Monetary and fiscal policy rules in a model with capital accumulation and potentially non-superneutral money Leopold von Thadden

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Deutsche Bundesbank, Wilhelm-Epstein-Strasse 14, 60431 Frankfurt am Main, Postfach 10 06 02, 60006 Frankfurt am Main

Tel +49 69 95 66-1 Telex within Germany 4 1 227, telex from abroad 4 14 431, fax +49 69 5 60 10 71

Please address all orders in writing to: Deutsche Bundesbank, Press and Public Relations Division, at the above address or via fax No. +49 69 95 66-30 77

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Abstract

We consider the properties of two monetary policy rules (monetary targeting, Taylor-type interest rate rule) in an intertemporal equilibrium model with capital accumulation and two outside assets (government bonds, fiat money). The paper shows that the long-run behaviour of the economy depends critically on whether under the monetary-fiscal regime the steady-state real interest rate is independent of inflation. If this is the case, there exists in our model a unique steady state with stable adjustment dynamics under either monetary policy rule. By contrast, if superneutrality fails, dynamics under the interest rate rule may suffer from global indeterminacy arising from multiple steady states which do not necessarily differ in terms of the 'activeness' of the interest rate feedback on inflation. This is ruled out under monetary targeting.

**Keywords:** Monetary Policy, Fiscal regimes, Overlapping generations

**JEL classification:** E52, E63, H62

#### Zusammenfassung

Diese Arbeit untersucht die Eigenschaften geldpolitischen Regeln von zwei (Geldmengensteuerung, Zinsregel vom Taylor-Typ) in einem intertemporalen Gleichgewichtsmodell mit Kapitalbildung und zwei staatlichen Aktiva (staatlichen Schuldtiteln, Fiatgeld). Es wird gezeigt, dass die langfristigen Eigenschaften der Modellökonomie wesentlich davon abhängen, ob unter dem monetären und fiskalischen Regime der langfristige Realzins unabhängig von der Inflationsrate ist. Wenn dies der Fall ist, besitzt unser Modell ein eindeutiges langfristiges Gleichgewicht mit stabiler Anpassungsdynamik unter beiden geldpolitischen Regeln. Ist jedoch Geld nicht superneutral, besteht bei der Zinsregel die Gefahr, dass die Dynamik des Systems global indeterminiert ist aufgrund multipler langfristiger Gleichgewichte, die sich nicht notwendigerweise in der Stärke des Feedback-Effektes bezüglich der Inflation in der Zinsregel Eine derartige Konstellation globaler Indeterminiertheit tritt bei der unterscheiden. Geldmengensteuerung nicht auf.

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# Monetary and fiscal policy rules in a model with capital accumulation and potentially non-superneutral money\*

#### 1 Introduction

Inspired by John Taylor's proposal of a simple interest rate rule (Taylor, 1993), there has emerged over the past decade a rich literature which strongly deemphasizes the role of money in the conduct of monetary policy and rather concludes that a 'good' policy should rely on an 'active' interest rate rule which reacts to inflationary pressures by raising the nominal interest rate by more than one-to-one. Such a policy is widely seen as a sufficient device to keep inflation close to some prespecified target level and, moreover, to ensure that the rule in itself is not a source of non-fundamental instability. More specifically, an active response has the desirable feature that it breaks inflationary expectations by raising the ex ante real interest rate, thereby dampening aggregate demand. By contrast, a 'passive' response, by reducing the ex ante real interest rate, suffers potentially from non-fundamental instability, as a rise in inflationary expectations is likely to be validated by an increase in aggregate demand and a subsequent rise in actual inflation.

More recently, without questioning the desirability of transparent rules, several studies have indicated that the stability properties of the Taylor principle are less robust than suggested in the early literature. In particular, Benhabib, Schmitt-Grohé, and Uribe (2001, 2002) argue that a Taylor-type feedback rule for the interest rate which respects the zero bound on nominal interest rates will be naturally associated with a second ('unintended') steady state with a lower, possibly negative inflation rate and a passive stance of the interest rate rule. From this perspective, the focus on the local stability properties of the target steady state seems to be misplaced, since it fails to acknowledge that dynamics tend to be indeterminate from a global perspective. To prevent the economy from spiralling into the unintended, second steady state, the authors suggest a couple of policy refinements. First, the interest rate rule may be augmented by an appropriate monetary or fiscal feedback mechanism which is triggered if the downward inflation dynamics become sufficiently severe.

<sup>\*</sup>Comments on a first version of this paper (entitled 'Fiscal criteria à la Maastricht and monetary policy rules') by Klaus Adam, Carsten Detken, Michael Devereux, Leo Kaas, Giovanni Lombardo, and Volker Wieland, as well as seminar participants at a Bundesbank workshop on 'Modelling monetary policy in dynamic general equilibrium models', the European Central Bank, the Bank of Finland, and the Universities of Bielefeld and Frankfurt are gratefully acknowledged. Leopold von Thadden: Research Centre, Deutsche Bundesbank, Wilhelm-Epstein-Strasse 14, 60431 Frankfurt/Main, Germany, e-mail: Leopold.von-Thadden@bundesbank.de.

<sup>&</sup>lt;sup>1</sup>For authoritative overviews, see Clarida, Gali, and Gertler (1999, 2000), and Woodford (2002).

In particular, as also stressed by Christiano and Rostagno (2001), the interest-rate rule may be embedded in a strategy of monetary targeting which is activated if the economy seems to slip towards the unintended steady state. Second, the feature of global indeterminacy may be avoided if the government maintains a globally passive interest rate rule in the first place.

This paper argues that, even if monetary policy is guided by a globally passive interest rate rule, dynamics can nevertheless be globally indeterminate. To derive this result, we argue that the danger of multiple steady states and globally indeterminate dynamics under a pure interest rate rule is much increased if one explicitly models the process of capital accumulation and allows for the possibility that the long-run real interest rate is not necessarily constant, but rather a function of the long-run inflation rate. To see intuitively why in a non-superneutral regime such a rule may well give rise to multiple steady states which do not exhibit a different stance of monetary policy (in terms of 'activeness' of the interest rate response), let  $\pi$  and  $r(\pi)$  denote the long-run inflation rate and real interest rate, respectively. Moreover, assume that the central bank follows a linear interest rate rule with feedback coefficient on inflation  $\gamma$ , nominal interest rate rule and the Fisher equation gives

$$i(\pi) = r^* + \pi^* + \gamma(\pi - \pi^*) = r(\pi) + \pi.$$

As graphed in Figure 1(b), for the case of superneutrality  $(r(\pi) = \text{constant})$ , which is always assumed in the above cited literature, there exists at best a unique steady state under a linear interest rate rule. However, the broad literature on money and growth is not conclusive enough, either from a theoretical or from an empirical perspective, to make this the only case of interest.<sup>2</sup> If the real interest rate depends negatively on the inflation rate (as discussed in contributions supporting the Tobin effect), then a globally passive Taylor rule  $(0 \le \gamma < 1)$  may well be consistent with the existence of multiple steady states (Figure 1(a)). Intuitively, a passive response accommodates a rise in inflation by lowering the real interest rate. If the longrun real interest rate itself depends negatively on inflation, multiple steady states may very well exist. Conversely, if the real interest rate depends positively on the inflation rate, as suggested by studies which favour the 'Anti-Tobin effect' (Gale (1983), Azariadis and Smith (1996)), then multiple steady states may well exist within the range of inflation outcomes in which the interest rate rule is active ( $\gamma > 1$ ), as graphed in Figure 1(c). By similar reasoning, one readily infers from Figure 1 that the scope for multiple steady states is considerably reduced if monetary policy

<sup>&</sup>lt;sup>2</sup>To summarize in any detail the findings of the literature on the (non)-superneutrality of money is beyond the scope of this paper. For classical studies which yield  $r'(\pi) = 0$ ,  $r'(\pi) < 0$ ,  $r'(\pi) > 0$ , respectively, see Sidrauski (1967), Tobin (1965), and Stockman (1981). For summaries of the theoretical and empirical literature, see Orphanides and Solow (1990) and Temple (2000).

 $<sup>^{3}</sup>$ Considering Figure 1(c), a truly global analysis would also have to address the discussion on

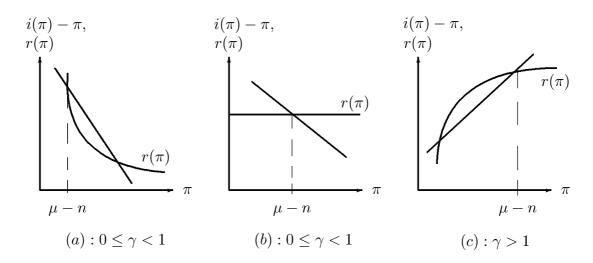


Figure 1: Monetary targeting vs. interest-rate rule if money is non-superneutral

follows a strategy of monetary targeting. In particular, assume that the steady-state real growth rate of the economy (n) is exogenous, consider a constant trend change in velocity (normalized to zero, for simplicity), and let  $\mu$  denote the constant growth rate of the money stock. Then, according to the principles of the quantity theory of money, monetary targeting tends to be associated with a unique steady state. Essentially, for a given value of the real growth rate of the economy, any choice of  $\mu$  acts as a sufficient anchor to pin down a unique long-run inflation rate, irrespective of the relationship between the real interest rate and the inflation rate.

The principle which underlies Figure 1 is a general one and, hence, potentially subject to many illustrations. Here, inspired by the debate on interactions between monetary and fiscal policies in European Monetary Union, we concentrate on an illustration which relates the superneutrality issue to a discussion of stylized fiscal and monetary policy rules in a small dynamic equilibrium model with two interest-bearing assets (physical capital, government bonds) and non-interest-bearing outside money. Depending on the details of the monetary and fiscal arrangement, in our model the long-run real interest rate may well be linked to steady-state inflation, and we show how this translates into globally indeterminate dynamics under a linear Taylor-type interest rate rule. By contrast, under monetary targeting, there exists always a unique steady state with locally determinate dynamics.

the zero bound on nominal interest rates summarized above. Moreover, though less relevant for the argument advanced in this paper, there are also recent contributions which question the stabilizing properties of active rules from a local perspective. Dupor (2001) presents a model in which capital enters as a sluggish state variable and in which the return equality between government bonds and capital makes steady states locally indeterminate. Carlstrom and Fuerst (2001) point out that in discrete-time models dynamic properties of steady states depend crucially on the timing of trading arrangements in goods and asset markets.

Before turning to the details of our analysis, we point out some general features of our modelling strategy. First, deviating from much of the literature on monetary policy rules, we use an overlapping generations economy of the Diamond type, similar to Schreft and Smith (1997, 1998). This choice is motivated by the feature that in economies of this type the long-run real interest rate is not solely pinned down by the economy's growth rate and the time preference of agents (as is the case under the modified golden rule implied by Ramsey economies), but rather as well by the details of the monetary and fiscal arrangement. Second, to keep our analysis analytically fully tractable at all stages, we develop a simple illustration of Figure 1 which embeds only constellations of type (a) and (b) as special cases, and not of type (c). Constellations of type (a) require, roughly speaking, that the substitution effects between interest-bearing and non-interest-bearing assets induced by a change in inflation predominate over the effect of inflation on overall savings. Our analysis does not elaborate on the savings rate channel, but rather focuses on the non-trivial substitution effects resulting from the inclusion of three assets, thereby biasing our results against a constellation of type (c). However, as we argue below, introducing the savings rate channel in our model will in general not suffice to establish the convenient benchmark result of superneutrality, although this extension changes the range of  $\gamma$  which yields multiple steady states. Finally, as forcefully stressed by Sargent and Wallace (1981), Leeper (1991), and, more recently, by the contributions to the 'fiscal theory of the price level', inflation dynamics may well be not fully under the control of the central bank, depending on the role assignment between fiscal and monetary policy. To facilitate a meaningful comparison of the two different monetary policies, we control for this criticism by choosing a set-up which is deliberately weighted in favour of the monetary agent. More specifically, monetary policy is assumed to predominate over fiscal policy in the sense that, for a given monetary policy rule (and irrespective of the initial price level), the fiscal agent is the residual actor which subordinates itself in a way that is consistent with the existence of a balanced growth path.<sup>4</sup>

Our model exhibits three simple intertemporal equilibrium conditions: an accumulation equation which specifies the total demand for all assets, a portfolio allocation rule, which describes how asset demands are allocated between interest-bearing assets and real balances, and the budget constraint of the government. In general, the dynamic behaviour of our economy is shaped by the interaction of the capital accumulation process, the debt dynamics of government bonds, and the forward-looking component of money demand. Given these features, we proceed in two steps.

