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# BANK OF FINLAND DISCUSSION PAPERS

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1 • 2001

Risto Herrala  
Research Department  
15.2.2001

## An assessment of alternative lender of last resort schemes

Suomen Pankin keskustelualoitteita  
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The views expressed are those of the author and do not necessarily reflect the views of the Bank of Finland.

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# An assessment of alternative lender of last resort schemes

Bank of Finland Discussion Papers 1/2001

Risto Herrala  
Research Department

## Abstract

We sketch a theoretical framework for comparing the properties of funded LOLR schemes. We construct an idealized lender of last resort and investigate how it formulates policy under alternative public and private governance structures. The alternatives are a (first-best) social utility maximizer that can dictate participation, and three voluntary schemes: a public lender of last resort, a mutual clearing house that formulates policy by voting, and a profit maximizing private LOLR scheme. We compare the policies formulated by these institutions from the viewpoint of social desirability. Our model targets the debate on free banking, in particular the issue of whether private institutions would fare well as lenders of last resort.

In our model, the first-best LOLR scheme always covers the whole banking sector and offers full insurance to the participants. We find that voluntary schemes succeed relatively well as lenders of last resort in situations where recipients of LOLR assistance can repay LOLR loans with interest. In this case, the LOLR can use interest rate policy to make the scheme attractive to banks of every quality and thus create incentives for comprehensive entry. In private schemes, policy tends to be distorted if the private scheme is the only possible scheme. However, competitive forces lead private institutions to approach the first-best outcome, which is the only contestable outcome.

The end result changes when we investigate a situation in which banks' ability to repay LOLR loans is limited. When lending is associated with losses for the LOLR, good quality banks will tend to stay out of the LOLR scheme and participation in voluntary schemes will always fall short of the first-best outcome. A compulsory scheme (such as a central bank that can impose a reserve requirement on banks) has an advantage over voluntary schemes.

Key words: liquidity, lender of last resort, banking, central banking, governance

JEL classification numbers: E 58, G 21

# Vaihtoehtoisen hätärahoitusjärjestelyjen teoreettinen vertailu

## Suomen Pankin keskustelualoitteita 1/2001

Risto Herrala  
Tutkimusosasto

### Tiivistelmä

Työssä arvioidaan eri tavoin organisoituja hätärahoitusjärjestelyjä sosiaalisen kokonaishyödyn kannalta. Tätä tarkoitusta varten rakennetaan malli hätärahoittajasta, jonka päätöksentekoa tarkastellaan erilaisten hallintorakenteiden vallitessa. Tarkasteltavia hallintoratkaisuja ovat 1) julkinen hätärahoitusorganisaatio, johon osallistuminen on vapaaehtoista, 2) pankkien ylläpitämä selvitystalo, jonka poliitikasta päätetään enemmistöpäätöksin, sekä 3) voittonsa maksimoiva hätärahoittaja. Näitä verrataan yhteiskunnan kokonaishyötyä maksimoivaan ratkaisuun, jossa osallistuminen on pakollista. Hätärahoitusjärjestelyyn osallistuminen edellyttää kaikissa tapauksissa pankeilta ennakkotalletusta hätärahoittajana toimivaan instituutioon.

Mallin avulla voidaan keskustella klassisesta kysymyksestä, tarvitseeko pankkijärjestelmä keskuspankkia hätärahoittajakseen, vai voivatko yksityiset ratkaisut toimia yhtä hyvin.

Mallin mukaan yhteiskunnan kokonaishyöty maksimoituu, kun kaikille pankeille annetaan kattava suoja likviditeettisokkeja vastaan. Optimi vaatii, että kaikki pankit osallistuvat järjestelmään.

Jos hätärahoituksen saajat kykenevät normaaliin takaisinmaksuun, vapaaehtoisilla hätärahoitusjärjestelyillä päästään suurimpaan kokonaishyötyyn. Vapaaehtoisissa järjestelyissä voidaan korkopolitiikan avulla saavuttaa osanottajien kannalta neutraali tilanne niin, että pankin kannattaa liittyä hätärahoitusjärjestelyyn riippumatta siitä, kuinka suuri riski sillä on ajautua hätärahoitukseen. Yksityisten hätärahoittajien noudattamaa politiikkaa vääristää sekä enemmistöpäätösten että monopoliaseman käyttö. Kilpailu kuitenkin ajaa yksityisiä hätärahoittajia kohti optimaalista politiikkaa.

Jos hätärahoituksen saajat eivät kykene maksamaan saamaansa rahoitusta takaisin, vapaaehtoiset hätärahoitusjärjestelyt eivät kykene houkuttelemaan yhteiskunnan hyödyn kannalta riittävää osanottajajoukkoa. Näissä olosuhteissa sellaiset pankit, joiden riskit ovat pieniä, jäävät vapaaehtoisten järjestelyjen ulkopuolelle, vaikka niiden osallistuminen olisi tehokkuuden kannalta tarkoituksenmukaista. Tässä tilanteessa pakolliset järjestelmät tarjoavat parhaan suojan.

Asiasanat: hätärahoitus, pankki, keskuspankki, hallinto

JEL luokittelu: E 58, G 21

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# 1 Introduction

We sketch a theoretical framework for comparing the properties of funded LOLR schemes from the point of view of social desirability.<sup>1</sup> We use the model to study the issue of whether banks need a public institution like a central bank as a lender of last resort. Alternatively, private institutions<sup>2</sup> may take up that role and make public involvement unnecessary.

In the historical record, both public and private lenders of last resort have assisted troubled banks. Central banks have often been the ones to act in the role of a lender of last resort. Sometimes the treasury has taken that role. A much used example of a private lender of last resort is the U.S. clearing house system in the 1800's, the shortcomings of which led to its replacement by the Federal Reserve System.<sup>3</sup>

The classic figure on the debate on LOLR, Walter Bagehot, was a 'free banker': he favoured a system, where the banking sector would carry the responsibility for sufficient collection of reserves. Only because of the special rights and privileges granted by the government,<sup>4</sup> must the Bank of England carry the LOLR responsibility according to Bagehot. The idea of free banking is very much alive in the debate on central banking still today. While much of the debate on free banking concerns the right of note issue, also the LOLR theme is an important part of the vision of a free banking system. Lawrence White (1995), a prominent voice for free banking, assigns the lender of last resort role to private clearinghouses.

At the other end of the spectrum of the current debate, Charles Goodhart (1995) proposes that there is a genuine call for a public institution such as the central bank to function as a lender of last resort to banks. He argues that bank failures have detrimental effects to the functioning of the economy. LOLR operations and ordinary monetary policy operations are just two alternative means to the same end, which is stable economic growth.<sup>5</sup>

An important dimension in the debate is what kind of LOLR facility the discussants envision. Walter Bagehot formulated the classical criteria for LOLR-assistance in the 19<sup>th</sup> century in connection with the discussion on The Bank of England's role as a LOLR. According to Bagehot, the Bank of England should give assistance to all solvent banks in need of liquidity.<sup>6</sup> The loan should be collateralized and a penalty rate should be applied. Charles Goodhart, in contrast, proposes that central banks should stand ready to assist even insolvent financial institutions in circumstances, where prospects for recovery of the injected funds are poor.

