

Khare, B. B.; Sinha, R. R.

**Article**

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## ESTIMATION OF PRODUCT OF TWO POPULATION MEANS BY MULTI-AUXILIARY CHARACTERS UNDER DOUBLE SAMPLING THE NON-RESPONDENTS

B. B. Khare<sup>1</sup>, R. R. Sinha<sup>2</sup>

### ABSTRACT

This paper considers the problem of estimating the product of two population means using the information on multi-auxiliary characters with double sampling the non-respondents. Classes of estimators are proposed for estimating  $P$  under two different situations [discussed by Rao (1986, 90)] using known population mean of multi-auxiliary characters. Further, this problem has been extended to the case when population means of the auxiliary characters are unknown and they are estimated on the basis of a larger first phase sample. In this situation, a class of two phase sampling estimators for estimating  $P$  is suggested using multi-auxiliary characters with unknown population means in the presence of non-response. The expressions of bias and mean square error of all the proposed estimators are derived and their properties are studied. An empirical study using real data sets is given to justify the theoretical considerations.

**Key words:** product, bias, mean square error, auxiliary characters, non-response.

### 1. Introduction

While conducting sample surveys in the field of agriculture, socio-economic and forest research, one may be interested in the estimation of the product of two population means. For example- if we want to estimate the total population of persons in a District using villages as the sampling units, then we will estimate the product of average number of occupied houses in a village and average number of persons in a house in that village and the population of the District can be obtained by multiplying the estimate of product to the number of villages. The auxiliary characters used in this circumstance may be the area, the number of cultivators, the number of agricultural labours, etc., of the village. Similarly one may use the amount of manure, water and seeds supplied to each plot as the auxiliary characters in the estimation of total yield of a crop within an agricultural field, which can be obtained by estimating the product of average area per plot

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<sup>1</sup> Department of Statistics, Banaras Hindu University, Varanasi, India. E-mail: bbkhare56@yahoo.com.

<sup>2</sup> Department of Mathematics, Dr. B. R. Ambedkar National Institute of Technology, Jalandhar, India. E-mail: raghawraman@gmail.com (Corresponding author).

and the average yield per plot and then multiplying this estimate of product by the number of plots in the agricultural field.

Many authors like Singh (1965, 67, 69), Shah & Shah (1978), Singh (1982a,b), Ray and Singh (1985), Khare (1987), Srivastativa *et al.* (1988), Khare (1990, 91) considered the problem of estimating the ratio and the product of two population means and suggested estimators/classes of estimators using the information of auxiliary character(s).

When the population means of the auxiliary characters are not known then in this case Singh (1982c) proposed generalized double sampling estimators for the ratio and product of population parameters using double sampling scheme. Further, Khare (1992) proposed the class of estimators for the product of two population means using multi-auxiliary characters with known and unknown population means.

In some cases, it happens that the information on the main characters and auxiliary characters may not be available in practice due to the occurrence of non-response for the selected units in the sample viz. while conducting the yield of a crop in an agricultural field, it may be possible that the data on yield of a crop as well as the area of the plot may be made available for some selected plots for which the information on the auxiliary characters may or may not be known due to lack of reporting the owner of the field. To reduce the effect of non-response in the estimation of parameter for a variable, Hansen and Hurwitz (1946) suggested a method of sub-sampling on the non-responding units and proposed an unbiased estimator for population mean. Further, following Hansen and Hurwitz (1946) strategies of sub-sampling the non-responding units, Khare and Sinha (2002 a, b, 2004 a, b, 2007, 2012 a, b), Singh *et al.* (2007), Singh and Kumar (2008) proposed some classes of estimators for the ratio/product of two population means using auxiliary character(s) in the presence of non-response. The objective of this paper is to suggest classes of estimators for estimating the product of two population means using information available on multi-auxiliary characters in the presence of non-response under different situations and study their theoretical and empirical properties.

## 2. Proposed classes of estimators

Let  $y_i (i = 1, 2)$  and  $x_j (j = 1, 2, \dots, p)$  be the main and auxiliary characters under study having non-negative  $k^{th}$  value  $Y_{ik}, X_{jk}$ ; ( $k = 1, 2, \dots, N$ ) with population means  $\bar{Y}_i (i = 1, 2)$  of study characters and  $\bar{X}_j (j = 1, 2, \dots, p)$  of auxiliary characters respectively. The whole population is supposed to be divided into two non-overlapping unknown strata of  $N_1$  responding and  $N_2$  non-responding units such that  $N_1 + N_2 = N$ . Let  $n$  be the size of the sample drawn from the population of size  $N$  using simple random sampling without replacement (SRSWOR) scheme of sampling and it has been observed that  $n_1$  units respond and  $n_2$  units do not respond in the sample of size  $n$ . We have considered that the responding and non-responding units are same for both the study and auxiliary characters. The stratum weights of responding and non-responding groups are given by  $W_1 = N_1/N$  and  $W_2 = N_2/N$  and their estimates are respectively given by  $\hat{W}_1 = w_1 = n_1/n$  and  $\hat{W}_2 = w_2 = n_2/n$ . Further, from the non-responding units  $n_2$ , we

draw a subsample of size  $m (= n_2 \delta^{-1}, \delta > 1)$  using SRSWOR technique of sampling and collect the information by the direct interview for  $y_i$  ( $i = 1, 2$ ). Using the methodology of Hansen and Hurwitz (1946), the unbiased estimator for  $\bar{Y}_i$  ( $i = 1, 2$ ) based on the information of  $(n_1 + m)$  units is given by

$$\bar{y}_{i(HH)} = w_1 \bar{y}_{i1} + w_2 \bar{y}_{i2}^*, \quad i = 1, 2 \tag{1}$$

where  $\bar{y}_{i1}$  and  $\bar{y}_{i2}^*$  are the sample means of  $y_i$  based on  $n_1$  and  $m$  units respectively.

The estimator  $\bar{y}_{i(HH)}$  is unbiased and has variance given by

$$V(\bar{y}_{i(HH)}) = V^{(i)} = \lambda S_{y_i}^2 + \lambda_\delta S_{y_{i(2)}}^2, \tag{2}$$

where  $S_{y_i}^2$  and  $S_{y_{i(2)}}^2$  are the population mean square of  $y_i$  for the entire and non-responding group of the population and  $\lambda = \frac{N-n}{Nn}$ ,  $\lambda_\delta = \frac{N_2}{Nn}(\delta - 1)$ .