<sup>&</sup>lt;sup>4</sup>For further details on the 'fiscal theory of the price level', see in particular the original contributions by Sims (1994) and Woodford (1994), the recent summaries in Carlstrom and Fuerst (2000), Christiano and Fitzgerald (2000), and Woodford (2001), the empirical analysis of Canzoneri, Cumby and Diba (2000), and the critical reviews from a 'monetarist' perspective by Buiter (1999) and McCallum (2001).

First, to establish some intuitive benchmark results, we restrict the analysis to a simple quantity-theory demand for money which specifies that real balances are proportional to current output and independent of future inflation. Despite this strong restriction, the dynamic behaviour of the economy depends critically on how fiscal policy subordinates itself to monetary policy. To illustrate this, we consider two fiscal rules. If fiscal policy follows a stock rule (defined as a constant ratio of the stock of government bonds to output), adjustments in the primary balance have the residual role of establishing the balance in the government's budget constraint. Due to this feature, debt dynamics are fully subordinated to the overall dynamics of the economy, superneutrality obtains, and the outcomes under the two monetary policy rules show only transitory differences. More specifically, we show that there exists, as long as the debt ratio remains below some feasibility bound, a unique steady state with stable adjustment dynamics under either monetary policy rule. By contrast, if fiscal policy follows a flow rule (defined as a constant deficit ratio), the residual role in the government's budget constraint is played instead by adjustments of the stock of government bonds. This imposes less discipline on the dynamics of the system, and, in line with panel (a) in Figure 1, the substitution possibilities between interestbearing and non-interest-bearing assets within the government's budget constraint now imply a negative relationship between the long-run real interest and inflation rate. Subject to a feasibility condition regarding the deficit ratio, monetary targeting still tends to be associated with a unique steady state, characterized by locally stable and determinate dynamics. By contrast, a passive Taylor rule may well give rise to two steady states. Under plausible assumptions, dynamics around the target steady state are saddlepath-stable, while the second steady state exhibits locally stable, though indeterminate dynamics. Depending on the realization of the initial price level, either of the two steady states can be reached.

Second, we make the money demand specification forward-looking.<sup>6</sup> Now, changes in the inflation rate induce also direct substitution effects in asset demands which tend to reinforce the results of the benchmark economy. In particular, we show that a policy of monetary targeting will still be associated with a unique steady state with determinate adjustment under either of the two fiscal rules. By contrast, under a Taylor-type rule the possibility of globally indeterminate dynamics is exacerbated, since now even under the fiscal regime of a stock rule, which imposes considerably more discipline on the fiscal agent, two steady states may well occur.

The remainder of the paper is structured as follows. Section 2 introduces the model.

<sup>&</sup>lt;sup>5</sup>The superneutrality issue is usually addressed within models in which capital and money are the only two assets, restricting the scope for substitution effects. By contrast, in this paper, even under a simple quantity-theory set-up, changes in inflation induce portfolio substitution effects, though indirectly, since changes in seigniorage revenue affect the mix between capital and bonds through the budget constraint of the government.

<sup>&</sup>lt;sup>6</sup>For a similar two-stage modelling strategy, see Sargent and Wallace (1981).

Section 3 discusses the general set of equilibrium conditions and the two monetary policy rules. Section 4 uses a simple quantity-theory demand for money and subsequently analyzes fiscal policies which follow a stock rule (Section 4.1) and a flow rule (Section 4.2). Section 5 discusses the robustness of our results if one allows for a forward-looking money demand specification. Section 6 offers some conclusions.

#### 2 The model

We consider a non-stochastic overlapping generations economy with production along the lines of Diamond (1965). Deviating from Diamond, to rationalize the coexistence of interest- and non-interest-bearing assets, we impose a cash-in-advance constraint which applies to a certain subset of consumption goods in the economy. Despite this rather ad-hoc specification, the reduced-form intertemporal equilibrium equations of our model are consistent with the analysis of Schreft and Smith (1997, 1998) which rigorously derives money holdings from a framework in which banks offer insurance against (random) liquidity needs of the private sector. Agents have rational expectations. Hence, the analysis relies on a simple forward-looking, deterministic framework. Let  $N_t$  denote the number of identical and two-period lived young agents in period t, with  $N_t/N_{t-1} = 1 + n > 1$  for all t = 0, 1, 2... Young agents are endowed with e=1 units of labour, to be offered inelastically, and agents have no labour endowment when being old. Young agents can invest in three distinct assets to transfer income into their second period of life. First, young agents can choose to hold real balances which simply act as an intergenerational trading device. Second, agents can invest in interest-bearing government bonds which mature after one period. Third, agents can invest in shares issued by firms, representing a claim on the return stream earned by physical capital in the production process. Given the pattern of labour endowments over the life cycle of agents, in any given period the output of the economy is jointly produced by young and old agents: while young agents offer the labour input, old agents own the capital stock resulting from last period's investment decisions. To keep track of the asset dynamics in the economy, beginning- and end-of-period stocks of assets need to be carefully distinguished. At the beginning of period t, the aggregate stocks of physical capital (K), nominal bonds (B) and nominal balances (M) are inherited from period t-1. More specifically, let  $k_{t-1} = K_{t-1}/N_t$ ,  $\widetilde{b}_{t-1} = \widetilde{B}_{t-1}/N_t$ ,  $\widetilde{m}_{t-1} = \widetilde{M}_{t-1}/N_t$  denote the predetermined stocks of capital, nominal bonds, and nominal money balances at the beginning of period t, measured per period-t young agent.

<sup>&</sup>lt;sup>7</sup>For a further discussion of the results of Schreft and Smith (1997, 1998), see Section 5.2. For a related cash-in-advance specification, see Hahn and Solow (1995).

#### 2.1 Production

The production side of the economy is described by a standard version of a neoclassical one-good economy. In particular, one unit of output, when invested, can be transformed into one unit of physical capital to be used one period later. Consumption goods, irrespective of whether exchanged against cash or credit, have identical properties in the production process. Assuming constant returns to scale and perfect competition in input and output markets, aggregate output (Y) is produced according to  $Y_t = F(K_{t-1}, N_t)$ . Let  $y = f(k) \equiv F(K, 1)$ , where k = K/N is the capital-labour ratio per young agent.

(A 1) The function f(k) satisfies:

(i) 
$$f(k) \ge 0$$
,  $f'(k) > 0$ ,  $f''(k) < 0$  for  $k \ge 0$ .  
(ii)  $\lim_{k \to 0} f'(k) = \lim_{k \to 0} \frac{f(k)}{k} = \infty$ ,  $\lim_{k \to \infty} f'(k) = \lim_{k \to \infty} \frac{f(k)}{k} = 0$ 

Capital depreciates at the rate  $\delta \in (0, 1)$ . Let  $\rho_t$  and  $w_t$  denote the rental rate and the wage rate to capital and labour in period t, respectively, with:

$$\rho_t = f'(k_{t-1}) \tag{1}$$

$$w_t = f(k_{t-1}) - f'(k_{t-1}) \cdot k_{t-1} = w(k_{t-1}). \tag{2}$$

We assume that factor incomes  $\rho_t$  and  $w_t$  are subject to a proportional tax at rate  $\tau \in (0, 1)$ . The after-tax return factor associated with a unit of capital, invested in period t and with pay-off in period t + 1, is

$$R_t = 1 - \delta + (1 - \tau) \cdot f'(k_t) = R(k_t). \tag{3}$$

#### 2.2 Government

We abstract from productive activities of the public sector and let  $\widetilde{G}_t$  denote the part of nominal output consumed by the government. With  $p_t$  describing the aggregate price level, the nominal primary deficit  $(\widetilde{D})$  in period t follows:  $\widetilde{D}_t = \widetilde{G}_t - \tau \cdot p_t \cdot Y_t$ . Let  $I_{t-1}$  represent the interest factor on bonds issued in period t-1.8 The nominal flow budget constraint of the government in period t can then be summarized as:

$$\widetilde{D}_t + I_{t-1} \cdot \widetilde{B}_{t-1} = \widetilde{B}_t + \widetilde{M}_t - \widetilde{M}_{t-1}. \tag{4}$$

<sup>&</sup>lt;sup>8</sup>For simplicity, interest earned on government bonds is assumed to be not taxed. However, this assumption could easily be relaxed.

Let  $\widetilde{d}_t = \widetilde{D}_t/N_t$  and introduce the real variables  $d_t = \frac{\widetilde{d}_t}{p_t}$ ,  $b_t = \frac{\widetilde{b}_t}{p_t}$  and  $m_t = \frac{\widetilde{m}_t}{p_t}$  to express (4) in real terms on a per capita basis

$$d_t + I_{t-1} \cdot \frac{p_{t-1}}{p_t} \cdot b_{t-1} = (1+n) \cdot b_t + (1+n) \cdot m_t - \frac{p_{t-1}}{p_t} \cdot m_{t-1}.$$
 (5)

Equation (5) says that the sum of the primary deficit and the principal and interest on government debt due in period t needs to be financed either from seigniorage revenues or the sale of newly issued bonds, maturing in period t + 1.

#### 2.3 Problem of the representative agent

Preferences are such that all net wage income  $(1-\tau) \cdot w_t$  received by young agents is saved. When deciding the composition of their portfolios, agents take all prices and return rates as given. Let asset demands of the representative agent be denoted by  $\hat{}$ -variables. Agents choose real money balances  $\hat{m}_t$  (with real return factor  $R_t^m = p_t/p_{t+1}$ ), government bonds  $\hat{b}_t$  (with real return factor  $I_t \cdot p_t/p_{t+1}$ ), and shares issued by firms  $\hat{k}_t$  ('capital', with real return factor  $R_t$ ). With government bonds and capital being perfect substitutes, the latter two return rates need to be identical. Moreover, in any equilibrium with non-zero production,  $I_t \geq 1$  needs to hold. Let preferences over cash goods  $(c_{CA})$  and credit goods  $(c_{CR})$  be represented by the utility function

(A 2)

$$U(c_{CR}, c_{CA}) = \frac{1}{1 - \varepsilon} \cdot \left[ z \cdot c_{CR}^{1 - \varepsilon} + (1 - z) \cdot c_{CA}^{1 - \varepsilon} \right] \quad \varepsilon \in (0, 1), \ z \in (0, 1),$$

where  $\varepsilon \in (0,1)$  implies that cash and credit goods are gross substitutes. Then, the decision problem of a young agent in period t can be summarized as

$$\begin{aligned} \max : & \frac{1}{1-\varepsilon} \cdot [z \cdot c_{CR,t+1}^{1-\varepsilon} + (1-z) \cdot c_{CA,t+1}^{1-\varepsilon}] \\ + \vartheta_t \cdot (R_t^m \cdot \widehat{m}_t + R_t \cdot (\widehat{b}_t + \widehat{k}_t) - c_{CR,t+1} - c_{CA,t+1}) \\ + \mu_t \cdot (R_t^m \cdot \widehat{m}_t - c_{CA,t+1}) \\ + \nu_t \cdot [(1-\tau) \cdot w_t - \widehat{m}_t - \widehat{b}_t - \widehat{k}_t], \end{aligned}$$

<sup>&</sup>lt;sup>9</sup>As noted in the introduction, the assumption of a constant savings rate (for simplicity, normalized to unity) biases our results in the following in favour of the Tobin effect. However, as long as cash transactions are only relevant in the second period, it is conceptually straightforward to generalize our analysis to a set-up with endogenous savings  $s_t = s(R_t(k_t), w(k_{t-1}))$ . Importantly, because agents are 'disconnected with the future' in our overlapping generations structure, this would not be a sufficient device to ensure superneutrality independently of the monetary and fiscal arrangement, as shown in Weil (1991).

