

Fershtman, Chaim; Seidman, Daniel J.

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Discussion Paper No. 845

STRIKES AND DEADLINE EFFECTS IN BARGAINING  
WITH ENDOGENOUS COMMITMENT

by

Chaim Fershtman\* and Daniel J. Seidmann\*\*

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ABSTRACT

Bilateral (sequential) negotiators delay agreements until a deadline if a player that rejects an offer is subsequently committed not to accept any poorer proposal and if the common discount factor is close enough to one. If the discount factor is lower then players agree at the outset. The empirically appealing U-shaped distribution of bargaining duration can therefore be explained naturally without an appeal to incomplete information.

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\*Department of Economics, Tel-Aviv University, Tel-Aviv 69978, Israel.

\*\*Trinity College, Dublin and visiting at the Department of Economics, Tel-Aviv University.

## STRIKES AND DEADLINE EFFECTS IN BARGAINING WITH ENDOGENOUS COMMITMENT

### Introduction

Strikes are generally regarded as socially wasteful delays in reaching agreement rather than, say, as intertemporal substitution.<sup>1</sup> This account seems to be particularly compelling when negotiations last until a deadline - an empirically common effect which was replicated experimentally by Roth, Murnighan and Schoumaker (1988). However, as Hicks (1963) recognized, it is difficult to explain the incidence of strikes when so viewed: for if some parties to the eventual agreement discount their payoffs, then all must recognize (at least retrospectively) that an earlier compromise would have made all parties better off. The same difficulty arises in explaining inefficient delay in other contexts, such as prolonged committee discussions. The problem of explaining inefficient delay is known in the labor literature as the Hicks Paradox; while the concentration of agreements in the first and last periods available for bargaining is known as the 'deadline effect'.

Rubinstein's (1982) influential model of two-person sequential bargaining with common knowledge of terminal payoffs formalized Hicks's argument. Rubinstein analyzed a repeated version of Nash's (1953) demand game in which one player proposes division of a fixed-size cake each period and the other player either accepts or rejects, the game ending when an offer is accepted; and demonstrated that the game has a

unique subgame-perfect equilibrium for any pair of discount factors (or per-period bargaining costs) in which agreement is reached in the first period.<sup>2</sup> The same result would also arise if the Rubinstein game were constrained to end after no more than  $T$  periods and disagreement payoffs were Pareto-inefficient.

The recent literature on delayed agreements has followed Hicks's (1963, p.147) suggestion that strikes occur because of private information about terminal agreement payoffs. One strand of the literature uses Myerson and Satterthwaite's (1983) result - that there may be no mechanism possessing ex post efficient, incentive compatible, individually rational Bayesian equilibria - by interpreting strikes as a means of reducing the cake for those pairs of types which cannot reach an efficient agreement in a given mechanism. (See Kennan (1986).) However, most of the literature has studied noncooperative sequential bargaining models based on Rubinstein (1982) - see, for example, Fudenberg, Levine and Tirole (1985), Grossman and Perry (1986) and Sobel and Takahashi (1983) - despite the well-known difficulty that delay may be arbitrarily short if offers are exchanged quickly enough.<sup>3</sup> As Hart (1989) noted, the literature has, nevertheless, continued to focus on incomplete information explanations of delay because of the widespread belief, engendered by Rubinstein (1982),<sup>4</sup> that delay is inconsistent with common knowledge of payoffs. This paper demonstrates, per contra, that deadline effects (and, ipso facto, delay) may occur when payoffs are commonly known in an alternative plausible sequential bargaining

model.

Our account is based on the interaction between two features. Firstly we suppose that bargaining must end by a finite deadline. Secondly, in contrast to the existing literature, we suppose that a player which has rejected an offer is subsequently committed to agree only to proposals which yield it a greater contemporaneous payoff. We call this feature 'endogenous commitment', as a player's commitments are determined by rejecting the other player's offers. Notice that players can propose divisions which they are committed to reject as offers - though in equilibrium they will not do so.