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<sup>1</sup> We concentrate our analysis specifically on the issue of policy formulation and abstract from the issue of efficiency in carrying out the policy.

<sup>2</sup> The CLS system for foreign exchange settlement ([www.intranet.com/cls.htm](http://www.intranet.com/cls.htm)), EBA-clearing ([www.abe.org](http://www.abe.org)), and the LiKobank (Liquiditäts-Konsortialbank GmbH) are examples of potential candidates.

<sup>3</sup> See Bordo (1990) for an overview of the classical debate and the historical record. See Gorton (1985), Garcia and Plautz (1988) on the U.S. system and Herrala (1999) on the Finnish one.

<sup>4</sup> The Bank of England enjoyed a position as the bank of the government. The government had come to the rescue of the Bank of England in the past, and was expected to do so in the future.

<sup>5</sup> See Goodhart and Huang (1999a and 1999b) for a formal treatment.

<sup>6</sup> See Bagehot (1873), our edition Bagehot (1910). See also Selgin (1996) and Glasner (1989) on modern expositions of the free banking idea and references.

Bordo (1990) observes a shift from classic Bagehotian LOLR criteria towards less restrictive criteria in central bank LOLR-operations since the 1930's. As a notable example of this, the FED has moved to allow LOLR assistance to insolvent depository institutions in some circumstances. There has also been a change in terminology. In the central banking community today, the key feature of lender of last resort operations, which distinguishes them from standard central bank operations, is considered to be that the terms of LOLR credit must be adjusted to account for the recipient's condition. Operations that fulfil the Bagehotian criteria would, in current terminology, be considered standard.<sup>7</sup> To be able to study both Bagehot's and Goodhart's position on the debate, we will use the term lender of last resort in a broad sense, which encompasses both positions. We will analyse separately a case, where the ability of recipients of liquidity to repay is guaranteed, and a case, where banks have limited possibility for repayment.

We study funded LOLR schemes meaning that, in our model, a LOLR cannot create liquidity on demand: liquid funds ('value') needs to be pre-deposited. Yet central banks do have the right to issue monetary units and, indeed, the proponents of free banking argue that also private institutions should have such a right.<sup>8</sup> Debate on the issue of nominal monetary units is beyond the scope of this paper. We would maintain on empirical grounds, that a LOLR responsibility appears to warrant pre-hoarding of reserves in any case. Private LOLR operators, but also central banks pre-hoard liquidity: they collect reserves in domestic assets and foreign denominated assets as buffers against liquidity drains. Money growth, exchange rate and inflation targets are examples of limitations, which tie the hands of monetary policy authorities as regards the use of nominal issue for liquidity creation.<sup>9</sup>

Our model takes as a starting point a situation, where banks benefit from pre-hoarding liquidity for future contingencies: there is a possibility that banks will experience a liquidity drain. To shield themselves, banks can hold liquid assets individually, 'in house', but they can also jointly benefit from economies of scale in reserve pooling, if they join a lender of last resort scheme. A lender of last resort is a pool of liquidity, the size of the pool being determined by the amount of reserve deposits collected from each member<sup>10</sup> and the size of membership. A lender of last resort supplies liquidity to members on demand as liquidity loans, when the members of the pool face liquidity drains. Banks differentiate LOLR schemes by the extent of cover they offer, and by the interest rate they charge on liquidity credit and pay on reserve deposits. Bank's decision to join a LOLR scheme is also effected by the average quality of membership.

In our model, the first best LOLR-scheme encompasses all banks, and it offers full insurance to them. Participation in private lender of last resort schemes would typically be voluntary, and we start our analysis by investigating the criteria under which banks would prefer to join a LOLR scheme. In the model banks differ privately in quality, by which we mean the probability that a bank

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<sup>7</sup> See Garcia and Plautz (1988).

<sup>8</sup> Bagehot (1873), White (1995).

<sup>9</sup> Our definition does rule out the use of taxation of liquidity from private agents. This possibility has been used by governments to finance LOLR operations. see Holmström and Tirole (1998) for discussion and a formal treatment of the issue.

<sup>10</sup> We will call 'members' such banks, which use the lender of last resort as their reserve.

encounters a liquidity shock.<sup>11</sup> A lender of last resort, which offers liquidity assistance at relatively low interest rates, is attractive to banks, which have low quality (ie a high probability for a liquidity drain). High quality banks, on the other hand, are attracted to schemes, which offer high returns on reserve deposits. It turns out, that there is only one interest rate policy, under which all banks voluntarily participate in a single LOLR scheme: the one where the rate on liquidity credit and the return on reserve deposits equals the rate on bank loans. Such a LOLR must refrain from collecting profits.

We investigate policy formulation by three voluntary lender of last resort schemes and compare their policy to the desirable policy, which leads to the first best outcome. The schemes under study are:

- A public lender of last resort: This scheme is an idealised public sector bank. This LOLR sets policy to maximise social utility.
- A mutual clearing house: This scheme is governed by its members, the banks which have created it, by voting.
- A profit maximising lender of last resort: This scheme is governed by a profit-maximising outsider.

We study the policies of these LOLRs' first under the premise, that banks can repay the liquidity assistance granted by the LOLR with interest. The analysis shows, that all schemes under study will offer full insurance for the participating banks. The public bank collects no profits and it sets the interest rate to attract maximum participation. By these means it will achieve the first best level of utility. The clearinghouse and the profit maximising lender of last resort will, in general, not achieve the first best if we allow only a single proposed scheme which banks can either join or not join. However, if we allow free entry of LOLR schemes, competition leads private operators towards first best policy, which is the only stable (contestable) outcome in the strategic situation.

The situation is different in the case, where the ability of recipients of LOLR assistance to repay is limited. Voluntary schemes will still offer full insurance, but the schemes will not be able to attract comprehensive participation. While LOLR schemes are attractive to bad quality banks in this scenario, good quality banks will tend to stay out of them. In our view, this is a plausible argument for why private schemes would not do well as LOLRs if we accept prof. Goodhart's view of how LOLR should operate. A central bank with the power to impose a reserve requirement on banks will be superior in terms of social desirability to the voluntary systems.

We start with an outline of the model. Then we investigate policy in individual banks. An analysis of first best policy and policy by the voluntary schemes follows. We conclude with some remarks about the relationship of our model to literature, and potential directions for further research.

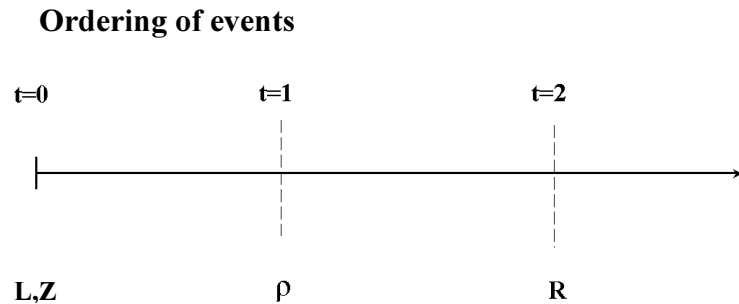
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<sup>11</sup> We impose no other restrictions except continuity on the quality distribution of banks.

