

Chandrasekaran, R.; Daughety, A.

Working Paper

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Discussion Paper, No. 278

Provided in Cooperation with:

Kellogg School of Management - Center for Mathematical Studies in Economics and Management
Science, Northwestern University

Suggested Citation: Chandrasekaran, R.; Daughety, A. (1977) : A Short Note on a Linear Equation Associated With Certain Economic Response Systems, Discussion Paper, No. 278, Northwestern University, Kellogg School of Management, Center for Mathematical Studies in Economics and Management Science, Evanston, IL

This Version is available at:

<https://hdl.handle.net/10419/220638>

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DISCUSSION PAPER NO. 278

A SHORT NOTE ON A LINEAR EQUATION
ASSOCIATED WITH CERTAIN ECONOMIC
RESPONSE SYSTEMS⁺

by

R. Chandrasekaran^{*}
A. Daughety^{**}

April 1977

⁺The authors thank John Roberts and Françoise Schoumaker for providing some of the applications contained herein. This work was supported in part by NSF Grant APR 75-16731.

^{*}School of Management and Administrative Sciences,
University of Texas at Dallas

^{**}Graduate School of Management,
Northwestern University

A Short Note on a Linear Equation
Associated with Certain Economic
Response Systems

We consider three problems:

Problem 1: Find the sign of $\frac{dp_i}{d\alpha}$ for the system of the equations [1]
(the equations represent demand and supply in a
service differentiated transport sector):

$$\begin{aligned} p_i(q_i, \alpha) &= R_1(\sum q_k) \\ p_i(q_j) &= R_j(\sum q_k) \quad j \neq i \\ MC_k^h(q_k^h) &= p_k(q_k) \quad k=1, \dots, n \\ & \quad h=1, \dots, H_k \\ q_k &= \sum_h q_k^h \end{aligned}$$

where suitable conditions are given for $\frac{\partial p_k}{\partial q_k}$, R'_k
and $\frac{\partial p_i}{\partial \alpha}$, and there are H_k firms providing mode k
service with n modes.

Problem 2: Does a solution exist (and what is it) for the follow-
ing system of first order conditions [4*]:

$$\varphi_{ih}(-2 + 2\gamma_i + \frac{1}{n}) + \bar{\varphi}_h(1 - 2\gamma_i) = -\Pi_{ih}(1 - \frac{1}{n})$$

where n is the number of agents, γ_i is the i^{th} agents
share of surplus from trade, Π_{ih} is the agents actual
marginal rate of substitution between good h and the
numeraire good while φ_{ih} is the announced marginal rate
substitution ($\bar{\varphi}_h = \frac{1}{n} \sum_i \varphi_{ih}$)?

* A related problem is in [3].

Problem 3: Given the $n-1$ demands $(m_j, j \neq 1)$ for a public good y ($y = \sum m_j$), consider a consumer who desires to minimize his Groves-Ledyard tax [2] C_i :

$$C_i = \alpha_i q y + \frac{\gamma}{2} \left[\frac{n-1}{n} (m_i - \hat{\mu}_i)^2 - \frac{1}{n-2} \sum_{j \neq i} (m_j - \hat{\mu}_i)^2 \right]$$

where q is the price of the good, $\gamma > 0$ is a coefficient set by the planning, α_i is the consumer's share of the cost of the good and

$$\hat{\mu}_i = \frac{1}{n-1} \sum_{j \neq i} m_j. \text{ If on the other hand all the } m_i$$

are to be found, is there a solution?

What these problems all have in common is that their solution depends on a system of linear equations that is considered in the following theorem:

Theorem: The system of equations $(R-BC)x = d$ with $|R| \neq 0$, $C = FEP$, all matrices $n \times n$, E the matrix of ones and x, d are $n \times 1$ vectors, has the solution:

$$x = R^{-1} \left[d + \frac{e' P R^{-1} d}{1 - e' P R^{-1} B F e} B F e \right]$$

(where e is an $n \times 1$ vector of ones) on the condition that $e' P R^{-1} B F e \neq 1$.

and where $k_1 = \frac{\partial p_1 / \partial q_1}{\partial p_1 / \partial q_1 - R_1'}$ and $r_j = \sum_h [1 / (MC_j^h)']$

$$d = \frac{-\partial p_1 / \partial \alpha}{\partial p_1 / \partial q_1 - R_1'} \cdot \begin{bmatrix} R_2' \\ \vdots \\ \vdots \\ R_n' \end{bmatrix}$$

and finally $dp = \begin{bmatrix} dp_2 / d\alpha \\ \vdots \\ \vdots \\ dp_n / d\alpha \end{bmatrix} = x$

In this case (I-MT) has an inverse (i.e. the conditions of the theorem are met).

In the second problem if we let $B = P = I$, and R and C be diagonal matrices with

$$r_{ii} = -2 + 2\gamma_i + \frac{1}{n}$$

and

$$c_{ii} = (1 - 2\gamma_i) / n$$

and if $\varphi_h = (\varphi_{1h}, \dots, \varphi_{nh})'$ is the $n \times 1$ solution vector while $d = (-\pi_{1h} / (1 - \frac{1}{n}), \dots, -\pi_{nh} / (1 - \frac{1}{n}))'$ then we can represent problem two as:

$$(R - CE)\varphi_h = d .$$

Here again we find the conditions of the theorem are met and the announced marginal rates of substitution (φ_h) can be expressed in terms of the actual marginal value of substitution (d).

Finally, in the third problem we find that the condition that $e'PR^{-1}BFe \neq 1$ is not met. The system of equations is

$$(nI - E)m = - \frac{qn}{\gamma} \alpha .$$

Letting $R = nI$, $B = P = F = I$ we see that $e'PR^{-1}BFe = 1$.

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