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OPTIMUM PRODUCT DIVERSITY AND THE INCENTIVES FOR ENTRY IN LARGE ECONOMIES

by

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Optimum Product Diversity and the Incentives for Entry In Large Economies[1]

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Larry E. Jones

The relationship between the collection of goods produced in a market economy and the collection that would be produced by a social planner has attracted a great deal of interest in recent years (e.g., Dixit and Stiglitz [2], Hart [5] and Spence [15].

Although the findings of most of these studies is that no universal conclusions can be drawn concerning the nature of the bias in this relationship (e.g., there are always too many or too few goods produced by the market), there has been some success in identifying the determinants of the direction of the bias.

At the same time, another body of literature has evolved concerning the relationship between monopolistically competitive and Walrasian (i.e., perfectly competitive) equilibria in large economies.

The two literatures are related, of course, in that it is natural to suspect that if the market equilibrium allocation is approximately Walrasian, the collection of goods produced by the market should be approximately correct.

The purpose of this paper is to provide a simple exposition of what some of the results from the second branch of literature mentioned above have to say about the question addressed by the first branch. In particular, we will show (Theorem 2) that, within the context of our model, if all goods in the economy are substitutes there is never too little commodity differentiation

in large economies. We show by means of example that there may be too much, however.

In contrast, if the goods are complementary, there may be too little differentiation even in large economies.

These examples and results all flow from a simple yet little-known property of continuously differentiable and strictly convex preferences (Theorem 1). This is that if the collection of goods produced by the market does not include all goods which would be produced in a Walrasian equilibrium and all of those goods which are produced are sold at their Walrasian prices, there is some unproduced good for which the marginal cost of production is exceeded by its marginal benefit. It follows that there is, when the market is large enough, an incentive for some firm to enter and produce that good.

The major conclusion that can be drawn from this fact is that it is impossible to end up (in large economies) in a situation in which the "wrong" set of goods is produced but they are sold at the "right" (i.e., Walrasian) prices. Thus, we are led to study situations in which prices are "wrong" for the goods which are produced.

Since prices can never be too low (firms would have negative profits in this case), it follows that the incentives to produce substitute goods are increased relative to the perfectly competitive case and the incentives to produce complements are decreased. It is this simple intuition that gives rise to the examples mentioned above.

The remainder of the paper is organized as follows. In section 2 the model and notation are introduced. In section 3,

the major results of the paper are stated and proved. In section 4 we present two examples, one in which the market produces too many goods, and one in which it produces too few. Finally, in section 5, a few concluding remarks are offered.

2. The Model and Notation

Since the aim of this paper is to try and relate the effects of possibly incorrect prices on market equilibrium product selection, we will limit discussion to the case where there are only finitely many potential goods. Most of the results generalize directly to economies in which there are a continuum of goods, the case more commonly considered in models of commodity differentiation (see the comments in Section 5 for the generalizations to this case).

Accordingly, we will assume that consumers have utility functions

$$U^h(L,x_1,\ldots,x_m),\ h = 1,\ldots,H$$

where H is the number of consumers, L is the level of consumption of leisure and x_i is the level of consumption of the i-th differentiated product.

Throughout what follows, we will assume that households are endowed with leisure and leisure only. Let L^h represent h's initial endowment of leisure.

Further, no firms will be allowed to produce leisure and so it is natural to use this good as the numeraire.

We will make the following assumptions concerning the U^h's:

Assumption A

(A1) U^h is continuous.

(A2) U^h is strictly increasing.

(A3) U^h is strictly convex if L is positive.

(A4) U^h is twice continuously differentiable on an open set containing R_+^{m+1}.

Given these assumptions, it follows that demand is well defined and unique given the availability of any subcollection of the k differentiated products and price for those goods. That is, if $K \subset T = \{1,\ldots,m\}$ is the set of goods available at prices, p_i, $i \in K$, define

$$\beta^h(K,p) = \{(L,x_1,\ldots,x_m) \mid \sum_{i\in K} p_i x_i + L \leq L_h \text{ and}$$

$$x_i = 0 \text{ if } i \notin K\}$$

Then,

$$\phi^h(K,p) = \{(L,x_1,\ldots,x_m) \mid (L,x_1,\ldots,x_m) \text{ maximizes}$$

$$U^h \text{ in } \beta^h(K,p)\}$$

is well defined and unique for all K and p.

Let

$$\Phi(K,p) = \sum_{h=1}^{H} \phi^h(K,p)$$

denote aggregate demand.

2.2 Firms and the Model

We will let N denote the number of potential producers and will assume throughout that N is larger than the number of potential products (i.e., there is free entry).

