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Revenue-Maximising Elasticities of Taxable Income in Multi-Rate Income Tax Structures

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John Creedy and Norman Gemmell

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Revenue-Maximising Elasticities of Taxable Income in Multi-Rate Income Tax Structures

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Abstract

The empirical literature on the elasticity of taxable income (ETI) sometimes questions whether estimated values are consistent with being on the revenue-increasing section of the Laffer curve, usually in the context of a single rate tax system or for top marginal rates. This paper develops conceptual expressions for this 'Laffer-maximum' or revenue-maximising ETI for the multi-rate income tax systems commonly used in practice. Using the New Zealand income tax system in 2010 to illustrate its properties, the paper demonstrates that a wide range of revenue-maximising ETI values can be expected across individual taxpayers, across tax brackets and in aggregate.

JEL Classification: H24; H31; H26

Keywords: Income Tax Revenue; Elasticity of taxable income; revenue elasticity, Laffer Curve

Executive Summary

The concept of the elasticity of taxable income (ETI) has become widely used in the literature on the incentive effects of income taxation. This elasticity is defined as the proportional change in taxable income divided by the net-of-tax rate (one minus the marginal tax rate), and is therefore positive. Discussions of tax revenue are often framed in terms of a Laffer curve, in which total tax revenue is related to the tax rate, within a tax system which is implicitly thought of as having a single constant marginal rate. However, in practice income tax structures typically have a number of marginal rates, and there are income ranges which reflect rate progression (an increasing marginal rate) or reductions in effective tax rates where means-tested benefit payments or benefits, such as Working for Families in NZ, are subject to abatement or taper rates. As a result, there is no single elasticity of taxable income that applies to all individuals at all income levels. This raises the question of whether estimates of the elasticity of taxable income exceed values which generate revenue-reducing responses to marginal tax rate increases.

The present paper seeks to answer this question by first establishing, in the context of a multi-rate income tax, expressions for the elasticity of taxable income, at any income level, above which an increase in the relevant marginal tax rate produces a decrease in tax revenue. This elasticity, consistent with the maximum point on the Laffer curve, is referred to as the *revenue-maximising* elasticity of taxable income. It is shown that it can take a wide range of values both for individuals and for groups of taxpayers such as those facing particular marginal tax rates.

This paper shows that identifying the revenue-increasing side of the Laffer curve (for individuals) involves a small number of key components of the revenue-maximising elasticity of taxable income, which can readily be calculated using only the details of the marginal tax rates and income thresholds describing the complete structure. Furthermore, with information on the complete distribution of taxable income, revenue-maximising ETI values at aggregate levels can be obtained.

Using the New Zealand income tax system in 2010 and 2011 for illustrative purposes, a wide range of values of the revenue-maximising ETI can be expected across individual taxpayers, across taxpayers within income brackets, and at aggregate levels. For the New Zealand system, where the lowest tax rate is non-zero (unlike many other countries' income tax regimes) the revenue-maximising ETI values are high absolutely, but relatively low compared to those expected in tax systems where the lowest marginal tax rate is zero. Though many individuals in the New Zealand system (as in any multi-rate structure) can be expected to have elasticities that produce revenue reductions for tax rate increases, the income tax structure as a whole seems unlikely to exhibit those properties. Nevertheless, for taxpayers in the highest bracket, and possibly for the second highest bracket, their revenue-maximising ETIs were well within the range of estimated ETIs frequently found in empirical studies across a number of countries, including NZ. This conclusion is reinforced when allowance is made for the additional features introduced by the New Zealand system of family tax credits which serves to lower substantially the revenue-maximising ETIs for taxpayers in families with children across a wide range of income levels, especially in the denser part of the taxpayer income distribution.

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Revenue-Maximising Elasticities of Taxable Income in Multi-Rate Income Tax Structures

1 Introduction

Popular discussions of tax revenues and rates are often framed in terms of the well-known Laffer curve, in which total tax revenue is related to the tax rate, within a tax system which is implicitly thought of as being linear and thus with a single constant marginal rate. In this case the condition under which a rate increase leads to an increase in total revenue is easily expressed in terms of the aggregate elasticity of taxable income (ETI) – the proportionate change in taxable income resulting from a given proportionate change in the net-of-tax rate. An economy is on the revenue-decreasing side of the Laffer curve if the elasticity of taxable income with respect to the *tax rate*, τ, is less than minus one. Hence, as Goolsbee (1999) and Hall (1999) indicate, this translates into an elasticity of taxable income with respect to the *net-of-tax rate*, $1-\tau$, greater than $(1-\tau)/\tau$. For a tax rate up to 0.5, this implies an elasticity of taxable income with respect to the net-of-tax rate greater than one before a revenue-negative response to a tax rate increase occurs.

In practice income tax structures typically have a number of marginal rates, and there are income ranges which reflect rate progression (an increasing marginal rate) or reductions in effective tax rates where means-tested benefit payments or tax credits, such as the US earned income tax credit, are subject to abatement or taper rates. As a result, there is no single elasticity of taxable income that applies to all individuals at all income levels.[1] This raises the question of whether, or under what circumstances, estimates of the elasticity of taxable income can be expected to exceed values which generate revenue-reducing responses to marginal tax rate changes.

The present paper seeks to answer this question by first establishing, in the context of a multi-rate income tax, expressions for the elasticity of taxable income, at any income level, above which an increase in the relevant marginal tax rate produces a decrease in tax revenue.[2] This elasticity, consistent with the maximum point on the Laffer curve, is referred to below as the *revenue-maximising* elasticity of taxable income, ETI^L. It is shown that it can take a wide range of values both for individuals and for groups of taxpayers

[1] Furthermore, it is likely to vary as the costs of income shifting, the ability to conceal income and the chances of being detected by the tax authorities, the ability to change hours of work in the short and long run, and so on, vary.

[2] There is an interesting comparison here with Cournot's (1838) pathbreaking discussion of demand curves. He showed that, for a producer of a good facing a falling demand curve, total revenue initially increases and then decreases as the price rises, with the maximum revenue being at the point of unit elasticity. He argued that, although it would be extremely difficult to identify a precise value of the elasticity at any time, it is important to know whether the producer is on the rising or falling side of the revenue curve.

such as those facing particular marginal tax rates.[3]

Establishing the values of revenue-maximising ETIs is important for a number of reasons. Firstly, despite the large number of empirical studies, it has proved difficult to obtain reliable estimates of the elasticity of taxable income, even where the focus of attention has been specific sub-sets of taxpayers such as those at the top of the income distribution: see Goolsbee (1999) for a detailed critique of the elasticity of taxable income concept and empirical estimates, and Giertz (2007, 2009, 2010) and Saez et al. (2009) for discussions of recent estimates and reviews of related literature. For the top marginal tax rate for example, Saez et al. (2009, p. 58) conclude that, 'the most reliable longer-run estimates range from 0.12 to 0.4, suggesting that the U.S. marginal top rate is far from the top of the Laffer curve'. In fact, the analysis in this paper suggests that such low ETI estimates are, at least in principle, quite consistent with revenue-reducing top marginal rate responses.

Secondly, Werning (2007), Saez et al. (2009) and others have argued that the set of welfare-improving tax reforms is closely related to whether an increase in a particular marginal tax rate is expected to produce an increase in revenue of some minimum amount. Werning (2007), for example, demonstrates that for a tax reform to generate a Pareto superior tax structure, it is required to reduce all tax rates but yield the same or more revenue overall, even though some taxpayers may respond in ways that reduce revenue while others' responses enhance revenues. Hence, Pareto efficiency requires the tax system to be on the revenue-increasing side of the Laffer curve.[4]

This paper shows that identifying the revenue-increasing side of the Laffer curve in this context is more complex than establishing where the elasticity of taxable income with respect to the tax rate equals minus one. However, the key components of the revenue-maximising elasticity of taxable income can readily be calculated using relatively little information, namely the details of the marginal tax rates and income thresholds describing the complete structure. Furthermore, with information on the complete distribution of taxable income, revenue-maximising ETI values at aggregate levels can be obtained.

The next section provides the relevant conceptual expressions for the revenue-maximising elasticity in a multi-rate system, applicable to individual taxpayers and in aggregate. Sections 3 and 4 then illustrate these based on the New Zealand income tax system which features a four-rate structure where all rates are non-zero and where major reforms in 2010 led to significant reductions in all four tax rates. The New Zealand system of income-contingent social transfers also allows their impact on ETI^L to be explored in section 3. Aggregate values are analysed in section 4 and brief conclusions are in section 5.

[3] Saez et al. (2009, p. 5) provide an expression for the revenue change due to behavioural responses to a change in the top tax rates.

[4] Trabandt and Uhlig (2009) attempt to assess empirically how far the existing systems of labour and capital income taxation in the US and a sample of European countries are on the 'wrong' side of the Laffer curve. Only the capital income taxes of Sweden and Denmark appear to fall into this category.

2 The ETI in Multi-rate Tax Structures

This section demonstrates, at the individual and aggregate levels, how the elasticity of individual or aggregate tax with respect to a bracket's marginal rate depends on characteristics of the tax structure, the relevant elasticity of taxable income, and (for aggregate values) the income distribution. For convenience, the distinction between gross income and taxable income is ignored, though this distinction is likely to be important for countries with extensive income tax deductions.[5] If there are endogenous, income-related deductions, the following analysis must be in terms of income after deductions have been made. In modelling revenue responses the analysis concentrates only on income tax, making no allowance for possible shifting to other lower-taxed income sources such as through incorporation or other tax-favoured entities. An analysis of *total* tax revenue responses would also need to consider consumption taxes: to the extent that taxable income reductions following income tax increases reflect 'real' rather than 'shifting' responses, consumption will also fall.