## 2 The model

In our model a continuum of banks operate for three periods,  $t \in \{0,1,2\}$ . Each banker has, on period zero, one unit of funds ( $\rho$  deposits and  $1-\rho$  own funds) at its disposal, which it can use to invest in loans (L) or reserves (Z). On period one banks encounter, with bank-specific probability  $p_i$ , a liquidity shock  $\rho$ ,<sup>12</sup> when deposits are withdrawn. The shock needs to be covered either by realisation of liquid reserves or liquidation of loans. On period two, the non-liquidated part of the loan portfolio matures and yields a profit  $R-1 > 0$  per one unit of matured loans for the banker. See chart 1 for an illustration of the ordering of events.

Chart 1.



Non-bank agents are risk neutral and have a return requirement equal to unity. In the economy, non-bank agents are divided into potential depositors, who put their savings in banks, and non-depositors, who prefer their savings in real wealth. This difference in behaviour may stem, for example, from differences in ability to monitor and understand the functioning of banks. Potential depositors have just enough funds at the start of the game to finance the banks' operations. A liquidity shock can be interpreted as a shift in aggregate wealth in the population of outsiders from potential depositors to non-depositors. Banks cannot leverage themselves up with outside funds on period 1 to clear the liquidity shock.<sup>13</sup>

We assume that loans are costly to liquidate: the loan stock needs to be diluted by  $q\rho$  ( $q > 1$ ) to cover the withdrawal of deposits  $\rho$ . To ensure that any bank will have sufficient funds to clear a withdrawal of deposits even if the bank does not hold reserves, we impose the condition:

$$0 < \rho \leq 1/q < 1 \tag{2.1}$$

(2.1) allows us to ignore the issue of bankruptcy, which is out of the focus of our interest.

In the model, reserve assets can be used 'one for one' to cover a liquidity shock so that (2.1) is a sufficient guarantee that banks can clear liquidity shocks by hoarding liquid reserves. A bank can create a liquidity reserve for itself on period zero by hoarding liquid assets in its portfolio. However, it can also join a

<sup>12</sup> We concentrate our analysis on the case where  $\rho$  is the same for all banks, only its probability varies. A varying  $\rho$  would complicate the policy of the LOLR so that we would not be able to use the median voter theorem for the analysis of the clearinghouse (see the discussion below).

<sup>13</sup> Diamond and Dybvig (1983) and Holmström and Tirole (1998) have, among others, shown how to rationalise the insufficient supply of liquidity from outsiders in a framework comparable to ours. For tractability, we abstract from such considerations.

lender of last resort arrangement (LOLR), which functions as a liquidity reserve for its member banks.

We use lower case letters to differentiate in between these two complementary ways, in which banks can invest in reserves. We define:

$$Z = z_1\rho + z_2\rho$$

where  $z_1\rho$  is ‘in house’ reserves while  $z_2\rho$  is a reserve deposit with the lender of last resort.

LOLRs are set up at  $t = 0$  at the same time as banks start operations. A LOLR is a pool of reserve assets, collected from member banks and committed under the policy of the LOLR. The policy of the LOLR defines 1) the reserve deposit rate  $z_2$  collected from each member at  $t = 0$ , 2) the gross interest rate  $r$  imposed on liquidity credit available for members of the LOLR at  $t = 1$ , and 3) the share of the pool  $\beta$  retained by the outsider who operates the LOLR as profit at  $t = 2$ .

We will study policy formulation in four alternative LOLR arrangements. The first best LOLR maximises social utility and can force participation. The voluntary LOLRs are a public bank, which maximises social welfare, a mutual clearinghouse, where member banks choose policy by majority voting, and a profit maximising scheme. We construct alternative scenarios to study the effects of competitive conditions and the members’ ability to repay LOLR credit. In the restricted entry scenario, a monopoly is imposed exogenously for one LOLR mechanism at the start of the game, while under free entry of LOLRs the number of LOLRs is not restricted. In a ‘limited ability to repay’ scenario we impose a cap on the interest rate that the LOLR can charge on liquidity credit.

To make liquidity pooling nontrivial, we wish to incorporate in the model the feature that the terms under which a LOLR grants credit to its members, cannot be conditioned on the quality of individual members. To this end we assume that banks have better information about their own quality than about the quality of other banks: the probability  $p$ , with which a bank encounters a liquidity shock on period  $t = 1$ , is private information of that bank alone. While banks do not know the probabilities with which other banks face liquidity shocks, they know that these probabilities are drawn from a known distribution, which is the same for all banks. We denote by  $f[p]$  the distribution function and by  $F[p]$  the cumulative distribution function of this distribution. For mathematical tractability we assume  $f[p]$  to be continuous on the relevant domain  $p \in (0,1)$ .

We will concentrate our analysis on a case where shocks of banks are independent. The independence assumption implies that the liquidity demand for any pool which covers a continuum of banks is foreseen with certainty already on period zero. The lack of aggregate uncertainty leads to an important simplification in analysis: it turns out that absent aggregate uncertainty all members of a LOLR arrangement will agree on the optimal level of cover. Such a simplification is necessary due to the limitations set by our methodology: the median voter theorem, which we use to evaluate the policy of the mutual clearinghouse, is applicable only in a vote over a single issue. In our case the vote will be about the terms of liquidity credit.<sup>14</sup>

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<sup>14</sup> As is well known, the problem in a multiple vote is that almost anything can happen depending on how the agenda is presented, see eg McKelvey (1990) for a discussion.

The demand for liquidity credit falls to zero after the return requirement  $r$  reaches the liquidation cost of loans  $qR$ . The lower bound for the interest rate of liquidity credit is  $r = 0$ . At this level the LOLR effectively gives the money away without imposing any repayment requirement. We thus have the following range for feasible values of  $r$ :

$$0 \leq r < qR \quad (2.2)$$

We must also impose the following restriction on parameters to ensure that banks can clear the necessary period 2 transactions:

$$R(1 - z_1\rho - z_2\rho) + z_2\rho(1 - \beta)r \geq r(\rho - z_1\rho) \quad (2.3)$$

Under (2.3), loan returns plus accrued interest on reserve deposits is greater or equal than the repayment requirement for liquidity credit. In what follows it will be of importance that (2.3) allows  $r > R$  for small enough  $\beta$ .

### 3 Policy of individual banks

#### 3.1 Autarky

We will now illustrate some key aspects of the problem in autarky, the simplest possible setting. Under autarky there is no lender of last resort, and each bank handles its own liquidity policy.