The model of firm interaction that we will examine is a special case (with the number of potential goods finite) of the model analyzed in Jones [8]. It differs from Cournotian models in several important ways. First, the individual firm production sets are unbounded. That is, the costs of the individual firms are a set-up cost plus constant marginal costs. Second, the basic strategic variables chosen by the firms will be price and product characteristics rather than quantity and product characteristics. Finally, the game that we will use to describe the strategic interaction of firms will proceed in two stages.

At the first stage, firms simultaneously choose product characteristics. At the second stage, firms set prices. The advantage of this two-step strategic formulation of the problem is that firms have the foresight to recognize the onset of direct price competition a la Bertrand [1] at the second stage if they choose the same product as some other firm at the first stage. Because of this, it will follow (Proposition 1) that, in equilibrium, firms will differentiate their products. (Note that this conclusion depends crucially on the constant marginal cost assumption.) Thus, the two stage nature of the interfirm interaction allows us to avoid the intuitively unappealling conclusion of perfect competition even when the number of firms is small.[2]

Thus, the cost function for all firms can be written as $C(q) = \bar{c} + q$. Thus, $\bar{c} = C(0)$ is a fixed cost and the marginal cost of output is constant at 1. Note that we are assuming that costs depend on the level of output but not its type. That is, the marginal cost of production is the same for all goods. While this seems like a severe assumption at first sight, it really amounts to a choice of normalization-- We will measure units of output in terms of their labor equivalents.

Let $Z=T\cup\{NP\}$, where NP signifies the decision by the firm not to produce any product at all.

Formally, firm i chooses a strategy $s_i = (z_i, p_i(z_1,\ldots,z_N))$ where $z^i \in Z$ is i's choice of product and p_i is i's choice of price given the product choices of all firms $(z_i,\ldots,z_N)$.

We will use the usual Nash notion of equilibrium.

Given an array of strategies $\underset{\sim}{s} = (s_1,\ldots,s_N)$, three concepts will be useful. The first, $T_s \subset T$ is the set of goods offered by the firms under the strategy s. Second, define $p_s(t)$ to be the lowest price offered by any firm choosing the product t at the first stage. Note that although it will follow that <u>in equilibrium</u> no two firms will choose the same product, this possibility is not ruled out a priori. Finally, for any t, let $n_s(t)$ be the number of firms both choosing t and offering to sell it at the lowest price, $p_s(t)$.

Given these definitions, we can define the profits of the ith firm as follows

$$\pi_i(s) \begin{cases} \frac{1}{n_s(t)}\,\phi(T_s,p_s;t)p_s(t) - C\left(\frac{1}{n_s(t)}\,\phi(t_s,p_s;t)\right) & \text{if } t_i = t \text{ and i's price equals } p_s(t) \\ -C(0) & \text{if } t_i = t \text{ and i's price is larger than } p_s(t) \\ 0 & \text{if } t_i = NP \end{cases}$$

From this description of profits, we can see several features of the model.

First, it follows that if several firms choose the same product and offer to sell it at the same lowest price, they split the market evenly.

Second, a firm choosing a product t in T and charging a price higher than $p_s(t)$ has losses of C(0). This implies that the fixed cost component of C is what is commonly referred to as sunk. Thus, it is useful to think of this cost as being paid at the first stage of the competition, before prices are announced and sales are realized.

The limiting results we will explore in the model are those in which C(0) is small. As can be readily seen, the limiting economy is one with a constant returns to scale production set and marginal cost 1 for all goods.

We turn now to a discussion of the limiting properties of the model.

3. Results

In this section, the results concerning the properties of the model outlined in section 2 will be presented.

First, we have the result that, as per design, firms

differentiate their products in equilibrium.

Proposition 1. In equilibrium, if $z_i \neq NP$ (i.e., firm i is producing) and $j \neq i$, $z_j \neq z_i$.

We will be interested in outlining systematic sources of product selection bias in large economies. To this end, it will be useful to set aside a special notation for the collection of goods which would be produced in the perfectly competitive equilibrium of the limiting constant returns to scale economy.

Let $T^* = \{t | \phi(T;p^*;t) > 0\}$ where p^* is that price function which is 1 for all t.

Then, we will be interested in the relationship between the collection of goods produced in the Nash equilibrium of the game outlined in section 2 (when C(0) is small) and T^*.

One's intuition might lead one to believe thatcomplementarities between goods might cause problems. That is, we might end up (when C(0) is small) in a situation where two complementary goods in T^*, s and t, are not produced. That is, it may not pay to produce s because t is not being produced and vice versa.