with the multipliers  $\vartheta_t$ ,  $\mu_t$ ,  $\nu_t$  being associated with the budget constraint in t+1, the additional cash constraint which applies to purchases of cash goods in t+1, and the portfolio constraint in t, respectively. Since cash and credit goods are homogenous in production, they will be sold to consumers at identical prices. Thus, from the consumers' perspective, there is an opportunity cost associated with the use of money whenever the nominal interest rate on bonds is positive. Because of this, as long as  $I_t > 1$ , money holdings in period t will be identical to the expected nominal value of cash purchases in period t+1. Concentrating on interior solutions with a strictly binding cash-in-advance constraint, we assume from now on: 10

$$I_t = R_t \cdot \frac{p_{t+1}}{p_t} > 1. \tag{6}$$

For  $(1-\tau)\cdot w_t$ ,  $p_t$ ,  $p_{t+1}$  being positive and finite, there exist uniquely defined asset demands  $\widehat{m}_t$  and  $(\widehat{b}_t + \widehat{k}_t)$ , as well as a uniquely determined, optimal consumption bundle in period t+1. However, at the individual level, the mix between  $\widehat{b}_t$  and  $\widehat{k}_t$  will be indeterminate. Differentiating with respect to  $c_{CR,t+1}$ ,  $c_{CA,t+1}$ ,  $\widehat{m}_t$ ,  $(\widehat{b}_t + \widehat{k}_t)$  and eliminating the multipliers gives the system of equations:

$$(1 - \tau) \cdot w_t = \widehat{m}_t + \widehat{b}_t + \widehat{k}_t \tag{7}$$

$$\frac{\frac{\partial U}{\partial c_{CA}}}{\frac{\partial U}{\partial c_{CR}}} = \frac{1-z}{z} \cdot \left(\frac{c_{CR,t+1}}{c_{CA,t+1}}\right)^{\varepsilon} = \frac{R_t}{R_t^m} = I_t \tag{8}$$

$$c_{CA,t+1} = R_t^m \cdot \widehat{m}_t \tag{9}$$

$$c_{CR,t+1} = R_t \cdot (\widehat{b}_t + \widehat{k}_t) \tag{10}$$

Evidently, the optimal solution to the portfolio problem of young agents requires the marginal rate of substitution between cash and credit goods to be equal to the price ratio of the goods as perceived by consumers. This price ratio is naturally given by the nominal inflation factor. Combining (8)-(10) yields a simple relationship, describing the demand for real balances:

$$\widehat{m}_t = \widetilde{z} \cdot I_t^{\frac{\varepsilon - 1}{\varepsilon}} \cdot (\widehat{b}_t + \widehat{k}_t), \quad \text{with: } \widetilde{z} = (\frac{1 - z}{z})^{1/\varepsilon}$$
(11)

The gross substitutability assumption, i.e.  $\varepsilon \in (0,1)$ , implies that the demand for real balances decreases in the interest rate. In the following we find it convenient to look also at the limiting case of logarithmic utility, obtained by  $\varepsilon \to 1$ :

 $<sup>^{10}</sup>$ A straightforward way to accommodate also the limiting case of  $I_t = 1$  is given by Hahn and Solow (1995) who assume that portfolio indifference is always broken in favour of  $\hat{k}_t + \hat{b}_t$ , i.e. in this special case money plays a residual role in asset demands.

(A 2')

$$U(c_{CR}, c_{CA}) = z \cdot \ln(c_{CR}) + (1 - z) \cdot \ln(c_{CA}).$$

Using (A 2') instead of (A 2) simplifies the portfolio allocation rule (11) considerably, implying that real balances and interest-bearing assets are demanded in constant proportions:

$$\widehat{m}_t = \frac{1-z}{z} \cdot (\widehat{b}_t + \widehat{k}_t). \tag{12}$$

# 3 Competitive equilibrium conditions

In a competitive equilibrium, all markets need to clear, and decisions need to result from optimizing behaviour for all periods from t = 0 onwards. Regarding the initial conditions in period t = 0, we assume that the economy inherits from the past some positive interest rate  $I_{-1}$  (relevant for any interest payments on government bonds in t = 0) and a positive price level  $p_{-1} > 0$ . Moreover, the initial generation of old agents, born in t = -1, is endowed with a positive capital stock, positive money holdings and a non-zero level of government bonds:

#### (A 3) Initial conditions

In the initial period t = 0, the economy inherits from the past

i) 
$$I_{-1} > 1$$
,  $p_{-1} > 0$ ,

ii) 
$$k_{-1} > 0$$
,  $\widetilde{m}_{-1} > 0$ ,  $\widetilde{b}_{-1} \neq 0$ .

Optimizing behaviour implies that from t=0 onwards the arbitrage equation (6) has to be satisfied. However, in t=0 the price level  $p_0$  is a potential jumping variable, implying that, given (A 3),  $p_0 = \frac{I_{-1} \cdot p_{-1}}{R(k_{-1})}$  is not required. Using per capita expressions and defining  $g_t = \widetilde{G}_t/(p_t \cdot N_t)$ , market clearing requires in t=0,1,2,...

Gov't bonds : 
$$\hat{b}_t = \frac{\widetilde{B}_t}{N_t \cdot p_t} = (1+n) \cdot b_t$$
 (13)

Capital: 
$$\hat{k}_t = \frac{K_t}{N_t} = (1+n) \cdot k_t$$
 (14)

Money: 
$$\widehat{m}_t = \frac{\widetilde{M}_t}{N_t \cdot p_t} = (1+n) \cdot m_t$$
 (15)

Output: 
$$f(k_{t-1}) = \frac{c_{CR,t} + c_{CA,t}}{1+n} + g_t + (1+n) \cdot k_t - (1-\delta) \cdot k_{t-1}$$
 (16)

Note the asymmetrical treatment of the labour market, which always clears at the full employment level according to equation (2).<sup>11</sup>

**Definition** Given the initial conditions specified in (A 3) and some value  $\tau \in (0, 1)$ , a competitive equilibrium is a sequence of prices  $\{p_t, I_t\}$ , nominal asset supplies  $\{\widetilde{B}_t, \widetilde{M}_t\}$  and quantities  $\{k_t, c_{CA,t+1}, c_{CR,t+1}, g_t\}$  such that in all periods

- i) markets clear according to (13)-(16),
- ii) the budget constraint of the government (5) is satisfied,
- iii) labour and capital receive competitive return rates according to (1) and (2),
- iv) consumption plans of agents are optimal under price-taking behaviour according to (7)-(10),
- v) return rates satisfy (6).

To facilitate the analysis of monetary and fiscal arrangements with balanced growth, we assume that factor incomes are proportional to output:

**(A 4)** 
$$k \cdot f'(k_t) = \alpha \cdot f(k_t), \ w(k_t) = (1 - \alpha) \cdot f(k_t).$$

Using (A 4) and consolidating equations (1)-(16) yields the following set of intertemporal equilibrium conditions, describing the evolution of the economy over time:

$$I_t = R(k_t) \cdot \frac{p_{t+1}}{p_t} > 1$$
 (17)

$$d_t + R(k_{t-1}) \cdot b_{t-1} = (1+n) \cdot b_t + (1+n) \cdot m_t - \frac{p_{t-1}}{p_t} \cdot m_{t-1}$$
 (18)

$$(1+n) \cdot k_t = (1-\tau) \cdot (1-\alpha) \cdot f(k_{t-1}) - (1+n) \cdot (m_t + b_t)$$
 (19)

$$m_t = \widetilde{z} \cdot I_t^{\frac{\varepsilon - 1}{\varepsilon}} \cdot [k_t + b_t] \tag{20}$$

The structure of the dynamic system (17)-(20) is rather simple. Equation (18) restates the budget constraint of the government. Equation (19) represents a standard

$$c_{CA, t} = \frac{p_{t-1}}{p_t} \cdot \widehat{m}_{t-1} = \frac{p_{t-1}}{p_t} \cdot (1+n) \cdot m_{t-1}$$

$$c_{CR, t} = R(k_{t-1}) \cdot (\widehat{k}_{t-1} + \widehat{b}_{t-1}) = R(k_{t-1}) \cdot (1+n) \cdot (k_{t-1} + b_{t-1})$$

$$(1+n) \cdot k_t = \widehat{k}_t = (1-\tau) \cdot w(k_{t-1}) - (1+n) \cdot (m_t + b_t)$$

$$g_t = \tau \cdot f(k_{t-1}) + (1+n) \cdot b_t - R(k_{t-1}) \cdot b_{t-1} + (1+n) \cdot m_t - \frac{p_{t-1}}{p_t} \cdot m_{t-1}$$

Then, adding up the three equations and using (2) and (3), one easily verifies that the output market needs to be in equilibrium as well.

<sup>&</sup>lt;sup>11</sup>By Walras' Law the market-clearing conditions (13)-(16) are not independent. Consider equations (5), (7), (9), and (10) and assume that the markets for government bonds, capital and money clear. Combining these conditions yields:

accumulation equation of Diamond-type overlapping generation economies, assuming a constant savings rate. Equation (20) summarizes the optimality conditions (8)-(10) on the part of consumers under a binding cash-in-advance constraint. Finally, the inequality (17) is necessary for money, bonds, and capital to be valued in equilibrium. For future reference it is convenient, by substituting out for  $b_t$  in (19) and (20), to establish the relation:

$$m_{t} = \frac{1}{1 + \widetilde{z}^{-1} \cdot I_{t}^{\frac{1-\varepsilon}{\varepsilon}}} \cdot \frac{(1-\tau) \cdot (1-\alpha)}{1+n} \cdot f(k_{t-1}). \tag{21}$$

Equation (21) represents a forward-looking money demand equation, stipulating that real balances are proportional to current-period output and declining in the nominal interest rate and, hence, expected inflation. Using (A 2') instead of (A 2), equation (21) becomes:

$$m_t = (1-z) \cdot \frac{(1-\tau) \cdot (1-\alpha)}{1+n} \cdot f(k_{t-1})$$
 (22)

Thus, by invoking the special properties of logarithmic preferences, the forward-looking element drops out and (21) turns into a simple version of the quantity theory of money with constant velocity.

As it stands, the system (17)-(20) is not determined without detailed assumptions regarding the monetary-fiscal arrangement.<sup>12</sup> While we leave the specifications of fiscal policies for the following sections, we abstract, regarding monetary policy, from commitment problems of the central bank. More specifically, we assume that the central bank, taking the initial values (A 3) as given, announces at the beginning of t = 0 a monetary policy rule, pertaining to all future periods, which is considered as perfectly credible by the private sector. As mentioned above, depending on the details of the rule, the initial price level  $p_0$  is a potential jumping variable. In period t = 0, with  $R(k_{-1})$  being replaced by the expression  $I_{-1} \cdot \frac{p_{-1}}{p_0}$ , the government's budget constraint (18) takes the general form:

#### Budget constraint of the government in (t=0):

$$\frac{I_{-1} \cdot \widetilde{b}_{-1} + \widetilde{m}_{-1}}{p_0} = (1+n) \cdot (b_0 + m_0) - d_0 \tag{23}$$

For the dynamic classification of equilibria in the subsequent sections, it will be important whether in equation (23) the real value of the nominal liabilities inherited from t = -1 (i.e. the LHS of (23)) is predetermined or not. Bearing this in mind,

<sup>&</sup>lt;sup>12</sup>For an early discussion of fiscal and monetary arrangements in the context of Diamond-type overlapping generations economies, see Gale (1983).

we consider two distinct rules. First, we assume that the monetary agent is a strict 'monetary targeter', in the sense that the aggregate money supply M grows in all periods at a constant rate  $\mu$ . To ensure non-negative seigniorage revenues, we impose for the remainder of this paper the condition  $\mu \geq 0$ .