Similarly, the estimator  $\bar{x}_{j(HH)}$  ( $j = 1, 2, \dots, p$ ) for the population mean  $\bar{X}_j$  ( $j = 1, 2, \dots, p$ ) is given by

$$\bar{x}_{j(HH)} = w_1 \bar{x}_{j1} + w_2 \bar{x}_{j2}^*, \quad j = 1, 2, \dots, p \tag{3}$$

where  $\bar{x}_{j1}$  and  $\bar{x}_{j2}^*$ ; ( $j = 1, 2, \dots, p$ ) are the sample means of the character  $x_j$  ( $j = 1, 2, \dots, p$ ) based on  $n_1$  and  $m$  units respectively.

Let  $\hat{P} (= \prod_{i=1}^2 \bar{y}_{i(HH)})$  denote conventional estimator for the product of two population means ( $P = \bar{Y}_1 \cdot \bar{Y}_2$ ) in the presence of non-response on study characters. Utilizing the information of auxiliary characters with known population means, the two different proposed classes of estimators for  $P$  under two different cases discussed by {Rao (1986), page 220} are as follows:

- **For the first case**, it is assumed that the population mean  $\bar{X}_j$  ( $j = 1, 2, \dots, p$ ) is known, and incomplete information occurred on  $y_i$  ( $i = 1, 2$ ) and  $x_j$  ( $j = 1, 2, \dots, p$ ) for the selected units in the sample of size  $n$ . In such situation, we propose a class of estimators  $t_p$  for the product of two population means ( $P$ ) using multi-auxiliary characters  $x_j$  ( $j = 1, 2, \dots, p$ ) with known population means as:

$$t_p = g(\prod_{i=1}^2 \bar{y}_{i(HH)}, (u_1, u_2, \dots, u_p)') = g(\varphi, \mathbf{u}'), \tag{4}$$

such that

$$g(P, \mathbf{e}') = P, \quad g_1(P, \mathbf{e}') = \left( \frac{\partial}{\partial \varphi} g(\varphi, \mathbf{u}') \right)_{(P, \mathbf{e}')} = 1 \tag{5}$$

where  $\mathbf{u}$  and  $\mathbf{e}$  denote the column vectors  $(u_1, u_2, \dots, u_p)'$  and  $(1, 1, \dots, 1)'$  respectively. We also denote  $\varphi = \prod_{i=1}^2 \bar{y}_{i(HH)}$  and  $u_j = \bar{x}_{j(HH)} / \bar{X}_j$ ;  $j = 1, 2, \dots, p$ .

- **For the second case**, it is assumed that the population mean  $\bar{X}_j$  ( $j = 1, 2, \dots, p$ ) is known, and incomplete information occurred on  $y_i$  ( $i = 1, 2$ ) only but complete information on  $x_j$  ( $j = 1, 2, \dots, p$ ) for the selected units in the sample of size  $n$ . In this situation, we propose a class of estimators  $t_p^*$  for the product of two population means ( $P$ ) using multi-auxiliary characters  $x_j$  ( $j = 1, 2, \dots, p$ ) with known population means as:

$$t_p^* = h\left(\prod_{i=1}^2 \bar{y}_{i(HH)}, (\omega_1, \omega_2, \dots, \omega_p)'\right) = h(\varphi, \boldsymbol{\omega}'), \quad (6)$$

such that

$$h(P, e') = P, \quad h_1(P, e') = \left(\frac{\partial}{\partial \varphi} h(\varphi, \boldsymbol{\omega}')\right)_{(P, e')} = 1 \quad (7)$$

where  $\boldsymbol{\omega}$  denotes the column vectors  $(\omega_1, \omega_2, \dots, \omega_p)'$  and  $\omega_j = \bar{x}_j / \bar{X}_j$ ;  $j = 1, 2, \dots, p$ .

For the expansion of the functions  $g(\varphi, \boldsymbol{u}')$  and  $h(\varphi, \boldsymbol{\omega}')$ , it is supposed that whatever be the sample chosen for any sampling design,  $(\varphi, \boldsymbol{u}')$  [or  $(\varphi, \boldsymbol{\omega}')$ ] assumes a value in a bounded closed convex subset  $D_p$  [or  $D_p^*$ ] of the  $(p + 1)$  dimensional real space containing the point  $(P, e')$ . In  $D_p$  [or  $D_p^*$ ], the function  $g(\varphi, \boldsymbol{u}')$  [or  $h(\varphi, \boldsymbol{\omega}')$ ] is continuous and bounded. The first and second partial derivatives of  $g(\varphi, \boldsymbol{u}')$  [or  $h(\varphi, \boldsymbol{\omega}')$ ] exist and are continuous and bounded in  $D_p$  [or  $D_p^*$ ].

Here,  $g_1(\varphi, \boldsymbol{u}')$  [or  $h_1(\varphi, \boldsymbol{\omega}')$ ] and  $g_2(\varphi, \boldsymbol{u}')$  [or  $h_2(\varphi, \boldsymbol{\omega}')$ ] denote the first partial derivatives of  $g(\varphi, \boldsymbol{u}')$  [or  $h(\varphi, \boldsymbol{\omega}')$ ] with respect to  $\varphi$  and  $\boldsymbol{u}'$  [or  $\boldsymbol{\omega}'$ ] respectively. The second partial derivatives of  $g(\varphi, \boldsymbol{u}')$ ,  $h(\varphi, \boldsymbol{\omega}')$  with respect to  $\boldsymbol{u}'$  and  $\boldsymbol{\omega}'$  are respectively denoted by  $g_{22}(\varphi, \boldsymbol{u}')$ ,  $h_{22}(\varphi, \boldsymbol{\omega}')$  and first partial derivative of  $g_2(\varphi, \boldsymbol{u}')$  [or  $h_2(\varphi, \boldsymbol{\omega}')$ ] with respect to  $\varphi$  is denoted by  $g_{12}(\varphi, \boldsymbol{u}')$  [or  $h_{12}(\varphi, \boldsymbol{\omega}')$ ].

Under the regularity conditions imposed on  $g(\varphi, \boldsymbol{u}')$  and  $h(\varphi, \boldsymbol{\omega}')$ , it may be seen that the bias and mean square error of the estimators  $t_p$  and  $t_p^*$  will always exist.

In order to derive the expressions for bias and mean square error of the estimators under large sample approximation, let us assume

$$\epsilon_{0i} = \frac{\bar{y}_i^* - \bar{y}_i}{\bar{y}_i}, \quad \epsilon_j = \frac{\bar{x}_j^* - \bar{x}_j}{\bar{x}_j}, \quad \epsilon'_j = \frac{\bar{x}_j - \bar{X}_j}{\bar{X}_j}, \quad \text{with } E(\epsilon_{0i}) = E(\epsilon_j) = E(\epsilon'_j) = 0$$

and  $|\epsilon_{0i}| < 1$ ,  $|\epsilon_j| < 1$ ,  $|\epsilon'_j| < 1 \quad \forall i = 1, 2; j = 1, 2, \dots, p$ .