As we shall see, however, given our assumptions on preferences, in particular that they are strictly convex and continuously differentiable, this type of phenomenon cannot occur if the prices are "right" (i.e., equal to marginal cost) for the set of goods which are produced. This is the content of Theorem 1.

Theorem 1. Suppose $\hat{T}$ is a collection of goods such that $T^* - \hat{T} \neq \emptyset$, then there is a good $t^* \in T^* - \bar{T}$, a consumer h, and an $\varepsilon > 0$ such that

$$\phi^h(\hat{T} \cup \{t^*\}, p^*; t^*) > 0$$

where

$$p^*(t) = \begin{cases} 1 & \text{if } t \neq t^* \\ 1 + \varepsilon & \text{if } t = t^* \end{cases}$$

That is, if the prices are right for the set of goods $\hat{T}$, it pays some producer to enter and produce the good t*.

Proof. Choose h so that when all goods are available at price 1, h buys some good outside of $\hat{T}$. This is always possible since $T^* - \hat{T} \neq \emptyset$. Let x^1 be h's demand when the goods $\hat{T}$ are available a price 1, $x^1 = \phi^h(\hat{T},1)$, and let T^1 be the set of goods, t, where $x_t^1 > 0$. Similarly, let $x^2 = \phi^h(T,1)$ and let T^2 be the set of t where $x_t^2 > 0$. Note that it necessarily folows that $\phi^h(\hat{T},1) = \phi^h(T^1,1)$, $\phi^h(T,1) = \phi^h(T^2,1)$, and $T^2 - T^1 \neq \emptyset$.

Finally, let MRS(t) be h's marginal rate of substitution between leisure and t when consumption is x^1.

Then, since $U^h(x^2) > U^h(x^1)$ and U^h is strictly concave and differentiable, it follows from the usual support arguments that

$$(L^2 - L^1) + \sum_{t \in T^1 \cup T^2} (x_t^2 - x_t^1)MRS(t) > 0$$

That is

$$L^2 + \sum_{t \in T^2} x_t^2 MRS(t) > L^1 + \sum_{t \in T^1} x_t^1 MRS(t).$$

From the fact that $x^1 = \phi^h(T^1,1)$, it follows that MRS(t) =

1 if $t \in T^1$. Thus, from the budget constraint it follows that

$$L^1 + \sum_{t\epsilon T^1} x_t^1 MRS(t) = L^1 + \sum_{t\epsilon T^1} x_t^1 1 = L^h,$$

Thus, it follows that

$$L^2 + \sum_{t\epsilon T^2} x_t^2 MRS(t) > L^h.$$

Now suppose that $MRS(t) \leq 1$ for all $t \epsilon T^2 - T^1$. Then, an argument similar to that used above gives

$$L^h = L^2 + \sum_{t\epsilon T^2} x_t^2 1 \geq L^2 + \sum_{t\epsilon T^2} x_t^2 MRS(t).$$

This contradiction implies that $MRS(t) > 1$ for some $t \epsilon T^2 - T^1$.

The argument that $\phi^h(\hat{T} \cup \{t^*\}, p^*; t^*) > 0$ is now straightforward. Q.E.D.

We are now in a position to state and prove the main result concerning our model. This is, that if all the goods in T are substitutes, there is never too little commodity differentiation when the economy is sufficiently large.

Before formally stating this result, we will need three definitions.

Definition: Define the notion of convergence on subsets of T by: $T^k \to \hat{T}$ if there exists K such that $k \geq K$ implies $T^k = \hat{T}$.[3]

Definition: The goods in T will be said to be (weak) substitutes if for all $\hat{T} \subset T$ and all $p^i : \hat{T} \to R^+$, $i = 1,2$ with $p^1(t) \geq p^2(t)$

$$\phi(\hat{T},p^1;t^*) \leq \phi(\hat{T},p^2;t^*)$$

That is, if the prices of all goods are raised (not decreased) and the price of t* is held constant, the demand for t* does not fall. Note that this is the definition of substitutes in the classical rather than the Slutsky sense since the requirement is on the uncompensated rather than the compensated demand curves.

Finally,

Definition: The equilibrium array of strategies,

$$(z_1,p_1(z_1,\ldots,z_N),\ldots,z_N,p_N(z_1,\ldots,z_N))$$

is said to be semi perfect if for all i and $(z_1,\ldots,z_N)$,

$$p_i(z_1,\ldots,z_N)\geq 1.$$

That is, an equilibrium is semiperfect if no firm threatens to cut price to below marginal cost. This is clearly a minimal dynamic consistency requirement. (In particular, it is satisfied by subgame perfect equilibria.)

