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## Fiscal sustainability: Must the problem be diminished before we can see it?

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*Erling Holmøy*

## **Fiscal sustainability: Must the problem be diminished before we can see it?**

**Abstract:**

Assessments of fiscal sustainability (FS) problems should be based on present values of government revenues and expenditures over an infinite horizon. The paper shows that realistic assumptions imply that the growth rate of government expenditure components may exceed both the steady state growth rate of the economy and the relevant discount rate, which makes the FS problem immeasurably large. The common practice of ad hoc exogenous alignment of government expenditures to the steady state growth path after some distant year may significantly diminish the FS problem, since the effective discounting is likely to remain low. Low effective discounting also makes the FS assessment highly non-robust, reducing its political relevance. It suggests that the fiscal sustainability should be improved by reducing the growth rates of government expenditures, a strategy followed in e.g. the Swedish pension reform.

**Keywords:** Fiscal sustainability, long run projections, discounting

**JEL classification:** H30, H55, H62

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# 1. Introduction

A vast literature, based on long run projections of government expenditures and tax bases, documents that most industrialized countries face severe problems financing their welfare schemes as their populations will age profoundly after 2010.<sup>1</sup> Kotlikoff (2001) clarifies that assessments of the fiscal sustainability (FS) problem should be based on comparisons of the present values of an exhaustive set of government primary revenues and expenditures entering the intertemporal government budget defined over an infinite horizon. This method makes it impossible to camouflage the true financial position of governments by creative fiscal language. It gives a key role to the component specific growth adjusted discount rates, i.e. the differences between the interest rate on government debt and the growth rates of the various components in the primary budget deficit.

Following this approach, this paper claims that an affirmative answer should be given to the question posed in the title for several economies: To be measurable in terms of present values, the FS problem must be diminished in several countries compared to the most plausible projections of the primary deficits. The following logic implications support this claim:

- 1) Two fundamental requirements must be met in order to compute the present value of each primary budget component over an infinite horizon:
  - a. The long run projections of government expenditure and revenue components must be consistent with steady state convergence of the economy, since alternative paths eventually will violate the existence conditions for equilibrium.
  - b. Existence of the present value integrals requires that the relevant discount rate exceeds the growth rate of each and every primary budget component.
- 2) Prolongation of the present policy, combined with realistic projections, imply that most expenditure components will grow at rate exceeding the steady state growth rate of the economy, i.e. violate 1a. Nor can it be ruled out that the most plausible growth rates of at least some important expenditure components exceed the relevant discount rate, i.e. violate 1b. In both cases the FS problem is immeasurably large.

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<sup>1</sup> See Chauveau and Loufir (1995), OECD (1998, 2000, 2001), the European Commission (2001), McMorrow and Roeger (2002) and Visco (2002) for international comparisons of quantitative assessments of the fiscal and macroeconomic consequences of ageing. Kotlikoff, Smetters and Walliser (2001), Kotlikoff (2001) and Feldstein (2005) estimate the fiscal gap in the US, and Kotlikoff and Burns (2004) makes the gloomy US fiscal perspectives visible for a wider public. Beetsma, Bettendorf and Broer (2003) and The Danish Welfare Commission (2004) estimate the need for raising the tax burden in the Netherlands and Denmark, respectively. Andersen, Jensen and Pedersen (2004) provide a review in English of the Danish Welfare Commission (2004). Heide, Holmøy, Solli and Strøm (2006) finds that the future fiscal stance in Norway looks strikingly gloomy, especially when compared to the impressive present fiscal stance.

- 3) The practice of *ad hoc* reductions after some distant year of the growth in expenditure components in order to meet conditions 1a and 1b, violates the basic assumption of prolongation of the present policy. It will also diminish the FS problem, possibly from an immeasurable order, which we want to assess as soberly as possible. The empirical importance for the FS assessment is likely to be large, since the growth adjusted discount rates are small. To my knowledge, these problems have not been given due attention in the FS literature.

The policy implications of low effective discounting are discouraging. FS assessments based on low growth adjusted discount rates will necessarily be highly non-robust, even to what happens in the unknown remote future. Policy makers cannot be blamed for ignoring results that can take on almost any number by slight changes in assumptions. Specifically, marginal policy adjustments affecting relevant growth rates will be sufficient to restore FS.<sup>2</sup>

Two possible objections should be rejected already at this stage. First, the argument above does not criticize the *method* of FS assessments based on present values as such. Rather, it implies that such assessments cannot be made when the lack of FS is "too large", in the sense that the growth adjusted discount rate is negative for at least one expenditure component. In such cases the policy response must make the *growth rate* of expenditures sustainable – any permanent level reduction will be insufficient. Second, the argument above should not be confused with dynamic inefficiency, which characterizes an economy growing at a higher rate than the interest rate. This is typically never seen in realistic long-term projections. However, dynamic efficiency of the aggregate economy does not exclude the possibility that the growth rate of specific government expenditures exceed the interest rate. For example, this may well be the case for government pension expenditures, given the present system, as a result of ageing.