#### (Strict) monetary targeting:

$$\widetilde{M}_t = (1+\mu) \cdot \widetilde{M}_{t-1} \Leftrightarrow \widetilde{m}_t = \frac{1+\mu}{1+n} \cdot \widetilde{m}_{t-1} \text{ for all } t \geqslant 0, \text{ with: } \mu \geq 0, \text{ given } \widetilde{M}_{-1}$$

Note that under the simple version of the quantity theory of money as given by (22), real balances  $m_0$  are a predetermined variable, since  $K_{-1}$  and, hence,  $k_{-1}$  are given. With  $\widetilde{m}_0$  being specified by the rule, both the price level  $p_0$  and the LHS of (23) are also predetermined. By contrast, under the forward-looking specification (21),  $m_0$  and  $p_0$  are not predetermined, implying that the real value of inherited government liabilities from t = -1 in (23) acts as a jumping variable. For further reference, under monetary targeting the government's budget constraint (18) turns into

$$d_t = (1+n) \cdot b_t - R(k_{t-1}) \cdot b_{t-1} + \frac{\mu}{1+\mu} \cdot (1+n) \cdot m_t.$$
 (24)

Second, we assume that the nominal interest rate  $I_t$  is set according to a Taylor-type interest rate rule.<sup>13</sup>

#### Interest rate rule:

$$I_t = R(k^*) \cdot (1 + \pi^*) + \gamma \cdot \left[\frac{p_{t+1}}{p_t} - (1 + \pi^*)\right] \quad \text{for all } t \geqslant 0, \text{ with } : \gamma \ge 0.$$
 (25)

In (25),  $I^* = R(k^*) \cdot (1 + \pi^*)$  is some long-run target value of the nominal interest factor, derived from an inflation target and a target real interest rate. The coefficient  $\gamma$  captures the feedback mechanism of how the central bank responds to deviations of (expected) inflation from the target inflation rate by changing the interest rate. Evidently, a policy of an interest rate peg  $(I_t = I)$  is a special case of this rule, with  $\gamma = 0$ . Regarding the initial conditions, under the simple quantity theory version of money the term  $m_0$  is again a predetermined variable. Yet, with neither the price level nor the nominal money supply in t = 0 being pinned down, the interest rate rule leads to nominal indeterminacy, quite consistent with the results established in Patinkin (1961) and Sargent and Wallace (1975). Consequently, the LHS of (23) will also be not predetermined, and this feature is also obtained if one uses instead

<sup>&</sup>lt;sup>13</sup>Taylor's original work of course stresses short-run frictions which are absent from this analysis. Despite this substantial difference, following the loose jargon in the literature, we refer to equation (25) occasionally as a Taylor-type interest rate rule.

the forward-looking money demand specification (21). Under the interest rate rule, dynamics within the budget constraint of the government are given by

$$d_t = (1+n) \cdot b_t - R(k_{t-1}) \cdot b_{t-1} + (1+n) \cdot m_t - \frac{p_{t-1}}{p_t} \cdot m_{t-1}$$
 (26)

$$\frac{p_{t-1}}{p_t} = \frac{R(k_{t-1}) - \gamma}{[R(k^*) - \gamma] \cdot (1 + \pi^*)]},$$
(27)

where the latter condition follows from combining the interest rate rule with the Fisher equation (17). Note that under the interest rate rule, seigniorage revenues of the government cannot be expressed solely in terms of contemporaneous real balances  $m_t$ , quite in contrast to the monetary targeting regime (24).

In the following sections, monetary policy predominates over fiscal policy in the sense that the central bank is free to choose one of the monetary policy rules without considering the implications of this choice for the combined budget constraint of the monetary and fiscal agent. By contrast, the fiscal agent takes the monetary policy rule as given and subordinates itself in a way which is consistent with the existence of a balanced growth path.<sup>14</sup>

# 4 Fiscal policies under a simple quantity-theory demand for money

To establish some fully tractable benchmark results, in this section we combine the simple quantity-theory version of money demand (22) with two different, stylized fiscal policies, the first combination leading to a constellation of type (b) in Figure 1, the second of type (a).

## 4.1 Fiscal policy according to a stock rule

We assume first that the fiscal agent follows a stock rule which specifies that in every period the end-of-period stock of government bonds is proportional to output  $y_t$ :

$$\widetilde{B}_t = \theta \cdot p_t \cdot f(k_{t-1}) \cdot N_t \Leftrightarrow (1+n) \cdot b_t = \theta \cdot f(k_{t-1})$$
(28)

By assumption, the constant debt ratio  $\theta$  must under either monetary policy be consistent with balanced growth paths. As a general feature, the stock rule implies that the residual adjustment burden in the government's budget constraint (18) falls on the primary balance  $d_t$ . More specifically, with the tax rate  $\tau$  being fixed, all adjustments occur through variations of government spending  $g_t$ .

<sup>&</sup>lt;sup>14</sup>Because of the overlapping generations structure, this strong commitment of the fiscal agent does not imply a Ricardian structure in which changes in fiscal policy would not matter. For a discussion, see Woodford (2001, Section 2).

#### 4.1.1 Monetary targeting

With  $b_t$  being effectively indexed to current-period output under the stock rule, rearranging of (18)-(20) yields a recursive structure in  $k_t$ ,  $m_t$ , and  $d_t$ :

$$k_t = [z \cdot (1-\tau) \cdot (1-\alpha) - \theta] \cdot \frac{f(k_{t-1})}{1+n}$$
 (29)

$$m_t = (1-z) \cdot \frac{(1-\tau) \cdot (1-\alpha)}{1+n} \cdot f(k_{t-1})$$
 (30)

$$d_{t} = \theta \cdot f(k_{t-1}) - R(k_{t-1}) \cdot \frac{\theta}{1+n} \cdot f(k_{t-2}) + \frac{\mu}{1+\mu} \cdot (1+n) \cdot m_{t}.$$
 (31)

The evolution of per capita real balances follows  $m_t = \frac{1+\mu}{1+n} \cdot \frac{p_{t-1}}{p_t} \cdot m_{t-1}$ , implying that in steady-state equilibrium (with  $m_t = m$ ) the inflation rate is uniquely pinned down by  $1 + \pi = (1 + \mu)/(1 + n)$ . Note that the restriction  $\mu \geq 0$  ensures that steady-state inflation is bounded from below by  $\pi \geq \frac{-n}{1+n} \approx -n$ . The key equations which characterize steady-state solutions can be summarized as:

$$\frac{1+\mu}{1+n} = 1+\pi \tag{32}$$

$$R(k) \cdot (1+\pi) > 1 \tag{33}$$

$$R(k) = 1 - \delta + (1 - \tau) \cdot \alpha \cdot \frac{1 + n}{z \cdot (1 - \tau) \cdot (1 - \alpha) - \theta}, \tag{34}$$

where (34) uses  $f'(k) = \alpha \cdot f(k)/k$ . The system (32)-(34) exhibits a complete dichotomy between the real and the nominal variables of the economy. In particular, similar to panel (b) in Figure 1, equation (34) implies that the steady-state real interest factor is independent of  $\pi$ .<sup>15</sup> Equation (34) has a unique solution k > 0, provided the debt ratio  $\theta$  chosen by the fiscal agent respects a certain feasibility restriction.

**Proposition 1** (Monetary targeting: existence of a steady state)
If

$$\theta < z \cdot (1 - \tau) \cdot (1 - \alpha) = \overline{\theta},$$
 (35)

there exists a unique solution  $k(\theta)$  to (34), with  $k(\theta) > 0$ . To ensure  $I(k(\theta)) > 1$ , let  $\underline{\theta}(\mu)$  denote the minimum value of  $\theta$  such that  $R(k(\theta)) \cdot (\frac{1+\mu}{1+n}) > 1$  for all  $\theta > \underline{\theta}(\mu)$ . Then, if  $\theta \in (\underline{\theta}(\mu), \overline{\theta})$ , there exists a unique value  $k(\theta) > 0$ , satisfying (33) and (34).

The Without loss of generality, in the subsequent analysis, in contrast to Figure 1, we use instead the return factors R(k) and  $1 + \pi$ , denoting 1 plus the respective return rate.

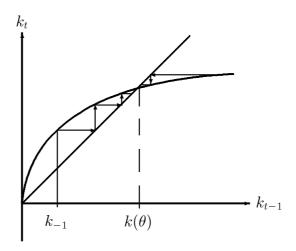


Figure 2: Stock rule: dynamics under monetary targeting

*Proof:* The proposition follows from the properties of f(k) stated in (A 1). If  $\theta \to \overline{\theta}$ ,  $k \to 0$ , and, hence, (32) will be satisfied. Note that  $\underline{\theta}(\mu)$  falls in  $\mu$ . In particular, if  $\theta \to -\infty$ ,  $k(\theta) \to \infty$  in (34), implying  $R(k) \to 1 - \delta$ . Hence,  $\underline{\theta}(\mu)$  takes on some finite, possibly negative value if  $\frac{1+\mu}{1+n} < 1/(1-\delta)$ ; otherwise,  $\underline{\theta}(\mu) \to -\infty$ .  $\Box$ 

Due to the one-dimensional dynamics of (29) and the recursive structure of (30) and (31), it is straightforward to verify that, combined with the nominal determinacy of  $p_0$ , dynamics around the steady state are locally stable and determinate:

#### **Proposition 2** (Monetary targeting: local dynamics)

Given  $k_{-1}$ , there exists locally a uniquely defined sequence  $k_t$  which converges to the steady-state level  $k(\theta)$  in a monotone manner. Associated with this, there are uniquely determined converging sequences of  $b_t$ ,  $m_t$ ,  $d_t$ , and  $g_t$  given by (28), (30), and (31). With  $\widetilde{m}_t$  being exogenously fixed under monetary targeting,  $p_t$  is also uniquely determined.

Remark: For an illustration of this result, see Figure 2.

<sup>&</sup>lt;sup>16</sup>Strictly speaking,  $\theta \in (\underline{\theta}(\mu), \overline{\theta})$  is not yet a sufficient condition for the existence of a steady state, since in (31) the implied steady-state value of government spending  $g = d(k(\theta)) + \tau \cdot f(k(\theta))$  must be feasible, i.e. non-negative and smaller than total output f(k). Also in the following local stability analysis, we assume that parameters are such that this mild condition is satisfied.

#### 4.1.2 Interest rate rule

The accumulation equation (29) and the money demand equation (30) remain unchanged, while the budget constraint of the government becomes:

$$d_{t} = \theta \cdot f(k_{t-1}) - R(k_{t-1}) \cdot \frac{\theta}{1+n} \cdot f(k_{t-2}) + (1+n) \cdot m_{t} - \frac{R(k_{t-1}) - \gamma}{[R(k^{*}) - \gamma] \cdot (1+\pi^{*})]} \cdot m_{t-1}.$$
(36)

Thus, the system (29), (30) and (36) has a similar recursive structure, and the critical equations summarizing the steady state are given by

$$R(k) = \gamma + (R(k^*) - \gamma) \cdot \frac{1 + \pi^*}{1 + \pi}$$
(37)

$$R(k) \cdot (1+\pi) > 1 \tag{38}$$

$$R(k) = 1 - \delta + (1 - \tau) \cdot \alpha \cdot \frac{1 + n}{z \cdot (1 - \tau) \cdot (1 - \alpha) - \theta}.$$
 (39)

Since (39) is identical to (34), the unique steady-state capital intensity  $k(\theta)$  is independent of  $\gamma$ , and the steady-state inflation rate  $\pi$ , reflecting the superneutrality of money, follows recursively from (37). Evidently, if the long-run target values of the interest rate rule are geared to the steady state established under monetary targeting, one obtains the result:

#### **Proposition 3** (Interest rate rule: existence of a steady state)

Consider the steady state of Proposition 1 and assume that the interest rate target  $I^*$  is derived from  $R(k^*) = R(k(\theta))$  and  $1 + \pi^* = \frac{1+\mu}{1+n}$ . Then, the interest rate rule is characterized by a unique steady state which is identical to the one in Proposition 1 under monetary targeting.

*Remark:* For an illustration of this result, see Figure 1(b).

Because of the recursive structure of the equations (29), (30), and (36) the dynamic properties of the steady state are very similar to the monetary targeting regime. More specifically, the steady state remains locally stable, but, owing to the nominal indeterminacy of the initial price level  $p_0$ , a subset of the real variables now becomes indeterminate as well:

#### **Proposition 4** (Interest rate rule: local dynamics)

Given the initial conditions (A 3), there exist uniquely defined sequences of  $k_t$ ,  $b_t$  and  $m_t$  converging to the respective steady-state levels. However, since  $p_0$  and, hence, the initial real value of inherited government liabilities are indeterminate under the

interest rate rule, the corresponding sequences  $g_t$  and  $d_t$  have indeterminate initial values  $g_0$  and  $d_0$ .<sup>17</sup>

Essentially, a high value of  $p_0$  acts like a tax on the consumption of the initial old generation, thereby making room for higher government purchases  $g_0$ . Thus, depending on the realization of  $p_0$ , adjustment paths under the interest rate rule differ (compared with monetary targeting) in the composition of consumption of initial old agents and the government in period 0, but not thereafter.

