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with Heterogenous Jobs and Workers**

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# On-the-Job Search in a Matching Model with Heterogenous Jobs and Workers

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## ABSTRACT

### **On-the-Job Search in a Matching Model with Heterogenous Jobs and Workers\***

This paper considers a matching model with heterogenous jobs (unskilled and skilled) and workers (low and high-educated) which allows for on-the-job search by mismatched workers. The latter are high-educated workers who transitorily accept unskilled jobs and continue to search for skilled jobs. Our findings show that on-the-job search introduces an additional source of between and within-group wage inequality. Furthermore, the higher quit rate of mismatched workers exerts a negative externality on unskilled jobs and weakens the labour market position of low-educated workers. This last feature changes the effects of skill-biased technological change and it alters the response of the labour market to shifts in the skill distribution.

JEL Classification: J63, J64

Keywords: job search, skills, unemployment, wage inequality

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# 1 Introduction

This paper develops a matching model with two-sided heterogeneity that allows for skill mismatch and introduces the novelty of on-the-job search. Firms have the choice between creating two types of jobs (skilled and unskilled) and the labour force consists of two kinds of workers (high-educated and low-educated).<sup>1</sup> High-educated workers have the advantage that they can perform both jobs, while low-educated workers can only be employed in unskilled jobs. Furthermore, in contrast to most of the literature on matching models with heterogeneous agents, we allow for on-the-job search by high-educated workers performing unskilled jobs. As a result, high-educated workers continue to accept unskilled jobs even when their productivity on skilled jobs is much higher. Moreover, given that high-educated workers will quit these jobs at a higher rate than low-educated workers, on-the-job search exerts a *negative externality* on firms with unskilled jobs, and this new channel tends to weaken the labour market position of the low-educated.

The job competition between mismatched workers and properly matched ones in the lower segment of the labour market has interesting implications concerning the issues of “overeducation” and “crowding-out” of low-educated workers by high-educated workers when both apply for the unskilled jobs.<sup>2</sup> As recently highlighted by Eurostat (2003), these phenomena are particularly relevant in some EU countries like the South Mediterranean ones.<sup>3</sup>

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<sup>1</sup>For notational convenience, we associate workers’ skills to their educational attainments. Moreover, the latter are exogenously given. Thus, any imperfect correlation that there might exist between education and skills is ignored in the paper.

<sup>2</sup>One of the first models of job competition and crowding-out can be found in Thurow (1975). In that model, the marginal product of labour is associated with the characteristics of jobs rather than of individuals. Hence, when there is a fixed amount of jobs and an excess of high-educated workers, low-educated workers get “crowded-out” from their traditional jobs and their unemployment rate will increase. Moreover, if the educational drive is sufficiently strong, despite being able to take unskilled jobs, the unemployment rate of high-educated workers may go up as well. A basic problem with this approach is that, by assuming a fixed supply of jobs, the supply and composition of vacancies does not adjust to the composition of workers.

<sup>3</sup>Using a slight variant of the “objective” procedure proposed by Verdugo and Verdugo (1989) to measure overeducation, whereby overeducated workers are those whose years of education are more than one standard deviation above the average years of education in the socioeconomic group where they belong, Oliver and Raymond (2003) show that the proportion of overeducated workers with a college degree in Spain (*Diplomados and Licenciados*) increased from 14% in 1985 to 21% in 1998. During this period, the share of

These countries have experienced an intense upgrading of tertiary education over the last fifteen years and this shift in the distribution of educational attainments seems to have outpaced the growth of skilled jobs, giving rise to shares between 15% and 25% of skill-mismatch at the highest educational level (ISCED 5-6 in terms of the educational categories of Eurostat). In this respect, Figure 1a illustrates this fact by showing that there is a positive correlation between the the intensity of the tertiary educational upgrading in thirteen EU countries over the last two decades and the degree of “over-education”.<sup>4</sup> Likewise, Figure 1b shows that overeducated workers have higher job mobility, as illustrated by the positive correlation between the degree of “over-education” and the percentage- point differences in the proportions of mismatched and correctly matched workers who are searching for another job. Both stylized facts constitute the main motivations for this paper.

The literature on matching models with heterogeneous agents is fairly recent, dating back to the influential contributions by Acemoglu (1999) and Mortensen and Pissarides (1999).<sup>5</sup> Assuming random search and a constant contact rate between unemployed workers and vacancies, Acemoglu (1999) offers a theory of how a *pooling* equilibrium (defined as a situation in which firms only create “middling” jobs in order to avoid screening costs) is replaced by a *separating* equilibrium when either the proportion of skilled workers or the skill differential is high enough. In the new equilibrium firms find it profitable to create skilled and unskilled jobs, and search for the appropriate candidates. This leads endogenously to changes in the unemployment rates of both types of workers and to an increase in between-group wage dispersion. Mortensen and Pissarides (1999), in turn, study a model with directed search

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the Spanish population (25-64) with a tertiary educational attainment raised from 15% to 23%. Alba-Ramírez (1993) and Dolado *et al.* (2000) have documented some of the stylised facts of overeducation and crowding-out in Spain since the mid-1980s.

<sup>4</sup>The data for Figures 1 a and b come from an ad hoc module carried out by Eurostat in the EU LFS 2000 designed to collect specific information on the transition from school to working life in EU countries. A job mismatch is defined as a job outside the field of education of school leavers which is considered to have lower skill requirements than those corresponding to the workers’ educational attainments. The educational upgrading is measured by the ratio of the fraction of the population aged 25-34 with tertiary education and the corresponding proportion of the population aged 45-54. Data correspond to 1999 (see *Education at the Glance*, OECD 2001).

<sup>5</sup>Other relevant contributions in this line of research are Burgess (1993), Pissarides (1994) and McKenna (1996).

and endogenous job destruction. In their model there is a perfect match between workers' skills and firms' skill requirements, and between and within-group wage dispersion arise from shocks to the productivities of the two types of jobs. Hence, although these papers set up the foundations of the role of search frictions in models with heterogeneity, they do not deal with the spillover effects of high-educated workers onto the creation and filling of unskilled job vacancies.

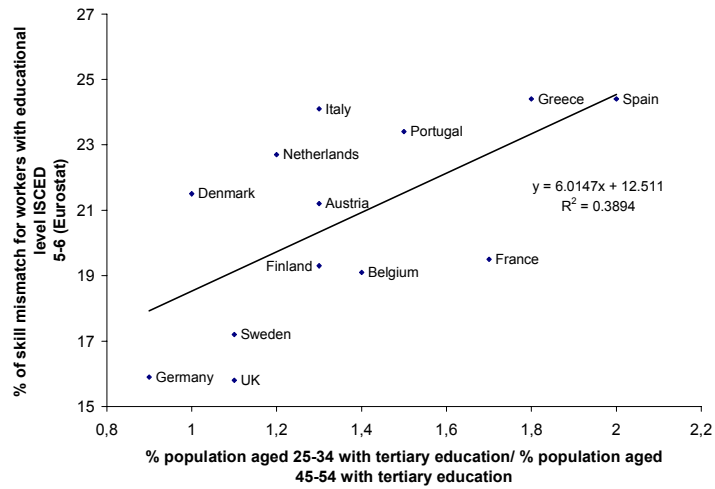


Figure 1a: Relationship between “educational upgrading” and mismatch

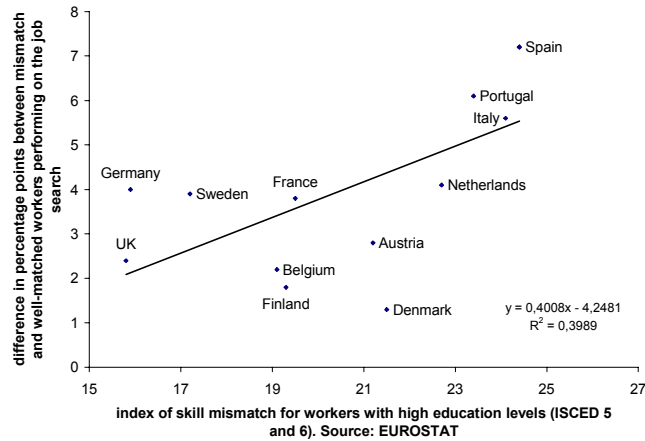


Figure 1b: Relationship between “over-education” and job search intensity

More recently, however, contributions by Gautier (2002) and Albrecht and Vroman (2002) have started to address directly this issue. In Gautier (2002), badly matched workers (high-skilled workers in simple jobs) continue to search on the job. Thus, his model is similar to ours, yet with two notable exceptions. First, rather than assuming that wages are determined by Nash bargaining, he adopts the simplifying assumption that firms and workers share their output in some fixed proportion. As a result, wages are unaffected by the overall labour market conditions. Second, in his model, high-skilled workers may be more productive on simple jobs than low-skilled workers. This second assumption implies that low-skilled workers may benefit from job competition by high-skilled workers if the latter are sufficiently more productive on unskilled jobs.<sup>6</sup>

Although there is anecdotal evidence about simple tasks that could be better performed by low-skilled workers (say, hamburger flipping or garbage collection) and viceversa (say, some clerical or services jobs which may require fluency in a foreign language), on average it seems sensible to assume that both types of workers are equally productive on simple jobs.<sup>7</sup> This is precisely the assumption made by Albrecht and Vroman (2002) (AV, henceforth), who analyse a model with endogenous skill requirements. They show that the labour market equilibrium may switch from one in which high-skilled workers match with both types of jobs (a *cross-skill matching* equilibrium) to another where they refuse unskilled jobs (an *ex-post segmentation* equilibrium). Like in Acemoglu (1999), when either the productivity differential in the two jobs or the proportion of high-educated workers in the population is sufficiently large, the equilibrium switches from the first to the second type with important implications for unemployment and wage dispersion.

In our paper, we adopt the basic structure of their model (skill differences across workers and job requirements, Nash bargaining approach and undirected search) but we introduce the novel feature of allowing for on-the-job search by mismatched workers. By incorporating on-the-job search, we show that the properties of this type of models change considerably.

*First*, with on-the-job search, the equilibrium always exhibits *cross-skill*

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<sup>6</sup>A third, less relevant, difference with our model is the assumption that simple and complex jobs are offered in different markets. Hence, unlike our model, high-skilled workers congest the market for low-skilled workers but not the other way around.

<sup>7</sup>Another relevant aspect which is not considered here is that of internal promotion, whereby “over-educated” workers may accept unskilled jobs as “stepping stones” for better jobs within the firm in the future (see, e.g. Brunello, 1996).



*matching.* The key difference with AV's model is that, by allowing mismatched workers to perform on-the-job search, they retain the option value from employment in a skilled job. It is therefore optimal for high-educated workers to accept an unskilled job if it pays more than their income during unemployment.