We incorporate these two properties in a three-period two-player sequential bargaining model with random choice of proposer each period; and demonstrate that for every (common) discount factor there is a unique subgame-perfect equilibrium. If the discount factor is low enough then the players reach immediate agreement. Otherwise agreement is delayed till the deadline.

Our finite period horizon assumption is quite standard in the related incomplete information literature and, as such, does not merit discussion. However, we will subsequently apply our model to negotiation failure in the shadow of interest arbitration; and on this interpretation it is the arbitrator which chooses between simultaneous proposals in the last period.

Endogenous commitment seems to be a plausible account of bargaining in a number of contexts: in labor negotiations the constraint may be

imposed on negotiators who are agents for the interested parties<sup>5</sup>; in committee discussions protocol (or the chair) often prevents repeated votes on a single proposal; and, more generally, casual empiricism suggests its psychological plausibility in two-person bargaining. Finally, note that the analysis would be unaffected if players could not endogenously commit, but an arbitrator was expected to split the difference between the most favorable offer that each player had rejected.<sup>6</sup>

Endogenous commitment adds structural dynamics to the analysis of bargaining. The game may change over time as players commit to rejecting some future offers. However, players can only commit by rejecting offers, so each player determines its rival's commitments; and any rejected concession (i.e., new commitment) changes the continuation of the game.

Our results are best understood by first explaining delay and then turning to the deadline effect. Firstly, consider a two-period version of the game. Notice that, were the last period to be reached, then payoffs would be determined by the best offer that each player had hitherto rejected. Consequently, in equilibrium a first-period offer by player 1 (say) is only accepted if the responder's (2's) return exceeds its expected discounted last period payoff conditional on rejecting the offer. Increases in the discount factor raise both the payoff required for an initial offer to be accepted and 1's expected discounted return from 'delaying' by making an offer which 2 will reject. If the

discount factor is close enough to unity, then 1 prefers to delay and therefore offers none of the cake, for any rejected concession would enable 2 to commit to reject some final period offers.

This intuition for delay also applies to games with more than two available bargaining periods. Clearly, the more distant the deadline from the initial period, the greater is the critical discount factor above which (in equilibrium) negotiators delay till the deadline.

We return to the three-period case to explain why the deadline effect holds in our model. Imagine, per contra, that there were an equilibrium in which 1's first period offer is rejected but a second-period offer is accepted. 1 would have to make a concession in such an equilibrium, for otherwise the second period proposer would also delay in equilibrium (as we have seen above). Moreover, in equilibrium a rejected concession must induce 2 to make an acceptable offer if it proposes in the second period for, were 1 selected again, it would regret that a concession had been made. However, the concession required to induce an acceptable offer from 2 is always so great that, if 1 delays at the outset, it does not make a concession. Thus, in equilibrium, 1 essentially chooses between an acceptable offer and delay till the last period. If the discount factor is low then 1 makes an acceptable offer; otherwise, the concession required for immediate acceptance is too costly, and agreement is delayed till the deadline.

We present our model and notation in Section 1. Section 2 states and discusses our main result, which we prove in section 3. In section 4 we show that the game may end in an intermediate period, or may end with a compromise agreement in the last period if at least one player is privately informed. We conclude in Section 5, briefly alluding to an application of our results to negotiation failures prior to arbitration.

## 1.2. Framework and Notation

Consider a pie of unit size, to be divided between two players. We follow the standard sequential bargaining approach (Rubinstein (1982)) in assuming that offers are made sequentially and that the game ends once an offer has been accepted. Furthermore players are impatient and  $0 \leq \delta \leq 1$  is their common impatience factor: players are indifferent between getting  $\delta x$  today and getting  $x$  in the next period.

Following Binmore (1987), we suppose:

**ASSUMPTION 1:** Each period a fair lottery determines which player makes the current offer.

In contrast to most of the complete information bargaining literature we assume that the game must end by a finite (commonly known) deadline. If no agreement has been reached by the deadline each player receives a payoff of zero. In the standard bargaining framework with A1 and a finite deadline there is a unique subgame perfect equilibrium in which the negotiations end in the first period, the proposer obtaining



$$1 - \frac{\delta}{2}.$$

We deviate from all of the previous literature by assuming the following.

