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## Unemployment Insurance and Labor Supply Decisions

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Discussion Paper No. 271

UNEMPLOYMENT INSURANCE AND LABOR SUPPLY DECISIONS <sup>1/</sup>

by

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## 1. INTRODUCTION

The purpose of the paper is twofold: to develop a more realistic model of job search than that currently available and to interpret recent empirical results on the side effects of unemployment insurance within the framework developed. The model embodies the following extensions: the worker is allowed to search while employed; the intensity of search is a choice variable; and the cost of search is viewed as the value of leisure foregone. Two institutional features of the unemployment insurance program in most states are also explicitly recognized. First, benefits are paid only for a specified duration rather than in every period of an unemployment spell. Second, workers who quit are not qualified for benefits.

In a market with imperfect wage information the job possibilities of an individual worker can be characterized as a distribution of possible wage offers. If this distribution is known and if a worker searches by sampling from this distribution in a sequential manner, then the optimal strategy is to accept the first offer obtained greater than some reservation wage. The reservation wage that maximizes the expected present value of the future earning stream is such that the cost of search equals the expected gain in future income attributable to search. The expected duration of search is the inverse of the probability per period of finding an acceptable wage offer. In all existing wage search models, unemployment compensation is viewed as a payment made contingent on being unemployed and the possibility of a future separation is typically ignored. Because this payment reduces the income foregone by searching while currently unemployed but does not affect future income, the expected search duration increases with the benefit rate. <sup>2/</sup>

Job turnover is the rule rather than the exception in every labor market. In those industries that experience significant fluctuations in demand, the typical worker, whether currently employed or not, expects that he or she will be laid off at some future date with positive probability. This expectation implies that the search behavior of those not currently eligible for benefits (new entrants, exhaustees, and quits) is influenced by the parameters of the unemployment insurance scheme nevertheless. Specifically, a worker either currently employed or unemployed but not receiving benefits will be eligible during any future employer initiated unemployment spell. Consequently, an improvement in either the benefit rate or the maximum benefit period makes current employment relatively more attractive. In response, an unqualified worker finds employment more quickly by both lowering his or her reservations wage and by searching more intensively. The same effect is also present in the case of an unemployed worker currently receiving benefits, although the fact that benefits received in the current unemployment spell are contingent on remaining unemployed tends to offset it. Nevertheless, the first effect dominates in the case of a worker about to exhaust the benefits obtainable during the current unemployment spell. These results imply that the predicted sign of the effect of an increase in benefits on unemployment duration is ambiguous.

A major caveat is in order. The analysis conducted in the paper is partial in the sense that the effects of changes in benefit parameters on the distribution of wage offers and on the probability of being laid off are ignored. The validity of the theoretical results are not affected but the interpretation of empirical relationships as though they were generated by the behavior modelled is highly questionable in general. However, we argue

that this objection is not pertinent in the case of K. Classen's study. <sup>3/</sup>

## 2. THE MODEL

An individual worker's preferences are representable by a utility function defined over the future sequence of consumption-leisure pairs. As in the standard labor supply analysis, assume that labor income is used to purchase market goods in every period. In addition, assume that the number of hours worked is fixed and the same in all jobs.<sup>4/</sup> To simplify still further, we restrict the analysis to the case in which the retirement date is random and independent of age. Finally, abstract from learning, another form of non-stationarity, by assuming that the distributions of all relevant random variables are known and constant over time. All of these assumptions are made so that dynamic issues that relate to the current date can be ignored. Specifically, they imply that the typical worker's decision problem is a Markov decision process.<sup>5/</sup>

The state space associated with this process is the set of possibilities concerning the worker's labor market status. These include not participating, searching while unemployed, and working. Searching while working is allowed. The nature of unemployment compensation legislation requires that we distinguish between situations in which the worker is qualified for the benefit from those in which the worker is not qualified.

Let  $y_j$  denote the goods purchased per period at date  $j$  with market income and let  $\ell_j$  denote the fraction of the interval  $(j, j+h)$  devoted to leisure. If utility is intertemporally separable, then the utility function is of the form

$$U_j = \frac{1}{1+rh} [h u(y_j, l_j) + U_{j+h}]$$

where  $u(\cdot)$  is the utility flow generated by  $(y_j, l_j)$  and  $r$  is the sum of the subject rate of time preference and the probability of retiring per period when  $u(0,1) = 0$ . In the sequel  $u(\cdot)$  is taken to be a Von Neumann-Morgenstern utility indicator. Because the origin of such an indicator is arbitrary, the assumption that the utility of specializing in leisure for an interval is zero can be maintained without loss of generality.

If during the current interval, the individual in question is not participating in the labor market, then  $(y, l) = (0, 1)$ . A searching but unemployed worker obtains the combination  $(b, 1 - s)$  if he is qualified for the benefit  $b$  where  $s$  is the fraction of the current period devoted to search. He obtains the combination  $(0, 1 - s)$  if not qualified. The worker, if employed at the wage  $w$  obtains the combination  $(y, l) = (w, l_0 - s)$  where  $l_0 < 1$  is the fraction of the period remaining after work.

In most states some prior period of employment is required to qualify for unemployment compensation. Typically workers who voluntarily quit do not qualify. Finally, there is generally a maximum number of periods during which a qualified worker is able to receive benefits. We incorporate the first two features by assuming that a worker qualifies if and only if he was previously employed and laid off of his last job. The length of the maximum benefit period is denoted by  $T$ . The length of the remainder of that period for a worker who is qualified but has been laid off for some time is denoted as  $t$ . In other words, a qualified unemployed worker with a remaining benefit period equal to  $t$  was laid off  $T - t$  periods ago.

The status of an unemployed worker is completely characterized by the value of  $t$ , the remaining length of his benefit eligibility. Hence, an unqualified worker is any for which  $t = 0$ . A worker is in this state, then, when he enters the labor market and when he has exhausted his benefits in any unemployment spell. The status of an employed worker is completely characterized by the wage earned since jobs are assured to be equivalent in every other respect.

The probability that the worker will be laid off in an interval of length  $h$  is assumed to be constant and independent of the workers action. Denote it by  $h\delta$ . The probability that a searching worker obtains an offer is assumed proportional to the fraction of the interval  $h$  that he devotes to search. Denote it as  $\alpha sh$ . If the interval is taken to be short enough, these two possibilities are approximately mutually exclusive. Finally, search is viewed as a process of randomly drawing samples in sequence from the wage offer distribution characterized by the known distribution function  $F(w)$ . Let  $\bar{w}$  denote the maximum attainable wage; i.e.,  $\bar{w}$  is the smallest  $w$  such that  $F(w) = 1$ .

In the sequel the parameters  $r$ ,  $\delta$ , and  $\alpha$  and the parameters of unemployment compensation  $b$  and  $T$  are positive. The distribution of offers,  $F(x)$ , is continuous. Finally, the utility function  $u(y, \ell)$  is strictly increasing and twice differentiable.