*Second,* on-the-job search introduces an interesting, additional source of within-group wage inequality. In AV's model, high-educated workers earn different wages on the two types of jobs, but they always earn a higher wage than low-educated workers. In our model, by contrast, over-qualified workers end up earning a lower wage than low-educated workers, despite having a higher outside option value. The explanation for this result lies in their higher quit rate. On-the-job search reduces the surplus value of jobs performed by mismatched workers and as a result they earn a lower wage than those low-educated workers who are correctly matched. The empirical evidence on this last issue is mixed. For instance, on the one hand, there is some evidence for the Netherlands (see, e.g. Hartog and Oosterbeek, 1988, and Gautier *et al.*, 2002, ) and for the U.S. (see, e.g., Sicherman, 1991) that over-educated workers earn more than correctly allocated workers but, on the other hand, Groot (1993, 1996), Verdugo and Verdugo (1989) and Alba-Ramírez and Blázquez (2003) obtain the opposite result for the U.K, the U.S and Spain, respectively.<sup>8</sup> Furthermore, the fact that this type of workers are being hired for simple jobs may be an indication of the existence of unobserved characteristics that make them more productive, as in Gautier (2002). Our model excludes this type of non-observable heterogeneity.

*Third,* in this respect, one of the striking implications of our model is that on-the-job search provides a novel mechanism to explain the widening of the wage distribution which has been observed notably in the US but also in many OECD countries since the 1980s, despite the rise in the relative supply of college graduates. In effect, even abstracting from the effects of a secular increase in the demand for skills induced by "skill-biased" technical change (e.g, Autor *et al.*, 1998) or in trade with LDCs (e.g. Borjas and Ramey, 1995), or a combination of both phenomena (e.g., Acemoglu, 2003), "skill-upgrading" gives rise to on-the-job search which, in turn, would produce an increase in both the average wage premium and within-group wage inequality

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<sup>8</sup>For example, in the influential study of Verdugo and Verdugo (1989) it is found that, while the wage return to an extra year of required education is 6.2%, the return to a year of overeducation is -8.0% in the U.S.

for workers with high educational attainments. This is so because a larger supply of high-educated workers end up being mismatched in unskilled jobs and since they are paid less than correctly-matched workers, both measures of wage inequality increase.

*Fourth*, while “skill-biased” technological change (an increase in the productivity of skilled jobs) always increases the unemployment rate of low-educated workers in models without on-the-job search, this is not necessarily the case in our framework. In general, when the cost of opening both types of jobs is the same, we find that the effect is ambiguous although for reasonable parameter values the traditional result holds. However, if skilled jobs are relatively scarce to start with, say, because the cost of opening skill vacancies is higher than the cost of opening unskilled vacancies, then the increase in the supply of skilled jobs due to “skill-biased” technological progress will lead to relatively more high-educated workers in skilled jobs and to a lower share of over-qualified job seekers. Given that firms opening unskilled job vacancies prefer to match with low-educated workers rather than with high-educated ones (who are equally productive but who with a higher quit probability), a higher productivity in skilled jobs may therefore help those firms in finding stable and appropriately-qualified workers, leading to a decrease of the unemployment rate of low-educated workers. Likewise, for a given productivity differential, the unemployment rate of low-educated workers will be lower the smaller is the (exogenous) job-destruction rate of skilled jobs relative to that of unskilled jobs. The explanation is again that the higher job tenure of adequately matched high-educated workers weakens the negatively externality that mismatched workers impose on firms creating unskilled vacancies. These properties suggest that there may be scope for differentiated labour market policies that reduce the turnover of high-educated workers. A full treatment of this issue is beyond the scope of this paper. However, it is easy to construct examples in which the combined effect of a reduction in the separation rate of skilled jobs and an increase in the cost of those jobs (an approximation of the effects of firing costs on skilled jobs) reduces the unemployment rate for both types of workers.

The above results contrasts with the ones obtained from an increase in the share of high-educated workers in the population. In this case, firms will also open more skilled jobs but, due to the “skill-upgrading”, mismatched workers represent a larger fraction of job seekers for unskilled vacancies, leading to lower profits for firms creating those jobs and a higher unemployment rate among the low-educated.

*Finally*, as discussed above, both AV’s and our model provide a useful analytical set-up to analyse the consequences of “overeducation” and “crowding-out”. Matching models with and without on-the-job search share the prediction that “skill-upgrading” will increase the unemployment rate of less-educated workers. However, while the unemployment rate of high-educated workers is invariant to changes in the skill distribution in AV’s model, we tend to find a negative correlation between the unemployment rate and the cohort size of high-educated workers. Hence, although a larger number of high-educated workers may end up accepting unskilled jobs, these workers do experience an improvement in their labour market position due to both the increase in the exit rate out of unemployment and the higher share of skilled job vacancies.<sup>9</sup> Thus, despite the sometimes popular view that “over-education”, under on a fixed supply of jobs like in Thurow (1975), may lead to an increase in the unemployment rates of both low and high-educated workers, our model yields the opposite result for the latter.

The plan for the rest of the paper is as follows. Section 2 presents the main characteristics of the model while section 3 discusses the properties of the steady-state equilibrium in terms of its existence, uniqueness, and the implications for the wage distribution. Section 4 focusses on the comparative statics stemming from two important changes in the parameters of the model: (i) an increase in the proportion of high-educated workers reflecting the “skill-upgrading” that has taken place in many countries, and (ii) an increase in the productivity of skilled jobs reflecting “skill-biased” technological progress. Section 5 discusses a few simulations of the proposed model which shed light on the size of the effects derived in the analytical sections. Finally, Section 6 concludes. Proofs of the main propositions and a slight generalization of the model entailing different separation rates are gathered in Appendices A and B, respectively.

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<sup>9</sup>According to our knowledge the only other paper that obtains this type of cohort-size effects is Shimer (2001). He analyses a model with young and old workers, where young workers change jobs more frequently. As a result, an inflow of young workers will be matched with a creation of additional jobs and this reduces the cyclical unemployment of both the young and old cohorts in the labour force.

## 2 The Model

### 2.1 Main assumptions

We consider an economy populated by a continuum of risk-neutral workers with measure normalised to unity. An exogenous fraction  $\mu \in (0, 1)$  of the workers is low-educated ( $l$ ) while the remaining fraction,  $1 - \mu$ , is high-educated ( $h$ ). All workers are infinitely-lived and time is continuous.

There are two types of jobs: skilled jobs ( $s$ ), and unskilled jobs ( $n$ ). Unskilled jobs can be performed by both types of workers, while a skilled job requires a high-educated worker. Furthermore, we assume that both types of workers are equally productive on unskilled jobs, while high-educated workers are more productive when matched to a skilled job. Formally, let  $y(i, j)$  denote the flow output of a job of type  $i$  ( $= s, n$ ) that is filled by a worker of type  $j$  ( $= h, l$ ). Our assumptions on the production technology can then be summarised as follows

$$y(s, h) = y(s) > y(n, h) = y(n, l) = y(n) > y(s, l) = 0.$$

For convenience, we assume that firms can open at most one job. The choice of the type of job is irreversible and the mass of each type of job is determined by a free-entry condition. Finally, job destruction is exogenous and follows a Poisson process with arrival rate  $s$  that is common to both jobs.<sup>10</sup> Whenever a job is destroyed, the worker becomes unemployed while the job becomes vacant. During unemployment workers receive a flow income  $b < y(n)$  which is to be interpreted as home production or leisure. The flow cost of maintaining a vacant job is denoted by  $c$  and is common to both types of jobs.

### 2.2 Matching

The labour market is characterised by matching frictions and on-the-job search is allowed for mismatched workers, namely, high-educated workers in unskilled jobs. As in AV, the meeting process between workers and vacancies is assumed to be *undirected*. This assumption allows us to capture the

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<sup>10</sup>Despite the common rate of job destruction rate,  $s$ , the effective job separation rate of unskilled jobs will be partially endogenous due to the on-the-job search by mismatched workers. On average workers in skilled jobs will therefore enjoy more stable employment relationships than workers on unskilled jobs, in line with the available empirical evidence.

idea that, given the overall labour market conditions, low-educated workers are better off the greater the fraction of unskilled vacancies, and firms offering unskilled vacancies benefit from a greater fraction of low-educated job seekers. A similar situation applies to both high-educated job seekers and firms opening skilled jobs. Specifically, the total number of matches between a worker and a firm is determined by a constant returns to scale matching function

$$m[v(n) + v(s), u(l) + u(h) + e(n, h)],$$

where  $u(j)$  is the mass of unemployed workers of type  $j$  ( $= n, s$ ),  $v(i)$  denotes the mass of vacancies of type  $i$  ( $= l, h$ ) and  $e(n, h)$  is the mass of high educated workers performing unskilled jobs. The total mass of unemployed workers is denoted as  $u$  while  $\tilde{u}(l)$  and  $\tilde{u}(h)$  denote the unemployment rates of low and high-educated workers, respectively. We assume that  $m[., .]$  is strictly increasing in both arguments and denote the “labour market tightness” by  $\theta = [v(n) + v(s)]/[u(l) + u(h) + e(n, h)]$ .

Since the mass of job seekers includes the mass of “mismatched” workers,  $e(n, h)$ , in the analysis of the transitions between job vacancies and workers it is important to distinguish between the shares of  $u(l)$  and  $u(h)$  in the pool of unemployed and in the pool of job seekers. For this purpose, we define the following two shares: (i)  $\phi$  ( $\phi = u(l)/[u(l) + u(h)]$ ) which denotes the proportion of low-educated unemployed workers in the mass of unemployed, and (ii)  $\psi$  ( $\psi = [u(l) + u(h)]/[u(l) + u(h) + e(n, h)]$ ) which represents the fraction of all unemployed workers in the mass of job seekers. With this notation, the share of low-educated unemployed workers in the total mass of job seekers becomes  $\phi\psi$  ( $= u(l)/[u(l) + u(h) + e(n, h)]$ ). Likewise, the shares of high-educated unemployed workers in the mass of unemployed and in the mass of job seekers are  $(1 - \phi)$  and  $\psi(1 - \phi)$ , respectively.

For the sake of realism we assume in the sequel that mismatched workers only change employer if they find a skilled job. Accordingly, the rate at which firms meet a job-seeker is equal to  $q(\theta) = m(1, \frac{1}{\theta})$ , but some unskilled jobs will meet a mismatched worker who will refuse to match. The effective matching rate of an unskilled job with a low-educated worker is therefore  $\psi\phi q(\theta)$ , while the corresponding rate with a high-educated worker is  $\psi(1 - \phi)q(\theta)$ .<sup>11</sup> Similarly, skilled jobs may meet a low-educated worker who is not

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<sup>11</sup>Had we assumed mismatched workers to be part of the mass of high-educated workers

qualified for the job. Thus, the matching rate of a skilled job can be written as  $(1 - \psi\phi)q(\theta)$ . To define the matching rates of workers we introduce the share  $\eta = v(n)/[v(n) + v(s)]$  which denotes the share of unskilled vacancies. Low-educated workers therefore exit unemployment at rate  $\eta\theta q(\theta)$ , while mismatched workers change employers at rate  $(1 - \eta)\theta q(\theta)$ .

Finally, the properties of the matching function imply that the matching rate of workers (firms) is increasing (decreasing) in  $\theta$ , and, conventionally, we assume that  $\lim_{\theta \rightarrow 0} q(\theta) = \lim_{\theta \rightarrow \infty} \theta q(\theta) = \infty$  and  $\lim_{\theta \rightarrow \infty} q(\theta) = \lim_{\theta \rightarrow 0} \theta q(\theta) = 0$ .