**ASSUMPTION 2** (Endogenous commitment): If at period  $\bar{t}$  a player refuses an offer, then at a later period  $t > \bar{t}$  he is forced to reject any subsequent poorer offer.

Endogenous commitment breaks with the stationary structure exploited in the existing literature (cf. Shaked and Sutton (1984)) since subgames starting at different periods may be distinguished by the path (i.e., rejected offers) leading to them. However, we will later show that endogenous commitment only affects equilibrium payoffs if there is a finite deadline: otherwise the players immediately agree to a share of  $\frac{\delta}{2}$  and  $1 - \frac{\delta}{2}$ . In sum, neither endogenous commitment nor a deadline alone can explain why agreements are reached in the last period available for bargaining. We will show that the combination of endogenous commitment and a deadline not only yields a deadline effect but also changes the offers that would be accepted immediately in equilibrium. Now deadline effects are only nontrivial if there are at least 3 periods available for bargaining. To simplify exposition we therefore assume:

**ASSUMPTION 3** (Deadline): Bargaining must end after no more than 3 periods.

We will analyze the bargaining game by characterizing its subgame perfect equilibrium, bearing in mind the commitment implied by previously rejected offers.

We index periods by  $t = 1, 2, 3$ . Define  $a_t(\beta_t)$  as the offer made by player 1 (2) in period  $t$ . All the offers in this paper are measured in terms of the share of player 1. Thus an offer  $\beta_t$  implies allocating  $\beta_t$  to player 1 and  $1 - \beta_t$  to player 2. The best offer that player 1(2) has rejected prior to period  $t$  is denoted by  $x_t(y_t)$  where

$$x_t = \text{Max}_{\tau < t}(\beta_\tau, 0)$$

$$y_t = \text{Min}_{\tau < t}(a_\tau, 1).$$

We let  $x_t^i(x_t, y_t)$  be player one's equilibrium payoffs in the subgame starting at period  $t$  when player  $i$  is chosen to offer and the players are committed to  $(x_t, y_t)$ ;  $y_t^i(x_t, y_t)$  is defined similarly for the second player.

Let

$$X_t(x_t, y_t) = \frac{1}{2}(x_t^1(x_t, y_t) + x_t^2(x_t, y_t))$$

$$Y_t(x_t, y_t) = \frac{1}{2}(y_t^1(x_t, y_t) + y_t^2(x_t, y_t))$$

such that  $X_t(x_t, y_t)$  ( $Y_t(x_t, y_t)$ ) is the first (second) player's expected payoff before the lottery determines which player proposes in period  $t$ .

## 2. The Main Results

**THEOREM 1:** For every  $\delta \in [0, 1]$  there is a unique subgame perfect equilibrium. There exists a  $\bar{\delta} \in (0, 1)$  such that

- (i) If  $\delta \in [0, \bar{\delta})$  then the game ends in the first period with the proposer and the responder respectively receiving shares of  $\frac{4(1-\delta)}{4-2\delta-\delta^2}$  and  $\frac{2\delta-\delta^2}{4-2\delta-\delta^2}$ .
- (ii) If  $\delta \in (\bar{\delta}, 1]$  then the game ends in the last period with the proposer receiving all the surplus.

The theorem states that the duration of bargaining has an empirically appealing U-shaped pattern, with agreements occurring in either the first or last periods; and, moreover, that there is a deadline effect if and only if the discount factor is close enough to 1. The delay that then occurs in equilibrium is Pareto-inefficient if  $\delta$  is less than 1: there are initial proposals whose terms both players strictly prefer to the ultimate agreement.<sup>7</sup> This argument (the Hicks Paradox) explains why delay cannot occur in standard complete information bargaining models; and the reason why Pareto-superior

offers are not made here goes to the heart of our bargaining model. In all periods but the last, the proposer in our model is disadvantaged relative to its counterpart in the literature since rejection of any offer strengthens a player's subsequent bargaining position by committing it to reject any future offer of a smaller share of the surplus. Any accepted offer must therefore be sufficiently favorable to outweigh the advantage of a subsequently strengthened bargaining position. If  $\delta$  is close to 1 then almost any concession would be rejected and therefore none can be made; while if  $\delta$  is close to zero then future bargaining power is of less value, and an initial offer is accepted. The theorem demonstrates that these two alternatives exhaust the equilibrium outcomes for any discount factor.