The worker maximized the expected future discounted utility stream at each date by choosing the appropriate strategy. Given the stationarity assumptions, the optimal strategy is an assignment of an action to each and every possible state; i.e. optimal actions do not depend on the current date. In the next section we show that the indirect utility of being employed,



given an optimal strategy, is a function of the expected utility associated with the possibility of being laid off in the future. Similarly, the indirect utility of being unemployed, given an optimal strategy, is a function of the expected utility of being laid off in some future job held, the benefit rate  $b$  and the length of the worker's remaining benefit period. These indirect utility functions, which define a preference ordering over the state space, are derived by solving the worker's choice problem in each labor market state. The method of solution is an application of Bellman's principle of dynamic optimality.<sup>6/</sup>

### 3. THE DECISION PROCESS

In the section we formulate the decision problem that a typical worker faces. Because the worker's current decision affects both well being in his current state and the probabilities of making transitions to other participation states, the problem is one of dynamic programming. In the section we apply Bellman's principle of dynamic optimality for the purpose of formulating the decision problem and of defining the indirect utility function associated with each participation state. Because of the stationarity assumptions, the optimal decisions define a Markov process. In principle, this process describes the dynamics of an individual's labor market history.

Let  $U$  denote the utility of the worker when employed and  $U_t$  denote the utility obtained when unemployed with  $t$  benefit periods remaining. These are all values of indirect utility functions to be specified below. However, the stationarity assumptions imply that none depend on the date. For this reason several useful initial results can be derived from a revealed preference argument.

For example, a potential worker would make the same participation decision at every date. Consequently, a participant is an individual with preferences such that

$$U_o > \frac{u(0,1)}{r} \quad (1)$$

because as a new entrant the worker must search for a first job without being qualified for unemployment compensation and because the present value of the future utility flow would equal  $u(0,1)/r$  were he or she not to participate.

A participant when a job is accepted reveals that the new state is preferred to the old. Hence, given the wage earned,

$$U \geq U_o \quad (2)$$

once employed, since the worker can always quit to become unemployed. Because the worker, if he or she were to quit, would not qualify for compensation, (2) also implies that no employed worker quits a current job to search for a new one as an unemployed worker.<sup>7/</sup> The worker may, however, search while employed.

If at some future date the worker is laid off, he or she searches for a new job as unemployed but qualified for unemployment compensation. Such a worker accepts a new job during the current period if and only if

$$U \geq U_t, \quad \text{for all } 0 \leq t \leq T \quad (3)$$

where  $T$  equals the maximum duration of benefits and  $t$  equals the remainder of that period as of the current date.

Let  $U = U(w, U_T)$  denote the indirect expected utility of being employed at a wage  $w$  where  $U_T$  is the indirect expected utility of being laid off at some future date. Similarly, let  $U_t = V(t, b, U_T)$  represent the indirect expected utility of being unemployed with a future benefit period of length  $t$  remaining during which benefit flow  $b$  will be received where  $U_T$  is the indirect utility of being laid off the worker's next job. Below we show that both functions are increasing in all arguments. Finally, let  $U_T = \theta(T, b)$ . The indirect utility of being laid off,  $\theta(\cdot)$ , is implicitly defined by the fact that

$$U_T = V(T, b, U_T). \quad (4)$$

As one might suspect, it can be shown that  $U_T$  is an increasing function of both the benefit rate and the length of the benefit period.

By definition, the indirect utility function given the worker's employment status, is the discounted future expected utility flow associated with the worker's optimal strategy. In other words, it is the largest possible discounted future expected utility stream. Because the worker's utility function is intertemporally separable, the indirect utility function associated with each employment state can be represented as the solution to a relatively simple functional equation by virtue of Bellman's principle of dynamic optimality. The equation relates the indirect utility of the worker's status at the beginning of any period to the indirect utility of every possible situation that the worker could be in at the end of the period given that he pursues an optimal strategy.

When employed, the worker's search strategy is characterized by his

choice of  $s$ , the fraction of a period devoted to search. Let  $h$  denote the length of the period. During this interval of time, the worker enjoys the utility flow  $hu(w, \ell_0 - s)$  given his choice of search intensity  $s$  where  $\ell_0$  is the fraction of the period remaining after work. If the length of the interval is sufficiently short, one of three mutually exclusive events will occur. Either the worker is laid off, he finds a higher paying job, or neither occurs. The probabilities are  $\delta h, \alpha s h \Pr\{x \geq w\}$  and  $1 - \delta h - \alpha s h \Pr\{x \geq w\}$  respectively where  $x$  denotes a randomly drawn offer from the known distribution  $F(x)$ ,  $\delta$  is the layoff frequency and  $\alpha s$  is the frequency with which offers are generated given the worker's search intensity. The expected utilities of the worker's possible end of period situations are  $U_T$  if laid off,  $E\{U(x, U_T) | x \geq w\}$  if a higher paying job is found, and  $U(w, U_T)$  otherwise.

Bellman's principle of dynamic optimality simply asserts that the worker's current choice maximizes the sum of the utility flow realized during the current period plus the mathematical expectations of the worker's discounted expected future utility flow given that an optimal strategy will be pursued in every future period. The indirect utility at the beginning of the period is the present value of the maximum of the sum. Consequently, when employed

$$\begin{aligned}
 U = U(w, U_T) = & \frac{1}{1 + rh} \text{Max}_{0 \leq s \leq \ell_0} [hu(w, \ell_0 - s) \\
 & + \delta h U_T + \alpha s h \Pr\{x \geq w\} E\{U(x, U_T) | x \geq w\} \\
 & + (1 - \delta h - \alpha s h \Pr\{x \geq w\})U(w, U_T)]
 \end{aligned}$$

where  $x$  is an offer randomly drawn from the distribution characterized by  $F(x)$ . Since

$$\Pr\{x \geq w\} = \int_w^{\bar{w}} dF(x) = 1 - F(w)$$

and

$$\Pr\{w \geq w\} E \{U(x, U_T) \mid x \geq w\} = \int_w^{\bar{w}} U(x, U_T) dF(x),$$

an equivalent representation follows:

$$U(w, U_T) = \frac{1}{1+rh} \text{Max}_{0 \leq s \leq \ell_0} [hu(w, \ell_0 - s) + U(w, U_T) + \delta h[U_T - U(w, U_T)] + \alpha sh \int_w^{\bar{w}} [U(x, U_T) - U(w, U_T)] dF(x)]. \quad (5)$$

The last two terms on the right side of (5) are respectively the expected loss in the future discounted stream attributable to being laid off and the expected gain in the present value of future utility attributable to finding a higher paying job during the current interval. As an implication of (5), note that the worker's optimal search intensity is such that the margin gain attributable to an increase in the time devoted to search, the derivative of the last term with respect to  $s$ , and the marginal utility of that time as leisure, the derivative of  $u(w, \ell_0 - s)$  with respect to its second argument, are equal.

The indirect utility function given employment  $U(w, U_T)$  is implicitly defined by (5). By substitution into (5), one can verify that the function is of the following additively separable class

$$U(w, U_T) = \frac{\omega(w)}{r+\delta} + \frac{\delta}{r+\delta} U_T \quad (6a)$$

where

$$\varphi(w) = \text{Max}_{0 \leq s \leq l_0} [ (w, l_0 - s) + \frac{\alpha s}{r + \delta} \int_w^{\bar{w}} [\varphi(x) - \varphi(w)] dF(x) ] \quad (6b)$$

The function  $\varphi(w)$  is the current utility flow plus the gain in the future expected utility stream attributable to current search discounted by the sum of the rate of time preference plus the probability of being laid off. The latter acts as a "depreciation rate" on the "return" attributable to current investment in search time for the obvious reason.<sup>8/</sup> The expected utility of working at a wage  $w$  is a weighted average of the present value of this flow,  $\varphi(w)/r$ , and the utility of being laid off at some future date,  $U_T$ . The weights reflect the probability of the latter event.