2.1 Effective Income Thresholds

The multi-step tax function depends on a set of income threshold, $a_k, ..., a_K$, and a corresponding set of marginal tax rates $\tau_k, ..., \tau_K$. Let the tax paid by individual i with income of y_i be denoted $T(y_i) = T(y_i, \tau_1, ..., \tau_K, a_1, ..., a_K)$. Tax revenue can be written as:

$$\begin{array}{rcl} T(y_i) & = & \tau_1 (y_i - a_1) \qquad\qquad\qquad\quad a_1 < y_i \leq a_2 \\ & = & \tau_1 (a_2 - a_1) \quad + \quad \tau_2 (y_i - a_2) \qquad a_2 < y_i \leq a_3 \end{array} \tag{1}$$

and so on. If y_i falls into the kth tax bracket, so that $a_k < y_i \leq a_{k+1}$, $T(y_i)$ can be expressed for $k \geq 2$ as:

$$T(y_i) = \tau_k (y_i - a_k) + \sum_{j=1}^{k-1} \tau_j (a_{j+1} - a_j) \tag{2}$$

This can be rewritten as:

$$T(y_i) = \tau_k (y_i - a_k^*) \tag{3}$$

where:

$$a_k^* = \frac{1}{\tau_k} \sum_{j=1}^{k} a_j (\tau_j - \tau_{j-1}) \tag{4}$$

and $\tau_0 = 0$. Thus the tax function facing any individual taxpayer in the kth bracket is equivalent to a tax function with a single marginal tax rate, τ_k, applied to income measured in excess of a single effective threshold, a_k^*. Therefore, unlike a_j, a_k^* differs across individuals depending on the marginal income tax bracket into which they fall.

[5] For discussion of the empirical importance of income-related deductions in personal income tax regimes in OECD countries, see Caminada and Goudswaard (1996) and Wagstaff and van Doorslaer (2001). For the US, Feldstein (1995, p. 675) estimated that total income tax deductions in 1993 amounted to about 60 per cent of estimated taxable income.

2.2 Changes in Individual Tax Payments

Consider a change in the individual's tax liability resulting from an exogenous increase in one of the marginal tax rates, with other rates and the thresholds unchanged. This gives rise to a behavioural response, so writing $T(y_i) = T_i$, rearranging the total derivative, $dT_i = \frac{\partial T_i}{\partial y_i} dy_i + \frac{\partial T_i}{\partial \tau} d\tau$, in elasticity form gives:

$$\eta_{T_i,\tau} = \eta'_{T_i,\tau} + \eta_{T_i,y_i}\eta_{y_i,\tau} \tag{5}$$

This uses the general notation, $\eta_{b,a} = \frac{a}{b}\frac{db}{da}$, to denote a 'total' elasticity, and $\eta'_{b,a} = \frac{a}{b}\frac{\partial b}{\partial a}$ to denote a partial elasticity. In the case where an income change does not lead to a movement across an income threshold, $\eta_{T_i,y_i} = \eta'_{T_i,y_i}$.[6]

The first term may be said to reflect a pure 'tax rate' effect of a rate change, with unchanged incomes, while the second term reflects the combined 'tax base' effect, resulting from the incentive effects on taxable income, and the revenue consequences of that income change (reflected in the revenue elasticity). When discussing the effect on total revenue of a change in the top income tax rate, Saez et al. (2009, p. 5) refer to the tax rate effect as 'mechanical' and the second term as the 'behavioural' effect respectively (they do not discuss the separate role of the revenue elasticity in this context).

The individual elasticity of taxable income, $\eta_{y_i,1-\tau}$, measures the behavioural response of taxable income to a change in a marginal net-of-tax rate, $1-\tau$, facing the individual. This can be applied to any particular tax rate (not simply the rate corresponding to the tax bracket in which the individual's income falls), and a subscript is omitted here for convenience. The elasticities $\eta_{y_i,1-\tau}$ and $\eta_{y_i,\tau}$ are related using $\eta_{y_i,1-\tau} = -\left(\frac{1-\tau}{\tau}\right)\eta_{y_i,\tau}$. Furthermore, the revenue elasticity is:

$$\eta_{T_i,y_i} = \frac{y_i}{y_i - a_k^*} \tag{6}$$

and the individual revenue elasticity must exceed unity. Within each threshold (for which the marginal rate is fixed) the elasticity declines as income increases. As an individual crosses an income threshold, the revenue elasticity takes a discrete upward jump, before gradually declining again.

Hence the elasticity of revenue with respect to the marginal rate faced by an individual in the kth tax bracket is given by:[7]

$$\eta_{T_i,\tau_k} = \eta'_{T_i,\tau_k} - \left(\frac{y_i}{y_i - a_k^*}\right)\left(\frac{\tau_k}{1-\tau_k}\right)\eta_{y_i,1-\tau_k} \tag{7}$$

The first term, η'_{T_i,τ_k} is the positively-signed mechanical effect elasticity of the rate change,

[6] For a proportional tax structure, with constant average and marginal rate, t, and where $\bar{y}$ is arithmetic mean income, $\frac{dT}{dt} = \bar{y} + \frac{td\bar{y}}{dt}$ and in terms of elasticities, $\eta_{T,t} = 1 + \eta_{\bar{y},t}$, giving the result mentioned in the introduction; namely, revenue is maximised where $\eta_{\bar{y},t} = -1$.

[7] Equation (7) demonstrates some similarities with the Saez *et al.* (2009, p. 5) expression for the aggregate revenue response to a change in the top marginal rate. Equation (7) provides a generalisation of the Saez *et al.* result to all marginal tax rates but applied to individuals. The expressions developed in subsection 2.3 for aggregate responses avoid a specific income distribution assumption, whereas Saez et al. (2009, p. 5) assume a Pareto distribution. The latter is less suitable for the whole distribution of taxpayers than for those facing the top marginal rate.

which differentiation of (2), and using (3), shows is:[8]

$$\eta^{'}_{T_i,\tau_k} = \frac{\tau_k \left(y_i - a_k\right)}{T(y_i)} = \frac{\left(y_i - a_k\right)}{\left(y_i - a_k^*\right)} \tag{8}$$

Individuals' mechanical elasticities therefore differ with their incomes and the tax structure, represented in (8) as differences between a_k and a_k^*. The second term in (7) combines the three elements that form the 'behavioural effect'. It can be seen that this comprises, in addition to the ETI, $\eta_{y_i,1-\tau_k}$, two terms associated with the tax structure and the individual's income level. The behavioural effect is larger the larger is the individual's ETI, the higher is the MTR, τ_k, and the closer is the taxpayer's income to the effective threshold, such that $y_i/(y_i - a_k^*)$ in (7) is larger. Since the first two elements (after the minus sign in (7)) of the behavioural effect are positive, the overall behavioural response reduces revenue, given that $\eta_{y_i,1-\tau_k} > 0$.

In view of the importance of the closeness of y_i to the effective income threshold, a_k^*, the following discussion refers to this as an 'income-threshold' effect though this is often referred to in the fiscal drag literature as a 'revenue elasticity' effect.[9] This terminology also reduces the number of references to elasticity measures. Importantly in this framework, because $y_i > a_k$ and $a_1 \geqslant 0$, equation (7) is undefined for $y_i = 0$; hence it cannot account for behavioural responses at the extensive margin such as where taxpayers exit the taxpaying population in response to a tax rate change.

Denoting the revenue-maximising elasticity of taxable income, ETI^L, by $\eta^L_{y_i,1-\tau_k}$, this is readily obtained by setting the left-hand-side of (7) to zero to yield:[10]

$$\eta^L_{y_i,1-\tau_k} = \eta^{'}_{T_i,\tau_k} \left(\frac{y_i - a_k^*}{y_i}\right) \left(\frac{1-\tau_k}{\tau_k}\right) \tag{9}$$

An observed or estimated value greater than ETI^L implies that any increase in the taxpayer's marginal tax rate reduces income tax revenue from that taxpayer. From (9), for a given value of individual income, y_i, ETI^L is lower when marginal tax rates, τ_k, are higher and income is close to the threshold, a_k^* (bearing in mind that a_k^* is a marginal tax rate weighted average of the a_ks).

Furthermore, since the terms on the right hand side of (9) are multiplicative, and $0 < \left(\frac{y_i - a_k^*}{y_i}\right) < 1$, but $\left(\frac{1-\tau_k}{\tau_k}\right)$ may be greater than one (for $\tau_k < 0.5$), then the behavioural components may act either to magnify or shrink the mechanical effect in determining the maximum ETI consistent with revenue maximization from the individual taxpayer. To assess likely magnitudes of the revenue-maximising elasticity it is clearly important also to know individual taxpayer's mechanical effect elasticities, $\eta^{'}_{T_i,\tau_k}$, which, as shown below, also vary considerably across individuals.

In fact, equation (7) can be used more generally to calculate maximum ETIs consistent with *any* particular value of η_{T_i,τ_k}, in addition to the specific revenue-maximising case of $\eta_{T_i,\tau_k} = 0$. For example, where tax revenue authorities wish to target a particular revenue increase via raising one or more marginal tax rates, it is important to know for

[8] The partial individual elasticity, η'_{T,τ_j}, for $j < k$ (that is, for changes in marginal tax rates below the tax bracket in which the individual falls) is given by $\eta'_{T,\tau_j} = \{\tau_j \left(a_{j+1} - a_j\right)\} / T\left(y\right)$, which is simply the tax paid at the rate, τ_j, divided by total tax paid by the individual.

[9] See, for example, Creedy and Gemmell (2002, 2006).

