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**Article**

## A NEW FAMILY OF ESTIMATORS OF THE POPULATION VARIANCE USING INFORMATION ON POPULATION VARIANCE OF AUXILIARY VARIABLE IN SAMPLE SURVEYS

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## A NEW FAMILY OF ESTIMATORS OF THE POPULATION VARIANCE USING INFORMATION ON POPULATION VARIANCE OF AUXILIARY VARIABLE IN SAMPLE SURVEYS

Housila P. Singh<sup>1</sup>, Surya K. Pal<sup>2</sup>

### ABSTRACT

This paper proposes a family of estimators of population variance  $S_y^2$  of the study variable  $y$  in the presence of known population variance  $S_x^2$  of the auxiliary variable  $x$ . It is identified that in addition to many, the recently proposed classes of estimators due to Sharma and Singh (2014) and Singh and Pal (2016) are members of the proposed family of estimators. Asymptotic expressions of bias and mean squared error ( $MSE$ ) of the suggested family of estimators have been obtained. Asymptotic optimum estimator ( $AOE$ ) in the family of estimators is identified. Some subclasses of estimators of the proposed family of estimators have been identified along with their properties. We have also given the theoretical comparisons among the estimators discussed in this paper.

ASM Classification: 62D05.

**Key words:** Auxiliary variable, Study variable, Bias, Mean squared error, Efficiency comparison.

### 1. Introduction

The problem of estimating the population variance assumes importance in various fields such as industry, agriculture, medical and biological sciences etc. In sample surveys, auxiliary information on the finite population under investigation is quite often available from previous experience, census or administrative databases. It is well known that the auxiliary information in the theory of sampling is used to increase the efficiency of the estimators of the parameters such as mean or total, variance, coefficient of variation etc. Out of many, ratio and regression methods of estimation are good examples in this context. In many

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situations of practical importance, the problem of estimating the population variance  $S_y^2$  of the study variable  $y$  deserves special attention. When the population parameters such as population mean, variance, coefficients of skewness and kurtosis of the auxiliary variable are known, several authors including Das and Tripathi (1978), Srivastava and Jhajj (1980), Isaki (1983), Prasad and Singh (1990, 1992), Kadilar and Cingi (2006), Shabbir and Gupta (2007), Gupta and Shabbir (2008), Singh and Solanki (2013a, b), Solanki and Singh (2013), Singh et al. (2013, 2014), Hilal et al. (2014), Sharma and Singh (2014), Solanki et al. (2015), Yadav et al. (2015) and Singh and Pal (2016) etc. have suggested various estimators and studied their properties.

The principal aim of this paper is to suggest a new family of estimators of the population variance  $S_y^2$  of the study variable  $y$  using information on population variance  $S_x^2$  of the auxiliary variable  $x$  along with its properties under large sample approximation.

Consider a finite population  $U = \{U_1, U_2, \dots, U_N\}$  of  $N$  units. Let  $y$  and  $x$  be the study and auxiliary variates respectively. We define the following parameters of the variates  $y$  and  $x$ :

$$\text{Population mean: } \bar{Y} = N^{-1} \sum_{i=1}^N y_i,$$

$$\text{Population mean: } \bar{X} = N^{-1} \sum_{i=1}^N x_i,$$

$$\text{Population variance /mean square: } S_y^2 = (N-1)^{-1} \sum_{i=1}^N (y_i - \bar{Y})^2,$$

$$\text{Population variance /mean square: } S_x^2 = (N-1)^{-1} \sum_{i=1}^N (x_i - \bar{X})^2.$$

$Q_i$  is the  $i^{\text{th}}$  quartile ( $i=1, 2, 3$ ) of the auxiliary variable  $x$ ,

$Q_r = Q_3 - Q_1$ : the population inter-quartile range of the auxiliary variable  $x$ ,

$Q_d = (Q_3 - Q_1)/2$ : the population semi-quartile range of the auxiliary variable  $x$ ,

$Q_a = (Q_3 + Q_1)/2$ : the population semi-quartile average of the auxiliary variable  $x$ .

It is desired to estimate the population variance  $S_y^2$  of the study variable  $y$  when the population variance  $S_x^2$  of the auxiliary variable  $x$  is known. For estimating population variance  $S_y^2$ , a simple random sample (SRS) of size  $n$  is drawn without replacement (WOR) from the population  $U$ . The conventional

unbiased estimators of the population parameters  $(\bar{Y}, S_y^2, \bar{X}, S_x^2)$  are respectively defined by

$$\bar{y} = n^{-1} \sum_{i=1}^n y_i, s_y^2 = (n-1)^{-1} \sum_{i=1}^n (y_i - \bar{y})^2, \bar{x} = n^{-1} \sum_{i=1}^n x_i$$

$$\text{and } s_x^2 = (n-1)^{-1} \sum_{i=1}^n (x_i - \bar{x})^2.$$

When the population variance  $S_x^2$  of the auxiliary variable  $x$  is known, Isaki (1983) suggested a ratio-type estimator for estimating population variance  $S_y^2$  defined by

$$t_1 = s_y^2 \left( \frac{S_x^2}{s_x^2} \right). \tag{1.1}$$

Upadhyaya and Singh (1986) suggested an alternative estimator for  $S_y^2$  as

$$t_2 = s_y^2 \left( \frac{s_x^{*2}}{S_x^2} \right), \tag{1.2}$$

where  $s_x^{*2} = (NS_x^2 - ns_x^2)/(N - n)$

$$= \{(1 + g)S_x^2 - gs_x^2\}$$

and  $g = n/(N - n)$ .

Das and Tripathi (1978) suggested a difference-type estimator for the population variance  $S_y^2$  as

$$t_3 = s_y^2 + d(S_x^2 - s_x^2), \tag{1.3}$$

where ‘ $d$ ’ is suitable chosen constant.

Shabbir (2006) suggested a class of estimators of  $S_y^2$  as

$$t_4 = \eta s_y^2 + (1 - \eta) s_y^2 \left( \frac{s_x^{*2}}{S_x^2} \right), \tag{1.4}$$

where  $\eta$  being suitable chosen constant.

It is well known that the estimator  $t_0 = s_y^2$  is an unbiased estimator of  $S_y^2$ . The variance  $MSE$  of  $s_y^2$  under  $SRSWOR$  to the first degree of approximation is given by

$$Var(t_0) = MSE(t_0) = f S_y^4 (\lambda_{40} - 1), \tag{1.5}$$

where

$$\lambda_{40} = \mu_{40} / \mu_{20}^2, \mu_{40} = (N - 1)^{-1} \sum_{i=1}^N (y_i - \bar{Y})^4,$$

$$\mu_{20} = (N - 1)^{-1} \sum_{i=1}^N (y_i - \bar{Y})^2 \text{ and } f = ((1/n) - (1/N)).$$

The biases and mean squared errors of the Isaki's (1983) estimator  $t_1$  and Upadhyaya and Singh's (1986) estimator  $t_2$  to the first degree of approximation are, respectively, given by

$$B(t_1) = S_y^2 f (\lambda_{40} - 1)(1 - C), \quad (1.6)$$

$$B(t_2) = -S_y^2 g f (\lambda_{04} - 1)C, \quad (1.7)$$

$$MSE(t_1) = S_y^4 f [(\lambda_{40} - 1) + (\lambda_{04} - 1)(1 - 2C)], \quad (1.8)$$

and

$$MSE(t_2) = S_y^4 f [(\lambda_{40} - 1) + g(\lambda_{04} - 1)(g - 2C)], \quad (1.9)$$

where  $\lambda_{pq} = \mu_{pq} / (\mu_{20}^{p/2} \mu_{02}^{q/2})$ ,  $\mu_{pq} = (N - 1)^{-1} \sum_{i=1}^N (y_i - \bar{Y})^p (x_i - \bar{X})^q$ ,  $(p, q)$  being non negative integers; and  $C = (\lambda_{22} - 1) / (\lambda_{04} - 1)$ .

It is easy to verify that

$$E(t_3) = S_y^2$$

which shows that the difference estimator  $t_3$  is unbiased for  $S_y^2$ .