It follows from the argument above that the main job for this paper is to argue convincingly that realistic assumptions imply that the growth adjusted discount rates of at least some government expenditure components most likely exceed the steady state growth rate of the economy. This is done for the Norwegian economy by a combined use of a disaggregated dynamic Computable General Equilibrium (CGE) model and special models of government expenditures. Such a combined use of models takes into account the maximum available information about the forces driving the growth in budget components. Although some of the determinants of the growth adjusted discount rates are

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<sup>2</sup> Negligible discount rates makes it easy to construct paradoxical trade-offs between policy actions now and far ahead, as pointed out in Nordhaus' (2006) critical comments to the Stern Review on climate changes.

country specific, the credo of this paper is that the detailed projections for Norway will demonstrate that the more fundamental problems pointed out above have general relevance for FS assessments.

This paper also introduces a methodological novelty, which may have a dramatic negative effect on the relevant growth adjusted discount rates. In the literature the discounting of government budget components have been based on the *pre-tax* interest rate. However, this paper shows that one should use the *after-tax* interest rate, and one should correct the tax revenue collected from interest income, when the economy as a whole faces an intertemporal constraint on its external debt accumulation. The paper shows that most growth adjusted discount rates will be negative if they are based on the after-tax interest rate.

It should be noted that there are other methods of FS assessments than the one adopted in this paper, which do not rely on calculations of present values over an infinite horizon. Cronin and McCoy (2000) claim the so-called OECD-method to be the conventional FS indicator. Drawing on Blanchard, Chouraqui, Hagemann and Sartor (1990), this method considers the conditions for stabilizing the ratio between government debt and GDP within a limited time horizon.<sup>3</sup> The rationale for considering this ratio must be a positive relationship between GDP and the primary budget surplus, which can be motivated by the positive correlation between GDP and the tax bases. The OECD-method requires neither complex modelling nor much data, and it is therefore easy to employ in cross-country comparisons. But simplicity has a price: The method ignores behavioural and interaction effects on government revenues and expenditures, as well as budget effects of population ageing. Most importantly in relation to the present paper, the OECD-method is by construction not able to capture disproportionate long run growth in the different budget components. Moreover, the implicit autonomous proportionality between GDP and the primary budget surplus can be questioned. For example Andersen and Pedersen (2006a,b) and Holmøy (2006) demonstrate that productivity growth in the private sector may have an adverse effect on FS.

Compared to the OECD-method, generational accounting takes FS assessments important steps further by including the fiscal effect of changes the age structure of the population, and the method bases the FS assessment on present value comparisons of the primary budget deficit calculated over an infinite time horizon.<sup>4</sup> Thus, the problems related to low effective discounting discussed in this paper are equally relevant to generational accounting as to assessments based on CGE models. However, the

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<sup>3</sup> See e.g. Roux (1993), Hemming and Miranda (1991), Cronin and McCoy (2000) and Goldfajn (2002).

<sup>4</sup> Bentz and Fetzer (2004) compares the OECD-method with Generational Accounting.

traditional generational accounting suffers from the following shortcomings: Age profiles of taxes, cash transfers and government spending on individual services are assumed to stay constant, and age- and gender specific values of these variables are prolonged quite mechanically, typically by the exogenous labour productivity growth. Moreover, generational accounting ignores endogenous behavioural and price effects on tax bases and expenditures. In principle, properly designed CGE models are able to account for all the effects ignored in generational accounting, as well as other potentially important effects for the FS assessment. On the other hand, sufficiently detailed CGE models are more costly to develop, it is hard for outsiders to check the results, and complexity in terms of behavioural responses and interaction effects may in practice be obtained at the cost of less accurate description of relevant population heterogeneity, tax rules and welfare schemes. However, the latter criticism cannot be raised against this paper due the disaggregated approach based on the combined use of several models.

The paper is organised as follows. Section 2 provides the empirical basis for the claim that realistic growth adjusted long run discount rates are low, maybe even negative. Section 3 demonstrates the non-robustness of FS assessments based on low effective discounting. Section 4 explains why the practice of exogenous steady state convergence of long run growth paths is likely to diminish the FS problem compared to the most realistic assessment. Section 5 concludes.

## 2. Realistic growth adjusted discount rates

### 2.1. Analytical framework

The government's intertemporal budget constraint is the conceptual point of departure for FS assessment. It requires a comparison of the present value of government expenditures and revenues calculated over an infinite horizon. To make a budget constraint defined over an infinite horizon operational, the present value of any positive or negative specific primary budget component,  $X_j$ , is decomposed as follows:

$$(1) \quad X_{pj0} \equiv \sum_{t=0}^{\infty} (1+i)^{-t} X_{jt} = \sum_{t=0}^{t'-1} (1+i)^{-t} X_{jt} + \frac{X_{jt'}(1+i)^{1-t'}}{i-g_j},$$

where  $X_{pj0}$  is the present value of  $X_j$ ,  $i$  is the nominal interest rate (assumed constant for expositional simplicity).  $t=t'$  is chosen so that the constant growth rate of  $X_j$  equals the constant  $g_j$  for  $t > t'$ .  $i - g_j$  is the (stationary) growth adjusted discount rate of  $X_j$ . Typically, the values for  $X_{0j}, \dots, X_{t'j}$  and  $g_j$  are found

from a model simulation. In this paper these variables are estimated by simulating a CGE model in tandem with other models through 2050. By then the relevant *individual* growth rates have become constant. I define the *fiscal gap*,  $F_{G0}$  as

$$(2) \quad F_{G0} = \sum_j X_{pj0} - B_{G0},$$

where  $B_{G0}$  is government financial wealth at the beginning of a year 0. The sum in (2) is referred to as the *required financial wealth* in Table 3.