We can now state the major result of this paper.

Theorem 2: Let T^n,p^n be a sequence of collections of goods and prices for a sequence of semiperfect equilibria with $C^n(0) \to 0$. Suppose that $T^n \to \hat{T}$ and $p^n \to p$ and that the goods are all substitutes. Then $T^* \subset \hat{T}$.

That is, asymptotically, there is never less product differentiation than would be produced in the Walrasian equilibrium.

Proof: If $T^* - \hat{T} \neq \emptyset$, choose a t^*,ε and h as in Theorem 1. Without loss of generality, assume that firm 1 is choosing NP at stage 1 of each game in the sequence. Consider the alternative

strategy by firm 1 in which it chooses the good t* at stage 1 and sets its price at $1 + \varepsilon$ (independent of the actions of the other firms).

Let p_n^1 denote the prices charged by the firms numbering 2,...,N when firm 1 enters and let p_n^2 be defined by

$$p_n^2(t) = \begin{cases} p_n^1(t) & \text{if } t \in T^n \\ 1 + \varepsilon & \text{if } t = t^* \end{cases}$$

Finally, let

$$p^*(t) = \begin{cases} 1 & \text{if } T \in T^n \\ 1 + \varepsilon & \text{if } t = t^* \end{cases}$$

Then p^2 gives the prices that would prevail in the market at the n-th stage if firm 1 adopts the strategy outlined above. In this case, the profits for firm 1 are:

$$\pi_1^n = \phi(T^n \cup \{t^*\}, p_n^2; t^*) \cdot (1 + \varepsilon) - C^n(\phi(T^n \cup \{t^*\}, p_n^2; t^*))$$

Since the equilibria are semi-perfect, it follows that $p_n^2(t) \geq 1$ for all n and all $t \in T^n$. Since the goods are substitutes it follows (due to the special form of C^n) that

$$\pi_1^n = \phi(T^n \cup \{t^*\}, p_n^2; t^*)(1 + \varepsilon) - C^n(\phi(T^n \cup \{t^*\}, p^*; t^*))$$

$$\geq \phi^h(T^n \cup \{t^*\}, p^*; t^*)(1 + \varepsilon) - C^n(\phi^n(T^n \cup \{t^*\}, p^*; t^*))$$

Now, as n goes to infinity the last term converges to (since ϕ^h is continuous)

$$\phi^h(\hat{T} \cup \{t^*\}, p^*; t^*) \varepsilon$$

which is strictly positive.

It follows that for n sufficiently large π_1^n is strictly positive under the proposed strategy. This contradicts the assumption that the original situation was an equilibrium since firm 1 was earning zero profits (since their first stage move was NP). Q.E.D.

Note that this result does not imply that the equilibrium allocations in this sequence of games converges to the Walrasian allocation since it need not be true that the prices converge to their perfectly competitive levels (i.e., marginal cost). Indeed, it is because of the fact that prices need not converge to their competitive levels that there can be too many varieties produced even in the limit. That is, it is quite possible that $\hat{T}$ contains T* plus more. That this can occur is shown in Example 1 of the next section. Example 2 shows that the opposite can occur if the goods are not all substitutes.

4. Examples of Product Selection Bias in Large Economies

The purpose of this section is to present, in some detail, two examples to show how product selection biases can arise in the context of our model. In the first, too many goods are produced; in the second, too few.

It should be emphasized that the examples are non-pathological in the sense that the utility functions are only slight variations of those commonly encountered in applied economic work.

Example 1: Too Many Goods.

This will be a simple one person economy with two goods other than labor. For simplicity in the presentation, let L denote the individual's consumption of labor and let x_1 and x_2 denote his consumption of the other two goods. We will assume that the individual is endowed with two units of labor and has the utility function

$$U(L,x_1,x_2) = \ln(x_1 + \frac{L}{10} + 1) + \ln(x_2 + 4)$$

This is only a slight modification of the usual Stone-Geary functional form (see Phlips [11] for a detailed presentation on this and the linear expenditure system). Under the standard interpretation of the Stone-Geary form, -4 is the subsistence level of good 2, etc. Thus, none of the three goods is a necessity.

Notice that this utility function as defined is at variance with our assumptions in several inessential ways. First, we have assumed that one unit of good 1 is a perfect substitute for 10 units of labor. This was done to guarantee that prices are bounded in such a way as to keep the computations as simple as possible. Note also that it is not strictly concave (because of the perfect substitutability mentioned above) and leisure is not essential. These differences are unimportant, however, as it will be evident that the example is quite robust.