The variance of the difference estimator  $t_3$  to the first degree of approximation is given by

$$MSE(t_3) = Var(t_3) = f S_y^4 [(\lambda_{40} - 1) + d(\lambda_{04} - 1)(d - 2C)] \quad (1.10)$$

which is minimum when

$$d = C. \quad (1.11)$$

Thus the resulting minimum  $MSE$  of  $t_3$  to the first degree of approximation is given by

$$\begin{aligned} \min.MSE(t_3) &= f S_y^4 [(\lambda_{40} - 1) - (\lambda_{04} - 1)C^2] \\ &= f S_y^4 (\lambda_{40} - 1)(1 - \rho^{*2}), \end{aligned} \quad (1.12)$$

where

$$\rho^* = \frac{Cov(s_y^2, s_x^2)}{\sqrt{Var(s_y^2)Var(s_x^2)}} = \frac{(\lambda_{22} - 1)}{\sqrt{(\lambda_{40} - 1)(\lambda_{04} - 1)}}.$$

To the first degree of approximation, the bias and  $MSE$  of the Shabbir's (2006) estimator  $t_4$  are respectively given by

$$B(t_4) = -S_y^2 g f (1 - \eta)(\lambda_{04} - 1)C, \quad (1.13)$$

$$MSE(t_4) = S_y^4 f [(\lambda_{40} - 1) + g(1 - \eta)(\lambda_{04} - 1)\{g(1 - \eta) - 2C\}]. \quad (1.14)$$

The  $MSE(t_4)$  is minimum when

$$\eta_{opt} = \left(1 - \frac{C}{g}\right). \tag{1.15}$$

Thus the resulting minimum  $MSE$  of Shabbir (2006) estimator  $t_4$  is given by

$$\min.MSE(t_4) = f S_y^4 (\lambda_{40} - 1)(1 - \rho^{*2}) \tag{1.16}$$

which equals to the minimum  $MSE$  of the difference-type estimator  $t_3$  [*i.e.*  $\min.MSE(t_4) = \min.MSE(t_3)$ ].

### 1.1. Sharma and Singh's (2014) estimators

Sharma and Singh (2014) have suggested three classes of estimators of the population variance  $S_y^2$  as:

$$t_5 = w_1 s_y^2 + w_2 (S_x^2 - s_x^{*2}), \tag{1.17}$$

$$t_6 = k_1 s_y^2 + k_2 (S_x^2 - s_x^{*2}) \left[ 2 - \left( \frac{s_x^{*2}}{S_x^2} \right) \right], \tag{1.18}$$

and 
$$t_7 = m_1 s_y^2 \left( \frac{s_x^{*2}}{S_x^2} \right) + m_2 (S_x^2 - s_x^{*2}), \tag{1.19}$$

where  $(w_1, w_2), (k_1, k_2)$  and  $(m_1, m_2)$  are suitable chosen scalars such that mean squared errors of  $t_5, t_6$  and  $t_7$  are respectively minimum. We note here that the minimum mean squared errors of the estimators  $t_5, t_6$  and  $t_7$  obtained by Sharma and Singh (2014) are incorrect. Therefore the first objective of the authors of the present paper is to give the correct expressions of the minimum mean squared errors of the estimators  $t_5, t_6$  and  $t_7$  proposed by Sharma and Singh (2014). The derivation of the correct expressions of the minimum mean squared errors of the estimators  $t_5, t_6$  and  $t_7$  proposed by Sharma and Singh (2014) are given in the following theorems.

**Theorem 1.1. (a):** The bias and  $MSE$  of the estimator  $t_5$  to the first degree of approximation, are respectively given by

$$B(t_5) = (w_1 - 1)S_y^2, \tag{1.20}$$

$$MSE(t_5) = S_y^4 [1 + w_1^2 \{1 + f(\lambda_{40} - 1)\} + w_2^2 r^2 g^2 f(\lambda_{04} - 1) + 2w_1 w_2 grf(\lambda_{22} - 1) - 2w_1], \tag{1.21}$$

where  $r = S_x^2 / S_y^2$  is the ratio of two variances.

**Proof:** To obtain the bias and  $MSE$  of  $t_5$ , we write

$$s_y^2 = S_y^2(1 + e_0), \quad s_x^2 = S_x^2(1 + e_1)$$

such that

$$E(e_0) = E(e_1) = 0$$

and to the first degree of approximation,

$$E(e_0^2) = f(\lambda_{40} - 1), \quad E(e_1^2) = f(\lambda_{04} - 1) \text{ and } E(e_0 e_1) = f(\lambda_{22} - 1).$$

Expressing  $t_5$  in terms of  $e$ 's we have

$$\begin{aligned} t_5 &= w_1 S_y^2(1 + e_0) + w_2 \{S_x^2 - (1 + g)S_x^2 + g(1 + e_1)S_x^2\} \\ &= w_1 S_y^2(1 + e_0) + w_2 g S_x^2 e_1 \end{aligned}$$

or

$$(t_5 - S_y^2) = w_1 S_y^2(1 + e_0) + w_2 g S_x^2 e_1 - S_y^2$$

or

$$(t_5 - S_y^2) = S_y^2 [w_1(1 + e_0) + w_2 g r e_1 - 1]. \quad (1.22)$$

Taking expectation of both sides of (1.22) we get the bias of  $t_5$  to the first degree of approximation as

$$B(t_5) = (w_1 - 1)S_y^2. \quad (1.23)$$

Squaring both sides of (1.22) we have

$$\begin{aligned} (t_5 - S_y^2)^2 &= S_y^4 [1 + w_1^2(1 + 2e_0 + e_0^2) + w_2^2 g^2 r^2 e_1^2 \\ &+ 2w_1 w_2 g r (e_1 + e_0 e_1) - 2w_1(1 + e_0) - 2w_2 g r e_1]. \end{aligned} \quad (1.24)$$

Taking expectation of both sides of (1.24) we get the  $MSE$  of  $t_5$  to first degree of approximation as

$$MSE(t_5) = S_y^4 [1 + w_1^2 \{1 + f(\lambda_{40} - 1)\} + w_2^2 g^2 r^2 f(\lambda_{04} - 1) + 2w_1 w_2 g r f(\lambda_{22} - 1) - 2w_1] \quad (1.25)$$

which proves the Theorem 1.1(a).

**Theorem 1.1. (b):** The optimum values of  $w_1$  and  $w_2$  that minimize the  $MSE(t_5)$  at (1.25) are respectively given by

$$w_{10} = [1 + f(\lambda_{40} - 1)(1 - \rho^{*2})]^{-1}, \quad (1.26)$$

$$w_{20} = -\frac{C}{gr[1 + f(\lambda_{40} - 1)(1 - \rho^{*2})]}, \quad (1.27)$$

and the resulting minimum  $MSE(t_5)$  is given by

$$\begin{aligned} \min.MSE(t_5) &= \frac{S_y^4 f(\lambda_{40} - 1)(1 - \rho^{*2})}{[1 + f(\lambda_{40} - 1)(1 - \rho^{*2})]}, \tag{1.28} \\ &= \frac{\min.MSE(t_3)}{\left[1 + \frac{\min.MSE(t_3)}{S_y^4}\right]}, \text{ [from (1.12)]} \end{aligned}$$

**Proof:** Proof is simple so omitted.

**Theorem 1.2. (a):** The bias and  $MSE$  of the estimator  $t_6$  to the first degree of approximation, are respectively given by

$$B(t_6) = S_y^2 [k_1 + k_2 g^2 r f(\lambda_{04} - 1) - 1] \tag{1.29}$$

and

$$\begin{aligned} MSE(t_6) &= S_y^4 [1 + k_1^2 \{1 + f(\lambda_{40} - 1)\} + k_2^2 r^2 g^2 f(\lambda_{04} - 1) \\ &\quad + 2k_1 k_2 g r f(\lambda_{04} - 1)(g + C) - 2k_1 - 2k_2 g^2 r f(\lambda_{04} - 1)]. \tag{1.30} \end{aligned}$$

**Proof:** Expressing the estimator  $t_6$  in terms of e's we have

$$\begin{aligned} t_6 &= k_1 S_y^2 (1 + e_0) + k_2 S_x^2 g e_1 [2 - (1 + g) + g(1 + e_1)] \\ &= k_1 S_y^2 (1 + e_0) + k_2 S_x^2 g e_1 (1 + g e_1) \end{aligned}$$

or

$$(t_6 - S_y^2) = S_y^2 [k_1 (1 + e_0) + k_2 g r (e_1 + g e_1^2) - 1]. \tag{1.31}$$

Taking the expectation of both sides of (1.31) we get the bias of  $t_6$  to the first degree of approximation as

$$B(t_6) = S_y^2 [k_1 + k_2 g^2 r f(\lambda_{04} - 1) - 1]. \tag{1.32}$$

Squaring both sides of (1.31) and neglecting terms of e's having power greater than two we have

$$\begin{aligned} (t_6 - S_y^2)^2 &= S_y^4 [1 + k_1^2 (1 + 2e_0 + e_0^2) + k_2^2 g^2 r^2 e_1^2 \\ &\quad + 2k_1 k_2 g r (e_1 + e_0 e_1 + g e_1^2) - 2k_1 (1 + e_0) - 2k_2 g r (e_1 + g e_1^2)]. \tag{1.33} \end{aligned}$$



Taking expectation of both sides of (1.33) we get the  $MSE$  of  $t_6$  to first degree of approximation as

$$MSE(t_6) = S_y^4 [1 + k_1^2 \{1 + f(\lambda_{40} - 1)\} + k_2^2 g^2 r^2 f(\lambda_{04} - 1) + 2k_1 k_2 g r f(\lambda_{04} - 1)(g + C) - 2k_1 - 2k_2 g^2 r f(\lambda_{04} - 1)] \quad (1.34)$$

which proves the Theorem 1.2(a).