## 2.2 The relevant discount rate

### *The risk free interest rate*

The interest rate is set to 5.5 percent. This should be interpreted as a nominal risk free interest rate. Specifying a real interest rate may be misleading as long as a unique deflator is not defined. In FS assessments it is a major point that the growth rates of the price components of various tax bases, cash transfers and government consumption differ. Of course, the FS assessment is invariant to deflating the interest rate and all prices, the wage rate and specific tax rates by an arbitrary uniform inflation rate. The 5.5 percent nominal interest rate is well in line with the "natural" Norwegian money market interest rate, estimated to 5.25 percent in Bernardsen and Gerdrup (2006). Assuming a 1.5 percent growth in world prices measured in NOK, and an optimum mix of assets in the Government pension, it is also consistent with the statutory estimate of 4.0 percent real rate of return (in terms of international purchasing power) on this fund, which underlies the fiscal policy rule adopted in Norway in 2001 (explained below). Compared to observed long run nominal interest rates on bonds the estimate of 5.5 percent is biased upwards. In December 2006 the nominal interest rate on 30-years bonds were slightly above 4 percent in the US and in Germany. The interest rate on Norwegian state 10 years bonds were 4.2 percent.

A risk free interest rate is chosen although the government pension fund invests 60 percent of its portfolio in shares in international companies and 40 percent in foreign bonds. Including the risk premium in the interest rate on government assets would not affect the FS assessment, if the cost associated with risk were added also on the expenditure side, as it should be.

The FS assessment for Denmark by Jensen, Nødgaard, Pedersen (2001) also assumes a 5.5 percent interest rate. Andersen and Pedersen (2006) set the nominal interest rate to 5.0 percent, but the return on equity is 8 percent. However, the return on equity has not been corrected for risk premium. Gokhale and Smetters (NBER 11060) set the real interest rate equals 3.65 percent. Their estimate on



the growth rate of government real expenditures is 2.8 percent, which implies an effective discount rate of aggregate expenditures as low as 0.85. The US Social Security Administration (2003) assumes a real interest rate of 3.1 percent, which is close to the rate used in the present paper for reasonable values of the deflator. An outlier in this context is the assumption of a 5.0 percent real interest rate in Beetsma, Bettendorf and Broer (2003).

*Should taxes be deducted in the discount rate?*

To my knowledge FS assessments base the present value calculations on the pre-tax interest rate. Compared to the after-tax interest rate this choice is not at all innocent when the growth adjusted discount rate  $0 < i - g_x$  is small. Tax rates levied on interest vary around 30 percent in most OECD countries. Given a nominal pre-tax interest rate of 5.5 percent, the *after-tax* interest rate is about 3.9 percent. The most important component in the growth rates of both government expenditures and revenues is the wage rate. As will be demonstrated below, plausible estimate on future nominal wage growth would be about 4.0 percent. Thus, even if wage growth were the only reason to growth in primary budget components, the growth adjusted discount rate would be negative. However, the negative margin would be larger because of the expenditure effect of ageing, standard improvements in old-age care and maturing public pension schemes.

It is not obvious whether the discount rate should be the pre- or the after-tax interest rate. A formal discussion is relegated to the appendix. The conclusions there can be summarized as follows:

1. In a closed economy government financial wealth must necessarily be equal to the financial debt held by the private sector (households and the corporate sector). The relevant discount rate is then the after-tax interest rate. Moreover, taxes collected from private interest income should be subtracted from the primary deficit. The intuition is that a primary deficit implies an equally large financial investment by the private sector. Consequently, the effective interest payments from the government are net of the tax on private interest income.
2. In an open economy the situation is less obvious since the government can finance a primary deficit by issuing debt to the foreign sector. To the extent that this is done without any effect on private debt, discounting of primary deficits should be based on the pre-tax interest rate.
3. If the open economy is subject to an intertemporal national budget constraint on net foreign debt, there exists an intertemporal constraint on the sum of government and private net debt. The relevant discount rate is then the after-tax interest rate. On the other hand, government revenues should include tax revenues imputed on the *national* interest income.

Since the interest paid by the private sector in Norway is relatively small compared to interest income of the government, the calculations in this paper follow the standard practice of using the pre-tax interest rate when discounting government budget components. However, since this choice is crucial, the practice should be examined more thoroughly.

### 2.3. Projecting government real expenditures<sup>5</sup>

#### *Demography*

The projections are based on the "middle alternative" in the official demographic projections from Statistics Norway (2002), which results from the most likely assumptions on deaths, births, migration etc.<sup>6</sup> Longevity is assumed to increase by about 7 years over this period. The ratio of those of working age 20-66 to those 67 and older decreases from 4.5 in 2002 to 2.5 in 2050. The demographic assumptions enter the detailed *dynamic micro simulation model*, MOSART, which simulates the life courses of a cross-section of the Norwegian population.<sup>7</sup> Specifically, MOSART captures transitions in and out of the labour market. The projected growth in the labour force is 11.9 percent from 2004 to 2050, whereas the ratio between the total number of pensioners and the labour force grows from 40 to 65 percent over this period, cf. Table 1. Maintaining the age specific transition rates from work to disability as observed in 2000, implies a 32 percent increase in the already large number of disability pensioners from 2004 to 2050.