[10] Fullerton (2008) gives the familiar revenue maximising tax rate for a proportional tax system, in terms of the ETI, as 1/(1+ETI). Using (9), and setting $\eta^{'}_{T_i,\tau_k} = 1$ and $a_k^* = 0$ for a proportional tax, rearrangement of (9) gives the revenue maximising tax rate, τ^L, as $\tau^L = \left(1 + \eta_{y_i,1-\tau}\right)^{-1}$.

which taxpayers or income groups this is likely to involve taxable income and/or revenue reductions. Where $\eta_{T_i,\tau_k} = b > 0$ is targeted, (9) becomes:

$$\eta^{b}_{y_i,1-\tau_k} = (\eta^{'}_{T_i,\tau_k} - b)\left(\frac{y_i - a^*_k}{y_i}\right)\left(\frac{1-\tau_k}{\tau_k}\right) \tag{10}$$

where $\eta^{b}_{y_i,1-\tau_k}$ denotes the maximum value consistent with the target $\eta_{T_i,\tau_k} = b$. In practice, if such a revenue target is set, it is likely to apply to aggregate revenue from all taxpayers. Nevertheless, (10) confirms that ETI^b is expected to be less than the revenue-maximising elasticity (for $b > 0$). As (10) makes clear, as b tends to $\eta^{'}_{T_i,\tau_k}$ (the pure mechanical effect), the maximum elasticity of taxable income consistent with this tends to zero such that the full mechanical effect is realised; see (7). Where $b > \eta^{'}_{T_i,\tau_k}$, the taxpayer's elasticity of taxable income would need to be (perversely) negative in order to generate a revenue increase in response to a tax rate rise.

2.3 Aggregation over Individuals

To aggregate over individuals, first convert (7) into changes, rather than elasticities:

$$\frac{dT_i}{d\tau_k} = \frac{\partial T_i}{\partial \tau_i} - \left(\frac{y_i}{y_i - a^*_k}\right)\frac{dy_i}{d\left(1-\tau_k\right)}\left(\frac{T_i}{y_i}\right) \tag{11}$$

Aggregating over $i = 1, ..., N_k$ individuals who are in the kth bracket:

$$\sum_{i=1}^{N_k}\frac{dT_i}{d\tau_k} = \sum_{i=1}^{N_k}\frac{\partial T_i}{\partial \tau_i} - \sum_{i=1}^{N_k}\left(\frac{y_i}{y_i - a^*_k}\right)\frac{dy_i}{d\left(1-\tau_k\right)}\left(\frac{T_i}{y_i}\right) \tag{12}$$

Suppose it is required that the total change in revenue from an increase in the rate τ_k is 0 (so that revenue from that rate is, *ceteris paribus*, a maximum). Furthermore, remembering that $T_i = \tau_k\left(y_i - a^*_k\right)$:

$$\sum_{i=1}^{N_k}\frac{\partial T_i}{\partial \tau_i} = \tau_k\sum_{i=1}^{N_k}\frac{dy_i}{d\left(1-\tau_k\right)} \tag{13}$$

Hence:

$$\frac{1}{\tau_k}\sum_{i=1}^{N_k}\frac{\partial T_i}{\partial \tau_i} = \frac{d\sum_{i=1}^{N_k} y_i}{d\left(1-\tau_k\right)} \tag{14}$$

and writing $\sum_{i=1}^{N_k} y_i = Y_k$, where $\eta^{L}_{Y_k,1-\tau_k}$ denotes the aggregate elasticity of taxable income for which aggregate revenue is unchanged, it is seen that:

$$\eta^{L}_{Y_k,1-\tau_k} = \frac{1}{Y_k}\left(\frac{1-\tau_k}{\tau_k}\right)\sum_{i=1}^{N_k}\frac{\partial T_i}{\partial \tau_i} \tag{15}$$

Using:

$$\frac{\partial T_i}{\partial \tau_i} = \left(\frac{\tau_k}{T_i}\frac{\partial T_i}{\partial \tau_i}\right)\frac{T_i}{\tau_k} = \frac{T_i}{\tau_k}\eta'_{T_i,\tau_k} \tag{16}$$

and substituting for $\eta'_{T_i,\tau_k} = \left(\frac{y_i}{y_i - a^*_k}\right)\left(\frac{\tau_k}{1-\tau_k}\right)\eta^{L}_{y_i,1-\tau_k}$ at maximum revenue, where $\eta^{L}_{y_i,1-\tau_k}$ is the elasticity of taxable income for individual i below which an increase in τ_k produces a larger tax payment, it can be seen that:

$$\eta^{L}_{Y_k,1-\tau_k} = \sum_{i=1}^{N_k}\left(\frac{y_i}{Y_k}\right)\eta^{L}_{y_i,1-\tau_k} \tag{17}$$

Hence the aggregate elasticity of taxable income in the kth tax bracket, such that the revenue from the bracket at the given tax rate is a maximum, is an income-share weighted average of individual elasticities. The above assumes that each individual does not move into a lower tax bracket as a result of the tax rate increase.[11]

3 Individual revenue-maximising ETIs in New Zealand

This section examines the revenue-maximising elasticities of taxable income and their components for individuals, under the New Zealand income tax system in 2010. As equations (9) and (10) reveal, calculating these elasticities only requires information on the tax schedule and income levels, from which the mechanical effect and behavioural response components can be obtained. Subsection 3.1 considers values for taxpayers in single earner households with no children. Subsection 3.2 considers the case of single earner households with children, where the New Zealand Family Tax Credit system is found to have a substantial impact on effective tax rates and thresholds.

3.1 Revenue-Maximising ETIs: Single Individuals without Children

Table 1 provides information about the marginal tax rates and income thresholds applying to households with a single earner and no children. The final column of the table reports the effective income threshold, a_k^*, for each income bracket.

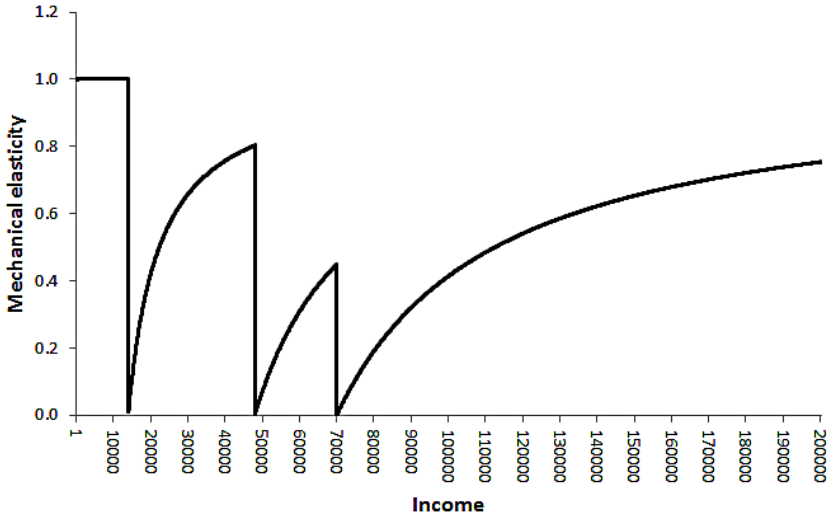
Table 1: New Zealand Income Tax Structure 2010: Single Earner with No Children

	Income threshold	Tax rate	Effective threshold
k	a_k	τ_k	a_k^*
1	1	0.125	1.0
2	14,000	0.21	5667.3
3	48,000	0.33	21061.0
4	70,000	0.38	27500.3

Figure 1 shows the variation in the mechanical elasticity as income increases. Each 'ratchet' in the diagram occurs at the corresponding income threshold. For incomes up to $200k, the mechanical elasticity varies between a high of 1.0 for taxpayers in the lowest tax bracket (up to $14k) and a low of zero for those taxpayers on the various tax thresholds where rates increase ($14k, $48k, $70k). The value of 1.0 in the lowest tax bracket arises here due to the absence of a tax-free income zone in the New Zealand system, such that a 1 percentage point increase in τ_0 induces a proportionate increase in revenue in the absence of any behavioural response. It can be seen that, for many individual income levels, a mechanical elasticity in the range 0 to 0.5 is quite common.

[11] Under the same assumption, Saez et al. (2009, p. 4) show that the actual or estimated ETI for the top tax bracket is an income-weighted average of ETIs for individuals in that bracket.

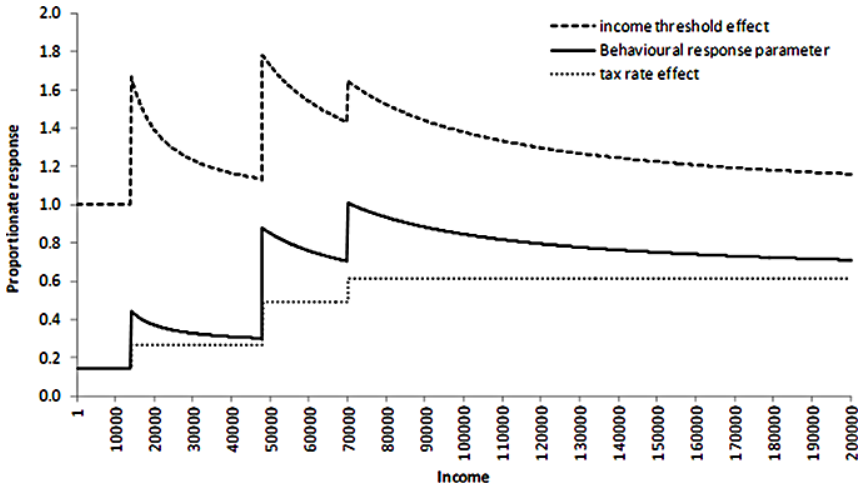
Figure 1: Mechanical Elasticities: Single Earner without Children



This illustrates the property of any progressive income tax system whereby the purely mechanical elasticity of tax payments with respect to changes in marginal tax rates faced by many individual taxpayers can be very small. It can also be seen that these elasticities rise with incomes above the top threshold. Hence these larger values have a larger impact on the income-weighted average mechanical effect across all taxpayers, that is likely to be of most relevance to revenue authorities. However, it also illustrates that adding an additional tax bracket towards the top of the income distribution would generate a new set of taxpayers with low values of the mechanical elasticity (those above but close to the new threshold).