Given that the consumer's labor endownment is 2 (and that the marginal cost of each of the two other goods is 1), it can be shown that his marginal rate of substitution between good 1 and

good 2 is strictly greater than 1 for all combinations of consumption in the relevant region. It follows that when both of the goods are priced at marginal cost, the consumer buys only good 1. Thus, the competitive equilibrium for this economy consists of pricing all three goods at 1 and the consumer setting $L = x_2 = 0$ and $x_1 = 2$.

That is, in the Walrasian equilibrium, only good 1 is produced.

We turn now to an analysis of the monopolistically competitive equilibria of this economy in the sense discussed in the previous sections.

We will consider a world with only two firms. As will be clear from what follows, the inclusion of additional firms would not affect the nature of the equilibrium in any way.

Although we will be interested in the subgame perfect equilibria of the game with demand as derived from this utility function, we should point out at this point that one semi-perfect equilibrium of this game is for producer one to enter and produce the first good. This is followed by setting the price equal to the monopoly price if no other firm enters and threatening to set price at marginal cost if anyone else enters. Since the MRS between goods 1 and 2 is always at least 1, this threat by firm 1 guarantees that demand for good 2 is zero at any price at least marginal cost. Thus, no firm would enter and produce good 2. Clearly, no firm can profitably enter and produce good 1. It follows then that under this threat by firm 1, no other firm enters and thus firm 1 is left as the only producing firm and charging the monopoly price.

This equilibrium is a little unsatisfactory, however, since the threat by firm 1 to cut his price to marginal cost is not credible if firm 2 decides to produce good 2. (It is credible if firm 2 chooses to produce good 1, however.) That is, although the equilibrium is semi-perfect, as will become clear in what follows, it is not subgame perfect.

To find a subgame perfect equilibrium for this game, we first need to find equilibria of second stage price game as a function of the first stage choices of the two firms.

(1) If both firms choose not to produce, there is no second stage and both firms earn zero profits.

(2) If both firms choose the same good (either good 1 or good 2), the usual Bertrand argument shows that an equilibrium is for both firms to set price at marginal cost. In this case, both firms earn profits of -C(0).

(3) If one firm chooses NP and the other chooses good 1, a simple calculation shows that the equilibrium price (i.e., the monopoly price) for good 1 is 10. In this case, the consumer spends all of his income on good 1 and the profits to the firm are 1.8 - C(0).

(4) If one firm chooses NP and the other chooses good 2 a calculation gives a price of 2 in the equilibrium. In this case, the entering firm earns profits of 1 - C(0).

(5) If both firms enter, but choose different goods, a lengthy calculation shows that a price equilibrium is for the firm producing good 1 to charge $p_1 = 2.50$ and for the firm producing good 2 to charge $p_2 = 1.06$. At these prices, the

consumer buys L =0, x_1 = .748 and x_2 = .1226. This gives profits of 1.122 - C(0) for the producer of good 1 and .0074 - C(0) for the producer of good 2.

From these considerations it follows that if C(0) < .0074, a subgame perfect equilibrium of this game is for firm 1 to enter, produce good one and announce the price strategy for the producer of good 1 as outlined in (2) through (5) above. In response to this, the best thing for firm 2 to do is to choose good 2 and follow the price strategy outlined for the producer of good 2 as outlined above. Of course, since the game is symmetric, there is another equilibrium with the roles of the two firms reversed.

Note that there is too much product differentiation in equilibrium. This is true in two senses.

First, there are more goods produced than in the Walrasian equilibrium of the limit economy. Note that this holds no matter how small C(0) is. In fact, the prescribed strategies still give an equilibrium even when C(0) = 0. This holds even if more firms are added. Note however that there are other equilibria when C(0) = 0 as well. In particular, if there are 4 firms, the Walrasian equilibrium is an equilibrium of the game with 2 firms entering and producing each of the goods.

Second, the allocation resulting from the equilibrium above is not Pareto optimal. In fact, a Pareto improvement can be had through having firm 2 quit producing and diverting those resources used in the production of good 2 in to the production of good 1. Enough surplus is created by this to compensate firm 2's loss of profits.

The intuition behind the inefficiency in this example is quite straightforward. Given that firm 1 is producing good 1, firm 2 has two options. It can either enter and produce good 1 or good 2. From the consumer's point of view, the preferable choice is that the firm produce good 1. From firm 2's point of view, this is an unattractive opportunity due to the impending price competition. Thus, it pays firm 2 to differentiate its product, avoid direct price competition a la Bertrand and earn positive profits. However, in the process, too many goods are produced.