**Table 1. Projected development in the number of pensioners, average annual benefits *ex ante* indexation and the labour force in the reference scenario. Thousand persons and thousand NOK, current prices**

	2004	2020	2050
Old age pensioners	639	873	1317
Average annual benefit	116	140	146
Disability pensioners	308	388	407
Average annual benefit	120	124	122
Widow pensioners	24	17	10
Total number of pensioners	971	1278	1734
Labour Force	2404	2564	2669
Pensioners in percent of labour force	40	50	65

<sup>5</sup> The projections of the government budget components are the same as those used in Heide, Holmøy, Solli and Strøm (2006).

<sup>6</sup> The most recent demographic projections, see Statistics Norway (2005) are not significantly different from the projections from 2002 with respect to the basic patterns of the age structure of the population.

<sup>7</sup> Fredriksen (1998) provides a detailed documentation of MOSART and examples of applications. The government uses the model regularly to compute individual pension benefits and total government pension expenditures.

### *Social security expenditures*

Social security transfers amounted to 17.9 percent of GDP in 2004. By including an accurate description of the pension rules, as well as a rich description of relevant population heterogeneity, MOSART is especially designed to compute individual pension entitlements, given individual income histories. Maintaining the present public pension system and the political intention of wage indexation of entitlements and benefits, government old-age expenditure is then projected to grow by 160 percent from 2004 to 2050, *ex ante* wage indexation of benefits. 26 percentage points of this increase can be attributed to maturing of the public pension system as more pensioners become entitled to supplementary benefits. An important force behind the remaining expenditure growth is the non-actuarial properties of the present pension system. The annual individual old-age benefits are independent of the number of years as a pensioner, and thereby of the expected increase in longevity. Given the age specific transition rates, ageing also entails a nearly 50 percent increase in government disability pension expenditures *ex ante* wage indexation of the benefits from 2004 to 2050.

### *Government consumption*

Government consumption amounted to 22.0 percent of GDP in 2004. Government consumption is divided into 1) *Defence*, 2) *Education*, 3) *Health and Social Care*, 4) *Administration etc.* The greatest expenditures are allocated to the provision of individual services within *Education* and *Health and Social Care*. Spending within each service category is decomposed into i) the price of the resources, ii) service standards measured by man-hours and other resources per user, iii) productivity growth, iv) the number of users, or the demand intensity, within different age groups, and v) the number of individuals in different age groups. Prolongation of present policy is interpreted as maintaining the service standards observed in 2004 within *Education* and *Health and Social Care*, as well as the 2004-levels of labour input within *Defence* and *Administration*. Moreover, the 2004-proportions of capital and intermediaries to labour remain constant in government sectors. Labour productivity grows by 0.5 percent annually in all government sectors, in accordance with the national accounting practice. Under these assumptions ageing makes it necessary to expand the employment in the *Health and social care* sector by 1-2 percent annually after 2015. In comparison total government employment grows annually by 0.6 percent from 2004 to 2025 and by 0.9 percent from 2026 to 2050.

## **2.4. Projecting tax bases and prices of government spending**

The large scale dynamic CGE model MSG6<sup>8</sup> captures complex endogenous mechanisms in the determination of a relatively detailed classification of all tax bases and prices of resources used for

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<sup>8</sup> MSG6 is described in Heide, Holmøy, Lerskau and Solli (2004).

government consumption. The detailed calculations of government use of resources and social security expenditures *ex ante* indexation enter MSG6 as exogenous inputs. Iterative model simulations ensure approximate consistency between the MOSART assumptions on individual earnings and average real income growth simulated by MSG6.

Prolongation of the present policy implies that all *ad valorem* tax rates are kept constant. Specific tax rates are kept constant in real terms with one exception: Until 2050 the payroll tax rate adjusts annually to meet the government budget constraint implied by the fiscal policy rule adopted since 2001. After 2050 the rule is ignored: The payroll tax rate is kept constant to its 2050-level, and the fiscal gap is determined residually. The fiscal policy rule restricts the average annual use of the petroleum wealth, measured by the primary budget deficit net of the petroleum cash flow accruing to the government sector, to the expected real return of the assets accumulated in the government petroleum fund. So far the real return has been set to 4 percent.<sup>9</sup>

MSG6 assumes the Norwegian economy to be too small to affect world prices and the international interest rate. The exchange rate is fixed. Consumers and producers are rational and equipped with model consistent expectations, and all agents have access to international capital markets. The economy as a whole obeys an intertemporal budget constraint on the accumulation of foreign debt. Goods and factors are perfectly mobile between industries. Supply equals demand in all markets in all periods. The representative consumer decides on labour supply and the composition of private consumption. Preference parameters are calibrated so that the uncompensated wage elasticity of total labour supply equals 0.1.<sup>10</sup> Most imported products are close but imperfect substitutes for the corresponding domestic products. Producers allocate output between the domestic and the foreign market. It is costly to redirect output between these two markets, and the production functions exhibit decreasing returns to scale.<sup>11</sup> Norwegian firms are price takers in the export markets and in the factor markets, but engage in monopolistic competition in most domestic markets.