Figure 2 shows values for the behavioural components on the right-hand-side of (7) across income levels: these are the income-threshold effect, $y_i/\left(y_i - a_k^*\right)$, the tax rate effect, $\tau_k/\left(1-\tau_k\right)$, and their combined 'response parameter'. Not surprisingly, the tax rate effect rises with the increasing marginal rate structure of the tax schedule, from around 0.14 for $k=1$ to 0.61 for $k=4$. With a top marginal rate of 0.38, the tax rate effect is relatively modest for higher income taxpayers compared to values expected in countries with higher top tax rates.[12] On the other hand, the non-zero lowest tax rate generates a non-trivial positive tax rate effect at lower income levels.

Figure 2: Components of Behavioural Response



[12] The value of 0.61 in 2010 fell to 0.492 in 2011 when the top tax rate was reduced to 0.33.

The income-threshold effect, on the other hand, demonstrates a 'ratchet effect' whereby values jump discretely as incomes cross tax thresholds, but then decline regularly and non-linearly between thresholds as incomes rise. The size of these discrete jumps is a function of both the sets of tax rates and thresholds, since a_k^* is determined by both. It can be seen that the combined effect, measured by the 'response parameter', displays a broadly similar pattern to the income-threshold effect, approaching it from below as tax rates rise. In a tax structure with a tax rate of 50 per cent, such that $\tau_k/(1-\tau_k) = 1$, the profile for the combined response parameter would exactly match, or overlay, the income-threshold effect profile (and, indeed, exceed it for $\tau_k > 0.5$).

Figure 3 shows the profile of ETI^L values across income levels associated with the mechanical elasticities and responses parameters for 2010 discussed above. A similar profile based on the 2011 tax settings is also shown (tax rates were reduced in 2011 to 0.105, 0.175, 0.30 and 0.33, with the same thresholds). These profiles demonstrate a similar but inverse ratchet effect whereby values rise with income levels (except for the first income bracket) and fall to zero at each income threshold. Hence the number and frequency of tax thresholds limits the magnitude of the revenue-maximising elasticities since each income threshold 're-sets' the elasticity to zero at that income level, reducing the subsequent amplitude of the profile.

Figure 3: Revenue-Maximising Elasticities: Single Earner without Children

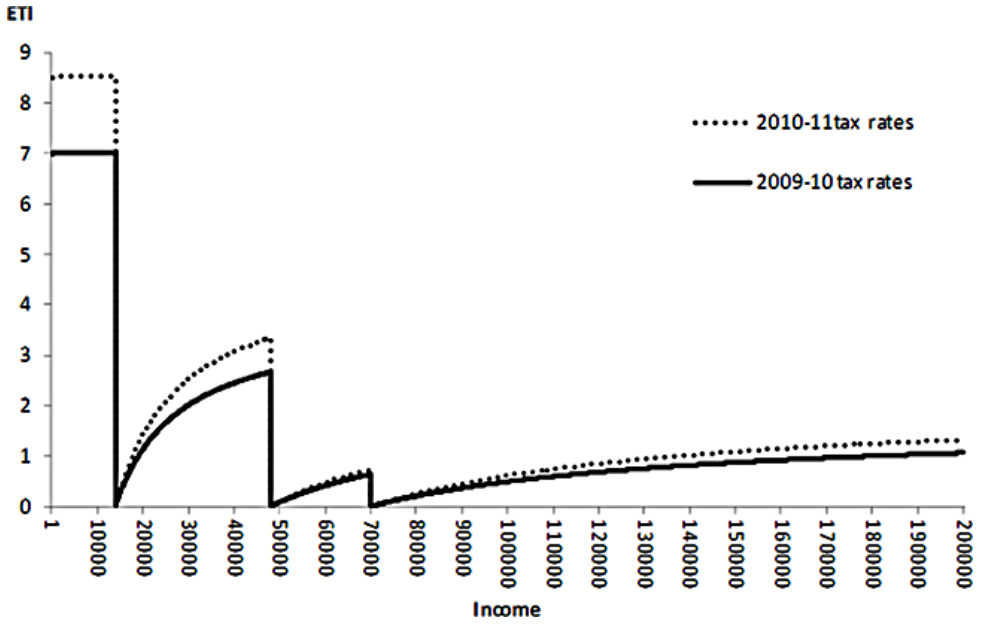


Figure 3 reveals the especially high, and constant, value of ETI^L in the lowest income bracket; that is, behavioural responses to increases in the lowest tax rate would have to be *extremely* large for increases in this rate to generate a revenue decline from those taxpayers. In fact, the values shown (7 in 2010; 8.5 in 2011) are relatively small compared to those associated with tax systems where $\tau_0 = 0$: ETI^L approaches infinity as the lowest rate is reduced to zero. An important caveat to these estimates is that, as noted earlier, they are based on a fixed taxpaying population where all $y_i > 0$,and hence cannot account for the possibility that changes in τ_0 may induce some taxpayers to reduce their incomes to zero (for example, by migrating or otherwise leaving the labour market).

Comparing the 2010 and 2011 profiles in Figure 3 also shows how reductions in tax rates across the schedule serve to increase the revenue-maximising ETIs. Not surprisingly this has the largest effect just below income thresholds; at an income of $48,000, for example, the reduced 2011 tax rates lead to an ETI$^L = 3.34$, compared with a value of 2.66 in

2010. Perhaps more importantly, at higher income levels where behavioural responses in practice are often estimated or alleged to be greater, the impact of the top tax rate reduction (from 0.38 to 0.33) is quite substantial. For example, the revenue-maximising elasticity at $200,000 is increased from 1.06 in 2010 to 1.32 in 2011, as a result of the tax cut. Claus et al. (2012) find that the range of estimates for the aggregate elasticity of taxable income, particularly in higher-income brackets, for New Zealand is similar to that generally found in other countries, of 0.3 to 1.0. This suggests that the likelihood of the top tax rate being on the revenue-decreasing side of the Laffer curve is much reduced as a result of the 2010 Budget reforms. It is clear from Figure 3 that values of ETI^L based on rules of thumb derived from proportional tax systems are likely to be very misleading.

3.2 Revenue-Maximising ETIs: Single-Earner Households with Children

The 2010 New Zealand system of social assistance included a set of refundable tax credits collectively known as Working for Families (WfF). This includes Family Tax Credits (FTCs) and In-Work Tax Credits (IWTCs) with the level of the payments determined by a complex mixture of number and age of children, household income, sole-parent credits, and hours worked by household members. These credits were withdrawn, or abated, at the rate of 20 cents per dollar of income, for household incomes in excess of a threshold. In 2010 this threshold was approximately $36,000 so that, in the income tax bracket $14–48k, a single-earner household with one or more children faced an effective marginal tax rate of 0.41 $(0.21 + 0.20)$ on his or her income between $36k and $48k. With a sufficient number of children in the household, the FTC may be received up to household incomes in excess of $100k before it is fully abated.[13] To illustrate the impact on ETI^Ls, the WfF system is simplified here, allowing the credits to abate from a household income of $36k, so that they are fully abated by $80k. Hence effective marginal rates become 0.41, 0.53, and 0.58 over the income ranges $36k–$48k, $48k–$70k and $70k–$80k respectively, and 0.38 above $80k. Table 2 gives these thresholds and effective marginal rates, along with the effective income thresholds, a_k^*. At the point where the effective marginal rate falls (that is, at $80k), the effective threshold drops substantially, as expected from equation (4).

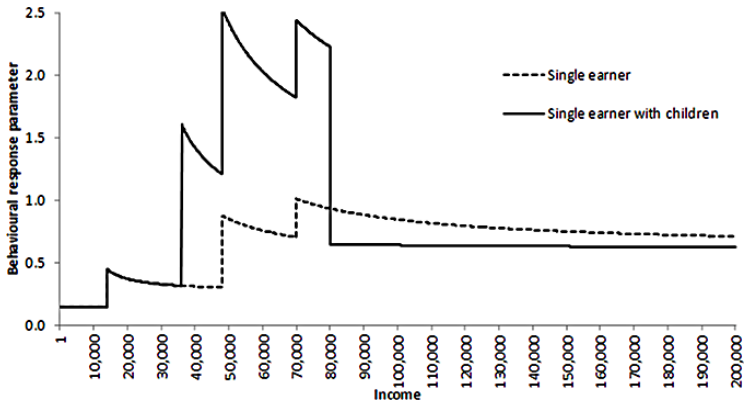
Table 2: New Zealand Income Tax Structure 2010: Single Earner Households with Children

k	Income threshold a_k	Tax rate τ_k	Effective threshold a_k^*
1	1	0.125	1.0
2A	14,000	0.21	5667.3
2B	36,000	0.41	19488.1
3	48,000	0.53	25943.6
4A	70,000	0.58	29741.6
4B	80,000	0.38	3289.8

Figure 4 shows that the existence of the means-tested benefits changes the behavioural

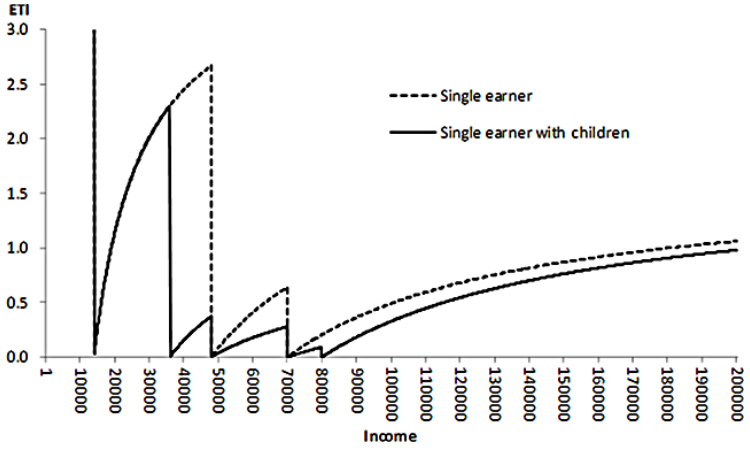
[13] For example, for a family with two children, FTC payment of $149 per week plus IWTC of $60 per week (if one or more parents is working at least part-time) would be received in 2011 when household annual income is below $36,827. With abatement of FTC and IWTC for incomes above this threshold level, some FTC continues to be received by households with incomes up to $74k, and up to $90k for IWTC. With three or more children, some IWTC continues to be received by households with incomes in excess of $100k. see http://www.ird.govt.nz/wff-tax-credits/.

