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Working Paper

Incorporating Prior Information into a GMM Objective for Mixed Logit Demand Systems

EAG Discussion Paper, No. EAG 12-1

Provided in Cooperation with:

Economic Analysis Group (EAG), Antitrust Division, United States Department of Justice

Suggested Citation: Romeo, Charles (2012) : Incorporating Prior Information into a GMM Objective for Mixed Logit Demand Systems, EAG Discussion Paper, No. EAG 12-1, U.S. Department of Justice, Antitrust Division, Economic Analysis Group (EAG), Washington, DC

This Version is available at:

<https://hdl.handle.net/10419/202394>

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Appendices B and C: Incorporating Prior Information into a GMM Objective for Mixed Logit Demand Systems

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February 2012

Appendix B: Outside Share and Instrumental Variables

B.1. Constructing the outside share

We assume that the market is all beer products, and given this we use three pieces of information in constructing the outside share. First, data from the 2008 edition of The U.S. Beer Market indicates that in 2007 each person, 21 and older, consumed an average 0.25308 cases of beer per week. Multiplying this figure by the 21 and over population in each market provides a first estimate of total market sales from which we subtract observed consumption recorded in our data to estimate outside beer consumption. Doing this yielded an average outside share of more than 85 percent.

The value of the outside share is that it brings the impact of outside competitors into the model. In the case of beer, however, this large outside share likely overstates the effect of outside competition. In large part outside competition results from the same brewers products being sold through other venues (liquor stores, convenience stores, mass merchandisers, and bars and restaurants). Allowing such a large outside share would understate the control and coordination that brewers likely exert in pricing across venues. In most localities, brewers license distributors who have exclusive territories and who serve all venues. We use this information about the structure of the beer market to reduce the outside share down to 25 percent of our first pass calculation and then renormalize shares yielding an overall average outside share of roughly 61.5 percent.

Our choice of 25 percent was simply a rough adjustment designed to keep to outside share large while attempting to account for coordination of brewer/distributor pricing across venues. Altering the size of the outside share affects the elasticity of demand for inside products. We evaluate the affect of the outside share size on the aggregate elasticity for unconstrained and product-level constrained results in Table B1.

We include aggregate elasticity estimates for two other levels of the outside share: 35.4 and 85.8 percent.¹ The results show substantial variation, with the aggregate elasticity varying

¹85.8 is the outside share that corresponds with a potential market equal to total beer consumption in the US; 35.4 is derived by multiplying 85.8*0.10 to shrink the potential market to 8.58 percent outside plus 14.2 percent inside which are then renormalized to equal 100 percent. This is approximately the smallest outside share consistent with estimation proceeding

from -0.506 to -1.203 for the unconstrained results and -0.916 to -1.908 for the product-level constrained results as the outside share increases. Better assessing the size of the outside share is left as a project for future research.

Table B1. The affect of outside share size on aggregate elasticity

mean outside share %	aggregate elasticity–global minima for models	
	demand only unconstrained	product-level constraints
35.4	-0.526	-0.916
61.5	-0.908	-1.135
85.8	-1.203	-1.908

The third piece of information that we use in setting the outside share is the fact that beer consumption is cyclical. Consumption follows a sinusoid that peaks in the summer and troughs in the winter. The amplitude of peaks and troughs is greater the further north one goes, and are nearly flat for cities along southern tier of the US. Tampa’s sine wave is an outlier among our 37 markets in that it peaks in the winter. We use this information to allow total market consumption to follow sine waves that we estimate separately for each market. For each market, we set per person consumption equal to the US average for the week in 2007 that includes the vernal equinox (the inflection point in most of the estimated sine waves), and allow total consumption to shift in accordance with our estimated sine waves.

B.2. Choosing and evaluating the strength of instruments

The focus of this section is to motivate the sets of instruments that we use in addition to our baseline set of exogenous product characteristics and mean demographics. We introduce four sets of instruments. The first is based on the own-product exchangeability argument in BLP. The second and third have intuitive bases: one uses lagged intertemporal market share differences, the other captures the effectiveness of past promotions. The fourth set are the means of included demographics discussed in Romeo (2011).

BLP formulate exchangeability arguments that apply to differentiated product demand systems in which the demand for a product does not depend on the ordering of rival products, having cost functions that depend only on own-product costs, and with a Nash-Bertrand pricing equilibrium. In this context, as discussed in BLP, the demand and cost functions are fully exchangeable, and the pricing function is partially exchangeable:

- (i) it is exchangeable in the order of the competing firms,
- (ii) for a given competitor, exchangeable in the order of that competitor’s products, and
- (iii) for a given product, exchangeable in the order of other products marketed by the same firm.

The value of exchangeability is that it restricts the basis of the set of optimal instruments.

smoothly. Smaller outside shares caused a dramatic increase in contraction mapping iterations.