Even if  $\delta$  is sufficiently low that the initial proposal is accepted, the responder earns more than  $\delta/2$ , its equilibrium share if there is no endogenous commitment (with or without a deadline). The ability to commit bolsters its threat to reject any inferior offer, and allows it to extract more rent from the initial proposer.

It is important to appreciate that the considerations of endogenous commitment and deadlines are entirely different from the explanation of delay in the related literature on bargaining with incomplete information, in which an uninformed player delays its concessions in order to sort its rival's types. Our model is a game of perfect information, so there is no scope for learning. Furthermore, when  $\delta$  is high enough for a deadline effect, then delay occurs with probability

one and no concessions are made. (We will return to these issues in Section 4, where we study extensions of our model which incorporate incomplete information.)

### 3. Proof of the Theorem and Analysis of the Equilibrium

We will prove the Theorem via a sequence of lemmata.

**LEMMA 1:** If play reaches period 3 with commitments of  $x_3$  and  $y_3$  then in the unique subgame perfect equilibrium players 1 and 2 respectively offer  $y_3$  and  $x_3$  and the offers are accepted.

**PROOF:** The proof is trivial since in the last period players make ‘take it or leave it’ offers.

□

Given the above equilibrium strategies the players’ expected payoffs are:

$$X_3(x_3, y_3) = \frac{1}{2}(x_3 + y_3); \quad Y_3(x_3, y_3) = 1 - \frac{1}{2}(x_3 + y_3). \quad (1)$$

We suppose, without loss of generality, that player 1 is (randomly) chosen to propose in period 1. Lemma 2 shows that it cannot offer  $y^* < 1$  (which we will call a ‘concession’) in period 1 unless player 2 accepts. Lemmata 3 and 4 will use this result to show that the game cannot end in period 2.

**LEMMA 2:** A necessary condition for player 1 to make a rejected offer of  $y^* < 1$  in period 1 is that a concession is required to induce player 2 to make an acceptable offer in period 2.

**PROOF:** After player 2 has rejected  $y^*$  there are two subgames, each of which occurs with probability  $\frac{1}{2}$ : (a) Player 1 offers in period 2; and (b) Player 2 offers in period 2. We will consider these seriatim.

(a) Granted that this subgame has been reached, player 1 weakly regrets that it offered  $y^*$  rather than 1: at best the concession does not affect the offers that 1 might make in period 2.

(b) If player 2 were to delay then player 1 would be strictly worse off by making a concession in period 1 (cf. Lemma 1). Hence player 2 must make an acceptable offer of  $x^*$  if player 1 is not to regret offering  $y^*$ . If player 2 also made an acceptable offer of  $x_1$  after rejecting  $y = 1$  in period 1, then  $x_1 > x^*$ : for player 2's offer would make player 1 indifferent between accepting and proceeding to period 3; and player 1's return in period 3 is strictly decreasing in the concessions it has made. Thus, in this subgame player 1 would strictly prefer to have made no concession if player 2 would make an acceptable offer after rejecting  $y = 1$ . Hence a necessary condition for player 1 to prefer  $y^*$  in this subgame is that player 2 makes an acceptable offer after rejecting  $y^*$  and delays after rejecting  $y = 1$ .

Subgames (a) and (b) each occur with probability  $\frac{1}{2}$  after player 2 has rejected a period 1 offer; so player 1's preferences over the set

of rejected offers are determined by its preferences in the two subgames. We have seen that player 1 never gains in (a) and is strictly worse off in (b) unless the conditions of the Lemma are satisfied. Consequently, these conditions are necessary for player 1 to offer  $y^* < 1$  in period 1.  $\square$

**LEMMA 3:** For every  $\delta \in [0,1]$  there is no equilibrium in which player 1 offers  $y < 1$  in period 1 which player 2 rejects.