Consider first a situation in which there is no LOLR and banks have no possibility to trade liquid assets at  $t = 1$  at inter bank markets. The program of bank  $i$  is:

$$\max_{z_1} U_i^A = R - 1 - z_{1,i}\rho(R - 1) - p_i\rho(1 - z_{1,i})(qR - 1) \quad (3.1)$$

The target function  $U_i^A$  is presented above in a form where the following interpretations can be given to its parts. ' $R-1$ ' denotes the banker's profit from a loan portfolio of size one. ' $z_{1,i}\rho(R-1)$ ' is the opportunity cost of holding reserves under autarky. ' $p_i\rho(1-z_{1,i})(qR-1)$ ', positive as  $q > 1$ , is the opportunity cost of a liquidity shock: the banker needs, with probability  $p_i$ , to realise  $\rho(1-z_{1,i})$  of the bank's loan portfolio at cost  $(qR-1)$  to cover the withdrawn deposits.

For further use we will give a shorthand  $C$  to the 'cost ratio' of the problem, which describes the ratio of the opportunity cost of holding reserve assets with the opportunity cost of liquidation of a unit of lending, ie:

$$C = \frac{(R - 1)}{(qR - 1)}, \quad 0 < C < 1 \quad \text{as} \quad q, R > 1$$

Note the following properties of the cost ratio. Firstly, the cost ratio is independent of  $Z$ , and due to this independence property we obtain only corner solutions for the choice of reserve levels: banks either collect full cover or no

cover at all. This independence property follows from the linearity of the target function:  $U_i^A$  is maximised at either  $z_{1,i} = 0$  or  $z_{1,i} = 1$  depending on whether the  $p_i < C$  or  $p_i > C$  respectively. Secondly, if the cost ratio were not below one, none of the banks would collect reserves on period zero: the costs related to the possibility of liquidation would be at least as low as the cost of holding reserves for all banks.

Choice under autarky, then, divides banks into two groups when there are no financial markets. Banks which have a shock probability below the cost ratio ( $p_i \leq C$ ), will not invest in reserves under autarky, while banks which have a shock probability above the cost ratio ( $p_i > C$ ) choose to invest in reserves for the whole amount of the potential shock under autarky. We will call banks in the first group ‘good banks’ while banks in the second group are referred to as ‘bad banks’.<sup>15</sup> To avoid confusion, the reader should keep in mind that the separating factor in between ‘good’ and ‘bad’ is the risk of encountering a shock, not the level of liquidity of the bank. In our terminology, good banks will have a low risk of encountering a liquidity shock, but they are less liquid than bad banks if they encounter a shock, because they choose not to invest to cover the risk. Bad banks, on the other hand, have high probability for encountering a shock, but they invest in liquidity to cover the risk.

The possibility to trade liquid assets at  $t = 1$  could be potentially valuable for banks, because some banks will end up having excess liquidity and some banks will be in short supply of liquidity in autarky. Denote by  $r_m$  the market price for a unit of liquidity at  $t = 1$ . Adjust program (3.1) to allow for a market for liquidity, and it gives the following characterisation for banks’ policy:

$$z_{1,i} = 0 \quad \text{for} \quad p_i < \frac{R - r_m}{r_m - 1}$$

$$z_{1,i} = 1 \quad \text{else}$$

Recall that aggregate demand for liquidity will be, with certainty,  $E[p]p$ . Liquidity supply equals demand at  $t = 1$  if:

$$G \left[ \frac{R - r_m}{r_m - 1} \right] = E[p]$$

This implies  $1 < r_m < R$ .<sup>16</sup>

In our model, this market solution to the banks’ liquidity problem fails for the following reason. If the price of liquidity is, at  $t = 1$ , above unity ( $r_m > 1$ ), then depositors of all banks would have an incentive to withdraw their deposits at  $t = 1$  and invest their funds at the market. There would be a ‘systemic crisis’. For this reason, banks want to make sure at  $t = 0$  that no market for liquidity arises at  $t = 1$ .

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<sup>15</sup> We will include the ‘border bank’ with  $p_i = C$ , for which the optimal choice of policy is indeterminate, in the ‘good bank’ -group.

<sup>16</sup> This is only one possible outcome for the bargaining game at  $t = 1$ . Banks that have excess liquidity are willing to trade all they have at any  $r_m \geq 1$ . Banks that are short of liquidity are willing to buy full cover for all  $r_m \leq qR$ .



In stead of going for the market solution, they go for the other option: they introduce a LOLR.<sup>17</sup>

## 3.2 Policy of banks when there is a lender of last resort

### A. An imposed LOLR monopoly

We will now discuss banks' policy in a setting where, at the start of the game, there is one LOLR, which enjoys a position of imposed monopoly. The banks' problem is to choose whether to join the LOLR arrangement, and how much own reserves to collect. Representative bank  $i$ 's target function is:

$$\begin{aligned}
 U_{p,i} = & R - 1 \\
 & - \rho z_{1,i} (R - 1) - \lambda_i \rho z_2 (R - (1 - \beta)r) \\
 & - p_i \rho \left( \left( 1 - z_{1,i} - \lambda_i \frac{z_2}{E_{\text{pool}}[p]} \right) (qR - 1) + \lambda_i (r - 1) \frac{z_2}{E_{\text{pool}}[p]} \right) \\
 \text{s.t. } & 0 \leq z_2 \leq E_{\text{pool}}[p]
 \end{aligned} \tag{3.2}$$

' $\lambda_i$ ' is the binary participation parameter of the bank ( $\lambda_i = 1$  if the bank joins the lender of last resort arrangement and  $\lambda_i = 0$  if it does not join). ' $E_{\text{pool}}[p]$ ', average quality of membership, is a shorthand for the expected shock probability of banks that are members of the lender of last resort. The terms on the second row in (3.2) denote, again, opportunity costs of reserve hoarding. ' $R - (1 - \beta)r$ ' is the return differential between a loan to firms and a deposit in the LOLR. The third row denotes costs that are conditional on the realisation of a liquidity shock (cost of liquidation of loans and cost of liquidity credit from the LOLR).  $z_{1,i}$  and  $\lambda_i$  are the choice variables for bank  $i$ .  $z_2$ ,  $\beta$  and  $r$  are determined by the lender of last resort. They are taken as given by the bank.

Notice the following property of (3.2). We observe that the average quality of membership in the LOLR arrangement (low  $E_{\text{pool}}[p]$  for high average quality membership and vice versa) will affect utility level of members. A LOLR with high average quality of membership needs to collect relatively less reserve deposits to achieve any given level of protection against liquidity shocks, compared to a LOLR with low average quality membership. A LOLR with low average quality of membership, on the other hand, returns more for depositors at any given level of lending rate and profits than a LOLR with high quality membership.