Now, using SRSWOR method of sampling, we have

$$E(\epsilon_{0i}^2) = \frac{V(\bar{y}_{i(HH)})}{\bar{y}_i^2} = \frac{1}{\bar{y}_i^2} \left[ \lambda S_{y_i}^2 + \lambda_\delta S_{y_i(2)}^2 \right], \quad E(\epsilon_j^2) = \frac{V(\bar{x}_{j(HH)})}{\bar{x}_j^2} = \frac{1}{\bar{x}_j^2} \left[ \lambda S_{x_j}^2 + \lambda_\delta S_{x_j(2)}^2 \right],$$

$$E(\epsilon_j'^2) = \frac{V(\bar{x}_j)}{\bar{x}_j^2} = \lambda \frac{S_{x_j}^2}{\bar{x}_j^2}, \quad E(\epsilon_{01}, \epsilon_{02}) = \frac{\text{Cov}(\bar{y}_{1(HH)}, \bar{y}_{2(HH)})}{\bar{y}_1 \bar{y}_2} = \frac{1}{\bar{y}_1 \bar{y}_2} \left[ \lambda S_{y_1 y_2} + \lambda_\delta S_{y_1 y_2(2)} \right],$$

$$E(\epsilon_{0i}, \epsilon_j) = \frac{\text{Cov}(\bar{y}_{i(HH)}, \bar{x}_{j(HH)})}{\bar{y}_i \bar{x}_j} = \frac{1}{\bar{y}_i \bar{x}_j} \left[ \lambda S_{y_i x_j} + \lambda_\delta S_{y_i x_j(2)} \right],$$

$$E(\epsilon_{0i}, \epsilon'_j) = \frac{\text{Cov}(\bar{y}_{i(HH)}, \bar{x}_j)}{\bar{y}_i \bar{x}_j} = \lambda \frac{S_{y_i x_j}}{\bar{y}_i \bar{x}_j}, \quad E(\epsilon'_j, \epsilon'_j) = \frac{\text{Cov}(\bar{x}_j, \bar{x}_j')}{\bar{x}_j \bar{x}_j'} = \lambda \frac{S_{x_j x_j'}}{\bar{x}_j \bar{x}_j'},$$

$$E(\epsilon_j, \epsilon_j') = \frac{\text{Cov}(\bar{x}_{j(HH)}, \bar{x}_j')}{\bar{x}_j \bar{x}_j'} = \frac{1}{\bar{x}_j \bar{x}_j'} \left[ \lambda S_{x_j x_j'} + \lambda_\delta S_{x_j x_j'(2)} \right].$$

(8)

The contribution of the terms involving the powers in  $\epsilon_{0i}$ ,  $\epsilon_j$  and  $\epsilon'_j$  of order higher than two in the bias and mean square error is assumed to be negligible.

### 3. Bias and mean square error (MSE) of $t_p$ and $t_p^*$

Expanding the functions  $g(\varphi, \mathbf{u}')$  about the point  $(P, \mathbf{e}')$  using Taylor's series up to the second order partial derivative, we get

$$t_p = g(P, \mathbf{e}') + (\varphi - P)g_1(P, \mathbf{e}') + (\mathbf{u} - \mathbf{e})'g_2(P, \mathbf{e}') + \frac{1}{2} [(\varphi - P)^2g_{11}(\varphi^*, \mathbf{u}^*) + 2(\varphi - P)(\mathbf{u} - \mathbf{e})'g_{12}(\varphi^*, \mathbf{u}^*) + (\mathbf{u} - \mathbf{e})'g_{22}(\varphi^*, \mathbf{u}^*)(\varphi - P)]$$

Using condition given in equation (5), we get

$$t_p = P + (\varphi - P) + (\mathbf{u} - \mathbf{e})'g_2(P, \mathbf{e}') + \frac{1}{2} [(\varphi - P)^2g_{11}(\varphi^*, \mathbf{u}^*) + 2(\varphi - P)(\mathbf{u} - \mathbf{e})'g_{12}(\varphi^*, \mathbf{u}^*) + (\mathbf{u} - \mathbf{e})'g_{22}(\varphi^*, \mathbf{u}^*)(\mathbf{u} - \mathbf{e})].$$

Similarly, expanding the  $h(\varphi, \boldsymbol{\omega}')$  about the point  $(P, \mathbf{e}')$  and using equation (7), we get

$$t_p^* = P + (\varphi - P) + (\boldsymbol{\omega} - \mathbf{e})'h_2(P, \mathbf{e}') + \frac{1}{2} \left[ (\varphi - P)^2h_{11}(\varphi, \boldsymbol{\omega}') + 2(\varphi - P)(\boldsymbol{\omega} - \mathbf{e})'h_{12}(\varphi^*, \boldsymbol{\omega}^*) + (\boldsymbol{\omega} - \mathbf{e})'h_{22}(\varphi^*, \boldsymbol{\omega}^*)(\boldsymbol{\omega} - \mathbf{e}) \right] \tag{9}$$

The expressions for bias and mean square error of  $t_p$  and  $t_p^*$  for any sampling design up to the terms of order  $n^{-1}$  are given by

$$Bias(t_p) = Bias(\varphi) + E(\varphi - P)(\mathbf{u} - \mathbf{e})'g_2(\varphi^*, \mathbf{u}^*) + \frac{1}{2}E(\mathbf{u} - \mathbf{e})'g_{22}(\varphi^*, \mathbf{u}^*)(\mathbf{u} - \mathbf{e}), \tag{10}$$

$$MSE(t_p) = MSE(\varphi) + 2E(\varphi - P)(\mathbf{u} - \mathbf{e})'g_2(P, \mathbf{e}') + E(g_2(P, \mathbf{e}'))'(\mathbf{u} - \mathbf{e})(\mathbf{u} - \mathbf{e})'g_2(P, \mathbf{e}') \tag{11}$$

$$Bias(t_p^*) = Bias(\varphi) + E(\varphi - P)(\boldsymbol{\omega} - \mathbf{e})'h_{12}(\varphi^*, \boldsymbol{\omega}^*) + \frac{1}{2}E(\boldsymbol{\omega} - \mathbf{e})'h_{22}(\varphi^*, \boldsymbol{\omega}^*)(\boldsymbol{\omega} - \mathbf{e}), \tag{12}$$

$$MSE(t_p^*) = MSE(\varphi) + 2E(\varphi - P)(\boldsymbol{\omega} - \mathbf{e})'h_2(P, \mathbf{e}') + E(h_2(P, \mathbf{e}'))'(\boldsymbol{\omega} - \mathbf{e})(\boldsymbol{\omega} - \mathbf{e})'h_2(P, \mathbf{e}'), \tag{13}$$

where  $\varphi^* = P + \theta_p(\varphi - P)$ ,  $\mathbf{u}^* = \mathbf{e} + \phi_1(\mathbf{u} - \mathbf{e})$  and  $\boldsymbol{\omega}^* = \mathbf{e} + \phi_2(\boldsymbol{\omega} - \mathbf{e})$ , such that  $0 < \theta_p, \phi_{1j}, \phi_{2j} < 1$  and  $\phi_1$  and  $\phi_2$  are  $(p \times p)$  diagonal matrix with  $j^{th}$  diagonal elements  $\phi_{1j}$  and  $\phi_{2j}$  respectively.