In a Nash-Bertrand context, prices respond explicitly to changes in characteristics and prices of other products owned by the same firm, and implicitly to changes in characteristics and prices of products owned by competing firms. Hence, the set of optimal instruments includes characteristic information from all competing firms. As discussed by Newey (1990) one can use a polynomial to approximate the set of optimal instruments. However, without exchangeability, the basis of the polynomial grows with sample size: in a market with J products each of which has K observed characteristics, the unrestricted basis of a first order polynomial is JK . Pakes (1997) shows that exchangeability imposes restrictions on the polynomial basis. For example, (ii) implies that for a given competitor, the sum across products for each characteristic of that competitor's products forms the polynomial basis, while (i) further restricts this basis by implying that we can also sum across competitors. In addition (iii) implies that own-product characteristics and the sum of characteristics across all other products owned by the same firm contribute to the restricted polynomial basis. As such, the first order terms of this restricted polynomial grow only in K .

We lose exchangeability types (i) and (ii) if the competition cannot be characterized by a unique Nash-Bertrand equilibrium. Research on the beer industry raises questions as to the reasonableness of this assumption relative to Stackelberg type leader-follower behavior or other forms of coordinated behavior. Greer (1998) and Tremblay and Tremblay (2005) provide a long history of statements by Anheuser-Busch executives to the media as evidence that it plays the role of industry price leader. Econometric evidence, however, has been less compelling. Using data on the U.K. brewing industry, Pinske and Slade (2004) cannot reject Nash-Bertrand pricing, and Slade (2004) does not find evidence to support coordinated effects. Rojas (2008) finds that Stackelberg behavior may explain the U. S. data better, but that the estimated differences between two Stackelberg variants that he tests and Nash-Bertrand are not likely to differ statistically. This evidence, while admittedly weak, does raise concerns about the validity of the exchangeability arguments related to the products of other firms. More specifically, if one knows the form of coordination, knows the identity of the participants to a coordination arrangement, and knows the degree of coordination, then (i) and (ii) still may be satisfied. The pricing function is exchangeable in:

- (ia') the order of firms with which a firm coordinates assuming the degree of coordination between firms is the same;
- (ib') the order of firms with which a firm competes;
- (ii') for a given competitor, exchangeable in the order of that firm's products if the degree of coordination with that firm is the same among all products.

Suppose two firms tacitly collude in the pricing of all their products. Then, assuming their identity is known and coordination is perfect, it is as though we have a new Nash-Bertrand equilibrium with the products of two firms treated as though they were produced by a single firm. Alternatively, if the identity of the colluders is unknown or the degree of coordination across products is unknown or variable then the conditions required for exchangeability cannot be satisfied. For example, suppose in the context of the beer industry, that Anheuser-Busch/InBev (AB/InBev) and MillerCoors tacitly coordinate on pricing, but that coordination is imperfect, and varies by product and market. It may be that AB/InBev leads with its Budweiser and Bud Light products in markets where it has the dominant market share, and that MillerCoors leads with Miller Genuine Draft, Miller Lite, and Coors Light in markets where its share

dominates. In both cases, the market leader might only respond to price moves by the competitors it deems most important.

Complications such as these make the conditions for exchangeability across competitors impossible to satisfy with any confidence. The alternatives are to use competitor characteristics in an unrestricted fashion, or to search out different instruments. We chose the latter path. In total we make use of four different instrument sets. Our “baseline” set is the set of all exogenous product characteristics in x . Our “own-brand” set use the characteristics of other own-firm products² based on exchangeability argument (iii).

We rely on intuition in introducing two additional instrument sets. The first focuses on the promotion variables. We recognize that retailers decide how much of a discount to pass through to consumers and they contribute to decisions on which products to feature and/or display. Retailers are likely to use the success of past promotion programs as a decision guide. We capture this as

$$\Delta_{sw} = \frac{1}{4} \sum_{\ell=t-k-4}^{t-k} \frac{s_{jm\ell} - s_{jm,\ell-1}}{w_{jm\ell} - w_{jm,\ell-1}}$$

where w is used to index feature, display, and discount promotions. Δ_{sw} measures the average one week change in share of product j relative to the one week change in promotional level of j .³ Effective past promotions will produce relatively high values of Δ_{sw} , ineffective ones will produce low or even negative values. In order to make Δ_{sw} predetermined and to reflect retailer decision processes, we set $k = 4, 52$, implying that the retailer reviews her experience from one month ago, and from one year ago.

The second additional instrument set also has an intuitive basis. One often hears on business reports or reads in the business pages how “same store sales” compare from month-to-month or year-to-year. This suggests that in setting prices, companies pay attention to intertemporal share changes. We form four share difference variables for each product in each market-time period: two one month differences, and two one year differences. Both the month and year differences are calculated using single week differences, and the difference of four week average shares. Suppressing j and m subscripts, share differences are calculated as follows:

²we construct own-brand package size/container type instruments, such that, in each market-week, for each brand and package size by both bottles and cans, we exclude the current observation and formulate the share of that brand’s products that are the same package size/container type.