**Theorem 1.2. (b):** The optimum values of  $k_1$  and  $k_2$  that minimizes the  $MSE(t_6)$  are respectively given by

$$k_{10} = \frac{[1 - gf(g + C)(\lambda_{04} - 1)]}{[1 + f\{(\lambda_{40} - 1) - (\lambda_{04} - 1)(g + C)^2\}]}, \quad (1.35)$$

$$k_{20} = \frac{[gf(\lambda_{40} - 1) - C]}{gr[1 + f\{(\lambda_{40} - 1) - (\lambda_{04} - 1)(g + C)^2\}]} \quad (1.36)$$

and the resulting minimum  $MSE(t_6)$  is given by

$$\min.MSE(t_6) = \frac{S_y^4 f(\lambda_{40} - 1)[1 - g^2 f(\lambda_{04} - 1) - \rho^{*2}]}{[1 + f\{(\lambda_{40} - 1) - (\lambda_{04} - 1)(g + C)^2\}]} \quad (1.37)$$

**Proof:** Differentiating (1.34) partially with respect to  $k_1$ ,  $k_2$  and equating to zero we get the following equations:

$$k_1 \{1 + f(\lambda_{40} - 1)\} + k_2 g r f(\lambda_{04} - 1)(g + C) = 1, \quad (1.38)$$

$$k_1 (\lambda_{04} - 1)(g + C) + k_2 g r (\lambda_{04} - 1) = g(\lambda_{04} - 1). \quad (1.39)$$

Solving (1.38) and (1.39) for  $k_1$  and  $k_2$ , we get the optimum values of  $k_1$  and  $k_2$ , respectively as given by (1.35) and (1.36).

Substituting the values of  $k_{10}$  and  $k_{20}$ , from (1.35) and (1.36) in (1.34) we get the resulting minimum  $MSE$  of  $t_6$  given by (1.37).

This proves the Theorem 1.2(b).

**Theorem 1.3. (a):** The bias and  $MSE$  of the estimator  $t_7$  to the first degree of approximation, are respectively given by

$$B(t_7) = S_y^2 [m_1 \{1 - gf(\lambda_{22} - 1)\} - 1], \quad (1.40)$$

$$\begin{aligned}
 MSE(t_7) &= S_y^4[1 + m_1^2\{1 + f[(\lambda_{40} - 1) + g(\lambda_{04} - 1)(g - 4C)]\}] \\
 &\quad + m_2^2 g^2 r^2 f(\lambda_{04} - 1) + 2m_1 m_2 g r f(\lambda_{04} - 1)(C - g) - 2m_1\{1 - gf(\lambda_{22} - 1)\}].
 \end{aligned}
 \tag{1.41}$$

**Proof:** Expressing the estimator  $t_7$  in terms of e's we have

$$\begin{aligned}
 t_7 &= S_y^2(1 + e_0)[1 + g - g(1 + e_1)] + m_2[S_x^2 - (1 + g)S_x^2 + gS_x^2(1 + e_1)] \\
 &= S_y^2(1 + e_0)(1 - ge_1) + m_2 g S_x^2 e_1
 \end{aligned}$$

or

$$(t_7 - S_y^2) = S_y^2[m_1(1 + e_0 - ge_1 - ge_0e_1) + m_2 gre_1 - 1].
 \tag{1.42}$$

Taking expectation of both sides of (1.42) we get the bias of  $t_7$  to the first degree of approximation as

$$B(t_7) = S_y^2[m_1\{1 - gf(\lambda_{22} - 1)\} - 1].
 \tag{1.43}$$

Squaring both sides of (1.42) and neglecting terms of e's having power greater than two we have

$$\begin{aligned}
 (t_7 - S_y^2)^2 &= S_y^4[1 + m_1^2(1 + 2e_0 - 2ge_1 + e_0^2 - 4ge_0e_1 + g^2e_1^2) + m_2^2 g^2 r^2 e_1^2 \\
 &\quad + 2m_1 m_2 g r(e_1 + e_0e_1 - ge_1^2) - 2m_1(1 + e_0 - ge_1 - ge_0e_1) - 2m_2 gre_1].
 \end{aligned}
 \tag{1.44}$$

Taking expectation of both sides of (1.44) we get the  $MSE$  of  $t_7$  to first degree of approximation as

$$\begin{aligned}
 MSE(t_7) &= S_y^4[1 + m_1^2\{1 + f[(\lambda_{40} - 1)] + g(\lambda_{04} - 1)(g - 4C)\}] \\
 &\quad + m_2^2 g^2 r^2 f(\lambda_{04} - 1) + 2m_1 m_2 g r f(\lambda_{04} - 1)(C - g) - 2m_1\{1 - gf(\lambda_{22} - 1)\}].
 \end{aligned}$$

This is same as given in (1.40). Thus the theorem is proved.

**Theorem 1.3. (b):** The optimum values of  $m_1$  and  $m_2$  that minimizes the  $MSE(t_7)$  given by

$$m_{10} = \frac{[1 - gf(\lambda_{04} - 1)C]}{[1 + f\{(\lambda_{40} - 1) - (\lambda_{04} - 1)C(2g + C)\}]},
 \tag{1.45}$$

$$m_{20} = -\frac{[1 - gf(\lambda_{04} - 1)C](C - g)}{gr[1 + f\{(\lambda_{40} - 1) - (\lambda_{04} - 1)C(2g + C)\}]}
 \tag{1.46}$$

and the resulting minimum  $MSE(t_7)$  is given by

$$\min.MSE(t_7) = \frac{S_y^4 f(\lambda_{40} - 1) [1 - \rho^{*2} \{1 + g^2 f(\lambda_{04} - 1)\}]}{[1 + f\{(\lambda_{40} - 1) - (\lambda_{04} - 1)C(2g + C)\}]} \quad (1.47)$$

**Proof:** Differentiating (1.40) partially with respects to  $m_1$ ,  $m_2$  and equating to zero we get the following equations:

$$\begin{bmatrix} \{1 + f[(\lambda_{40} - 1) + g(\lambda_{04} - 1)(g - 4C)]\} & grf(\lambda_{04} - 1)(C - g) \\ grf(\lambda_{04} - 1)(C - g) & g^2 r^2 f(\lambda_{04} - 1) \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \end{bmatrix} = \begin{bmatrix} 1 - gf(\lambda_{04} - 1)C \\ 0 \end{bmatrix} \quad (1.48)$$

Solving (1.48) we get the optimum values of  $m_1$  and  $m_2$  as given in (1.44) and (1.45) respectively. Substituting the optimum values of  $m_{10}$  and  $m_{20}$  of  $m_1$  and  $m_2$  respectively in the  $MSE(t_7)$  at (1.41), we get the minimum  $MSE$  of  $t_7$  as given by (1.47).

Thus the theorem is proved.

## 1.2. Efficiency comparison

This section compares some existing known estimators of the population variance  $S_y^2$ .

From (1.5), (1.8), (1.9), (1.12) and (1.16) we have

$$Var(t_0) - \min.Min(t_j) = f S_y^4 (\lambda_{40} - 1) \rho^{*2}, \quad (1.49)$$

$$j = 3, 4 \quad \geq 0,$$

$$MSE(t_1) - \min.Min(t_j) = f S_y^4 [\rho^* \sqrt{(\lambda_{40} - 1)} - \sqrt{(\lambda_{04} - 1)}]^2 \quad (1.50)$$

$$j = 3, 4 \quad \geq 0,$$

$$MSE(t_2) - \min.Min(t_j) = f S_y^4 [\rho^* \sqrt{(\lambda_{40} - 1)} - g \sqrt{(\lambda_{04} - 1)}]^2 \quad (1.51)$$

$$j = 3, 4 \quad \geq 0.$$

It follows from (1.49) to (1.51) that the difference estimator  $t_3$  [Das and Tripathi (1978)] and Shabbir (2006) estimator  $t_4$  (at optimum condition) are better than the usual unbiased estimator  $s_y^2$ , Isaki's (1983) estimator  $t_1$  and Upadhyaya and Singh's (1986) estimator  $t_2$ .