Since the derivative of the right side of (6b) with respect to  $s$  is zero given an optimal action, a complete differentiation of (6b) yields

$$\frac{d\varphi(w)}{dw} \left[ 1 + \frac{\alpha s}{r + \delta} \int_w^{\bar{w}} dF(x) \right] = u_1(w, l_0 - s)$$

where  $u_1(\cdot)$  represents the marginal utility of income, formally the first partial derivative of  $u(\cdot)$ . Because  $u_1(\cdot)$  is positive (6a) implies

$$\frac{\partial U(w, U_T)}{\partial w} = \frac{1}{1+r} \frac{d\varphi(w)}{dw} > 0 \quad (7a)$$

of course,

$$\frac{\partial U(w, U_T)}{\partial U_T} = \frac{\delta}{r + \delta} > 0 \quad (7b)$$

as well. Because the possibility of being laid off is an event that can

occur only in the future, given current employment, an increase in the utility of being laid off does not increase the indirect utility of being employed by an equal amount due to time preference. However, the magnitude of the effect tends to unity as the frequency of being laid off  $\delta$  increases without bound, as one's intuition might suggest.

An analogous approach yields a characterization of the same worker's indirect utility function when unemployed, denoted as  $V(t, b, U_T)$ . During the current time interval of length  $h$ , the worker enjoys the utility flow  $hu(b, 1 - s_t)$  if qualified for benefits. Here  $s_t$  denotes the proportion of the current interval devoted to search given that the remaining future benefit period is  $t$ . An offer  $x$  is generated by search with probability  $\alpha s_t h$  and is accepted if and only if it is at least as large as the worker's current reservation wage,  $w_t$ . An unemployed worker chooses both  $s_t$  and  $w_t$  to maximize the expected future discounted utility flow given  $t$ .

If an acceptable offer is found, the worker is employed at the end of the current interval and can expect to enjoy the future discounted utility stream  $U = U(x, U_T)$  given that the optimal search strategy is followed once employed. Because the remaining benefit period is of length  $t - h$  at the end of the current interval,  $U_{t-h} = V(t - h, b, U_T)$  is the indirect utility of being unemployed at the end given optimal search behavior as an unemployed worker. By virtue of Bellman's principle of dynamic optimality, the indirect utility of being unemployed at the beginning of the current interval is the present value of the maximum utility flow obtained in the current interval plus the expected end of interval utility, the weighted average of  $U$  and  $U_{t-h}$  with weights equal to the probability of becoming

employed and remaining unemployed respectively. Formally, this statement can be expressed as follows:

$$\begin{aligned}
 U_t &= V(t, b, U_t) \\
 &= \frac{1}{1+rh} \underset{w_t \geq 0}{\text{M a x}}_{0 \leq s_t \leq 1} [hu(b, 1 - s_t) + (1 - \alpha s_t h \Pr\{x \geq w_t\})V(t - h, b, U_T)] \\
 &\quad + \alpha s_t h \Pr\{x \geq w_t\} E\{V(x, U_T) \mid x \geq w_t\}
 \end{aligned}$$

for every  $0 < t \leq T$ .

An equivalent but more convenient representation follows:

$$\begin{aligned}
 V(t, b, U_T) &= \frac{1}{1+rh} \underset{w_t \geq 0}{\text{M a x}}_{0 \leq s_t \leq 1} [hu(b, 1 - s_t) + V(t - h, b, U_T)] \\
 &\quad + \alpha s_t h \int_{w_t}^{\bar{w}} [U(x, U_T) - V(t - h, b, U_T)] dF(x)] \\
 &\quad \text{for every } 0 < t \leq T. \tag{8}
 \end{aligned}$$

The last term on the right side of (8) is the total expected future indirect utility gain attributable to the possibility of finding a job. An optimal choice of the reservation wage maximizes this gain by virtue of (8). In addition, the optimal search intensity equates the marginal future utility gain attributable to the time spent searching and the forgone marginal value of that time as leisure.

The argument just presented is that the indirect utilities associated with a remaining benefit period of various lengths are all elements of a solution to the difference equation (8). Because an exhaustee and a new entrant receive no compensation, it follows immediately that

$$U_0 = V(t, 0, U_T) = V(t-h, 0, U_T) = V(0, 0, U_T) . \tag{9}$$



This initial condition determines the particular solution to (8) that is of interest. Elsewhere (8) and (9) are used to derive the following properties: <sup>9/</sup>

$$\frac{\partial V(\cdot)}{\partial t} > 0 \quad \text{if } b > 0 \quad (10.a)$$

$$\frac{\partial V(\cdot)}{\partial b} > 0 \quad \text{if } t > 0 \quad (10.b)$$

$$0 < \frac{\partial V(\cdot)}{\partial U_T} < \frac{\delta}{r+\delta} = \frac{\partial U(\cdot)}{\partial U_T} \quad \text{if } \delta > 0 \quad \text{and } \alpha > 0. \quad (10.c)$$

Condition (10.a) and (10.b) respectively imply that the indirect utility of being unemployed is an increasing function of the length of the benefit period remaining and the benefit rate, a conclusion that is not surprising. Condition (10.c) has the following interpretation. The indirect utility of both employment and unemployment increase with the indirect utility of being laid off because being laid off is a future possible event given either state. However, because a worker currently unemployed must first find a job before he or she can be laid off and because doing so requires time, the response in the indirect utility of being employed to an increase in  $U_T$  is larger than the response of the indirect utility of being unemployed to the same increase.

Finally, the indirect utility of being laid off,  $U_T$ , is a function of the two benefit parameters,  $b$  and  $T$ , as noted earlier. This function  $\vartheta(T,b)$  is implicitly defined by equation (4). The fact that  $\vartheta(\cdot)$  is increasing in both the benefit rate and the maximum benefit period can now be established as a corollary of (10). Specifically, a complete differentiation of (4) and the properties represented as (10) imply

$$\frac{\partial \theta(\cdot)}{\partial b} = \left[ \frac{\partial V(\cdot)}{\partial t} / \left(1 - \frac{\partial V(\cdot)}{\partial U_T}\right) \right]_{t=T} > 0 \quad (11.a)$$

and

$$\frac{\partial \theta(\cdot)}{\partial T} = \left[ \frac{\partial V(\cdot)}{\partial b} / \left(1 - \frac{\partial V(\cdot)}{\partial U_T}\right) \right]_{t=T} > 0 \quad (11.b)$$

where the notation requires evaluation at  $t = T$ . Because the indirect utility associated with an optimal strategy is an increasing function of  $U_T$  in every state, a worker's utility increases with both benefit parameters whether employed or not. The implications of this fact for search behavior are elaborated in the next section.

#### 4. SEARCH BEHAVIOR AND THE DEMAND FOR LEISURE

An obvious purpose of unemployment insurance is to reduce the income loss that would otherwise occur when a worker loses his job. Because benefit payments are terminated when a worker is reemployed, existing search models imply that the duration of unemployment will increase as a side effect. Our extended model suggests two possible offsetting incentive effects.