³As noted above, we do not observe promotional levels by product, but only by brand. Hence, $w_{jmt} = w_{j' mt}$, for all $j, j' \in B$, where B indexes a brand.

$$\Delta s_{t-5,t-9} = s_{t-5} - s_{t-9}$$

$$\Delta s_{t-5,t-57} = s_{t-5} - s_{t-57}$$

$$\Delta \bar{s}_{month} = \frac{1}{4} \sum_{\ell=t-8}^{t-5} s_{\ell} - \frac{1}{4} \sum_{\ell=t-12}^{t-9} s_{\ell}$$

$$\Delta \bar{s}_{year} = \frac{1}{4} \sum_{\ell=t-8}^{t-5} s_{\ell} - \frac{1}{4} \sum_{\ell=t-60}^{t-57} s_{\ell}$$

We chose month and year differences based on single week and monthly average shares in an attempt to capture the full range of historical share information firms would review in updating prices. Lagging the differences has the practical effect of making it more likely that they will be valid instruments.

In using each instrument set, we form orthonormal polynomials of the own-brand, promotion effect, and share difference instrument sets and their square.⁴ Our first step in using these instruments is to evaluate the relevance and strength of each set of instruments and mean demographics. We do this by evaluating partial R² measures developed by Shea (1997) and F-statistics based on these measures. Shea (1997) develops partial R²s to evaluate instrument relevance in contexts with multiple endogenous variables. A simple example of how his measure works is given by considering the regression $Y = X\beta + \varepsilon$, where Y is $T \times 1$ and X is $T \times 2$ and both X s are endogenous. Suppose now that two instruments vectors Z_1 and Z_2 are available and that Z_1 is highly correlated with both X s while Z_2 is uncorrelated with either X . Reduced form regressions $X_k = f(Z_1, Z_2)$, $k = 1, 2$ would produce high R²s even though β is unidentified. In contrast, Shea's measure captures the fact that there are fewer relevant instruments than endogenous variables and produces a partial R² = 0 for both regressions.⁵

Table B2 contains the results of these evaluations. All the partial R² estimates are from reduced form regressions of each of our five endogenous variables (price, discount, feature, display, and feature and display) on the baseline instrument set plus at least one additional instrument set. Columns 1 - 3 include, respectively, results for regressions with the own-brand, share difference, and promotion effect instrument sets. The partial R² estimates indicate that our

⁴We have found that standardizing all data dramatically improves the condition numbers of the input matrices and that this and orthonormalizing the instrument polynomials substantially improves numerical performance. For example, the condition numbers for the instrument matrix in specification (iii) using instrument set (1) with mean demographics (see below) were $\text{cond}(z) = 819$ and $\text{cond}((z - \bar{z})/\sigma_z) = 4.58$. This degree of improvement was typical.

⁵There are three steps to forming Shea's partial R². First, evaluate \tilde{X}_k , the component of X_k orthogonal to X_{-k} . Second, form \bar{X}_k , the component of X_k 's projection on Z , orthogonal to X_{-k} 's projection on Z . Third, partial R² = $\text{corr}(\tilde{X}_k, \bar{X}_k)^2$.

baseline instruments combined with each of these other instrument sets are relevant for all five endogenous variables. The F-statistics in the next block of the table exclude the baseline instruments and examine the joint significance of the own-brand, share difference, and promotion effect instruments respectively in each column. These F-statistics are based on the partial R^2 s in order to capture the explanatory power of each set of instruments for a given endogenous variable that is orthogonal to that instrument set's ability to explain the other endogenous variables. The results indicate that all three sets of instruments are strong for all five endogenous variables.

Romeo (2011) makes the argument that mean demographics can better control for price shifts across markets than can exogenous product characteristics or the own-brand instruments: the set of products in each market is nearly identical yielding these instruments little power to explain cross-market price level differences. Columns 4 - 7 evaluate empirically the relevance and strength of mean demographics as additional instruments. The regressions in Columns 4 - 6, are the same as those in Columns 1 - 3 respectively, but with mean demographics as additional instruments, while in Column 7 all sets of instruments are included. The F-tests for these columns evaluate only the joint significance of the mean demographics. Comparisons of the partial R^2 estimates in Columns 1 and 4, 2 and 5, and 3 and 6 indicate that mean demographics are relevant for all five endogenous variables as the partial R^2 s increase substantially in all comparisons, and the F-tests indicate mean demographics to be jointly significant. In Column 7 the F-test evaluates the strength of mean demographics in a regression that includes all the other sets of instruments. Mean demographics are shown to be strong instruments even in this case, thereby lending empirical support to the argument that they provide information that is orthogonal to that provided by all the other instruments.

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Table B2. Partial R² and F-statistics for first stage regressions of price, discount, feature, display, and feature and display on one or more instrument sets.