### 2.3 Bargaining

In equilibrium, our model considers three types of matches: (i) high-educated workers on skilled jobs, (ii) high-educated workers on unskilled jobs, and (iii) low-educated workers on unskilled jobs. In each of these matches, the firm-worker pair divides the surplus of the match according to the asymmetric Nash bargaining solution. The exogenous surplus share of workers is denoted by  $\beta \in (0, 1)$ . Moreover, we adopt the following standard notation:  $U(j)$  denotes the value of unemployment for a worker of type  $j$ ,  $V(i)$  denotes the value of a vacant job of type  $i$ ,  $W(i, j)$  denotes the value of employment for a worker of type  $j$  on a job of type  $i$  and  $J(i, j)$  denotes the value to the firm of filling a job of type  $i$  with a worker of type  $j$ . Accordingly, the surplus of a match between a job of type  $i$  and a worker of type  $j$  can be expressed as  $S(i, j) = W(i, j) + J(i, j) - V(i) - U(j)$  and, when a match is consummated, the wage  $w(i, j)$  satisfies the standard Nash bargaining condition<sup>12</sup>

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seeking an unskilled job, then the matching rate would become  $(1 - \psi\phi)q(\theta)$  instead of  $\psi(1 - \phi)q(\theta)$ . The latter would imply an even lower stability of unskilled jobs and this would exacerbate the negative search externality of mismatched workers on low-educated workers.

<sup>12</sup>We are grateful to Robert Shimer for pointing out to us in a note (see Shimer, 2003) that in models with on-the-job search, the equivalence between the Nash bargaining solution and the linear sharing rule in (1) may not be valid since firms may prefer to pay a higher efficiency wage in order to reduce worker turnover. However, his result obtains when employed workers pay a search cost ( $\sigma$ ) which allows them to search with the same efficiency as unemployed workers. Nonetheless, our assumption that  $\sigma = 0$  implies that the efficiency wage ( $w^*$ ) would be  $w^* = w(s, h)$  and, since  $w(s, h) > y(n)$  in equilibrium, firms will not find profitable to do so. In reality,  $\sigma = 0$  can be justified by the fact that on-the-job seekers, despite having less time to search than unemployed workers, may have more contacts to find an alternative job.

$$(1 - \beta)[W(i, j) - U(j)] = \beta[J(i, j) - V(i)]. \quad (1)$$

Below we concentrate on steady states and we assume that high-educated workers accept both types of jobs. The proof that this strategy is optimal is provided in Section 3. Formally, a *cross-skill matching* equilibrium can be summarised by a vector  $\{\theta, \eta, \phi, \psi, u\}$  that satisfies the following conditions: (i) match formation is voluntary, (ii) the expected profit of each type of job is equal to zero, and (iii) the state variables  $u(l) = \phi u$ ,  $u(h) = (1 - \phi)u$  and  $e(n, h)$  satisfy the appropriate steady-state conditions defined below.

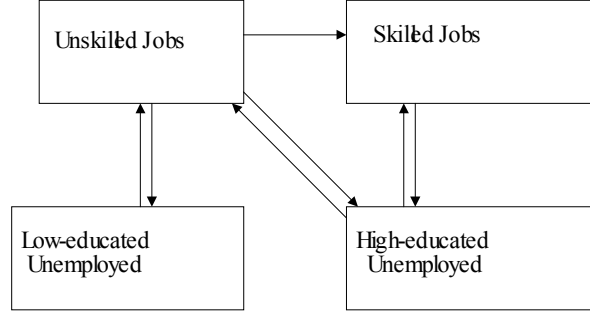


Figure 2: The flow diagram

## 2.4 Flow equations

Figure 2 illustrates the flows of workers between the three possible states. At each moment in time a flow  $\eta\theta q(\theta)\phi u$  of low-educated unemployed find employment and this flow is equal in steady state to the flow of low-educated workers into unemployment,  $s(\mu - \phi u)$ . Similarly, the flow  $\theta q(\theta)(1 - \phi)u$  of high-educated workers exiting unemployment equals the flow into unemployment. A share  $1 - \eta$  of these workers encounters a skilled job, while the remaining fraction  $\eta$  is employed in an unskilled job and will continue to search on the job. Finally, the flows into and out of unemployment of correctly-matched workers and of mismatched workers are the same.

Accordingly, the steady state condition for  $u(l) = \phi u$  is

$$\eta\theta q(\theta)\phi u = s(\mu - \phi u). \quad (2)$$

Similarly, for  $u(h) = (1 - \phi)u$ , we have

$$\theta q(\theta)(1 - \phi)u = s[1 - \mu - (1 - \phi)u], \quad (3)$$

whereas the mass of mismatched workers  $e(u, h)$  satisfies

$$\eta\theta q(\theta)(1 - \phi)u = [s + \theta q(\theta)(1 - \eta)]e(n, h). \quad (4)$$

## 2.5 Asset values

### 2.5.1 Workers

The asset value of a low-educated unemployed,  $U(l)$ , satisfies

$$rU(l) = b + \theta q(\theta)\eta[W(n, l) - U(l)]. \quad (5)$$

Similarly, given the assumption that high-educated workers accept both types of jobs, the asset value of a high-skill unemployed,  $U(h)$ , verifies

$$rU(h) = b + \theta q(\theta) [\eta(W(n, h) - U(h)) + (1 - \eta)(W(s, h) - U(h))], \quad (6)$$

while the asset values of low-educated and high-educated workers in unskilled and skilled jobs, respectively, satisfy

$$rW(n, l) = w(n, l) + s(U(l) - W(n, l)), \quad (7)$$

$$rW(s, h) = w(s, h) + s(U(h) - W(s, h)). \quad (8)$$

Finally, the asset value of employment for mismatched workers verifies

$$rW(n, h) = w(n, h) + s[U(h) - W(n, h)] + \theta q(\theta)(1 - \eta)[W(s, h) - W(n, h)], \quad (9)$$

where the term  $\theta q(\theta)(1 - \eta)[W(s, h) - W(n, h)]$  corresponds to the expected return from successful on-the-job search.



### 2.5.2 Firms

As regards firms, the values of opening unskilled and skilled vacancies are given, respectively, by

$$rV(n) = -c + \psi q(\theta) [\phi(J(n, l) - V(n)) + (1 - \phi)(J(n, h) - V(n))], \quad (10)$$

$$rV(s) = -c + q(\theta)(1 - \psi\phi)[J(s, h) - V(s)], \quad (11)$$

whereas the values to the employer of filling those vacancies with workers of the required type, verify

$$rJ(n, l) = y(n) - w(n, l) + s[V(n) - J(n, l)], \quad (12)$$

$$rJ(s, h) = y(s) - w(s, h) + s[V(s) - J(s, h)]. \quad (13)$$

Lastly, the value to a firm that fills an unskilled vacancy with a high-educated (mismatched) worker is

$$rJ(n, h) = y(n) - w(n, h) + s[V(n) - J(n, h)] + \theta q(\theta)(1 - \eta)[(V(n) - J(n, h))]. \quad (14)$$

## 3 Equilibrium

Below we derive the equilibrium. Following the strategy in AV we express all equilibrium relations in terms of the labour market tightness,  $\theta$ , and the share of unemployed workers with low education,  $\phi$ .

### 3.1 Worker flows

We start with the equilibrium flow equations. From equations (2) and (3) we can solve for  $u$  and  $\eta$  as a function of  $\theta$  and  $\phi$  (plus the exogenous variable  $\mu$ ). This yields

$$u(\theta, \phi; \mu) = \frac{s}{s + \theta q(\theta)} \frac{1 - \mu}{1 - \phi}, \quad (15)$$

$$\eta(\theta, \phi; \mu) = \frac{(1 - \phi)\theta q(\theta)\mu + s(\mu - \phi)}{\theta q(\theta)\phi(1 - \mu)}. \quad (16)$$

For given  $\mu$  and  $\phi$ , the unemployment rate of high-educated workers ( $\tilde{u}(h) = (1 - \phi)u/(1 - \mu) = \frac{s}{\theta q(\theta) + s}$ ) is decreasing in  $\theta$ , whilst the unemployment rate of low-educated workers ( $\tilde{u}(l) = \phi u/\mu = \frac{s}{\eta \theta q(\theta) + s}$ ) is decreasing in both  $\theta$  and  $\eta$ . It is also straightforward to verify that  $\eta$  is decreasing in  $\phi$ , and increasing in  $\theta$  as  $\phi > \mu$  in a cross-skill matching equilibrium.<sup>13</sup>

Next, since  $e(n, h)/u$  is equal to  $(1 - \psi)/\psi$ , equation (4) yields the equilibrium value of  $\psi$  as a function of  $\theta$ ,  $\phi$  and  $\eta$ ,

$$\psi(\theta, \eta; \mu) = \frac{1}{1 + \frac{\eta(1 - \phi)\theta q(\theta)}{s + (1 - \eta)\theta q(\theta)}} \quad (17)$$

with  $\partial\psi(\cdot)/\partial\theta < 0$ ,  $\partial\psi(\cdot)/\partial\eta < 0$ . Intuitively, at a higher value of the labour market tightness there will be less unemployed workers and more mismatched workers, reducing  $\psi$ . Similarly, an increase in the fraction of unskilled jobs reduces  $u(l)$  and increases  $e(n, h)$ , resulting in a drop of  $\psi$ . Finally, substituting equation (16) into the above expression yields a new function  $\psi(\theta, \phi; \mu)$  with the same arguments as  $u(\theta, \phi; \mu)$  and  $\eta(\theta, \phi; \mu)$ . This is our third equilibrium expression which yields  $\partial\psi(\cdot)/\partial\theta < 0$  and  $\partial\psi(\cdot)/\partial\phi > 0$ .<sup>14</sup>

>From the above results we can conclude that the effective matching rate of unskilled jobs,  $\psi q(\theta)$  decreases unambiguously in  $\theta$  (for given values of  $\phi$ ). By contrast, in the case of skilled jobs, the changes in  $q(\theta)$  and  $\psi(\cdot)$  have an opposite effect on the effective matching rate  $(1 - \psi\phi)q(\theta)$ . On the one hand, there is the well-known congestion effect, captured by

<sup>13</sup>Formally, since low-educated workers have a lower exit rate out of unemployment than high-educated workers, they must be relatively over-represented in the mass of unemployed as the inflow into unemployment is the same for both types of workers.

<sup>14</sup>>From the signs of those derivatives, it is straightforward to obtain that  $\frac{\partial\psi\phi}{\partial\theta} < 0$ ,  $\frac{\partial\psi\phi}{\partial\phi} > 0$  and  $\frac{\partial\psi(1 - \phi)}{\partial\theta} < 0$ . Moreover, we assume that  $\frac{\partial\psi(1 - \phi)}{\partial\phi} < 0$ . The sign of those derivatives is used to prove equilibrium uniqueness in Appendix A.

$\partial q(\theta)/\partial\theta < 0$ , which makes it more difficult to find a worker and, on the other, there is composition effect of opposite sign, captured by  $\partial\psi(\cdot)/\partial\theta < 0$ , as the proportion of high-educated workers searching for a good job increases. Nonetheless, it can be shown that for sufficiently high values of  $\mu$  the change in the indirect composition effect turns out to be dominated by the direct congestion effect. In what follows we shall therefore assume that, for given values of  $\phi$ , an increase in  $\theta$  reduces  $(1 - \psi\phi)q(\theta)$ .<sup>15</sup>

## 3.2 Equilibrium wages

We now proceed with a derivation of the equilibrium wages. These solutions are needed to obtain the two free entry conditions.