We will discuss the choice of in house reserves first. Maximising (3.2) w.r.t. in house reserves ( $z_1$ ) shows that banks are divided in their policy in the same way as under autarky. Good banks will not invest in in-house reserves to supplement the cover supplied by the lender of last resort. Bad banks will supplement the

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<sup>17</sup> While our solution to the 'market failure' (banks want to avoid creating inter bank markets altogether) is extreme and empirically refutable, we would maintain that the problem is real. As has been discussed above, the failure of markets could be rationalised also via a moral hazard argument or a co-ordination failure argument.

cover supplied by the lender of last resort by investing in in-house reserves until they achieve full cover. Formally:

$$z_{1,i} = \begin{cases} 0 & \text{for } p_i \leq C \\ 1 - \lambda_i \frac{z_2}{E_{\text{pool}}[p]} & \text{for } p_i > C \end{cases} \quad (3.3)$$

Inserting (3.3) into (3.2), we can investigate the banks' (discrete) choice in membership and autarky. Banks' choice is characterised by:

$$\begin{aligned} \text{for } p_i \leq C & \begin{cases} \lambda_i = 1 & \text{if } p_i > p_L = \frac{R - (1-\beta)r}{qR - r} E_{\text{pool}}[p] \\ \lambda_i = 0 & \text{otherwise} \end{cases} \\ \text{for } p_i > C & \begin{cases} \lambda_i = 1 & \begin{cases} \text{if } p_i < p_U = \frac{R-1}{r-1} - \frac{R - (1-\beta)r}{r-1} E_{\text{pool}}[p] \text{ and } r > 1 \\ \text{if } R - 1 - (R - (1-\beta)r)E_{\text{pool}}[p] > 0 \text{ and } r = 1 \\ \text{if } p_i > p_U \text{ and } r \leq 1 \end{cases} \\ \lambda_i = 0 & \text{otherwise} \end{cases} \end{cases} \quad (3.4)$$

An interesting issue that arises is that, under (3.4), the willingness of a bank to join a LOLR arrangement may correlate negatively with the quality of membership. One might think that new entrants would always be more willing to join a pool that has high quality (ie a low  $E_{\text{pool}}$ ) than a pool, which has low quality. This, however, is only the case when the LOLR follows a low return policy, which satisfies  $(1-\beta)r < R$ . If the pool follows a high return policy, which satisfies  $(1-\beta)r > R$ , then good quality banks will be 'draining' the pool, ie they receive high rates of interest from it while contributing little in return. In this case, a high concentration of good banks in the LOLR deters entry. A decision to join a LOLR that follows a return neutral policy ( $(1-\beta)r = R$ ) is independent of the quality of membership.

We can utilise (3.4) more generally to examine, how participation in the lender of last resort arrangement varies with changes in policy of the LOLR. For further reference, we will give the main results of this analysis in two of corollaries below.

**Corollary 1**      on voluntary participation in the LOLR –arrangement

All good banks will voluntarily participate if  $(1-\beta)r \geq R$ . All bad banks will voluntarily participate if  $(1-(1-\beta)E_{\text{pool}})r \geq (1-E_{\text{pool}})R$ . All banks voluntarily participate in a lender of last resort arrangement which follows  $(r, \beta) = (R, 0)$ .

**Proof:** The result, which is feasible under (2.2), and (2.3), follows from the definitions of  $p_L$  and  $p_U$  given above:  $(r, \beta) = (R, 0)$  is the only policy for which  $p_L \leq 0$  and  $p_U \geq 1$ .

It is perhaps surprising that there is only one combination of the interest rate parameter and the profit parameter, under which all banks voluntarily participate in the LOLR arrangement. This combination  $(r, \beta) = (R, 0)$  equates both the return on reserve deposits with the return on loans, and the opportunity cost of hoarding ‘in house’ reserves with the opportunity cost of obtaining liquidity from the LOLR. Under this policy even very good banks, which have a very small (indeed even zero) probability for the realisation of a liquidity shock, will be willing to join. Also very bad banks, which encounter a liquidity shock for certain, would be indifferent in between using the LOLR as a reserve as compared to investing in own reserves.

Corollary 2      on effects of policy on pool borders under voluntary participation

$$\begin{cases} \frac{\partial p_L}{\partial r} \leq 0 & \frac{\partial p_L}{\partial \beta} \geq 0 \\ \frac{\partial p_U}{\partial r} \leq 0 & \frac{\partial p_U}{\partial \beta} \leq 0 \end{cases}$$

$$\text{for } |J| = 1 + \frac{R - (1 - \beta)r}{(qR - r)(r - 1)} \left( (qR - r) \frac{\partial E_{\text{pool}}}{\partial p_U} - (r - 1) \frac{\partial E_{\text{pool}}}{\partial p_L} \right) > 0$$

indeterminate for  $|J| = 0$

signs reversed for  $|J| < 0$

Proof: the results in corollary 2 can be obtained from (3.4) by use of the implicit function theorem.

Corollary 2 illustrates that for pools that follow a low return or high return policy (but not for pools that are return neutral), changes in the interest rate parameter and the profit parameter may lead to ‘counterintuitive’ changes in participation. One might think that an increase in the interest rate parameter would always tend to drive out bad banks and bring in more good banks, and an increase in the profit parameter would drive out both types of banks. Corollary 2 states that this holds always when  $|J| > 0$ , ie for return neutral pools, and for pools that are close to return neutral or when the concentration of banks is not too unbalanced at the borders.

The Jacobian determinant can, in this context, be interpreted as a measure of ‘aggregate feedback’, the effect of the change in the quality of membership, on individual bank’s membership decision. When  $|J| < 0$ , policy responses are ‘counter intuitively’ reversed because the quality criterion dominates over the direct effect of the policy variable: aggregate feedback reverses the behaviour of the population. To illustrate, take as an example a high return pool, where all good banks participate and the concentration of bad banks is very high at the border  $p_U$ . With a high return pool we now have a situation, where the pool ‘drains’ the bad banks, and ‘floods’ the good banks, and entry to the pool depends negatively on

the quality of the pool. An increase in the rate of interest leads to more entry from the population of bad banks, as the favourable effects of the deterioration in the quality of membership in the pool suffice to counterbalance the unfavourable effects of the change in the interest rate.

## B. Many proposed LOLR schemes

We now move to consider a situation, where there are multiple proposed LOLR schemes at the start of the game. Those schemes, which get nonzero participation, will be implemented.

The relative preference of bank  $i$  between any two proposed LOLR arrangements (say pool  $a$  and pool  $b$ ), each of which covers some continuum of banks, is governed by the condition:

bank  $i$  weakly prefers pool  $a$  to pool  $b$  if

$$\begin{aligned}
 & -z_1^a(R-1) - z_2^a(R - (1-\beta^a)r^a) \\
 & - p_i \left( -z_1^a - \frac{z_2^a}{E^a[p]} \right) (qR-1) - p_i (r^a-1) \frac{z_2^a}{E^a[p]} \\
 & \geq -z_1^b(R-1) - z_2^b(R - (1-\beta^b)r^b) \\
 & - p_i \left( -z_1^b - \frac{z_2^b}{E^b[p]} \right) (qR-1) - p_i (r^b-1) \frac{z_2^b}{E^b[p]}
 \end{aligned} \tag{3.5}$$

To get a grasp of what is going on, consider, first, a situation in which the two pools under comparison ( $a$  and  $b$ ) are return neutral. It is, then, straightforward to show (by simplifying (3.5) accordingly) that

1. Among return neutral pools, all banks prefer the pool with the lowest level of profits ( $\beta \geq 0$ ), if the pools offer the same level of cover ( $0 \leq z_2/E \leq 1$ ) and apply the same interest rate.
2. Among return neutral pools, all banks prefer the pool with the highest cover given that the pools retain the same level of profits and apply the same interest rate.