Instrument sets used:	(1)	(2)	(3)	(4)	(5)	(6)	(7)
baseline	yes	yes	yes	yes	yes	yes	yes
own-brand	yes	no	no	yes	no	no	yes
share differences	no	yes	no	no	yes	no	yes
promotion effect	no	no	yes	no	no	yes	yes
mean demographics	no	no	no	yes	yes	yes	yes
1 st stage partial R ² :							
price	0.506	0.486	0.487	0.531	0.505	0.504	0.539
discount	0.049	0.030	0.026	0.155	0.148	0.150	0.159
feature	0.033	0.022	0.019	0.078	0.072	0.069	0.086
display	0.103	0.081	0.064	0.255	0.245	0.241	0.262
feature and display	0.082	0.053	0.042	0.147	0.138	0.132	0.163
1 st stage F-stats:							
price	209.48	65.23	40.11	242.05	107.80	30.45	77.78
discount	127.55	87.40	23.11	34.61	23.89	10.03	22.24
feature	79.74	62.46	17.99	52.91	73.37	8.40	72.47
display	217.41	228.01	22.85	167.12	84.36	2.57	91.09
feature and display	225.05	188.68	35.22	129.24	171.67	13.36	177.91
degrees of freedom	18	8	16	6	6	6	6
P(F > f) = 0.05	1.60	1.94	1.645	2.10	2.10	2.10	2.10
IV set evaluated in F-test:							
own-brand	yes	no	no	no	no	no	no
share differences	no	yes	no	no	no	no	no
promotion effect	no	no	yes	no	no	no	no
mean demographics	no	no	no	yes	yes	yes	yes

Appendix Tables C1 - C8

Table C1. Unconstrained minimums 2, 3 and 4 for mixed logit estimates for beer demand

variables	unconstrained estimates					
	minimum 2		minimum 3		minimum 4	
	means ($\bar{\alpha}, \theta$)	std devs Y	means ($\bar{\alpha}, \theta$)	std devs Y	means ($\bar{\alpha}, \theta$)	std devs Y
price	-4.461 (0.690) [†]	0.027 (0.044)	-4.799 (0.760) [†]	0.008 (0.015)	-5.936 (0.531) [†]	4e-8 (7e-10) [†]
light	-1.306 (0.203) [†]	0.125 (0.104)	-1.389 (0.302) [†]	0.117 (0.089)	-0.183 (0.078) [*]	0.00001 (6e-6)
Corona	0.348 (0.177) [*]	0.0004 (0.0004)	0.315 (0.370)	0.167 (0.104) ^{**}	0.606 (0.164) [†]	0.038 (0.030)
discount	-0.944 (0.415) [*]	0.0003 (0.0002)	-0.992 (0.405) [†]	0.0001 (0.0001) [*]	-0.996 (0.271) [†]	0.0002 (3.5e-5) [†]
feature	0.519 (0.151) [†]		0.433 (0.176) [*]		0.304 (0.157) ^{**}	
display	0.718 (0.107) [†]		0.642 (0.113) [†]		0.639 (0.104) [†]	
feature and display	0.983 (0.146) [†]		0.880 (0.165) [†]		0.897 (0.131) [†]	
demographic interactions						
price:	income	-0.064 (1.390)	-0.052 (1.476)	0.003 (0.775)		
	income ²	-0.027 (2.867)	-0.231 (3.107)	-0.080 (2.158)		
	age 21 - 34	-0.049 (1.848)	-0.085 (1.935)	2.654 (0.868)		
	age 35 - 54	-2.856 (0.957) [†]	-3.485 (0.896) [†]	3.005 (0.433) [†]		
light:	age 21 - 34	-0.167 (0.581)	-0.305 (0.631)	3.263 (0.693) [†]		
	age 35 - 54	6.396 (1.214) [†]	-7.184 (1.572) [†]	3.811 (0.694) [†]		
Corona:	%Hispanic	-2.497 (0.486) [†]	-2.465 (0.517) [†]	-2.643 (0.374) [†]		
discount:	income	0.730 (0.489)	0.821 (0.520)	1.265 (0.498) [†]		
	income ²	-3.298 (2.048)	-3.528 (1.992) ^{**}	-4.670 (1.150) [†]		

(Standard deviations in parentheses)

*Significant at the 10% level; **significant at the 5% level; †significant at the 1% level.