First, an important cost of search is the opportunity value of the time required that could otherwise be allocated to leisure activities. Because leisure and income are not perfect substitutes in general, the cost of search depends on the size of the benefit payment received in the case of a qualified unemployed worker. In this section, we show that an increase in the benefit rate stimulates the demand for leisure as a consequence if and only if income and leisure are complements in household production.

Second, an increase in either the benefit rate or the maximum benefit period induces an increase in the indirect utility of being laid off in the

future as well as the indirect utility of remaining unemployed during a current spell in the case of a qualified worker. Because employment is more attractive as a consequence of the first effect, it tends to offset the increase in the incentive to remain unemployed implied by the second. Indeed, the first effect dominates if the worker is near the end of his or her benefit period or has exhausted benefits receivable during the current spell. Only the first effect operates in the case of an unqualified unemployed worker.

In the remainder of this section, these results are stated more formally and are discussed in greater detail. For this purpose it is useful to define two concepts: the notions of escape rate and complementarity in household production.

Given random search, the probability that an unemployed worker finds an acceptable job in a time interval of length  $h$  is equal to the product of the probability that an offer will arrive during the interval,  $\alpha sh$ , and the probability that such an offer is acceptable,  $\Pr\{x \geq w\} = 1 - F(w)$ . In other words, the probability that an unemployed worker will make a transition from unemployment to employment in the interval is  $hq$  where

$$q = \alpha s [1 - F(w)]$$

is the escape rate, the expected frequency with which acceptable offers are found. Obviously, the escape rate increases with the time allocated to search because the chance of obtaining an offer increases with  $s$ . An increase in the reservation wage reduces the escape rate because the probability that a random offer will be acceptable declines with the minimum acceptable wage.

Viewing non-market time and market goods as inputs in a household process that produces the consumer services enjoyed is often insightful.<sup>10/</sup> From this perspective, the utility function  $u(y, \ell)$  is regarded as a "production

function" whose arguments are inputs in the process. Real income and leisure are complements in the production theory sense, then, if and only if the "marginal product" of one is a non-decreasing function of the other; i.e. the cross partial derivative of  $u(\cdot)$  is non-negative.

If the optimal strategy of an unemployed worker is to search ( $s > 0$ ), demand leisure ( $s < 1$ ) and require a positive wage ( $w > 0$ ),<sup>11/</sup> then condition (8) implies that an optimal reservation wage and search intensity combination  $(w, s)$  satisfies

$$U(w, U_T) = V(t, b, U_T) \quad (13.a)$$

and

$$u_2(b, 1-s) = \alpha \int_w^{\bar{w}} [U(x, U_T) - V(t, b, U_T)] dF(x) \quad (13.b)$$

where  $u_2(\cdot)$  is the partial derivative of  $u(\cdot)$  with respect to leisure. The worker is indifferent between employment at his reservation wage and remaining unemployed according to (13.a). The optimal search intensity equates, given (13.b), the marginal indirect utility gain attributable to the time allocated to search and the marginal utility of its alternative use in leisure activity. The sufficient second order condition is diminishing marginal utility of leisure,  $u_{22}(\cdot) < 0$ . This condition is maintained in the sequel.

Note that the reservation wage unambiguously increases with the current benefit payment,  $b$ , holding constant  $U_T$ , the indirect utility of being laid off in the future, since  $U(\cdot)$  increases in  $w$  from (7.a) and  $V(\cdot)$  increases in  $b$  by virtue of (10.b). This is the usual disincentive effect highlighted in the search literature. However,  $U_T$  increases with the benefit rate also from (11.a). Since an increase in  $U_T$  increases the indirect utility of employment by more than it increases the indirect utility of unemployment by virtue of (10.c), the net effect of an increase in  $b$  on the reservation

wage is ambiguous in general. For the same reason, the effect of an increase in the benefit rate on the optimal search intensity is ambiguous given (13.b) if  $u(y,l)$  is additively separable. If not separable, the confusion is only complicated since the cross partial derivative,  $u_{12}(\cdot)$ , can take on either sign in principle.

The equations of (13) also imply that a qualified worker's optimal action depends on the length of the worker's remaining benefit period,  $t$ . Because the worker's realized duration to date equals  $V = T-t$ , if  $t \geq 0$ , and because the indirect utility of being unemployed increases with  $t$  from (10.a), a qualified worker's reservation wage falls with realized unemployment duration until the current benefit period is exhausted.<sup>12/</sup> In the case of an exhaustee and an unqualified worker, the reservation wage is independent of duration at the margin because  $V(0,0,U_T)$  replaces the right side of (13.a).

Because the derivative of the right side of (13.b) with respect to  $w$  is zero given (13.a), because the right side of (13.b), the return to search, decreases with  $t$  by virtue of (10.c) and because the cost of search, the left side of (13.b), decreases with  $s$  given the second order condition, the time allocated to search decreases with  $t$ . In other words, qualified workers search more intensely as their benefits run out. But, again the time allocated to search is independent of realized duration in the case of an unqualified worker or exhaustee. These results together with (12) clearly imply the following consequences.

Proposition 1: In the case of a qualified worker who has not yet exhausted his or her unemployment benefits, the escape rate increases with realized unemployment duration. The escape probability is independent of realized duration at the margin if the worker is either not qualified to receive

benefits during the current unemployment spell or has already exhausted them.

These results are illustrated graphically in Figure 1 for the case of a qualified worker. In the figure  $q_T$  denotes the escape rate at the moment the worker is laid off and  $q_0$  represents the escape rate once the benefits are exhausted. The same worker, if unqualified, would escape from unemployment at the rate  $q_0$  independent of duration.

Due to the complementarity between income and leisure, the marginal utility of leisure during a period in which benefits payments are received is different from the same in a period in which payments are not received given an identical allocation of time to search in the two period. Specifically, the opportunity cost of search is higher when payments are received if and only if income and leisure are strict complements in household production; i.e. for the same  $s$ ,

$$u_2(b, 1-s) \begin{matrix} > \\ < \end{matrix} u_2(0, 1-s) \text{ as } u_{12}(\cdot) \begin{matrix} > \\ < \end{matrix} 0.$$

As a consequence, it can be shown that the time allocated to search jumps up in a discontinuous manner at the moment benefits are exhausted if income and leisure are strict complements.<sup>13/</sup> Because the reservation wage is continuous in realized duration in all cases, the following interesting fact follows from (12).

Proposition 2: At the moment benefits are exhausted, the escape rate jumps up (down) if income and leisure are strict complements (substitutes) in household production.

Obviously, the case in which the marginal utility of leisure is independent of income is illustrated in Figure 1.

Earlier we noted that the effect of a change in the benefit rate on a

worker's optimal action is ambiguous in general. This ambiguity can be resolved if one has more information about the worker's current status. Consider, for example, a worker just laid off; i.e.,  $t = T$ . Since  $U_T = V(T, b, U_T)$  from (4), the equations of (13) may be rewritten as

$$U(w, U_T) = U_T \quad (14.a)$$

$$u_2(b, 1-s) = \alpha \int_w^{\bar{w}} [U(x, U_T) - U_T] dF(x) \quad (14.b)$$

Because a unit increase in  $U_T$  increases the indirect utility of being employed by less than a unit and the latter increases with the wage, (14.a) implies that the reservation wage of a worker just laid off unambiguously increases with both benefit parameters. In other words, the traditional disincentive effect dominates. Similarly, it is obvious that an increase in either parameter reduces the marginal return to search time, the right side of (14.b). If an increase in the benefit rate does not reduce the cost of search, i.e.  $u_{12}(\cdot) \geq 0$ , then the time allocated to search also falls with both parameters. In sum, the following holds.