Differentiating equations (11) and (13) partially with respect to  $g_2(P, \mathbf{e}')$  and  $h_2(P, \mathbf{e}')$  respectively and equating them to zero, we get the conditions for the minimum value of the mean square error of  $t_p$  and  $t_p^*$

$$g_2(P, \mathbf{e}') = -[E(\varphi - P)(\mathbf{u} - \mathbf{e})' / E(\mathbf{u} - \mathbf{e})(\mathbf{u} - \mathbf{e})'] \tag{14}$$

and  $h_2(P, \mathbf{e}') = -[E(\varphi - P)(\boldsymbol{\omega} - \mathbf{e})' / E(\boldsymbol{\omega} - \mathbf{e})(\boldsymbol{\omega} - \mathbf{e})'] \tag{15}$

respectively.

Now, putting the value of  $g_2(P, \mathbf{e}')$  from equation (14) to (11) and  $h_2(P, \mathbf{e}')$  from equation (15) to (13), the minimum value of the mean square error  $t_p$  and  $t_p^*$  will be given by

$$MSE(t_p) \Big|_{\min} = MSE(\varphi) - E(\varphi - P)(\mathbf{u} - \mathbf{e})' [E(\mathbf{u} - \mathbf{e})(\mathbf{u} - \mathbf{e})']^{-1} E(\varphi - P)(\mathbf{u} - \mathbf{e}) \quad (16)$$

$$MSE(t_p^*) \Big|_{\min} = MSE(\varphi) - E(\varphi - P)(\boldsymbol{\omega} - \mathbf{e})' [E(\boldsymbol{\omega} - \mathbf{e})(\boldsymbol{\omega} - \mathbf{e})']^{-1} E(\varphi - P)(\boldsymbol{\omega} - \mathbf{e}) \quad (17)$$

Considering SRSWOR method of sampling, let us define two  $p \times p$  positive definite matrices  $\mathcal{A} = [a_{jj}']$  and  $\mathcal{A}_0 = [a_{0jj'}(2)]$  such that

$$a_{jj'} = \lambda a_{0jj'} + \lambda_\delta a_{0jj'}(2) \quad \forall j \neq j' = 1, 2, \dots, p$$

where  $a_{0jj'} = \rho_{x_j x_{j'}} C_{x_j} C_{x_{j'}}$ ,  $a_{0jj'}(2) = \rho_{x_j x_{j'}(2)} C_{x_{j(2)}} C_{x_{j'(2)}}$ ,  $C_{x_j}^2 = S_{x_j}^2 / \bar{X}_i^2$ ,  
 $C_{x_{j(2)}} = S_{x_{j(2)}}^2 / \bar{X}_i^2$

$\rho_{x_j x_{j'}}$  - correlation coefficient between  $x_j$  and  $x_{j'}$  for entire population,

$\rho_{x_j x_{j'}(2)}$  - correlation coefficient between  $x_j$  and  $x_{j'}$  for non-responding group of population.

Then the expressions for bias and mean square error of  $t_p$  and  $t_p^*$  up to the terms of order  $(n^{-1})$  in the case of SRSWOR method of sampling are given by

$$Bias(t_p) = Bias(\varphi) + P(\lambda \boldsymbol{\mathfrak{b}} + \lambda_\delta \boldsymbol{\mathfrak{b}}_{(2)})' g_{12}(\varphi^*, \mathbf{u}^*) + \frac{1}{2} trace \mathcal{A} g_{22}(\varphi^*, \mathbf{u}^*), \quad (18)$$

$$MSE(t_p) = MSE(\varphi) + (g_2(P, \mathbf{e}'))' \mathcal{A} g_2(P, \mathbf{e}') + 2P(\lambda \boldsymbol{\mathfrak{b}} + \lambda_\delta \boldsymbol{\mathfrak{b}}_{(2)})' g_2(P, \mathbf{e}'), \quad (19)$$

$$Bias(t_p^*) = Bias(\varphi) + \lambda \left\{ P \boldsymbol{\mathfrak{b}}' h_{12}(\varphi^*, \boldsymbol{\omega}^*) + \frac{1}{2} trace \mathcal{A}_0 h_{22}(\varphi^*, \boldsymbol{\omega}^*) \right\}, \quad (20)$$

$$\text{and } MSE(t_p^*) = MSE(\varphi) + \lambda \left\{ (h_2(P, \mathbf{e}'))' \mathcal{A}_0 h_2(P, \mathbf{e}') + 2P \boldsymbol{\mathfrak{b}}' h_2(P, \mathbf{e}') \right\} \quad (21)$$

where  $\boldsymbol{\mathfrak{b}} = (\mathfrak{b}_1, \mathfrak{b}_2, \dots, \mathfrak{b}_p)'$  and  $\boldsymbol{\mathfrak{b}}_{(2)} = (\mathfrak{b}_{1(2)}, \mathfrak{b}_{2(2)}, \dots, \mathfrak{b}_{p(2)})'$  are two column vectors such that

$$\mathfrak{b}_j = \frac{S_{x_j}}{\bar{X}_j} \left\{ \rho_{y_1 x_j} \frac{S_{y_1}}{\bar{Y}_1} + \rho_{y_2 x_j} \frac{S_{y_2}}{\bar{Y}_2} \right\} \text{ and } \mathfrak{b}_{j(2)} = \frac{S_{x_{j(2)}}}{\bar{X}_j} \left\{ \rho_{y_1 x_{j(2)}} \frac{S_{y_1(2)}}{\bar{Y}_1} + \rho_{y_2 x_{j(2)}} \frac{S_{y_2(2)}}{\bar{Y}_2} \right\},$$

$$Bias(\varphi) = P \left\{ \lambda \rho_{y_1 y_2} \frac{S_{y_1}}{\bar{Y}_1} \frac{S_{y_2}}{\bar{Y}_2} + \lambda_\delta \rho_{y_1 y_2(2)} \frac{S_{y_1(2)}}{\bar{Y}_1} \frac{S_{y_2(2)}}{\bar{Y}_2} \right\}, \quad (22)$$

$$MSE(\varphi) = P^2 \left\{ \lambda \left( \frac{S_{y_1}^2}{\bar{Y}_1^2} + \frac{S_{y_2}^2}{\bar{Y}_2^2} + 2\rho_{y_1 y_2} \frac{S_{y_1} S_{y_2}}{\bar{Y}_1 \bar{Y}_2} \right) + \lambda_\delta \left( \frac{S_{y_1(2)}^2}{\bar{Y}_1^2} + \frac{S_{y_2(2)}^2}{\bar{Y}_2^2} + 2\rho_{y_1 y_2(2)} \frac{S_{y_1(2)} S_{y_2(2)}}{\bar{Y}_1 \bar{Y}_2} \right) \right\}, \tag{23}$$