Now, we present the comparison of the estimators  $t_5$ ,  $t_6$  and  $t_7$  due to Sharma and Singh (2014) with that of Das and Tripathi's (1978) difference estimator  $t_3$  and Shabbir (2006) estimator  $t_4$ . From (1.12), (1.16), (1.28), (1.37) and (1.47) we have

$$\min.MSE(t_j) - \min.Min(t_5) = f S_y^4(\lambda_{40} - 1)(1 - \rho^{*2}) \left[ 1 - \frac{1}{\{1 + f(\lambda_{40} - 1)(1 - \rho^{*2})\}} \right] \tag{1.52}$$

$$j = 3,4 \quad \geq 0,$$

$$\min.MSE(t_j) - \min.Min(t_6) = f S_y^4(\lambda_{40} - 1) \left[ (1 - \rho^{*2}) \left( 1 - \frac{1}{D} \right) + \frac{g^2 f(\lambda_{04} - 1)}{D} \right] \tag{1.53}$$

$$j = 3,4 \quad \geq 0,$$

$$\min.MSE(t_j) - \min.Min(t_7) = f S_y^4(\lambda_{40} - 1) \left[ (1 - \rho^{*2}) \left( 1 - \frac{1}{D^*} \right) + \frac{\rho^{*2} g^2 f(\lambda_{04} - 1)}{D^*} \right] \tag{1.54}$$

$$j = 3,4 \quad \geq 0,$$

where

$$D = [1 + f\{(\lambda_{40} - 1) - (\lambda_{04} - 1)(g + C)^2\}] \quad \text{and}$$

$$D^* = [1 + f\{(\lambda_{40} - 1) - (\lambda_{04} - 1)C(2g + C)\}].$$

It follows from (1.52) to (1.54) that the estimators  $t_5, t_6$  and  $t_7$  due to Sharma and Singh (2014) are better than Das and Tripathi's (1978) difference estimator  $t_3$  and Shabbir (2006) estimator  $t_4$  and hence better than the usual unbiased estimator  $s_y^2$ , Isaki's (1983) estimator  $t_1$  and Upadhyaya and Singh's (1986) estimator  $t_2$ .

## 2. The suggested class of estimators for the population variance $S_y^2$

Keeping the form of Das and Tripathi's (1978) difference type estimator, Isaki's (1983) ratio-type estimator, Upadhyaya and Singh's (1986) estimators, Singh et al.'s (1988) estimator, Shabbir's (2006) estimator, Kadilar and Cingi's (2006, 2007) estimators, Shabbir and Gupta's (2007) estimator, Singh and

Solanki’s (2013a, b) estimator, Solanki and Singh (2013) estimator, Singh et al.’s (2013, 2014) estimator, Sharma and Singh’s (2014), Solanki et al. (2015) estimator and Singh and Pal (2016) estimators in view, we define a generalized class of estimators for  $S_y^2$  as:

$$t_{SP} = \left[ \phi_1 s_y^2 \left\{ \xi + (1 - \xi) \left( \frac{S_{x(a,b)}^2}{S_x^2} \right)^\alpha \lambda \right\} + \phi_2 (S_x^2 - s_{x(a,b)}^2) \left\{ \phi + (1 - \phi) \left( \frac{S_{x(a,b)}^2}{S_x^2} \right)^\delta \right\} \right] \times \left[ \theta + (1 - \theta) \left( \frac{s_{x(a,b)}^2}{S_x^2} \right)^p \right] \times \exp \left\{ \frac{q(S_x^2 - s_{x(a,b)}^2)}{(S_x^2 + s_{x(a,b)}^2)} \right\}, \quad (2.1)$$

where  $s_{x(a,b)}^2 = (aS_x^2 + bS_x^2)/(a + b)$ ,  $(\phi_1, \phi_2)$  being suitable chosen constants,  $(\phi, \xi, \theta, \lambda)$  are suitable chosen scalars such that  $0 \leq (\phi, \xi, \lambda) \leq 1$ ,  $\lambda$  may be equal to  $\tau = (1 + \psi C_{xy}) / (1 + \psi C_x^2)$  with  $\psi = (1 - f) / n$  and  $f = n / N$ ;  $(\alpha, \delta, p, q)$  are scalars taking real values to generate ratio and product- type acceptable estimators; and  $(a, b)$  are either real numbers or the functions of the known parameters of the study variable  $y$  such as  $C_y$  coefficient of variation, (see Searls (1964), Lee (1981) and Singh (1986)),  $\beta_2(y) (= \lambda_{40})$  (coefficient of kurtosis of the study variable  $y$  see Singh et al. (1973) and Searls and Intarapanich (1990)), coefficient of skewness  $\beta_1(y) (= \lambda_{30}^2)$  of  $y$ ,  $\Delta(y) = (\beta_2(y) - \beta_1(y) - 1)$  (see, Sen (1978), Upadhyaya and Singh (1984) and Singh and Agnihotri (2008)) or the functions of auxiliary variable  $x$  such as population mean  $\bar{X}$ , coefficient of variation  $C_x$ , coefficients of skewness  $\beta_1(x) (= \lambda_{03}^2)$  and kurtosis  $\beta_2(x) (= \lambda_{04})$  and the parameter  $\Delta(x) = (\beta_2(x) - \beta_1(x) - 1)$  or the population correlation coefficient  $\rho$  between the study variable  $y$  and the auxiliary variable  $x$ .

We would like to remark that for various values of the parameters in (2.1), we get some existing known estimators as shown in Table 2.1. Many other estimators can also be generated from the proposed family of estimators  $t_{SP}$  for suitable values of scalars  $(\phi_1, \xi, \alpha, \lambda, a, b, \phi_2, \phi, \delta, \theta, p, q)$ .

**Table 2.1.** Some known members of proposed class of estimators

Values of the constants	Estimator
$(\lambda, \phi_1, \phi_2, \theta, \xi, \phi, \alpha, \delta, p, q, a, b)$	
$(1, 1, 0, -, -, 0, 0, 0, 0, -, -)$	$t_{SP(1)} = s_y^2$
$(-, 1, d, -, 1, -, -, 0, 0, 0, 0, -)$	$t_{SP(2)} = s_y^2 + d(S_x^2 - s_x^2)$ Das and Tripathi’s (1978) estimator

(1, 1, 0, -, 0, -, - $\alpha$ , -, 0, 0, 0, -)	$t_{SP(3)} = s_y^2 \left( \frac{S_x^2}{s_x^2} \right)^\alpha$ Das and Tripathi's (1978) estimator
(1, 1, 0, -, 0, -, -1, -, 0, 0, 1-b, b)	$t_{SP(4)} = s_y^2 \left( \frac{S_x^2}{s_x^2 + b(s_x^2 - S_x^2)} \right)$ Das and Tripathi's (1978) estimator
(1, 1, 0, -, 0, -, -1, -, 0, 0, 0, -)	$t_{SP(5)} = s_y^2 (S_x^2 / s_x^2)$ Isaki's (1983) ratio estimator
(1, 1, 0, -, 0, -, -1, -, 0, 0, (1 + g), -g)	$t_{SP(6)} = s_y^2 (s_x^{*2} / S_x^2)$ Upadhyaya and Singh's (1986) estimator
(1, $\phi_1$ , $\phi_2$ , -, -, -, 0, 0, 0, 0, 0, -)	$t_{SP(7)} = \phi_1 s_y^2 + \phi_2 (S_x^2 - s_x^2)$ Singh et al.'s(1988) estimator
(1, $\phi_1$ , 0, -, 0, -, -1, -, 0, 0, 0, -)	$t_{SP(8)} = \phi_1 s_y^2 (S_x^2 / s_x^2)$ Prasad and Singh's (1990) estimator
(1, 1, 0, -, 0, -, -1, -, 0, 0, $\frac{\beta_2(x)}{S_x^2}$ , 1)	$t_{SP(9)} = s_y^2 \left( \frac{S_x^2 + \beta_2(x)}{s_x^2 + \beta_2(x)} \right)$ Upadhyaya and Singh's (1999) estimator
(1, 1, 0, -, A, -, -1, -, 0, 0, $\frac{\beta_2(x)}{S_x^2}$ , 1)	$t_{SP(10)} = A s_y^2 + (1 - A) s_y^2 \left( \frac{S_x^2 + \beta_2(x)}{s_x^2 + \beta_2(x)} \right)$ Chandra and Singh's(2001) estimator
(1, 1, 0, -, 1 + w, -, -1, -, 0, 0, $\frac{\beta_2(x)}{S_x^2}$ , 1)	$t_{SP(11)} = (1 + w) s_y^2 - w s_y^2 \left( \frac{S_x^2 + \beta_2(x)}{S_x^2 + \beta_2(x)} \right)$ Chandra and Singh's(2001) estimator
(1, 1, 0, -, 0, -, -1, -, 0, 0, $-\frac{C_x}{S_x^2}$ , 1)	$t_{SP(12)} = s_y^2 \left( \frac{S_x^2 - C_x}{s_x^2 - C_x} \right)$ Kadilar and Cingi's (2005) estimator
(1, 1, 0, -, 0, -, -1, -, 0, 0, $-\frac{\beta_2(x)}{S_x^2}$ , 1)	$t_{SP(13)} = s_y^2 \left( \frac{S_x^2 - \beta_2(x)}{s_x^2 - \beta_2(x)} \right)$ Kadilar and Cingi's (2005) estimator
(1, 1, 0, -, 0, -, -1, -, 0, 0, $-\frac{C_x}{S_x^2}, \beta_2(x)$ )	$t_{SP(14)} = s_y^2 \left( \frac{S_x^2 \beta_2(x) - C_x}{s_x^2 \beta_2(x) - C_x} \right)$ Kadilar and Cingi's (2005) estimator
(1, 1, 0, -, 0, -, -1, -, 0, 0, $-\frac{\beta_2(x)}{S_x^2}, C_x$ )	$t_{SP(15)} = s_y^2 \left( \frac{S_x^2 C_x - \beta_2(x)}{s_x^2 C_x - \beta_2(x)} \right)$ Kadilar and Cingi's (2005) estimator
(1, 1, 0, -, $\xi$ , -1, -, 0, 0, (1 + g), -g)	$t_{SP(16)} = \xi s_y^{*2} + (1 - \xi) s_y^2 \left( \frac{s_x^{*2}}{S_x^2} \right)$ Shabbir's (2006) estimator
(1, 1, 0, -, $\xi$ , -, -1, -, 0, 0, 0, -)	$t_{SP(17)} = \xi s_y^2 + (1 - \xi) s_y^2 \left( \frac{S_x^2}{s_x^2} \right)^\tau$ Kadilar and Cingi's (2006) estimators