Proposition 3: In the case of a newly laid off worker, the escape rate decreases with the maximum benefit period. If goods and leisure are complements in household production, then an increase in the benefit rate also decreases the escape rate.

Our description of the effect of unemployment benefits on the probability of escaping unemployment is completed by considering a worker who is not currently receiving benefit payments. In this case, the equations of (13) can be rewritten as

$$U(w, U_T) = V(0, 0, U_T) \quad (15.a)$$

$$u_2(0, 1-s) = \alpha \int_w^{\bar{w}} [U(x, U_T) - V(0, 0, U_T)] dF(x) \quad (15.b)$$

Obviously, a change in either benefit parameter affects this worker's optimal action only through its effect on the utility of being laid off in the future. Using the arguments previously presented, the reader can now verify for himself the final result.

Proposition 4: In the case of an unemployed worker not currently receiving benefit payments, an increase in either benefit parameter increases the escape probability.

The implications of Propositions 1 through 4 in the case in which the marginal utility of leisure is independent of income are illustrated in Figure 2. The dashed line in the figure represents the original relationship between the escape rate and the realized unemployment duration. The solid line is the same relationship after benefit liberalization. Consider a sample of qualified unemployed workers who are identical except for realized duration in the current unemployment spell. The expected proportion of those in any duration interval of length  $h$  who escape in a time period of equal length equals  $h q$ . Although a smaller proportion of those recently laid off become employed given an increase in either benefit parameter, a larger proportion of those near the end of the benefit period and beyond it escape unemployment. The two effects, then, act in opposite directions on the aggregate flow into employment as indicated by the "twist" in the relationship between the escape rate and realized duration induced by benefit liberalization.

##### 5. EXPECTED UNEMPLOYMENT DURATION: THEORY AND EVIDENCE

Because of the uncertainties implicit in the search process, the length of the period required to find an acceptable job is a random variable. Its



expectation is the theoretical counterpart to the average duration of unemployment per spell for a sample of identical workers all of whom face the same market conditions. Most existing empirical studies attempt to measure the statistical relationship between measures of average duration and the unemployment benefit paid per week, the benefit rate. The purpose of this section is to interpret the results of some of these studies within the framework of our theoretical model.

Consider a worker not qualified for compensation during his or her current unemployment spell; i.e., the worker is either a new entrant or has quit a previous job. In this case, the worker's escape rate is  $q_0$ , a constant independent of unemployment duration to date. Consequently, the probability distribution of the realized spell duration,  $v$ , is a negative exponential with expectation  $1/q_0$ . Specifically,

$$D_0 = \int_0^{\infty} v q_0 e^{-q_0 v} dv = 1/q_0. \quad (16)$$

By virtue of Proposition 4, an increase in either benefit parameter reduces the duration of unemployment in the case of a worker unqualified if the worker is covered in any future employer initiated unemployment spell.

The expected unemployment duration of a new entrant or quitter falls given a liberalization of benefits in the model because the indirect utility of employment is increased by the resulting reduction in the cost of being laid off in the future and because the disincentive effect of the contingent nature of the payment scheme is not present when the worker is not currently receiving a benefit payment. This hypothesis arises only in an explicitly dynamic framework; one that distinguishes between current payments and future eligibility. This influence is not tested directly in any of the existing empirical studies since all are restricted to samples of workers who receive

benefits during the observed unemployment spell. Nonetheless, the studies do provide indirect evidence suggesting that the distinction is important.

To interpret the existing evidence, one must first consider the theoretical case of a worker who does receive benefits, up to the maximum allowable, during his or her current unemployment spell. In this case our theory suggests that the escape rate depends on a worker's unemployment duration to date, a fact that complicates the derivation of the probability distribution for completed spells.

Let  $q_t$  denote the escape rate given that the length of the current benefit period remaining is equal to  $t$ . For  $t > 0$ , the length of the spell to date is equal to  $v = T-t$ . Let  $q(v)$  denote the escape rate given that the duration to date is  $v$ . From Proposition 1 we know that

$$q(v) = \begin{cases} q_t & \text{if } v = T-t < T \\ q_0 & \text{if } v \geq T \end{cases} \quad (17)$$

Let  $G(v)$  denote the probability that the worker will find an acceptable job within a period of length  $v$ ; i.e. it is the distribution function for the random length of the unemployment spell. The probability of finding an acceptable job in the interval  $(v, v+dv)$  is the product of the escape probability during that interval,  $q(v)dv$  and the probability that the worker is not yet employed as of the beginning of the interval,  $1-G(v)$ ; i.e.

$$dG(v) = q(v) [1-G(v)]dv.$$

Consequently,

$$G(v) = 1 - \exp \left\{ - \int_0^v q(z)dz \right\} \quad (18)$$

Graphically, then,  $G(v)$  is a strictly increasing function of the area under the curve relating the escape rate and duration, that illustrated in Figure 1. In other words, a shift in the curve that results in an increase in the area for a given  $v$  increases  $G(v)$ .

The expected duration of a spell in the case of a currently qualified worker is the expectation of  $v$  with respect to the probability distribution  $G(v)$ ; i.e.,

$$D = E\{v\} = \int_0^{\infty} v dG(v). \quad (18)$$

In principle, several of the existing empirical studies attempt to estimate the effect of unemployment compensation by regressing spell duration on benefits received. In practice, however, the weeks of compensation rather than weeks unemployed are often used as the dependent variable. Holen's study is a good example. <sup>14/</sup>

By truncating the dependent variable in this way, the appropriate theoretical counterpart of average observed duration is

$$D_1 = \Pr\{v \leq T\} E\{v | v \leq T\} + (1 - \Pr\{v \leq T\})T$$

since all exhaustees are "assigned" a spell duration equal to the length of the maximum compensation period. Since  $\Pr\{v \leq T\} = G(T)$  and

$$E\{v | v \leq T\} = \frac{1}{G(T)} \int_0^T v dG(v),$$

substitution and integration by parts yields

$$D_1 = T + \int_0^T (v-T) dG(v) = T - \int_0^T G(v) dv. \quad (19)$$

In other words, Holen's measured effect is an estimate of the partial derivative of  $D_1$  with respect to  $b$ .

In Figure 2, the shift in the relationship between the escape rate and unemployment duration to date induced by an increase in the benefit rate is illustrated. The solid line represents the relationship associated with the higher benefit rate, other things equal, by virtue of Propositions 3 and 4. Since  $G(v)$  is an increasing function of the area under the curve, given  $v$ , from (18), an increase in the benefit rate decreases  $G(v)$  for small values of  $v$  but increases  $G(v)$  for large values. Theoretically, then, the response in  $D_1$  to an increase in  $b$  is ambiguous.