Table C2. Unconstrained joint demand-supply minimums 2, 3 and 4 for mixed logit estimates for beer demand

variables	unconstrained joint estimates					
	minimum 2		minimum 3		minimum 4	
	means ($\bar{\alpha}, \theta$)	std devs Y	means ($\bar{\alpha}, \theta$)	std devs Y	means ($\bar{\alpha}, \theta$)	std devs Y
price	-5.870 (0.284) [†]	3.5e-8 (2.4e-10) [†]	-6.836 (0.248) [†]	0.004 (0.001) [†]	-6.360 (0.283) [†]	0.008 (0.003) [†]
light	-0.378 (0.023) [†]	1.1e-5 (3.2e-6) [†]	-0.160 (0.023) [†]	0.004 (0.001) [†]	-0.477 (0.021) [†]	0.269 (0.064) [†]
Corona	0.718 (0.014) [*]	0.065 (0.011) [†]	0.750 (0.014) [*]	0.002 (0.001) ^{**}	0.787 (0.014) [*]	0.031 (0.012) [†]
discount	-1.539 (0.043) [*]	2.6e-6 (7.0e-7)	-1.232 (0.042) [*]	0.011 (0.004) ^{**}	-1.114 (0.041) [*]	0.0001 (5.4e-5) ^{**}
feature	1.240 (0.068) [†]		0.791 (0.067) [†]		0.901 (0.066) [†]	
display	-0.056 (0.033) [*]		0.406 (0.033) [*]		0.272 (0.032) [*]	
feature and display	1.116 (0.051) [†]		1.002 (0.048) [†]		0.943 (0.049) [†]	
demographic interactions						
price:	income	-0.294 (0.219)	-0.701 (0.124) [†]		-0.985 (0.130) [†]	
	income ²	-0.115 (0.626)	-0.629 (0.585)		1.386 (0.529) [†]	
	age 21 - 34	0.828 (0.161) [†]	1.522 (0.152) [†]		-0.945 (0.269) [†]	
	age 35 - 54	-0.090 (0.231)	0.871 (0.205) [†]		-1.053 (0.195) [†]	
light:	age 21 - 34	0.892 (0.202)	1.230 (0.226) [†]		-0.719 (0.221) [†]	
	age 35 - 54	1.205 (0.272) [†]	1.944 (0.331) [†]		-1.330 (0.204) [†]	
Corona:	%Hispanic	0.520 (0.032) [†]	-2.345 (0.301) [†]		-1.906 (0.302) [†]	
discount:	income	1.132 (0.252) [†]	1.260 (0.274) [†]		1.060 (0.226) [†]	
	income ²	-3.725 (0.527) [†]	-4.364 (0.578) [†]		-3.487 (0.772) [†]	

(Standard deviations in parentheses)

*Significant at the 10% level; **significant at the 5% level; †significant at the 1% level.

Table C3. Brand level constrained minimums 2, 3 and 4 for mixed logit estimates for beer demand

variables	Brand level constraints					
	minimum 2		minimum 3		minimum 4	
	means ($\bar{\alpha}, \theta$)	std devs Y	means ($\bar{\alpha}, \theta$)	std devs Y	means ($\bar{\alpha}, \theta$)	std devs Y
price	-8.495 (0.358) [†]	0.0008 (0.0003) ^{**}	-7.690 (0.233) [†]	4e-8 (4e-10)	-7.226 (0.210) [†]	0.097 (0.055) [*]
light	-1.081 (0.048) [*]	0.0001 (5e-5) [†]	-0.234 (0.172)	1e-5 (3e-6) [†]	-0.090 (0.079)	0.164 (0.045) [*]
Corona	0.473 (0.142) [†]	0.0002 (4e-5) [†]	0.510 (0.090) [†]	0.320 (0.012) [†]	0.362 (0.137) [†]	4e-5 (2e-5)
discount	-0.226 (0.170)	0.0003 (0.0002) [*]	-0.003 (0.160)	3e-6 (9e-7) [†]	-0.672 (0.141) [†]	0.019 (0.007) [†]
feature	0.689 (0.100) [†]		0.489 (0.096) [†]		0.651 (0.099) [†]	
display	0.295 (0.075) [†]		0.627 (0.069) [†]		0.448 (0.065) [†]	
feature and display	0.923 (0.089) [†]		0.802 (0.089) [†]		0.702 (0.083) [†]	
demographic interactions						
price:	income	1.179 (0.469) ^{**}	-0.717 (0.407) [*]		-0.430 (0.319)	
	income ²	-6.618 (1.243) [†]	1.467 (0.960)		0.923 (1.037)	
	age 21 - 34	0.839 (0.422) ^{**}	-0.147 (0.266)		-0.574 (0.346) [*]	
	age 35 - 54	0.785 (0.388) ^{**}	-1.978 (0.240) [†]		-2.617 (0.203) [†]	
light:	age 21 - 34	0.515 (0.205) ^{**}	2.351 (0.293) [†]		0.162 (0.308)	
	age 35 - 54	0.625 (0.233) [†]	1.965 (0.288) [†]		-0.426 (0.224) [*]	
Corona:	%Hispanic	-3.436 (0.385) [†]	-3.321 (0.258) [†]		-3.424 (0.363) [†]	
discount:	income	0.190 (0.433)	0.269 (0.355)		0.557 (0.358)	
	income ²	-0.957 (1.060)	-1.004 (0.939)		-2.485 (0.785) [†]	

(Standard deviations in parentheses)

*Significant at the 10% level; ** significant at the 5% level; †significant at the 1% level.