$(1, \phi_1, \phi_2, -, -, -, 0, 0, 0, 1, 0, -)$	$t_{SP(18)} = [\phi_1 s_y^2 + \phi_2 (S_x^2 - s_x^2)] \exp \left\{ \frac{S_x^2 - s_x^2}{S_x^2 + s_x^2} \right\}$ Shabbir and Gupta's (2007) estimator
$(1, \phi_1, \phi_2, 2, -, -, 0, 0, p, 0, 0, -)$	$t_{SP(19)} = [\phi_1 s_y^2 + \phi_2 (S_x^2 - s_x^2)] \left[ 2 - \left( \frac{s_x^2}{S_x^2} \right)^p \right]$ Gupta and Shabbir's (2008) estimator
$(1, \phi_1, \phi_2, 2, -, -, 0, 0, 1, 0, 0, -)$	$t_{SP(20)} = [\phi_1 s_y^2 + \phi_2 (S_x^2 - s_x^2)] \left[ 2 - \left( \frac{s_x^2}{S_x^2} \right) \right]$ Gupta and Shabbir's (2008) estimator
$(1, \phi_1, \phi_2, 2, -, -, 0, 0, -1, 0, 0, -)$	$t_{SP(21)} = [\phi_1 s_y^2 + \phi_2 (S_x^2 - s_x^2)] \left[ 2 - \left( \frac{s_x^2}{S_x^2} \right) \right]$ Gupta and Shabbir's (2008) estimator
$(1, 1, 0, -, 0, -, -1, -, 0, 0, \frac{\eta(1-\alpha^*)S_x^2 - \nu}{S_x^2}, \alpha^* \eta)$	$t_{SP(22)} = s_y^2 \frac{(\eta S_x^2 - \nu)}{\{\alpha^* (\eta S_x^2 - \nu) + (1 - \alpha^*) (\eta S_x^2 - \nu)\}}$ Yadav and Pandey's (2012) type estimator and Singh and Malik's (2014) type estimator
$(1, k, 0, -, 0, -, -1, -, 0, 0, \frac{\eta(1-\alpha^*)S_x^2 - \nu}{S_x^2}, \alpha^* \eta)$	$t_{SP(23)} = k s_y^2 \frac{(\eta S_x^2 - \nu)}{\{\alpha^* (\eta S_x^2 - \nu) + (1 - \alpha^*) (\eta S_x^2 - \nu)\}}$ Yadav and Pandey's (2012) type estimator
$(1, 1, 0, -, 0, -, -1, -, 0, 0, \frac{Q_1}{S_x^2}, 1)$	$t_{SP(24)} = s_y^2 \left( \frac{S_x^2 + Q_1}{s_x^2 + Q_1} \right)$ Subramani and Kumarapandiyani's (2012b) estimator
$(1, 1, 0, -, 0, -, -1, -, 0, 0, \frac{Q_3}{S_x^2}, 1)$	$t_{SP(25)} = s_y^2 \left( \frac{S_x^2 + Q_3}{s_x^2 + Q_3} \right)$ Subramani and Kumarapandiyani's (2012b) estimator
$(1, 1, 0, -, 0, -, -1, -, 0, 0, \frac{Q_2}{S_x^2}, 1)$	$t_{SP(26)} = s_y^2 \left( \frac{S_x^2 + Q_2}{s_x^2 + Q_2} \right)$ Subramani and Kumarapandiyani's (2012b) estimator
$(1, 1, 0, -, 0, -, -1, -, 0, 0, \frac{Q_d}{S_x^2}, 1)$	$t_{SP(27)} = s_y^2 \left( \frac{S_x^2 + Q_d}{s_x^2 + Q_d} \right)$ Subramani and Kumarapandiyani's (2012b) estimator
$(1, 1, 0, -, 0, -, -1, -, 0, 0, \frac{Q_a}{S_x^2}, 1)$	$t_{SP(28)} = s_y^2 \left( \frac{S_x^2 + Q_a}{s_x^2 + Q_a} \right)$ Subramani and Kumarapandiyani's (2012b) estimator
$(1, 1, 0, -, 0, -, -1, -, 0, 0, \frac{M_d}{S_x^2}, 1)$	$t_{SP(29)} = s_y^2 \left( \frac{S_x^2 + M_d}{s_x^2 + M_d} \right)$ Subramani and Kumarapandiyani's (2012a) estimator

$(1, w_1, w_2 \frac{(a+b)}{b}, 0, -, -, 0, 0, -1, 0, \frac{h}{S_x^2}, b)$	$t_{SP(30)} = [w_1 s_y^2 + w_2 (S_x^2 - s_x^2)] \left( \frac{bS_x^2 + h}{bs_x^2 + h} \right)$ Singh and Solanki's (2013a) estimator
$(1, 1, 0, -, 0, -, -1, -, 0, 0, \frac{C_x}{S_x^2}, 1)$	$t_{SP(31)} = s_y^2 \left( \frac{S_x^2 + C_x}{s_x^2 + C_x} \right)$ Singh and Solanki's (2013a) estimator
$(1, 1, 0, -, 0, -, -1, -, 0, 0, \frac{C_x}{S_x^2}, \beta_2(x))$	$t_{SP(32)} = s_y^2 \left( \frac{S_x^2 \beta_2(x) + C_x}{s_x^2 \beta_2(x) + C_x} \right)$ Singh and Solanki's (2013a) estimator
$(1, 1, 0, -, 0, -, -1, -, 0, 0, \frac{\beta_2(x)}{S_x^2}, C_x)$	$t_{SP(33)} = s_y^2 \left( \frac{S_x^2 C_x + \beta_2(x)}{s_x^2 C_x + \beta_2(x)} \right)$ Singh and Solanki's (2013a) estimator
$(1, 1, 0, -, 0, -, -1, -, 0, 0, \frac{Q_3}{S_x^2}, \rho)$	$t_{SP(34)} = s_y^2 \left( \frac{S_x^2 \rho + Q_3}{s_x^2 \rho + Q_3} \right)$ Khan and Shabbir's (2013) estimator
$(1, 1, 0, -, 0, -, -1, -, 0, 0, \frac{\mathcal{Y}_i^2}{S_x^2}, 1)$ $\gamma$ being a constant such that $0 \leq \gamma \leq 1$	$t_{SP(35)} = s_y^2 \left( \frac{S_x^2 + \mathcal{Y}_i^2}{s_x^2 + \mathcal{Y}_i^2} \right) i=1$ to 6, $L_1 = Q_1$ , $L_2 = Q_2$ , $L_3 = Q_3$ , $L_4 = Q_r$ , $L_5 = Q_d$ , $L_6 = Q_a$ , where $Q_1$ (first quartile), $Q_2$ (second range), $Q_3$ (third quartile), $Q_d = (Q_3 - Q_1) / 2$ , $Q_a = (Q_3 + Q_1) / 2$ . Singh et al.'s (2013) estimator
$(1, w_1, w_2, -, -, -, 0, 0, 0, 0, (1+g), -g)$	$t_{SP(36)} = w_1 s_y^2 + w_2 (S_x^2 - s_x^{*2})$ Sharma and Singh's (2014) estimator
$(1, k_1, k_2, -, -, 2, 0, 1, 0, 0, (1+g), -g)$	$t_{SP(37)} = k_1 s_y^2 + k_2 (S_x^2 - s_x^{*2}) [2 - (s_x^{*2} / S_x^2)]$ Sharma and Singh's (2014) estimator
$(1, m_1, m_2, -, 0, -, 1, 0, 0, 0, (1+g), -g)$	$t_{SP(38)} = m_1 s_y^2 (s_x^{*2} / S_x^2) + m_2 (S_x^2 - s_x^{*2})$ Sharma and Singh's (2014) estimator
$(1, w_1, w_2 \frac{(a+b)}{b}, 0, -, -, 0, 0, -1, 0, \frac{\eta^* L^2}{S_x^2}, \delta^*)$	$t_{SP(39)} = [w_1 s_y^2 + w_2 (S_x^2 - s_x^2)] \left( \frac{\delta^* S_x^2 + \eta^* L^2}{\delta^* s_x^2 + \eta^* L^2} \right)$ Singh and Pal's (2016) estimator
$(1, w_1, w_2 \frac{(a+b)}{b}, -, -, -, 0, 0, 0, 1, \frac{\eta^* L^2}{S_x^2}, \delta^*)$	$t_{SP(40)} = [w_1 s_y^2 + w_2 (S_x^2 - s_x^2)] \exp \left( \frac{\delta^* (S_x^2 - s_x^2)}{\delta^* (S_x^2 + s_x^2) + 2\eta^* L^2} \right)$ Singh and Pal's (2016) estimator