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<sup>9</sup> The strict interpretation of the fiscal policy rule can be explained more precisely by decomposing the accumulation of government financial assets,  $B$ :  $B_t - B_{t-1} = iB_{t-1} + P_t - D_t$ , where  $i$  is the international nominal interest rate,  $P$  is the cash flow from the petroleum sector to the government, and  $D$  is the non-petroleum primary fiscal deficit. Let  $\pi$  denote international inflation. According to the rule  $D = (i - \pi)B_{t-1}$ , which represents a constraint on  $D$ . Contingent on  $P$ , the annual growth in  $B$  depends purely on exogenous variables:  $B_t - B_{t-1} = \pi B_{t-1} + P_t$ . As  $P$  declines, the annual consumption of the petroleum wealth converges to the permanent income associated with this wealth after conversion to financial assets.

<sup>10</sup> This choice is consistent with the results in Aaberge, Dagsvik and Strøm (1995).

<sup>11</sup> The scale elasticities range from 0.85 - 1.00. Evidence of decreasing returns to scale at the firm level is presented in Klette (1999).

Table 1 summarizes the growth picture in our baseline scenario. The GDP growth is basically driven by exogenous growth in total factor productivity (TFP) of 1.3 percent in private industries, which is in line with historical trends. Private consumption can grow faster than GDP, mainly because of relatively slow growth in government consumption. Petroleum revenues amounted to 41 percent of government revenues in 2005. The price and production forecasts of oil and gas are the same as in Ministry of Finance (2004). Compared to petroleum prices observed in 2006, the assumption of a real oil price of 25 dollars per barrel is low. However, the basic conclusions in this paper are robust to large changes in the oil and gas prices. Although the wage rate determination is part of the large simultaneous model structure, one may still say that the wage rate adjustment is the basic mechanism that ensures that the economy meets the intertemporal external balance constraint. The nominal growth in unit labour cost is 4.1 percent. This slightly exceeds the prediction of the Scandinavian Model of Inflation, i.e. the world price growth rate plus the growth rate of labour productivity, due to the petroleum wealth.<sup>12</sup>

**Table 2. Macroeconomic development in the baseline scenario. Average annual growth rates. Percent**

	2004-2025	2026-2050
GDP	2.0	1.6
Employment	0.3	0.1
Labour cost per hour	4.1	4.1
Consumer real wage rate	2.5	1.7
Net foreign wealth relative to GDP	6.0	0.2
Private consumption	3.0	2.1
Government consumption	0.7	1.1

## 2.5. Fiscal gap

Table 3 shows the growth rates of the main government budget components at the end of the simulation period, as well as the simulated nominal values in 2050. The classification of components in the table is aggregated compared to the model classification, and aggregate growth rates may vary over time when detailed items grow at constant but different rates. The component specific growth adjusted discount rates are obtained by subtracting the reported long run growth rates from the 5.5 percent nominal interest rate. Most of the resulting effective discount rates are close to 1 percent. The exception is the petroleum revenues, which are gradually depleted.

<sup>12</sup> See Holmøy and Heide (2005) for an analysis of how decreasing returns to scale in private industries affects the equilibrium growth in unit labour cost compared to the Scandinavian Model of Inflation.

**Table 3. Main components of government revenues and expenditures based on present tax rates in 2050. Nominal interest rate = 0.055.**

	Nominal values in terms of nominal GDP in 2050	Long run growth rates ( $g_j$ )	Growth adjusted discount rate ( $0.055 - g_j$ )	Nominal present values in terms of nominal GDP in 2050
Non-interest revenues, of which	0.526	0.0448	0.0102	50.501
Petroleum revenues	0.013	-0.0275	0.0825	0.152
Indirect taxes	0.127	0.0353	0.0197	6.435
Direct taxes, excl. petroleum revenues	0.386	0.0464	0.0086	44.955
Non-interest expenditures, of which	0.612	0.0447	0.0103	59.639
Pensions and transfers to households	0.308	0.0437	0.0113	27.266
Government consumption	0.256	0.0464	0.0086	29.887
Miscellaneous	0.048	0.0357	0.0193	2.486
Required financial wealth				8.109
Accumulated financial wealth				1.015
Fiscal gap				7.082

*Interpreted literally*, Table 3 shows that the present policy is very far from being sustainable, although this policy would have generated a government financial net wealth of 1.015 nominal GDP by the end of 2050 in the base line projection. The required financial wealth in 2050 would have to be as much as about eight times higher in order to finance the primary deficits after 2050. In this perspective the Norwegian petroleum wealth is not large: In 2050 the accumulated assets in the Government petroleum fund plus the present value of the remaining petroleum revenues amount to slightly less than 2 percent of the present value of government consumption and pension expenditures.

### 3. The non-robustness of fiscal sustainability assessments

Although the fiscal gap estimate reported in Table 3 is derived from sound principles and the maximum available relevant information, it is not sufficiently robust to be used face value in a final FS assessment of maintaining the present policy. The reason is the low growth adjusted discount rates, lying in the vicinity of 0.01 for the most important budget components.  $i - g_j = 0.01$  implies that the present value of  $X$  over an infinite horizon is  $100X$ . The sensitivity can be illustrated by some striking examples. Raising the growth rate of non-interest government revenues from 0.0448 to 0.050 would, *cet. par*, increase the present value of the revenues after 2050 by 108.2 percent (from 410 409 to 854 520 billions NOK). The fiscal gap in terms of nominal GDP in 2050 would turn from a deficit of 7.1 to a surplus of 105.1. As another example, assume that the reported growth rates are approximations based on 4 decimals, and that the "true" stationary growth rates of non-interest revenues and expenditures are, respectively, 0.04484 (instead of 0.04480) and 0.04465 (instead of 0.04470). Then the fiscal gap in 2050 is reduced by 14 billions NOK.

