

Incorporating prediction and estimation risk in point-in-time credit portfolio models

Alfred Hamerle

(Universität Regensburg)

Michael Knapp

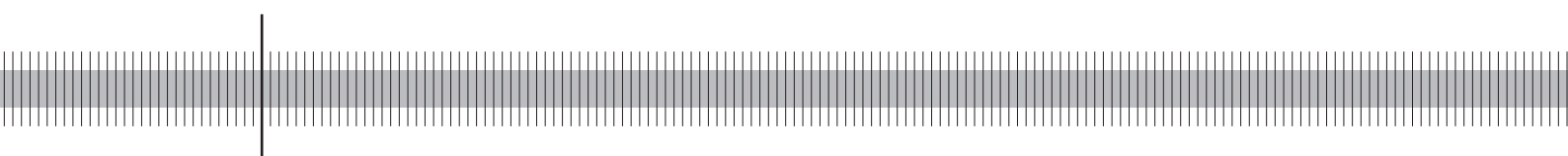
(Universität Regensburg)

Thilo Liebig

(Deutsche Bundesbank)

Nicole Wildenauer

(Universität Regensburg)



Discussion Paper
Series 2: Banking and Financial Studies
No 13/2005

Editorial Board:

Heinz Herrmann
Thilo Liebig
Karl-Heinz Tödter

Deutsche Bundesbank, Wilhelm-Epstein-Strasse 14, 60431 Frankfurt am Main,
Postfach 10 06 02, 60006 Frankfurt am Main

Tel +49 69 9566-1

Telex within Germany 41227, telex from abroad 414431, fax +49 69 5601071

Please address all orders in writing to: Deutsche Bundesbank,
Press and Public Relations Division, at the above address or via fax +49 69 9566-3077

Reproduction permitted only if source is stated.

ISBN 3-86558-101-3

Abstract

In this paper we focus on the analysis of the effect of prediction and estimation risk on the loss distribution, risk measures and economic capital. When variables for the determination of probability of default and loss distribution have to be predicted because they are not available at the time the prediction is made, the prediction is prone to errors. The model parameters for the estimation of probability of default or asset correlation are not available, and usually have to be estimated using historical data. The incorporation of prediction and estimation risk generally leads to broader loss distributions and therefore to rising values of risk parameters such as Value at Risk or Expected Shortfall. The level of economic capital required may be strongly underestimated if prediction and estimation risk are ignored.

Keywords: probability of default, PD, credit risk, default correlation, asset correlation, point in time, value at risk, estimation risk, credit portfolio models, credit risk management

JEL Codes: C1, G21

Non-technical Summary

The present paper studies the impact of the incorporation of estimation and prediction risk in credit portfolio models. It particularly addresses potential changes in the loss distribution and the risk variables, such as Value at Risk (VaR) or Expected Shortfall, and in the economic capital. Generally speaking, the input parameters of a credit portfolio model, such as probability of default (PD) and default correlations, must be estimated from empirical data. The resulting estimated values are prone to uncertainty and errors and are generally inconsistent with the “true” values. Moreover, the PD models often contain predictions for risk factors, eg macroeconomic factors, which can contain prediction errors. This article develops methods to incorporate estimation and prediction errors adequately in loss distribution simulations. A data set of the Deutsche Bundesbank is used to demonstrate the practical implementation. This set comprises the master, financial accounting and default data of some 60,000 West German firms for the years 1989 to 2003. The key results of the study are as follows.

1. The inclusion of estimation and prediction risk generally leads to a broader loss distribution and to an (in some cases distinct) increase in VaR and Expected Shortfall.
2. Required economic capital may be strongly underestimated if prediction and estimation risk are ignored.
3. A relatively long data history and sizeable portfolios are advantageous as, in those cases, risk parameters can be estimated more precisely, thereby reducing, *ceteris paribus*, the estimation risk.
4. For the risk parameter loss given default (LGD), which is assumed to be a constant here, estimation and prediction risk need to be incorporated as well.

Nichttechnische Zusammenfassung

Der vorliegende Beitrag untersucht die Auswirkungen der Einbeziehung von Schätz- und Prognoserisiken in Kreditportfoliomodellen. Insbesondere geht es dabei um mögliche Veränderungen der Schadensverteilung sowie von Risikokennzahlen wie Value at Risk oder Expected Shortfall und des ökonomischen Eigenkapitals. In der Regel müssen die Inputparameter eines Kreditportfoliomodells wie z.B. Ausfallwahrscheinlichkeiten und -korrelationen aus empirischen Daten geschätzt werden. Die resultierenden Schätzwerte sind mit Schätzfehlern behaftet und stimmen im Allgemeinen nicht mit den „wahren“ Werten überein. Ferner enthalten die Modelle für Ausfallwahrscheinlichkeiten häufig Prognosen für Risikofaktoren, z.B. makroökonomische Faktoren, die ebenfalls fehlerbehaftet sein können. In diesem Artikel werden Verfahren entwickelt, wie Schätz- und Prognosefehler bei der Simulation der Schadensverteilung adäquat berücksichtigt werden können. Die praktische Umsetzung wird anhand eines Datensatzes der Deutschen Bundesbank demonstriert. Dieser umfasst Stamm-, Bilanz- und Ausfalldaten von bis zu 60.000 westdeutschen Unternehmen für die Jahre 1989 bis 2003. Die zentralen Ergebnisse der Studie sind:

1. Die Einbeziehung von Schätz- und Prognoserisiken führt im Allgemeinen zu breiteren Verlustverteilungen und zu einer (in einigen Fällen deutlichen) Erhöhung der Risikokennzahlen Value at Risk bzw. Expected Shortfall.
2. Werden Schätz- und Prognoserisiken ignoriert, kann dies eine deutliche Unterschätzung des ökonomisch notwendigen Eigenkapitals zur Folge haben.
3. Längere Datenhistorien und größere Portfolien sind von Vorteil, da in diesen Fällen die Risikoparameter genauer geschätzt werden können, wodurch ceteris paribus das Schätzrisiko verringert wird.
4. Für den Risikoparameter LGD, der hier als konstant angenommen wird, sind ebenfalls Schätz- bzw. Prognoserisiken zu berücksichtigen.

Content

1	Introduction	1
2	Model approach	3
2.1	Simultaneous modelling of default probabilities and correlations	3
2.2	Dynamic obligor-specific modelling	6
2.3	Parameter estimation	8
2.4	Predicting the loss distribution	9
2.5	Incorporating prediction risk	10
2.6	Incorporating estimation risk	11
3	Empirical results	12
3.1	The data	12
3.2	Models	12
3.2.1	Obligor-specific and macroeconomic risk factors	12
3.2.2	Model specifications	14
3.3	Simulation of the loss distribution	15
3.4	Simulation results and interpretation	16
4	Conclusions	21
5	References	23

Incorporating Prediction and Estimation Risk in Point-in-Time Credit Portfolio Models

1 Introduction

In recent years, financial researchers and banks have focused increasingly on credit risk measurement and management. The new capital adequacy framework drawn up by the Basel Committee on Banking Supervision (BCBS) decisively fostered this development. Under this framework, credit institutions are permitted to use internal estimates of the relevant determinants of obligors' credit risk, such as the probability of default (PD), loss given default (LGD) and exposure at default (EAD). Default correlations represent another key credit risk variable when assessing a credit portfolio. Modelling default correlations and correlations in the changes in credit ratings also poses a great challenge, and further research needs to be done on this topic.

A number of models exist to capture and quantify credit portfolio risk. Of these, banks most commonly use the CreditMetrics, CreditRisk+, Moody's KMV and CreditPortfolioView models. At first glance, these models have different structures and loss distribution results often vary substantially from model to model. This may be attributed above all to the different distribution assumptions and the diverse procedures used to parametrise the input parameters. However, it can be shown (see Koyluoglu and Hickman (1998), Gordy (2000) and Hamerle and Rösch (2005)) that the distribution moments of default rates in all three models can be transformed into each other and that the predicted loss distributions then no longer differ significantly, provided that the specification of input parameters is comparable.

Prediction and estimation risk, which have hardly been analysed in connection with credit risk models up to now (see, however, Knapp (2002) and Löffler (2003)), represent a further source of uncertainty.

