellis RP, McGuire TG. Provider behaviour under prospective reimbursement: Cost sharing and supply. *Journal of Health Economics* 1986; **5**: 129-152.
- [15] Barros PP, Olivella P. Waiting lists and patient selection. *Journal of Economics & Management Strategy* 2005; **14**: 623-646.
- [16] Gonzalez P. On a policy of transferring public patients to private practice. *Health Economics* 2005; **14**: 513-527.
- [17] Biglaiser G, Ma C-tA. Moonlighting: Public service and private practice. Mimeo, 2003.
- [18] Brekke KR. Competition or coordination in hospital markets with unionised labour. *International Journal of Health Care Finance and Economics* 2004; **4**: 65-89.
- [19] Calem PS, Dor A, Rizzo JA. The welfare effects of mergers in the hospital industry. *Journal of Economics and Business* 1999; **51**: 197-213.
- [20] Kesteloot K, Voet N. Incentives for cooperation in quality improvement among hospitals – The impact of the reimbursement system. *Journal of Health Economics* 1998; **17**: 701-728.
- [21] Siciliani L. Does more choice reduce waiting times? *Health Economics* 2005; **14**: 17-23.
- [22] Anderson SP, de Palma A, Thisse J-F. *Discrete Choice Theory of Product Differentiation*. MIT Press: Cambridge, Massachusetts, 1992.
- [23] Cullis J, Jones P, Propper, C. Waiting lists and medical treatment: Analysis and policies. In Culyer A, Newhouse J (eds). *Handbook of Health Economics*, Chapter 17, Amsterdam, Elsevier: 1201-1249, 2000.
- [24] Tirole J. *The Theory of Industrial Organization*. MIT Press: Cambridge, Massachusetts, 1988.

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