Figure 4: Behavioural Responses: Single Earner with Children



response parameters substantially, and hence the ETI^L values, as shown in Figure 5, over the relevant ranges of income. Notably, in the income range where a large fraction of family incomes are located (roughly \$30–80k) the response parameter is substantially increased and, equivalently, the ETI^{L}s fall considerably from values around 2.5 at incomes in the \$40–48k range for single earners without children, to values between zero and 0.37 for single earners with children. This large fall is partly due to the mechanical effect falling to zero at the new effective threshold at \$36k, resulting from the operation of WfF. The benefit system might therefore be expected substantially to increase the potential for revenue-reducing responses in this income range if effective marginal rates were to be raised either via changes to statutory tax rates or benefit abatement rates.[14]

Figure 5: Revenue-Maximising Elasticities



4 Aggregate Values of ETI^L

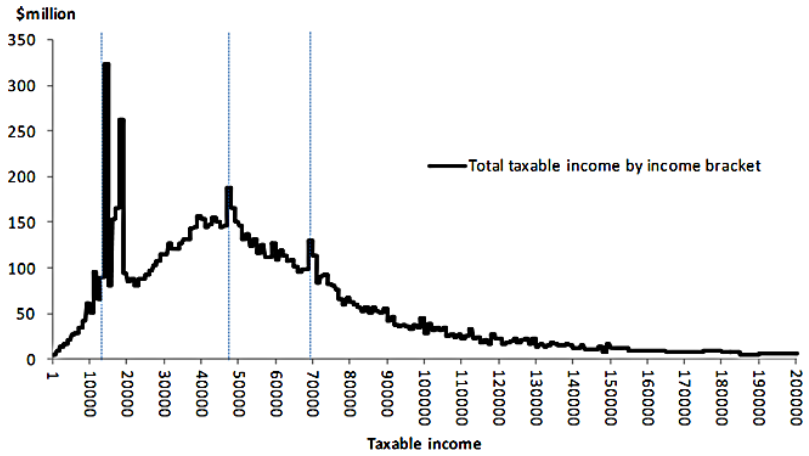
The revenue-maximising elasticities at the individual level, reported in the previous section, give a good indication of the range of values possible for individual taxpayers but give no sense of values that might be expected in aggregate across all, or groups of, taxpayers. To assess this requires information on the distribution of taxable income and information on the distribution of taxpayers by effective marginal tax rate (including transfer abatement

[14] In fact, following the reductions to all statutory tax rates in May 2010, the WfF abatement rate was raised in May 2011, increasing over time in small steps up to 0.25 (at September 2013 it is 0.2125).

rates), which in turn requires knowledge of taxpayer household characteristics such as numbers and age of children. Without such taxpayer and household details, this section does not attempt to measure actual aggregate ETI^Ls for New Zealand based on effective MTRs (that is, including abatement rates) but rather calculates values based on the statutory income tax structure in Table 1 which applies to all income taxpayers, and to which the available taxable income distribution data relate.

Figure 6 shows the 2010 New Zealand distribution of taxable income by $100 income bands.[15] The vertical axis on this chart measures (instead of population frequencies within the bands) the total amount of taxable income earned by taxpayers whose incomes fall within the relevant $100 band. Hence, for example, the income band ranging from $47,100 to $47,200 has a local maximum of $188 million of taxable income. The horizontal axis shows the bands of taxable income up to $200k. For convenience the figure also shows the income tax thresholds at $14k, $48k and $70k.

Figure 6: Taxable Income Distribution: New Zealand 2010



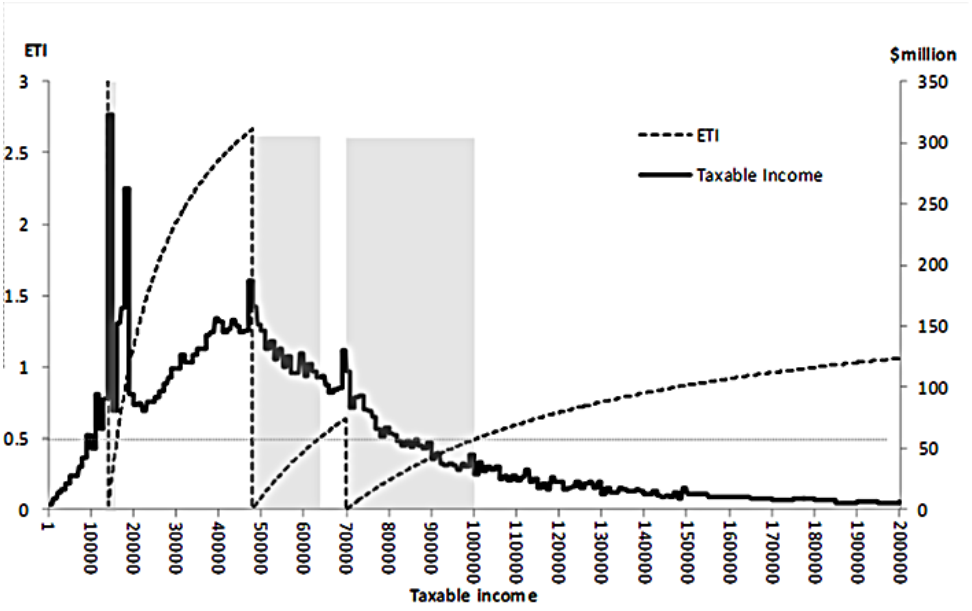
In addition to the usual skewed and approximately lognormal shaped distribution, the figure reveals a number of distinct spikes. The spikes at around $18,000 and $14,000 are unlikely to reflect behavioural factors since these are largely determined by New Zealand's universal state pension, set at approximately those levels for individuals over 65 years in single or couple households respectively. The spikes at, or just below, $48k and $70k are suggestive of behavioural responses to the positioning of those two income thresholds, with a global maximum (ignoring the pension spikes at $14k and $18k) of taxable income declared just below the lower threshold at $48k and a local maximum just below the higher-rate threshold at $70k.[16] More generally it can be seen in Figure 6 that the bulk of total taxable incomes are earned by individuals earning roughly in the $20k–$70k income range (mean taxable income in 2010, excluding those with zero or negative income, was approximately $35.7k and median income was around $24.5k).

Figure 7 combines the taxable income distribution information with the ETI^L profile for single earners in Figure 4 over the same range of incomes. This reveals, for example, that

[15] New Zealand Inland Revenue data, available at: http://www.ird.govt.nz/aboutir/external-stats/revenue-refunds/inc-dist-of-ind/. The data shown in the chart, in $100 bands, have been derived from the published Inland Revenue data where incomes are split into $1000 or $5000 bands up to $250,000. Hence, for example, between $47,000 and $48,000 total taxable income is $1,887 million; that is, $188 million per $100 within this $1000 range.

[16] Similar distributions for earlier years reveal spikes at $60k but not at $70k when the upper threshold (prior to 2008-09) was set at $60k. A comparable, if more muted, pattern emerges when the lower threshold rose from $38k to $48k in 2008-09. The New Zealand system provides strong incentives, and legal opportunity, for the self-employed or small business owner to allocate up to $70k of earnings to a non-working or part-time working spouse where the primary earner's income exceeds this amount.

Figure 7: Revenue-Maximising Elasticities and Taxable Incomes



there are three ranges of taxable incomes where $\text{ETI}^L < 0.5$: these are approximately between \$14k–\$16k, \$48k–\$64k and \$70k–\$101k. Given the relatively large fraction of taxable income accounted for by these income ranges (over 35%), and the finding that estimated elasticities of taxable income are frequently around 0.5, this suggests that a non-trivial fraction of taxpayer incomes could be vulnerable to revenue-reducing responses when marginal tax rates are increased.

Figure 8: Cumulative Distribution of Revenue-Maximising Elasticities

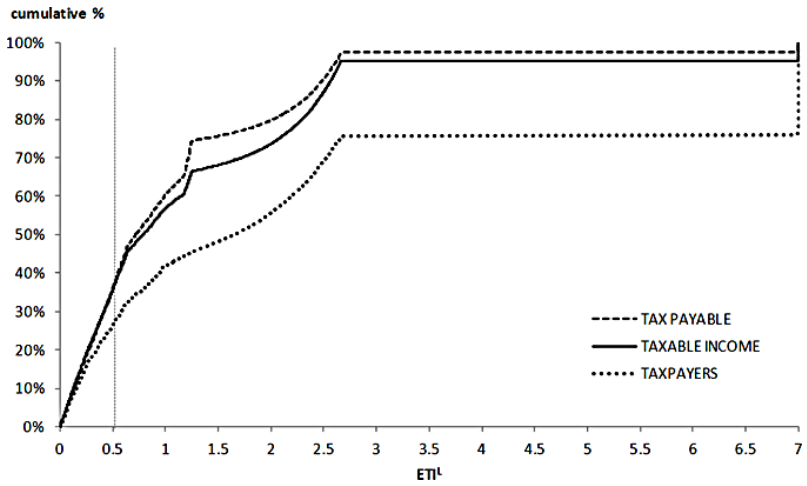
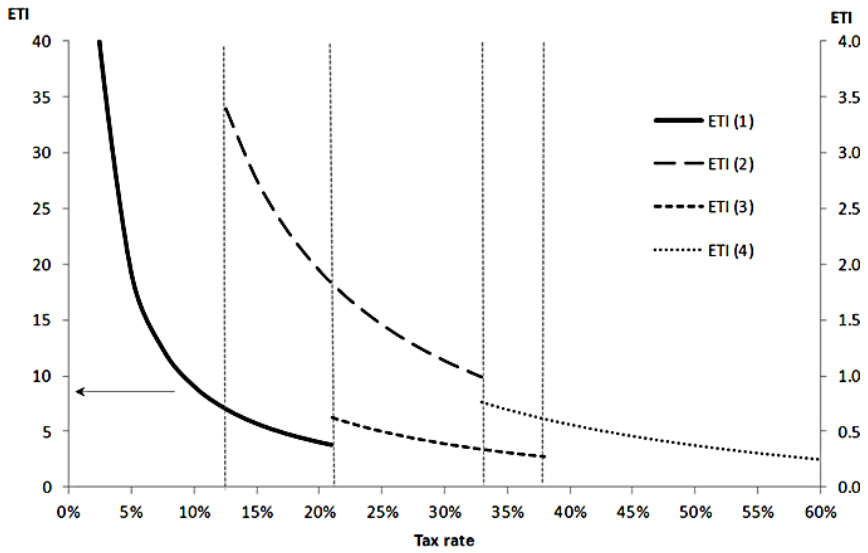


