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## Working Paper Monetary policies to counter the zero interest rate: an overview of research

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# Seppo Honkapohja

# Monetary Policies to Counter the Zero Interest Rate: an Overview of Research



Bank of Finland Research Discussion Papers 18 • 2015

# Monetary Policies to Counter the Zero Interest Rate: an Overview of Research<sup>\*</sup>

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July 10, 2015

#### Abstract

Many central banks have lowered their interest rates close to zero in response to the crisis since 2008. In standard monetary models the zero lower bound (ZLB) constraint implies the existence of a second steady state in addition to the inflation-targeting steady state. Large scale asset purchases (APP) have been used as a tool for easing of monetary policy in the ZLB regime. I provide a theoretical discussion of these issues using a stylized general equilibrium model in a global nonlinear setting. I also review briefly the empirical literature about effects of APP's.

JEL Classification: E63, E52, E58.

Keywords: Adaptive Learning, Monetary Policy, Inflation Targeting, Zero Interest Rate Lower Bound

## 1 Introduction

Financial crises in many advanced market economies since 2008 have lead to major changes in the monetary policy practices in the United States and Europe. Drastic lowering of interest rates was the first policy response of central banks to the rapidly deterioration of economic and financial developments. Interest rates reached very low levels, popularly called the zero lower

<sup>\*</sup>Keynote lecture in the June 2015 Meeting of the Austrian Economic Association. Any views expressed are those of the author and do not necessarily reflect the views of the Bank of Finland. I am very grateful to Adam Gulan for assistance and comments.

bound (ZLB), and the ZLB became a new constraint for monetary policy. The ZLB constraint led to the introduction of some new tools in monetary policy. In particular, large-scale asset purchase programs (APP), also called quantitative easing, were introduced first in Japan and subsequently in the United States of America and United Kingdom. The extended asset purchase program of the ECB is the latest example of such a program. Other new monetary policies such as forward guidance were also introduced.

The Japanese experience with the ZLB regime inspired new economic research already at the end of the 1990's. For prominent early analyses emphasizing the signalling channel of monetary policy, see e.g. Krugman (1998), Eggertsson and Woodford (2003) and Svensson (2003). These and subsequent papers view the ZLB and liquidity trap as a shift in the usual rational expectations (RE) equilibrium. A fairly long-lasting fundamental shock can make the ZLB constraint binding. For example, a negative shock in financial intermediation can make the real equilibrium policy rate negative, see Woodford (2011). Another example, suggested in Christiano, Eichenbaum, and Rebelo (2011), is a shock to intertemporal preferences which changes the subjective discount factor in a way that leads to a zero nominal interest rate.

The liquidity trap in these models can be called a **fundamental liquidity trap** as it is caused by an adverse fundamental shock. There is a sizeable literature on the effectiveness of different monetary and fiscal policy measures in a fundamental liquidity trap.<sup>1</sup> Methodologically, the analysis is these liquidity traps is straightforward as it involves a shift of the targeted RE equilibrium.

The ZLB and liquidity trap are, however, a more fundamental problem of global indeterminacy of equilibria created by the ZLB constraint. The steady state interest rate in the liquidity trap may have reached the zero (or a very close to zero) level as a permanent or long-lasting state of the economy. If monetary policy is conducted in accordance with an interest rate rule such as a Taylor rule, multiple steady states to the economy necessarily exist even without any adverse shocks to fundamentals.

In a standard NK model a key equation for a nonstochastic steady state is the Fisher equation  $R = \beta^{-1}\pi$ , where R is the gross interest rate,  $\beta$  is the subjective discount factor and  $\pi$  is the gross inflation rate. Usually the interest rate rule incorporates a specified target inflation rate  $\pi^* \geq 1$  (and

<sup>&</sup>lt;sup>1</sup>For monetary policy dynamics with the ZLB constraint, e.g., see Fuhrer and Madigan (1997) and Wollman (1998). More recent literature is discussed later.

an associated output level) as a steady state. Since the policy rule faces a constraint  $R \geq 1$  and is a continuous function, then a second steady state necessarily exists as indicated by the second intersection of the interest rate rule and the Fisher equation in Figure 1. This illustrates the problem of multiplicity of steady states when monetary policy is described by a Taylor rule. In this situation there can also exist sunspot equilibria.

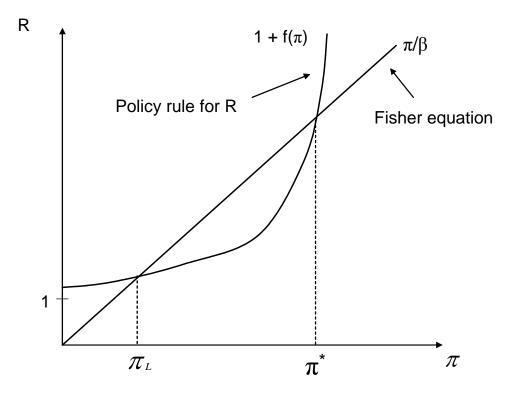


Figure 1: Multiple steady states with ZLB and Taylor rule

It should be noted that many models of fundamental liquidity traps in the papers referenced above ignore this global indeterminacy problem. These papers focus attention on shifts that make the targeted equilibrium move into the ZLB-constrained state and the existence of the second steady state is excluded from the analysis. However, note that a sufficiently long-lasting fundamental shock to the targeted equilibrium would imply non-existence of the fundamental liquidity trap.<sup>2</sup>

 $<sup>^{2}</sup>$ One could question the use of the term liquidity trap in this setting, as the assumed

The low steady state illustrated in Figure 1 can be called an **expectations driven liquidity trap.**<sup>3</sup> The existence of multiple equilibria can be viewed as reflecting problems of confidence that can adversely affect economic behavior. With multiple equilibria a shock to expectations (or confidence) may lead to a process whereby the economy moves from a state not that far from the inflation target equilibrium to a situation in which the ZLB for the interest rates becomes a binding or almost binding constraint creating a second steady state. This process results in low output and inflation, which in turn can lead to significant easing of monetary policy with the policy interest rate reaching the ZLB constraint.

Figure 2 shows how data for Japan and possibly the Euro area may be consistent with the model of an expectations driven liquidity trap.<sup>4</sup> The framework with multiple equilibria is methodologically challenging. My first objective in this lecture is to consider basic aspects of the expectations-driven liquidity traps and the role of monetary policies to combat them.

shock disappears in finite time.

 $<sup>^{3}</sup>$ The classification of liquidity traps into fundamental and expectations driven traps is suggested in Mertens and Ravn (2014).

<sup>&</sup>lt;sup>4</sup>Bullard (2010) shows an analogous figure with U.S. and Japanese data.