In summary, if fiscal policy adheres to a stock rule, this implies strong adjustment requirements for the primary deficit and thereby disciplines effectively the potentially destabilizing debt dynamics of the economy. This discipline, combined with a simple quantity-theory demand for money and assuming a feasible debt burden, ensures that there exists a unique steady-state real interest rate which is identical under monetary targeting and the interest rate rule and is not affected by changes in the inflation target. Moreover, under either monetary regime, dynamics around this steady state are locally stable. Under monetary targeting, with the initial price level being determinate, the dichotomy between the real and the nominal sector of the economy is complete, ensuring that dynamics are locally determinate. By contrast, under the interest rate rule, adjustment requirements for the primary deficit depend on the realization of  $p_0$  (which is indeterminate), implying that a subset of variables is indeterminate in the transition towards the steady state (restricted, in fact, to the first period). Thus, the two monetary policy rules exhibit a large amount of symmetry. As we show in the following section, this symmetry is not robust to changes in the specification of the fiscal regime.

### 4.2 Fiscal policy according to a flow rule

Rather than targeting the debt ratio, we now assume that the fiscal agent follows a flow rule, specifying that in each period the government deficit (inclusive of interest payments) is proportional to output:

$$\widetilde{D}_{t} + I_{t-1} \cdot \widetilde{B}_{t-1} = \xi \cdot p_{t} \cdot f(k_{t-1}) \cdot N_{t} \Leftrightarrow d_{t} + R(k_{t-1}) \cdot b_{t-1} = \xi \cdot f(k_{t-1}). \tag{40}$$

Accordingly, the government budget constraint turns into:

$$\xi \cdot f(k_{t-1}) = (1+n) \cdot b_t + (1+n) \cdot m_t - \frac{p_{t-1}}{p_t} \cdot m_{t-1}$$
(41)

With the deficit ratio  $\xi$  being constant over time, the residual role within the government's budget constraint needs now to be played by adjustments in the stock of

<sup>&</sup>lt;sup>17</sup>As a caveat, see footnote 16, i.e. the proposition is subject to the qualification that, for a given value of  $\gamma$ , the initial real value of government liabilities implied by  $p_0$  must lead to a feasible path of  $g_t$  in (36).

bonds. Again, we restrict the fiscal agent to choices of  $\xi$  which are consistent with a balanced growth path.

#### 4.2.1 Monetary targeting

As shown in (24), the seigniorage term in (41) can be expressed solely as a function of contemporaneous money balances  $m_t$ . As a result of this feature, the set of equations (18)-(20) is still recursive under monetary targeting, despite the somewhat looser specification of fiscal policy under the flow rule which no longer indexes  $b_t$  directly to  $y_t$ . Upon isolating  $b_t$  in (18) and substituting in (41), combined with  $m_t = \frac{1+\mu}{1+n} \cdot m_{t-1}$ , one obtains for the government budget constraint, the money market equilibrium, and the accumulation equation, respectively:

$$k_t = \left[ \left( 1 - \frac{1-z}{1+\mu} \right) \cdot (1-\tau) \cdot (1-\alpha) - \xi \right] \cdot \frac{f(k_{t-1})}{1+n} \tag{42}$$

$$m_t = (1-z) \cdot \frac{(1-\tau) \cdot (1-\alpha)}{1+n} \cdot f(k_{t-1})$$
 (43)

$$b_t = \frac{(1-\tau)\cdot(1-\alpha)}{1+n}\cdot f(k_{t-1}) - k_t - m_t \tag{44}$$

Exploiting this recursive structure, the critical steady-state conditions are given by:

$$\frac{1+\mu}{1+n} = 1+\pi \tag{45}$$

$$R(k) \cdot (1+\pi) > 1 \tag{46}$$

$$R(k) = 1 - \delta + \frac{(1 - \tau) \cdot \alpha \cdot (1 + n)}{(1 - \frac{1 - z}{(1 + n)(1 + \pi)}) \cdot (1 - \tau) \cdot (1 - \alpha) - \xi}$$
(47)

The system of equations (45)-(47) implies that the steady-state real interest rate now depends negatively on the inflation rate - similar to panel (a) in Figure 1 and quite in contrast to the otherwise similar system (32)-(34) discussed under the stock rule. This negative relationship is largely due to the substitution possibilities between interest-bearing bonds and outside money in the government's budget constraint. In particular, since money demand is assumed to be not forward-looking, a shift to a higher inflation rate leads on impact at unchanged real balances to a rise in seigniorage revenue. This reduces, at a given deficit, the need for bond financing and induces a crowding-in of capital, which depresses the real interest rate. The upper bound on the deficit ratio  $\xi$ , ensuring the existence of a steady state, is now itself a function of the inflation objective of the central bank. Yet, under monetary targeting, according to the logic underlying Figure 1(a), there exists a unique steady state which supports this inflation objective:

**Proposition 5** (Monetary targeting: existence of a steady state) Define

$$(1 - \frac{1-z}{1+\mu}) \cdot (1-\tau) \cdot (1-\alpha) = \overline{\xi}(\mu). \tag{48}$$

Let  $\underline{\xi}(\mu)$  denote the minimum value such that  $R(k(\xi)) \cdot \frac{1+\mu}{1+n} > 1$  for  $\xi > \underline{\xi}(\mu)$ . Then, if  $\xi \in (\xi(\mu), \overline{\xi}(\mu))$ , there exists a unique value  $k(\xi) > 0$ , satisfying (46) and (47).

*Remark:* The proof is similar to Proposition 1. Again,  $\underline{\xi}(\mu)$  falls in  $\mu$ , and beyond some threshold value of  $\mu$ ,  $\xi(\mu) \to -\infty$ .

Owing to the recursive structure of (42)-(44) it is straightforward to characterize the local dynamics of the system around the steady state. In particular,  $\xi < \overline{\xi}(\mu)$  implies that the one-dimensional dynamics of (42) in  $k_t$  are locally stable and, similar to Proposition 2, determinate.

#### **Proposition 6** (Monetary targeting: local dynamics)

Given the initial conditions (A 3), there exist uniquely defined sequences of  $k_t$ ,  $b_t$ ,  $m_t$ ,  $p_t$ ,  $g_t$  and  $d_t$  converging to the respective steady-state levels.

Remark: The result follows from the structure of (42), which could be graphed as in Figure 2.  $\Box$ 

#### 4.2.2 Interest rate rule

This section shows that the monetary authority, by choosing a globally passive interest rate rule, may well accommodate the (indirect) substitution effects between money and interest-bearing assets in a way which leads to a second, unintended steady state at which inflation deviates from its target value. As discussed in Section 3, under the interest rate rule the seigniorage term in (41) depends on  $m_{t-1}$  and  $m_t$ , implying that the dynamics of the system are no longer one-dimensional as in the previous section. Instead, the equations (42) and (43) are replaced by the two-dimensional system in  $k_t$  and  $m_t$ :<sup>18</sup>

$$m_t = (1-z) \cdot \frac{(1-\tau) \cdot (1-\alpha)}{1+n} \cdot f(k_{t-1})$$
 (49)

$$\xi \cdot f(k_{t-1}) = \frac{1+n}{1-z} \cdot m_t - (1+n) \cdot k_t - \frac{R(k_{t-1}) - \gamma}{(R(k^*) - \gamma) \cdot (1+\pi^*)} \cdot m_{t-1}$$
 (50)

<sup>&</sup>lt;sup>18</sup>Equation (44) remains unchanged, i.e. the system is block-recursive in the dynamics of  $k_t$  and  $m_t$  and the bond dynamics  $b_t$ .

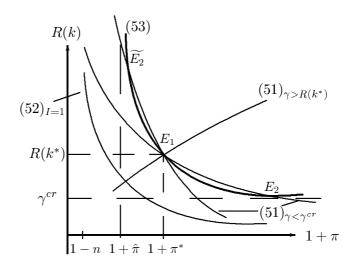


Figure 3: Flow rule: multiple steady states under the interest rate rule Steady states are characterized by:

$$R(k) = \gamma + (R(k^*) - \gamma) \cdot \frac{1 + \pi^*}{1 + \pi}$$
(51)

$$R(k) \cdot 1 + \pi > 1 \tag{52}$$

$$R(k) = 1 - \delta + \frac{(1 - \tau) \cdot \alpha \cdot (1 + n)}{(1 - \frac{1 - z}{(1 + n)(1 + \pi)}) \cdot (1 - \tau) \cdot (1 - \alpha) - \xi},$$
 (53)

where (53) follows from combining the steady-state versions of (49) and (50). Note that (53) is identical to (47). As graphed in Figure 3, the two equations (51) and (53) describe hyperbolas in  $(1 + \pi) - R(k)$ —space. Equation (53), independently of  $\gamma$ , slopes downward near the steady state  $\{1 + \pi^* = \frac{1+\mu}{1+n}, R(k^*) = R(k(\theta))\}$ . By contrast, the slope of (51) depends on how  $\gamma$  relates to  $R(k^*)$ . Evidently,  $\gamma > R(k^*)$  ( $\gamma = R(k^*)$ ) implies that (51) is increasing (horizontal) in  $(1 + \pi) - R(k)$ —space, ruling out the existence of a second steady state with non-negative activity ( $k \ge 0$ ). If  $\gamma < R(k^*)$ , equation (51) is downward-sloping, and a second steady state in addition to the target steady state  $E_1$  may well exist. Appealing to the geometry of Figure 3, one easily confirms that a necessary condition for a second steady state is given by  $\gamma < \gamma^{cr}$ , with

$$\gamma^{cr} = \lim_{\pi \to \infty} RHS(53) = 1 - \delta + \frac{(1 - \tau)\alpha(1 + n)}{(1 - \tau)(1 - \alpha) - \xi} < R(k^*),$$

where the latter inequality is clearly always satisfied. For  $\gamma$  close to (and smaller than)  $\gamma^{cr}$ , the second intersection  $E_2$  has  $\pi > \pi^*$  and  $R(k) < R(k^*)$ , while for

<sup>&</sup>lt;sup>19</sup>For  $R(k^*)$  and  $1 + \pi^*$  sufficiently close to unity, Figures 3 and 1 are equivalent, since the latter one uses the approximation  $r \cdot \pi \approx 0$ .

 $\gamma$  becoming sufficiently small the second intersection may well cross  $E_1$ , leading to  $\widetilde{E}_2$  with  $\pi < \pi^*$ ,  $R(k) > R(k^*)$ . Recognizing that at any second steady state the inequalities (52) and  $\pi \geq -n$  need to hold as well, the following proposition states a particularly simple sufficient (not necessary) condition which guarantees the existence of two steady states whenever  $0 \leq \gamma < \gamma^{cr}$ .

#### **Proposition 7** (Interest rate rule: existence of steady states)

Consider the steady state under monetary targeting of Proposition 5 and assume that the interest rate target  $I^* > 1$  is derived from  $1 + \pi^* = \frac{1+\mu}{1+n}$  and  $R(k^*) = R(k(\xi))$ .

- 1) If  $\gamma > \gamma^{cr}$ , the steady state  $\{1 + \pi^*, R(k^*)\}$  is unique.
- 2) If  $0 \le \gamma < \gamma^{cr}$ , (51) and (53) have generically a second intersection with k > 0, always satisfying (52). For this intersection to constitute a second steady state,  $\pi \ge -n$  needs to hold as well.
- 2a) if  $\gamma$  is close to  $\gamma^{cr}$ , the second intersection  $E_2$  has  $\pi > \pi^*$ ,  $R(k) < R(k^*)$  and always satisfies  $\pi \geqslant -n$ ;
- 2b) at  $\underline{\gamma} < \gamma^{cr}$ ,  $E_2$  passes through  $E_1$  and turns into  $\widetilde{E_2}$ , with  $\pi < \pi^*$ ,  $R(k) > R(k^*)$ . If  $\gamma < \underline{\gamma}$ ,  $\pi \geqslant -n$  will be satisfied if there exists a value  $1 + \widehat{\pi} \in [1 n, \frac{1+\mu}{1+n})$  such that  $\lim_{\pi \mid \widehat{\pi}} RHS(53) = \infty$ .

