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**Working Paper**

## Density Forecasts of Polish Industrial Production: a Probabilistic Perspective on Business Cycle Fluctuations

Institute of Economic Research Working Papers, No. 75/2017

**Provided in Cooperation with:**

Institute of Economic Research (IER), Toruń (Poland)

*Suggested Citation:* Mazur, Blazej (2017) : Density Forecasts of Polish Industrial Production: a Probabilistic Perspective on Business Cycle Fluctuations, Institute of Economic Research Working Papers, No. 75/2017, Institute of Economic Research (IER), Toruń

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**Density Forecasts of Polish Industrial Production: a Probabilistic Perspective on Business Cycle Fluctuations**

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**Article prepared and submitted for:**

**9<sup>th</sup> International Conference on Applied Economics Contemporary Issues in Economy, Institute of Economic Research, Polish Economic Society Branch in Toruń, Faculty of Economic Sciences and Management, Nicolaus Copernicus University, Toruń, Poland, 22-23 June 2017**

**Toruń, Poland 2017**

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## Density Forecasts of Polish Industrial Production: a Probabilistic Perspective on Business Cycle Fluctuations<sup>1</sup>

**JEL Classification:** *E37; C53*

**Keywords:** *density forecasts; Bayesian inference; business cycle; Dynamic Conditional Score models; Generalized  $t$  distribution.*

### Abstract

**Research background:** Current approaches used in empirical macroeconomic analyses use the probabilistic setup and focus on evaluation of uncertainties and risks, also with respect to future business cycle fluctuations. Therefore, forecast-based business conditions indicators should be constructed using not just point forecasts, but rather density forecasts. The latter represent whole predictive distribution and provide relevant description of forecast uncertainty.

**Purpose of the article:** We discuss a problem of model-based probabilistic inference on business cycle conditions in Poland. In particular we consider a model choice problem for density forecasts of Polish monthly industrial production index and its selected sub-indices. Based on the results we develop indicators of future economic conditions constructed using probabilistic information on future values of the index.

**Methodology/methods:** In order to develop a relevant model class we make use of univariate Dynamic Conditional Score models with Bayesian inference methods. We assume that the conditional distribution is of the generalized  $t$  form in order to allow for heavy tails. Another group of models under consideration relies on the idea of business cycle modelling using the Flexible Fourier Form. We compare performance of alternative models based on ex-post evaluation of density forecasting accuracy using such criteria as Log-Predictive Score (LPS) and Continuous Ranked Probability Score (CRPS).

**Findings & Value added:** The assessment of density forecasting performance for Polish industrial production index turns out to be difficult since it depends on the

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<sup>1</sup> This research was supported by the Polish National Science Center based on decision number DEC-2013/09/B/HS4/01945

choice of verification window. The pre-2013 data supports the deterministic cycle model whereas more recent observations can be explained by a very simple mean-reverting Gaussian AR(4) process. This provides an indirect evidence indicating the change of pattern of Polish business cycle fluctuations after 2013. A probabilistic indicator of business conditions is also sensitive to details of its construction. The results suggest application of forecast pooling strategies as a goal for further research.

## **Introduction**

The purpose of the paper is to set up a methodology that allows for practical predictive business cycle analysis based on an industrial production index. Inference about future evolution of business cycle conditions should be model-based and take into account the estimation and the prediction uncertainty. In the paper we make an effort to present such a model and apply it to Polish data on industrial production. In order to do so we consider a menu of alternative specifications and discuss their properties as well as out-of-sample predictive performance. We make use of the models to construct a probabilistic index describing future prospects as to the growth rate of the industrial production index.

The shift in attention from point forecasts to probabilistic (or density) forecasts is quite widespread in the recent econometric literature (see e.g. Clark & Ravazzolo (2015)). An influential paper summarizing the state-of-the-art as to formal, statistical evaluation of density forecasts was given by Gneiting and Raftery (2007). The paper contains references to so-called proper scoring rules and strictly proper scoring rules that should be used for *ex-post* evaluation of density forecasts. Such criteria are log-predictive score (LPS) and continuous ranked probability score (CRPS) which are used in the empirical part of the paper.

Another related issue is that of structural change. It is not impossible that the underlying economic process driving the business cycle fluctuations is not time-homogenous. Hence it is necessary to consider the problem of possible changes in adequacy of the competing models. In the case of Polish economy there is a number of reasons that support the view that some sort of structural change might underlie the economic growth. For example it is not obvious that the pattern of business cycle fluctuations that has been identified for the Polish economy before say 2013 can be still considered adequate afterwards.