Figure 8 provides more complete information about the distribution of revenue-maximising elasticities, based on the 2010 set of income tax rates and thresholds. It shows the cumulative percentage of taxpayers, total taxable income and assessed tax liability (vertical axis), associated with each elasticity (horizontal axis). For example, it indicates that almost 40 per cent of total taxable income (and assessed tax) is earned by individuals with $\text{ETI}^L < 0.5$, representing around 25 per cent of taxpayers. Hence, the graph can indicate, for any estimated or hypothetical value of the ETI, the fraction of income or taxpayers for which this elasticity is likely to imply a revenue-reducing behavioural response. Furthermore, the profiles reach close to 100 per cent of taxable income (and 75 per cent of taxpayers) at $\text{ETI}^L = 2.7$, reflecting the fact that this is approximately the maximum ETI^L except for the 25 per cent of taxpayers earning less than \$14k, for whom $\text{ETI}^L = 7$.

4.1 Simulating Changes in Income Tax Rates

Since the value of ETI^L is sensitive to the structure of the income tax, it is interesting to consider how these values may vary in response to income tax reforms. ETI^{L}s for groups of taxpayers, such as those facing each τ_k, can also be examined using equation (17). These are shown in Figure 9. This simulation varies τ_k, while holding all $\tau_{l\neq k}$ constant at their 2010 values. For each combination of rates, the new revenue-maximising values are computed. The simulations shown are: τ_1: 0.025 to 0.21; τ_2: 0.125 to 0.33; τ_3: 0.21 to 0.38; τ_4: 0.33 to 0.60; that is, the τ_k for those taxpayers observed within each of the 2010 income brackets is allowed to take values between those of the neighbouring MTRs (except for the bottom rate, where the minimum was set to 0.025 to avoid excessive ETI^L values).

Figure 9: Simulating Revenue-Maximising Elasticities by Income Bracket



Using subscripts $k = 1$ to 4 to refer to the aggregate revenue-maximising elasticity, ETI^L_k, within each income bracket (from lowest to highest), it can be seen that raising τ_1 has the expected large impact on ETI$^L_1$: it falls from around 40 to 3.8 as τ_1 is increased from 0.025 to 0.21 (left-hand axis of Figure 9). A similarly sharp fall occurs with ETI$^L_2$ though for much smaller values: falling from 3.4 to 1.0 as τ_2 is raised from 0.125 to 0.33 (right-hand axis). These ETI^L values for taxpayers in the lowest two income brackets would appear to be relatively high suggesting that, at least between those thresholds ($1-$48k), a revenue-negative response to tax rate increases would be surprising; that is, it would require quite substantial behavioural responses not normally associated with lower income (mainly wage-earning) taxpayers.

However, for taxpayers in the top two income brackets (above $48k), the simulations suggest much lower ETI^L values, with ETI$^L_3$ varying between 0.63 and 0.27 as τ_3 is increased, and ETI$^L_4$ falling from 0.77 at $\tau_4 = 0.33$, to 0.25 at $\tau_4 = 0.60$. These values are certainly within the range of ETI estimates reported by various studies, at least for relatively high income taxpayers, and they could therefore be indicative of potential revenue-negative aspects of New Zealand's top tax rates were these to be increased.[17] In fact, both the two highest rates were reduced in 2011 to 0.30 and 0.33, though it is too early to identify

[17] The curves in Figure 10 are similar to that traced out by Giertz (2009) for changes in the top tax rate in the US. Giertz (2009, p. 130) shows that different assumed empirical values for the ETI generate a decline in revenue from the top income tax bracket in response to a marginal tax rate increase, as the ETI rises from 0.2 to 1.0.

revenue responses.[18] These ETI_k^L simulations demonstrate that values are highly variable within and across income/tax brackets such that simple rules of thumb (for example, based on ETI^Ls for single rate income taxes or top marginal rates) are unlikely to be a reliable guide to 'true' threshold values between revenue-positive and revenue-negative responses.

To consider aggregation over the complete distribution requires a modification of equation (17), such that, where Y is aggregate income:

$$\eta^L_{Y,1-\tau} = \sum_{k=1}^{K} \left(\frac{Y_k}{Y} \right) \eta^L_{Y_k,1-\tau_k} \tag{18}$$

This shows the threshold elasticity in the case where aggregate revenue, over all income brackets, increases as a result of an equal percentage-point increase in all tax rates. Thus, using the same 2010 taxable income distribution and tax schedule information, it is possible to calculate the aggregate ETI^L by taking an income-weighted average of all threshold values within each tax bracket (equivalent to an income-weighted average of all individual taxpayers' ETI^Ls) using equation (18) above. For the 2010 set of tax rates and thresholds this turns out to be 1.32. Reassuringly, at least for the New Zealand revenue authorities, the 2010 tax system appears to require a relatively high aggregate elasticity of taxable income overall before raising tax rates above their 2010 values would be expected to be revenue-negative overall. If, as seems likely, the actual aggregate ETI is less than 1.32, then the 2010 income tax as a whole is not on the revenue-decreasing side of the Laffer curve. Of course, as shown above, this does not preclude certain features of the system, such as the top marginal rate bracket generating revenue-negative components.[19]

Figure 10: Simulating Changes in all Tax Rates

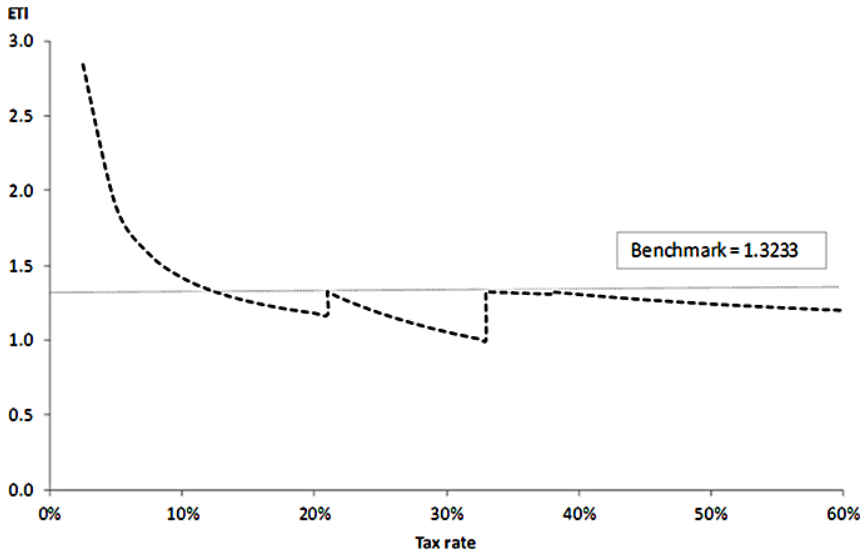


Figure 10 shows a profile for the aggregate ETI^L obtained from simulations which sequentially vary each τ_k, from the lowest to the highest rate, while holding the remaining three τ_ks at their actual 2010 values of 0.125, 0.21, 0.33, and 0.38. In each case the rate is increased until it reaches the 2010 value of the next highest rate. It is then re-set at its 2010 value and the marginal rate above is then increased. Hence for each rate indicated on the horizontal axis that corresponds to the actual rate in the corresponding bracket in the 2012 tax structure, the aggregate revenue-maximising elasticity is equal to the actual

[18] Such an exercise would also be complicated by the simultaneous effects of the global recession that arguably reduced New Zealand incomes over a sustained period from 2008, independently of the 2010 tax changes.

[19] As emphasised earlier, these simulations also take no account of the impact of the abatement of family tax credits which tend to lower aggregate ETI^L values.

or 'benchmark' of 1.32, mentioned above. Figure 10 shows, for example, that as the lowest rate, τ_1, is increased from around 0.025, the aggregate ETI^L drops from around 2.7, reaching 1.17 when $\tau_1 = 0.21$ (that is, when τ_1 is equal to the 2010 value of τ_2).

Re-setting τ_1, to 0.125 and raising the second lowest rate, τ_2, from 0.21 to 0.33, can be seen to reduce the ETI^L from1.32 to 1.00. Raising τ_3 and τ_4 appears to have little impact on the aggregate ETI^L presumably because of the smaller proportions of taxpayers involved. These simulations also indicate that the aggregate ETI^L remains above unity for all tax rates simulated (the Figure shows a maximum τ_4 of 0.60; further simulation indicates that at $\tau_4 = 0.80, 0.95$, ETI$^L = 1.15, 1.12$). These values greater than one are not inevitable but simply reflect the particular parameter settings of the 2010 New Zealand income tax; for example, different results would be obtained if *all* MTRs were to be raised, rather than the simulations in Figure 10 which raise each τ_k one at a time.