*Proof*: As a general feature, variations in  $\gamma$  induce a rotation of (51) around  $\{1+\pi^*, R(k^*)\}$ . Combining (51) and (53) gives a quadratic equation in R(k), ensuring that there are not more than two intersections. For  $\xi$  being fixed, there exists, if  $\pi$  gets close to -1, some value  $\widehat{\pi}$  such that RHS(53)  $\to \infty$  if  $\pi \to \widehat{\pi}$ , and RHS(53) < 0 if  $\pi < \widehat{\pi}$ . Evidently,  $k \geq 0$  requires  $\pi \geq \widehat{\pi}$ . If  $\widehat{\pi} \geqslant -n$ , as assumed in 2b), there must exist a second solution of (51) and (53) with  $\pi > \widehat{\pi} \geqslant -n$  and k > 0, whenever  $0 \leq \gamma < \gamma^{cr}$ , since  $\lim_{\pi \to \infty} \text{RHS}(51) = \gamma < \lim_{\pi \to \infty} \text{RHS}(53) = \gamma^{Cr}$  and  $\lim_{\pi \downarrow \widehat{\pi}} \text{RHS}(51) < \lim_{\pi \downarrow \widehat{\pi}} \text{RHS}(53) = \infty$ . While (52) holds at  $E_1$  by construction, it also holds along (53) as  $\pi \to \widehat{\pi}$  and  $\pi \to \infty$ , and, hence, at any  $E_2$  or  $\widehat{E_2}$ .

Remark: Depending on the location of  $\{1 + \pi^*, R(k^*)\}$  along (53),  $\underline{\gamma} < 0$  is certainly possible. Hence, the restriction  $\gamma \geq 0$  may well imply that the second steady state is always of the  $E_2$ -type for all  $\gamma \in [0, \gamma^{cr})$ .

The intuition behind this result is as follows. Assume, again, that, at a given deficit, inflation rises, leading on impact to higher seigniorage revenue and a reduced supply of government bonds. For this process to trigger a crowding-in of capital, monetary policy needs to accommodate it by lowering the real return rate on interest-bearing assets, requiring  $\gamma < R(k^*)$ . If this is the case, the activity level increases, and in the long run the economy may well settle down at a different steady state, characterized by a higher deficit in absolute terms, higher seigniorage revenue and a lower interest burden on outstanding bonds. Conversely, if  $\gamma > R(k^*)$  the monetary agent raises

the real interest rate, thereby precluding the build-up of a higher capital stock required at the second steady state.

Regarding the stability properties of steady states, the system (49)-(50) has one predetermined variable  $(k_{-1})$ , while the indeterminacy of  $p_0$  effectively turns  $m_{-1}$  into a jumping variable.<sup>20</sup> Local dynamics around the steady states can be classified as follows:

#### **Proposition 8** (Interest rate rule: local dynamics)

The local stability behaviour of the steady states established in Proposition 7 depends on how  $\gamma$  relates to two critical values  $\gamma, \overline{\gamma}$  with  $\gamma < \gamma^{cr} < R(k^*) < \overline{\gamma}$ .

1) Target steady state  $(E_1)$ 

If  $\gamma \in (\underline{\gamma}, \overline{\gamma})$ ,  $E_1$  is a saddle. If  $\gamma < \underline{\gamma}$  or  $\gamma > \overline{\gamma}$ ,  $E_1$  is a sink, as long as  $\xi < \widehat{\xi} < \overline{\xi}(\mu)$ , otherwise a source.

2) Consider a constellation with a second steady state, requiring  $\gamma < \gamma^{cr}$ .

If  $\gamma > \underline{\gamma}$ ,  $E_2$  is a sink, as long as  $\xi < \widehat{\xi} < \overline{\xi}(\mu)$ , otherwise a source. If  $\gamma < \underline{\gamma}$ ,  $\widetilde{E_2}$  is a saddle.

For a proof and the derivation of the critical values  $\underline{\gamma}$ ,  $\overline{\gamma}$ , and  $\hat{\xi}$ : see appendix  $\Box$ 

Remark: Again, since  $\gamma < 0$  is certainly possible, the restriction  $\gamma \geqslant 0$  may imply that, for all  $\gamma \in [0, \gamma^{cr})$ ,  $E_1$  is a saddle, while the second steady state  $E_2$  is always a sink (as long as  $\xi < \overline{\xi} < \overline{\xi}(\mu)$ , otherwise a source).

Essentially, Proposition 8 says that, as long as the target steady state  $E_1$  is unique and  $\gamma$  is sufficiently close to  $R(k^*)$ , there exists locally a uniquely defined saddlepath converging to the steady state. For values of  $\gamma$  sufficiently far away from  $R(k^*)$ , the target steady state  $E_1$  turns either into a source or a sink. However, the steady state will always turn into a sink (and hence exhibit locally stable, though indeterminate local dynamics) if the deficit ratio  $\xi$  remains below a certain threshold value  $\hat{\xi}$ , thereby ruling out unpleasant debt dynamics. In the appendix we show that this restriction is rather mild. In particular, we show that even in the limiting case of  $\mu = 0$  (in which monetary policy restricts fiscal policy in the strongest possible way, since seigniorage is zero) one obtains under plausible parametrizations  $\hat{\xi} > 0$ , ensuring that, for example, a balanced budget rule ( $\xi = 0$ ) satisfies  $\xi < \hat{\xi}$ . Moreover, the same restriction ensures that in a constellation with two steady states the low inflation steady state will be a saddle, while the high inflation steady state will be a sink. Which of these two local force fields predominates depends on the initial

<sup>&</sup>lt;sup>20</sup>More precisely, according to our discussion of (A 3), in the t = 0- version of (50) the coefficient in front of  $m_{-1}$  is not predetermined.

<sup>&</sup>lt;sup>21</sup>For a discussion of how the magnitude of the feedback coefficient interacts with the determinacy of equilibria, see also Bernanke and Woodford (1997).

position of the system, which in turn depends on the initial value of  $p_0$ . As a general feature, in a constellation with a saddle and a sink, either of the two steady states can be approached if the initial position of the system is sufficiently close to it, leading to globally indeterminate dynamics.

# 5 Fiscal policies under a forward-looking demand for money

Certainly, a more realistic money demand specification should include a forward-looking component as exhibited by equation (21). Under this specification, real balances depend negatively on the current nominal interest rate and, hence, on expected inflation, which introduces direct substitution effects between interest- and non-interest-bearing assets, along the lines of Tobin (1965). As a result, the overall dynamics of the economy become more involved, and, in particular, under monetary targeting dynamics cease to be one-dimensional (under either fiscal rule). However, we show in this section that the inclusion of the additional substitution effects tends to reinforce the results of the benchmark economy. More specifically, a policy of monetary targeting is still associated with a unique steady state with determinate adjustment, regardless of whether the fiscal agent subordinates itself by means of the stock or a flow rule. By contrast, under a Taylor-type rule the possibility of globally indeterminate dynamics is exacerbated in the sense that now even under the fiscal regime of a stock rule, which imposes considerably more discipline on the fiscal agent, two steady states may well occur.

## 5.1 Fiscal policy according to a stock rule

#### 5.1.1 Monetary targeting

Given a monetary targeting regime,  $I_t$  evolves as

$$I_t = R(k_t) \cdot \frac{p_{t+1}}{p_t} = R(k_t) \cdot \frac{m_t}{m_{t+1}} \cdot \frac{1+\mu}{1+n},$$

implying that for an announced path of nominal balances  $\widetilde{m}_t$  the current demand for real balances depends on the path of expected future prices. Due to this feature, replacing (30) by (21) in (29)-(31) breaks the recursive structure and leads to a two-dimensional dynamic system in  $k_t$  and  $m_t$ :

$$k_t = [(1-\tau) \cdot (1-\alpha) - \theta] \cdot \frac{f(k_{t-1})}{1+n} - m_t$$
 (54)

$$m_{t} = \frac{1}{1 + \widetilde{z}^{-1} \cdot [R(k_{t}) \cdot \frac{m_{t}}{m_{t+1}} \cdot \frac{1+\mu}{1+n}]^{\frac{1-\varepsilon}{\varepsilon}}} \cdot \frac{(1-\tau) \cdot (1-\alpha)}{1+n} \cdot f(k_{t-1}), \quad (55)$$

with  $d_t$  recursively determined in (31). In steady state, (54) and (55) combine to

$$R(k) = 1 - \delta + (1 - \tau) \cdot \alpha \cdot \frac{1 + n}{\frac{(1 - \tau) \cdot (1 - \alpha)}{1 + \tilde{z} \cdot [R(k) \cdot (1 + \pi)] \frac{\varepsilon - 1}{\varepsilon}} - \theta}.$$
 (56)

Thus, in contrast to the simple quantity-theory specification introduced in Section 4.1.1, in (56) superneutrality no longer holds and the long-run real interest rate is rather a function of the target inflation rate. More specifically, (56) defines implicitly, for k > 0, a downward sloping locus in  $(1 + \pi) - R(k)$ -space. With long-run inflation being uniquely pinned under monetary targeting by  $(1 + \mu)/(1 + n)$ , there will still be at best a unique steady state, given some feasibility condition regarding  $\theta$ . Because of the two-dimensional structure of (54) and (55), the local dynamics around this steady state are a priori less obvious. Yet, one can show that the essence of Proposition 2 carries over to the forward-looking specification. With  $k_{t-1}$  being the only predetermined variable in (54) and (55) and, following our discussion in section 3,  $m_t$  acting now as a jumping variable, locally determinate dynamics require one stable and one unstable eigenvalue. This will always be the case, as the following extension of Proposition 2 shows:

#### **Proposition 9** (Monetary targeting: local dynamics)

Consider the fiscal regime of a stock rule under a forward-looking money demand, as given by (54) and (55). Then, dynamics around the unique steady state are locally determinate and exhibit monotone adjustment, since the eigenvalues follow the pattern  $0 < \lambda_1 < 1, \lambda_2 > 1$ .

Proof: see appendix. 
$$\Box$$

#### 5.1.2 Interest rate rule

If the monetary authority follows instead an interest rate rule, the relevant steadystate conditions are given by

$$R(k) = \gamma + (R(k^*) - \gamma) \cdot \frac{1 + \pi^*}{1 + \pi}$$
(57)

$$R(k) \cdot 1 + \pi > 1 \tag{58}$$

$$R(k) = 1 - \delta + (1 - \tau) \cdot \alpha \cdot \frac{1 + n}{\frac{(1 - \tau) \cdot (1 - \alpha)}{1 + \tilde{z} \cdot [\gamma \cdot (\pi - \pi^*) + R(k^*) \cdot (1 + \pi^*)] \frac{\varepsilon - 1}{\varepsilon}} - \theta}, \quad (59)$$

where (59) follows from substituting (57) into (56) and collapses for the special case  $\varepsilon = 1$  into (34). Equation (59) shows that the direct substitution effects introduced by the forward-looking money demand specification will not be operating, quite intuitively, under a policy of an interest rate peg ( $\gamma = 0$ ), which ensures that the RHS of

(59) is constant and the dynamics are as in Section 4.1.2. By contrast, if the interest rate rule has a strictly positive feedback coefficient ( $\gamma > 0$ ), (59) slopes downward in  $(1+\pi)-R(k)$ -space close to the target steady state, similar to Figures 1(a) and 3. A complete analysis of the system (57)-(59) with associated two-dimensional dynamics in  $m_t$  and  $k_t$  along the lines of Section 4.2.2., while conceptually straightforward, gives few additional insights. Instead, we simply indicate for a particular example how, in addition to the target steady state discussed above, an additional steady state emerges if the substitution effects and, hence,  $\gamma$  become sufficiently large. More specifically, consider a unique steady state under monetary targeting with  $\theta = 0$ , i.e. assume a zero debt ratio. Let  $\gamma_{\min} = 1 - \delta + \frac{(1-\tau)\cdot\alpha\cdot(1+n)}{(1-\tau)(1-\alpha)} < R(k^*)$ . Then, if  $\gamma \in (\gamma_{\min}, R(k^*))$ , (57) and (59) have by continuity at least two intersections, since  $\lim_{\pi\to\infty} RHS(57) = \gamma > \lim_{\pi\to\infty} RHS(59) = \gamma_{\min}$  and  $\lim_{\pi\to-1} RHS(57) = \gamma_{\min}$  $\infty > \lim_{\pi \to -1} RHS(59)$ , with the latter term being strictly finite. Moreover, similar to the proof of Proposition 7 one easily verifies that (58) and  $\pi \geq -n$  will be satisfied by construction at the second steady state if this is the high-inflation steady state. Hence, under the natural assumption that forward-looking asset demands give rise to direct substitution effects along the lines of Tobin (1965), multiple steady states can emerge under a passive Taylor rule, even if the fiscal agent exercises maximum restraint regarding the issuance of government bonds by setting  $\theta = 0$ .