where  $(A, w, \alpha^*, h, w_1, w_2, k_1, k_2)$  are suitable chosen constants and  $(\delta^*, L)$  are either real constants or function of known parameter of an auxiliary variable  $x$  and  $\eta^*$  being a constant such that  $|\eta^*| \leq 1$ .



### 2.1. Derivation of the expressions of bias and mean squared error (MSE) of the class of estimators $t_{SP}$

To obtain the bias and MSE of the class of estimators  $t_{SP}$  at (2.1) in terms of  $e$ 's we have

$$t_{SP} = [\phi_1 S_y^2 (1 + e_0) \{ \xi + (1 - \xi)(1 + b^* e_1)^\alpha \lambda \} - \phi_2 b^* S_x^2 e_1 \{ \phi + (1 - \phi)(1 + b^* e_1)^\delta \}] \\ \times \{ \theta + (1 - \theta)(1 + b^* e_1)^p \} \times \exp \left\{ - \left( \frac{b^* q}{2} \right) e_1 \left( 1 + \frac{b^*}{2} e_1 \right)^{-1} \right\}, \quad (2.2)$$

where  $b^* = b/(a + b)$ .

We assume that  $|b^* e_1| < 1$  so that  $(1 + b^* e_1)^\alpha$ ,  $(1 + b^* e_1)^\delta$ ,  $(1 + b^* e_1)^p$  and  $\left(1 + \frac{b^*}{2} e_1\right)^{-1}$  are expandable. Now expanding the right hand side of (2.2), we have

$$t_{SP} = \left[ \phi_1 S_y^2 (1 + e_0) \left\{ \xi + (1 - \xi) \lambda \left[ 1 + \alpha b^* e_1 + \frac{\alpha(\alpha - 1)}{2} b^{*2} e_1^2 + \dots \right] \right\} \right. \\ \left. - \phi_2 b^* S_x^2 e_1 \left\{ \phi + (1 - \phi) \left[ 1 + \delta b^* e_1 + \frac{\delta(\delta - 1)}{2} b^{*2} e_1^2 + \dots \right] \right\} \right] \\ \times \left\{ \theta + (1 - \theta) \left[ 1 + p b^* e_1 + \frac{p(p - 1)}{2} b^{*2} e_1^2 + \dots \right] \right\} \\ \times \left\{ 1 - \frac{b^* q}{2} e_1 \left( 1 + \frac{b^*}{2} e_1 \right)^{-1} + \frac{b^{*2} q^2}{8} e_1^2 \left( 1 + \frac{b^*}{2} e_1 \right)^{-2} - \dots \right\} \\ = S_y^2 \left[ \phi_1 (1 + e_0) \left\{ \xi_0 + (1 - \xi) \lambda \alpha b^* \left( e_1 + \frac{(\alpha - 1) b^*}{2} e_1^2 + \dots \right) \right\} \right. \\ \left. - \phi_2 b^* e_1 \left\{ 1 + (1 - \phi) \delta b^* \left( e_1 + \frac{(\delta - 1) b^*}{2} e_1^2 + \dots \right) \right\} \right] \times \left\{ 1 + (1 - \theta) p b^* \left( e_1 + \frac{(p - 1) b^*}{2} e_1^2 + \dots \right) \right\} \\ \times \left\{ 1 - \frac{b^* q}{2} e_1 + \frac{b^{*2} q(q + 2)}{8} e_1^2 - \dots \right\} \\ = S_y^2 \left[ \phi_1 \left\{ \xi_0 (1 + e_0) + (1 - \xi) \lambda \alpha b^* (e_1 + e_0 e_1) + \frac{(1 - \xi) \lambda \alpha (\alpha - 1) b^{*2}}{2} e_1^2 + \dots \right\} \right.$$

$$\begin{aligned}
 & -\phi_2 b^* r \{e_1 + (1-\phi)\delta b^* e_1^2 + \dots\} \times \left\{ 1 + (1-\theta)pb^* e_1 + (1-\theta)\frac{p(p-1)b^{*2}}{2} e_1^2 \right. \\
 & \quad \left. - \left(\frac{b^*q}{2}\right)e_1 + \left(\frac{b^{*2}pq(1-\theta)}{2}\right)e_1^2 + \frac{b^{*2}q(q+2)}{8} e_1^2 + \dots \right\} \\
 = & S_y^2 \left[ \phi_1 \left\{ \xi_0(1+e_0) + (1-\xi)\lambda ab^*(e_1 + e_0e_1) + \frac{(1-\xi)\lambda\alpha(\alpha-1)}{2} b^{*2} e_1^2 + \dots \right\} \right. \\
 & \quad \left. - \phi_2 b^* r \{e_1 + (1-\phi)\delta b^* e_1^2 + \dots\} \times \left\{ 1 + u_1 b^* e_1 + \frac{b^{*2}u_2}{2} e_1^2 + \dots \right\} \right] \\
 = & S_y^2 \left[ \phi_1 \left\{ \xi_0(1+e_0) + (1-\xi)\lambda ab^*(e_1 + e_0e_1) + \frac{(1-\xi)\lambda\alpha(\alpha-1)}{2} b^{*2} e_1^2 + \xi_0 b^* u_1 (e_1 + e_0e_1) \right. \right. \\
 & \quad \left. \left. + (1-\xi)\lambda ab^{*2} u_1 (e_1^2 + e_0e_1^2) + \frac{b^{*2}u_2}{2} \xi_0 (e_1^2 + e_0e_1^2) + \dots \right\} - \phi_2 b^* r \{e_1 + (1-\phi)\delta b^* e_1^2 + b^* u_1 e_1^2 + \dots\} \right] \\
 = & S_y^2 \left[ \phi_1 \left\{ \xi_0 + \xi_0 e_0 + b^* \xi^* e_1 + b^* \xi^* e_0 e_1 + \left(\frac{b^{*2}\theta^*}{2}\right) e_1^2 + b^{*2} \left( (1-\xi)\lambda\alpha u_1 + \frac{u_2 \xi_0}{2} \right) e_0 e_1^2 + \dots \right\} \right. \\
 & \quad \left. - \phi_2 b^* r \{e_1 + b^* [(1-\phi)\delta + u_1] e_1^2 + \dots\} \right], \tag{2.3}
 \end{aligned}$$

where

$$\xi_0 = [\xi + (1-\xi)\lambda], \quad \xi^* = [\xi u_1 + (1-\xi)\lambda(\alpha + u_1)], \quad u_1 = \left[ (1-\theta)p - \frac{q}{2} \right],$$

$$\theta^* = [\alpha(\alpha-1)(1-\xi)\lambda + \xi_0 u_2], \quad u_2 = \left[ (1-\theta)p(p-q-1) + \frac{q(q+2)}{4} \right],$$

$$\phi^* = [(1-\phi)\delta + u_1] = [(1-\phi)\delta + (1-\theta)p - (q/2)], \quad r = S_x^2 / S_y^2,$$

$$b^* = b/(a+b).$$

Neglecting terms of e's in (2.3) having power greater than two, we have

$$t_{SP} \cong S_y^2 \left[ \phi_1 \left\{ \xi_0 + \xi_0 e_0 + b^* \xi^* e_1 + b^* \xi^* e_0 e_1 + \left(\frac{b^{*2}\theta^*}{2}\right) e_1^2 \right\} - \phi_2 b^* r \{e_1 + b^* \phi^* e_1^2\} \right]$$

or

$$(t_{SP} - S_y^2) \cong S_y^2 \left[ \phi_1 \left\{ \xi_0 + \xi_0 e_0 + b^* \xi^* e_1 + b^* \xi^* e_0 e_1 + \left(\frac{b^{*2}\theta^*}{2}\right) e_1^2 \right\} - \phi_2 b^* r \{e_1 + b^* \phi^* e_1^2\} - 1 \right], \tag{2.4}$$