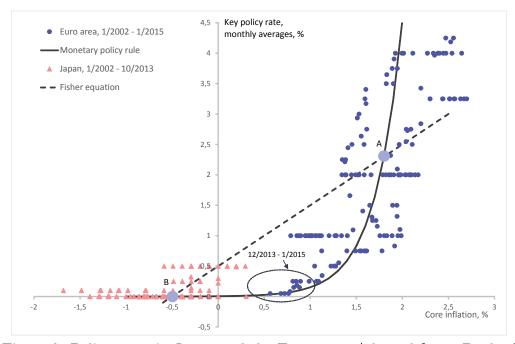


Figure 2: Policy rates in Japan and the Euro area. Adapted from: Bank of Finland Bulletin 1/2015.

Reifschneider and Williams (2000) show the possibility of multiple equilibria for a backward looking model. RE models of the ZLB and liquidity trap include Benhabib, Schmitt-Grohe, and Uribe (2001), Benhabib, Schmitt-Grohe, and Uribe (2002), Mertens and Ravn (2014) and Aruoba, Cuba-Borda, and Schorfheide (2014). There is also a sizeable literature employing models of learning that emphasize significant uncertainties faced by private agents when planning their economic activities.<sup>5</sup> This literature has major implications for the analysis of liquidity traps and design of economic policies, see e.g. Evans and Honkapohja (2010), Benhabib, Evans, and Honkapohja (2014) and the references therein.

In this paper I first analyze expectations driven liquidity traps under the general assumption of imperfect knowledge and learning. I use the adaptive learning approach which is increasingly used in the literature to which I have myself contributed. This view differs from the existing literature that is predominantly based on the RE hypothesis. RE is a very strong assumption

<sup>&</sup>lt;sup>5</sup>There has recently been interest in relaxing the RE hypothesis in the context of macroeconomic policy analysis, see e.g. Taylor and Williams (2010) and Woodford (2013).

about the agents' knowledge as under RE agents are continuously able to predict perfectly the future path of the economy, except for the effects of unforecastable random shocks.<sup>6</sup>

Sections 2-4 develop a standard New Keynesian model and review the basic aspects of adaptive learning. Section 5 analyzes global learning dynamics under a Taylor rule and the effectiveness of an asset purchase program using the model. Section 6 provides a commentary about other topics in macro policies to combat the liquidity trap and about empirical research on the effects of policies on financial and macroeconomic variables.

## 2 A New Keynesian Model

The analytical framework I use is a standard New Keynesian model.<sup>7</sup> A continuum of household-firms produce each a differentiated consumption good under monopolistic competition and price-adjustment costs. The government uses monetary policy, buys a fixed amount of output, finances spending by lump-sum taxes, and issues public debt.

The objective for agent s is to maximize expected, discounted utility subject to a standard flow budget constraint (in real terms):

$$Max \ \hat{E}_0 \sum_{t=0}^{\infty} \beta^t U_{t,s} \left( c_{t,s}, \frac{M_{t-1,s}}{P_t}, h_{t,s}, \frac{P_{t,s}}{P_{t-1,s}} - 1 \right)$$
(1)

st. 
$$c_{t,s} + m_{t,s} + b_{t,s} + \Upsilon_{t,s} = m_{t-1,s}\pi_t^{-1} + R_{t-1}\pi_t^{-1}b_{t-1,s} + \frac{P_{t,s}}{P_t}y_{t,s},$$
 (2)

where  $c_{t,s}$  is the consumption aggregator,  $M_{t,s}$  and  $m_{t,s}$  denote nominal and real money balances,  $h_{t,s}$  is the labor input into production, and  $b_{t,s}$  denotes the real quantity of risk-free one-period nominal bonds held by the agent at the end of period t.  $\Upsilon_{t,s}$  is the lump-sum tax collected by the government,  $R_{t-1}$  is the nominal interest rate factor between periods t-1 and t,  $P_{t,s}$  is the price of consumption good s,  $y_{t,s}$  is output of good s,  $P_t$  is the aggregate price level, and the inflation rate is  $\pi_t = P_t/P_{t-1}$ . The subjective discount factor is denoted by  $\beta$ .  $\hat{E}_0$  denotes a subjective expectations operator.

<sup>&</sup>lt;sup>6</sup>There are some papers that generalize the usual RE setup to include more significant imperfections in information than just exogenous shocks.

 $<sup>^{7}</sup>$ The framework was developed in Evans, Guse, and Honkapohja (2008). It is also employed in Evans and Honkapohja (2010), Benhabib, Evans, and Honkapohja (2014) and Honkapohja and Mitra (2015).

The utility function has the parametric form

$$U_{t,s} = \frac{c_{t,s}^{1-\sigma_1}}{1-\sigma_1} + \frac{\chi}{1-\sigma_2} \left(\frac{M_{t-1,s}}{P_t}\right)^{1-\sigma_2} - \frac{h_{t,s}^{1+\varepsilon}}{1+\varepsilon} - \frac{\gamma}{2} \left(\frac{P_{t,s}}{P_{t-1,s}} - 1\right)^2,$$

where  $\sigma_1, \sigma_2, \varepsilon, \gamma > 0$ . The final term displays the cost of adjusting prices in the spirit of Rotemberg (1982). This formulation rather than the Calvo model of price stickiness is used as it enables us to study global dynamics in the nonlinear system. The household decision problem is also subject to the usual "no Ponzi game" (NPG) condition.

Production function for good s is given by

$$y_{t,s} = h^{\alpha}_{t,s},$$

where  $0 < \alpha < 1$ . Output is differentiated and firms operate under monopolistic competition and face a downward-sloping demand curve

$$P_{t,s} = \left(\frac{y_{t,s}}{y_t}\right)^{-1/\nu} P_t.$$
(3)

Here  $P_{t,s}$  is the profit maximizing price set by firm s consistent with its production  $y_{t,s}$ . The parameter  $\nu$  is the elasticity of substitution between two goods and is assumed to be greater than one.  $y_t$  is aggregate output which is exogenous to the firm.

The government's flow budget constraint in real terms is

$$b_t + m_t + \Upsilon_t = g_t + m_{t-1}\pi_t^{-1} + R_{t-1}\pi_t^{-1}b_{t-1}, \qquad (4)$$

where  $g_t$  denotes government consumption of the aggregate good,  $b_t$  is the real quantity of government debt, and  $\Upsilon_t$  is the real lump-sum tax collected. We assume that fiscal policy follows a linear tax rule for lump-sum taxes as in Leeper (1991)

$$\Upsilon_t = \kappa_0 + \kappa b_{t-1},\tag{5}$$

where we will assume that  $\beta^{-1} - 1 < \kappa < 1$ . Thus fiscal policy is kept "passive" in the terminology of Leeper (1991).