#### 5.2 Fiscal policy according to a flow rule

#### 5.2.1 Monetary targeting

Introducing the forward-looking money demand specification in (42) and (43) gives

$$k_t + \frac{1}{1+\mu} \cdot m_t = [(1-\tau) \cdot (1-\alpha) - \xi] \cdot \frac{f(k_{t-1})}{1+n}$$
 (60)

$$m_t = \frac{(1-\tau)\cdot(1-\alpha)}{1+\widetilde{z}^{-1}\cdot[R(k_t)\cdot\frac{m_t}{m_{t+1}}\cdot\frac{1+\mu}{1+n}]^{\frac{1-\varepsilon}{\varepsilon}}}\cdot\frac{f(k_{t-1})}{1+n},$$
 (61)

with  $b_t$  recursively determined by (44). Thus, similar to the analysis in Section 5.1.1., dynamics become two-dimensional in  $k_t$  and  $m_t$ . For brevity, let  $\tilde{z} \cdot [R(k) \cdot (1+\pi)]^{\frac{\varepsilon-1}{\varepsilon}} = A(R(k), 1+\pi)$ . Then, upon substituting out for m in (60) and (61), the critical steady-state relation for the flow rule is given by

$$R(k) = 1 - \delta + (1 - \tau) \cdot \alpha \cdot \frac{1 + n}{\frac{1 + (1 - \frac{1}{(1 + n)(1 + \pi)}) \cdot A}{1 + A}} \cdot (1 - \tau) \cdot (1 - \alpha) - \xi$$

$$A = A(R(k), 1 + \pi) = \widetilde{z} \cdot [R(k) \cdot (1 + \pi)]^{\frac{\varepsilon - 1}{\varepsilon}}, \quad A_k > 0, \ A_{\pi} < 0.$$
(62)

Equation (62) implicitly specifies a downward-sloping locus in  $(1+\pi)-R(k)$ -space, as long as k>0. Hence, again, there exists under monetary targeting at best a

unique steady state with determinate adjustment dynamics, i.e. Proposition 6 is robust upon introducing a forward-looking money demand specification.

#### **Proposition 10** (Monetary targeting: local dynamics)

Consider the fiscal regime of a flow rule under a forward-looking money demand, as given by (60) and (61). Then, dynamics around the unique steady state are locally determinate and exhibit monotone adjustment, since the eigenvalues follow the pattern  $0 < \lambda_1 < 1, \lambda_2 > 1$ .

Proof: see appendix.

#### 5.2.2 Interest rate rule

In Section 4.2.2 it has been shown that (62), when combined with a Taylor rule, gives rise to globally indeterminate dynamics if one restricts the analysis to the special value of  $\varepsilon = 1$ . Relaxing this assumption is certainly not a recipe to remove such unpleasant dynamics, as one easily confirms in simulations. Since no qualitatively new results emerge from such simulations, we rather conclude this section by discussing the robustness of the strong findings under monetary targeting (Propositions 9 and 10) regarding a different specification of fiscal policy.

#### 5.2.3 Discussion: a primary deficit rule

A straightforward (equally stylized) alternative to the fiscal rules studied in this paper is a regime in which not the stock of bonds or the gross deficit but rather the primary deficit is treated as a constant fraction of current output, with  $d_t =$  $\phi \cdot f(k_{t-1})$ . By indexing the primary deficit  $d_t$  rather than the stock of bonds  $b_t$  or the gross deficit  $d_t + R(k_{t-1}) \cdot b_{t-1}$  to current output, this imposes even less discipline on the dynamics of the system. In particular, Schreft and Smith (1997, 1998) have carefully studied the special case of a balanced primary budget (i.e.  $\phi = 0$ ) in a related overlapping generations model. Since under this particular specification seigniorage revenues are not used to fund non-interest government expenditures, the two papers essentially isolate the open-market effects of monetary policy and address the general equilibrium consequences of different liquidity degrees of government liabilities. Owing to the less restrictive dynamics under a primary deficit rule, not only does a pure interest rate peg tend to be associated with multiple steady states and globally indeterminate dynamics, as shown in Schreft and Smith (1998)), but, in contrast to the results derived in this paper, so also does a policy of monetary targeting (Schreft and Smith, 1997).<sup>22</sup>

 $<sup>^{22}</sup>$ Schreft and Smith (1997) use a money demand with simple quantity-theory properties, similar to our specification in Section 4. By contrast, Schreft and Smith (1998) use a forward-looking

#### 6 Conclusion

The main purpose of this paper is to show that the scope for globally indeterminate dynamics under a simple Taylor rule is considerably increased if one explicitly models the process of capital accumulation and allows for the possibility that the longrun interest rate is not invariant to changes in the inflation rate. In particular, extending previous results by Benhabib, Schmitt-Grohé and Uribe (2001, 2002), we argue that, if money is non-superneutral, a Taylor rule may well give rise to globally indeterminate dynamics, arising from multiple steady states which do not exhibit a different stance of monetary policy (in terms of 'activeness' of the interest rate response). To this end, we develop a small exogenous growth model with two interest-bearing assets (capital, government bonds) and non-interest-bearing fiat money. Using an overlapping generations structure, the long-run real interest rate in our set-up is not completely pinned down by parameters describing the real side of the economy but rather depends on the details of the monetary and fiscal arrangement. Owing to this feature, even under otherwise 'monetarist' assumptions, multiple steady states and globally indeterminate dynamics can occur under a linear interest rate rule for a wide parameter range of the feedback coefficient on inflation. We illustrate this by discussing two stylized fiscal policy rules. First, we consider a fiscal policy which prescribes a constant deficit ratio over time. Under this rule, unpleasant global dynamics may well occur, even under a strictly monetarist demand for money, which specifies that real balances are proportional to current output and independent of expected inflation. Second, more restrictively, we consider a fiscal policy which prescribes a constant ratio of the stock of government bonds to output in all periods. While this policy imposes greater discipline on the evolution of the economy, unpleasant global dynamics can still occur if one allows for a forwardlooking money demand specification. By contrast, as long as velocity follows a predictable long-run path, a policy of monetary targeting prevents unpleasant global dynamics of this type under either fiscal rule and leads instead to a unique steady state with determinate adjustment dynamics. The main reason for this result is that monetary targeting pins down a unique long-run inflation rate in models with exogenous growth, irrespective of the relationship between the long-run real interest rate and the inflation rate.<sup>23</sup>

In summary, by analyzing the interaction between capital formation and the dynamics of government debt in a monetary growth model, this paper stresses 'long-run' features which have escaped attention in much of the recent literature on monetary

specification in which, because of risk aversion, the money demand depends positively on the interest rate, in contrast to the assumptions made in this section. Another key difference to this paper, apart from the different money demand and fiscal policy specifications, is that the two papers do not consider an interest rate rule, which allows for  $\gamma > 0$ .

<sup>&</sup>lt;sup>23</sup>In monetary growth models with endogenous growth, this assessment may need to be modified.

policy rules. We are aware that this bias towards the long run comes at the cost that our analysis is silent on many of the short-run frictions which are typically discussed in this literature. In particular, under the assumption of some price stickiness, the leverage of the central bank to influence the short-run movements of the economy by changing nominal interest rates increases, thereby strengthening the effectiveness of the interest rate instrument. Because of such omissions, there is much room for future research to balance more evenly short- and long-run elements in related frameworks. Similarly, to assess the quantitative importance of our results compared with those obtained in prototype Ramsey economies, simulations should probably rather use an overlapping generations framework of perpetual youth along the lines of Blanchard (1985) and Weil (1991). When conducting such an exercise, savings should be made fully endogenous. Correspondingly, particular attention should be paid to how this extension affects the long-run real interest rate and, hence, the range of the feedback coefficient on inflation which yields multiple steady states. Moreover, recent work by Leith and Wren-Lewis (2000) indicates how such frameworks can be amended to study in more detail the set of monetary and fiscal policy rules currently in place under European Monetary Union. Such extensions are important and we leave them for future work. However, the qualitative results established in this paper should be robust to such extensions. These results indicate that a policy which tries to steer the long-run course of the economy solely by an interest rate rule without reference to some monetary aggregate may not be immune to the unpleasant features of globally indeterminate dynamics.

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# **Appendix**

#### **Proof of proposition 8:**

To assess the dynamic properties of steady states we linearize (49) and (50)

$$m_{t} = (1-z)\frac{(1-\tau)(1-\alpha)}{1+n}f(k_{t-1})$$

$$\xi f(k_{t-1}) = \frac{1+n}{1-z}m_{t} - (1+n)k_{t} - \frac{R(k_{t-1}) - \gamma}{(R(k^{*}) - \gamma)(1+\pi^{*})}m_{t-1}$$

around the steady state by means of the first-order approximation

$$\begin{bmatrix} d \, m_t \\ d \, k_t \end{bmatrix} = J \cdot \begin{bmatrix} d \, m_{t-1} \\ d \, k_{t-1} \end{bmatrix},$$
 
$$J = \begin{bmatrix} 0 & \frac{(1-\tau)(1-\alpha)(1-z)f^{'}(k)}{1+n} \\ -\frac{R(k)-\gamma}{(R(k^*)-\gamma)(1+\pi^*)(1+n)} & \frac{[(1-\tau)\cdot(1-\alpha)-\xi]f^{'}(k)}{1+n} - \frac{R^{\prime}(k)}{(R(k^*)-\gamma)(1+\pi^*)(1+n)}m \end{bmatrix}.$$

Substituting out for m and using  $\frac{R(k)-\gamma}{(R(k^*)-\gamma)(1+\pi^*)} = \frac{1}{1+\pi}$  from (51), gives

$$Det(J) = \frac{(1-\tau)(1-\alpha)(1-z)}{(1+\pi)(1+n)} \frac{f'(k)}{1+n} > 0$$

$$Tr(J) = [(1-\tau)(1-\alpha) - \xi] \frac{f'(k)}{1+n} - \frac{x(k)}{R(k^*) - \gamma},$$

where  $x(k) = R'(k) \frac{f(k)}{1+n} \frac{(1-\tau)(1-\alpha)(1-z)}{(1+\pi^*)(1+n)} < 0$ . Using  $f'(k) = \alpha f(k)/k$  in (53), we derive for further reference,

$$1 + Det(J) - Tr(J) = 1 - \alpha + \frac{x(k)}{R(k^*) - \gamma}$$

$$1 + Det(J) + Tr(J) = \frac{\left[ (1 + \frac{1-z}{(1+n)(1+\pi)})(1-\tau)(1-\alpha) - \xi \right]f'(k)}{1+n} - \frac{x(k)}{R(k^*) - \gamma}$$

1) Consider first the target steady state  $(1 + \pi^*, R(k^*))$ . Note that Tr(J) depends on  $\gamma$ , while Det(J) is simply a constant, which can be rewritten as

$$Det(J) = \frac{\alpha}{\frac{(1-\tau)(1-\alpha)-\xi}{(1-\tau)(1-\alpha)\frac{1-z}{1+u}} - 1} > 0,$$

with Det(J) > 0 following from Proposition 5. For further reference, we need to know how Det(J) relates to unity. Note that Det(J) rises in  $\xi$ , with  $Det(J) \to 0$  if  $\xi \to -\infty$ , while Det(J) < 1 is not necessarily satisfied. Let

$$\widehat{\xi} = (1 - \tau)(1 - \alpha)[1 - (1 + \alpha)\frac{1 - z}{1 + \mu}] < \overline{\xi}(\mu)$$
(63)

denote the critical value such that Det(J) = 1, implying that 0 < Det(J) < 1 if  $\xi < \widehat{\xi}$ . Considering Tr(J), note that, at  $(1 + \pi^*, R(k^*))$ , i) if  $\gamma < R(k^*)$ , Tr(J) > 0, ii) if  $\gamma \to R(k^*)$ , Tr(J) passes from  $\infty$  to  $-\infty$ , and iii) if  $\gamma \to \infty$ ,  $Tr(J) \to [(1 - \tau)(1 - \alpha) - \xi] \frac{f'(k^*)}{1+n} > 0$ .