### 3.2.1 The wage of low-educated workers

Substituting (5), (7), (10) and (12) into equation (1) and imposing the free-entry condition for unskilled vacancies,  $V(n) = 0$ , we obtain that the match surplus of a low-educated worker,  $S(n, l) \equiv J(n, l) + W(n, l) - U(l) - V(n)$ , satisfies

$$(r + s)S(n, l) = y(n) - rU(l),$$

implying that  $w(n, l)$  is given by

$$w(n, l) = rU(l) + \beta[y(n) - rU(l)]. \quad (18)$$

### 3.2.2 The wage of high-educated workers on skilled jobs

Likewise, substitution of equations (6), (8), (11) and (13) into equation (1) and the free-entry condition for skilled vacancies,  $V(s) = 0$ , yields the match surplus of a high-educated worker,  $S(s, h) \equiv J(s, h) + W(s, h) - U(h) - V(s)$ , which verifies

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<sup>15</sup>Compared to a standard matching model this is the only additional restriction that we impose on the matching technology. Moreover, although a precise sufficient condition for  $\partial(1 - \psi\phi)q(\theta)/\partial\theta < 0$  is cumbersome to obtain, in our numerical simulations we find that the above derivative is always negative for values of  $\mu \geq 0.5$ .

$$(r + s)S(s, h) = y(s) - rU(h),$$

so that the wage of a high-educated worker in a skilled job is given by

$$w(s, h) = rU(h) + \beta[y(s) - rU(h)]. \quad (19)$$

### 3.2.3 The wage of mismatched workers

The derivation of the wage of mismatched workers is slightly more complicated and can be obtained from substitution of equations (6), (9), (10) and (14) into equation (1) which, together with  $V(n) = 0$ , implies that the match surplus of a mismatched worker,  $S(n, h) \equiv J(n, h) + W(n, h) - U(h) - V(n)$ , satisfies

$$[r + s + \theta q(\theta)(1 - \eta)]S(n, h) = y(n) - rU(h) + \theta q(\theta)(1 - \eta)\beta \left[ \frac{y(s) - rU(h)}{r + s} \right],$$

leading to the following expression for their wage

$$w(n, h) = rU(h) + \beta[y(n) - rU(h)] - (1 - \beta)\theta q(\theta)(1 - \eta)\beta \left[ \frac{y(s) - rU(h)}{r + s} \right]. \quad (20)$$

According to (20), mismatched workers earn less than a share  $\beta$  of the flow surplus  $y(n) - rU(h)$ . The reason is that their wages are reduced by the amount  $(1 - \beta)$  times the capital gain from successful on-the-job search, namely, the firm's share of the surplus that the worker creates by searching on the job.<sup>16</sup>

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<sup>16</sup>This result was previously obtained by Pissarides (1994) in a model with two types of jobs and a homogenous workers where on-the-job search by mismatched workers takes place only at short tenure in the bad jobs. However, that paper does not yield a closed-form comparison of both wages and we shall do in section 3.5.3.

### 3.3 Match surpluses

The equilibrium expressions for the match surpluses can now be obtained in two steps. First, we obtain the equilibrium asset value of unemployed workers by substituting the expressions for  $S(n, l)$ ,  $S(n, h)$  and  $S(s, h)$  into (5) and (6). This yields

$$rU(l) = \frac{(r+s)b + \theta q(\theta)\eta\beta y(n)}{r+s + \theta q(\theta)\eta\beta} \quad (21)$$

$$rU(h) = \frac{(r+s)\lambda_3 b + \theta q(\theta)\beta [\eta(r+s)y(n) + (1-\eta)\lambda_2 y(s)]}{\lambda_1 \lambda_2}, \quad (22)$$

where  $\lambda_1 = r+s + \theta q(\theta)(1-\eta)\beta$ ,  $\lambda_2 = r+s + \theta q(\theta)(1-\eta + \eta\beta)$ , and  $\lambda_3 = r+s + \theta q(\theta)(1-\eta)$ .

Next, substituting (21) and (22) back into the surplus expressions yields the closed-form solutions

$$S(n, l) = \frac{y(n) - b}{r+s + \theta q(\theta)\eta\beta}, \quad S(n, h) = \frac{y(n) - b}{r+s + \theta q(\theta)(1-\eta + \eta\beta)}.$$

from which it follows that  $S(n, h) < S(n, l)$ . Furthermore, since mismatched workers retain the option value of employment in skilled jobs,  $S(n, h)$  is independent of  $y(s)$ .<sup>17</sup>

### 3.4 Free-entry conditions

Finally, using expression (1) for the Nash bargaining solution, we can write the free-entry conditions for the two types of jobs as follows

$$rV(n) = -c + \psi q(\theta)(1-\beta) [\phi S(n, l) + (1-\phi)S(n, h)] = 0,$$

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<sup>17</sup>Formally, with on-the-job-search the increase in the wage  $w(n, h)$  due to the rise in the outside option of high-educated workers,  $rU(h)$ , is exactly offset by the increase in the future surplus in a skilled job, which leads to a fall in  $w(n, h)$ . Without on-the-job search, the second effect is absent leading to a negative relation between  $S(n, h)$  and  $y(s)$ .

$$rV(s) = -c + q(\theta)(1 - \psi\phi)(1 - \beta)S(s, h) = 0,$$

and substituting in the closed-form solutions for  $S(n, l)$ ,  $S(n, h)$  and  $S(s, h)$  derived above, yields

$$\frac{c}{q(\theta)} = \psi(1 - \beta) \left[ \phi \frac{y(n) - b}{r + s + \theta q(\theta)\eta\beta} + (1 - \phi) \frac{y(n) - b}{\lambda_2} \right], \quad (23)$$

$$\frac{c}{q(\theta)} = (1 - \beta)(1 - \psi\phi) \left[ \frac{y(s) - b}{\lambda_1} - \theta q(\theta)\eta\beta \frac{y(n) - b}{\lambda_1\lambda_2} \right]. \quad (24)$$

Equations (23) and (24) are the free-entry conditions for unskilled and skilled jobs, respectively, which henceforth, for given  $\eta$  and  $\psi$ , will be denoted in implicit form as  $F_N(\theta, \phi) = 0$  and  $F_S(\theta, \phi) = 0$ . Note that the right-hand side of these equations defines the expected future profits when the job is filled whereas the left-hand side represents the expected cost of keeping a vacancy unfilled. Thus, equations (23) and (24), together with the expressions in (15), (16) and (17) for  $u(\theta, \phi; \mu)$ ,  $\eta(\theta, \phi; \mu)$  and  $\psi(\phi, \eta; \mu)$ , defines the system of equations determining the equilibrium of our model.

## 3.5 Properties of the Equilibrium

### 3.5.1 Existence

We start the analysis by deriving the conditions under which a *cross-skill* matching equilibrium exists. First of all, as in AV, we need to rule out the corner solution in which firms only create unskilled jobs. A sufficient condition to ensure that firms are willing to create skilled jobs is that  $V(s) > 0$  if  $\eta = 1$ . This condition can be easily derived.

When  $\eta = 1$ , the outside option value of workers (common to both types) would simplify to

$$rU = \frac{(r + s)b + \beta\theta q(\theta)y(n)}{r + s + \beta\theta q(\theta)}.$$

Substituting this value into the free-entry condition  $V(n) = 0$  when no skilled jobs are available, i.e.  $J(n) = c/q(\theta)$ , we obtain

$$\frac{c}{q(\theta)} = (1 - \beta) \frac{y(n) - b}{r + s + \beta\theta q(\theta)}.$$

The above equation yields a unique solution for  $\theta$ , denoted as  $\theta^*$ . The necessary condition to rule out the corner solution with  $\eta = 1$ , can thus be written as  $V(s) > V(n) = 0$  which is equivalent to the condition that  $(1 - \mu)[y(s) - rU] > [y(n) - rU]$  which gives <sup>18</sup>

$$(1 - \mu) \frac{(r + s)(y(s) - b) + \theta^* q(\theta^*) \beta (y(s) - y(n))}{(r + s)(r + s + \theta^* q(\theta^*) \beta)} > \frac{y(n) - b}{r + s + \theta^* q(\theta^*) \beta}.$$

Finally, rearranging terms, we obtain

$$y(s) - b > \left[ 1 + \frac{\mu(r + s)}{(1 - \mu)(r + s + \beta\theta^* q(\theta^*))} \right] (y(n) - b), \quad (25)$$

which is equivalent to the existence condition given in AV. Hence, according to equation (25), skilled jobs need to be more productive than unskilled jobs and the required productivity differential increases with  $\mu$ .

Second, to conclude the proof of existence, we need to show that high-educated workers accept unskilled jobs when firms create both types of jobs. This result requires that the match surplus  $S(n, h)$  is positive when (25) is satisfied. Since  $S(n, h) = \frac{y(n) - b}{r + s + \theta q(\theta)(1 - \eta + \eta\beta)}$ , this is equivalent to assuming that  $y(n) > b$ . Hence, as anticipated in Section 2, in our economy high-educated workers never find it optimal to refuse unskilled jobs.

**Proposition 1** *In any equilibrium high-educated workers accept unskilled jobs.*

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<sup>18</sup>Notice that this condition is derived under the assumption that workers do not engage in on-the-job search unless there is some strictly positive mass of skilled jobs. This assumption is natural given our assumption of purely random search. Furthermore, when  $\eta = 1$  it is necessarily true that  $\phi = \mu$ .

The intuition behind the above result is rather simple. Since workers with high education can search as efficiently during employment as during unemployment, they will accept any job that offers a wage above  $b$ .<sup>19</sup> A *cross-skill* matching equilibrium is therefore the only possible type of non-trivial equilibrium with two types of jobs. Furthermore, given Proposition 1, equation (25) is both a necessary and a sufficient condition to ensure existence of equilibrium.

### 3.5.2 Uniqueness

Since we are interested in the comparative statics properties of the model, a necessary preliminary step in the analysis should focus on the conditions for uniqueness. As shown in Appendix A, uniqueness is guaranteed under the following three sufficient conditions: (i) low-educated workers are a majority of the population, (ii) workers obtain at least half of the surplus of any match, and (iii) the productivity differential  $\{y(s) - y(n)\}$  exceeds a certain threshold value,  $y^*$  (defined in Appendix A). Formally, these restrictions on the parameter space can be stated as follows

$$\text{A.1 } \mu \geq 0.5, \beta \geq 0.5 \text{ and } \{y(s) - y(n)\} \geq y^*.$$

To obtain uniqueness, we substitute the solutions in equations (16) and (17) for  $\eta(\theta, \phi; \mu)$  and  $\psi(\theta, \eta; \mu)$  into the free-entry conditions (23) and (24). This yields a system of two equations in two unknowns, namely, the labour market tightness ( $\theta$ ) and the fraction of low-educated unemployed workers ( $\phi$ ). Furthermore, when  $\mu \geq 0.5$  and  $\beta \geq 0.5$ , we show in Appendix A that the profits of unskilled jobs increase with  $\phi$  and decrease with  $\theta$ . The free-entry condition for unskilled jobs is therefore associated with an upward-sloping locus in the space  $(\theta, \phi)$ . Intuitively, an increase in the labour market tightness makes it more difficult to fill any job. Thus, the profits of unskilled jobs will decrease unless the fraction of low-educated workers in the mass of unemployed workers and in the mass of job seekers increases enough.<sup>20</sup> Likewise,

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<sup>19</sup>By contrast, in AV mismatched workers do not engage in on-the-job search. By accepting an unskilled job high-educated workers therefore forego the option of employment in a skilled job. Consequently, beyond some threshold level of  $y(s)$ , these workers will prefer to refuse unskilled jobs and continue to search, giving rise to an *ex-post segmentation* matching equilibrium.