We assume that  $g_t$  is constant and given by  $g_t = \bar{g}$ . From market clearing we have

$$c_t + \bar{g} = y_t. \tag{6}$$

Monetary policy is based on inflation targeting (IT) and is, for simplicity, specified using a forward-looking Taylor rule, which is nonlinear due to the ZLB constraint.

$$R_t - 1 = f\left(\pi_{t+1}^e\right). \tag{7}$$

For brevity, it is assumed that private agents know the interest rate rule, i.e. monetary policy is transparent. For simplicity, I have dropped the dependence of interest rate on output expectations. The function  $f(\pi)$  is taken to be positive and non-decreasing. We assume the existence of  $\pi^*$  and  $R^*$  such that  $R^* = \beta^{-1}\pi^*$  and  $f(\pi^*) = R^* - 1$ . Here  $\pi^*$  can be viewed as the inflation target of the Central Bank. and it is assumed that  $\pi^* \ge 1$ . In the numerical analysis we will use the functional form

$$f(\pi) = (R^* - 1) \left(\frac{\pi}{\pi^*}\right)^{AR^*/(R^* - 1)},$$
(8)

which implies the existence of a steady state at  $(\pi^*)$ . Using  $R^* = \pi^*\beta^{-1}$ , we obtain  $f_{\pi^*}(\pi^*) = AR^*/\pi^* = A\beta^{-1}$ . We assume that A > 1. Equations  $g_t = \bar{g}$ , (5) and (7) constitute "normal policy".

Next, I summarize the key behavioral equations. The basic framework is directly adopted from earlier papers including Evans and Honkapohja (2010) and Benhabib, Evans, and Honkapohja (2014). The derivation of these equations is outlined in the Appendix.

It is assumed that (i) utilities are logarithmic, (ii) expectations are identical and (iii) price-setters have seen that in the past their price has been equal to the aggregate price. With these assumptions the optimal price setting decisions imply an infinite horizon Phillips curve in which current decisions depend on expectations over the infinite future.<sup>8</sup>

$$Q_{t} = \frac{\nu}{\alpha\gamma} y_{t}^{(1+\varepsilon)/\alpha} - \frac{\nu-1}{\gamma} y_{t} (y_{t} - \bar{g})^{-1} +$$

$$\frac{\nu}{\gamma} \sum_{j=1}^{\infty} \alpha^{-1} \beta^{j} \left( y_{t+j}^{e} \right)^{(1+\varepsilon)/\alpha} - \frac{\nu-1}{\gamma} \sum_{j=1}^{\infty} \beta^{j} \left( \frac{y_{t+j}^{e}}{y_{t+j}^{e} - \bar{g}} \right)$$

$$\equiv \tilde{K}(y_{t}, y_{t+1}^{e}, y_{t+2}^{e}...), \text{ where}$$

$$Q_{t} = \pi_{t} (\pi_{t} - 1).$$

$$(10)$$

<sup>&</sup>lt;sup>8</sup>The formulation of infinite horizon learning in New Keynesian models is emphasized by Preston (2005) and Preston (2006), and is used in Evans and Honkapohja (2010) and Benhabib, Evans, and Honkapohja (2014) to study the properties of a liquidity trap.

It is also assumed that households act in a Ricardian way, i.e. they impose the intertemporal budget constraint (IBC) of the government  $\lim_{T\to\infty} D_{t,t+T}b_{t+T} =$ 0 in conjunction with (4), evaluated at the agents' own expectations. Here  $D_{t,t+T}$  is the discount factor from t to t+T and is formally defined in the Appendix. Using the consumer budget constraint and the iterated form of the Euler equation yields an intermediate consumption function (see the Appendix).<sup>9</sup> Then combining the household and government intertemporal budget constraints, yields the consumption function

$$c_t = (1 - \beta) \left( y_t - \bar{g} + \sum_{j=1}^{\infty} (D^e_{t,t+j})^{-1} (y^e_{t+j} - \bar{g}) \right).$$
(11)

## **3** Learning and Temporary Equilibrium

The formulation of learning needs to be discussed next. When learning, each agent is assumed to have a model for perceived dynamics of state variables, called the perceived law of motion (PLM), to make his or her forecasts of relevant variables. The PLM parameters are estimated using available data and the estimated PLM is used for forecasting in each period. When new data becomes available in the next period, the PLM parameters are re-estimated and the agent employs the re-estimated PLM in forecasting.

A common formulation is to postulate that the PLM is a linear regression model where endogenous variables depend on intercepts, observed exogenous variables and possibly lags of endogenous variables and a standard estimation method is employed.<sup>10</sup> Note that the current model is forward-looking, so there are no lagged endogenous variables. If the exogenous variables are not autocorrelated, then regressing on them does not add any information to forecasting. The procedure then amounts regressions on an unknown constant, which is equivalent to assuming that agents form expectations using so-called steady state learning.<sup>11</sup> A further justification for using steady-state learning is that the regression formulation cannot be applied in non-stochastic

 $<sup>^{9}</sup>$ Evans, Honkapohja, and Mitra (2012) state the assumptions under which Ricardian Equivalence holds along a path of temporary equilibria with learning if agents have an infinite decision horizon.

<sup>&</sup>lt;sup>10</sup>See Sargent (1993), Evans and Honkapohja (2001), Sargent (2008), and Evans and Honkapohja (2009) for discussions and techniques to study learning dynamics.

<sup>&</sup>lt;sup>11</sup>See Evans, Guse, and Honkapohja (2008), pp.1443-4 for more details.

settings.<sup>12</sup>

Steady-state learning with point expectations is formalized as

$$s_{t+j}^e = s_t^e \text{ for all } j \ge 1, \text{ and } s_t^e = s_{t-1}^e + \omega_t (s_{t-1} - s_{t-1}^e)$$
 (12)

for variables  $s = y, \pi$ . It should be noted that expectations  $s_t^e$  refer to future periods and not the current one. It is assumed that the newest available data point is  $s_{t-1}$ , i.e. expectations are formed in the beginning of the current period and current-period values of endogenous variables are not known at that moment.  $\omega_t$  is called the "gain sequence," and measures the extent to which the estimates adjust to the most recent forecast error. In stochastic systems one typically assumes "decreasing gain" learning  $\omega_t = t^{-1}$  which corresponds to least-squares updating. The case  $\omega_t = \omega$  for  $0 < \omega \leq 1$ , called "constant gain" learning, is also widely used. In this case it is assumed that  $\omega$  is small and theoretical stability conditions are stated so that they hold for all sufficiently small gains.