However, Holen provides other pertinent information, an estimate of the effect of benefit payments on the probability of exhausting. This probability is positively associated with higher benefit rates in Holen's sample. Because it can be shown that  $G(v)$  decreases with  $b$  if  $G(T)$  does, given  $v < T$ , (19) and Holen's estimate imply that the duration measure  $D_1$  should also be positively associated with higher benefit rates in her sample. Holen's estimate of this effect is consistent with this inference. Specifically, a ten (1970) dollar increase in weekly benefits is associated with a one week increase in duration as measured by  $D_1$ . Holen's estimate of the effect of an increase in  $T$  on  $D_1$  is also positive and highly significant in the statistical sense. This result is hardly surprising given the form of (19).

The relationship between Holen's duration measure and the expected duration of a covered unemployment spell is easily derived. In general, the latter is

$$D = \Pr\{v \leq T\}E\{v|v \leq T\} + (1 - \Pr\{v \leq T\})E\{v|v \geq T\}.$$

Holen's measure is obtained by replacing  $E\{v|v \geq T\}$ , the expected duration of an exhaustee with  $T$ , the maximum benefit period. Because in

our framework the escape rate once benefits are exhausted is the constant  $q_0$  from (17), one can show that

$$E\{v|v \geq T\} = D_0 + T.$$

Consequently, the difference between the average spell duration and compensated duration,

$$D - D_1 = (1 - G(T))D_0, \quad (20)$$

equals the probability of exhausting multiplied by the expected duration of a worker not receiving benefits.

The bias in Holen's estimates as measures of the effect of the benefit parameters on unemployment duration is equal to the partial derivative of this difference with respect to the parameter in question. Since Holen's evidence suggests that  $1 - G(T)$  increases with  $b$  but our theory predicts that  $D_0$  decreases with  $b$ , the direction of bias is uncertain in the case of the benefit rate. The situation is different in the case of the maximum benefit period however. Since Holen's evidence implies that the probability of exhausting falls as  $T$  rises and our theory implies that  $D_0$  is decreasing in  $T$ , her estimate of the effect of an increase in the maximum benefit period on duration is biased upward.

The estimates of the effect of the benefit rate on duration reported by Ehrenberg and Oaxaca do not suffer from truncation bias of this form.<sup>15/</sup> The dependent variable is the full unemployment spell in their studies. Although our framework does not contain a qualitative inference concerning the effect of an increase in the benefit rate on average duration,  $D$ ,

the evidence presented by Ehrenberg and Oaxaca suggests that it is positive. At least this inference is valid if one accepts the premise that the authors are able to control for differences in labor market conditions, the layoff probability and the wage offer distribution, and for differences in the unemployment insurance programs faced by workers in their sample, who reside in virtually all states in the Union.

Interestingly the magnitudes of the measured effects are generally smaller than those reported by Holen. This fact and (20) imply that  $D_0$  falls in response to an increase in the benefit rate as our theory predicts, given Holen's estimate of the effect on the probability of exhausting.

The unique feature of Classen's data is that they pertain to workers in the same state during roughly the same period of time. <sup>16/</sup> Consequently, the maintained hypothesis that all face the same market conditions is more reasonable. The source of variation in the benefit rate arises from the fact that the time period surrounds a date at which the benefit rate was increased. Specifically, some members of the sample received the old (lower) benefit rate during the observed unemployment spell while others, those who lost their jobs after the date at which the increase took place, received the new (higher) benefit rate. This fact, in our framework, suggests an interpretation of Classen's results that is different from the author's.

Classen's result is that workers in the subsample receiving the higher benefit rate have longer unemployment durations on average. An interpretation of this result as evidence for the proposition that an increase in the benefits rate increases the expected spell durations  $D$  is fallacious

in our framework for the following reason. Members of both subsamples, although they receive different benefit rates during the observed spells, are all eligible for the higher benefit payment in any future unemployment spell. In other words, the indirect utility associated with the possibility of being laid off the next job,  $U_T$ , is the same for two otherwise identical workers. Consequently, differences in search behavior is due only to the difference in benefit payments received during the observed spell. This fact implies that Classen's estimate is biased upwards.

Formally, we demonstrate the bias by showing that the theory predicts Classen's result even though there is no qualitative implication concerning the partial derivative of  $D$ , expected duration, as defined in (18) with respect to  $b$ , the benefit rate. Let  $b$  and  $\hat{b}$  denote the new and old benefit rates respectively so that  $b > \hat{b}$ . Consider two workers who are identical except that one lost his job before and the other after the date at which the benefit rate was increased. Let  $(\hat{w}_t, \hat{s}_t)$  denote the reservation wage-search intensity combination chosen by the former and let  $(w_t, s_t)$  denote the combination chosen by the latter given that each has a remaining benefit period equal to  $t$ . Since  $U_T = \theta(b, T)$  for both workers but  $\hat{b}$  replaces  $b$  in the equations of (13) in the case of the worker who receives payments in the observed spell at the old rate, the argument outlines in detail above implies  $\hat{w}_t < w_t$  in general and  $\hat{s}_t > s_t$  if income and leisure are complements in household production. Consequently, the escape rates for the two workers are such that  $\hat{q}_t > q_t$  for all  $t > 0$  by virtue of equation (12). This fact, equation (17) and equation (18) imply  $\hat{G}(v) > G(v)$  for any  $v \leq T$ . Finally, because the measure of duration used by Classen is the number of compensated weeks, the theoretical counter-

part of the difference between the average durations of the two samples is

$$D_1 - \hat{D}_1 = \int_0^T [\hat{G}(v) - G(v)]dv > 0 \quad (21)$$

by virtue of equation (19).

The same analysis is applicable when comparing any two groups, one which receives benefits during the observed spell while the other does not, provided that members of both groups qualify for benefits in subsequent employer initiated unemployment spells. For example, a comparison of new entrants and/or job quitters with laid off workers qualifies. In making such a comparison, Marston finds a small but significant difference in the predicted direction. 17/

The analysis supporting the inequality (21) presumes that income and leisure are complements in household production. Marston's evidence that the escape rate jump up discontinuously at the exhaustion date strongly supports this hypothesis given Proposition 2. The hypothesis that the reservation wage declines with unemployment duration has a history of support dating at least from the evidence reported by Kasper. 18/



6. SUMMARY

There are two major conclusions to be drawn from the analysis. First, the effect of unemployment insurance benefits on measured search unemployment is theoretically ambiguous once the institutional features of most states' programs are taken into account. Specifically, because the potential benefit period per unemployment spell is limited and because only workers experiencing employer initiated unemployment qualify, workers receiving no benefits (new entrants, exhaustees and those who quit) have an incentive to become employed more rapidly than would otherwise be the case. Second, although this theoretical influence has not been tested directly, existing empirical evidence is consistent with it.

The evidence for a positive benefit effect on duration presented by Holen <sup>19/</sup> is inconclusive because the duration measure used is weeks of compensation rather than weeks of unemployment. Indeed, the theory predicts Holen's result when the durations of exhaustees is truncated. Classen's result is also predicted by the model. <sup>20/</sup> However, her estimate of the effect on duration is biased upward if the members of the sample not receiving the increase during the observed unemployment spell are encouraged by it to become employed more rapidly as our theory suggests. The evidence for a positive effect on duration reported by Ehrenberg and Oaxaca <sup>21/</sup> can not be questioned on these grounds. Nevertheless, it does not imply that measured unemployment increases with unemployment compensation for the following reasons.