a) Consider  $\gamma > R(k^*)$ . Then, 1 + Det(J) - Tr(J) > 0, regardless of  $\gamma$ . Moreover, 1 + Det(J) + Tr(J), which is continuous in  $\gamma$ , must be negative if  $\gamma$  sufficiently close to  $R(k^*)$  by ii); similarly, by iii), it must be positive if  $\gamma$  becomes sufficiently large. Thus, by continuity there must exist some value  $\overline{\gamma} \in (R(k^*), \infty)$  at which 1 + Det(J) + Tr(J) = 0. Hence, the steady state is a saddle if  $\gamma \in (R(k^*), \overline{\gamma})$ ; if

close to  $R(k^*)$  by ii); similarly, by iii), it must be positive if  $\gamma$  becomes sufficiently large. Thus, by continuity there must exist some value  $\overline{\gamma} \in (R(k^*), \infty)$  at which 1 + Det(J) + Tr(J) = 0. Hence, the steady state is a saddle if  $\gamma \in (R(k^*), \overline{\gamma})$ ; if  $\gamma > \overline{\gamma}$ , the steady state is a sink if  $\xi < \widehat{\xi}$  (ensuring Det(J) < 1), otherwise a source. b) Consider  $\gamma < R(k^*)$ . If  $\gamma \to R(k^*)$ , 1 + Det(J) - Tr(J) < 0, but for  $\gamma$  being small, this may no longer hold. More specifically, let  $\underline{\gamma} = R(k^*) + \frac{x(k^*)}{1-\alpha} < R(k^*)$ . Then,  $1 + Det(J) - Tr(J) < 0 \Leftrightarrow \gamma > \underline{\gamma}$ . Hence, the steady state is a saddle if  $\gamma \in (\underline{\gamma}, R(k^*))$ ; if  $\gamma < \underline{\gamma}$ , the steady state is a sink if  $\xi < \widehat{\xi}$ , otherwise a source.

2) To relate  $\underline{\gamma}$  to a constellation with two steady states (requiring  $\gamma < \gamma^{cr} < R(k^*)$ ), combine (51) and (53):

$$\frac{f(k)}{k} = \frac{1+n}{q(k)-\xi}, \text{ with:}$$

$$q(k) = \left(1 - \frac{(R(k)-\gamma)(1-z)}{(R(k^*)-\gamma)(1+\pi^*)(1+n)}\right)(1-\tau)(1-\alpha).$$
(64)

The LHS(64) and the RHS(64) decline in k. One easily verifies that, whenever there are two steady states (with k > 0), at the low-inflation steady state  $L_1$  needs to hold  $\frac{\partial LHS(64)}{\partial k} > \frac{\partial RHS(64)}{\partial k} \iff \frac{(1-\alpha)f(k)/k}{k} < \frac{(1+n)q'(k)}{[q(k)-\xi]^2} \Leftrightarrow \gamma > R(k^*) + \frac{x(k)}{1-\alpha}$ . Conversely, the high-inflation steady state  $L_2$  has  $\gamma < R(k^*) + \frac{x(k)}{1-\alpha}$ . Moreover, at  $L_1$  and  $L_2$ , Det(J) > 0, Tr(J) > 0, and  $1 + Det(J) - Tr(J) < 0 \iff \gamma > R(k^*) + \frac{x(k)}{1-\alpha}$ . Hence, the low-inflation steady state  $L_1$  is always a saddle, and  $L_2$  is a sink if Det(J) < 1. Combining this with Proposition 7, gives  $L_1 = E_1$  and  $L_2 = E_2$  if  $\gamma > R(k^*) + \frac{x(k^*)}{1-\alpha} = \underline{\gamma}$ . At  $E_2$ , Det(J) < 1 is implied by  $\xi < \widehat{\xi}$ , since Det(J) falls in  $\pi$  and k. Conversely, if  $\gamma < \underline{\gamma}$ ,  $L_1 = \widetilde{E_2}$  and  $L_2 = E_1$ , and, as established above,  $E_1$  is a sink if  $\xi < \widehat{\xi}$ , otherwise a source.

Remark: To see that the bound  $\hat{\xi}$  defined by (63) is not restrictive, consider the limiting case  $\mu=0$ , i.e. monetary policy restricts fiscal policy in the strongest possible way, since seigniorage is zero. Then,  $\hat{\xi}>0$  requires  $z\gtrsim \alpha$ , where z denotes the share of credit goods in total purchases and  $\alpha$  denotes the share of capital in factor incomes. Certainly,  $z\gtrsim \alpha$  holds for plausible parametrizations, implying that even for  $\mu=0$  a balanced budget rule ( $\xi=0$ ), for example, leads to a constellation with a saddle and a sink.

#### Proof of proposition 9:

To assess the dynamics of (54)-(55) around the steady state (56), we replace, for analytical convenience, (55) by (20). Thus:

$$m_t = \widetilde{z} \cdot I_t^{\frac{\varepsilon - 1}{\varepsilon}} \cdot [k_t + \frac{\theta}{1 + n} \cdot f(k_{t-1})]$$
 (65)

$$k_t = [(1-\tau)\cdot(1-\alpha)-\theta]\cdot\frac{f(k_{t-1})}{1+n}-m_t,$$
 (66)

with  $I_t = R(k_t) \cdot (1 + \mu)/(1 + n) \cdot m_t/m_{t+1}$ . Let  $\widetilde{z} \cdot I_t^{\frac{\varepsilon-1}{\varepsilon}} = A(k_t, m_t, m_{t+1})$ , with partial derivatives  $A_{k_t} > 0$ ,  $A_{m_{t+1}} > 0$ ,  $A_{m_t} = -A_{m_{t+1}}$ . Since  $m_{t+1}$  appears only in (65), dynamics of (65) and (66) are of second order and can be assessed from<sup>24</sup>:

$$\left[\begin{array}{c} d\,m_{t+1} \\ d\,k_t \end{array}\right] = J \cdot \left[\begin{array}{c} d\,m_t \\ d\,k_{t-1} \end{array}\right],$$

$$J = \begin{bmatrix} 1 + \frac{1 + A + A_{k_t} \cdot (k + \frac{\theta}{1 + n} f(k))}{A_{m_{t+1}} \cdot (k + \frac{\theta}{1 + n} f(k))} & -\frac{[(1 - \tau)(1 - \alpha) - \theta][A + A_{k_t} \cdot (k + \frac{\theta}{1 + n} f(k))] + A\theta}{A_{m_{t+1}} \cdot (k + \frac{\theta}{1 + n} f(k))} \cdot \frac{f'(k)}{1 + n} \\ -1 & [(1 - \tau) \cdot (1 - \alpha) - \theta] \cdot \frac{f'(k)}{1 + n} \end{bmatrix}$$

Exploiting the steady-state relations  $f'(k) = \alpha \cdot \frac{1+n}{\frac{(1-\tau)(1-\alpha)}{1+A} - \theta}$  and  $k + \frac{\theta}{1+n} \cdot f(k) = m/A > 0$ , one obtains

$$Det(J) = \frac{\alpha \cdot (1+A)}{A_{m_{t+1}} \cdot (k + \frac{\theta}{1+n} \cdot f(k))} + [(1-\tau) \cdot (1-\alpha) - \theta] \cdot \frac{f'(k)}{1+n} > 0,$$

$$Tr(J) = 1 + \frac{1 + A + A_{k_t} \cdot (k + \frac{\theta}{1+n} \cdot f(k))}{A_{m_{t+1}} \cdot (k + \frac{\theta}{1+n} \cdot f(k))} + [(1-\tau) \cdot (1-\alpha) - \theta] \cdot \frac{f'(k)}{1+n} > 0,$$

with 
$$1 + Det(J) - Tr(J) = \frac{(\alpha - 1)(1 + A) - A_{k_t} \cdot (k + \frac{\theta}{1 + n} \cdot f(k))}{A_{m_{t+1}} \cdot (k + \frac{\theta}{1 + n} \cdot f(k))} < 0$$
. Thus, eigenvalues of  $J$  obey  $0 < \lambda_1 < 1, \lambda_2 > 1$ .

#### Proof of proposition 10:

Using again  $\tilde{z} \cdot I_t^{\frac{\varepsilon-1}{\varepsilon}} = A(k_t, m_t, m_{t+1})$ , let B = A/(1+A), with partial derivatives  $B_{k_t} = A_{k_t}/(1+A)^2 > 0$ ,  $B_{m_t} = A_{m_t}/(1+A)^2 < 0$ ,  $B_{m_{t+1}} = -B_{m_t} > 0$ . Moreover, for brevity let  $\varphi = \frac{1+\frac{\mu}{1+\mu}A}{1+A}(1-\tau)(1-\alpha)-\xi$ . Similar to the previous proof, dynamics

 $<sup>\</sup>overline{\phantom{a}^{24}\text{Upon recursive substitution for } m_t \text{ and } m_{t+1} \text{ one could equivalently obtain a second-order difference equation in } k_{t+1}, k_t, \text{ and } k_{t-1}, \text{ with one predetermined value } k_{t-1}.$ 

are of second order and we approximate (60) and (61)

$$k_{t} + \frac{1}{1+\mu} \cdot m_{t} = [(1-\tau) \cdot (1-\alpha) - \xi] \cdot \frac{f(k_{t-1})}{1+n}$$

$$m_{t} = \frac{(1-\tau) \cdot (1-\alpha)}{1+\widetilde{z}^{-1} \cdot [R(k_{t}) \cdot \frac{p_{t+1}}{p_{t}}]^{\frac{1-\varepsilon}{\varepsilon}}} \cdot \frac{f(k_{t-1})}{1+n},$$

around the steady state (62) by

$$\left[\begin{array}{c} d\,m_{t+1} \\ d\,k_t \end{array}\right] = J \cdot \left[\begin{array}{c} d\,m_t \\ d\,k_{t-1} \end{array}\right],$$

$$J = \begin{bmatrix} 1 + \frac{1}{1+\mu} \frac{B_{k_t}}{B_{m_{t+1}}} + \frac{\varphi/k}{(1-\tau)(1-\alpha)B_{m_{t+1}}} & -\frac{B_{k_t}}{B_{m_{t+1}}} \frac{[(1-\tau)(1-\alpha)-\xi]f'(k)}{1+n} - \frac{(1-\tau)(1-\alpha)B\alpha/k}{(1-\tau)(1-\alpha)B_{m_{t+1}}} \\ -\frac{1}{1+\mu} & [(1-\tau)(1-\alpha)-\xi] \frac{f'(k)}{1+n} \end{bmatrix},$$

where we have used the steady-state relations  $\varphi/k = (1+n)/f(k), (1+n)m/f(k) = (1-\tau)(1-\alpha)B$ . After some tidying-up, one obtains

$$Det(J) = \frac{[(1-\tau)(1-\alpha)-\xi]f'(k)}{1+n} + \alpha \frac{(1+n)/f(k)}{(1-\tau)(1-\alpha)B_{m_{t+1}}} > 0,$$

$$Tr(J) = 1 + \frac{[(1-\tau)(1-\alpha)-\xi]f'(k)}{1+n} + \frac{(1+n)/f(k)}{(1-\tau)(1-\alpha)B_{m_{t+1}}} - \frac{m}{1+\mu} \frac{R'(k)}{R(k)} > 0,$$

with 
$$1 + Det(J) - Tr(J) = (\alpha - 1) \frac{(1+n)/f(k)}{(1-\tau)(1-\alpha)B_{m_{t+1}}} + \frac{m}{1+\mu} \frac{R'(k)}{R(k)} < 0$$
. Hence,  $0 < \lambda_1 < 1$ ,  $\lambda_2 > 1$ .

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