When using a credit risk model, all input parameters must generally be estimated or predicted. For example, the approximate PD of obligors in a particular rating class may be identified via the default rate (or average default rates). Alternatively, a statistical default model that delivers individual PD predictions may be specified. All estimates and predictions are uncertain and are prone to errors. This is also true for the other risk

parameters, eg default correlations, LGD etc. This paper examines the adequate integration of estimation and prediction risks into credit portfolio models.

To simultaneously assess the PD and default correlations, statistical default models containing systematic and unsystematic risk components are defined as factor models. The systematic risk component is split into an observable and an unobserved part. The observable part is captured by means of macroeconomic risk factors, the unobserved part is modelled by means of random time effects. In addition, the models also contain obligor-specific information, for example, in the form of a rating score. The unknown model parameters are estimated using the maximum likelihood method. Maximum likelihood theory also helps to provide the variances and covariances of the parameter estimates that are required to factor in estimation and prediction risk to determine the loss distribution. As this analysis does not focus on the risk parameters EAD and LGD, they are specified deterministically. In a next step, the loss distributions are simulated. Several model specifications are compared including and excluding estimation and prediction risk.

The remainder of the paper is organised as follows:

Section 2 describes methods for estimating obligors' PD and loss distribution and integrating estimation and prediction risk. Section 3 presents the data set of the Deutsche Bundesbank used for the analysis. We present the four estimation and prediction models for PD that were used for the evaluation and the empirical model results. The simulation approach applied to determine the predicted loss distribution is also depicted in this section. This section closes with the analysis of the empirical loss distribution and an interpretation of the models and their results and conclusions. Section 4 summarises the results of this paper and concludes with an outlook.

2 Model approach

2.1 Simultaneous modelling of default probabilities and correlations

The states “default” and “non-default” of obligor i in period t – in most applications one year – are modelled using the indicator variable D_{it} , ie

$$D_{it} = \begin{cases} 1 & \text{borrower } i \text{ defaults in period } t \\ 0 & \text{otherwise} \end{cases}$$

$i \in N_t$, $t = 1, \dots, T$.

N_t denotes the “risk set” consisting of all obligors who are not defaulted at the beginning of period t .

Alternatively, the default event may be triggered in the model when a metric variable R_{it} ($i \in N_t$, $t = 1, \dots, T$) falls below a prescribed threshold at a particular point in time (in the observed period) t . The seminal works of Merton (1974, 1977) and Black and Scholes (1973) laid the fundament for this approach.

The firm value model relies on a level of company information that is rarely available in practice. Accurate information about the capital structure must be available, and it must be possible to determine the assumed tradable firm value of the company at all times. However, this is usually not the case for small and medium-sized enterprises, which account for the lion's share of banks' portfolios. Therefore, the random variable R_{it} ($i \in N_t$, $t = 1, \dots, T$) which triggers the default event is assumed to be latent and unobservable. Only the default event itself can be observed. By analogy to the firm value model, the default event is assumed to occur when R_{it} falls below a threshold c_{it} , ie

$$R_{it} < c_{it} \Leftrightarrow D_{it} = 1. \quad (1)$$

As in the case of index or factor models used to determine capital market risk, we assume that default risk can be split into systematic and unsystematic risk components.

The systematic risk components F_t ($t = 1, \dots, T$) apply to all firms in a particular risk segment (eg a sector) during period t and hence cannot be diversified. Unsystematic risks U_{it} ($i \in N_t$, $t = 1, \dots, T$) only affect a single firm.

A simple single-factor model is assumed for random variable R_{it} , which triggers the default event, in equation (2), ie

$$R_{it} = \sqrt{\rho} F_t + \sqrt{1-\rho} U_{it} \quad (2)$$

($i \in N_t$, $t = 1, \dots, T$), where $F_t \sim N(0,1)$ represents standard normally distributed, systematic risk components, ie components that have an impact on all firms at a specified time and thus are not diversifiable. Unsystematic firm-specific (and hence diversifiable) risk drivers U_{it} are also assumed to have a standard normal distribution.

Moreover, the unsystematic risks of different firms are assumed to be independent of each other and of the systematic risk factors F_t .

Equation (2) represents the CreditMetrics (default mode) model. It also corresponds to the specification of the Basel II model. The correlation of the threshold values R_{it} and R_{jt} of two companies is given by ρ . By analogy to the firm value model, these variables may be interpreted as normalised returns of the firm value. Therefore ρ is frequently referred to as the asset correlation. It is assumed that the value of ρ is always the same within a risk segment, for example, a sector, whereas another single-factor model with a different value of ρ may apply within a different sector.¹

Together with equation (1) and given threshold c_{it} , we obtain the conditional PD given the systematic risk factor F_t

$$\lambda_{it}(F_t) = P(R_{it} < c_{it}) = P\left(U_{it} < \frac{c_{it} - \sqrt{\rho} F_t}{\sqrt{1-\rho}}\right) = \Phi\left(\frac{c_{it} - \sqrt{\rho} F_t}{\sqrt{1-\rho}}\right). \quad (3)$$

For a given realisation f_t of the systematic factor of a risk segment, only unsystematic risk, which is assumed to be independent, is effective. Accordingly, joint default by two obligors in period t is conditionally independent with the respective (conditional) probabilities (3).

¹ By contrast, the Basel II model assumes one single-factor model for the entire credit portfolio.

The determination of PDs according to (3) is implicitly contingent on the additional condition that the obligor has not defaulted in the preceding periods. Thus, the probabilities in (3) may also be seen as "time-discrete hazard rates" (see eg Hamerle and Tutz (1989)). The models examined here are simple, time-discrete versions of the intensity-based models (see, for example, Duffie and Singleton (1999) and Jarrow and Turnbull (1995)) frequently used in risk neutral valuation of defaultable bonds or credit derivatives. Intensity-based models are formulated in continuous time and represent generalisations of time-discrete hazard rate models.

The unconditional PD is obtained as the expected value of the distribution of F_t , ie

$$\lambda_{it} = E(\lambda_{it}(F_t)) = \int_{-\infty}^{\infty} \lambda_{it}(f_t) \varphi(f_t) df_t, \quad (4)$$

where $\varphi(z)$ indicates the density function of the standard normal distribution.

In the unconditional approach, the defaults occurring during a period are correlated. It follows that

$$\rho(D_{it}, D_{jt}) = \frac{\lambda_{ijt} - \lambda_{it} \lambda_{jt}}{\sqrt{\lambda_{it}(1 - \lambda_{it})} \sqrt{\lambda_{jt}(1 - \lambda_{jt})}}, \quad (5)$$

where

$$\lambda_{ijt} = \int_{-\infty}^{\infty} \lambda_{it}(f_t) \lambda_{jt}(f_t) \varphi(f_t) df_t = \Phi_2(c_{it}, c_{jt}, \rho)$$

is the joint PD of obligors i and j in period t . $\Phi_2(z_1, z_2, \rho)$ denotes the distribution function of the two-dimensional standard normal distribution with the correlation parameter ρ . The default correlations between pairs of obligors generally differ, as the default thresholds may be obligor-specific.

Examining the joint defaults of N_t obligors in a given risk segment assuming a given value of f_t in period t , we obtain

$$P(D_{1t} = d_{1t}, \dots, D_{N,t} = d_{N,t} | f_t) = \prod_{i \in N_t} \lambda_{it}(f_t)^{d_{it}} [1 - \lambda_{it}(f_t)]^{(1-d_{it})} \quad (6)$$

where $d_{it} \in \{0,1\}$.

Finally, the unconditional joint probability of default in period t is

$$P(D_{1t} = d_{1t}, \dots, D_{N,t} = d_{N,t}) = \int_{-\infty}^{\infty} \prod_{i \in N_t} \lambda_{it}(f_t)^{d_{it}} [1 - \lambda_{it}(f_t)]^{(1-d_{it})} \varphi(f_t) df_t. \quad (7)$$

2.2 Dynamic obligor-specific modelling

In a next step, obligor-specific and systematic risk factors which are observable in dynamic modelling are integrated into the modelling approach.

The systematic risk components are split into an observable and an unobserved part. The observable components capture changes in the macroeconomic environment, specifically cyclical developments. Macroeconomic indicators such as interest rates, the unemployment rate, the GDP growth rate, new orders or the external value of the euro are the key variables. Furthermore, it has been determined that the major macroeconomic risk drivers affect PD with a delay of about one or two years; see also Rösch (2003) and Hamerle, Liebig and Rösch (2003). Consequently, modelling and prediction of PD may be based on the known values of these risk factors.