## **Method of the Research**

The methodological part of the paper deals with the following problems. Firstly, the model classes under consideration are introduced. The models can be divided into two groups: the ones that explicitly account for the cyclical properties of the data and the ones that rely on more sophisticated stochastic properties. The first group of models make use of so-called Flexible Fourier Form in order to represent the business cycle fluctuations. However, the models have rather simple stochastic properties, relying on the autoregressive formula with conditionally Gaussian observations. The other group of models make use of the idea of Dynamic Conditional Score (DCS) approach of Harvey (2013) and make use of more flexible conditional distribution, being the generalized  $t$  distribution. The basic structure of the models is recalled and references are given for more detailed description.

The Bayesian model for analysis of deterministic cycle used here is described by Lenart & Mazur (2016) and its application for in-sample business cycle analysis is considered by Lenart et. al. (2016). The underlying idea is close to that of cyclostationarity. It is assumed that short-term deviations from the time-varying mean  $\mu_t$  are represented by a Gaussian autoregressive process denoted by  $v_t$ :

$$y_t = \mu_t + v_t, \quad v_t = \psi_1 v_{t-1} + \dots + \psi_p v_{t-p} + \varepsilon_t, \quad \varepsilon_t \sim iiN(0, \sigma^2)$$

where  $y_t$  represents the observed series of year-on-year growth rates. The Flexible Fourier time-varying mean is given by:

$$\mu_t = \sum_{f=1}^F (\alpha_{1,f} \sin(t\phi_f) + \alpha_{2,f} \cos(t\phi_f)).$$

The parameters denoted by  $\phi_f \in (\phi_L, \phi_U) \subseteq (0, \pi)$  represent frequencies of the fluctuations and the fixed lower and upper bounds (denoted by  $\phi_L$  and  $\phi_U$ ) can be used in order to restrict attention to cyclical fluctuations of specific period length (i.e. to exclude fluctuations with a period that is either too long or too short). The flexibility of the cyclical part of the model depends on the number of the Fourier components denoted by  $F$ . An interesting feature of the model is that it allows for  $F > 1$  which implies that the business cycle fluctuations are driven by components with just more than one empirically important frequency. In practical applications we restrict  $F$  not to exceed 3 as higher values might lead to overfitting issues. Statistical inference in such a model within the Bayesian setup is rather complicated – it is described in detail by Lenart & Mazur (2016).

The generalized  $t$  distribution used here is described by Theodossiou (1998), see also the discussion by Harvey & Lange, 2016. Its probability density function has the following form:

$$p(y) = \frac{1}{\sigma} K(\nu, \gamma) \left[ 1 + \frac{1}{\nu} \left( \frac{y - \mu}{\sigma} \right)^\gamma \right]^{-\frac{(1+\nu)}{\gamma}}$$

with:

$$K(\nu, \gamma) = \frac{\gamma}{2\nu^{1/\gamma}} \frac{1}{B\left(\frac{\nu}{\gamma}, \frac{1}{\gamma}\right)}$$

The distribution given above has location parameter  $\mu$ , scale parameter  $\sigma$  and two shape parameters:  $\nu$  and  $\gamma$ . An interesting feature of this symmetric probability distribution is that it allows for heavy tails and nests a number of known distributions as nested or limiting cases. For example as  $\gamma = 2$ , it becomes Student- $t$  with  $\nu$  degrees of freedom. On the other hand, with  $\nu \rightarrow \infty$ , the limiting case is GED( $\gamma$ ), denoting the generalized error distribution, see e.g. Harvey & Lange (2016) for more detailed discussion. Such a distribution is quite flexible and therefore capable of capturing many empirically relevant situations, especially related with occurrence of rare events.

The class of Dynamic Conditional Score (DCS) models have been presented in detail by Harvey (2013). The model class is closely related to Generalized Autoregressive Score models of Creal et. al. (2013). In the paper we follow the formulation by Harvey. However, our contribution is in developing a methods of Bayesian inference for the models.

The structure of Dynamic Conditional Score modelling reflects the idea that for the given conditional distribution of the data some of its features can be dynamically updated. In the case of DCS models the updating mechanism explicitly depends on the score of the conditional distribution (i.e. partial derivative of the log-density w.r.t. to the parameter under consideration). In other words, properties of the updating mechanism depend on properties of the conditional distribution, which is a very appealing concept. The general idea together with a number of application is described by Harvey (2013). Here we assume that the following formulation holds:

$$g_t = \eta_1 g_{t-1} + \dots + \eta_p g_{t-p} + \phi_1 s_{t-1} + \dots + \phi_q s_{t-q}$$

where  $g_t$  represents the deviation of the feature under consideration from its average (or seasonally changing) state:  $g_t = f_t - \delta_t$ , with  $\delta_t$  being either time invariant ( $\delta_t = \delta$ ) or seasonal ( $\delta_t = \delta_s$ ) with the initial conditions described by  $g_0 \dots g_{-p+1}$ . Moreover,  $s_t$  is the value of the score at the point corresponding to the realized observation ad time  $t$ .