I now list the temporary equilibrium equations with steady-state learning under policy transparency, i.e., when the monetary policy rule is known. Under the IT regime agents substitute current values and their own forecasts of inflation and output in the policy rule. The key equations are:

1. The aggregate demand

$$y_t = \bar{g} + (\beta^{-1} - 1)(y_t^e - \bar{g}) \left(\frac{\pi_t^e}{1 + f(\pi_t^e) - \pi_t^e}\right).$$
(13)  
$$\equiv Y(y_t^e, \pi_t^e, R_t).$$

2. The nonlinear Phillips curve

$$\pi_t = Q^{-1}[\tilde{K}(y_t, y_t^e, y_t^e...)] \equiv Q^{-1}[K(y_t, y_t^e)] \equiv \Pi(y_t, y_t^e)],$$
(14)

where  $\tilde{K}(.)$  is defined in (9),  $Q(\pi_t) \equiv (\pi_t - 1) \pi_t$  in (10) and

$$K(y_t, y_t^e) \equiv \frac{\nu}{\gamma} \left( \alpha^{-1} y_t^{(1+\varepsilon)/\alpha} - (1-\nu^{-1}) \frac{y_t}{(y_t - \bar{g})} \right)$$
(15)

$$+\frac{\nu}{\gamma}\left(\beta(1-\beta)^{-1}\left(\alpha^{-1}(y_t^e)^{(1+\varepsilon)/\alpha}-\left(1-\nu^{-1}\right)\frac{y_t^e}{y_t^e-\bar{g}}\right)\right)$$

<sup>&</sup>lt;sup>12</sup>There would be asymptotic perfect multicollinearity. See Evans and Honkapohja (1998) or Section 7.2 of Evans and Honkapohja (2001) for discussions of learning in deterministic and stochastic models.

3. Money demand

$$m_t = \chi \beta \frac{R_t}{R_t - 1} c_t. \tag{16}$$

4. Bond dynamics

$$b_t + m_t = g - \Upsilon_t + \frac{R_{t-1}}{\pi_t} b_{t-1} + \frac{m_{t-1}}{\pi_t}.$$
(17)

The state variables are  $b_{t-1}$ ,  $m_{t-1}$ , and  $R_{t-1}$ . With Ricardian consumers the dynamics for bonds and money do not influence the dynamics of the endogenous variables, though the evolution of  $b_t$  and  $m_t$  is influenced by the dynamics of inflation and output.

Under IT and transparency the system has two expectational variables, output  $y_t^e$  and inflation  $\pi_t^e$ . We now assume that evolution of expectations is given by

$$y_t^e = y_{t-1}^e + \omega(y_{t-1} - y_{t-1}^e), \qquad (18)$$

$$\pi_t^e = \pi_{t-1}^e + \omega(\pi_{t-1} - \pi_{t-1}^e).$$
(19)

A fixed point of the system (18)-(19) is said to be **expectationally stable** if there exists  $\bar{\omega} > 0$  such that for all  $\omega \in (0, \bar{\omega})$  there is convergence to the fixed point from all initial conditions sufficiently near the fixed point. Dynamics and stability of fixed points of the system (18)-(19) can be studied by examining the corresponding aspects of dynamics from an associated Estability differential equation system

$$\frac{dy^e}{d\tau} = G_1(y^e, \pi^e) - y^e \frac{d\pi^e}{d\tau} = G_2(y^e, \pi^e) - \pi^e,$$

where  $\tau$  is virtual time. In particular, the local stability conditions from the E-stability differential equation are identical to the expectational stability condition for the learning system (18)-(19). We remark that this E-stability technique can be applied quite generally to adaptive learning systems that are based on least squares estimation of a forecasting model. See the references in footnote 4.

## 4 Steady States and Expectation Dynamics

A non-stochastic steady state  $(y, \pi, R)$  must satisfy the Fisher equation  $R = \beta^{-1}\pi$ , the interest rate rule (7), and the steady-state form of the equations for output and inflation (13) and (14).

One steady state clearly obtains when the actual inflation rate equals the inflation target. Then  $R = R^*$ ,  $\pi = \pi^*$  and  $y = y^*$ , where  $y^*$  is the unique solution to the equation

$$\pi^* = \Pi(Y(y^*, \pi^*, R^*, R^*), y^*)].$$

Figure 1 already illustrated that the targeted steady state under IT is not unique:

**Lemma 1** Under the Taylor rule (7), there exists a second steady state  $R_L$ ,  $\pi_L$ , and  $y_L$ , where  $y_L$  solves the equation

$$\pi_L = \Pi(Y(y_L, \pi_L, R_L, R_L), y_L).$$
(20)

The result states that IT with a Taylor rule suffers from global indeterminacy. The proof of the result is straightforward in view of the properties of the interest rate rule (7). Near but below  $\pi^*$  we have  $\beta^{-1}\pi > 1 + f(\pi)$  as the Taylor rule is locally active. In the limit  $\pi \to 0$  we have  $\beta^{-1}\pi \to 0$  while  $1 + f(\pi) \to 1$ . By continuity of  $f(\pi)$  there must be value  $\pi_L < \pi^*$  such that  $\beta^{-1}\pi_L = 1 + f(\pi_L)$ .

Next, I consider local expectational stability of the steady states under the learning dynamics (18)-(19).

**Proposition 2** The targeted steady state  $\pi^*$  is expectationally stable. The low steady state defined by  $\pi_L$  is a saddle point and thus is not expectationally stable.

The proof of this proposition is given in Evans and Honkapohja (2010).

I now begin to analyze global aspects of the learning dynamics using the phase diagram for the two-dimensional E-stability differential equation system. First, a standard calibration of the model is introduced following Benhabib, Evans, and Honkapohja (2014). The next two figures are constructed with the following parameter values A = 2.5,  $\pi^* = 1.02$ ,  $\beta = 0.99$ ,  $\alpha = 0.7$ ,  $\gamma = 350$ , v = 21,  $\varepsilon = 1$ , and g = 0.2.

While A = 1.5 is the usual value for the interest rate rule in the literature, we choose A = 2.5 to clearly separate the intended and unintended steady states in the numerical analysis (our results are robust to using A = 1.5). The calibrations of  $\pi^*$ ,  $\beta$ ,  $\alpha$ , and the approximate GDP share of government spending, g are standard. The labor supply elasticity is set to  $\varepsilon = 1$ . To calibrate  $\gamma$ , we exploit the relation of the Rotemberg and the Calvo models of costly and sticky price adjustments via their reduced form implications for the linearized Phillips curve. According to Keen and Wang (2007),  $\gamma =$  $\frac{(\nu-1)\phi}{(1-\phi)(1-\beta\phi)}$  where  $1-\phi$  is the fraction of firms changing their price during the quarter. Using Basu and Fernald (1997) we calibrate  $\nu = 21$ , implying a conservatively estimated 5% markup. The calibration  $\phi = 0.7916$  follows from the estimate of Kehoe and Midrigan (2010), p. 8 for the frequency of regular price changes from BLS data (excluding temporary changes in price) to be 14.5 months or 4.8 quarters. Using the formula above we obtain  $\gamma \approx 350$  for our calibration. The literature contains a range of estimates for the value of  $\nu$  and  $\phi$  and our qualitative results are robust to different calibrations.