Taking expectation of both sides of (2.4) we get the bias of  $t_{SP}$  to the first degree of approximation as

$$B(t_{SP}) = S_y^2 \left[ \phi_1 \left\{ \xi_0 + f(\lambda_{04} - 1) b^* \left( \frac{b^* \theta^*}{2} + \xi^* C \right) \right\} - \phi_2 r b^{*2} \phi^* f(\lambda_{04} - 1) - 1 \right]. \tag{2.5}$$

Squaring both sides of (2.4) and neglecting terms of  $e$ 's having power greater than two, we have

$$(t_{SP} - S_y^2)^2 = S_y^4 [1 + \phi_1^2 \{ \xi_0^2 (1 + 2e_0 + e_0^2) + 2b^* \xi_0 \xi^* (e_1 + 2e_0 e_1) + b^{*2} (\xi^{*2} + \theta^* \xi_0) e_1^2 \} + \phi_2^2 r^2 b^{*2} e_1^2 - 2\phi_1 \phi_2 b^* r \{ \xi_0 (e_1 + e_0 e_1) + b^* (\xi^* + \phi^* \xi_0) e_1^2 \} - 2\phi_1 \{ \xi_0 (1 + e_0) + b^* \xi^* (e_1 + e_0 e_1) + ((b^{*2} \theta^*) / 2) e_1^2 \} + 2\phi_2 b^* r (e_1 + b^* \phi^* e_1^2)]. \tag{2.6}$$

Taking expectation of both sides of (2.6) we get the *MSE* of  $t_{SP}$  to the first degree of approximation as

$$MSE(t_{SP}) = S_y^4 [1 + \phi_1^2 A_1 + \phi_2^2 A_2 - 2\phi_1 \phi_2 A_3 - 2\phi_1 A_4 + 2\phi_2 A_5], \tag{2.7}$$

where

$$\begin{aligned} A_1 &= [\xi_0^2 + f \{ \xi_0^2 (\lambda_{40} - 1) + b^* (\lambda_{04} - 1) [b^* (\xi^{*2} + \theta^* \xi_0) + 4\xi_0 \xi^* C] \}], \\ A_2 &= r^2 b^{*2} f(\lambda_{04} - 1), \\ A_3 &= b^* r f(\lambda_{04} - 1) [b^* (\xi^* + \phi^* \xi_0) + \xi_0 C], \\ A_4 &= [\xi_0 + b^* f(\lambda_{04} - 1) ((b^* \theta^* / 2) + \xi^* C)], \\ A_5 &= b^{*2} \phi^* r f(\lambda_{04} - 1), \\ f &= ((1/n) - (1/N)). \end{aligned}$$

Differentiating (2.7) partially with respect to  $\phi_1$  and  $\phi_2$  and equating to zero, we have

$$\begin{bmatrix} A_1 & -A_3 \\ -A_3 & A_2 \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = \begin{bmatrix} A_4 \\ -A_5 \end{bmatrix}. \tag{2.8}$$

Solving (2.8) we get the optimum values of  $\phi_1$  and  $\phi_2$  as

$$\left. \begin{aligned} \phi_1 &= \frac{(A_2 A_4 - A_3 A_5)}{(A_1 A_2 - A_3^2)} = \phi_{10} (say) \\ \phi_2 &= \frac{(A_3 A_4 - A_1 A_5)}{(A_1 A_2 - A_3^2)} = \phi_{20} (say) \end{aligned} \right\}. \tag{2.9}$$

Putting (2.9) in (2.8) we get the resulting minimum MSE of  $t_{SP}$  as

$$\min .MSE(t_{SP}) = S_y^4 \left[ 1 - \frac{(A_2 A_4^2 - 2A_3 A_4 A_5 + A_1 A_5^2)}{(A_1 A_2 - A_3^2)} \right]. \tag{2.10}$$

Thus we established the following theorem.

**Theorem 2.1:** Up to the first degree of approximation,

$$MSE(t_{SP}) \geq S_y^4 \left[ 1 - \frac{(A_2 A_4^2 - 2A_3 A_4 A_5 + A_1 A_5^2)}{(A_1 A_2 - A_3^2)} \right]$$

with equality holding if

$$\begin{aligned} \phi_1 &= \phi_{10}, \\ \phi_2 &= \phi_{20}, \end{aligned}$$

where  $\phi_{i0}$  's ( $i=1, 2$ ) are defined in (2.9).

**2.2. Special Case-I** ( $\lambda = 1$ )

Putting  $\lambda = 1$  in (2.1) we get the class of estimators of  $S_y^2$  as

$$\begin{aligned} t_{SP}^{(1)} &= \left[ \phi_1 S_y^2 \left\{ \xi + (1 - \xi) \left( \frac{S_{x(a,b)}^2}{S_x^2} \right)^\alpha \right\} + \phi_2 (S_x^2 - s_{x(a,b)}^2) \left\{ \phi + (1 - \phi) \left( \frac{S_{x(a,b)}^2}{S_x^2} \right)^\delta \right\} \right] \\ &\times \left\{ \theta + (1 - \theta) \left( \frac{S_{x(a,b)}^2}{S_x^2} \right)^p \right\} \times \exp \left\{ \frac{q(S_x^2 - s_{x(a,b)}^2)}{(S_x^2 + s_{x(a,b)}^2)} \right\}. \end{aligned} \tag{2.11}$$

Substitution of  $\lambda = 1$  in (2.5) and (2.7) yields the bias and MSE of the class of estimators to the first degree of approximation, respectively as

$$B(t_{SP}^{(1)}) = -S_y^2 \left[ 1 - \phi_1 \left\{ 1 + f(\lambda_{04} - 1) b^* \left( \frac{b^* \theta_1^*}{2} + \xi_1^* C \right) \right\} + \phi_2 r b^{*2} \phi^* f(\lambda_{04} - 1) \right] \tag{2.12}$$

and

$$MSE(t_{SP}^{(1)}) = S_y^4 [1 + \phi_1^2 A_1^* + \phi_2^2 A_2 - 2\phi_1 \phi_2 A_3^* - 2\phi_1 A_4^* + 2\phi_2 A_5], \tag{2.13}$$

where

$$A_1^* = [1 + f\{(\lambda_{40} - 1) + b^*(\lambda_{04} - 1)[b^*(\xi_1^{*2} + \theta_1^*) + 4\xi_1^* C]\}],$$

$$\begin{aligned}
 A_3^* &= b^* rf(\lambda_{04} - 1)[b^* (\xi_1^* + \phi^*) + C], \\
 A_4^* &= [1 + b^* f(\lambda_{04} - 1)(b^* \theta_1^* / 2) + \xi_1^* C], \\
 \xi_0 &= 1, \xi_1^* = [u_1 + \alpha(1 - \xi)], \theta_1^* = [\alpha(\alpha - 1)(1 - \xi) + u_2].
 \end{aligned}$$

The  $MSE(t_{SP}^{(1)})$  is minimized for

$$\left. \begin{aligned}
 \phi_1 &= \frac{(A_2 A_4^* - A_3^* A_5)}{(A_1^* A_2 - A_3^{*2})} = \phi_{10}^{(1)} \\
 \phi_2 &= \frac{(A_3^* A_4^* - A_1^* A_5)}{(A_1^* A_2 - A_3^{*2})} = \phi_{20}^{(1)}
 \end{aligned} \right\}. \tag{2.14}$$

Thus the resulting minimum  $MSE(t_{SP}^{(1)})$  is given by

$$\min .MSE(t_{SP}^{(1)}) = S_y^4 \left[ 1 - \frac{(A_2 A_4^{*2} - 2A_3^* A_4^* A_5 + A_1^* A_5^2)}{(A_1^* A_2 - A_3^{*2})} \right]. \tag{2.15}$$

Thus we established the following corollary.

**Corollary 2.1:** Up to the first degree of approximation,

$$MSE(t_{SP}^{(1)}) \geq S_y^4 \left[ 1 - \frac{(A_2 A_4^{*2} - 2A_3^* A_4^* A_5 + A_1^* A_5^2)}{(A_1^* A_2 - A_3^{*2})} \right]$$

with equation holding if

$$\begin{aligned}
 \phi_1 &= \phi_{10}^{(1)}, \\
 \phi_2 &= \phi_{20}^{(1)}.
 \end{aligned}$$

From (2.10) and (2.15) we have

$$\begin{aligned}
 \min .MSE(t_{SP}^{(1)}) - \min .MSE(t_{SP}) \\
 = S_y^4 \left[ \frac{(A_2 A_4^2 - 2A_3 A_4 A_5 + A_1 A_5^2)}{(A_1 A_2 - A_3^2)} - \frac{(A_2 A_4^{*2} - 2A_3^* A_4^* A_5 + A_1^* A_5^2)}{(A_1^* A_2 - A_3^{*2})} \right]
 \end{aligned}$$

which is positive if

$$\frac{(A_2 A_4^2 - 2A_3 A_4 A_5 + A_1 A_5^2)}{(A_1 A_2 - A_3^2)} > \frac{(A_2 A_4^{*2} - 2A_3^* A_4^* A_5 + A_1^* A_5^2)}{(A_1^* A_2 - A_3^{*2})}. \tag{2.16}$$

Thus the proposed family of estimators  $t_{SP}$  would be more efficient than the family of estimators  $t_{SP}^{(1)}$  as long as inequality (2.16) is satisfied.