We assume that the feature being updated ( $f_t$ ) corresponds to the conditional location or the conditional scale. These represent the respective parameters of the Generalized  $t$  distribution introduced above. Consequently, the model is not formulated in terms of conditional moments, though due to

symmetry of the distribution the relationship between scale and variance is not that complicated. For the scale parameter it is necessary to add so-called linking function that maps its values into the real line. When more than one feature is being updated, it is possible to consider a matrix version of the dynamic equation. However, the path is not pursued here. We assume that the updating mechanism is diagonal, i.e. works separately for each feature (however, there exists a relationship between the expression for score for the scale and the location parameters, see Harvey & Lange, 2016).

The standard criteria for *ex-post* evaluation of the forecasts include RMSFE and MAE. However, the quantities apply to the point forecasts only and hence convey some information about adequacy of the location of the forecast, but completely ignore its dispersion (or other features of the distribution). However, from the decision-making point of view it is quite obvious, that such a strong information reduction might be innocuous under very special conditions only. Here we assume that it is necessary to include other criteria for forecast evaluation as well.

Finally, we propose an index of future business conditions that relies on full predictive distribution of the year-on-year growth rates. The index is intended to be evaluated using monthly data (in order to keep the inflow of new information). However, the y-on-y growth rates at monthly frequency often display considerable short-term variation (even in the case of calendar-adjusted data). We therefore assume that the index represents the probability that the average growth rate for the period covering  $t+4$ ,  $t+5$ ,  $t+6$  is greater than the average growth rate for  $t+1$ ,  $t+2$  and  $t+3$ . In other words it measures the probability of the tendency for the growth rates to increase during the next six months (on average). Alternatively, it could be computed with 6M or 12M base period (instead of 3M). However, it must be emphasized that the index does not convey information as to the magnitude of the growth, dealing just with the direction of change in the growth rate dynamics. However, it might be interpreted as reflecting future prospects as to the growth cycle, taking into account prediction uncertainty and dynamic dependence between forecasts for various horizons.

## **Empirical analysis of Polish industrial production data**

The dataset under consideration i.e. year-to-year growth rates of Polish industrial production (in per-cents, monthly data, adjusted for calendar effects, not seasonally adjusted, 1997M01-2016M12,  $T = 240$ ) is depicted in Figure 1.

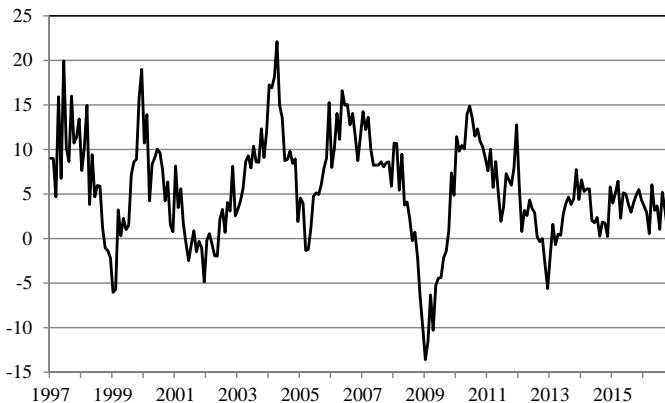
We treat the first 120 observations (10 years) as a training sample and verify the out-of-sample predictive performance of alternative models on

the remaining 120 data points. The forecasts are generated within the expanding-window setup. No effort is made to mimic real-time data flow – the most recent readouts available are used.

We consider only the direct forecasts of y-o-y growth rates. This is because a preliminary analysis using various DCS models indicates that such specifications are unable to deliver non-trivial forecasts for horizons greater than 12 months (for higher horizons the sequence of point forecasts implied y-o-y forecast is practically flat, which is obviously inadequate).

Consequently, in what follows we consider the models estimated on y-o-y data only and use the direct approach for the sake of prediction. We make use of two Gaussian autoregressive models, one with 4 lags and one with 22 lags (labeled AR(4) and AR(22)).