I also assume that interest rate expectations  $r_{t+j}^e = R_{t+j-1}/\pi_{t+j}^e$  revert to the steady state value  $\beta^{-1}$  for  $j \ge T$ . This truncation is needed for technical reasons to prevent agents forecasting negative real interest rates indefinitely, which would imply unbounded consumption. In Figures 3 and 4 we use T = 28.

Figure 3 illustrates the global adaptive dynamics for the calibrated model. The two vertical lines constitute the isocline  $\frac{dy^e}{d\tau} = 0$ , while the upward sloping curve is the isocline  $\frac{d\pi^e}{d\tau} = 0$ . Local stability of the targeted steady state at  $\pi^e = \pi^*$  and the saddle-point property of  $\pi_L \approx 0.993$  are evident from Figure 3. The "liver-shaped" area inside curve emanating from the low steady state is the domain of attraction of the target steady state: there is convergence to the target from all initial conditions inside the "corridor of stability" Leijonhufvud (1973).

It is seen that convergence to  $\pi^*$  is cyclical and that there is a heteroclinic orbit from the low steady state to the target steady state. All starting points outside the corridor of stability eventually lead to a deflation trap in which inflation and output expectations as well as their actual values gradually decline to lower levels unless economic policies are somehow changed from the normal policies.

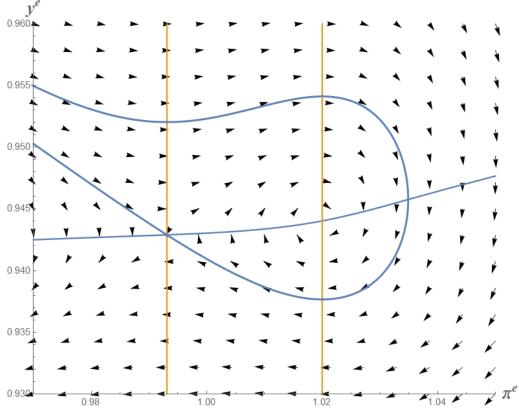


Figure 3: Global learning dynamics – the Ricardian case.

## 5 Avoiding Liquidity Traps

The preceding analysis poses a challenge for macroeconomic policy. If big shocks can result in unstable dynamics of expectations and actual variables leading to the ZLB regime, what are appropriate policies to avoid convergence to a liquidity trap? Related, what kind of policies provide escape from the trap after the economy has fall into it?

Before proceeding, it can be noted as a first observation that as consumers are Ricardian, the increasing financial wealth of consumers does not lead to a sufficiently strong response in consumption as in this case there are no Pigou or wealth effects from financial wealth (money and bonds). Pigou (1943) and Patinkin (1965) argued for real balance or wealth effects as a mechanism that can avoid the liquidity trap. Benhabib, Evans, and Honkapohja (2014) have recently shown that with non-Ricardian consumers the economy can escape a liquidity trap, provided that fiscal policy is set in an appropriate way. They also show that resulting adjustment paths are cyclical and involve wide fluctuations in inflation and output.

#### 5.1 Monetary Policy and Asset Purchases

What about monetary policies against the expectations driven liquidity trap? One possibility would be to switch away from a Taylor rule to a money supply rule when inflation falls to a very low level. Using a learning framework, Evans and Honkapohja (2005) suggest that in a flexible price economy switching to a money supply rule at a low threshold level of inflation can succeed in avoiding the liquidity trap. Benhabib, Schmitt-Grohe, and Uribe (2002) derive a similar result under RE. These results are, however, of limited significance as the case of sticky prices was not considered.

In the standard model developed above, a switch to a money supply rule can be viewed as an asset purchase program since the issue of additional money leads to a corresponding reduction in the issue of government bonds held by the private agents.<sup>13</sup> Recalling the government flow budget constraint (4), I now interpret the stock of bonds consisting of those held by the households  $b_t^H$  and by the central bank  $b_t^{CB}$ , so that

$$b_t = b_t^H + b_t^{CB}$$

or

$$B_t = B_t^H + B_t^{CB}$$

in nominal terms. Flows relating to money and bonds in the budget constraint for the central bank are linked together by

$$M_t - M_{t-1} = B_t^{CB} - R_{t-1}B_{t-1}^{CB} = B_t^{CB} - B_{t-1}^{CB} - (R_{t-1} - 1)B_{t-1}^{CB},$$

where  $R_{t-1}$  is gross nominal interest rate, so that an increase in the money stock  $M_t - M_{t-1}$  increases the bond stock held by the central bank by the amount  $B_t^{CB} - B_{t-1}^{CB} - (R_{t-1} - 1)B_{t-1}^{CB}$ .

<sup>&</sup>lt;sup>13</sup>The simple model cannot take account of other aspects of asset purchases that focus on e.g. improving financial intermediation.

For concreteness, it is assumed that the asset purchase program is introduced at some low threshold level for actual inflation and that this program increases the nominal money supply at some constant rate (the so-called Friedman rule)<sup>14</sup>

$$M_{t+1} = (1 + dM)M_t.$$

Using the money-consumption optimality condition (16), the implied interest rate is

$$R_t = \frac{m_t}{m_t - \chi(1-\beta)(y_t^e - \bar{g})\left(\frac{\pi_t^e}{R_t}\right)\left(\frac{\pi_t^e}{R_t^e - \pi_t^e}\right)}.$$
(21)

The key mechanism here is that a switch to the money supply rule may lower the nominal interest rate further even though it has already been at a low positive level (compare Figure 1).

Here the future values of the interest rate, denoted as  $R_t^e$ , can be formally solved from the equation (21) after setting  $R_t = R_t^e$  and  $m_t = m_t^e$ . The solution for temporary equilibrium also requires specifying future values for real balances  $m_t^e$ . As real balance dynamics depend on price level developments, it is assumed that agent forecast future real balances in accordance with

$$m_t^e = (1 + dM)^2 m_{t-1} / (\pi_t^e)^2.$$
(22)

Equation (22) involves the assumption that for future periods agents have steady-state expectations of inflation.