<sup>20</sup>Formally, for a given value of  $\theta$  an increase in the share of low-educated job seekers,  $\phi\psi$ , requires an increase in  $\phi$  as  $\partial(\phi\psi)/\partial\phi = \psi + \phi\partial\psi/\partial\phi > 0$ .



when the productivity differential  $\{y(s) - y(n)\}$  exceeds  $y^*$ , the profits of skilled jobs decrease both with  $\theta$  and  $\phi$ . This results in a downward-sloping curve that intersects the free entry curve of unskilled jobs at most once.

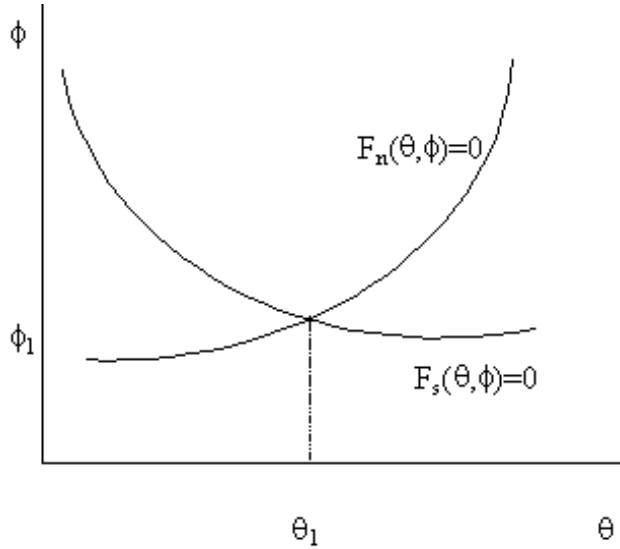


Figure 3: Uniqueness

The unique equilibrium is depicted in Figure 3. The reason why we need restrictions on  $\beta$  and  $\mu$  is related to the opposite effect of changes in the share of unskilled vacancies,  $\eta$ , on the outside option values of skilled and unskilled workers. While  $U(l)$  raises with  $\eta$ , since low-educated workers will match more often with their suitable jobs,  $U(h)$  decreases with  $\eta$ , since high-educated workers have a higher chance of being mismatched. Thus, consider for instance an increase in  $\phi$ . For a given value of  $\theta$ , a lower higher value of  $\phi$  requires a reduction in the share of skilled vacancies,  $\eta$ , and as a result  $U(h)$  will increase while  $U(l)$  will decrease. These two changes have opposite effects on the profits of unskilled jobs but, under Assumption A.1, the net effect on profits from the fall in  $U(l)$  and the rise in  $U(h)$  is always positive.

### 3.5.3 Wage distribution

Having derived the equilibrium, we can now analyze the effects of on-the-job search on the equilibrium wage distribution.

In our economy all workers are equally productive on unskilled jobs. Nonetheless, firms with unskilled jobs prefer to hire low-educated workers.

The reasons for this outcome are twofold. *First*, low-educated workers have a lower outside option than high-educated workers because they cannot perform skilled jobs. *Second*, firms anticipate that mismatched workers will quit an unskilled job whenever they have located a firm with a skilled vacancy. From equation (22), it follows that the first effect tends to raise the wage of mismatched workers, while the second effect tends to reduce it. Thus, the feature that high-educated workers have access to better jobs introduces two opposite effects on the wage of mismatched workers.

Nonetheless, in Appendix A we show that the negative effect always dominates. The explanation is that the wage differential,  $w(n, h) - w(n, l)$ , has the same sign as the surplus differential,  $S(n, h) - S(n, l)$ , which is negative as shown in section 3.3.

**Proposition 2** *In any cross-skill matching equilibrium,  $w(n, l) > w(n, h)$ .*

Proposition 2 contrasts with the findings of AV who obtain the opposite result. Without on-the-job search, the effective separation rate of high-educated workers is the same as the one of low-educated workers, so that the inequality  $w(n, h) > w(n, l)$  always holds because  $U(h) > U(l)$ . Finally, it is easy to show that in both models  $w(s, h)$  is always larger than  $w(n, l)$  as high-educated workers have a better outside option and because  $y(s) > y(n)$ .

In sum, our results clearly indicate that on-the-job search leads to a widening of the within-group wage inequality for high-educated workers relative to the case in which mismatched workers refrain from searching. In Section 5 we shall evaluate the contribution of this additional channel for the overall wage inequality using a calibrated version of the model.

## 4 Comparative statics

In this section we present some interesting comparative statics on the effects of: (i) an increase in the fraction of high-educated individuals in the population,  $1 - \mu$ , referred to as “skill upgrading”, and (ii) an increase in the productivity of high-educated workers,  $y(s)$ , for given  $y(n)$ , referred to as “skill-biased” technological change. In the sequel, we will concentrate on the effects of those two changes on both the cohort-specific unemployment rates and the overall degree of labour market tightness in the economy.

## 4.1 “Skill-upgrading”

What happens if the share of high-educated workers increases? To answer this question, we consider the effects of an inflow of high-educated workers, raising the share of these workers in the population to some value  $1 - \mu' > 1 - \mu$ .

Immediately after this change, the share of high-educated workers in the pool of job seekers  $1 - \phi\psi$ , increases. Skilled jobs will therefore match more frequently with appropriately qualified workers and firms will respond to this change by creating more skilled vacancies. By contrast, unskilled jobs will now meet more frequently with over-qualified workers. Some of these workers are unemployed and will accept the job offer while others are already employed on unskilled jobs. Yet, in both cases the increase in  $1 - \phi\psi$  tends to exert a negative effect on the profits of unskilled jobs as  $S(n, h) < S(n, l)$ . Firms will therefore respond to the increase in the share of high-educated workers by creating less unskilled jobs. In the sequel, we shall refer to this *negative search externality* as *NSE*.

Any increase in the proportion of high-educated workers is thus accompanied by an increase in the mass of skilled vacancies,  $v(s)$ , and a reduction in the mass of unskilled vacancies,  $v(n)$ .<sup>21</sup> Notice as well that these changes give rise to an unambiguous fall in the proportion of unskilled vacancies,  $\eta$ , while the effect on the labour market tightness,  $\theta$ , is unclear. The total number of jobs will increase when the rise in  $v(s)$  exceeds the fall in  $v(n)$ . But this need not translate into a higher value of  $\theta$ , since the distributions of educational levels and jobs also change the elements in the denominator of  $\theta$ , i.e.  $u(l)$ ,  $u(h)$  and  $e(n, h)$ . First, given the shift towards skilled jobs, the unemployment rate of low-educated workers,  $\tilde{u}(l)$ , tends to increase. Second, in the new equilibrium we have a larger share of high-educated workers and, since  $\tilde{u}(h) < \tilde{u}(l)$ , this tends to reduce the overall unemployment rate. Hence, in general it is thus impossible to predict the change in  $\theta$  whereas  $\phi$  unambiguously decreases.

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<sup>21</sup>This statement is true for the absolute number of unskilled jobs,  $v(n)$ , and for the ratio between the number of unskilled jobs and the number of low-educated job seekers. The latter is obviously a more meaningful statistic as the mass of low-educated workers falls.

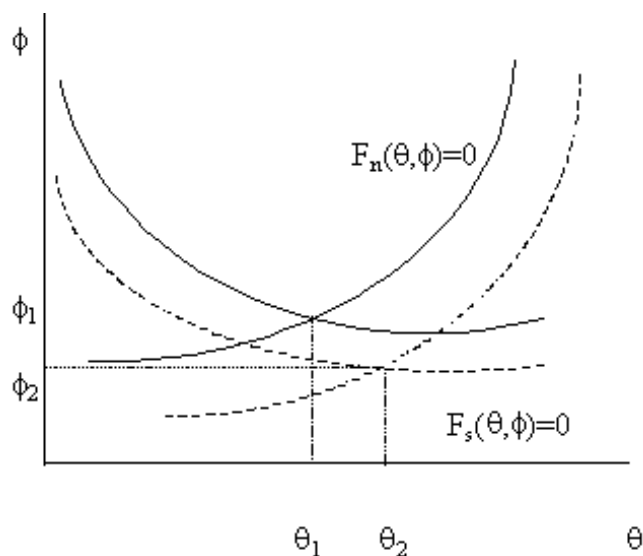


Figure 4: The effects of skill upgrading

Figure 4 offers an illustration of the previous effects for parameter configurations that satisfy A.1, where  $\theta$  increases. From (16) we obtain that the increase in  $1 - \mu$  reduces  $\eta(\theta, \phi; \mu)$  for any given value of  $(\theta, \phi)$ . According to equations (19) and (20), this shift in the distribution of jobs tends to reduce the outside option value of low-educated workers,  $U(l)$ , while it improves the corresponding option value of high-educated workers  $U(h)$ . Under Assumption A.1 the net effect on the profits of unskilled jobs is positive. Furthermore, the reduction in  $\eta$  goes in parallel with an increase in the fraction of unemployed in the pool of job seekers,  $\psi$ , which exerts a further positive effect on the profits of unskilled jobs. Hence, in order to restore zero profits, the  $F_N(\theta, \phi) = 0$  locus needs to shift to the right. Similarly, the increase both in  $U(h)$  and  $\psi$  tends to reduce the profits of skilled jobs for given values of  $(\theta, \phi)$ . Thus, the  $F_S(\theta, \phi) = 0$  locus will shift to the left.

Accordingly, the overall effect of an increase in the share of high-skill workers is thus a fall in  $\phi$ , and an ambiguous effect on  $\theta$  and  $\phi\psi$ , given the opposite move of  $\phi$  and  $\psi$ . From this we cannot draw unambiguous conclusions for the changes in the unemployment rates of high- and low-educated workers,  $\tilde{u}(h)$  and  $\tilde{u}(l)$ . Nonetheless, in our numerical experiments we always find an increase in  $\theta$ , as in Figure 6, and a fall in  $\eta$ . Thus, an increase in the cohort-size of high-educated workers tends to reduce their cohort-specific unemployment rate  $\tilde{u}(h)$ . That is, each high-educated job seeker enjoys a

higher exit rate out of unemployment and a larger share of the job offers are skilled jobs. Conversely, for low-educated workers we obtain an increase in  $\tilde{u}(l)$ . In the new steady-state equilibrium they match at a higher rate, but a larger proportion of these jobs are skilled jobs, resulting in a reduction of the overall matching rate  $\eta\theta q(\theta)$ .

These cohort-size effects for high-educated workers are absent in AV. In their model the overall labour market tightness,  $\theta$ , is invariant to changes in  $\mu$  and/or  $y(s)$ . Thus, any increase in  $v(s)$  is offset by an equivalent reduction in  $v(n)$ . The only variable that changes is therefore  $\eta$  and this variable affects  $\tilde{u}(l)$  but leaves invariant  $\tilde{u}(h)$ . On the contrary, allowing for on-the-job search leads to cohort-size effects for both types of workers.<sup>22</sup>

## 4.2 “Skill-biased” technological change

Let us now consider a situation in which technological change is biased towards high-educated workers, resulting in an increase of their productivity to  $y'(s) > y(s)$ , for given  $y(n)$ . From (24), it follows immediately that the increase in  $y(s)$  raises the profits of skilled jobs. The  $F_S(\theta, \phi) = 0$  locus will shift to the right. Furthermore, since  $S(n, h)$  is independent of  $y(s)$ , the  $F_N(\theta, \phi) = 0$  locus remains unchanged.