Example 2: Too Few Goods

The purpose of this example is to show that if the substitutes condition of Theorem 2 fails to hold, the strict containment of $\hat{T}$ in T* is possible. That is, too few goods can be produced in equilibrium even asymptotically if compelementarities are present.

The intuition underlying the example is quite straightforward. There are two households, and, as in Example 1, three goods. Again, we will let the consumption levels of three goods be denoted by the L, x_1 and x_2. It is convenient to think of good 1 as gin and good 2 as vermouth. The utility functions of the two individuals are such that at the Walrasian prices, consumer 1 mixes gin and vermouth (i.e., he makes martinis) and consumer 2 likes only gin. However, when consumer 1's consumption of gin is sufficiently low, he drinks only gin. Further, consumer 2 is much more wealthy than consumer 1.

Thus, in equilibrium, one firm enters and produces gin. There is no incentive for a firm to enter and produce vermouth

because in the resulting configuration the gin producer drives the price up in order to extract the surplus from consumer 2. In the process, this drives consumer 1's consumption of gin down to the point where his demand for vermouth is zero. Thus, in equilibrium, only gin is produced even though at the Walrasian prices, both goods are produced.

All that remains is to check that the details of the above story are substantiated in the example.

Suppose that the utility functions of the two individuals are:

$$U_1(L,x_1,x_2) = [L^{-1} + x_1^{-1} + (x_2+ 1)^{-1}]^{-1}$$

and

$$U_2(L,x_1,x_2) = L^{1/2}(x_1 + 1)^{1/2}$$

Further, we will assume that consumer 1 is endowed with 3 units of labor and consumer 2 is endowed with 20 units of labor. Thus, U_1 is a slightly altered version of the CES utility function and U_2 is of the Stone-Geary variety.

To see that one pure strategy subgame perfect equilibrium of this game has the properties described above, it is sufficient to show that if a firm enters and produces good 2, there is an equilibrium in the subsequent price game in which this producer does not earn positive profits. (Note: There may be other equilibria as well.)

Assume that firm 1 has chosen to produce good 1 and that firm 2 has chosen to produce good 2. Then, for our purposes, it

is enough to show that if firm 2 sets price at 1 (and hence earns zero profits on its sales) the best response by firm 1 is to set its price at a level such that if firm 2 charges a price higher than 1 it has no sales. This implies that it is an equilibrium of the subgame for firm 2 to charge marginal cost (and hence earn no profits).

Intuitively, the reason one might expect this to be the case is the following: When firm 1 raises its price, two things happen. First, through consumer 1 there is both a direct and an indirect effect. The direct effect reduces consumer 1's demand for the usual reasons. This is reinforced by the accompanying reduction in consumption of the complementary good, good 2. Thus, we would expect consumer 1's demand to be fairly elastic. On the other hand, consumer 2 is much wealthier and his demand for good 1 is relatively inelastic at low price levels. Thus, it pays firm 1 to drive up the price and capture more of consumer 2's wealth. In the process, this drives consumer 1 out of the market for good 2 altogether.

Given the parameters we have selected, the usual Kuhn-Tucker arguments show that the demand for good 1 when the price of good 2 is 1 is given by

$$\Phi_1(p_1,1)=\begin{cases} (5/p_1-\frac{1}{2}) + \left(\dfrac{4}{p_1^{1/2}(2+p_1^{1/2})}\right) & \text{if } p_1 \le 4 \\ (5/p_1-\frac{1}{2}) + \dfrac{3}{p_1^{1/2}+p_1} & \text{if } 4 \le p_1 \le 10 \\ 0 + \dfrac{3}{p_1^{1/2}+p_1} & \text{if } p \ge 10. \end{cases}$$

Throughout, the first term represents the demand by consumer 2 and the second is the demand of consumer 1.

Given this, it can be shown that firm 1 maximizes its profits by setting $p_1 = 7$. At this level, the profits to firm 1 are 16.009.

Given that the price of good 1 is at $p_1 = 7$, the demand for good 2 is given by:

$$\Phi_2(7,p_2) = \begin{cases} 0 \text{ if} & p_2 \geq \left(\frac{3}{1+\sqrt{7}}\right)^2 = .677 \\ \frac{3+p_2}{\sqrt{p_2}(\sqrt{p_2}+\sqrt{7}+1)} - 1 & \text{if } p_2 \leq \left(\frac{3}{1+\sqrt{7}}\right)^2 \end{cases}$$

Thus, firm 2 cannot earn positive profits if firm 1 sets $p_1 = 7$. Hence, a best response by firm 2 to $p_1 = 7$ is to set $p_2 = 1$. This minimizes his losses (and gives him zero sales).