This paper chooses a different method to demonstrate how prediction risk can be accounted for. Predictions are based on a multi-factor model.² Using this approach, we model sectoral insolvency rates as dependent on the macroeconomic factors and cyclical indicators described above. As a rule, the macroeconomic factors chosen have an impact with a delay of one to two years.

A very simplified model is estimated in this paper. We consider only a single sector. In a linear regression model, the transformed³ insolvency rate

² See Knapp and Hamerle (1999).

³ This transformation is often used in econometrics and ensures predicted insolvency rates to be between 0 and 1.

$$y_t = \ln \frac{1 - z_{t,MFM}}{z_{t,MFM}}$$

($z_{t,MFM}$: insolvency rate of year t in the construction sector⁴) can be estimated as a function of public sector gross fixed investment (construction investment) in the preceding year and the construction sector's business climate index in the preceding year.⁵

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 z_{t-1,investment} + \alpha_3 z_{t-1,climate} + \varepsilon_t. \quad (8)$$

The estimation results for model (8) are presented in section 3.2.

On the basis of model (8), predictions \hat{y}_t of the transformed insolvency rate can be made first. Next, by means of inverse transformation, predictions $\hat{z}_{t,MFM}$ of the current period t may be estimated and then integrated into the estimate for λ_{it} . The corresponding regression parameter in the PD model is denoted by γ , see equations (9) and (10), respectively. Additionally, some unobserved systematic risk remains in the model which is captured by the random effect variable F_t . This risk may be caused by contemporary systematic risk factors whose values are unknown at the time the rating or the PD prediction is calculated.

Obligor-specific risk factors may enter in the form of rating information, for instance, rating classes or individual rating scores. The size, legal form or age of the firm may represent additional factors. The information is assumed to have been recorded at time $t-1$ and is summarised in vector \mathbf{x}_{it-1} . The corresponding parameter vector is denoted by $\boldsymbol{\beta}$.

Hence, individual, time-dependent default thresholds c_{it} are determined for each obligor by means of dynamic individual modelling according to

$$c_{it} = \beta_0 + \boldsymbol{\beta}' \mathbf{x}_{it-1} + \gamma \hat{z}_{t,MFM}. \quad (9)$$

⁴ The abbreviation MFM denotes the multi-factor model used to predict the insolvency rate.

⁵ Several macroeconomic variables, eg business climate index, unemployment rate and interest rate were included in the model. (8) shows the best model for the estimation and prediction of the insolvency rate in the construction sector.

Taking (3) into account, probit specification leads to

$$\lambda_{it}(f_t) = \Phi\left(\frac{\beta_0 + \boldsymbol{\beta}' \mathbf{x}_{it-1} + \gamma \hat{z}_{t,MFM} - \sqrt{\rho} f_t}{\sqrt{1-\rho}}\right). \quad (10)$$

2.3 Parameter estimation

A central assumption in modelling the default distribution of a given number N_t of obligors in a segment (eg a sector) during time t is that unsystematic risk is independent. For given realisations f_t of the unobserved systematic risk components as well as of \mathbf{x}_{it-1} and $\hat{z}_{t,MFM}$ the following relationship holds

$$P(D_{1t} = d_{1t}, \dots, D_{N_t t} = d_{N_t t} | f_t) = \prod_{i \in N_t} \lambda_{it}(f_t)^{d_{it}} [1 - \lambda_{it}(f_t)]^{(1-d_{it})} \quad (11)$$

where $d_{it} \in \{0,1\}$ and $\lambda_{it}(f_t)$ are given by probit specification (10).

As f_t is unobserved, (11) cannot be used to estimate the parameters by the maximum likelihood method. Instead, the unconditional joint probabilities of default must be used for estimation. According to (7), for given values of \mathbf{x}_{it-1} and $\hat{z}_{t,MFM}$ it follows that

$$P(D_{1t} = d_{1t}, \dots, D_{N_t t} = d_{N_t t}) = \int_{-\infty}^{\infty} \prod_{i \in N_t} \lambda_{it}(f_t)^{d_{it}} [1 - \lambda_{it}(f_t)]^{(1-d_{it})} \varphi(f_t) df_t .$$

If all periods $t = 1, \dots, T$ are taken into account, we obtain for the log likelihood function of the overall model

$$\begin{aligned} & l(\beta_0, \boldsymbol{\beta}, \gamma, \rho) \\ &= \sum_{t=1}^T \ln \left\{ \int_{-\infty}^{\infty} \prod_{i \in N_t} \lambda_{it}(f_t)^{d_{it}} [1 - \lambda_{it}(f_t)]^{(1-d_{it})} \varphi(f_t) df_t \right\} \end{aligned} \quad (12)$$

where $\lambda_{it}(f_t)$ is given by (10).

The integral on the right-hand side of (12) can be evaluated using numerical methods such as the Gauss-Hermite procedure.

Variances and covariances of the parameter estimates are required to take account of the estimation risk. These estimates can be derived from the theory of maximum likelihood estimation. While the result is only an asymptotical formula, it may serve as a very useful proxy considering the large samples usually available in credit risk modelling.

2.4 Predicting the loss distribution

Once the unknown parameters have been estimated for the given model specification, these estimates can be used to calculate predictions. If the estimation period covers T periods, the predictions refer to period $T+1$. The values \mathbf{x}_{iT} and $\hat{\mathbf{z}}_{T+1,MFM}$ are then used in the dynamic individual approach. This allows us to obtain the individual predictions of PD.

$$\hat{\lambda}_{iT+1} = \hat{\lambda}(\mathbf{x}_{iT}, \hat{\mathbf{z}}_{T+1,MFM}).$$

For credit risk models the predicted loss distribution for the next period $T+1$ (for example, one year) must be calculated. The loss distribution may then be used to calculate risk measures such as expected loss, unexpected loss, Value at Risk and Expected Shortfall.

This paper assumes a loss (rate) given default of 45% at all times, as the analysis does not focus on this risk parameter.⁶ Moreover, various EAD values are specified. These values are based on a German bank's actual loan portfolio. The values span a range of 21.94 to 340.31 monetary units, with an average of 101.89 monetary units. The total exposure comes to 60,930.27 monetary units.

As the predicted loss in the next period $T+1$, we obtain

$$L_{T+1} = \sum_{i \in N_{T+1}} D_{iT+1} EAD_{iT+1} LGD_{iT+1},$$

where N_{T+1} indicates the number of obligors in the portfolio at the beginning of period $T+1$.

⁶ The value of 45% is chosen in line with the LGD value assigned under the foundation internal ratings based (FIRB) approach applicable to senior claims on corporates, sovereigns and banks not secured by recognised collateral corporate, bank and sovereign senior unsecured exposures (see Basel Committee on Banking Supervision (2004), paragraph 287).

Calculating a “relative” predicted loss expressed as a percentage of the total exposure, we obtain

$$L_{T+1}^* = \frac{L_{T+1}}{\sum_{i \in N_{T+1}} EAD_{iT+1}}.$$

Since EAD and LGD are deterministic, the joint distribution (7) for the predicted defaults is the basis for calculating the predicted loss distribution. This distribution is above all based on the parameter estimates to predict $\hat{\lambda}_{iT+1}$ as well as the parameter estimates for the distribution of the unobserved systematic risk component F_t . We obtain

$$\begin{aligned} & \hat{P}(D_{1T+1} = d_{1T+1}, \dots, D_{N_{T+1}T+1} = d_{N_{T+1}T+1}) \\ &= \int_{-\infty}^{\infty} \prod_{i \in N_t} \hat{\lambda}_{iT+1}(f_{T+1})^{d_{iT+1}} [1 - \hat{\lambda}_{iT+1}(f_{T+1})]^{(1-d_{iT+1})} \varphi(f_{T+1}) df_{T+1}. \end{aligned} \quad (13)$$

In the case of dynamic individual modelling, simulations generally have to be used to predict the distribution, as $2^{N_{T+1}}$ different probabilities would have to be calculated to obtain the precise predicted distribution of loss L_{T+1} .

2.5 Incorporating prediction risk

The values $\hat{z}_{T+1,MFM}$ which are entered into the predicted loss distribution are forecasts. These forecasts are uncertain and contain prediction errors, meaning that the actual insolvency rate of the next year may deviate from the predicted rate. To account for the uncertainty, new predictions are produced in each simulation run using model (8) by first generating the error term ε_{T+1} randomly and then using the result to calculate the (realised) prediction $z_{T+1,MFM}$.⁷ The procedure for simulating the predicted loss distribution is described in detail in section 3.3.