- $\rho_{y_1 y_2}$  - correlation coefficient between  $y_1$  and  $y_2$  for the entire population,
- $\rho_{y_1 x_j}$  - correlation coefficient between  $y_1$  and  $x_j$  for the entire population,
- $\rho_{y_2 x_j}$  - correlation coefficient between  $y_2$  and  $x_j$  for the entire population,
- $\rho_{y_1 y_2(2)}$  - correlation coefficient between  $y_1$  and  $y_2$  for the non-responding group of population,
- $\rho_{y_1 x_j(2)}$  - correlation coefficient between  $y_1$  and  $x_j$  for the non-responding group of population,
- $\rho_{y_2 x_j(2)}$  - correlation coefficient between  $y_2$  and  $x_j$  for the non-responding group of population.

The conditions for which  $MSE(t_p)$  and  $MSE(t_p^*)$  will attain the minimum values are given by

$$g_2(P, e') = -P(\lambda \mathbf{b} + \lambda_\delta \mathbf{b}_{(2)}) \mathbf{A}^{-1} \tag{24}$$

and 
$$h_2(P, e') = -P \mathbf{b} \mathbf{A}^{-1} \tag{25}$$

respectively. And, the values of the minimum mean square error for  $t_p$  and  $t_p^*$  are given by

$$MSE(t_p) \Big|_{min} = MSE(\varphi) - P^2 \left\{ (\lambda \mathbf{b} + \lambda_\delta \mathbf{b}_{(2)})' \cdot \mathbf{A}^{-1} \cdot (\lambda \mathbf{b} + \lambda_\delta \mathbf{b}_{(2)}) \right\} \tag{26}$$

and 
$$MSE(t_p^*) = MSE(\varphi) - P^2 \lambda \mathbf{b}' \mathbf{A}_0^{-1} \mathbf{b}. \tag{27}$$

#### 4. Extension of the proposed class of estimator to the case when population means of the auxiliary characters are unknown

In the case when the population means of the auxiliary characters  $\bar{X}_j$  ( $j = 1, 2, \dots, p$ ) are unknown but sampling frame is available, we use two phase sampling technique to estimate the unknown population means of the auxiliary characters  $x_1, x_2, \dots, x_p$ . In two phase sampling scheme, we first select a larger sample of size  $n'$  from  $N$  using SRSWOR method of sampling and collect information regarding the auxiliary characters and estimate  $\bar{X}_j$  ( $j = 1, 2, \dots, p$ ) based on  $n'$  units by  $\bar{x}'_j$  ( $j = 1, 2, \dots, p$ ). Again, a second phase sample of size  $n$  ( $n < n'$ ) is drawn from  $n'$  units by SRSWOR method of sampling and observe the study characters  $y_i$  ( $i = 1, 2$ ). For the study characters  $y_i$  ( $i = 1, 2$ ), we observe that only  $n_1$  units are responding and  $n_2$  units are not responding in the sample of size  $n$ . Now, to reduce the effect of non-response, the information is collected by the direct interview on the sub-sampled units of size  $m$  ( $= n_2 \delta^{-1}$ ,  $\delta > 1$ ) for  $y_i$  ( $i = 1, 2$ ) and following Hansen and Hurwitz (1946), the unbiased estimator



$\bar{y}_{i(HH)}$  [given in section 2 equation (1)] is considered for  $\bar{Y}_i$  ( $i = 1, 2$ ) based on the information of  $(n_1 + m)$  units.

When the population means  $\bar{X}_1, \bar{X}_2, \dots, \bar{X}_p$  are unknown but estimated by  $\bar{x}'_1, \bar{x}'_2, \dots, \bar{x}'_p$ , which is based on larger first phase sample of size  $n'$  and we have incomplete information on  $y_i$  ( $i = 1, 2$ ) but complete information on  $x_j$  ( $j = 1, 2, \dots, p$ ) for the sample of size  $n$  ( $n < n'$ ), then we propose a class of estimators  $t_p^{**}$  for  $P$  which is given by

$$t_p^{**} = f(\prod_{i=1}^2 \bar{y}_{i(HH)}, (z_1, z_2, \dots, z_p)') = f(\varphi, \mathbf{z}'), \quad (28)$$

such that

$$f(P, \mathbf{e}') = P, \quad f_1(P, \mathbf{e}') = \left( \frac{\partial}{\partial \varphi} f(\varphi, \mathbf{z}') \right)_{(P, \mathbf{e}')} = 1 \quad (29)$$

where  $\mathbf{z}$  denotes the column vectors  $(z_1, z_2, \dots, z_p)'$  and  $z_j = \bar{x}_j / \bar{x}'_j$ ;  $j = 1, 2, \dots, p$ .

The function  $f(\varphi, \mathbf{z}')$  satisfies all the necessary regularity conditions similar to those given for the functions  $g(\varphi, \mathbf{u}')$  and  $h(\varphi, \boldsymbol{\omega}')$ .

Now, expanding the function  $f(\varphi, \mathbf{z}')$  about the point  $(P, \mathbf{e}')$  by using Taylor's series up to the second order derivatives, the expressions for bias and mean square error of the estimator  $t_p^{**}$  for any sampling design up to the terms of order  $(n^{-1})$  are given by

$$\text{Bias}(t_p^{**}) = \text{Bias}(\varphi) + E(\varphi - P)(\mathbf{z} - \mathbf{e}')' f_{12}(\varphi^*, \mathbf{z}^{*'}) + \frac{1}{2} E(\mathbf{z} - \mathbf{e}')' h f_{22}(\varphi^*, \boldsymbol{\omega}^{*'}) (\boldsymbol{\omega} - \mathbf{e}), \quad (30)$$

and

$$\text{MSE}(t_p^{**}) = \text{MSE}(\varphi) + 2E(\varphi - P)(\mathbf{z} - \mathbf{e}')' f_2(P, \mathbf{e}') + E(f_2(P, \mathbf{e}'))' (\mathbf{z} - \mathbf{e})(\mathbf{z} - \mathbf{e}')' f_2(P, \mathbf{e}'), \quad (31)$$

where  $\mathbf{z}^* = \mathbf{e} + \phi_3(\mathbf{u} - \mathbf{e})$ ;  $0 < \phi_{3j} < 1$  and  $\phi_3$  is  $(p \times p)$  diagonal matrix having diagonal elements  $\phi_{3j}$  ( $j = 1, 2, \dots, p$ ).