The overall effects of an increase in  $y(s)$  are illustrated in Figure 5. The rightward shift of the  $F_S(\theta, \phi) = 0$  locus leads to an increase in  $\theta$  and  $\phi$ . From the increase in  $\theta$ , we can immediately conclude that “skill-biased” technological change reduces  $\tilde{u}(h)$ , while the effect on  $\tilde{u}(l)$  is *a priori* unclear. For a given value of  $\theta$ , the share of unemployed workers with low skills,  $\phi$ , can only increase if  $\eta$  decreases. But this effect is at least partially offset by the increase in  $\theta$  which tends to raise  $\eta$  for given values of  $\phi$ .

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<sup>22</sup>As mentioned earlier, this result is in the same spirit as the one obtained by Shimer (2001). However, his result refers to cyclical/transitional unemployment rates and not to equilibrium unemployment, as in our case.

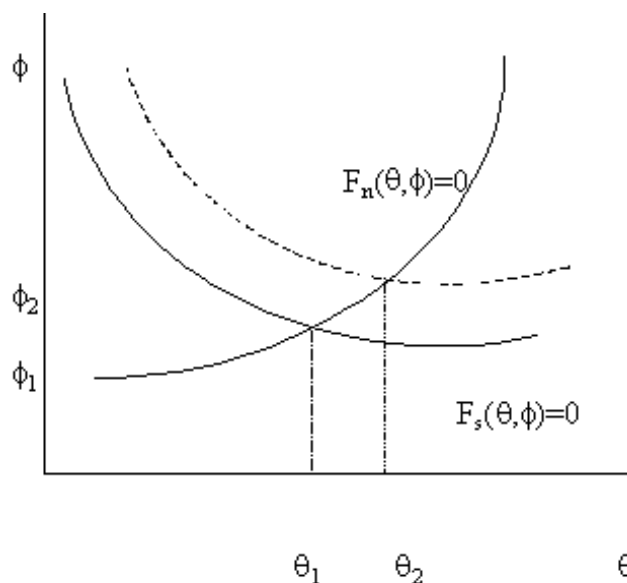


Figure 5: The effects of skill-biased technological change

**Proposition 3** *Under Assumption A.1, “skill-biased” technological change reduces  $\tilde{u}(h)$  while the effect on  $\tilde{u}(l)$  is ambiguous.*

The above Proposition is again clearly different from the results in AV who obtain that  $\partial\tilde{u}(h)/\partial y(s) = 0$  and  $\partial\tilde{u}(l)/\partial y(s) > 0$ . A first difference is therefore that “skill-biased” technological change may improve the employment prospects of high-educated workers.

It should be noted that this result does not depend on our assumptions about the common cost of vacancy creation,  $c$ , and unemployment income,  $b$ . In our model technological change would be neutral if  $y(s)$ ,  $y(n)$ ,  $b$  and  $c$  all grew at the same rate. Nonetheless, if skill-biased technological change implies different growth rates of  $y(s)$  and  $y(u)$ , it would still reduce  $\tilde{u}(h)$  when the cost of skilled vacancies and the unemployment income of high-educated are indexed to  $y(s)$ . What does seem to depend on the values of  $c$  and  $b$  is the effect of changes in  $y(s)$  on  $\tilde{u}(l)$ . In our numerical simulations below we find that skill-biased technological change raises  $\tilde{u}(l)$  for common values of  $b$  and  $c$ . Interestingly enough, however, when we allow for a higher cost of skilled vacancies it is easy to construct examples in which skill-biased technological change reduces  $\tilde{u}(l)$ .

This latter result points at an important difference between “skill-biased” technological change and “skill-upgrading”. Both changes induce the creation of more skilled jobs which exerts a negative congestion externality on the unskilled jobs in the market. On top of that skill-upgrading also results in an increase in the share of high-educated job seekers which further reduces the profits of unskilled jobs. By contrast, in the case of skill-biased technological change, more high-educated workers are drawn into skilled jobs. As a result, the share of low-educated job seekers,  $\phi\psi$ , tends to go up which exerts a positive effect on the profits of unskilled jobs. When the cost of vacancy creation are the same for both jobs, the congestion effect dominates over the composition effect and  $\tilde{u}(l)$  goes up. However, when skilled jobs are sufficiently more expensive to create, so that a large share of high-educated workers end up in unskilled jobs, the positive composition effect generated by a higher productivity differential may actually dominate so that  $\tilde{u}(l)$  goes down.

## 5 Simulations

In this section we perform a few illustrative simulations with the model. Our aim is to gauge quantitatively the comparative-statics effects of the model following the two changes discussed in the previous section, namely, *skill upgrading* and *skill-biased technological change*.

The model is calibrated using a standard Cobb-Douglas matching function,  $m = [v(n) + v(s)]^{1/2} [u(l) + u(h) + e(n, h)]^{1/2}$ , together with the following parameter configuration:  $\beta = 0.5$ ,  $r = 0.03$ ,  $c = 0.5$ ,  $y(n) = 1$  (equal productivity in unskilled jobs),  $y(s) = 1.5$ ,  $s = 0.1$ ,  $b = 0.1$ , and  $\mu = 0.75$ . Thus, in the baseline version of the model, the proportion of low-educated workers is 75% of the (unit mass) population. Under this choice of parameters, Table 1 reports the equilibrium values of the vector of unknowns  $\{\theta, \eta, \phi, \psi, u\}$  plus the unemployment rates of both types of workers,  $\tilde{u}(l)$  and  $\tilde{u}(h)$ .

The equilibrium value of  $\theta$  is 1.435 which implies an unemployment duration of 10 months, in line with the average duration of unemployment in some European countries, like Spain, where unemployment hysteresis has been strong (see Bentolila and Jimeno, 2003). Accordingly, the unemployment rates for low and high-educated workers are 11.1% and 7.8%, respectively, giving rise to an overall unemployment rate ( $u$ ) of 10.2%, in line with the average EU rate during the last fifteen years. The proportion of mis-

matched workers in the pool of job seekers,  $1 - \psi$ , is 23.7%, while the share of high-educated workers applying for unskilled jobs,  $1 - \psi\phi$ , equals 38.3%. This fraction captures the *NSE* on firms creating those vacancies. Likewise, the share of unskilled vacancies in total vacancies,  $\eta$ , is 67.4% and the fraction of low-educated workers in the pool of searchers,  $\phi$ , is 81.0%. Further,  $w(s, h)$  turns out to be 1.36 while  $w(n, l)$  and  $w(n, h)$  are 0.86 and 0.78, respectively. Thus, in agreement with Proposition 1, mismatched workers get paid less than low-educated workers in unskilled jobs.<sup>23</sup> Henceforth, we use the ratio  $w(s, h)/w(n, h)$  to capture *within-group wage inequality* (*WGI* henceforth), and the ratio between a weighted average of  $w(s, h)$  and  $w(n, h)$  and  $w(n, l)$  to represent the *average skill premium by education* (*AWI* henceforth).<sup>24</sup> Note that *WGI* is a good proxy for the penalty to over-education. In our baseline estimation *WGI* is 1.74 while *AWI* is 1.37.

Table 1  
Equilibrium values in Baseline Model

Variable	Estimate
$\theta$	1.435
$\eta$	0.674
$\phi$	0.810
$\psi$	0.763
$u$	0.102
$\tilde{u}(l)$	0.111
$\tilde{u}(h)$	0.078

Figure 6 displays four panels illustrating the effects of *skill upgrading*, captured by a continuous fall in  $\mu$  from 0.75 to 0.50, on  $\tilde{u}(l)$  and  $\tilde{u}(h)$ , *NSE*, *WGI* and *AWI*, respectively. Note that, in order to follow the correct direction of changes as  $\mu$  decreases, the graphs should be looked from right to left since the horizontal axis displays increasing values of  $\mu$ . Although, as argued in section 4,  $\tilde{u}(h)$  can go up or down since the effect of a change of  $\mu$  on  $\theta$  cannot be unambiguously signed, we find that for our choice of parameters it declines from 7.8% to 7.3% when  $\mu$  goes down from 75% to 50%, illustrating in this way the cohort-size effect discussed before. By contrast,  $\tilde{u}(l)$  increases

<sup>23</sup>Had we allowed for  $y(n, h) > y(n, l)$ , then we could have obtained that  $w(n, h) \geq w(n, l)$ .

<sup>24</sup>The corresponding weights are  $\frac{1-\mu-u(h)-e(n,h)}{1-\mu-u(h)}$  and  $\frac{e(n,h)}{1-\mu-u(h)}$ .



from 11.1% to 13.8%. Regarding  $NSE$ , we find that it increases from 38.3% to almost 50%. Finally, both  $WGI$  and  $AWI$  increase, the reason being that the reduction in the share of unskilled jobs,  $\eta$ , decreases the outside option value of low-educated workers,  $U(l)$ , whereas the corresponding increase in  $1 - \eta$  increases  $U(h)$ . Thus, for unchanged productivities,  $w(s, h)$  typically increases with “skill-upgrading” whilst both  $w(n, l)$  and  $w(n, h)$  decrease, leading to the observed widening of both  $WGI$  and  $AWI$ .

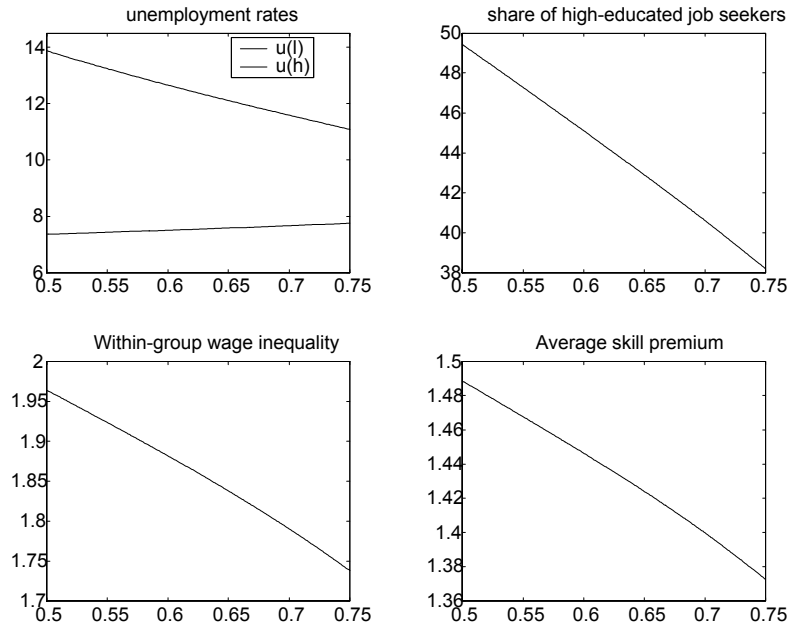


Figure 6: The effects of skill upgrading

Figure 7, in turn, shows the effects of *skill biased technical progress*, captured by a continuous increase in  $y(s)$  from 1.5 to 2.0. In accord with Proposition 2,  $\tilde{u}(h)$  falls to 7.2% whereas  $\tilde{u}(l)$  goes up to 11.5%. Similarly,  $NSE$  drops by almost six percentage points reflecting the reduction in the proportion of mismatched workers in unskilled jobs. As with *skill upgrading*, both  $WGI$  and  $AWI$  increase for similar reasons as above.