Finally, Table 3 shows how the ETI$_k^L$ can be affected by the family tax credit system. The third column of the table shows, by income bracket, the benchmark values of ETI$_k^L$, for $k = 1...4$, for the 2010 tax rates shown in Figure 9, for individuals without children and who therefore face the statutory marginal income tax rates. Column 4 of Table 3 reports equivalent ETI$_k^L$ values but based on the effective marginal tax rates for single earners with two children. In practice of course, the actual distribution of taxable income data (and on which Table 3 values are based) relates to a range of household compositions but the differences between columns 3 and 4 of the table serve to illustrate how household composition, especially the presence of children, can substantially change ETI$_k^L$ values. For example, the ETI$_4^L$ value for taxpayers earning over $70k is 0.616, based on a statutory top tax rate of 0.38 applied to all taxpayers with incomes over $70k. When using the effective rates and thresholds applicable to single earners with two children this becomes 0.04 over $70k-80k and 0.621 above $80k. A similar comparison for the second tax bracket yields ETI$_2^L = 1.82$ (no children) compared with 1.375 and 0.213 (2 children) over the $14k-36k and $36k-48k income ranges. Therefore the introduction into the tax schedule of additional rates and thresholds associated with income contingent transfers would appear to have the potential to substantially alter revenue-maximising ETIs for such taxpayers.

Table 3: Revenue-Maximising Elasticities by Income Bracket; Households with and without Children

k	Income threshold a_k	ETI$_k^L$ using τ_ks for: no children	2 children
1	1	7.0	7.0
2A	14,000	1.822	1.375
2B	36,000		0.213
3	48,000	0.338	0.148
4A	70,000	0.616	0.044
4B	80,000		0.621
ALL		1.323	0.892

4.2 Revenue-Raising ETIs

Governments seeking to raise income tax revenue efficiently are likely to be wary of initiating tax rate rises that induce large adverse behavioural responses even where these are not extreme enough to generate a net reduction in revenue. As mentioned in Section 2, equation (10) allows maximum elasticities of taxable income to be identified in association

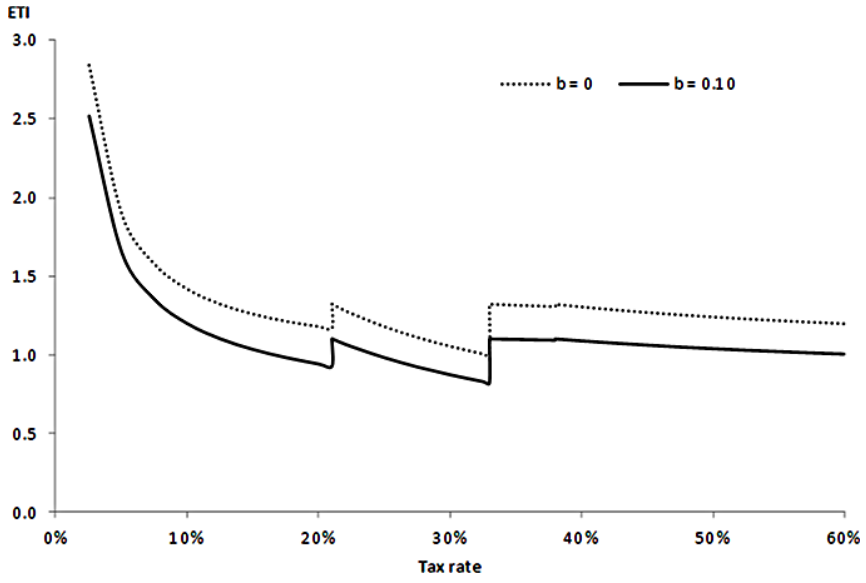
with any particular net revenue objective; that is, they identify the threshold elasticity between achieving and failing to achieve a particular revenue response to a specified change in τ_k. The maximum possible would of course be the mechanical response.

To demonstrate the sensitivity of these threshold elasticities to different net revenue objectives first requires a sense of plausible orders of magnitude for b in equation (10). This, in turn, requires knowledge of the size of the mechanical effect elasticity, η'_{T,τ_k}. Figure 1 showed the value of this elasticity for taxpayers at different income levels, typically ranging from 0.0 to 0.8 and Appendix A shows values that η'_{T,τ_k} across groups of taxpayers in different income brackets generally ranges from 0.2 to 0.6; see Table 4].

Simulations reported here set $b = 0.1$ in (10) which may be compared with the previous revenue-maximising case of $b = 0$. This value of $b = 0.1$ still implies quite strong behavioural effects since the net revenue response of 0.10 to a change in τ_k, though positive, is well below the positive net revenue response expected if only pure mechanical effects operate.

Figure 11 compares the aggregate ETI^L (that is, $b = 0$) for the Laffer case, reproduced from Figure 10, with the equivalent ETI^b for $b = 0.1$. This shows that the aggregate ETI^b always lies below the ETI^L values, typically around 0.2 to 0.3 less. For the 2010 tax rates, for example, ETI$^{b=0.1} = 1.10$ compared with ETI$^L = 1.32$.

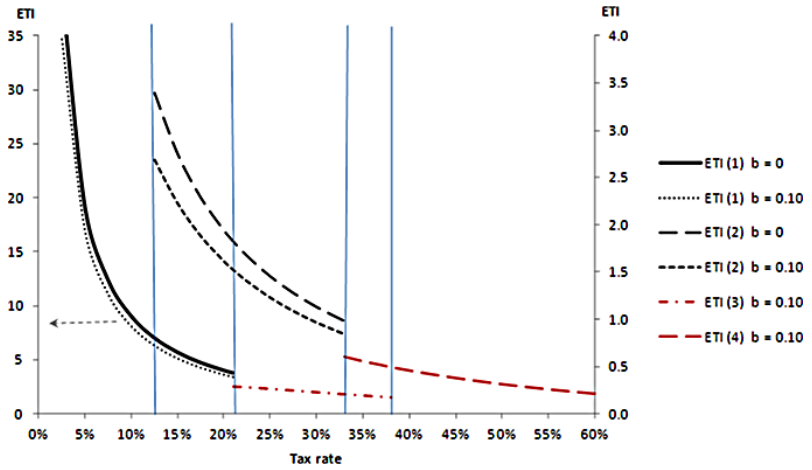
Figure 11: Revenue-Maximising Elasticities for Positive Revenue Changes



Considering the effect on the values of the aggregate ETI$^{b=0.1}$ for groups of taxpayers within each tax bracket, Figure 12 shows the profiles for ETI$_k^{b=0.1}$, $k = 1...4$. For the two lowest tax brackets ($k = 1, 2$) the Laffer equivalents, ETI$_k^L$, are also shown. This reveals the small impact on ETI$_1^b$; for example at $\tau_1 = 0.125$, ETI$_1^b = 6.31$ compared with ETI$_1^L = 7.00$. For ETI$_2^b$, the drop is also modest at 1.52 compared to 1.82 when $\tau_2 = 0.21$. Comparing the equivalent changes for τ_3 and τ_4, (compare Figures 9 and 12) reveals that, whereas ETI$_3^L$ ranges from 0.63 to 0.27 (as τ_3 increases from 0.21 to 0.38), the equivalent values of ETI$_3^{b=0.1}$ are 0.54 to 0.26. The impact of a positive value of b on ETI$_3^b$ is therefore greatest at lower values of τ_3.

For the highest tax rate, τ_4, when $b = 0.1$, ETI$_4^{b=0.1}$ now ranges from $0.60 - 0.21$ ($\tau_4 = 0.33 - 0.60$) compared to ETI$_4^L = 0.77 - 0.25$. Though still not large, these reductions in the critical threshold elasticities could nevertheless potentially be significant in practice. For example, at the benchmark top tax rate of 0.38, ETI$_4^L = 0.62$ while ETI$_4^{b=0.1} = 0.49$. That

Figure 12: Revenue-Maximising Elasticities by Income Bracket for Positive Revenue



is, if the actual ETI_4 exceeds 0.62 a 10 per cent increase in the top tax rate (from 0.38 to 0.418) will reduce revenue, but if ETI_4 exceeds 0.49, the rate change will fail to deliver even a 1 per cent increase in revenue as a result of a 10 per cent rise in the top tax rate.

From Table 4 in Appendix A, the mechanical elasticity associated with a rise in τ_4 is 0.24, implying that $\text{ETI}_4^{b=0.24} = 0$. Thus a net revenue increase that is substantially below the mechanical-only increase seems quite possible, since an increase that is only around 40 per cent (0.1/0.24) of the mechanical effect will occur if the actual ETI for taxpayers in the top bracket is around 0.49.

5 Conclusions

Recent empirical literature on the elasticity of taxable income (ETI) has been concerned with whether an estimated ETI is likely to exceed a threshold value consistent with the revenue-maximising point on the Laffer curve. This has been explored in the context of a single marginal rate system or with respect to the top marginal rate only. For the multi-rate income tax systems commonly used in practice, this paper has developed conceptual expressions for this revenue-maximising elasticity, ETI^L. It has shown both that the values of ETI^L can be expected to vary widely within and across income tax brackets, and that approximations based on a proportional income tax, or top marginal rate, are likely to be highly inaccurate. Expressions for the ETI^L in a multi-rate income tax are composed of three elements: a mechanical effect, an income threshold effect and a tax rate effect. Each of these three elements varies across taxpayers within a given tax structure and across tax structures. They are highly sensitive to the number and frequency of tax rates and thresholds. It was further shown that the ETI^L can be generalised to accommodate *any* specified revenue change, as well as the revenue-maximising 'zero revenue change' case. A similar method is used in Appendix B to derive the associated revenue-maximising tax rate, t_k^L, for individual taxpayers within each income bracket and in aggregate.