The growth in the most important expenditure components, such as public pensions and care for the elderly, is determined by demographic trends and the welfare schemes. Thus,  $i < g_j$  is not a theoretical curiosity, even if the economy is dynamically efficient. As pointed out above, most growth adjusted discount rates would be negative if the discounting were based on the after-tax interest rate. The nominal pre-tax interest rate of 0.055 and the Norwegian tax rate on interest income of 0.28 implies an after-tax interest equal to 0.0396, which is lower than the growth rates of most nominal primary budget components.

The small growth adjusted discount rate implies large effects of reducing the interest rate. As an illustration, consider the effects of reducing it from 5.5 to 5.0 percent. This reduces the government financial wealth accumulated by the end of 2050 by 6.8 percent compared to the reference path. However, a much stronger deterioration of the fiscal stance comes from lower discounting of future budget deficits. Whereas the present value computed in 2050 of non-interest revenues increases by 94 percent, the corresponding increase in non-interest expenditures is 121.4 percent. Most of the latter increase is due to a huge increase in the present value of the government wage bill, which grows at a stationary rate of 4.71 percent. Reducing the interest rate by 0.5 points raises the present value of 1 (dollar) over an infinite horizon from 126.6 to 344.8, i.e. by 172 percent! Compared to the effect of weaker discounting, the positive equilibrium effects on the tax bases are almost negligible.

As a less technical example of non-robustness, consider using the average growth rate for government consumption expenditures in the computation of the corresponding present value, rather than adding the present values of the different components of government consumption. In 2050 the aggregate government consumption expenditure has reached a growth rate of 4.27 percent. In terms of nominal GDP in 2050 the corresponding present value in 2050 becomes 20.8 (169 244 billions NOK), which is 9.1 (73 638 billions) less than the "correct" estimate! This aggregation error is mainly a consequence of the government wage bill – by far the largest government consumption component - growing significantly faster in the long run than other components (4.99 percent). The true *average* growth rate is therefore not stationary in 2050 and onwards, but increases as the weight of wage costs grows.

#### **4. Steady state convergence diminishes the FS problem**

The previous section demonstrates that reliable FS estimates require very detailed models of government revenues and expenditures, since low effective discount rates make it important to account for differences in the long run growth rates of budget components. Even the quite detailed classification in our model framework can be criticised for being too crude. The most important oversimplification is probably the description of man-hours as homogenous with a common wage rate. FS assessments for Norway are particularly sensitive to aggregation errors due to the importance of the non-renewable petroleum resources. The projected petroleum revenues *fall* at an annual rate of 2.75 percent after 2050, whereas pensions and other transfers to households grow by 4.34 percent per year. Non-uniform growth is also the case for cash transfers to households, since some of these are indexed to the consumer price index rather than the wage rate. For the same reason the revenue from indirect taxes grows significantly slower than the revenue from direct taxes.

Thus, the practice of imposing assumptions which imply convergence towards a steady state growth path, defined as stationary relative prices and a uniform growth rate of all real variables, would overrule aspects of reality and yield highly misleading results. The conditions for steady state convergence are highly unrealistic, including e.g. homothetic preferences, uniform labour augmenting productivity growth in all sectors and uniform growth in all world prices. Specifically, the growth trends of government consumption and welfare transfers results from policy, and steady state convergence would violate the fundamental premise of FS assessments that the present policy is maintained. For example, enforcing steady state convergence from above of public pension expenditures implicitly presupposes cost saving pension reforms or a more favourable demographic development. However, growth paths without steady state convergence cannot be sustained over an



infinite horizon, since the fastest growing sector eventually absorb the whole economy. Such paths have entered the unrealistic domain long before they violate mathematical existence conditions.

The practical solution is to impose *ad hoc* steady state convergence of the economy, including the growth in government expenditure components, after a “sufficiently distant” year. With “normal” discounting such a practice typically has a minor quantitative effect on the growth path in the nearest decades and the FS assessment. But this is not the case when the effective discount rates are as low as about 1 percent. Both the assumptions determining the steady state growth rate, as well as the timing of the exogenous steady state convergence may have significant impact on present values of budget components. In economies facing problems of fiscal sustainability if the present policy is prolonged, several government expenditure components are likely to grow faster than the plausible steady state growth of the total economy. In this case the exogenous adjustments of welfare schemes and demographic trends that ensure general steady state convergence will reduce the present value of the government primary deficit. The improvement of the fiscal stance will be greater the earlier in the projection these adjustments start. However, without steady state convergence the lack of FS is immeasurably large. This justifies the title of this paper: In order to be measured the FS problem must be diminished.

## **5. Conclusions**

The correct method of assessing the FS of the present fiscal policy is to compare present values over an infinite horizon of an exhaustive set of primary government budget component. The method requires 1) steady state convergence of the economy, including all government primary budget components, and 2) the resulting unique steady state growth adjusted discount rate must be positive. Taking the rich welfare state Norway as an example, this paper has demonstrated that the most realistic projections of the growth in government expenditures may well be more rapid than the growth of the economy as a result of population ageing and generous welfare schemes. Even if the economy is dynamically efficient, it is not unlikely that the long run growth rates of important expenditure components also exceed the discount rate used to calculate the present values of the budget components. Such a situation is much more likely when the economy as a whole faces a constraint on the accumulation of foreign debt, since the relevant discount rate then is the after-tax interest rate. When the long run growth rate in at least one expenditure component exceeds the steady state growth rate of the rest of the economy and/or the relevant discount rate, the lack of FS is immeasurably large.