Table C4. Model statistics for minimums 2, 3, and 4, for both the unconstrained and brand level constrained model

statistic	unconstrained		
	minimum 2	minium 3	minimum 4
own-elasticities: # (%) > 0	0 (0)	0 (0)	0 (0)
# (%) ≤ -1	97,013 (74.3)	68,534 (52.5)	103,544 (79.3)
cross-elasticities: # (%) < 0	0 (0)	0 (0)	0 (0)
aggregate elasticity	-0.887	-0.791	-0.951
Lerner bound violations: # (%) < 0	0 (0)	0 (0)	0 (0)
# (%) > 1	50,228 (38.4)	65,866 (50.4)	40,158 (30.7)
unconstrained: joint demand-supply			
own-elasticities: # (%) > 0	0 (0)	0 (0)	0 (0)
# (%) ≤ -1	109,908 (84.1)	124,428 (95.2)	122,397 (93.7)
cross-elasticities: # (%) < 0	0 (0)	0 (0)	0 (0)
aggregate elasticity	-0.969	-1.059	-1.029
Lerner bound violations: # (%) < 0	0 (0)	0 (0)	0 (0)
# (%) > 1	36,480 (27.9)	19,696 (15.1)	23,505 (18.0)
brand-level constraints			
own-elasticities: # (%) > 0	0 (0)	0 (0)	0 (0)
# (%) ≤ -1	129,727 (99.3)	128,901 (98.7)	128,152 (98.1)
cross-elasticities: # (%) < 0	0 (0)	0 (0)	(0)
aggregate elasticity	-1.118	-1.047	-1.099
Lerner bound violations: # (%) < 0	0 (0)	0 (0)	0 (0)
# (%) > 1	7,004 (5.4)	5,761 (4.4)	6,927 (5.3)

Notes for Tables C5-C8: Cincinnati elasticities and merger simulations:

At first glance, the elasticities in Table C5 suggest that Coors and Miller have reasonable ability to internalize the effects of price increases post-merger; Coors Light and Keystone Light's shares are responsive to price changes in Miller High Life Light and Milwaukee's Best Light, and vice-versa. However, when we examine the margins in the Bertrand merger simulation in Table C6, we observe that the products that offer the greatest opportunities to internalize the effects of price increases all have uniformly low margins. The four highest margin products: Miller Genuine Draft, Miller High Life, Miller Lite and Milwaukee's Best all show very low cross-elasticities with Coors products. This tends to mute price increases. As a result, the simulation predicts tiny price increases for the four high margin products, and small price increases for the other products. Only Coors and Keystone Light, both of which had minuscule pre-merger shares, show substantial price increases.

Turning to the Stackelberg analysis, we first go back to elasticity Table C5. The lower left shows the effect on Anheuser-Busch product shares from Coors and Miller product price increases. In general, Anheuser-Busch's light beer product shares (Busch Light, Bud Light, Michelob Ultra and Natural Light) respond strongly to price increases in Coors' and Miller's light beer products. Anheuser-Busch takes this into account in our Stackelberg merger simulations increasing its responsiveness to follower light beer price changes post-merger. This induces larger price increases for light beer products that Anheuser-Busch leads with in both Stackelberg settings. Miller/Coors responds by raising its prices above the Bertrand optimum as well.

Table C5. Cincinnati brand level elasticity matrix for model with product level constraints