Here,  $f_1(\varphi, \mathbf{z}')$  and  $f_2(\varphi, \mathbf{z}')$  denote the first partial derivatives of  $f(\varphi, \mathbf{z}')$  with respect to  $\varphi$  and  $\mathbf{z}'$  respectively. The second partial derivative of  $f(\varphi, \mathbf{z}')$  with respect to  $\mathbf{z}'$  is denoted by  $f_{22}(\varphi, \mathbf{z}')$  and the first partial derivative of  $f_2(\varphi, \mathbf{z}')$  with respect to  $\varphi$  is denoted by  $f_{12}(\varphi, \mathbf{z}')$ .

The estimator  $t_p^{**}$  will attain the minimum value of the mean square error for

$$f_2(P, \mathbf{e}') = -[E(\varphi - P)(\mathbf{z} - \mathbf{e}')' / E(\mathbf{z} - \mathbf{e})(\mathbf{z} - \mathbf{e}')'] \quad (32)$$

and, the minimum value of mean square error of  $t_p^{**}$  is given by

$$\text{MSE}(t_p^{**})|_{\min} = \text{MSE}(\varphi) - E(\varphi - P)(\mathbf{z} - \mathbf{e}')' [E(\mathbf{z} - \mathbf{e})(\mathbf{z} - \mathbf{e}')']^{-1} E(\varphi - P)(\mathbf{z} - \mathbf{e}). \quad (33)$$

To obtain the expressions of bias and the mean square error of  $t_p^{**}$  under SRSWOR method of sampling, we assume  $\epsilon_j'' = \frac{\bar{x}'_j - \bar{X}_j}{\bar{X}_j}$ , such that  $E(\epsilon_j'') = 0, |\epsilon_j''| < 1$  and, therefore, we have

$$E(\epsilon_j''^2) = \frac{V(\bar{x}'_j)}{\bar{X}_j^2} = \lambda' \frac{S_{\bar{x}'_j}^2}{\bar{X}_j^2}, \quad E(\epsilon_j'', \epsilon_{j'}'') = \frac{Cov(\bar{x}'_j, \bar{x}'_{j'})}{\bar{X}_j \bar{X}_{j'}} = \frac{1}{\bar{X}_j \bar{X}_{j'}} [\lambda' S_{x_j x_{j'}}]; \quad j \neq j' = 1, 2, \dots, p$$

$$E(\epsilon_{0i}, \epsilon_j'') = \frac{Cov(\bar{y}_{i(HH)}, \bar{x}'_j)}{\bar{Y}_i \bar{X}_j} = \lambda' \frac{S_{y_i x_j}}{\bar{Y}_i \bar{X}_j}, \quad E(\epsilon_j', \epsilon_j'') = \frac{Cov(\bar{x}_j, \bar{x}'_j)}{\bar{X}_j^2} = \lambda' \frac{S_{\bar{x}_j}^2}{\bar{X}_j^2},$$

where  $\lambda' = \frac{N-n'}{N n'}$ .

Now, the expressions for bias and the mean square error of  $t_p^{**}$  up to the order  $n^{-1}$  under SRSWOR method of sampling are given by

$$Bias(t_p^{**}) = Bias(\varphi) + \lambda' \left\{ P \mathcal{B}' f_{12}(\varphi^*, \mathbf{z}^{*'}) + \frac{1}{2} trace \mathcal{A}_0 f_{22}(\varphi^*, \mathbf{z}^{*'}) \right\} \tag{34}$$

$$and \quad MSE(t_p^{**}) = MSE(\varphi) + \lambda' \left\{ (f_2(P, \mathbf{e}'))' \mathcal{A}_0 f_2(P, \mathbf{e}') + 2 P \mathcal{B}' f_2(P, \mathbf{e}') \right\} \tag{35}$$

The conditions for which  $MSE(t_p^{**})$  will attain the minimum value are given by

$$f_2(P, \mathbf{e}') = P \mathcal{A}_0^{-1} P \tag{36}$$

and the minimum mean square error of  $t_p^{**}$  is given by

$$MSE(t_p^{**}) = MSE(\varphi) - P^2 \lambda' \mathcal{B}' \mathcal{A}_0^{-1} \mathcal{B}. \tag{37}$$

### 5. Concluding remarks

- i) The proposed classes of estimators  $t_p, t_p^*$  and  $t_p^{**}$  have a wider class of estimators. Following the strategies of Raj (1965), Singh (1967), Abu-Dayyeh *et al.* (2003), Kadilar and Cingi (2005), Perri (2005) and many more, a large number of estimators may be formed, some of them for  $t_p, t_p^*$  and  $t_p^{**}$  are given in Table 1.

**Table 1.** Members of classes of estimators  $t_p, t_p^*$  and  $t_p^{**}$

$t_p$	$t_p^*$	$t_p^{**}$
$T_{P1} = \varphi \prod_{j=1}^p u_j^{\theta_{1j}}$	$T_{P1}^* = \varphi \prod_{j=1}^p \omega_j^{\theta_{1j}^*}$	$T_{P1}^{**} = \varphi \prod_{j=1}^p z_j^{\theta_{1j}^{**}}$
$T_{P2} = \varphi \prod_{j=1}^p u_j^{\theta_{2j}} + \sum_{j=1}^p c_j (\bar{X}_j - \bar{x}_{j(HH)})$	$T_{P2}^* = \varphi \prod_{j=1}^p \omega_j^{\theta_{2j}^*} + \sum_{j=1}^p c_j^* (\bar{X}_j - \bar{x}_{j(HH)})$	$T_{P2}^{**} = \varphi \prod_{j=1}^p z_j^{\theta_{2j}^{**}} + \sum_{j=1}^p c_j^{**} (\bar{X}_j - \bar{x}_{j(HH)})$
$T_{P3} = \varphi \sum_{j=1}^p w_j u_j^{\theta_{3j}/w_j}, \quad \sum_{j=1}^p w_j = 1$	$T_{P3}^* = \varphi \sum_{j=1}^p w_j \omega_j^{\theta_{3j}/w_j}, \quad \sum_{j=1}^p w_j = 1$	$T_{P3}^{**} = \varphi \sum_{j=1}^p w_j z_j^{\theta_{3j}/w_j}, \quad \sum_{j=1}^p w_j = 1$