The inclusion of prediction risk described above has the same impact as an increase in the variance of the random time effect F_t and hence the same impact as an increase in the asset correlation.

⁷ In CreditPortfolioView a comparable procedure is used (see Wilson (1997a, b)).

2.6 *Incorporating estimation risk*

Incorporating estimation risk into the calculation of the predicted loss distribution of credit portfolio models is an important extension to the model. Jorion (1996) pointed out the issue of integrating estimation risk in the context of measuring market risk. Very little literature is available on integrating estimation risk in credit risk assessment. Löffler (2003) provides some examples of the possible effects of estimation risk on the risk measurement of loan portfolios. Knapp (2002) also provides examples of how to take estimation risk into account.⁸

The parameter estimates contained in (13) are point estimates which, in turn, may be subject to estimation errors. Especially if the portfolios are small or the data history is short, or both, these errors may well be substantial. As a consequence, the values of risk measures such as Value at Risk or Expected Shortfall must be considered as estimates themselves and may deviate more or less substantially from true values. In such cases, confidence intervals containing the true values may be specified. This paper specifies the distributions of Value at Risk and Expected Shortfall by means of simulations. The upper and lower bounds of the confidence intervals can then be derived.

The joint probability distribution of the maximum likelihood estimates of the model parameters is needed to simulate the distribution of the risk measures. It follows from the theory of maximum likelihood estimation that the respective maximum likelihood estimates are asymptotically normally distributed. In addition, the covariance matrix can be estimated.⁹ The resulting (approximative) multivariate normal distribution may be used for random sampling of different realisations of parameter estimations. In a next step, a complete loss distribution is simulated for every realised combination. The simulation procedure is described in detail in section 3.3.

⁸ See Knapp (2002), p 178 ff.

⁹ See, for example, Fahrmeier, Hamerle and Tutz (1996), section 2.3.

3 Empirical results

3.1 The data

Empirical evaluations are based on a data set of the Deutsche Bundesbank. This data set comprises the master, financial accounting and default data of up to 60,000 West German firms from 1989 to 2003. Insolvency as defined by the Insolvency Code (Insolvenzordnung) in effect from 1 January 1999 was taken as the default criterion.¹⁰ Prior to 1999 the default criterion for West Germany was insolvency or over indebtedness as outlined in the bankruptcy code.¹¹ In this paper our analyses focus on the construction sector.

As the data is limited to companies that submitted bills to the Deutsche Bundesbank for rediscounting, both the sectoral and the period insolvency rates of the original data are lower than the insolvency rate of West German enterprises as a whole from 1989 to 2003. Therefore, the insolvency rate of the original data set was adjusted to the sectoral insolvency rate in West Germany.¹² Figure 1 in section 3.2.1 shows the insolvency rate of the construction sector in West Germany.

3.2 Models

3.2.1 Obligor-specific and macroeconomic risk factors

We used the model presented in equation (10) in section 2.2. Only a single obligor-specific risk factor is used in this paper, ie a rating score, specified as $x_{i,t,Score}$. This score was developed on the basis of the available data and incorporates a variety of key balance sheet indicators.¹³ Lower scores indicate a higher creditworthiness.

The estimation results of the multi-factor model for the construction sector are shown in Table 1.

¹⁰ The default criterion was defined as D_{it} in section 2.1.

¹¹ See Scheule (2003), p 113.

¹² For additional reasons for this calibration, see Scheule (2003), p 115 f.

¹³ The score for the construction sector includes the following key indicators: the ratio of bank liabilities to assets, the ratio of liabilities arising from goods and services (trade creditors) to net turnover, the ratio of staff costs to net turnover and the ratio of the profit or loss on ordinary activities to operational income. See also Falke (2005).

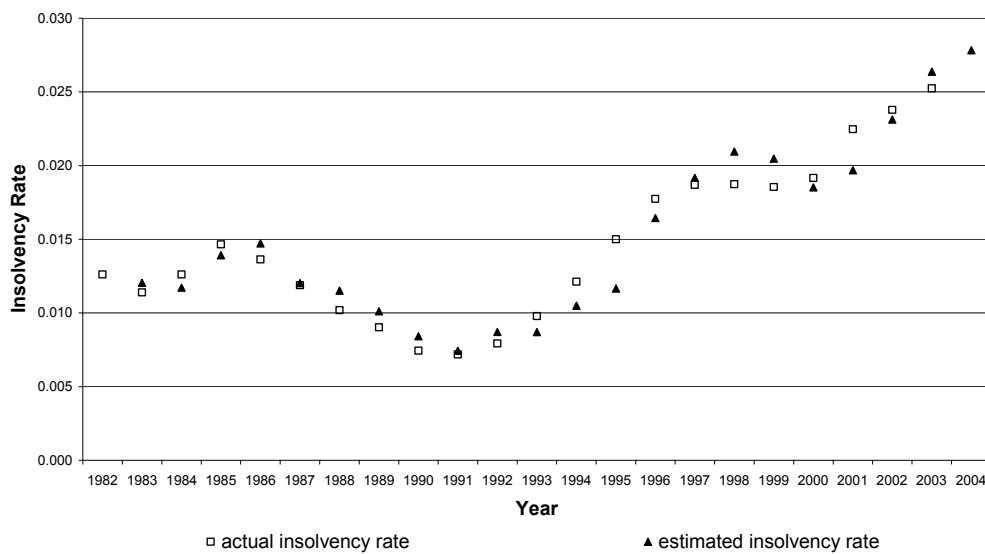
Table 1: Results for model (8)

Parameter	Estimate	Standard error	p-value
α_0	0.199695	0.334006	0.5578
α_1	0.511316	0.190627	0.0157
α_2	0.121410	0.041478	0.0094
α_3	0.008513	0.004747	0.0907

Thus, if last year's insolvency rate or public sector gross fixed capital formation (construction investment) or the construction sector's business climate index rise, the insolvency rate of the construction sector declines, respectively.¹⁴ These relationships are significant at the 10% level. The estimated value of R^2 is 88.8%, and the estimated error variance of the regression model (8) is $\hat{\sigma}_e^2 = 0.0125$.

The prediction of the annual insolvency rate of the construction sector (West Germany) for 2004 is given by $\hat{z}_{2004,MFM} = 2.78\%$. A real-fit-diagram of actual and estimated insolvency rates is shown in Figure 1.

Figure 1: Actual and estimated insolvency rates in the construction sector



¹⁴ This negative relation results from the transformation of the insolvency rate in the model.

3.2.2 Model specifications

The following models are estimated:

Model 1 (neither prediction nor estimation risk included)

In addition to the constant, this model contains the obligor-specific score¹⁵ of the previous year, $x_{i,t-1,Score}$, the sectoral prediction $\hat{z}_{t,MFM}$ of the multi-factor model and a random time effect, f_t . For a given value of f_t we obtain:

$$\lambda_{it}(f_t) = \Phi \left(\frac{\beta_0 + \beta_{Score} x_{i,t-1,Score} + \gamma \hat{z}_{t,MFM} - \sqrt{\rho} f_t}{\sqrt{1-\rho}} \right), \quad (15)$$

where $\Phi(z)$ is the distribution function of the standard normal distribution.

Model 2 (prediction risk included)

In addition to model 1 this model allows for prediction uncertainty of the insolvency rate. The procedure is described in more detail in section 3.3.

Model 3 (estimation risk included)

This model extends model 1 by taking into account the estimation risk. To this end, the joint distribution of parameter estimates $\hat{\beta}_0$, $\hat{\beta}_{Score}$, $\hat{\gamma}$ and $\hat{\rho}$ is used. The procedure is described in more detail in section 3.3.

Model 4 (both prediction and estimation risk included)

Both prediction uncertainty and estimation risk are taken into account in model 4. For more details see section 3.3.

¹⁵ This score was modelled according to Falke (2005).

3.3 Simulation of the loss distribution

A Monte Carlo simulation is used to determine the predicted loss distribution. The loss distribution for 2004 is predicted for 598 non defaulted construction companies represented in the data set of the Deutsche Bundesbank for 2003.

We first describe the most complex situation below, in which both prediction and estimation risk are taken into account. Exclusion of these risk types simplifies the procedure accordingly.