*Ad hoc* exogenous enforcement of steady state convergence of all government expenditures and other primary budget components from an arbitrary distant year diminishes the FS problem to be within a measurable range. This common practice violates the basic FS assumption of maintaining the present policy. With “normal” discount rates, such an overruling of the most plausible expenditure projections can be justified, because it has a negligible impact on present values as long as it sets in after a sufficiently distant year. But low growth adjusted discount rates implies that both the timing of the steady state convergence, as well as the determinants of the steady state growth rate, will have a large empirical effect on the FS assessment.

It will not be fair to blame policy makers for ignoring FS estimates based on small growth adjusted discount rates of government expenditures, since they will be highly non-robust to exogenous assumptions and events in a very remote unknown future. Apparently "innocent" changes in model design, aggregation and computational errors will be greatly magnified. These deceptive conclusions do not imply any kind of criticism of the present value based method of FS assessment. The conclusions are important just because this method is the correct one. Low long run growth adjusted interest rates are not likely to be a peculiarity of Norway, although the figures reported in this paper probably are most representative for countries with a large labour intensive government production sectors and non-actuarial public pension systems with wage indexation of benefits.

A constructive way of looking at the problem of immeasurable lack of FS or highly non-robust FS estimates is to pay more attention to policy reforms which affect the growth rates of government expenditures, rather than shifts in expenditure levels. FS estimates have typically been translated into the once-and-for-all increase (decrease) in the *level* of one or several tax rates (cash transfers or spending components). Low or negative growth adjusted discount rates of government expenditures indicate that restoring FS requires a reduction of the *growth rate* of the exploding expenditure components. Such policy actions have already been taken. Good examples are the public pension reforms implemented in Sweden and proposed in Norway, in which an automatic mechanism neutralizes the effect of increased longevity on the average present value of public pension benefits.

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## Should discounting be based on the pre- or after-tax interest rate?

The accumulation of private and government financial wealth is given by the differential equations

$$1) \quad \dot{B}_P = i(1 - \tau)B_P + S_P,$$

$$2) \quad \dot{B}_G = iB_G + S_G + \bar{a}B_P,$$

where  $\dot{B}_P$  and  $\dot{B}_G$  are the time derivatives of, respectively, private wealth,  $B_P$ , and government wealth  $B_G$ .  $i$  is the pre-tax interest rate,  $\tau$  is the tax rate on interest income, both assumed to be constant.  $S_P$  is private savings, and  $S_G$  is the primary budget surplus net of the tax revenue collected on private interest income. Define the after-tax interest rate  $i_N = i(1 - \tau)$ . Without losing the main points, savings in both sectors are assumed to grow at constant rates. The solution of (1) then becomes

$$3) \quad B_P(t) = B_{P0}e^{i_N t} + \frac{S_{P0}(e^{(g_P - i_N)t} - 1)}{g_P - i_N} e^{i_N t}.$$

Inserting (3) the solution of (2) can be written

4)

$$B_G(t) = B_{G0}e^{it} + B_{P0}(e^{it} - e^{i_N t}) + \frac{S_{G0}(e^{g_G t} - e^{it})}{g_G - i} + S_{P0} \left[ \frac{e^{i_N t}}{g_P - i_N} + \frac{\bar{a}e^{g_P t}}{(g_P - i_N)(g_P - i)} - \frac{e^{it}}{g_P - i} \right] \\ + S_{P0} \left[ \frac{e^{i_N t}}{g_P - i_N} + \frac{\bar{a}e^{g_P t}}{(g_P - i_N)(g_P - i)} - \frac{e^{it}}{g_P - i} \right].$$

Except for the tax revenue collected from private interest income, no restrictions have been imposed that implies inter-dependency between  $B_P$  and  $B_G$ . The Non-Ponzi-Game (NPG) condition associated with  $B_G$  is

$$5) \quad \lim_{t \rightarrow \infty} B_G(t)e^{-it} = 0.$$

Inserting (4) it takes the form

$$\lim_{t \rightarrow \infty} \left( B_{G0} + B_{P0} (1 - e^{-\pi}) + \frac{S_{G0} (e^{(g_G - i)t} - 1)}{g_G - i} + S_{P0} \left[ \frac{e^{-\pi}}{g_P - i_N} + \frac{\bar{u} e^{(g_P - i)t}}{(g_P - i_N)(g_P - i)} - \frac{1}{g_P - i} \right] \right) = 0.$$

We shall henceforth assume that  $g_P < i_N$  and  $g_O < i_N$ . The expression then simplifies to

$$6) \quad B_{G0} + B_{P0} + \frac{S_{G0}}{i - g_G} + \frac{S_{P0}}{i - g_P} = 0$$

The fiscal gap is defined as

$$7) \quad F_{G0} = - \left( \frac{S_{G0}}{i - g_G} + \frac{S_{P0}}{i - g_P} + B_{P0} \right) - B_{G0}.$$

Thus, without any further restrictions the discounting of all relevant flow variables should be based on the pre-tax interest rate when calculating the fiscal gap. This corresponds to the situation of an economy in which both the government and the private sector have unlimited access to the international capital market.