This indicates that, given return neutrality, that proposed scheme, which is closest to zero profits and full cover will be chosen by banks in equilibrium. When we relax return neutrality, the comparison of alternative schemes becomes more complicated, because the average quality of membership starts to effect the participation decision. In this case we can have equilibria of a type, where a high profit pool and a low profit pool both get members, even though both pools offer the same amount of cover and apply a similar interest rate on liquidity credit. If such ‘competing’ arrangements coexist, then there must be sufficient quality differential in between them.

We may, however, utilise (3.5) to derive the general result, that if a return neutral alternative with zero profits and full cover is proposed at the start of the game, then that alternative will dominate in equilibrium. We present this result in Corollary 3.

Corollary 3

on relative preference over alternative voluntary LOLR arrangements

If pool b follows a policy  $(z_2^b, r^b, \beta^b) = (E_{\text{pool}}^b[p], R, 0)$  and pool a does not follow a policy  $(z_2^a, r^a, \beta^a) = (E_{\text{pool}}^a[p], R, 0)$ , then:

1. pool a is unstable or it covers only one point in the continuum of banks
2. pool b is stable.<sup>18</sup>

Proof: We will concentrate on 3.1 first. Assume, on the contrary, that pool a is stable. By standard optimization methods it can be verified, that good banks will choose  $z_1 = 0$  and bad banks will choose  $z_1 = 1 - z_2/E_{\text{pool}}[p]$ . Set the appropriate values for  $z_1$  in (3.5), insert the policy  $(z_2^b, r^b, \beta^b) = (E_{\text{pool}}^b[p], R, 0)$ , and replace  $z_2^a$  by  $\alpha E_{\text{pool}}[p]$ ,  $0 < \alpha \leq 1$ . We get that bank  $i$  prefers pool a to pool b if:

$$\begin{cases} p_i(R - \alpha r^a - qR(1 - \alpha)) \\ \geq E_{\text{pool}}^a[p]\alpha(R - r^a + \beta^a r^a) & \text{when } p_i \leq C \\ p_i(-(R - 1) + \alpha(R - r^a)) \\ \geq E_{\text{pool}}^a[p]\alpha(R - r^a + \beta^a r^a) & \text{when } p_i > C \end{cases} \quad (3.6)$$

Under stability (3.6) would need to hold for all banks in pool a. At minimum, pool a consist of the (finite number of banks) for which  $p_i = E_{\text{pool}}^a[p]$ . When we inspect whether (3.6) holds for such an average bank we observe that this leads to

$$\begin{cases} \Rightarrow (1 - \alpha)(R - qR) \geq \alpha\beta^a r^a & \text{when } p_i \leq C \\ \Rightarrow -(R - 1) \geq \alpha\beta^a r^a & \text{when } p_i > C \end{cases} \quad (3.7)$$

We observe that (3.7) can never hold for bad banks as the left side is negative by assumption, while the right side is positive. For the good banks, (3.7) holds with equality if  $\alpha = 1$  and  $\beta = 0$  (a offers full cover and extracts no profits). Inserting this policy back into (3.6) we observe, that the condition holds only for pools a for which  $p_i = E_{\text{pool}}[p]$ . The members in these unitary pools would be indifferent in between a and b.

We thus conclude, that pool a, which covers a continuum of banks, cannot exist such that all members in that pool prefer

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<sup>18</sup> By a stable pool we mean one, where all members of that pool weakly prefer that pool to any other pool.

pool a to pool b. That a stable pool b exists under these conditions, can be shown by reversing the inequality sign in (3.5) and applying the same steps.

We, therefore, conclude that a proposed LOLR arrangement  $(z_2, r, \beta) = (E_{\text{pool}}, R, 0)$  is special in that all banks will voluntarily join it and that it has the unique stability property which indicates that no other pool, which follows a different policy, can attract a continuum of members. Corollary 3 does not rule out the existence of specialised LOLRs for good banks with identical risks. Could such specialised LOLRs co-exists with the large aggregate LOLR? Recall that any LOLR that has only a finite number of participants will encounter aggregate uncertainty. In the model, banks will be averse to such uncertainty, because payoffs are asymmetric for an unlucky and a lucky outcome ( $qR$  and  $R$  respectively). We would thus maintain that, on the assumption that there can only be a finite number of identical banks, also good banks would prefer to join the single aggregate LOLR rather than set up small, specialised clubs.

To conclude, we have in this section established the reaction functions of banks to LOLR policy. We will now go on to study the policy of the alternative LOLR's, given the reaction functions they encounter.

## 4 Policy of the lender of last resort

### 4.1 The first best

We define that the first best lender of last resort maximises the sum of aggregate utilities of all banks and the profit, which accrues to the LOLR. In contrast to other lender of last resort schemes considered here, the first best has the power to force participation in the LOLR –arrangement. The first best LOLR need not worry about bank's reaction to it's policy.

The program to be solved for the choice of policy can be expressed as:

$$\begin{aligned}
\max_{p_L, p_U, z_2, r, \beta} U^{\text{FB}} &= R - 1 - \int_0^{p_L} \{p\rho(qR - 1)\}f[p]dp \\
&- \int_{p_L}^C \left\{ z_2\rho(R - (1 - \beta)r) + p \left( \rho(qR - 1) \left( 1 - \frac{z_2}{E_{\text{pool}}[p]} \right) + (r - 1) \frac{z_2}{E_{\text{pool}}[p]} \right) \right\} f[p]dp \\
&- \int_C^{p_U} \left\{ \left( 1 - \frac{z_2}{E_{\text{pool}}[p]} \right) \rho(R - 1) + (R - (1 - \beta)r)\rho z_2 + p\rho(r - 1) \frac{z_2}{E_{\text{pool}}[p]} \right\} f[p]dp \\
&- \int_{p_U}^1 \{ \rho(R - 1) \} f[p]dp \\
&+ \int_{p_L}^{p_U} \beta r z_2 p f[p]dp
\end{aligned} \tag{4.1}$$

Corollary 4 summarises the optimal policy of the first best LOLR.

Corollary 4 on the policy of the first best lender of last resort

4.1 The first best LOLR will include all banks in the LOLR -arrangement, ie

$$p_L = 0 \text{ and } p_U = 1$$

4.2 The first best lender of last resort will offer full cover against liquidity shocks, ie:

$$z_2 = E[p] = \int_0^1 pf[p]dp .$$

4.3 The planner is indifferent in between alternative liquidity loan rates ( $r$ ) and profit rates ( $\beta$ ) (it is indifferent on the distribution of wealth).