Since the estimators  $(T_{P1}, T_{P2}, T_{P3}), (T_{P1}^*, T_{P2}^*, T_{P3}^*)$  and  $(T_{P1}^{**}, T_{P2}^{**}, T_{P3}^{**})$  are the members of  $t_p, t_p^*$  and  $t_p^{**}$  and they satisfy accordingly the conditions (5), (7) and

(29), the values of constants involved in  $(T_{P_1}, T_{P_2}, T_{P_3})$ ,  $(T_{P_1}^*, T_{P_2}^*, T_{P_3}^*)$  and  $(T_{P_1}^{**}, T_{P_2}^{**}, T_{P_3}^{**})$  can be calculated by the equations (14), (15) and (32) respectively. In the case when the values of parameters in the optimum value of the constants are not known one may estimate it on the basis of the sample values or may use past data. Srivastava and Jhajj (1983) shown that such values do not affect the mean square error of the estimator up to the terms of order  $n^{-1}$  while Reddy (1978) shown that such values are stable over time and region. So the proposed class of two phase sampling estimator is preferred in large scale sample survey.

ii) On comparing the estimators  $t_p$ ,  $t_p^*$  and  $t_p^{**}$  with  $\varphi$  in terms of precision, we find

a)  $MSE(t_p) < MSE(\varphi)$  if

$$MSE(\varphi) < (g_2(P, e'))' \mathcal{A} g_2(P, e') + 2 P(\lambda \mathcal{B} + \lambda_\delta \mathcal{B}_{(2)})' g_2(P, e') < 0$$

b)  $MSE(t_p^*) < MSE(\varphi)$  if

$$MSE(\varphi) < \lambda \{ (h_2(P, e'))' \mathcal{A}_0 h_2(P, e') + 2 P \mathcal{B}' h_2(P, e') \} < 0$$

c)  $MSE(t_p^{**}) < MSE(\varphi)$  if

$$MSE(\varphi) < \lambda' \{ (f_2(P, e'))' \mathcal{A}_0 f_2(P, e') + 2 P \mathcal{B}' f_2(P, e') \} < 0.$$

iii) When there is complete information on the study characters  $y_i (i = 1, 2)$  and the auxiliary character and  $x_j (j = 1, 2, \dots, p)$ , i.e.  $W_2 = 0$ , then we find that the estimators  $t_p$  and  $t_p^*$  are equally efficient for the class of estimators proposed by Khare (1992) for  $P$  using known population means of auxiliary characters. Similarly, the estimator  $t_p^{**}$  is also equally efficient to for class of two phase sampling estimators proposed by Khare (1992) for unknown population means of auxiliary characters.

iv) Due to the involvement of various parameters, in the mean square error of  $t_p$  and  $t_p^*$ , it is very difficult to find the condition for superiority of  $t_p$  over  $t_p^*$ . However, in the case of one auxiliary character, it has been obtained that relative efficiency of  $t_p$  with respect to  $t_p^*$  increases by increasing the values of  $\frac{\rho_{y_1 x_1(2)}}{\rho_{y_1 x_1}}$  and  $\frac{\rho_{y_2 x_1(2)}}{\rho_{y_2 x_1}}$  and decreasing the value of  $\frac{\rho_{y_2 x_1}}{\rho_{y_1 x_1}}$ . Hence, one may use the estimators  $t_p$  and  $t_p^*$  depending upon the values of  $\frac{\rho_{y_1 x_j(2)}}{\rho_{y_1 x_j}}$ ,  $\frac{\rho_{y_2 x_j(2)}}{\rho_{y_2 x_j}}$  and  $\frac{\rho_{y_2 x_j}}{\rho_{y_1 x_j}}$  for all  $j = 1, 2, \dots, p$ .

## 6. An empirical study

**Source:** Police-station – Baria, Tahasil – Champua, District Census Handbook-1981, Orissa, Govt. of India. Total number of villages 109, 25% villages (i.e. 27 villages) from the bottom are considered as the non-responding group of the population. In this data set, study characters and auxiliary characters are as follows:

$y_1$ -Number of occupied residential houses in the village,

$y_2$ -Average number of persons in the house in the village,

- $x_1$ -Number of cultivators in the village,
- $x_2$ -Area (in hectares) of the village,
- $x_3$ -Number of main workers in the village.

The values of the parameters of the population under study are as follows :

$\bar{Y}_1= 88.3670$	$\bar{Y}_2= 5.5832$	$\bar{X}_1= 100.5505$	$\bar{X}_2= 256.3331$	$\bar{X}_3= 165.2661$	
$S_{y_1}= 59.3208$	$S_{y_2}= 0.6024$	$C_{x_1}= 0.7314$	$C_{x_2}= 0.6105$	$C_{x_3}= 0.6828$	
$S_{y_1(2)}= 45.2704$	$S_{y_2(2)}= 0.5025$	$C_{x_1(2)}= 0.5678$	$C_{x_2(2)}= 0.4944$	$C_{x_3(2)}= 0.5769$	
$\rho_{y_1x_1}= 0.795$	$\rho_{y_1x_2}= 0.854$	$\rho_{y_1x_3}= 0.907$	$\rho_{y_2x_1}= -0.084$	$\rho_{y_2x_2}= -0.117$	$\rho_{y_2x_3}= -0.136$
$\rho_{y_1x_1(2)}= 0.658$	$\rho_{y_1x_2(2)}= 0.759$	$\rho_{y_1x_3(2)}= 0.891$	$\rho_{y_2x_1(2)}= 0.092$	$\rho_{y_2x_2(2)}= 0.199$	$\rho_{y_2x_3(2)}= 0.109$
$\rho_{x_1x_2}= 0.715$	$\rho_{x_1x_3}= 0.841$	$\rho_{x_2x_3}= 0.796$	$\rho_{x_1x_2(2)}= 0.541$	$\rho_{x_1x_3(2)}= 0.785$	$\rho_{x_2x_3(2)}= 0.657$
	$\rho_{y_1y_2}= -0.194$	$\rho_{y_1y_2(2)}= 0.023$			

To compare the efficiency of the proposed classes of estimators  $t_p$ ,  $t_p^*$  and  $t_p^{**}$  with respect to the conventional estimator  $\varphi [= \prod_{i=1}^2 \bar{y}_{i(HH)}]$  through an empirical study based on real data set, their respective members  $T_{P_1} = \varphi \prod_{j=1}^p u_j^{\theta_{1j}}$ ,  $T_{P_1}^* = \varphi \prod_{j=1}^p \omega_j^{\theta_{1j}^*}$  and  $T_{P_1}^{**} = \varphi \prod_{j=1}^p z_j^{\theta_{1j}^{**}}$  are considered.