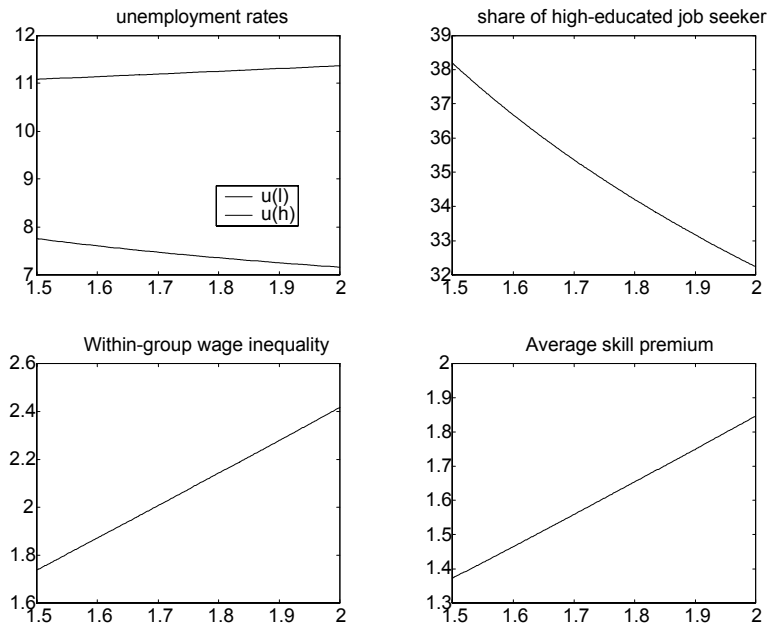


Figure 7: The effects of “skill-biased” technological change

So far, the numerical results corroborate our theoretical predictions. In the remainder we shall consider two slight generalisations of the model. In the first case we allow for different costs of vacancy creation,  $c(s) \neq c(n)$ , while the second example considers job-specific separation rates,  $s(s) \neq s(n)$ .<sup>25</sup>

Different costs of vacancy creation are a plausible assumption. In particular, it seems reasonable to assume that skilled job openings are more costly to create than unskilled job openings. Furthermore, from our discussion in the previous section we know that this assumption may change the response of  $\tilde{u}(l)$  to changes in  $y(s)$ . This feature is illustrated in Figure 8. In this example the cost of skilled vacancies,  $c(s)$ , is assumed to be 1, while the cost of unskilled jobs,  $c(n)$ , is kept constant at its benchmark value of 0.5. As can be seen, in this case  $\tilde{u}(l)$  follows a U-shaped pattern in  $y(s)$ . It initially falls up to values of  $y(s)$  around 1.80, beyond which it starts to increase again. The reason for this increase is that for, for high values of  $y(s)$ , the congestion effect dominates over the weaker *NSE*. Nonetheless for  $y(s) = 2$ , the case in which the cost of job creation is perfectly indexed to  $y(s)$ , the unemployment rate of low-educated workers is still below the value

<sup>25</sup>Appendix B contains the derivation of the equations determining equilibrium in this more general setup.

at  $y(s) = 1.5$ . Hence, in economies with relatively high costs of skilled job creation, skill-biased technological change may therefore help to reduce the unemployment of low-educated workers over a certain range of productivity differentials .

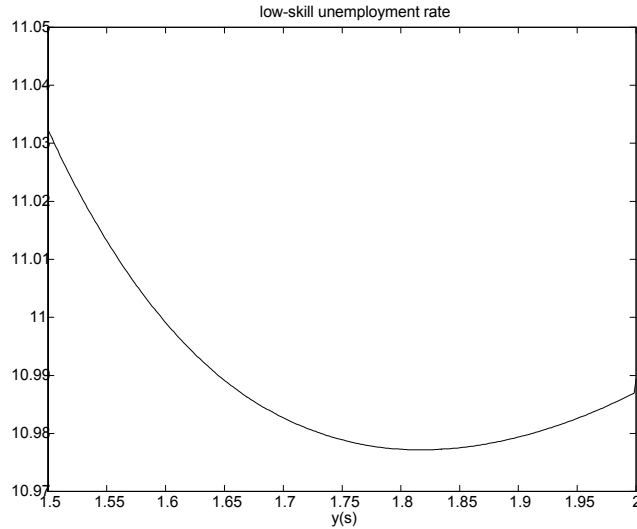


Figure 8: Different job creation costs

Next, “skill-biased” technological change in our model reduces the average tenure on unskilled jobs, while the tenure on skilled jobs is fixed. Nonetheless, if we were to consider fully endogenous job-destruction rates, the increase in the relative profits of skilled jobs would give rise to a lower separation rate on them. To capture this effect in our set-up, we allow for a reduction in the separation rate of skilled jobs,  $s(s)$ , from a value of 0.1 to 0.075 while keeping  $s(n)$  constant at its benchmark value of 0.1. Figure 9 illustrates the effect of this change on  $\tilde{u}(l)$ .<sup>26</sup> It turns out that a drop in the rate of turnover on skilled jobs reduces  $\tilde{u}(l)$  for any given value of  $y(s)$ , although in contrast to what happened with the increase in  $c(s)$ ,  $\tilde{u}(l)$  keeps on being increasing in  $y(s)$  . The explanation is somewhat similar to the argument for the cohort-size effects. When skilled jobs are more stable than unskilled jobs, high-educated workers return less frequently to the unemployment pool. Other things equal, the pool of searchers is therefore comprised by a larger share of low-educated workers and this exerts a positive effect on the profits

<sup>26</sup>The effect of such a change on  $\tilde{u}(h)$  are not reported since it obviously falls for given values of  $y(s)$ .

of unskilled jobs. Improving the stability of skilled jobs might therefore be beneficial for both types of workers in the presence of “skill-biased” technical change.

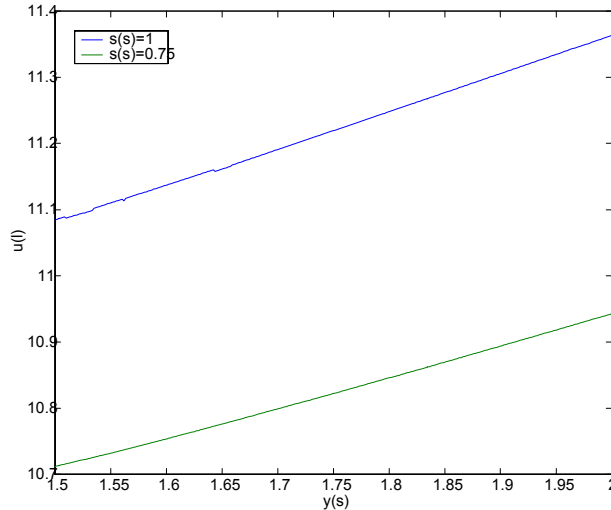


Figure 9: Different separation rates (a)

This last observation seems to point out to the need for differential labour market policies. In particular, in order to avoid the *NSE* from frequent unemployment spells of high-educated workers on low-educated workers, there may be scope for employment protection legislation that is differentiated across worker and/or job type. To illustrate somewhat the implications of this selective policies, Figure 10 shows the combined effects of both an increase in  $c(s)$ , from 0.5 to 1, and a reduction in  $s(s)$ , from 0.1 to 0.075. The change in  $c(s)$  would capture less jobs creation whereas the drop in  $s(s)$  would imply less job destruction in the skilled sector, in accord with the standard effects of employment protection legislation which makes both hiring and firing of workers more expensive. As can be observed, when  $y(s)$  increases, both unemployment rates fall relative to their benchmark values. The explanation is that, for the chosen configuration of parameter values, the net effect of both changes reduces  $\tilde{u}(h)$  (i.e., the effect of the reduction in  $s(s)$  is stronger than the rise in  $c(s)$ ) whilst, at the same time, it makes  $\tilde{u}(l)$  fall as well (i.e., the effect of the lower *NSE* implied by both the increase in  $c(s)$  and the drop in  $s(s)$  dominates over the direct negative effect on  $\tilde{u}(l)$  stemming from the increase in  $y(s)$ ). Of course, our example cannot be generalised to any value of the productivity differential and the explicit

introduction of firing cost in this model exceeds the scope of this paper. This issue is left for future research.

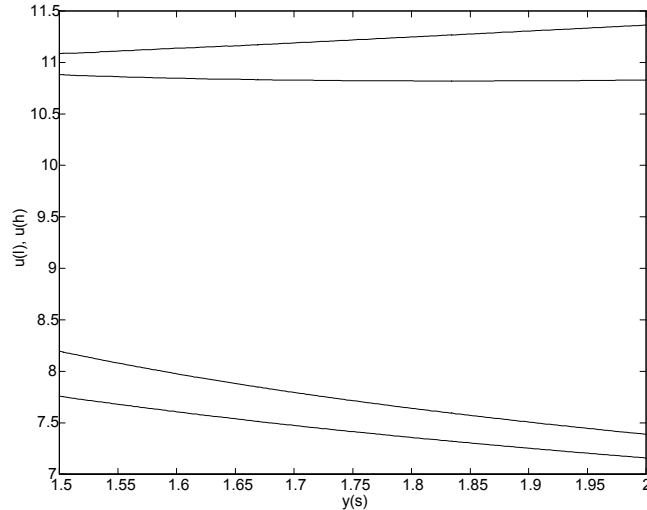


Figure 10: The effects of firing costs in skilled jobs

## 6 Conclusions

In this paper, we have analysed the properties of a matching model with two types of workers (low and high-educated) and two types of jobs (unskilled and skilled). High-educated workers can perform both jobs while the low-educated ones are only productive in unskilled jobs. A skilled job performed by a high-educated worker produces the highest output and both types of workers are equally productive in unskilled jobs. The novel element in the model is to allow for on-the-job search by mismatched high-educated workers while keeping a Nash bargaining rule to share the surpluses within each market. We show that this model can account for some of the stylised facts of the labour market in countries where a large tertiary educational upgrading which has taken place over the last decades.

In particular, we show that, with on-the-job search, *skill upgrading* generally decreases the unemployment rate of high-educated workers whereas it increases the unemployment rate of low educated workers. Thus we get a cohort-size effect whereby, in a market with search frictions, an increase in the supply of high-education ends up increasing the demand for skilled jobs by so much that the unemployment rate of the high-educated workers falls. Conversely, *skill-biased technical change*, while decreasing the unemployment

rate of the high-educated workers, can have ambiguous effects on the unemployment rate of the low-educated workers. The intuition is that mismatched workers in unskilled jobs create a negative search externality on firms opening unskilled vacancies which hampers the creation of these jobs. In situations where skilled vacancies are more expensive to open than unskilled jobs, and therefore skilled jobs are relatively scarce, *skill-biased technical change* can lead to a large improvement in the profitability of those jobs, and a reduction in the unemployment rate of the low-educated workers. Finally, under the assumption of equal productivity of both types of workers in unskilled jobs, our result that mismatched workers get a lower wage than correctly matched low-educated workers, offers a new channel to explain a widening of within-group wage inequality and a higher average skill premium which has been observed in many OECD countries.