Proof: These results are solutions to program (4.1)

The first best LOLR wants to include all banks in the arrangement and offer full cover because it wants to avoid costly liquidation of loans, and because pooling is a more efficient way to hold reserves than in-house reserve holding.<sup>19</sup> The liquidity loan rate and profits are ambiguous because the effect of changing these parameters is just to redistribute wealth.

## 4.2 Voluntary LOLR schemes

We will study the case of restricted entry in subsections A.–C. Under restricted entry the scheme under study will be the only possible scheme. We will assume that the ability of banks to repay is limited by (2.3) only, ie banks can pay interest on liquidity credit at least up to  $r = R$ . Analysis of the competing schemes scenario is in subsection D, and the scenario where banks have limited ability to repay in subsection E.

### A. The voluntary public LOLR

We define that the public agent proposes a LOLR scheme that maximises the aggregate utility of all banks in a situation where participation in the lender of last resort scheme is voluntary. The maximisation problem of the voluntary public lender of last resort thus becomes:

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<sup>19</sup> The fact that the first best includes all banks means that the pool does not need to be able to differentiate in between the participants to implement the first best. We therefore need not assume that the first best scheme requires better information about the quality of banks from the pool management than the voluntary schemes do.

$$\begin{aligned}
\max_{z_2, r, \beta} U^P &= R - 1 - \int_0^{p_L} \{p\rho(qR - 1)\}f[p]dp \\
&- \int_{p_L}^C \left\{ z_2\rho(R - (1 - \beta)r) + p \left( \rho(qR - 1) \left( 1 - \frac{z_2}{E_{\text{pool}}[p]} \right) + (r - 1) \frac{z_2}{E_{\text{pool}}[p]} \right) \right\} f[p]dp \\
&- \int_C^{p_U} \left\{ \left( 1 - \frac{z_2}{E_{\text{pool}}[p]} \right) \rho(R - 1) + (R - (1 - \beta)r)\rho z_2 + p\rho(r - 1) \frac{z_2}{E_{\text{pool}}[p]} \right\} f[p]dp \\
&- \int_{p_U}^1 \{p(R - 1)\}f[p]dp \\
&+ \int_{p_L}^{p_U} \beta r z_2 \rho f[p]dp
\end{aligned} \tag{4.2}$$

In (4.2), the limits of the pool  $p_L$  and  $p_U$  are endogenous (as defined in 3.4). The policy of the public facility is summarised in the following corollary.

**Corollary 5**      on the policy of the public LOLR

5.1    The public lender of last resort will offer full cover against liquidity shocks ie:

$$z_2 = E_{\text{pool}}[p] = \frac{1}{F[p_U] - F[p_L]} \int_{p_L}^{p_U} pf[p]dp.$$

5.2    The level of liquidity loan rate equals return on real investment

$$r = R$$

5.3    The level of profits is zero

$$\beta = 0$$

5.4    All banks will voluntarily participate ( $p_L = 0$ ,  $p_U = 1$ )

Proof: 5.1, 5.2 and 5.3 are solutions to program (4.2). 5.4 follows from 5.2, 5.3 and corollary 1. The solution is within the limits set by 2.2 and 2.3. The public scheme is stable under corollary 3.

We observe that the public facility will use the interest rate parameter as the means to attract as much participation as possible under the full security offered by the lender of last resort. For the same reason it refrains from collecting profits. This ensures that all participate and wasteful liquidation of loans never occurs.



## B. The mutual clearinghouse

In a clearinghouse, policy is formulated by majority vote of member banks. Under this scheme, profits constitute a dividend payment to members (an equal payment to each member) on period two, which makes interest payments on reserve deposits and dividends indistinguishable from the point of view of banks. Without loss of generality we will, therefore, set profits to zero.

The opinion of member bank  $i$  on policy can be solved from the program

$$\begin{aligned} \max_{z_2, r} U_i = & R - 1 - \rho z_1 (R - 1) - \rho z_2 (R - r) \\ & - \rho \left( \left( 1 - z_{1,i} - \frac{z_2}{E_{\text{pool}}[p]} \right) (qR - 1) + (r - 1) \frac{z_2}{E_{\text{pool}}[p]} \right) \end{aligned} \quad (4.3)$$

Corollary 6 on the opinion of the members of the clearinghouse on the policy of the clearinghouse

6.1 All members agree that the clearinghouse should offer full cover against liquidity shocks, ie

$$z_2 = E_{\text{pool}}[p]$$

6.2 The preferred policy of member  $i$  on liquidity credit rate  $r$  satisfies:

$$(r - R) \frac{\partial E_{\text{pool}}[p]}{\partial r} + E_{\text{pool}}[p] - p_i = 0$$

Proof: These are solutions to program (4.3)

Corollary 6 implies that banks disagree only on the liquidity credit rate, and this issue needs to be subjected to vote. By the median voter theorem,<sup>20</sup> in a single majority vote the view of the median voter will prevail. The median voter theorem and corollary 6.2 give the following decision making rule for the LOLR:

$$(r - R) \frac{\partial E_{\text{pool}}[p]}{\partial r} + E_{\text{pool}}[p] - \text{Median}_{\text{pool}}[p] = 0 \quad (4.4)$$

It is possible to utilise 4.4 to investigate the conditions under which a mutual clearing house can reach the stable outcome, which gives first best level of utility.

Corollary 7 on the policy of the clearinghouse

The mutual clearing house will equalise the rate of liquidity credit with the return on real investment ( $r = R$ ) if the median of the quality distribution of banks equals the mean, ie if the distribution of banks is symmetric.

<sup>20</sup> See eg McKelvey in Ichiishi, Neyman, Rauman (1990)

Proof: This follows directly from 4.4 and the fact that this policy gives full participation under corollary 2.

We thus obtain the result that full participation can occur only in the special case where the quality distribution of banks is symmetric. For monotonous distributions, for which the median in all subintervals of the distribution is greater than the mean (ie whenever the distribution of banks is skewed to the left), the clearinghouse will set  $r > R$ . The opposite applies in situations, where the quality distribution is skewed towards the bad banks. It is intuitively appealing that that when the distribution banks is skewed towards the good banks, a clearinghouses tends to follow policy which favours the good banks excessively and drives out some of the bad banks, and vice versa. Such schemes are vulnerable to competition from other schemes that are close to first best.

### C. The profit maximising LOLR

The program of the profit maximising agent is simply

$$\begin{aligned} \max_{z_2, r, \beta} \Pi &= z_2 r \beta \rho \int_{p_L}^{p_U} f[p] dp \\ \text{s.t. } 0 &\leq z_2 \leq E_{\text{pool}}[p] \end{aligned} \quad (4.5)$$

Corollary 8 on the policy of the profit maximising LOLR

8.1 The profit maximiser will set the level of reserves to offer full cover against liquidity shocks, ie

$$z_2 = E_{\text{pool}}[p]$$

8.2 The profit maximiser's choice of lending rate satisfies.

$$F[p_U] - F[p_L] + r \left( \frac{\partial p_U}{\partial r} f(p_U) - \frac{\partial p_L}{\partial r} f(p_L) \right) = 0$$

8.3 The profit maximiser's choice of profits satisfies:

$$F[p_U] - F[p_L] + \beta \left( \frac{\partial p_U}{\partial \beta} f(p_U) - \frac{\partial p_L}{\partial \beta} f(p_L) \right) = 0$$

Proof: These are solutions to program (4.5).