Is this kind of asset purchase program effective? The answer depends on the state of the economy at the point of switching to the asset purchase program / money supply rule.

Figure 4 imposes a dashed curve on Figure 3 indicating the combinations of y(0) and  $\pi(0)$  that lie the boundary between convergence to and divergence from the target steady state for dynamics under the money supply rule. There is convergence to the target steady state from any starting point in the area to the right of the dashed curve.<sup>15</sup>

 $<sup>^{14}</sup>$ See Evans and Honkapohja (2003) for local analysis of the learning dynamics near the target steady state for the Friedman rule. Their analysis uses a linearized stochastic model.

<sup>&</sup>lt;sup>15</sup>The calibration is as before, to which specifications  $\chi = 0.03$ ,  $\kappa_0 = 0.005$  and  $\kappa = \beta^{-1} - 1 + 0.05$  are added. Regarding initial conditions it is assumed that  $R(0) = R_L + 0.0001$ ,  $y^e(0) = y(0)$  and  $\pi^e(0) = \pi(0)$ .

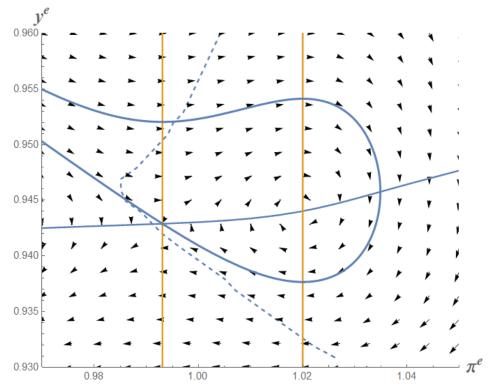


Figure 4: Boundary of effectiveness for asset purchase program

It can be seen that this area of convergence covers most of the domain of attraction for the Taylor rule (except some unlikely parts of the state space), so in this sense it is an improvement to the inflation targeting policy with a Taylor rule. However, an asset purchase program is not a fool-proof solution to the problem of the liquidity trap. In particular, the asset purchase program must be started before the inflation rate of the economy gets too low.

### 5.2 Targeting of Prices or Nominal GDP

One limitation of inflation targeting is that a Taylor rule with an announced constant inflation target tells very little about future policies apart from the target. In other words, it does not provide much forward guidance which idea has been employed as a form of unconventional monetary policy during the crisis. Price level or nominal GDP targeting with an announced target path for the price level or nominal GDP in the future is a specific way for giving additional forward guidance.<sup>16</sup> Recently, some policy makers have suggested price level or nominal GDP targeting as an alternative to inflation targeting; see Carney (2012) and Evans (2012).

Using the learning approach Honkapohja and Mitra (2015) have recently shown that either price-level or nominal GDP targeting can provide a good way to avoid or get out of a expectations-driven liquidity trap. Their major finding is that the additional forward guidance in price level and nominal GDP targeting can be used to overcome the trap. The key condition for this result is that monetary policy is credible in the sense that private agents incorporate the future path of the target variable into the formation of expectations.

## 6 Other Topics: Comments on the Literature

### 6.1 Fiscal Policy

The use of fiscal policy is a commonly suggested response to the expectations driven liquidity trap. One strand of the literature relies firmly on the RE assumption. As already noted, much of the applied literature has focused on the case of a fundamental liquidity trap caused by a fundamental shock.

A number of papers argue that fiscal policy has large spending multipliers in the ZLB regime and that cuts in marginal taxes are not effective, see e.g. Christiano, Eichenbaum, and Rebelo (2011), Eggertsson (2010), Woodford (2011) and Coenen et al. (2012). These papers employ the concept of the fundamental liquidity trap. The literature on fiscal policies in expectations driven liquidity traps is less extensive. The latter are caused by an expectational shock and there need not be any change in the fundamentals of the economy. A key observation is that the impact of fiscal policy depends on the nature of the liquidity trap: multipliers tend to be small and tax cuts can effective in an expectations-driven liquidity trap, see Aruoba, Cuba-Borda, and Schorfheide (2014) and Mertens and Ravn (2014). This is a major difference to the literature on fundamental liquidity traps.<sup>17</sup>

<sup>&</sup>lt;sup>16</sup>For price-level targeting see for example Svensson (1999) and Vestin (2006). For nominal income targeting see Hall and Mankiw (1994), Jensen (2002) and Mitra (2003). A recent overview of nominal income targeting is in Bean (2009).

<sup>&</sup>lt;sup>17</sup>The literature referenced relies heavily on the RE assumption. One could also con-

A different viewpoint is to consider the possibilities for designing fiscal and monetary policies that have potential to lift the economy out of such a liquidity trap. Using the RE assumption, Benhabib, Schmitt-Grohe, and Uribe (2002) suggest an inflation sensitive tax policy to rule out  $\pi_L$  as a steady state via mechanism of violating of the transversality condition at  $\pi_L$ . Werning (2012) constructs a combination of fiscal and monetary optimal policies that under RE lead the economy out of the fundamental liquidity trap.

The learning view suggests a different view about fiscal policy as a tool for avoiding (expectations-driven) liquidity traps when knowledge of agents and the government is imperfect. Government spending can be used as defence against too low levels of inflation, so that the spending reacts to a low inflation threshold as trigger, see e.g. Evans, Guse, and Honkapohja (2008), Evans and Honkapohja (2010) and Benhabib, Evans, and Honkapohja (2014). Appropriately designed fiscal switching rule can be a "fool-proof" policy against the expectations driven liquidity trap.

#### 6.2 Financial Intermediation and Quantitative Easing

Major worsening of financial intermediation has been one of the characteristics of the global financial crisis. It has been argued that various types of financial frictions or constraints emerged as part of the crisis and alleviation of these constraints has been one objective in the APP's of central banks. Such frictions or constraints can help by creating portfolio balance adjustments when large-scale APP's have been executed. The effectiveness and implications of APP's have been studied using extended NK models when the RE equilibrium of the economy is a fundamental liquidity trap. The policies usually try to alleviate financial constraints or frictions.

These frictions has been in the focus of a sizeable research effort in a number of papers that extend standard NK models include financial intermediation problems. Gertler and Karadi (2011) and Gertler and Karadi (2013) consider a few different unconventional monetary policies taking the form of direct lending to firms from proceeds from sales of government bonds, purchases of private assets and purchases of government bonds. The model

sider fiscal policy and multipliers during a disequilibrium adjustment process in a learning framework. For this in an RBC framework see Mitra, Evans, and Honkapohja (2013) and Mitra, Evans, and Honkapohja (2015).

of Gertler and Kiyotaki (2011) emphasizes financial constraints faced by financial intermediaries. They consider the consequences of direct lending to firms, offers of credit to banks at penalty rate and direct equity injections.