**Figure 1.** Growth rates of Polish industrial production index (y-o-y, in [%]).



Source: Eurostat

The models are chosen to represent complicated and simple structures of the autocorrelation function. Moreover, we consider a deterministic cycle model, with  $F=3$ , frequency parameters restricted to the  $(0.052, 0.52)$  interval and 22 lags in the autoregressive part (labeled AR(22)-F(3)). The last specification under consideration is a DCS model with  $p = q = 6$  for the location parameter and  $p = q = 2$  for the log-scale parameter. The model allows for asymmetric response to the score (following Harvey & Lange, 2016) and its conditional distribution is of the generalized  $t$  form (asy-Gt-DCS(6,6;2,2), labeled DCS for short).

Table 1 contains basic summary characteristics for *ex-post* evaluation of point and density forecasts obtained from the models mentioned above. The



results are reported for horizons of 12, 18 and 24 months ahead. Moreover, the results are also calculated using the last 36 realized forecasts only (the last observation used for the purpose of evaluation is that representing 2017M01).

**Table 1.** *Ex-post* evaluation of point and density forecasts for 12, 18 and 24 months ahead. LPS is computed using natural logs (cumulated), CRPS is in positive orientation (the lower the better, averaged).

	$h = 12$			$h = 18$			$h = 24$		
	RMSE	LPS	CRPS	RMSE	LPS	CRPS	RMSE	LPS	CRPS
full sample analysis									
AR(4)	6.46	-359.47	3.52	6.5	-340.84	3.58	5.58	-314.52	3.3
AR(22)	5.83	-351.57	3.3	5.87	-332.48	3.32	5.12	-308.72	3.1
AR(22)-F(3)	6.72	-363.96	3.96	6.8	-347.25	3.97	6.36	-325.44	3.79
DCS	6.77	-370	3.82	6.83	-350.14	3.85	6.13	-323.79	3.61
last 36 observations									
AR(4)	2.67	-103.44	1.94	2.48	-104.77	1.95	2.73	-106.05	2.06
AR(22)	4.79	-109.62	2.82	4.53	-109	2.69	3.81	-107.66	2.39
AR(22)-F(3)	8.14	-131.54	5.02	7.81	-130.48	4.73	7.08	-128.48	4.21
DCS	6.08	-114.43	3.57	5.93	-113.68	3.35	4.93	-110.7	2.85

Source: own computations.

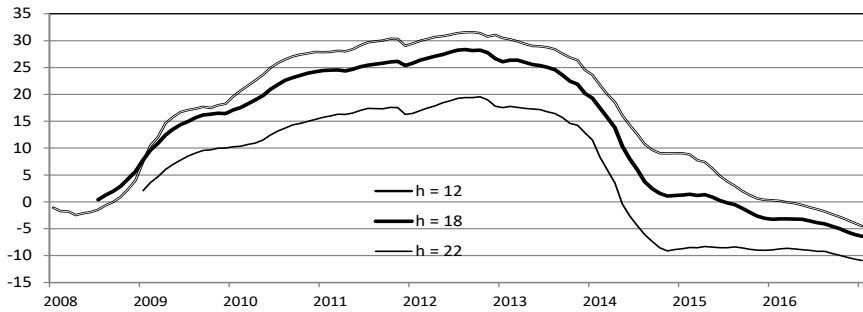
Analysis of Table 1 seems to lead to a very simple conclusion. The overall predictive performance is dominated by the Gaussian AR(22) model and if one restricts attention to the last 3 years, the results are dominated by a simple AR(4) model. In particular neither the stochastically sophisticated DCS specification nor the business-cycle oriented AR(22)-F(3) lead to satisfactory results. The conclusion is unanimously supported by point and density criteria.

One might want to conduct more detailed analysis and to decompose the e.g. the difference in cumulated LPS between certain models into the contribution of individual observation. Such a decomposition for the two winning models (AR(22) and AR(4)) against the AR(22)-F(3) specification is presented in Figure 2A and 2B (with positive values supporting AR(22)-F(3)). The figures reveal the fact that throughout most of the verification window the data provide strong and systematic support in favor of the business-cycle-oriented model AR(22)-F(3).