Estimation uncertainty is captured by taking into account the joint distribution of the parameter estimates $\hat{\beta}_0$, $\hat{\beta}_{Score}$, $\hat{\gamma}$ and $\hat{\rho}$. In accordance with the maximum likelihood theory, a multivariate normal distribution is assumed where the covariance matrix is estimated by means of the maximum likelihood estimates.¹⁶ The vector of the maximum likelihood estimates is entered as the mean vector. A realisation of estimates of β_0 , β_{Score} , γ and ρ is sampled from this distribution in an outer loop. Next, the complete loss distribution for this realisation is generated, taking into account the prediction risk. We proceed as follows:

- Step 1: A random realisation f_{T+1} of the systematic risk factor is drawn from a standard normal distribution.
- Step 2: Using the values of $x_{i,T,Score}$ and $\hat{z}_{T+1,MFM}$ for 2003 and 2004, respectively, we compute the predictions of the conditional PD $\hat{\lambda}_{iT+1}(f_{T+1})$, $i \in N_{T+1}$, according to the specifications of models 1 and 2 and according to the parameter estimates obtained from the outer loop.
- The prediction $\hat{z}_{T+1,MFM} = 0.0278$ is used in models 1 and 3. Models 2 and 4, which incorporate prediction risk, are sampled from a distribution of the prediction value $\hat{z}_{T+1,MFM}$. To this end, the transformed insolvency rate y_{T+1} is predicted by sampling the distribution of the error term ε_{T+1} ($\sim N(0;0.0125)$). Using the inverse transformation, we obtain the realisation of the predicted insolvency rate $\hat{z}_{T+1,MFM}$.

¹⁶ The calculation of the covariance matrix estimate is performed with NLMIXED procedure using SAS statistical software.

- Step 3: According to the conditional independence, N_{T+1} Bernoulli events are generated with these conditional PDs. Taking into account the relevant EADs and an LGD of 45% the portfolio loss realisation is calculated.
- Steps 1 through 3 are repeated 10,000 times, and the loss distribution of the portfolio is calculated.

The simulation program then returns to the beginning of the outer loop and generates a further realisation of the parameter estimates. For the new realisation, a complete (conditional) loss distribution is simulated, and so forth. The cycle is repeated 2,000 times.

For each of these 2,000 loss distributions risk measures such as VaR or Expected Shortfall may be calculated in order to generate estimates of the probability distributions for these risk measures. Using the distribution of the risk measures appropriate confidence intervals may be calculated.

On the other hand, the (conditional) loss distributions may be aggregated into a single unconditional loss distribution.

If no estimation risk is taken into account, ie in model 1 and 2, the outer loop of the simulation is not applied.

3.4 Simulation results and interpretation

First, the loss distributions and some of their indicators are provided for all four models, with the risk parameters given as percentage shares of total exposure.

Table 3: Loss distribution risk parameters for models 1 through 4

Portfolio loss	Model 1 (excluding prediction risk, excluding estimation risk)	Model 2 (including prediction risk, excluding estimation risk)	Model 3 (including estimation risk, excluding prediction risk)	Model 4 (including prediction risk, including estimation risk)
Expected loss	0.98128%	0.99664%	0.98891%	1.00266%
99% quantile	1.78260%	1.91625%	1.84433%	2.00234%
99.9% quantile	2.09167%	2.31395%	2.18248%	2.45691%
VaR (99%)	0.80132%	0.91960%	0.85541%	0.99968%
VaR (99.9%)	1.11040%	1.31730%	1.19356%	1.45426%
Expected Shortfall (99%)	1.92113%	2.09323%	1.99356%	2.20157%
Standard deviation	0.30888%	0.34408%	0.32581%	0.36303%

The loss distributions of models 1 through 4 may be represented graphically as follows:

Figure 2: Loss distribution model 1 (excluding prediction and estimation risk)

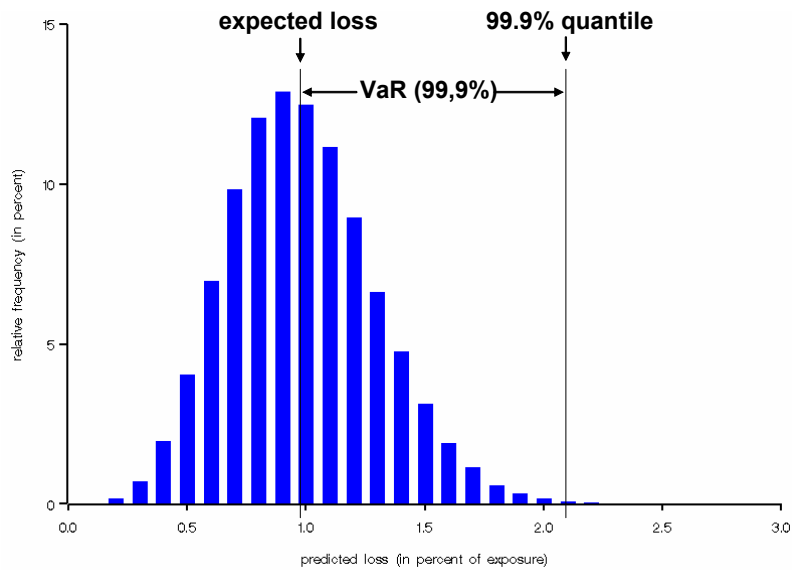


Figure 3: Loss distribution model 2 (including prediction risk, excluding estimation risk)

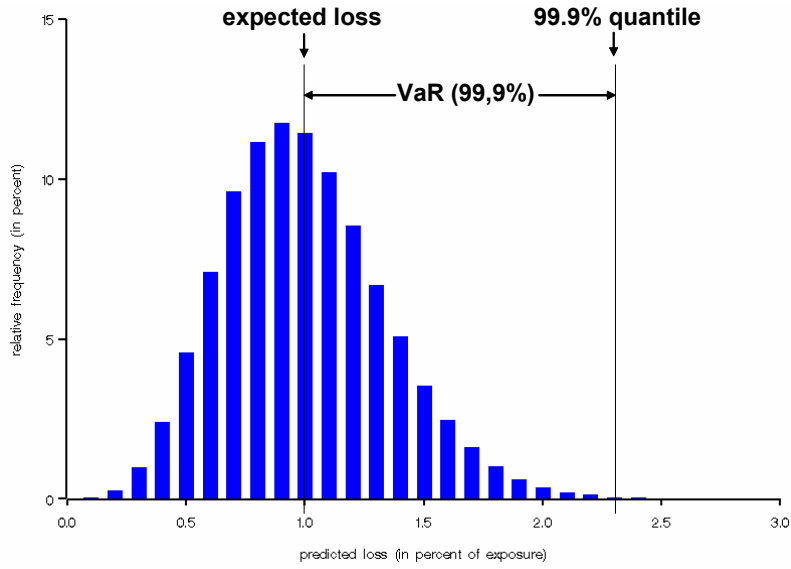


Figure 4: Loss distribution model 3 (including estimation risk, excluding prediction risk)