price change →	Coors		Miller						Anheuser-Busch							Grpo - Modlo	Heinkn
	Light	Keyst Light	Gen Draft	High Life	High Life Light	Light	Milwaukee's		Busch	Busch Light	Bud Select	Bud	Bud Light	Mich Ultra	Nat Light	Corona	Heinkn
							Best	Best Light									
Coors Light	-2.930	0.125	<u>0.001</u>	0.006	0.038	0.004	0.008	0.123	0.004	0.181	<u>0.001</u>	<u>0.004</u>	0.667	0.060	0.098	0.022	0.014
Keystone Light	0.138	-2.642	<u>0.001</u>	<u>0.005</u>	0.037	<u>0.003</u>	<u>0.007</u>	0.130	<u>0.003</u>	0.182	<u>0.001</u>	<u>0.004</u>	0.630	0.060	0.101	0.023	0.014
Miller Genuine Draft	0.006	0.005	-2.194	0.084	0.004	0.376	0.070	0.015	0.073	0.012	0.057	0.189	0.034	0.008	0.012	0.042	0.034
Miller High Life	0.009	0.006	0.024	-2.168	0.005	0.356	0.070	0.022	0.071	0.016	0.053	0.173	0.045	0.011	0.015	0.041	0.033
Miller High Life Lgt	0.123	0.106	0.002	0.012	-2.835	0.008	0.016	0.113	0.007	0.158	0.003	0.009	0.587	0.058	0.087	0.021	0.013
Miller Light	<u>0.002</u>	<u>0.001</u>	0.028	0.094	<u>0.001</u>	-1.970	0.065	0.004	0.081	<u>0.004</u>	0.064	0.218	<u>0.010</u>	0.002	<u>0.003</u>	0.047	0.039
Milwaukee: Best	0.011	0.008	0.021	0.075	0.007	0.279	-2.070	0.028	0.065	0.020	0.046	0.144	0.059	0.014	0.020	0.037	0.030
Best Lgt	0.096	0.100	0.002	0.010	0.027	0.006	0.013	-2.702	0.006	0.140	0.002	0.007	0.431	0.070	0.122	<u>0.016</u>	<u>0.012</u>
Busch	0.007	0.004	0.025	0.085	0.004	0.371	0.071	0.016	-2.260	0.012	0.055	0.183	0.034	0.008	0.011	0.042	0.035
Busch Light	0.131	0.119	0.002	0.009	0.036	0.006	0.011	0.124	0.005	-2.802	0.002	0.007	0.592	0.059	0.101	0.022	0.013
Bud Select	0.004	0.003	0.027	0.089	0.002	0.408	0.069	<u>0.009</u>	0.077	0.007	-2.222	0.202	0.020	<u>0.005</u>	0.007	0.045	0.036
Budweiser	0.004	0.003	0.027	0.088	0.002	0.417	0.067	<u>0.009</u>	0.077	0.008	0.061	-2.160	0.021	<u>0.005</u>	0.008	0.045	0.037
Bud Light	0.140	0.120	<u>0.001</u>	0.007	0.038	0.004	0.009	0.120	0.004	0.171	<u>0.001</u>	0.005	-2.459	0.060	0.097	0.022	0.013
Michelob Ultra	0.099	0.095	0.002	0.012	0.029	0.008	0.016	0.159	0.007	0.139	0.003	0.009	0.464	-2.828	0.112	0.017	<u>0.012</u>
Natural Light	0.102	0.101	0.002	0.011	0.029	0.007	0.014	0.163	0.007	0.150	0.003	0.009	0.469	0.068	-2.705	0.018	<u>0.012</u>
Corona	0.045	0.041	0.018	0.063	0.013	0.278	0.051	0.037	0.055	0.059	0.041	0.137	0.208	0.019	0.031	-2.479	0.029
Heineken	0.037	0.034	0.020	0.068	0.011	0.301	0.055	0.039	0.059	0.049	0.044	0.146	0.170	0.018	0.029	0.039	-2.552
Outside	0.046	0.043	0.007	0.028	0.016	0.081	0.030	0.086	0.022	0.069	0.014	0.045	0.217	0.037	0.058	0.019	0.014

Outside share = 57.565 percent. Aggregate brands (Craft, Import, Sub-Premium, Premium, and Super-Premium) and brands with share < 0.50 percent are excluded from table.

In each column: largest cross-elasticity is in **bold**; smallest is underlined.

Table C6. Cincinnati Bertrand merger simulation results for Miller/Coors merger. Estimated under product-level constraints.

Brewer	brand	%price change	%CMCR	pre-merger share	share change	%pre-merger margin	change in %margin
Coors	Coors	11.45	15.46	0.16	-0.14	44.44	5.71
	Coors Light	0.55	2.75	1.79	-0.19	36.29	0.35
	Keystone	23.33	15.41	0.03	-0.03	44.32	10.53
	Keystone Light	0.78	3.04	1.82	-0.25	39.91	0.46
Miller	Miller Genuine Draft	0.08	0.66	0.44	0.00	60.03	0.03
	Miller Genuine Draft Light	0.83	1.71	0.13	-0.02	33.25	0.56
	Miller High Life	0.08	0.72	1.59	0.00	57.54	0.04
	Miller High Life Light	0.72	3.50	0.60	-0.09	37.57	0.45
	Miller Lite	0.07	0.53	5.27	-0.01	55.54	0.04
	Milwaukee's Best	0.09	0.76	1.61	0.00	60.90	0.03
	Milwaukee's Best Light	1.65	2.99	2.80	-0.84	37.81	1.01
Anheuser-Busch	Busch	0.05	0.00	1.26	0.01	49.92	0.03
	Busch Light	0.10	0.00	2.60	0.03	51.53	0.05
	Bud Select	0.05	0.00	0.89	0.00	51.58	0.03
	Budweiser	0.05	0.00	2.81	0.01	49.69	0.03
	Bud Light	0.15	0.00	8.27	-0.02	50.98	0.07
	Michelob Ultra	0.19	0.00	1.24	-0.01	49.71	0.10
	Natural	0.06	0.00	0.17	0.00	54.71	0.03
	Natural Light	0.17	0.00	2.04	0.00	51.01	0.08
Grupo-Modelo	Corona	0.00	0.00	0.98	0.02	40.99	-0.01
Heineken	Heineken	0.02	0.00	0.71	0.01	39.70	0.02

Outside good: initial share = 57.57; share change = 1.49. Aggregate brands (Craft, Import, Sub-Premium, Premium, and Super-Premium) and brands with pre-merger share < 0.10 percent excluded from table.