If we impose the NPG-condition on private wealth, i.e.  $\lim_{t \rightarrow \infty} B_P(t) e^{-i_N t} = 0 \Leftrightarrow B_{P0} (g_P - i_N) = S_{P0}$ , the fiscal gap expression simplifies to

$$8) \quad F_{G0} = - \left( \frac{S_{G0}}{i - g_G} + \frac{\bar{u} B_{P0}}{i - g_P} \right) - B_{G0},$$

but the discounting of all relevant flow variables should still be based on the pre-tax interest rate.

### Special case 1: No international capital movements

In this case we must have  $B_P + B_G = 0$ . The government cannot issue debt to any other sector than the rest of the economy, i.e. the private sector. The accumulation of government wealth then becomes

$$\dot{B}_G = i_N B_G + S_G, \text{ which implies}$$

$$9) \quad B_G(t) = e^{i_N t} \left[ B_{G0} - \left( \frac{S_{G0}}{i_N - g_G} \right) (e^{(g_G - i_N)t} - 1) \right].$$

The Non-Ponzi-Game (NPG) condition associated with  $B_G$  now takes the form

$$10) \quad \lim_{t \rightarrow \infty} B_G(t) e^{-i_N t} = \left[ B_{G0} + \frac{S_{G0}}{i_N - g_G} \right] = 0.$$

The fiscal gap becomes

$$11) \quad F_{G0} = \frac{-S_{G0}}{i_N - g_G} - B_{G0}.$$

Three conclusions can be drawn in this case:

1. Future primary budget surpluses should be discounted with the after-tax interest rate when calculating the fiscal gap.
2. The relevant definition of the primary budget surplus excludes taxes collected on private interest income.
3. Provided that  $B_G > 0$ , both point 1 and 2 makes the government budget constraint stricter compared to an analysis which ignores the tax rate on interest income.

### **Special case 2: Foreign debt constrained by a national budget constraint**

In this case the government and the private sector have access to the international capital market, but the foreign sector behaves as one institutional sector with an intertemporal budget constraint. By definition, the financial wealth of the foreign by definition corresponds to the national foreign debt of the “home” economy we consider. Consequently, the government sector cannot accumulate debt independent of the debt accumulation in the private sector (and vice versa).

Define national financial wealth as  $B_N = B_G + B_P$ . It grows according to

$$12) \quad \dot{B}_N = \dot{B}_G + \dot{B}_P = i(1 - \tau)B_P + S_P + iB_G + S_G + \bar{\alpha}B_P = iB_N + S_N$$



where  $S_N \equiv S_P + S_G$ . The relevant interest rate for discounting government budget components will be independent of the time paths for  $S_N$ . For expositional transparency, it is therefore assumed that  $S_N(t) = S_{N0}e^{g_N t}$ , where  $g_N < i$ . The solution for  $B_N$  can then be written

$$13) \quad B_N(t)e^{-it} = B_{N0} + \int_{s=0}^t S_{N0}e^{(g_N-i)s} ds = B_{N0} + \left( \frac{S_{N0}}{g_N - i} \right) (e^{(g_N-i)t} - 1).$$

The NPG-condition imposed on  $B_N$  implies  $\lim_{t \rightarrow \infty} B_N(t)e^{-it} = 0 \Leftrightarrow B_{N0} = -\frac{S_{N0}}{i - g_N}$ . Given the NPG-condition, the wealth accumulation degenerates to

$$14) \quad B_N(t) = -\left( \frac{S_{N0}}{i - g_N} \right) e^{g_N t} = B_{N0}e^{g_N t}.$$

Substituting  $B_P = B_N - B_G$  into (2) yields

$$15) \quad \dot{B}_G = iB_G + S_G + \bar{\alpha}(B_N - B_G) = i_N B_G + S_G + \bar{\alpha}B_N = i_N B_G + S_{G0}e^{g_G t} + \bar{\alpha}B_{N0}e^{g_N t}.$$

Since the after-tax interest rate is the coefficient associated with  $B_G$ , it follows directly that the relevant discount rate when calculating the fiscal gap. The solution now becomes

$$16) \quad B_G(t)e^{-i_N t} = B_{G0} + \left( \frac{S_{G0}}{i_N - g_G} \right) (1 - e^{(g_G-i_N)t}) + \left( \frac{\bar{\alpha}B_{N0}}{i_N - g_N} \right) (1 - e^{(g_N-i_N)t}).$$

The fiscal gap formula becomes

$$17) \quad F_{G0} = \lim_{t \rightarrow \infty} \left[ \left( \frac{S_{G0}}{i_N - g_G} \right) (1 - e^{(g_G-i_N)t}) + \left( \frac{\bar{\alpha}B_{N0}}{i_N - g_N} \right) (1 - e^{(g_N-i_N)t}) \right] - B_{G0}$$

$$= \frac{S_{G0}}{i_N - g_G} + \frac{\bar{\alpha}B_{N0}}{i_N - g_N} - B_{G0}.$$

Two conclusions can be drawn from (17):

1. In the case of a national intertemporal constraint on the sum of government and private financial wealth, discounting of all government expenditures and revenues should be based on the after-tax interest rate. Compared to fiscal gap estimates based on the pre-tax interest rate, the effect in the first point makes the government budget constraint,  $F_{G0} = 0$ , stricter, provided that  $B_{G0} > 0$ .
2. On the other hand, revenues should include a capital tax revenue imputed on the national interest income.