The mean square error (*MSE*) of  $T_{P_1}$  and  $T_{P_1}^*$  along with their optimum value of constants (*OV*C) and their percentage relative efficiency (*PRE*) with respect to  $\varphi$  for different values of sub-sampling fraction ( $1/\delta$ ) are shown in Table 2, while the *MSE*( $T_{P_1}^{**}$ ) and *PRE*( $T_{P_1}^{**}$ ) with respect to  $\varphi$  in the case of fixed sample sizes, i.e.  $n' = 70$  and  $n = 40$  for different values of sub-sampling fraction ( $1/\delta$ ) are given in Table 3.

## 7. Discussion and conclusion

From Table 2, it has been observed that the estimators  $T_{P_1}$  and  $T_{P_1}^*$  are more efficient than  $\varphi$  [i.e.  $\hat{P}$  ] for the different values of the sub-sampling fraction. We also observe that the mean square error of  $T_{P_1}$  and  $T_{P_1}^*$  decreases while the relative efficiency of  $T_{P_1}$  and  $T_{P_1}^*$  with respect to  $\varphi$  increases as the number of auxiliary characters and sub-sampling fraction increase. From the Table 2, it has also been observed that the estimator  $T_{P_1}$  is more efficient than  $T_{P_1}^*$  and the efficiency is increasing with the increase in the number of auxiliary characters and sub-sampling fraction.

From Table 3, we observe that the estimator  $T_{P_1}^{**}$  is more efficient than  $\varphi$  for the different values of the sub-sampling fraction ( $1/\delta$ ). The relative efficiency of  $T_{P_1}^{**}$  with respect to  $\varphi$  increases while *MSE*( $T_{P_1}^{**}$ ) decreases as the number of auxiliary characters and sub-sampling fractions increase. Hence, we conclude that the efficiency of the estimators  $T_{P_1}$ ,  $T_{P_1}^*$  and  $T_{P_1}^{**}$  with respect to  $\varphi$  can be increased by increasing the number of the auxiliary characters as well by increasing the values of the sub-sampling fractions.

**Table 2.** Percentage relative efficiency [*PRE* (-)] with respect to  $\varphi$  at different values of  $\delta$

Estimators	Auxiliary character(s)	$N = 109, n = 40$		
		$1/\delta$		
		1/4	1/3	1/2
$\varphi$	-	100.00 (2905.7121)*	100.00 (2494.6591)	100.00 (2083.6062)
$T_{P1}$	$x_1$	220.05 (1320.4901)	229.52 (1086.8873)	244.43 (852.4396)
	<i>OVC</i>	$\theta_{11} = -0.6702$	$\theta_{11} = -0.6802$	$\theta_{11} = -0.6941$
	$x_1, x_2$	411.70 (705.7874)	428.12 (582.7021)	453.38 (459.5749)
	<i>OVC</i>	$\theta_{11} = -0.3343,$ $\theta_{12} = -0.6326$	$\theta_{11} = -0.3369,$ $\theta_{12} = -0.6311$	$\theta_{11} = -0.3408,$ $\theta_{12} = -0.6286$
	$x_1, x_2, x_3$	691.16 (420.4097)	701.38 (355.6792)	717.60 (290.3568)
	<i>OVC</i>	$\theta_{11} = -0.015, \theta_{12} =$ $-0.382, \theta_{13} = -0.575$	$\theta_{11} = -0.028, \theta_{12} =$ $-0.385, \theta_{13} = -0.565$	$\theta_{11} = -0.048, \theta_{12} =$ $-0.388, \theta_{13} = -0.549$
$T_{P1}^*$	$x_1$	157.09 (1849.7092)	173.40 (1438.6562)	202.76 (1027.6033)
	<i>OVC</i>	$\theta_{11}^* = 0.714$	$\theta_{11}^* = 0.714$	$\theta_{11}^* = 0.714$
	$x_1, x_2$	185.13 (1569.5338)	215.34 (1158.4808)	278.77 (747.4279)
	<i>OVC</i>	$\theta_{11}^* = -0.348,$ $\theta_{12}^* = -0.347$	$\theta_{11}^* = -0.348,$ $\theta_{12}^* = -0.347$	$\theta_{11}^* = -0.348,$ $\theta_{12}^* = -0.347$
	$x_1, x_2, x_3$	199.42 (1457.1192)	238.48 (1046.0662)	328.12 (635.0133)
	<i>OVC</i>	$\theta_{11}^* = -0.079, \theta_{12}^* =$ $-0.391, \theta_{13}^* = -0.523$	$\theta_{11}^* = -0.079, \theta_{12}^* =$ $-0.391, \theta_{13}^* = -0.523$	$\theta_{11}^* = -0.079, \theta_{12}^* =$ $-0.391, \theta_{13}^* = -0.523$

\*Mean square error of the estimators (·) is shown in the parenthesis.

**Table 3.** Percentage relative efficiency [*PRE* ( $T_{P1}^{**}$ )] with respect to  $\varphi$  at different values of  $\delta$  for fixed  $n'$  and  $n$

Estimators	Auxiliary character(s)	$n' = 70, n = 40$		
		$1/\delta$		
		1/4	1/3	1/2
$\varphi$	-	100.00 (2905.7121)*	100.00 (2494.6591)	100.00 (2083.6062)
$T_{P1}^{**}$	$x_1$	132.74 (2188.9752)	140.31 (1777.9222)	152.44 (1366.8693)
	<i>OVC</i>	$\theta_{11}^{**} = -0.717$	$\theta_{11}^{**} = -0.717$	$\theta_{11}^{**} = -0.717$
	$x_1, x_2$	144.89 (2005.4099)	156.47 (1594.3569)	176.08 (1183.3040)
$T_{P1}^{**}$	<i>OVC</i>	$\theta_{11}^{**} = -0.346,$ $\theta_{12}^{**} = -0.622$	$\theta_{11}^{**} = -0.346,$ $\theta_{12}^{**} = -0.622$	$\theta_{11}^{**} = -0.346,$ $\theta_{12}^{**} = -0.622$
	$x_1, x_2, x_3$	150.58 (1929.7012)	164.27 (1518.6482)	188.12 (1107.5953)
	<i>OVC</i>	$\theta_{11}^{**} = -0.067, \theta_{12}^{**} =$ $-0.382, \theta_{13}^{**} = -0.538$	$\theta_{11}^{**} = -0.067, \theta_{12}^{**} =$ $-0.382, \theta_{13}^{**} = -0.538$	$\theta_{11}^{**} = -0.067, \theta_{12}^{**} =$ $-0.382, \theta_{13}^{**} = -0.538$

\*Mean square error of the estimators is shown in the parenthesis.

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