Table C7. Cincinnati Stackelberg merger simulation results for Miller/Coors merger. Estimated under product-level constraints: Anheuser-Busch leads with Budweiser and Bud Light.

Brewer	brand	%price change	100*Stackelberg conjecture		pre-merger share	share change	%pre-merger margin	%margin change
			pre-merger	post-merger				
Coors	Coors	11.35	-	-	0.16	-0.14	44.44	5.66
	Coors Light	0.55	-	-	1.79	-0.17	36.29	0.35
	Keystone	23.03	-	-	0.03	-0.03	44.32	10.42
	Keystone Light	0.81	-	-	1.82	-0.24	39.91	0.48
Miller	Miller Genuine Draft	0.09	-	-	0.44	0.00	60.03	0.03
	Miller Genuine Draft Light	0.88	-	-	0.13	-0.02	33.25	0.59
	Miller High Life	0.08	-	-	1.59	-0.01	57.54	0.04
	Miller High Life Light	0.72	-	-	0.60	-0.08	37.57	0.45
	Miller Lite	0.05	-	-	5.27	-0.01	55.54	0.03
	Milwaukee's Best	0.10	-	-	1.61	0.00	60.90	0.04
	Milwaukee's Best Light	1.71	-	-	2.80	-0.84	37.81	1.05
Anheuser-Busch	Busch	0.10	-	-	1.26	-0.01	49.94	0.04
	Busch Light	0.05	-	-	2.60	0.11	51.70	0.02
	Bud Select	0.10	-	-	0.89	-0.01	51.59	0.05
	Budweiser	-0.23	0.01	-0.06	2.81	0.18	49.76	-0.11
	Bud Light	0.49	0.17	0.41	8.27	-0.65	51.67	0.24
	Michelob Ultra	0.10	-	-	1.24	0.04	49.86	0.05
	Natural	0.06	-	-	0.17	0.00	54.76	0.03
	Natural Light	0.09	-	-	2.04	0.06	51.17	0.04
Grupo-Modelo	Corona	-0.01	-	-	0.98	0.02	40.99	-0.01
Heineken	Heineken	0.04	-	-	0.71	0.01	39.70	0.03

Outside good: initial share = 57.57; share change = 1.77. Aggregate brands (Craft, Import, Sub-Premium, Premium, and Super-Premium) and brands with pre-merger share < 0.10 percent excluded from table.

Table C8. Cincinnati Stackelberg merger simulation results for Miller/Coors merger. Estimated under product-level constraints: Anheuser-Busch leads with all products.

Brewer	brand	%price change	100*Stackelberg conjecture		pre-merger share	share change	%pre-merger margin	%margin change
			pre-merger	post-merger				
Coors	Coors	11.50	-	-	0.16	-0.14	44.61	5.71
	Coors Light	0.58	-	-	1.79	-0.15	36.41	0.37
	Keystone	23.54	-	-	0.03	-0.03	44.48	10.58
	Keystone Light	0.85	-	-	1.82	-0.23	40.03	0.51
Miller	Miller Genuine Draft	0.12	-	-	0.44	0.00	60.25	0.05
	Miller Genuine Draft Light	0.97	-	-	0.13	-0.02	33.31	0.64
	Miller High Life	0.11	-	-	1.59	-0.01	57.74	0.05
	Miller High Life Light	0.74	-	-	0.60	-0.08	37.69	0.46
	Miller Lite	0.08	-	-	5.27	-0.01	55.76	0.04
	Milwaukee's Best	0.14	-	-	1.61	0.00	61.09	0.05
	Milwaukee's Best Light	1.80	-	-	2.80	-0.85	37.90	1.10
Anheuser-Busch	Busch	0.06	-0.06	-0.05	1.26	0.01	48.49	0.03
	Busch Light	0.41	0.09	0.24	2.60	-0.11	53.06	0.20
	Bud Select	0.07	-0.04	-0.04	0.89	0.01	50.00	0.03
	Budweiser	0.07	-0.14	-0.14	2.81	0.01	48.16	0.04
	Bud Light	0.54	0.29	0.78	8.27	-0.63	52.56	0.25
	Michelob Ultra	0.59	0.04	0.10	1.24	-0.09	50.96	0.29
	Natural	0.02	0.00	0.00	0.17	0.01	53.67	0.01
	Natural Light	0.57	0.06	0.17	2.04	-0.14	52.35	0.27
Grupo-Modelo	Corona	0.00	-	-	0.98	0.03	41.13	0.00
Heineken	Heineken	0.05	-	-	0.71	0.01	39.85	0.03

Outside good: initial share = 57.57; share change = 2.41. Aggregate brands (Craft, Import, Sub-Premium, Premium, and Super-Premium) and brands with pre-merger share < 0.10 percent excluded from table.