Curdia and Woodford (2011) analyze policies that manipulate bank reserves through purchases of treasuries and credit easing and can alleviate imperfections in private finance. These policies can be effective especially if the interest rate is bound by the ZLB. The model of Del Negro, Eggertsson, Ferrero, and Kiyotaki (2013) has both borrowing and resale constraints on the private sector, where the latter is meant capture liquidity problems. Del Negro et al argue that collapse of liquidity for private assets was central in the Great Recession. Araujo, Schommer, and Woodford (2015) emphasize constraints that stem from collateral requirements for private assets and argue that central bank policies are irrelevant when collateral is abundant but can influence markets when collateral constraints are present.

### 6.3 Empirical Studies

A sizeable number of studies have focused on empirical estimation of the impact of a large-scale APP. These studies use different methodologies including estimated DSGE models and pure econometric models. The focus of these studies has been on the impact of APP's on financial and macro-economic variables. Impact of APP's on long-term interest rates has been considered in a number of studies.<sup>18</sup> The studies indicate that estimated effects on 10-year interest rates in the US have been in the range of 0.3-1.0 percentage units.<sup>19</sup> Similar results have been obtained for the UK.

There are only a handful of studies on the impact of APP's on GDP growth and inflation though new studies frequently appear. Here are examples of these studies. Chung, Laforte, Reifschneider, and Williams (2012) use a range of statistical and structural models to examine the likelihood of ZLB episodes as constraints on central bank policies and the impact of federal Reserve APP's. Chen, Curdia, and Ferrero (2012) use a DSGE model with bond market segmentation to estimate the impacts of the Fed's second APP on growth and inflation. Gambacorta, Hofmann, and Peersman (2014) uses

<sup>&</sup>lt;sup>18</sup>See e.g. Cecioni, Ferrero, and Secchi (2011) for a review and Krishnamurty, Nagel, and Vissing-Jorgensen (2014) for a recent study.

<sup>&</sup>lt;sup>19</sup>To calibrate the impact, it can be noted that a 0.5 percent reduction in the interest rate would require a two percentage units reduction in the policy rate.

a panel VAR model with monthly data from eight advanced economies to evaluate the macroeconomic effects of unconventional monetary policies.

These studies are in broad agreement about the qualitative effects, so that APP's tend to raise GDP growth and increase the inflation rate for a period. However, the quantitative estimates of the macroeconomic effects vary a great deal between the studies.

## 7 Conclusions

I have discussed the recent literature on the interest-rate zero lower bound and liquidity traps. When coupled with the zero lower bound interest rate, monetary policy rules can create a problem of multiple equilibria, so that a steady state with low inflation or even deflation exists in addition to the usual targeted steady state with positive inflation. One part of the literature has ignored the equilibrium multiplicity problem as the liquidity trap is formally treated as resulting a shift in the targeted equilibrium to near the ZLB constraint caused by a fundamental shock. This shock must not be too persistent to avoid non-existence of the targeted equilibrium.

Another part of the literature takes the equilibrium multiplicity problem seriously and looks for ways to avoid problems created by the low-inflation equilibrium. My overview has focused on the latter literature. Moreover, I have emphasized the learning approach to economic dynamics that, if left unchecked, may take the economy to the ZLB regime and deflation. Such a process can arise from an sufficiently large initial negative shock to expectations and confidence.

I discussed two monetary policies to counter the liquidity trap. These are monetary easing through an asset purchase program and or a move to pricelevel or nominal GDP targeting. The former can be successful, provided it is initiated sufficiently early, so that the deflationary process has not gone very far. The latter can be a fool-proof way to cure a liquidity trap that has arisen from confidence problems but is effective only if the forward guidance from the nominal level targeting is credible in the minds of the private agents.

I have also given a brief summary, with references, on other topics of interest in connection with liquidity traps. These include fiscal policy, policies to improve financial intermediation and empirical studies.

## Appendix

## A Deriving the Phillips Curve and the Consumption Function

As in Evans, Guse, and Honkapohja (2008), the first-order conditions for an optimum yield

$$0 = -h_{t,s}^{\varepsilon} + \frac{\alpha \gamma}{\nu} (\pi_{t,s} - 1) \pi_{t,s} \frac{1}{h_{t,s}}$$

$$+ \alpha \left( 1 - \frac{1}{\nu} \right) y_t^{1/\nu} \frac{y_{t,s}^{(1-1/\nu)}}{h_{t,s}} c_{t,s}^{-\sigma_1} - \frac{\alpha \gamma \beta}{\nu} \frac{1}{h_{t,s}} E_{t,s} (\pi_{t+1,s} - 1) \pi_{t+1,s}.$$
(23)

$$c_{t,s}^{-\sigma_1} = \beta R_t E_{t,s} \left( \pi_{t+1}^{-1} c_{t+1,s}^{-\sigma_1} \right)$$
(24)

and

$$m_{t,s} = (\chi\beta)^{1/\sigma_2} \left( \frac{\left(1 - R_t^{-1}\right) c_{t,s}^{-\sigma_1}}{E_{t,s} \pi_{t+1}^{\sigma_2 - 1}} \right)^{-1/\sigma_2},$$
(25)

where  $\pi_{t+1,s} = P_{t+1,s}/P_{t,s}$ .

#### A.1 The Nonlinear Phillips Curve

Equation (23) is the nonlinear New Keynesian Phillips curve describing the optimal price-setting by firms. The term  $(\pi_{t,s} - 1) \pi_{t,s}$  arises from quadratic adjustment costs, and this expression is increasing in  $\pi_{t,s}$  over the allowable range  $\pi_{t,s} \geq 1/2$ . The first term on the right-hand side is the marginal disutility of labor while the third term can be viewed as the product of the marginal revenue from an extra unit of labor with the marginal utility of consumption. The terms involving current and future inflation arise from the price-adjustment costs.

Defining

$$Q_{t,s} = (\pi_{t,s} - 1) \,\pi_{t,s},\tag{26}$$

and using the appropriate root  $\pi \geq \frac{1}{2}$  for any given  $Q \geq -\frac{1}{4}$ , the production function and the demand curve, we can iterate the Euler equation (23) as

$$Q_{t,s} = x_{t,s} + \sum_{j=1}^{\infty} \beta^j E_{t,s} x_{t+j,s},$$
(27)

provided that the transversality condition  $\beta^j E_{t,s} x_{t+j,s} \to 0$  as  $j \to \infty$  holds.<sup>20</sup> Here

$$x_{t,s} \equiv \frac{\nu}{\alpha\gamma} (P_{t,s}/P_t)^{-(1+\varepsilon)\nu/\alpha} Y_t^{(1+\varepsilon)/\alpha} - \frac{\nu-1}{\gamma} y_t (P_{t,s}/P_t)^{-(\nu-1)} c_{t,s}^{-\sigma_1}.$$

The variable  $x_{t+j,s}$  is a mixture of aggregate variables and the agent's own future decisions.