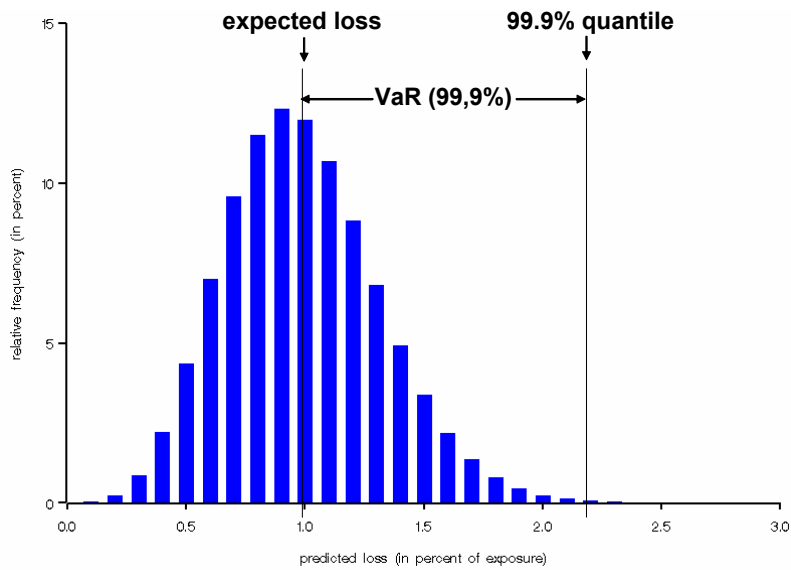
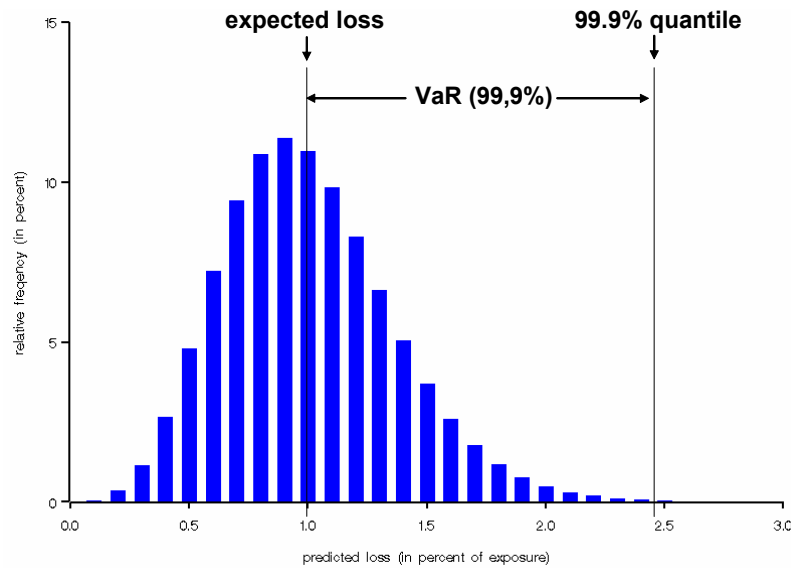


Figure 5: Loss distribution model 4 (including prediction and estimation risk)



A comparison of models 1 and 2 shows that accounting for prediction uncertainty causes expected loss to rise. This result follows from Jensen's inequality, as the convex course of the lower part of the probit transformation curve indicates. For the same reason, expected loss rises slightly in models 3 and 4.

Figures 2 through 5 and table 3 illustrate that the inclusion of prediction and estimation uncertainty leads to “broader” loss distributions. Accordingly, the relevant VaR values (the difference between the respective loss distribution quantile and expected loss) and the Expected Shortfall value increase, in some cases considerably. In particular, the VaR (99.9%) rises by about 7% if estimation risk is included and by 18% if prediction risk is included. If prediction and estimation risk are captured, it rises by some 30% against the benchmark model (which excludes both prediction and estimation risk).¹⁷

Finally, we consider the distributions of the risk parameters VaR (99%) and Expected Shortfall (99%). The empirical distributions can be calculated from the 2,000 realised conditional loss distributions. Table 4 shows selected indicators of both distributions, and Figures 6 and 7 contain the corresponding graphical representations.

¹⁷ The comparison for models 3 and 4 is based on the unconditional loss distribution data. Taking into account the risk measure distributions results in different figures (see the explanation below).

Table 4: Selected VaR (99%) and Expected Shortfall (99%) distribution parameters

	Model 4 VaR (99%)	Model 4 Expected Shortfall (99%)
Mean	0.92736%	2.10129%
Median	0.92136%	2.08965%
90% quantile	1.04651%	2.37637%
95% quantile	1.08711%	2.46289%
99% quantile	1.18271%	2.68772%

Figure 6: VaR distribution (99%) for model 4 (including prediction and estimation risk)

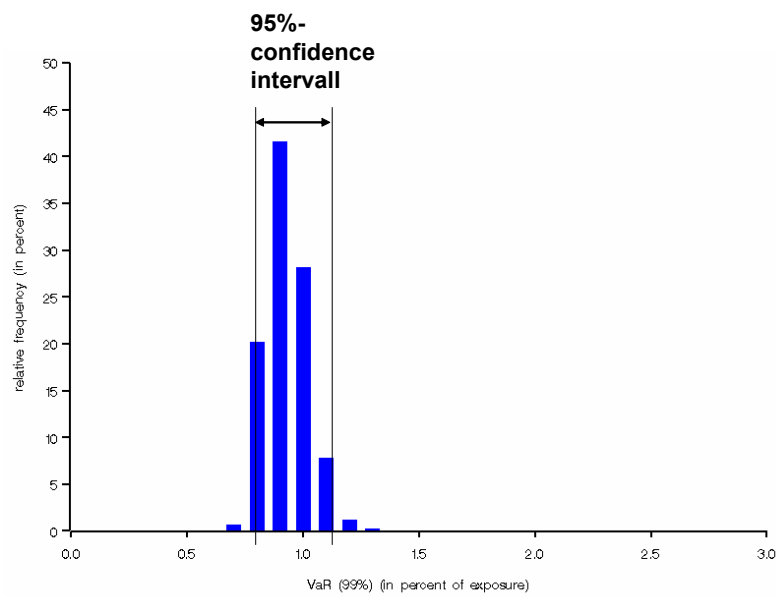
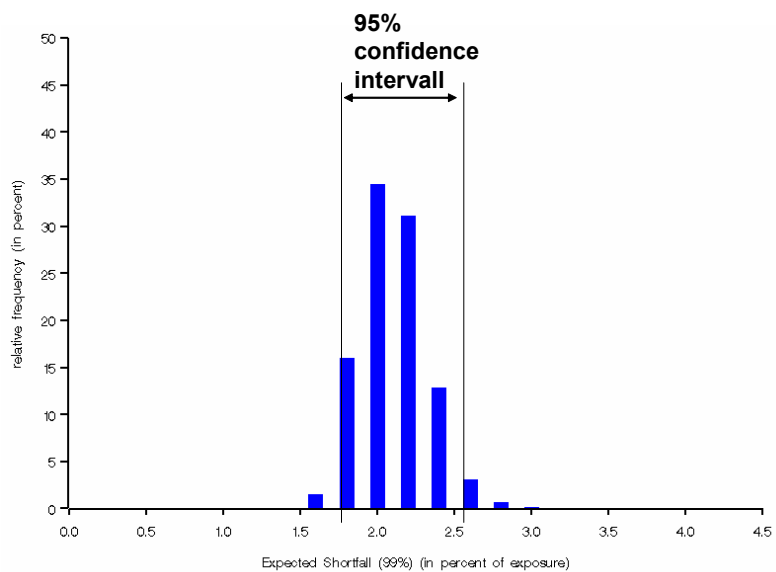


Figure 7: Expected Shortfall distribution (99%) for model 4 (including prediction and estimation risk)



Both the VaR distribution and the Expected Shortfall distribution are considerably right skewed. A 95% confidence interval for the VaR is [0.77109%; 1.11950%] for model 4, a 95% confidence interval for the Expected Shortfall for the same model is [1.72509%; 2.56071%]. Both distributions provide useful information if additional estimation risks are taken into account when calculating capital requirements. If a conservative approach is chosen, the upper bound of the confidence interval, for example, may serve as a risk measure. If the values of VaR (99%) of model 1 and model 2 (0.80132% and 0.91960%) are compared with the upper bound of the 95% confidence interval of the value of VaR (99%) in model 4 (1.11950%), the capital requirements geared to the VaR would rise accordingly by 39% or by 22%.

The Expected Shortfall (99%) also rises, taking into account the estimation risk. If the upper bound of the 95% confidence interval is chosen, Expected Shortfall rises by 33% compared to Expected Shortfall in model 1 and by 22% compared to model 2.

The impact of estimation risk on the loss distribution depends on the quality of the estimates. Better estimates of the model parameters – ie lower variances of the parameter estimates– lead to a smaller impact on the loss distribution.

4 Conclusions

This paper analyses the effects of prediction and estimation risk on the loss distribution of credit portfolio models. The empirical analysis is based on a data set from the Deutsche Bundesbank containing default data of West German firms in the construction sector.

A statistical default model is estimated to predict individual probabilities of default (PDs) of the firms. The model is an individual, dynamic extension of the Basel II probit specification. In addition to a rating score based on key balance sheet indicators, the model takes macroeconomic information into account. The prediction of the insolvency rate in the construction sector serves as a macroeconomic risk factor. We use a multi-factor model based on the lagged values of sectoral insolvency rate, public sector construction investment and the business climate index of the construction sector to predict the insolvency rate of the construction sector.