**PROOF:** Player 2 would only make an acceptable offer in period 2 after rejecting  $y$  if there were an  $x \geq \delta Y_3(x, y)$  and  $1 - x \geq \delta Y_3(0, y)$ .

Equation (1) implies that player 1 would accept any  $x \geq \delta y / (2 - \delta)$ , and therefore that player 2 makes an acceptable offer in period 2 after rejecting  $y$  if and only if

$$y \leq \frac{2(1-\delta)(2-\delta)}{\delta^2}. \quad (2)$$

In particular, player 2 would make no concession after rejecting  $y = 1$  only if

$$\delta \geq 3 - \sqrt{5}. \quad (3)$$

In sum, player 2 only makes an acceptable offer in period 2 after rejecting  $y < 1$  but not after rejecting  $y = 1$  if (2) and (3). If

(3) is satisfied, then player 1 can earn  $\frac{1}{2}\delta^2$  by offering no concession in period 1; and, from (3):

$$\frac{1}{2}\delta^2 \geq 7 - 3\sqrt{5}. \quad (4)$$

Furthermore, in equilibrium player 1's (period 1) return from having an offer of  $y$  rejected is bounded above by  $y$ . Consequently, a necessary condition for player 1 to make a rejected concession in period 1 is:

$$7 - 3\sqrt{5} \leq 2(1 - \delta)(2 - \delta)/\delta^2. \quad (5)$$

Trivial manipulations confirm that (5) is satisfied iff  $\delta \leq \delta^*$  where  $\delta^* \equiv \{3 - [14 - 3\sqrt{5}]^{1/2}\}/\{3\sqrt{5} - 5\}$ .

However, we have seen that player 2 will make an acceptable offer after rejecting  $y = 1$  if  $\delta \leq 3 - \sqrt{5}$ ; so player 1's rejected concession is only profitable if  $\delta > 3 - \sqrt{5}$ . The Lemma is therefore proved by noting that  $\delta^* < 3 - \sqrt{5}$ .

□

**LEMMA 4:** In equilibrium the game does not end in period 2.

**PROOF:** If  $\delta > 3 - \sqrt{5}$  then we know from (3) that the period 2 proposer would delay if  $y = 1$  were rejected in period 1. We can



therefore prove the Lemma by showing that player 1 would make an acceptable offer in period 1 if  $\delta \leq 3 - \sqrt{5}$  (when the game would always end in period 2 after  $y = 1$  is rejected).

Notice, firstly, that if  $\delta < 3 - \sqrt{5}$  then the game would end in period 2 whatever the offer rejected in period 1. To see this, observe that player 1 would accept no less than  $\delta y / (2 - \delta)$  in period 2 after offering  $y$  in period 1. Hence player 2 makes an acceptable offer after  $y$  is rejected if for all  $y$

$$\delta Y_3(0, y) = \delta - \frac{1}{2}\delta y \leq 1 - \delta y / (2 - \delta)$$

viz.

$$1 - \delta \geq \delta^2 / (2 - \delta).$$

This condition is equivalent to  $\delta \leq 3 - \sqrt{5}$ . On the other hand, the best offer that player 2 would accept is  $\min[y, (2 - 2\delta) / (2 - \delta)]$ . If  $y < (2 - 2\delta) / (2 - \delta)$  then player 1 obviously prefers to offer  $y$  in period 2; otherwise, player 1 would make an acceptable offer if

$$\delta X_3(0, y) = \frac{1}{2}\delta y \leq (2 - 2\delta) / (2 - \delta),$$

which is again equivalent to  $\delta \leq 3 - \sqrt{5}$ .