Note that corollary 8 and corollary 1 imply that the profit maximising LOLR scheme will always have limited participation, because the profit parameter will always be positive. To which level the interest rate parameter and the profit parameter settle, is specific to the quality distribution in the banking sector.

#### D. Free entry of LOLRs

The analysis in sections 4.2A–C refers to the case, where each proposed scheme was the only one on the table. We saw that, in such a situation, private schemes tend to choose policies that are non-optimal from the social point of view. Corollaries 5 and 6 give a rather bleak picture of the scope for private LOLR schemes. We would argue, however, that this result is due to our assumption that entry of LOLRs is restricted. The analysis does make the point, that even non-profit private schemes should be expected to fail to implement the socially optimal outcome, if there are barriers of entry.

If there are no barriers to entry, then one can argue that competition leads towards the first best outcome. To establish this, we use a heuristic argument. Consider an initial situation, where the first best stable scheme is not implemented. Then new entrants could drive others out of the LOLR market, by going for the stable policy except that they charge an infinitesimal profit. Such a profitable entry possibility exists until the stable scheme is implemented.

#### E. Limited scope for repayment

The previous analysis considers a case in which banks, which receive liquidity from the LOLR, are able to repay the liquidity with interest. The historical record on LOLR shows, that LOLR-assistance is often granted to insolvent institutions in circumstances where prospects for recovery appear poor. We will now analyse such a scheme based on the results obtained in the previous chapters. Notice, that by corollary 4, first best still always covers the whole banking sector.

We have studied this case simply by imposing a cap  $\bar{r}$  on the interest rate parameter  $r$ . We use a moral hazard argument to rationalise the cap. Suppose that the realisation of  $R$  requires a private monitoring effort  $b$  per unit of loans by the banker (without the effort, the loan stock returns 0). The banker has an incentive to induce the effort if project returns diluted by cost of liquidity credit is greater than the private effort cost, ie

$$r \leq \frac{(R - b)L}{\rho} \equiv \bar{r}$$

Consider first, what happens in light of our assumptions, when the cap on the interest rate parameter starts to fall and takes  $r$  with it. We can interpret this as a situation where the private monitoring cost of firms increases. By corollary 1, we observe that when  $r$  falls below  $R$ , then good banks will start to leave the scheme. All bad banks will participate. By corollary 2 the amount of good banks that participate in general falls with  $r$ , except under very special circumstances, which we discussed in chapter 3. As the interest rate approaches zero, there will still be some good banks in the voluntary scheme along with the bad banks on the condition that profits are not too large.

As  $\bar{r}$  approaches zero, the policies of the voluntary public bank and the clearing house will converge: both offer full cover by corollaries 5 and 7, and collect zero interest rates. By corollary 8, the profit maximising LOLR will offer

full cover, but it will set a nonzero rate of profits so that participation in this scheme and aggregate utility provided by this scheme will be smaller than in the other schemes.

Corollary 9 on the special facilities ( $0 \leq r \leq \bar{r} < R$ .)

#### 9.1 the public LOLR

The public facility will offer full cover and sets the interest rate to obtain as comprehensive participation as possible. The scheme will not achieve the first best level of utility because some good banks will stay out of the scheme.

#### 9.2 the mutual clearinghouse

The clearinghouse will offer full cover. When the ability of banks to repay the LOLR goes to zero, the policy of the clearinghouse becomes identical to the public bank irrespective of the quality distribution of banks.

#### 9.3 profit maximising LOLR

A profit maximising lender of last resort will offer full cover. It will always set a positive level of profits and, therefore, it never achieves the utility level of the public bank.

What this analysis, in our view, illustrates is the potential gains obtainable by the possibility to dictate participation in LOLR facilities. In contrast to schemes, where recipients of LOLR assistance could repay the credit, voluntary participation never leads to first best when the LOLR incurs losses. As regards the debate on LOLR, the analysis suggests that banks could never agree on a comprehensive LOLR scheme, which gives assistance in situations where prospects for recovery are poor. Banks which consider themselves to be in less risky position than other banks would tend to stay out of such schemes. Central banks with a right to impose reserve requirements on banks, could implement such an scheme for the banking sector.

## 5 Concluding remarks

Our goal has been to introduce governance considerations to the theoretical debate on the lender of last resort,<sup>21</sup> and thus shed light on the classical debate on free banking. We study policy formulation of two alternative private LOLR-schemes, a mutual clearing house and a profit maximising scheme. Our analysis leads us to propose, that private schemes are likely to function well as lenders of last resort only, if entry is non-restricted, and if recipients of LOLR assistance can repay

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<sup>21</sup> For an extensive review of the literature see Bank of England 1999.

with interest the LOLR credit which they obtain. Under other conditions private schemes fail to reach the first best because they fail to attract comprehensive entry.

Our model utilises a three period structure that is common in ‘liquidity models’.<sup>22</sup> In our view, our framework offers some promise for the analysis of banking and insurance, as both banks and insurance companies are basically liquidity pools. Our model could be used relatively straightforwardly to rationalise the view that some insurance schemes (health insurance) are likely to work better on compulsory rather than voluntary basis. Our model could be enriched to study competition in financial services.

Hart and Moore’s work on governance,<sup>23</sup> especially on outside provision vs. co-operatives, has inspired our choice of methodology for separating the alternative lender of last resort arrangements. The method relies on the applicability of the median voter theorem, and this limits the usefulness of the framework in analysing situations with multidimensional decision spaces.

Due to this limitation, the paper leaves a number of interesting issues for further study. In reality, private operators would be likely to use more complex pricing schemes than the one we have utilised. Aggregate uncertainty could be introduced to study the choice of aggregate liquidity risk by the LOLR.<sup>24</sup> Agency costs could be introduced for the public LOLR to facilitate a more realistic trade-off of public and private schemes. As it is, the model is tractable and, in our view, it still gives interesting insights to the classical debate. For us, the main lesson is that in the classical debate on public versus private LOLR, the main opposing views appear both to be right, given the LOLR concept which they utilise. This clears the way for an analysis of the issue, of whether and how insolvent banks should get LOLR assistance.

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<sup>22</sup> Since Diamond and Dybvig’s seminal paper (1983), there has been a growing literature on the economics of liquidity. See eg Holmström and Tirole (1998) for references.

<sup>23</sup> See Hart and Moore (1996, 1998).

<sup>24</sup> We study instrument choice and the implications of aggregate uncertainty for a LOLR in a forthcoming paper.

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