These two facts together imply that $p_1 = 7$, $p_2 = 1$ is an equilibrium of the price game when firm one produces good 1 and firm 2 produces good 2. Thus, if firm 1 is producing good 1, firm 2 has no incentive to enter and produce good 2.

More formally, it follows that the following is a pure strategy equilibrium in the two stage game:

Firm 1

(i) Enters and produces good 1.

(ii) If firm 2 does not enter, firm 1 changes the monopoly price for good 1.

(iii) If firm 2 enters and produces good 1, the two firms revert to the Bertrand price equals marginal cost equiilbrium and

both firms lose their fixed costs.

(iv) If firm 2 enters and produces good 2, firm 1 sets his price at $p_1 = 7$. Given this, firm 2, at best, loses its fixed costs.

Firm 2: Chooses NP at the first stage.

Thus, in equilibrium, only good 1 is produced while in the Walrasian equilibrium (as can easily be checked), both goods 1 and 2 are consumed in positive amounts.

Section 5. Related Remarks

We will conclude the paper with a few related remarks.

(A). The first thing that should be pointed out is that in a sense the model is a bit contrived. This is because T is assumed to be finite. That is, the result in Proposition 1 that firms differentiate their products in equilibrium and the assumption that T is finite together imply that we would never expect to converge to the competitive equilibrium as fixed costs converge to zero since each producing firm will have a monopoly in the type of good it selects.

The restriction to the case of finite T was done in order to simplify the mathematics and concentrate on the incentives for entry.

Similar models with infinite T have been studied extensively in the literature. Examples include many forms of the location model (e.g., Hotelling [7] and Eaton and Lipsey [3]) and the quality models of Shaked and Sutton ([13] and [14]).

In terms of the results of this paper, it has been shown in Jones [8] that these go through in a straightforward way to the case of infinite T as long as demand satisfies assumptions

similar to those made here.

As far as the examples of section 4 are concerned, examples similar to Example 1 have been presented in Shaked and Sutton [13] and Jones [8].

The example of Shaked and Sutton is one of quality. The set of goods is the unit interval [0,1] with higher indices interpreted as goods of higher quality. There are a continuum of individuals with identical preferences but different incomes. In contrast to the approach adopted here, consumption choice is assumed to be completely indivisible. Thus, consumers choose only a quality level, the quantities being deterministic. Since it is assumed that the marginal cost of quality is zero, in the Walrasian equilibrium only the highest quality is produced. In the monopolistically competitive equilibrium that they calculate, however, two goods, one the highest quality and one slightly lower, are produced. Thus, too many goods are produced. Again this configuration remains an equilibrium no matter how low the fixed costs are.

The example in Jones [8] is similar in that again the monopolistically competitive equilibria do not converge to the Walrasian equilibrium as fixed costs are decreased. That example also has [0,1] as the set of goods and again the index can be interpreted as quality. As in the Shaked and Sutton example, in the Walrasian equilibrium only the highest quality good is produced. In the monopolistically competitive equilibrium, only the highest quality good is produced no matter how low the fixed costs are. No firm will enter and produce a lower quality since the resulting price competition drives prices so low that the

lower quality firm cannot make positive profits.

In Jones [8], a set of sufficient conditions are given for the convergence of the monopolistically competititve equilibria to the Walrasian equilibrium. In addition to the assumptions made here, there are two important assumptions. The first is that there are no isolated points in the set of goods produced in the Walrasian equilibrium. The second is that all the goods are substitutes. Note that the first of these assumptions rules out the finite T case studied here as well as the two examples discussed above. The role of the second of these two assumptions is to guarantee that the analog of Theorem 2 holds in the infinite T case.

Finally, as the Shaked and Sutton example discussed above shows, Example 1 has a direct analog in the infinite T case. At this point it is not known if Example 2 also has an infinite T analog. Of course, the approach to demand adopted by Shaked and Sutton implies that all goods are necessarily substitutes and therefore can be of no help in this regard.

(B) The notion that complementarities might cause problems for the convergence of monopolistically competitive equilibria to the perfectly competitive equilibrium as the economy grows has appeared in the literature before. This problem has been emphasized in connection with Cournot quantity setting models by both Hart [5] and Makowski [9]. In these models, strict complementarities are ruled out.

It is easy to see that the presence of strict complementarities would give rise to a situation in which too few products were produced in our model as well. That is, good 1

would not be produced because good 2 was not, and conversely.