The unknown parameters of the statistical default model are estimated with the maximum likelihood method. As the distribution of the estimates can be derived from the maximum likelihood theory, the estimation risk can be integrated into the simulation of the predicted loss distribution. The distribution of the insolvency rates can be determined from the estimation of the multi-factor model. EAD is specified on the basis of a bank's actual loan portfolio, and LGD is set at 45% at all times.

It turns out that the credit risk of loan portfolios is substantially underestimated if prediction and estimation risk are not taken into account. If these additional sources of risk are included, risk measures such as VaR or Expected Shortfall increase noticeably, in turn increasing economic capital requirements.

Prediction and estimation risk will gain more and more importance in the future, particularly in the analysis of credit portfolio models with a time horizon of several years.

5 References

Basel Committee on Banking Supervision (2004), "International Convergence of Capital Measurement and Capital Standards: A Revised Framework", June.

Black, Fischer; Scholes, Myron (1973): The Pricing of Options and Corporate Liabilities. In: *Journal of Political Economy*, 637-654.

Duffie, Darrell; Singleton, Kenneth J. (1999): Modeling Term Structures of Defaultable Bonds. In: *Review of Financial Studies*, 12, 687-720.

Fahrmeir, Ludwig; Hamerle, Alfred; Tutz, Gerald (1996): *Multivariate statistische Verfahren*, Berlin, New York.

Falke, Martin (2005): Sektorale Modellierung unternehmensspezifischer Risiken am Beispiel deutscher Unternehmen. *Working Paper*, February.

Gordy, Michael B. (2000): A comparative anatomy of credit risk models. In: *Journal of Banking and Finance*, 24, 119-149.

Hamerle, Alfred; Liebig, Thilo; Rösch, Daniel (2003): Credit Risk Factor Modeling and the Basel II IRB Approach. In: *Discussion Paper Series 2: Banking and Financial Supervision*, 02/2003, Deutsche Bundesbank.

Hamerle, Alfred; Rösch, Daniel (2005): Bankinterne Parametrisierung und empirischer Vergleich von Kreditrisikomodellen. In: *Die Betriebswirtschaft*, 65, 179-196.

Hamerle, Alfred; Tutz, Gerhard (1989): *Diskrete Modelle zur Analyse von Verweildauern und Lebenszeiten*, Frankfurt/Main, New York.

Jarrow, Robert A.; Turnbull, Stuart M. (1995): Pricing Derivatives of Financial Securities Subject to Credit Risk. In: *Journal of Finance*, 50, 53-85.

Jorion, Philippe (1996): Risk²: Measuring the Risk in Value at Risk. In: *Financial Analysts Journal*, November/December, 47-56.

Knapp, Michael (2002): Zeitabhängige Kreditportfoliomodelle, Wiesbaden, Dt. Univ.-Verl., Gabler Edition Wissenschaft.

Knapp, Michael; Hamerle, Alfred (1999): Multi-Faktor-Modelle zur Bestimmung segmentspezifischer Ausfallwahrscheinlichkeiten für die Kredit-Portfolio-Steuerung. In: *Wirtschaftsinformatik*, 138-144.

Koyluoglu, H. Ugur; Hickman, Andrew (1998): A Generalized Framework for Credit Risk Portfolio Models. *Working Paper*, Oliver, Wyman & Co., September.

Löffler, Gunter (2003): The Effect of Estimation Error on Measures of Portfolio Credit Risk. In: *Journal of Banking and Finance*, 27, 1427-1453.

Merton, Robert (1974): On the Pricing of Corporate Debt: The Risk Structure of Interest Rates. In: *Journal of Finance*, 29, 449-470.

Merton, Robert (1977): On the Pricing of Contingent Claims and the Modigliani-Miller Theorem. In: *Journal of Financial Economics*, 5, 241-249.

Rösch, Daniel (2003): Correlations and Business Cycles of Credit Risk: Evidence from Bankruptcies in Germany. In: *Financial Markets and Portfolio Management*, 17, No 3, 309-331.

Scheule, Harald (2003): Prognose von Kreditausfallrisiken, Bad Soden/Taunus, Uhlenbruch Verlag.

Wilson, Tom (1997a): Portfolio Credit Risk I. In: *Risk*, September.

Wilson, Tom (1997b): Portfolio Credit Risk II. In: *Risk*, September.

The following Discussion Papers have been published since 2004:

Series 1: Economic Studies

1	2004	Foreign Bank Entry into Emerging Economies: An Empirical Assessment of the Determinants and Risks Predicated on German FDI Data	Torsten Wezel
2	2004	Does Co-Financing by Multilateral Development Banks Increase “Risky” Direct Investment in Emerging Markets? – Evidence for German Banking FDI	Torsten Wezel
3	2004	Policy Instrument Choice and Non-Coordinated Monetary Policy in Interdependent Economies	Giovanni Lombardo Alan Sutherland
4	2004	Inflation Targeting Rules and Welfare in an Asymmetric Currency Area	Giovanni Lombardo
5	2004	FDI versus cross-border financial services: The globalisation of German banks	Claudia M. Buch Alexander Lipponer
6	2004	Clustering or competition? The foreign investment behaviour of German banks	Claudia M. Buch Alexander Lipponer
7	2004	PPP: a Disaggregated View	Christoph Fischer
8	2004	A rental-equivalence index for owner-occupied housing in West Germany 1985 to 1998	Claudia Kurz Johannes Hoffmann
9	2004	The Inventory Cycle of the German Economy	Thomas A. Knetsch
10	2004	Evaluating the German Inventory Cycle Using Data from the Ifo Business Survey	Thomas A. Knetsch
11	2004	Real-time data and business cycle analysis in Germany	Jörg Döpke

12	2004	Business Cycle Transmission from the US to Germany – a Structural Factor Approach	Sandra Eickmeier
13	2004	Consumption Smoothing Across States and Time: International Insurance vs. Foreign Loans	George M. von Furstenberg
14	2004	Real-Time Estimation of the Output Gap in Japan and its Usefulness for Inflation Forecasting and Policymaking	Koichiro Kamada
15	2004	Welfare Implications of the Design of a Currency Union in Case of Member Countries of Different Sizes and Output Persistence	Rainer Frey
16	2004	On the decision to go public: Evidence from privately-held firms	Ekkehart Boehmer Alexander Ljungqvist
17	2004	Who do you trust while bubbles grow and blow? A comparative analysis of the explanatory power of accounting and patent information for the market values of German firms	Fred Ramb Markus Reitzig
18	2004	The Economic Impact of Venture Capital	Astrid Romain, Bruno van Pottelsberghe
19	2004	The Determinants of Venture Capital: Additional Evidence	Astrid Romain, Bruno van Pottelsberghe
20	2004	Financial constraints for investors and the speed of adaption: Are innovators special?	Ulf von Kalckreuth
21	2004	How effective are automatic stabilisers? Theory and results for Germany and other OECD countries	Michael Scharnagl Karl-Heinz Tödter

22	2004	Asset Prices in Taylor Rules: Specification, Estimation, and Policy Implications for the ECB	Pierre L. Siklos Thomas Werner Martin T. Bohl
23	2004	Financial Liberalization and Business Cycles: The Experience of Countries in the Baltics and Central Eastern Europe	Lúcio Vinhas de Souza
24	2004	Towards a Joint Characterization of Monetary Policy and the Dynamics of the Term Structure of Interest Rates	Ralf Fendel
25	2004	How the Bundesbank really conducted monetary policy: An analysis based on real-time data	Christina Gerberding Andreas Worms Franz Seitz
26	2004	Real-time Data for Norway: Challenges for Monetary Policy	T. Bernhardsen, Ø. Eitrheim, A.S. Jore, Ø. Røisland
27	2004	Do Consumer Confidence Indexes Help Forecast Consumer Spending in Real Time?	Dean Croushore
28	2004	The use of real time information in Phillips curve relationships for the euro area	Maritta Paloviita David Mayes
29	2004	The reliability of Canadian output gap estimates	Jean-Philippe Cayen Simon van Norden
30	2004	Forecast quality and simple instrument rules - a real-time data approach	Heinz Glück Stefan P. Schleicher
31	2004	Measurement errors in GDP and forward-looking monetary policy: The Swiss case	Peter Kugler Thomas J. Jordan Carlos Lenz Marcel R. Savioz