The model could be extended in a number of ways. One extension is to endogenise the skill distribution by allowing workers to invest in education. A second extension would be to consider a model of directed search. In that environment workers can target their search to different types of jobs but nonetheless high-educated may consciously decide to apply for unskilled jobs (with some probability). Finally, one could consider the possibility of allowing for multiple meetings so that the model can address issues of ranking of applicants by firms.

## 7 Appendix A : Proofs

### Proof of $w(n, h) < w(n, l)$

From (18) and (20), the wage differential  $w(n, h) - w(n, l)$  can be expressed as

$$w(n, h) - w(n, l) = (1 - \beta) [rU(h) - rU(l) - \theta q(\theta)(1 - \eta)S(s, h)],$$

where  $rU(h) = b + \theta q(\theta)\beta [\eta S(n, h) + (1 - \eta)S(s, h)]$  and  $rU(l) = b + \theta q(\theta)\beta \eta S(n, l)$ .

Then, replacing  $rU(l)$  and  $rU(h)$  into the wage differential yields

$$rU(h) - rU(l) = \theta q(\theta)\beta [\eta(S(n, h) - S(n, l)) + (1 - \eta)S(s, h)],$$

implying that

$$w(n, h) - w(n, l) = (1 - \beta)\theta q(\theta)\beta \eta [S(n, h) - S(n, l)].$$

Hence

$$\text{sign} [w(n, h) - w(n, l)] = \text{sign} [S(n, h) - S(n, l)].$$

This, together with  $S(n, h) - S(n, l) < 0$ , yields the required inequality

$$w(n, h) - w(n, l) < 0. \blacksquare$$

### Proof of Uniqueness

Substituting  $\eta = \eta(\theta, \phi; \mu)$  and  $\psi = \psi(\theta, \phi; \mu)$  from equations (16) and (17) into the free entry conditions yields two equations in two unknowns,  $\theta$  and  $\phi$ , denoted in implicit form by  $F_S(\theta, \phi) = 0$  and  $F_N(\theta, \phi) = 0$ .

**Skilled jobs:** To show that the  $F_S(\theta, \phi) = 0$  locus has a negative slope, it is sufficient to show that

$$\frac{d\phi}{d\theta} = -\frac{\partial F_S / \partial \theta}{\partial F_S / \partial \phi} < 0.$$

Taking the derivative of  $F_S(., .)$  with respect to  $\theta$ , dividing all terms by  $\Delta = (1 - \beta)(y(n) - b)$  and denoting the ratio  $\frac{y(s) - b}{y(n) - b}$  by  $R$ , yields

$$\begin{aligned}
\frac{1}{\Delta} \frac{\partial F_S}{\partial \theta} &= \frac{q'(\theta)}{\lambda_1} (1 - \psi\phi) \left[ R - \frac{\beta\eta\theta q(\theta)}{\lambda_2} \right] \\
&\quad - \frac{q(\theta)}{[\lambda_1]^2} (1 - \psi\phi) \left[ R - \frac{\beta\eta\theta q(\theta)}{\lambda_2} \right] \frac{\partial \lambda_1}{\partial \theta} \\
&\quad - \frac{q(\theta)}{\lambda_1} (1 - \phi) \frac{\beta\eta}{\lambda_2} \cdot \frac{\partial \theta q(\theta)}{\partial \theta} + \frac{q(\theta)}{\lambda_1} (1 - \phi) \frac{\beta\eta\theta q(\theta)}{[\lambda_2]^2} \cdot \frac{\partial \lambda_2}{\partial \theta} \\
&\quad + \frac{q(\theta)}{\lambda_1} \left[ R - \frac{\beta\eta\theta q(\theta)}{\lambda_2} \right] \frac{\partial(1 - \psi\phi)}{\partial \theta}.
\end{aligned}$$

Since  $q'(\theta) < 0$ ,  $\frac{\partial \lambda_1}{\partial \theta} > 0$ , and  $\frac{\partial \theta q(\theta)}{\partial \theta} > 0$  the first three terms of the above expression are negative whereas the last two terms, given that  $\frac{\partial \lambda_2}{\partial \theta} > 0$ ,  $\frac{\partial(1 - \psi\phi)}{\partial \theta} > 0$ , are positive. Nonetheless, combining the the third and fourth terms yield a negative term whilst our assumption that  $\frac{\partial(1 - \psi\phi)q(\theta)}{\partial \theta} < 0$ , ensures that a combination of the first and fifth terms is also negative. Thus,  $\partial F_S / \partial \theta < 0$ .

The corresponding expression for  $\partial F_S(\cdot, \cdot) / \partial \phi$  is:

$$\begin{aligned}
\frac{1}{\Delta} \frac{\partial F_S}{\partial \phi} &= -\frac{q(\theta)}{[\lambda_1]^2} (1 - \psi\phi) \left[ R - \frac{\beta\eta\theta q(\theta)}{\lambda_2} \right] \frac{\partial \lambda_1}{\partial \phi} \\
&\quad + \frac{q(\theta)}{[\lambda_1]^2} \left[ R - \frac{\beta\eta\theta q(\theta)}{\lambda_2} \right] \cdot \frac{\partial[(1 - \psi\phi)]}{\partial \phi} \\
&\quad + \frac{q(\theta)(1 - \psi\phi)}{[\lambda_2]^2} \cdot \beta\eta\theta q(\theta) \cdot \frac{\partial \lambda_2}{\partial \phi} \\
&\quad - \frac{\beta q(\theta)}{\lambda_1 \lambda_2} \frac{\partial \eta \theta q(\theta)}{\partial \phi}
\end{aligned}$$

The first two terms are negative since  $\frac{\partial \lambda_1}{\partial \phi} > 0$  and  $\frac{\partial(1 - \psi\phi)}{\partial \phi} < 0$  whereas the last two terms are positive since  $\frac{\partial \lambda_2}{\partial \phi} > 0$  and  $\frac{\partial \eta \theta q(\theta)}{\partial \phi} < 0$ . However, if  $R$  is sufficiently large, then the negative sign of the first two terms will dominate. Thus,  $\partial F_S / \partial \phi < 0$  when the productivity differential  $\{y(s) - y(n) \geq y^*\}$ , namely, a threshold value such that, in absolute value, the sum of the first two terms exceeds the remaining two terms (which do not depend on  $R$ ) in the above derivative.



**Unskilled jobs:** To show that the  $F_N(\theta, \phi) = 0$  locus has a positive slope, it is sufficient to show that

$$\frac{d\phi}{d\theta} = -\frac{\partial F_N/\partial\theta}{\partial F_N/\partial\phi} > 0.$$

As before, taking the derivative of  $F_N(.,.)$  with respect to  $\theta$  and dividing all terms by  $\Delta = (1 - \beta)(y(n) - b)$  yields

$$\begin{aligned} \frac{1}{\Delta} \frac{\partial F_N}{\partial\theta} &= q'(\theta)\psi \left[ \frac{\phi}{r + s + \theta q(\theta)\eta\beta} + (1 - \phi)\frac{1 - \phi}{\lambda_2} \right] \\ &+ \left[ \frac{q(\theta)\phi}{r + s + \theta q(\theta)\eta\beta} + \frac{q(\theta)(1 - \phi)}{\lambda_2} \right] \cdot \frac{\partial\psi}{\partial\theta} \\ &- \left[ \frac{\beta q(\theta)\psi\phi}{[r + s + \theta q(\theta)\eta\beta]^2} \right] \cdot \frac{\partial\eta\theta q(\theta)}{\partial\theta} + \frac{\psi\theta q(\theta)(1 - \phi)}{[\lambda_2]^2} \cdot \frac{\partial\lambda_2}{\partial\theta}, \end{aligned}$$

which is unambiguously negative since  $q'(\theta) < 0$ ,  $\frac{\partial\psi}{\partial\theta} < 0$  and  $\frac{\partial\lambda_2}{\partial\theta} < 0$ .

Similarly, the derivative  $\partial F_N(.,.)/\partial\phi$  is given by

$$\begin{aligned} \frac{1}{\Delta} \frac{\partial F_N}{\partial\phi} &= \left[ \frac{q(\theta)}{r + s + \theta q(\theta)\eta\beta} - \frac{q(\theta)}{\lambda_2} \right] \cdot \frac{\partial\psi\phi}{\partial\phi} \\ &- \frac{\beta\psi\phi q(\theta)}{[r + s + \theta q(\theta)\eta\beta]^2} \cdot \frac{\partial\eta\theta q(\theta)}{\partial\phi} \\ &+ \frac{q(\theta)}{\lambda_2} \cdot \frac{\partial\psi}{\partial\phi} - \frac{q(\theta)\psi(1 - \phi)}{[\lambda_2]^2} \cdot \frac{\partial\lambda_2}{\partial\phi}, \end{aligned}$$

where the first three terms are positive since  $\frac{\partial\psi\phi}{\partial\phi} > 0$ ,  $\frac{\partial\eta\theta q(\theta)}{\partial\phi} < 0$ ,  $\frac{\partial\psi}{\partial\phi} > 0$  and  $\lambda_2 > r + s + \theta q(\theta)\eta\beta$  whilst the last term is negative since  $\frac{\partial\lambda_2}{\partial\phi} > 0$ . However, if  $\beta \geq 0.5$  and  $\mu \geq 0.5$ , so that  $\phi \geq 0.5$ , the second term dominates the fourth term in absolute value. Therefore  $\partial F_N/\partial\phi > 0$ . ■

## 8 Appendix B: Equilibrium with unequal separation rates

In Section 5 we simulate the model with different job-separation rates for skilled and unskilled jobs, denoted, respectively, by  $s(s)$  and  $s(n)$ , with  $s(s) <$

$s(n)$ . In this case the steady state conditions for  $u(l)$ ,  $u(h)$  and  $e(n, h)$  can be written as

$$\eta\theta q(\theta)\phi u = s(n)(\mu - \phi u)$$

$$\eta\theta q(\theta)(1 - \phi)u = [s(n) + \theta q(\theta)(1 - \eta)] e(n, h)$$

$$(1 - \eta)\theta q(\theta) [(1 - \phi)u + e(n, h)] = s(s) [1 - \mu - (1 - \phi)u - e(n, h)].$$

To determine the equilibrium value of  $\{\theta, \eta, \phi, \psi, u\}$ , the equations above are need to be complemented by the two free-entry conditions which, following the same arguments as in the derivation of equations (23) and (24), become

$$\frac{c}{q(\theta)} = \psi(1 - \beta) \left[ \phi \frac{y(n) - b}{r + s(n) + \theta q(\theta)\eta\beta} + (1 - \phi) \frac{y(n) - b}{\varsigma_2} \right]$$

$$\frac{c}{q(\theta)} = (1 - \beta)(1 - \psi\phi) \left[ \frac{y(s) - b}{\varsigma_1} - \theta q(\theta)\eta\beta \frac{y(n) - b}{\varsigma_1\varsigma_2} \right],$$

where  $\varsigma_1 = r + s(s) + \theta q(\theta)(1 - \eta)\beta$ ,  $\varsigma_2 = r + s(n) + \theta q(\theta)(1 - \eta + \eta\beta)$ .

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