Using the New Zealand income tax system in 2010 and 2011 to illustrate the properties of the ETI^L, the paper demonstrated that a wide range of values can be expected across individual taxpayers, across taxpayers within income brackets, and at aggregate levels.

For the New Zealand system, where the lowest tax rate is non-zero (unlike many other countries' income tax regimes) ETI^L values were found to be high absolutely, but relatively low compared to those expected in tax systems where the lowest marginal tax rate is zero. Though many individuals in the New Zealand system (as in any multi-rate structure) can be expected to have ETI^{L}s below likely actual values (and hence potentially revenue-reducing) for those taxpayers, the income tax structure as a whole seems unlikely to exhibit those properties. Nevertheless, examining ETI^{L}s across groups of taxpayers associated with each of the four income brackets suggested that for taxpayers in the highest bracket, and possibly for the second highest bracket, ETI^{L}s were well within the range of estimated ETIs frequently found in empirical studies across a number of countries. This conclusion is reinforced when allowing for the additional features introduced by the New Zealand system of family tax credits which serves to lower substantially ETI^{L}s for taxpayers in families with children across a wide range of income levels, especially in the denser part of the taxpayer income distribution.

Appendix A: Mechanical Elasticities

Figure 1 demonstrated how the elasticity capturing the mechanical response to a marginal tax rate change in a multi-rate system varies across taxpayers, as determined by equation (8). For the income tax system as a whole, mechanical effect elasticities can be calculated for taxpayers in each income/tax bracket and for a variety of tax rate changes. Table 4 shows the values of η'_{T,τ_k} for each tax rate and bracket. That is, the total revenue response to a change in τ_k is decomposed into the revenue from the kth bracket and revenue from taxpayers in marginal rate brackets above k. (Clearly there is no additional revenue from tax brackets below the kth bracket).

The table shows the effect of a one percentage point increase in each of the four tax rates, and in all four tax rates simultaneously. In this latter case, Table 4 confirms that the 1 percentage point tax rate increases raise (mechanical) revenue by 1 per cent: $\eta'_{T,\tau_k} = 1.0$ ($k = 1, 2, 3, 4$). The numbers on the diagonal in the table show the values of the mechanical elasticity for those in bracket k when τ_k is changed, ranging from values of 1.0 for τ_1 (the tax schedule is proportional up to \$14k), to 0.60 ($k = 2$), 0.26 ($k = 3$), and 0.52 ($k = 4$). Additional mechanical elasticities for taxpayers in higher brackets are shown below the diagonal, with the overall mechanical effect for each tax rate shown in the final row. Those in the final row are relatively small (0.20, 0.41, 0.16, 0.24 for τ_1 to τ_4 respectively) reflecting the limited proportionate impact on *total* revenues from a change in *one* MTR, but suggest that target values of b would also be relatively small even where only pure mechanical effects are expected.

Table 4: New Zealand Income Tax Structure 2010: Mechanical Elasticities

Tax bracket	Income range	Tax rates 1	2	3	4	All
1	\$1-14k	**1.00**				**1.74**
2	\$14k-48k	0.40	**0.60**			**1.31**
3	\$48k-70k	0.15	0.59	**0.26**		**1.04**
4	>\$70k	0.05	0.21	0.22	**0.52**	**0.75**
All	>\$1	0.20	0.41	0.16	0.24	**1.00**

Appendix B: The Revenue-Maximising Tax Rate

As mentioned in section 2, Fullerton (2008) discusses the derivation of the revenue maximising tax rate for a proportional income tax. Saez *et al*. (2009, p. 6) derive an analogous expression for the top tax rate which maximises revenue from taxpayers in the top income tax bracket. This is also readily obtained for the multi-rate system described in equation (1). Setting the change in revenue in (7) to zero and rearranging to give, instead of the revenue-maximising ETI corresponding to a given value of τ_k, the revenue-maximising or revenue-maximising tax rate, τ_k^L, in terms of a given elasticity of taxable income, it can be shown that:

$$\tau_k^L = \left[\left(\frac{y_i}{(y_i - a_k^*)} \right) \left(\frac{\eta_{y_i, 1-\tau_k}}{\eta'_{T_i, \tau_k}} \right) + 1 \right]^{-1} \tag{B.1}$$

Substituting for the mechanical elasticity in (B.1) using $\eta'_{T_i,\tau_k} = \frac{(y_i - a_k)}{(y_i - a_k^*)}$ from (8), further rearrangement shows that:

$$\tau_k^L = \frac{(y_i - a_k)}{y_i(1 + \eta_{y_i, 1-\tau_k}) - a_k} \tag{B.2}$$

For a proportional income tax, where $a_k = a_k^* = 0$, equation (B.2) yields the Fullerton special case of $\tau_k^L = (1 + \eta_{y_i, 1-\tau_k})^{-1}$.[20]

Table 5: New Zealand Income Tax 2010: Revenue Maximising Tax Rates

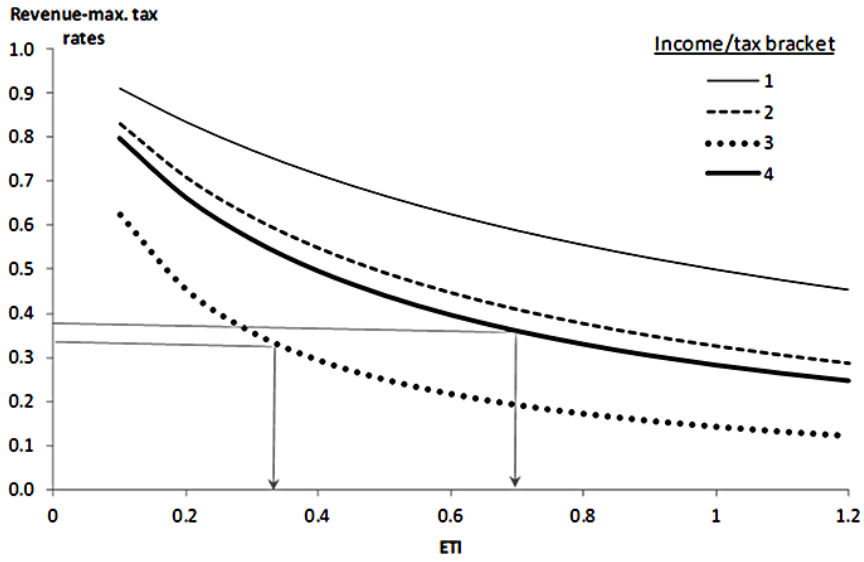
Tax bracket	Average income	Elasticity of Taxable Income: 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1	$6,948	0.91	0.83	0.77	0.71	0.67	0.62	0.59	0.56	0.53	0.50
2	$27,079	0.83	0.71	0.62	0.55	0.49	0.45	0.41	0.38	0.35	0.33
3	$57,546	0.62	0.45	0.36	0.29	0.25	0.22	0.19	0.17	0.16	0.14
4	$115,418	0.80	0.66	0.57	0.50	0.44	0.40	0.36	0.33	0.30	0.28
All	$35,725	0.86	0.75	0.67	0.60	0.55	0.50	0.46	0.43	0.40	0.38

Using (B.2) it is straightforward to calculate values for τ_k^L for individuals in New Zealand across the four income brackets. For example, Table 5 shows values for τ_k^L for individuals at mean incomes within each tax bracket, and at mean income across all brackets. These, and intermediate values of the elasticity of taxable income, are also displayed in Figure 13. Each curve in Figure 13 (or row in Table 5) shows the predicted decline in the revenue-maximising tax rate as the elasticity of taxable income increases. The table reveals that values of τ_1^L and τ_2^L for individuals at mean incomes for brackets 1 and 2 are in excess of 0.50 and 0.33 respectively (for elasticity values up to 1.0); that is they substantially exceed the actual 2010 tax rates of 0.125 and 0.21 respectively. Thus, ETI values would need to (considerably) exceed 1.0, for the revenue-maximising tax rates for mean income taxpayers in those brackets to be lower than the actual 2010 rates.[21]

[20] The Saez *et al*. expression for the revenue-maximising *top* tax rate is given by $\tau_K^L = (1 + \alpha\eta_{y_i, 1-\tau_K})^{-1}$, where α is a measure of average income in the top bracket relative to the top threshold income. It can be shown that this is equivalent to equation (B.2), where, in the present case, $\alpha = y_i / (y_i - a_K)$, for $y_i > a_K$.

[21] It can be shown that τ_2^L is below the actual rate of 0.21 for ETI > 1.9. The equivalent case for τ_1^L requires much higher values of the ETI.

Figure 13: Revenue Maximising Tax Rates



For τ_3^L and τ_4^L, however, it can be seen that these rates lie below the actual 2010 equivalents of 0.33 and 0.38 for ETI values equal to, or greater than, approximately 0.35 and 0.7 respectively. That is, the critical threshold value of τ^L occurs at a lower ETI value for the second highest tax rate than for the highest tax rate. This reflects the fact that mean income in the third bracket ($57,546) is closer to the relevant threshold ($48,000), than is the case for the highest bracket ($115,418 versus $70,000).

It is generally believed that ETIs for the self-employed and small businesses tend to be higher than for otherwise equivalent employees. The degree to which the actual 2010 tax rates are likely to be above the relevant revenue-maximising rates may therefore depend on how the incomes of the self-employed and small business owners are distributed across the income brackets. The suggestion in Figure 6 of some bunching of taxpayers immediately below income thresholds is consistent with some taxpayers, especially those close to but otherwise above the threshold, responding in a revenue-reducing manner to the increase in statutory rate associated with the higher income bracket. However, that is also consistent with some taxpayers, such as secondary earners in self-employed households, *increasing* their declared incomes and tax payments, in response to the higher tax rate as two or more earner households reallocate income within the household. This is also likely to undermine attempts to estimate ETIs that fail to account for such intra-household behaviour.

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