32	2004	Estimating Equilibrium Real Interest Rates in Real Time	Todd E. Clark Sharon Kozicki
33	2004	Interest rate reaction functions for the euro area Evidence from panel data analysis	Karsten Ruth
34	2004	The Contribution of Rapid Financial Development to Asymmetric Growth of Manufacturing Industries: Common Claims vs. Evidence for Poland	George M. von Furstenberg
35	2004	Fiscal rules and monetary policy in a dynamic stochastic general equilibrium model	Jana Kremer
36	2004	Inflation and core money growth in the euro area	Manfred J.M. Neumann Claus Greiber
37	2004	Taylor rules for the euro area: the issue of real-time data	Dieter Gerdesmeier Barbara Roffia
38	2004	What do deficits tell us about debt? Empirical evidence on creative accounting with fiscal rules in the EU	Jürgen von Hagen Guntram B. Wolff
39	2004	Optimal lender of last resort policy in different financial systems	Falko Fecht Marcel Tyrell
40	2004	Expected budget deficits and interest rate swap spreads - Evidence for France, Germany and Italy	Kirsten Heppke-Falk Felix Hüfner
41	2004	Testing for business cycle asymmetries based on autoregressions with a Markov-switching intercept	Malte Knüppel
1	2005	Financial constraints and capacity adjustment in the United Kingdom – Evidence from a large panel of survey data	Ulf von Kalckreuth Emma Murphy

2	2005	Common stationary and non-stationary factors in the euro area analyzed in a large-scale factor model	Sandra Eickmeier
3	2005	Financial intermediaries, markets, and growth	F. Fecht, K. Huang, A. Martin
4	2005	The New Keynesian Phillips Curve in Europe: does it fit or does it fail?	Peter Tillmann
5	2005	Taxes and the financial structure of German inward FDI	Fred Ramb A. J. Weichenrieder
6	2005	International diversification at home and abroad	Fang Cai Francis E. Warnock
7	2005	Multinational enterprises, international trade, and productivity growth: Firm-level evidence from the United States	Wolfgang Keller Steven R. Yeaple
8	2005	Location choice and employment decisions: a comparison of German and Swedish multinationals	S. O. Becker, K. Ekholm, R. Jäckle, M.-A. Muendler
9	2005	Business cycles and FDI: evidence from German sectoral data	Claudia M. Buch Alexander Lipponer
10	2005	Multinational firms, exclusivity, and the degree of backward linkages	Ping Lin Kamal Saggi
11	2005	Firm-level evidence on international stock market comovement	Robin Brooks Marco Del Negro
12	2005	The determinants of intra-firm trade: in search for export-import magnification effects	Peter Egger Michael Pfaffermayr

13	2005	Foreign direct investment, spillovers and absorptive capacity: evidence from quantile regressions	Sourafel Girma Holger Görg
14	2005	Learning on the quick and cheap: gains from trade through imported expertise	James R. Markusen Thomas F. Rutherford
15	2005	Discriminatory auctions with seller discretion: evidence from German treasury auctions	Jörg Rocholl
16	2005	Consumption, wealth and business cycles: why is Germany different?	B. Hamburg, M. Hoffmann, J. Keller
17	2005	Tax incentives and the location of FDI: evidence from a panel of German multinationals	Thiess Buettner Martin Ruf
18	2005	Monetary Disequilibria and the Euro/Dollar Exchange Rate	Dieter Nautz Karsten Ruth
19	2005	Berechnung trendbereinigter Indikatoren für Deutschland mit Hilfe von Filterverfahren	Stefan Stamford
20	2005	How synchronized are central and east European economies with the euro area? Evidence from a structural factor model	Sandra Eickmeier Jörg Breitung
21	2005	Asymptotic distribution of linear unbiased estimators in the presence of heavy-tailed stochastic regressors and residuals	J.-R. Kurz-Kim S.T. Rachev G. Samorodnitsky
22	2005	The Role of Contracting Schemes for the Welfare Costs of Nominal Rigidities over the Business Cycle	Matthias Pastian
23	2005	The cross-sectional dynamics of German business cycles: a bird's eye view	J. Döpke, M. Funke S. Holly, S. Weber

24	2005	Forecasting German GDP using alternative factor models based on large datasets	Christian Schumacher
25	2005	Time-dependent or state-dependent price setting? – micro-evidence from German metal-working industries –	Harald Stahl
26	2005	Money demand and macroeconomic uncertainty	Claus Greiber Wolfgang Lemke
27	2005	In search of distress risk	J. Y. Campbell, J. Hilscher, J. Szilagyi
28	2005	Recursive robust estimation and control without commitment	Lars Peter Hansen Thomas J. Sargent
29	2005	Asset pricing implications of Pareto optimality with private information	N. R. Kocherlakota Luigi Pistaferri
30	2005	Ultra high frequency volatility estimation with dependent microstructure noise	Y. Aït-Sahalia, P. A. Mykland, L. Zhang
31	2005	Umstellung der deutschen VGR auf Vorjahrespreisbasis – Konzept und Konsequenzen für die aktuelle Wirtschaftsanalyse sowie die ökonomische Modellierung	Karl-Heinz Tödter
32	2005	Determinants of current account developments in the central and east European EU member states – consequences for the enlargement of the euro area	Sabine Herrmann Axel Jochem
33	2005	An estimated DSGE model for the German economy within the euro area	Ernest Pytlarczyk
34	2005	Rational inattention: a research agenda	Christopher A. Sims

35	2005	Monetary policy with model uncertainty: distribution forecast targeting	Lars E.O. Svensson Noah Williams
36	2005	Comparing the value relevance of R&D reporting in Germany: standard and selection effects	Fred Ramb Markus Reitzig
37	2005	European inflation expectations dynamics	J. Döpke, J. Dovern U. Fritsche, J. Slacalek
38	2005	Dynamic factor models	Sandra Eickmeier Jörg Breitung
39	2005	Short-run and long-run comovement of GDP and some expenditure aggregates in Germany, France and Italy	Thomas A. Knetsch
40	2005	A “wreckers theory” of financial distress	Ulf von Kalckreuth

Series 2: Banking and Financial Studies

1	2004	Forecasting Credit Portfolio Risk	A. Hamerle, T. Liebig, H. Scheule
2	2004	Systematic Risk in Recovery Rates – An Empirical Analysis of US Corporate Credit Exposures	Klaus Düllmann Monika Trapp
3	2004	Does capital regulation matter for bank behaviour? Evidence for German savings banks	Frank Heid Daniel Porath Stéphanie Stolz
4	2004	German bank lending during emerging market crises: A bank level analysis	F. Heid, T. Nestmann, B. Weder di Mauro, N. von Westernhagen
5	2004	How will Basel II affect bank lending to emerging markets? An analysis based on German bank level data	T. Liebig, D. Porath, B. Weder di Mauro, M. Wedow
6	2004	Estimating probabilities of default for German savings banks and credit cooperatives	Daniel Porath
1	2005	Measurement matters – Input price proxies and bank efficiency in Germany	Michael Koetter
2	2005	The supervisor's portfolio: the market price risk of German banks from 2001 to 2003 – Analysis and models for risk aggregation	Christoph Memmel Carsten Wehn
3	2005	Do banks diversify loan portfolios? A tentative answer based on individual bank loan portfolios	Andreas Kamp Andreas Pfingsten Daniel Porath
4	2005	Banks, markets, and efficiency	F. Fecht, A. Martin

5	2005	The forecast ability of risk-neutral densities of foreign exchange	Ben Craig Joachim Keller
6	2005	Cyclical implications of minimum capital requirements	Frank Heid
7	2005	Banks' regulatory capital buffer and the business cycle: evidence for German savings and cooperative banks	Stéphanie Stolz Michael Wedow
8	2005	German bank lending to industrial and non-industrial countries: driven by fundamentals or different treatment?	Thorsten Nestmann
9	2005	Accounting for distress in bank mergers	M. Koetter, J. Bos, F. Heid C. Kool, J. Kolari, D. Porath
10	2005	The eurosystem money market auctions: a banking perspective	Nikolaus Bartzsch Ben Craig, Falko Fecht
11	2005	Financial integration and systemic risk	Falko Fecht Hans Peter Grüner
12	2005	Evaluating the German bank merger wave	Michael Koetter
13	2005	Incorporating prediction and estimation risk in point-in-time credit portfolio models	A. Hamerle, M. Knapp, T. Liebig, N. Wildenauer