**2.3. Special Case-II**  $(\phi_1, \lambda) = (1,1)$

Putting  $(\phi_1, \lambda) = (1,1)$  in (2.1) we get class of estimators of  $S_y^2$  as

$$t_{SP}^{(2)} = \left[ s_y^2 \left\{ \xi + (1 - \xi) \left( \frac{S_{x(a,b)}^2}{S_x^2} \right)^\alpha \right\} + \phi_2 (S_x^2 - s_{x(a,b)}^2) \left\{ \phi + (1 - \phi) \left( \frac{S_{x(a,b)}^2}{S_x^2} \right)^\delta \right\} \right] \times \left[ \theta + (1 - \theta) \left( \frac{S_{x(a,b)}^2}{S_x^2} \right)^p \right] \times \exp \left\{ \frac{q(S_x^2 - s_{x(a,b)}^2)}{(S_x^2 + s_{x(a,b)}^2)} \right\}. \tag{2.17}$$

Inserting  $(\phi_1, \lambda) = (1,1)$  in (2.5) and (2.7) yield the bias and  $MSE$  of  $t_{SP}^{(2)}$  to the first degree of approximation, respectively given by

$$B(t_{SP}^{(2)}) = S_y^2 b^* f(\lambda_{04} - 1) \left[ \left( \frac{b^* \theta_1^*}{2} \right) + \xi_1^* C - \phi_2 r b^* \phi^* \right] \tag{2.18}$$

and

$$MSE(t_{SP}^{(2)}) = S_y^4 [1 + A_1^* - 2A_4^* + \phi_2^2 A_2 - 2\phi_2 (A_3^* - A_5)]. \tag{2.19}$$

The  $MSE(t_{SP}^{(2)})$  is minimized for

$$\phi_2 = \frac{(A_3^* - A_5)}{A_2} = \phi_{20}^* \text{ (say)}. \tag{2.20}$$

Thus the resulting minimum  $MSE(t_{SP}^{(2)})$  is given by

$$\begin{aligned} \min.MSE(t_{SP}^{(2)}) &= S_y^4 \left[ 1 - A_1^* - 2A_4^* - \frac{(A_3^* - A_5)^2}{A_2} \right], \tag{2.21} \\ &= fS_y^4 (\lambda_{40} - 1)(1 - \rho^{*2}) \end{aligned}$$

which equals to the minimum  $MSE$  of the difference estimator  $t_3$  defined in (1.3).

Now, we state the following corollary.

**Corollary 2.2:** Up to the first degree of approximation,

$$MSE(t_{SP}^{(2)}) \geq fS_y^4 (\lambda_{40} - 1)(1 - \rho^{*2})$$

with equality holding if

$$\phi_2 = \phi_{20}^* .$$

From (2.15) and (2.21) we have

$$\min .MSE(t_{SP}^{(2)}) - \min .MSE(t_{SP}^{(1)}) = \frac{S_y^4[A_3^*(A_3^* - A_5) - A_2(A_1^* - A_4^*)]^2}{A_2(A_1^*A_2 - A_3^{*2})} \tag{2.22}$$

which is always positive. It follows that the proposed family of estimators  $t_{SP}^{(1)}$  is better than the family of estimators  $t_{SP}^{(2)}$  and the difference type estimators  $t_3$  in (1.3) at their optimum conditions.

**2.4. Special Case-III** ( $\phi_1 = 1$ )

For  $\phi_1 = 1$ , the suggested class of estimators  $t_{SP}$  reduces to the class of estimators

$$t_{SP}^{(3)} = \left[ s_y^2 \left\{ \xi + (1 - \xi) \left( \frac{s_{x(a,b)}^2}{S_x^2} \right)^\alpha \right\} + \phi_2 (S_x^2 - s_{x(a,b)}^2) \left\{ \phi + (1 - \phi) \left( \frac{s_{x(a,b)}^2}{S_x^2} \right)^\delta \right\} \right] \times \left\{ \theta + (1 - \theta) \left( \frac{s_{x(a,b)}^2}{S_x^2} \right)^p \right\} \times \exp \left\{ \frac{q(S_x^2 - s_{x(a,b)}^2)}{(S_x^2 + s_{x(a,b)}^2)} \right\}. \tag{2.23}$$

Putting  $\phi_1 = 1$  in (2.5) and (2.7) we get the bias and  $MSE$  of the estimator  $t_{SP}^{(3)}$  to the first degree of approximation, respectively given by

$$B(t_{SP}^{(3)}) = -S_y^2 \left[ 1 - \left\{ \xi_0 + fb^*(\lambda_{04} - 1) \left( \frac{b^*\theta^*}{2} + \xi^*C \right) \right\} + \phi_2 r b^{*2} \phi^* f(\lambda_{04} - 1) \right] \tag{2.24}$$

and

$$MSE(t_{SP}^{(3)}) = S_y^4 [1 + A_1 - 2A_4 + \phi_2^2 A_2 - 2\phi_2(A_3 - A_5)]. \tag{2.25}$$

The  $MSE(t_{SP}^{(3)})$  is minimized when

$$\phi_2 = \frac{(A_3 - A_5)}{A_2} = \phi_{20}^{*(1)} \text{ (say)}. \tag{2.26}$$

Thus the resulting minimum  $MSE(t_{SP}^{(3)})$  is given by

$$\min .MSE(t_{SP}^{(3)}) = S_y^4 \left[ 1 + A_1 - 2A_2 - \frac{(A_3 - A_5)^2}{A_2} \right]. \tag{2.27}$$

Now, we state the following corollary.

**Corollary 2.3.** To the first degree of approximation,

$$MSE(t_{SP}^{(3)}) \geq S_y^4 \left[ 1 + A_1 - 2A_2 - \frac{(A_3 - A_5)^2}{A_2} \right]$$

with equality holding if

$$\phi_2 = \phi_{20}^{*(1)}.$$

From (2.10) and (2.27) we have

$$\begin{aligned} \min .MSE(t_{SP}^{(3)}) - \min .MSE(t_{SP}) &= \frac{S_y^4 [A_2(A_1 - A_4) - A_3(A_3 - A_5)]^2}{A_2(A_1A_2 - A_3^2)} \\ &\geq 0 \end{aligned} \tag{2.28}$$

which clearly indicates that the  $t_{SP}$  family of estimators is more efficient than that of the  $t_{SP}^{(3)}$  family of estimators .

### Concluding remarks

This paper intends to suggest a new family of estimators for the variance  $S_y^2$  of the variable  $y$  of interest when the population variance  $S_x^2$  of the auxiliary variable  $x$  is known. The proposed family generalizes that of the several estimators ( $t_{SP(i)}$ ,  $i= 1$  to 38) as listed in Table 2.1. We have obtained the bias and mean squared error ( $MSE$ ) expressions up to first order of approximation in simple random sampling without replacement ( $SRSWOR$ ). From the bias and  $MSE$  expressions of the suggested family, one can easily derive the bias and  $MSE$  expressions of existing known estimators as well as those of potential new proposals. The present study unifies several results at one place.

The family is certainly not exhaustive but it can act as different against the proliferation of equivalent proposals that could be appearing in the future. Three subclasses of the proposed family are identified and their properties are studied. We have also given the comparisons among the proposed class of estimators and the three subclasses of estimators. It has been theoretically shown that the proposed class of estimators is more efficient than the difference type estimator  $t_3$  due to Das and Tripathi (1978) and hence the usual unbiased estimator  $s_y^2$  and Isaki (1983) ratio estimator  $t_1$  and several other estimators. This paper also provides the correct  $MSE$  expressions of the estimators ( $t_5, t_6, t_7$ ) recently proposed by Sharma and Singh(2014). Indeed, improvement upon the difference type estimators  $t_3$  as well as upon other estimators can be achieved when the theoretical expressions of the minimum mean squared error are considered. These expressions are based on the knowledge of population parameters which can be



obtained either through past data or experience gathered in due course of time. For more discussion on this issue, the reader is referred to Das and Tripathi (1978) and Srivastava and Jhaji (1980). However, more light on this study can be focused if one would have included an empirical study. Overall this study is of academic interest as well as of practical importance, see, Diana et al. (2011), Singh et al. (2013) and Singh and Solanki (2013 a, b), Solanki and Singh(2013), Singh et al. (2013), Singh et al.(2014), Solanki et al. (2015) and Singh and Pal (2016) etc.

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