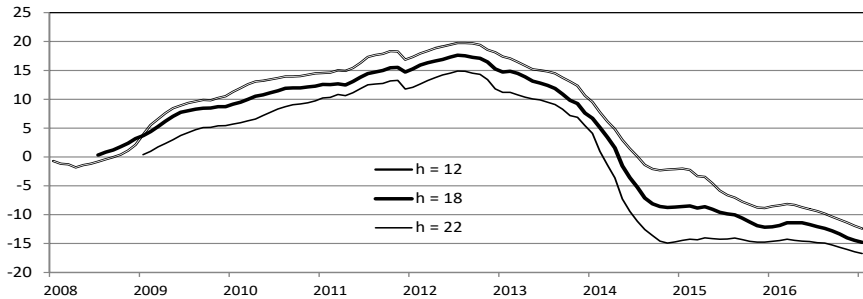
However, in 2013 the pattern breaks down and the performance of the model deteriorates quickly. As a consequence, the last three years of the data bring strong evidence against the model, and the empirical support shifts towards the AR(4) specification. The abrupt change might suggest that that the business cycle properties of the Polish industrial production

growth rates have changed after 2013 in such way that the previously observed pattern seems no longer valid. Importantly, the model with the best forecasting performance in the recent period, namely the AR(4), generates quite trivial forecasts with quick mean reversion.

**Figure 2.** Differences in cumulated LPS between AR(22)-F(3) and other models evolving in time (positive values in favor of AR(22)-F(3))



A: differences vs. AR(4)



B: differences vs. AR(22)

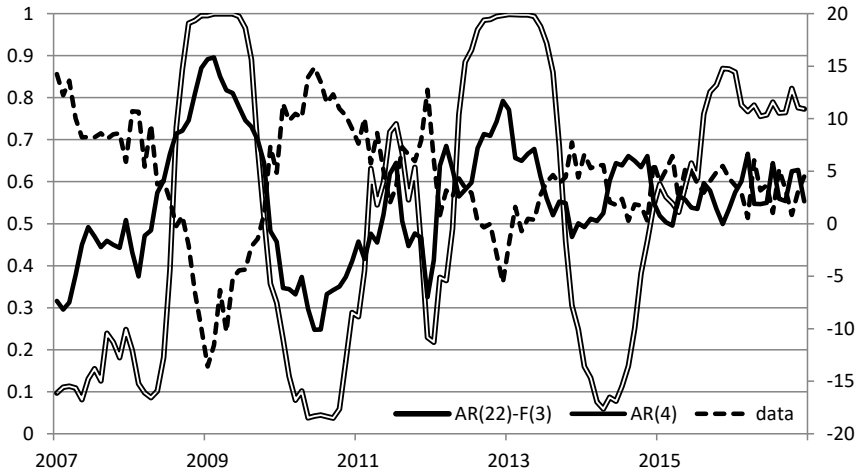
Source: Own calculations

Finally in Figure 3 we present values of the probabilistic indicator of future economic conditions. Based on the results of the predictive comparison we pick the results for AR(4) and AR(22)-F(3). Moreover, the version presented refers to one-year-ahead forecast. At each point in time the value of the indicator represents the probability that the average growth rate for the period  $t+7 \dots t+12$  will exceed its counterpart compared for the period  $t+1 \dots t+6$ .

One might notice that for most of the time the probabilistic index obtained from AR(22)-F(3) provides clear-cut signals, being close to either 0

or 1. Moreover, it seems to provide adequate information (i.e. it leads actual changes), at least in the first part of the sample. Closer to the sample end the fluctuations of industrial production growth rates dampen, so the adequacy is not easy to verify.

**Figure 3.** Probabilistic index of future economic conditions (6/6 months ahead) – left axis (obtained from two models, AR(22)-F(3) and AR(4) and the data (right axis).



Source: own computations.

## Conclusions

In the paper we compare density predictive performance of alternative model specification with application to y-on-y growth rates of Polish industrial production. In particular we consider two kinds of models. Models of the first kind focus on business cycle fluctuations using the deterministic approach based on Flexible Fourier Form (see Lenart & Mazur, 2016). Models of the second kind are more general in terms of stochastic specification. The models belong to the Dynamic Conditional Score class and allow for heavy tailed conditional distribution (of the generalized  $t$  class) and time-varying conditional scale. We generate density forecasts for horizons up to 24 months ahead. Evaluation of the forecasts (based on CRPS and LPS criteria) seems to indicate that the DCS-type models do not generate additional predictive power despite their sophistication. Closer examination reveals the fact that up to approximately 2013 the best performing model was that of deterministic cycle, while in the more recent period the

evidence shifts toward a trivial AR(4) specification with quick mean reversion. It might suggest that there was a change in the process underlying Polish business cycle fluctuations. A practical suggestion (and a direction for future research) is the use of dynamic prediction pools of density forecasts for the sake of probabilistic forecasting of Polish industrial production series.

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