Suppose that the expected unemployment duration of qualified workers (D) increases in response to an increase in compensation. The stock of such workers in a steady state is

$$U = \delta ND$$

where  $\delta$  is the layoff rate and  $N$  is the number of employed workers. If new entrants equal a constant proportion ( $\alpha$ ) of the labor force ( $L$ ) and the flow of those who quit equals a constant proportion ( $\beta$ ) of the employed stock ( $N$ ), then the steady state stock of unemployed workers not currently receiving compensation is

$$U_o = \alpha LD_o + \beta ND_o$$

where  $D_o$  denotes the common expected duration. Since  $L = N + U_o + U$ , the steady state unemployment rate,

$$\frac{(U_o + U)}{L} = \frac{(\alpha + \beta)D_o + \delta D}{1 + \beta D_o + \delta D}, \quad (22)$$

is an increasing function of both  $D_o$  and  $D$ . If  $D_o$  decreases with benefits as our theory suggests, the net effect on the unemployment rate depends on the magnitudes of  $\alpha$  and  $\beta$ , the entry and quit propensities, relative to  $\delta$ , the layoff propensity, as well as the absolute sizes of the responses in the durations of the two groups to an increase in benefits.

Finally, equation (22) illustrates one of the limitations of the partial supply side approach taken in this and related studies. For example, Feldstein has recently presented a convincing argument and supporting evidence for the proposition that larger benefits induce a larger layoff rate. <sup>22/</sup> The integration of Feldstein's approach with the search theoretic supply model is an important topic for future research. The effects of unemployment compensation on entry and quitting are also issues that need to be studied in detail.

7. MATHEMATICAL APPENDIX

Formal proofs of selected results asserted in the text are presented. Throughout the existence of an optimal strategy and an associated differentiable value function are presumed. The procedure followed in the appendix is to restate each result as it appears in the text and then to present the proof without further comment.

The properties of the value function  $V(t,b,U_T)$  include

$$\frac{\partial V(\cdot)}{\partial t} > 0 \quad \text{if } b > 0 \quad (10.a)$$

$$\frac{\partial V(\cdot)}{\partial b} > 0 \quad \text{if } t > 0 \quad (10.b)$$

$$0 < \frac{\partial V(\cdot)}{\partial U_T} < \frac{\delta}{r+\delta} \quad \text{if } (\delta, \alpha) > 0 \quad (10.c)$$

Proof: Let  $z(t) = V(t,b,U_T)$  and define the function  $f(\cdot)$  as follows:

$$f(b, U_T, z) = \max_{\substack{0 \leq s \leq 1 \\ w \geq 0}} [u(b, 1-s) + \alpha s \int_w^{\bar{w}} [U(x, U_T) - z] dF(x) - rz] \quad (A.1)$$

Because  $u(\cdot)$  is strictly increasing in  $b$  and  $U(\cdot)$  is strictly increasing in  $U_T$ ,  $f(\cdot)$  is strictly increasing in  $z$  by virtue of the Envelope Theorem. Moreover, a unique non-negative value of  $z$  exists solving  $f(b, U_T, z) = 0$  for all  $(b, U_T) \geq 0$ .

By rearranging (8) in the appropriate manner and then by letting  $h \rightarrow 0$ , the following first order differential equation is generated

$$\dot{z} = f(b, U_T, z) \quad (A.2)$$

where of course  $\dot{z} = \frac{\partial v(\cdot)}{\partial t}$ . The fact that  $f(\cdot)$  is strictly decreasing in  $z$  implies that  $z$  converges monotonically to  $z^*$ , a number defined by  $f(b, U_T, z^*) = 0$ , given any initial condition  $z_0$ . By virtue of (8) and (9), the particular initial condition of interest satisfied  $f(0, U_T, z_0) = 0$ . Since  $b > 0$ , the properties of  $f(\cdot)$  imply  $z_0 < z^*$ . Consequently,  $f(b, U_T, z(t)) > 0$  for all  $t > 0$  as asserted in (10.a).

In integral form (A.2) is

$$z(t) = z_0 + \int_0^t f(b, U_T, z(v)) dv.$$

Since  $f(\cdot)$  is strictly increasing in  $b$  and  $U_T$  and  $f(0, U_T, z_0) = 0$  that  $z_0$  is non-decreasing in both,  $z(t)$  is strictly increasing in both for all  $t > 0$  follows as (10.b) and (10.c) assert. Moreover,

$$\frac{\partial z(t)}{\partial U_T} = \frac{\partial z_0}{\partial U_T} + \int_0^t \frac{\partial f}{\partial U_T} dv < \frac{\partial z^*}{\partial U_T}.$$

The proof is completed by observing that a differentiation of  $f(b, U_T, z^*) = 0$ , holding  $b$  constant results in

$$\frac{\partial z^*}{\partial U_T} = \frac{\alpha s^* \int_{w^*}^{\bar{w}} \frac{\partial U(\cdot)}{\partial U_T} dF}{r + \alpha s^* \int_{w^*}^{\bar{w}} dF} = \frac{\delta}{r + \delta} \cdot \frac{\alpha s^* (1 - F(w))}{r + \alpha s^* (1 - F(w^*))} < \frac{\delta}{r + \delta}$$

by virtue of (A.1), the Envelope Theorem and (7.b). Q.E.D.

The escape rate is defined as

$$g(t) = \alpha s(t) [1 - F(w(t))] \tag{A.3}$$

where  $(s(t), w(t))$  is a solution to the maximization problem in (A.1) given  $z = z(t)$ , the solution to (A.2). The term in square brackets is

strictly concave in  $(x,w)$  by appropriate assumption so that a unique optimal choice is assured. The fact that  $z(t)$  is continuous in  $t$  implies that  $s(t)$  and  $w(t)$  are as well. The pair  $(s_0, w_0)$ , however, is the solution to the same problem given  $z = z_0$ , the initial value for (A.2),  $b = 0$ , and  $q_0$ , the associated escape rate. Finally interior solutions are assumed.

Proposition 2: At the moment benefits are exhausted, the escape rate jumps up (down) if income and leisure are strict complements (substitutes) in household production; i.e.,

$$\lim_{t \rightarrow 0} q(t) \begin{matrix} > \\ < \end{matrix} q_0 \text{ as } u_{12}(\cdot) \begin{matrix} > \\ < \end{matrix} 0.$$

Proof: By virtue of (A.1),  $U(w_0, U_T) = z_0$  and  $U(w(t), U_T) = z(t)$ . Since  $U(\cdot)$  and  $z(\cdot)$  is continuous,  $\lim_{t \rightarrow 0} w(t) = w_0$ . The source of the discontinuity is  $s(t)$ . For  $t > 0$ , the first order condition is

$$u_2(b, 1-s(t)) = \alpha \int_{w(t)}^{\bar{w}} [U(x, U_T) - z(t)] dF(x).$$

However,

$$u_2(0, 1-s_0) = \alpha \int_{w_0}^{\bar{w}} [U(x, U_T) - z_0] dF(x).$$

The continuity of  $u_2(\cdot)$  and the facts that  $(z(t), w(t))$  limits to  $(z_0, w_0)$  as  $t \rightarrow 0$  clearly imply

$$u_2(b, 1 - \lim_{t \rightarrow 0} s(t)) = u_2(0, 1-s_0).$$

where  $u_2(\cdot)$  is the partial of  $u(\cdot)$  with respect to its second argument. Since the second partial,  $u_{22}(\cdot)$  is negative and  $b > 0$ , the assertion follows.