Note that the assumption of continuously differentiable preferences rules out this possibility in the model analyzed here. Thus, the problem with complementarities encountered here is much more severe. In particular, note that the phenomenon exhibited in Example 2 is of a profoundly different nature in that it is not that one good is not produced and hence it does not pay to produce the other. In fact, in the example one of the goods is produced. Thus, the reason that good 2 is not produced is not because good 1 is not produced. Rather, as can be seen from the proof of Theorem 2, it is because good 1 is priced "incorrectly" (i.e., above marginal cost)[4].

(C) A remark on the distinction between subgame perfection and semiperfection is in order. Note that in the model analyzed here, subgame perfection implies semiperfection so that the main result, Theorem 2,is stronger than the corresponding result assuming subgame perfection of the equlibria.

However, semiperfect equilibria do allow for the possibility of some irrational (i.e., non-equilibrium) threats in the price subgame. The only place that this distinction really matters is in the examples of Section 4. They would have considerably less bite if they were examples based on equilibria which were semi-, but not subgame perfect. This is not the case, however, since in both examples the equilibria are in fact subgame perfect.

(D) One question that is especially relevant to Example 2 is that of the importance of our assumption that firms are allowed to choose only one good. In particular, can we get rid

of the phenomenon exhibited in Example 2 by allowing firms to produce more than one good?

It is easy to see that changing the model to allow for multiproduct firms would destroy the equilibrium of Example 2. That is, given that firm 1 is producing good 1, rather than choosing NP, firm 2 can do better by entering at both goods. This follows since the resulting price competition at good 1 will cause the price to fall to 1. Given this, firm 2 can make positive profits from the sale of good 2.

It is easy to see that this is not an equilibrium as well since given the price competition for good 1, firm 1 can do better by choosing NP at the first stage. This leaves firm 2 alone producing both goods. Given this, the optimal pricing policy is to price so that demand for good 2 is driven to zero.[5] This implies that firm 2, given that firm 1 is choosing NP, should avoid the duplication of fixed costs and produce only good 1. This cannot be an equilibrium as the above argument shows.

In short, there is no equilibrium in pure strategies for this game.

It is easy to change the game slightly in such a way that equilibrium will exist and multiproduct firms are allowed. This is to follow Prescott and Visscher [12] and have firms make their product selections sequentially. Thus, firm 1 first chooses all of its products, then firm 2, etc.

It is easy to see that in the single product case Theorem 2 remains valid for subgame perfect equilibria and that the equilibria presented in the two examples of section 4 are still equilibria under this new strategic form (this need not be true

in general).

It also follows that in the multiproduct case one subgame perfect pure strategy equilibrium with demand as in Example 2 is for firm 1 to choose both goods and price so as to drive the demand for good 2 to zero. In response to this, firm 2's strategy is: choose NP if 1 chooses both 1 and 2; choose goods 1 and 2 if 1 chooses only good 1,; choose good 1 if 1 chooses only good 2; and choose good 1 if firm 2 chooses NP. Prices are as given in Section 4.

In this case it does not pay firm 1 to drop 2 from its product portfolio because it realizes that if it did firm 2 would react by choosing both goods 1 and 2 thereby driving 1's profits to zero. Thus, it pays firm 1 to pay the fixed costs for both goods and keep firm 2 out of the market altogether. This holds even though firm 1 realizes that ultimately it will price so that no good 2 is purchased.

Note that although in this equilibrium both goods are selected by a firm, only one is produced and so there is still less diversity than would be provided in the Walrasian equilibrium.

FOOTNOTES

[1]I would like to thank V. V. Chari, Beth Hayes and Nancy Stokey for their assistance in the construction of the examples and the National Science Foundation for financial assistance in the form of grant number SES-8308446.

[2]It should be pointed out that one particularly appealing

interpretation of the model presented in Dixit and Stiglitz [2] is that it is an analysis of the second stage of a two stage game such as the one presented here. This, along with the argument presented here justifies their implicit assumption of the differentiation of products by firms.

[3]Note that this is just the usual notion of closed convergence of closed subsets of a metric space (see Hildenbrand [6] for definitions) restricted to the case where T is finite.

[4]It seems that the only reason that strict complements cause problems in the quantity setting model is that their presence implies that inverse demand is not continuous. It is this continuity which is the crucial issue in that setting (see Mas-Colell [10], for example).

[5]Since the only way to get any demand for good 2 is to price good 1 so that the surplus from the sale of good 1 to consumer 1 is not effectively exploited.

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