We now make some further adaptive learning assumptions. First, agents are assumed to have point expectations. Second, agents are assumed to have learned from experience that  $P_{t,s}/P_t = 1$  in temporary equilibrium, so that they set  $(P_{t+j,s}/P_{t+j})^e = 1$ . Third, agents have learned from experience that  $c_{t,s} = y_t - g_t$  in per capita terms and thus agents impose in their forecasts that  $c_{t+j,s}^e = y_{t,t+j}^e - g_{t,t+j}^e$ . With no fiscal policy changes this becomes  $c_{t+j,s}^e = y_{t+j,s}^e = g_t^e$ .

Next, the representative agent assumption is invoked so that all agents s have the same initial money and debt holdings and also make the same forecasts  $c_{t+1,s}^e \pi_{t+1,s}^e$ , as well as forecasts of other relevant variables. Under these assumptions  $h_{t,s} = h_t$ ,  $y_{t,s} = y_t$ ,  $c_{t,s} = c_t$  and  $\pi_{t,s} = \pi_t$ , and all agents make the same forecasts. For convenience, the utility of consumption and of money is also taken to be logarithmic ( $\sigma_1 = \sigma_2 = 1$ ).

Then (27) takes the form

$$Q_{t} = \frac{\nu}{\alpha\gamma} y_{t}^{(1+\varepsilon)/\alpha} - \frac{\nu-1}{\gamma} y_{t} (y_{t} - \bar{g})^{-1} +$$

$$\frac{\nu}{\gamma} \sum_{j=1}^{\infty} \alpha^{-1} \beta^{j} \left( y_{t+j}^{e} \right)^{(1+\varepsilon)/\alpha} - \frac{\nu-1}{\gamma} \sum_{j=1}^{\infty} \beta^{j} \left( \frac{y_{t+j}^{e}}{y_{t+j}^{e} - \bar{g}} \right)$$

$$\equiv \tilde{K}(y_{t}, y_{t+1}^{e}, y_{t+2}^{e}...),$$

$$(28)$$

where  $Q_t = (\pi_t - 1) \pi_t$ . The expectations are formed at time t where at the time of forecasting variables at time t are not in the information set of the agents. Period t variables are known to agents at the moment of current decision making. (28) is the price-setting equation that determines  $\pi_t$  for given expectations  $\{y_{t+j}^e\}_{j=1}^\infty$ .

 $<sup>^{20}</sup>$ The condition is an implication of the necessary transversality condition for optimal price setting, see Benhabib, Evans, and Honkapohja (2014).

#### A.2 The Consumption Function

Equation (24) is the standard Euler equation giving the intertemporal firstorder condition for the consumption path. Equation (25) is the money demand function. To derive the consumption function one defines the asset wealth  $a_t = b_t + m_t$  and the flow budget constraint

$$a_t + c_t = y_t - \Upsilon_t + r_t a_{t-1} + \pi_t^{-1} (1 - R_{t-1}) m_{t-1}, \qquad (29)$$

where  $r_t = R_{t-1}/\pi_t$ . Note that I assume  $(P_{jt}/P_t)y_{jt} = y_t$ . Iterating (29) forward and imposing

$$\lim_{j \to \infty} (D_{t,t+j}^{e})^{-1} a_{t+j}^{e} = 0, \text{ where}$$

$$D_{t,t+j}^{e} = \frac{R_{t}}{\pi_{t+1}^{e}} \prod_{i=2}^{j} \frac{R_{t+i-1}^{e}}{\pi_{t+i}^{e}}$$
(30)

with  $r_{t+i}^e = R_{t+i-1}^e/\pi_{t+i}^e$ , I obtain the life-time budget constraint of the house-hold<sup>21</sup>

$$0 = r_t a_{t-1} + \Phi_t + \sum_{j=1}^{\infty} (D^e_{t,t+j})^{-1} \Phi^e_{t+j}$$
(31)

$$= r_t a_{t-1} + \phi_t - c_t + \sum_{j=1}^{\infty} (D_{t,t+j}^e)^{-1} (\phi_{t+j}^e - c_{t+j}^e), \qquad (32)$$

where

$$\Phi_{t+j}^{e} = y_{t+j}^{e} - \Upsilon_{t+j}^{e} - c_{t+j}^{e} + (\pi_{t+j}^{e})^{-1} (1 - R_{t+j-1}^{e}) m_{t+j-1}^{e}, \qquad (33)$$

$$\phi_{t+j}^{e} = \Phi_{t+j}^{e} + c_{t+j}^{e} = y_{t+j}^{e} - \Upsilon_{t+j}^{e} + (\pi_{t+j}^{e})^{-1} (1 - R_{t+j-1}^{e}) m_{t+j-1}^{e}.$$

Invoking the relations

$$c_{t+j}^e = c_t \beta^j D_{t,t+j}^e, \tag{34}$$

which are an implication of the consumption Euler equation (24), and using (25) I obtain the consumption function

$$c_t \frac{1+\chi\beta}{1-\beta} = r_t b_{t-1} + \frac{m_{t-1}}{\pi_t} + y_t - \Upsilon_t + \sum_{j=1}^{\infty} (D_{t,t+j}^e)^{-1} (y_{t+j}^e - \Upsilon_{t+j}^e).$$

<sup>&</sup>lt;sup>21</sup>Here all expectations are formed in period t, which is indicated in the notation for  $D_{t,t+j}^e$  but is omitted from the other expectational variables.

Assuming that households act in a Ricardian way, i.e. they impose the intertemporal budget constraint (IBC) of the government, modifies the consumption function as in Evans and Honkapohja (2010). From (4) and assuming  $\lim_{T\to\infty} D_{t,t+T}b_{t+T} = 0$ , one obtains

$$0 = r_t b_{t-1} + \Delta_t + \sum_{j=1}^{\infty} D_{t,t+j}^{-1} \Delta_{t+j}, \qquad (35)$$

where  $\Delta_t = \bar{g} - \Upsilon_t - m_t + m_{t-1}\pi_t^{-1}$ . Combining the two budget constraints, iterating forward and using (34) yields the consumption function

$$c_t = (1 - \beta) \left( y_t - \bar{g} + \sum_{j=1}^{\infty} (D_{t,t+j}^e)^{-1} (y_{t+j}^e - \bar{g}) \right).$$
(36)

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