FOOTNOTES

- 1/ The author was aided in writing this paper by conversations with Kenneth Burdett. Indeed, the model developed is a utility maximizing extension of that analyzed by Burdett in "Employee Search and Quits," forthcoming in the American Economic Review. The author also acknowledges the useful and extensive suggestions for revision of the original draft offered by the editors and a referee. Remaining errors and confusing passages are mine.
- 2/ See Dale T. Mortensen, "Job Search, The Duration of Unemployment, and the Phillips Curve," American Economic Review 60, Dec. 1970, pp. 847 - 862 for example.
- 3/ Kathy Classen, "Effects of Unemployment Insurance on Job Search: Evidence from Arizona and Pennsylvania," Symposium on the Economics of Unemployment Insurance, this journal and issue.
- 4/ In an earlier draft, employed workers are allowed to choose hours worked. All the results reported here are true in this case as well.
- 5/ See Steven A. Lippman and John J. McCall, "The Economics of Job Search: A Survey," Economic Inquiry XIV, June, 1976, pp. 155-189, for a discussion of Markov decision processes and their relationship to job search models.
- 6/ See Richard E. Bellman, Dynamic Programming. Princeton, N. J.: Princeton University Press, 1957.
- 7/ This implication is consistent with the observation that only a small fraction of those who quit become unemployed. See P. Matilla, "Job

Quitting and Frictional Unemployment," American Economic Review 64, 1974, pp. 255-339.

8/ Specifically, the capital value of any job to the worker is lost when permanently laid off.

9/ The proofs are contained in an unpublished mathematical appendix.

10/ This view is suggested by Gary S. Becker, "A Theory of the Allocation of Time," Economic Journal 75, Sept. 1965, pp. 493-517, among others.

11/ Specialization in search is ruled out as uninteresting and an unemployed worker not searching is a non-participant by definition. The existence of a non-trivial reservation wage is studied by Burdett, "Employee Search and Quits," in the context of a similar model.

12/ See Lippman and McCall, "The Economics of Job Search: A Survey," for a review of the alternative explanations of a falling reservation wage.

13/ See the unpublished mathematical appendix.

14/ Arlene Holen, "Effects of Unemployment: Insurance Entitlement on Duration and Job Search Outcome," Symposium on the Economics of Unemployment Insurance, this journal and issue.

15/ Ronald G. Ehrenberg and Ronald L. Oaxaca, "Unemployment Insurance, Duration of Unemployment, and Subsequent Wage Gain," American Economic Review 66, Dec. 1976, pp. 754-766.

16/ Classen, "Effects of Unemployment Insurance on Job Search".

- 17/ Steven T. Marston, "The Impact of Unemployment Insurance on Job Search," Brookings Papers on Economic Activity, 1, 1975.
- 18/ Herschel Kasper, "The Asking Price of Labor and the Duration of Unemployment," Review of Economics and Statistics, May 1967, pp. 165-172.
- 19/ Holen, "Effects of Unemployment Insurance Entitlement on Duration and Job Search Outcomes."
- 20/ Classen, "Effects of Unemployment Insurance on Job Search."
- 21/ Ehrenberg and Oaxaca, "Unemployment Insurance, Duration of Unemployment, and Subsequent Wage Gain."
- 22/ Martin S. Feldstein, "The Effects of Unemployment Insurance on Temporary Layoff Unemployment," Harvard University, Department of Economics, Nov. 1976.



FIGURE #1

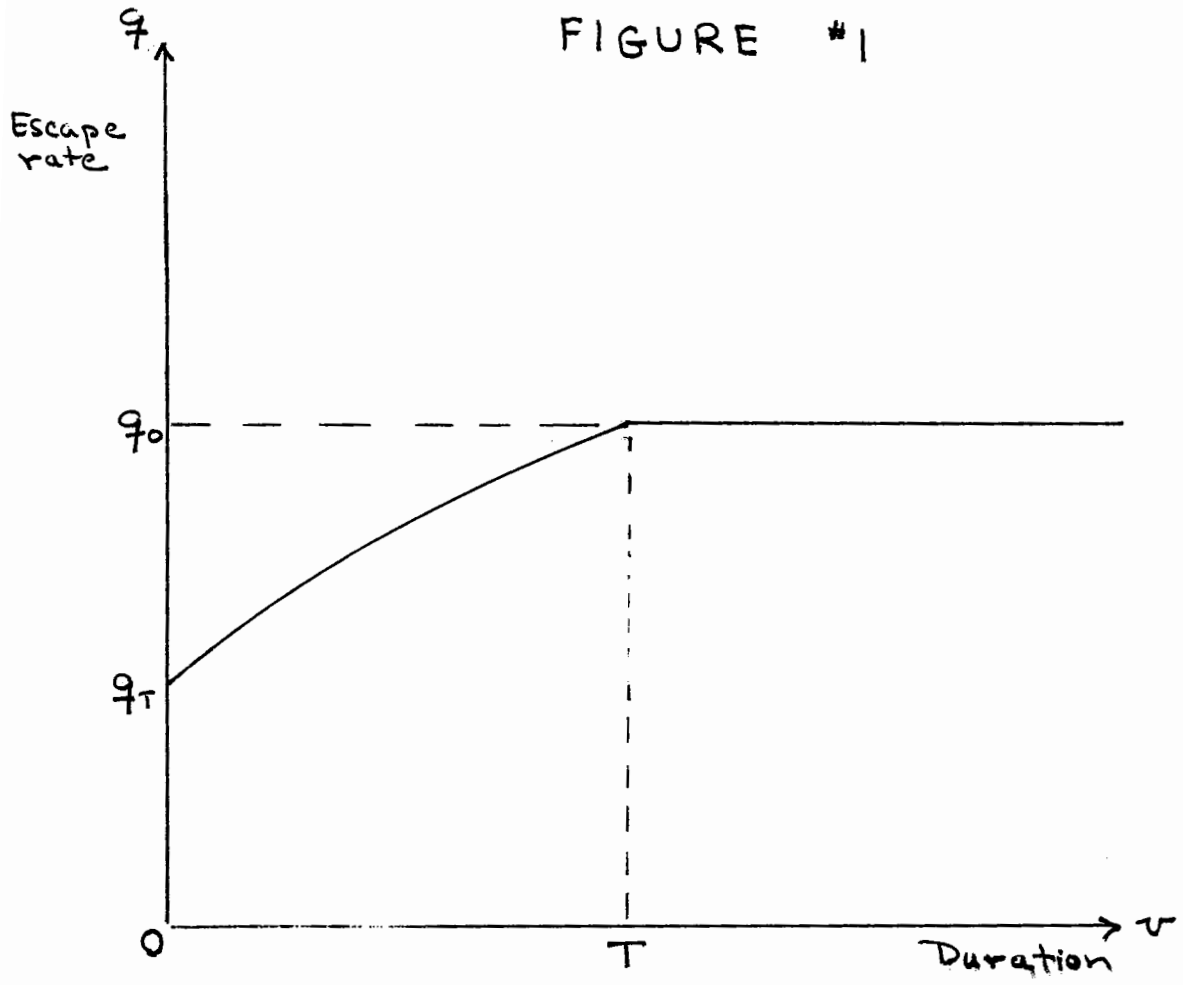


FIGURE #2