Consider a subgame starting in period 2 after  $y \leq 1$  is rejected. If  $y \geq (2 - 2\delta) / (2 - \delta)$  then players 1 and 2 would respectively accept

offers of  $\delta y/(2-\delta)$  and  $(2-2\delta)/(2-\delta)$  in period 2; so  $Y_2(0,y) = (2-\delta y)/(4-2\delta)$ . On the other hand, player 1's return from delaying and offering no concession in period 1 would be  $\delta/2$ . Hence player 1 would make an acceptable offer in period 1 if there is a  $y \geq \max\{\frac{1}{2}\delta, (2-2\delta)/(2-\delta)\}$  such that  $1-y \leq \delta(2-\delta y)/(4-2\delta)$ .

By supposition, the game always ends in period 2, so  $\delta \leq 3 - \sqrt{5}$ , which implies that  $\frac{1}{2}\delta \leq (2-2\delta)/(2-\delta)$ . It is easy to confirm that  $y < 1 - \delta Y_2(0,y)$  if  $y = (2-2\delta)/(2-\delta)$ . To see this, note that

$$\begin{aligned} 1 - \delta Y_2(0, (2-2\delta)/(2-\delta)) &= 1 - \delta \frac{2-\delta \frac{2-2\delta}{2-\delta}}{4-2\delta} \\ &= \frac{4-6\delta+3\delta^2-\delta^3}{(2-\delta)^2} \end{aligned}$$

Therefore  $\frac{2-2\delta}{2-\delta} \geq 1 - \delta Y_2(0, \frac{2-2\delta}{2-\delta})$  iff

$$\frac{2-2\delta}{2-\delta} \geq \frac{4-6\delta+3\delta^2-\delta^3}{(2-\delta)^2}$$

i.e.,  $(2-2\delta)(2-\delta) = 2(1-\delta)(2-\delta) \geq (1-\delta)(\delta^2 - 2\delta + 4) = 4-6\delta + 3\delta^2 - \delta^3$

which is a contradiction for all  $\delta < 1$ .

Consequently, player 1 would prefer to deviate by offering  $(2-2\delta)/(2-\delta)$ ; from which the Lemma follows. □

Lemmata 2 and 3 imply that, in equilibrium, either player 1 makes an acceptable offer or it makes no concession and the game ends in period 3 with no concession being offered in period 2.

We now show that player 1 must offer  $y \leq y^* \equiv 4(1-\delta)/(4-2\delta-\delta^2)$  in order to secure acceptance from player 2 in period 1. If  $\delta < 3 - \sqrt{5}$  then both players make acceptable offers following any  $y$  that is rejected in period 1; so

$$\delta Y_2(0,y) = \delta(2-\delta y)/(4-2\delta) \quad (5)$$

for any  $y \geq 2(1-\delta)/(2-\delta)$ . Under these circumstances player 2 would accept any  $y \leq y^*$ , which is always less than  $2(1-\delta)/(2-\delta)$ .

If  $\delta > 3 - \sqrt{5}$  then player 1 makes an acceptable offer whenever player 2 does so. To see this, note that player 2 would accept  $z = \min(y, 2(1-\delta)/(2-\delta))$  in period 2. It is obvious that player 1 prefers an acceptable offer of  $y$  if  $z = y$ . Using inequality (2), player 1's return from delaying in period 2 ( $\frac{1}{2}\delta y$ ) is bounded above by  $(\delta^2 - 3\delta + 2)/\delta$ , which exceeds  $2(1-\delta)/(2-\delta)$  whenever  $\delta > 3 - \sqrt{5}$ . Hence player 1 would also offer  $2(1-\delta)/(2-\delta)$  were player 2 to make an acceptable offer after rejecting  $y$ .

Using (2) again, we can confirm that there is a  $\delta^* > 3 - \sqrt{5}$  such that player 2 (and therefore player 1) would make an acceptable offer in period 2 following rejection of  $y^*$  whenever  $\delta \in [0, \delta^*]$ ; where  $\delta^*$  is in  $[0, 1]$  and is a solution of

$$\delta^3 - 2\delta^2 - 8\delta + 8 = 0. \quad (6)$$

On the other hand, there is a  $\delta^+ \in (3 - \sqrt{5}, 1]$  such that player 1 prefers receiving  $y^*$  to delaying till period 3 and making no concession if  $\delta \in [0, \delta^+)$ ; where  $\delta^+$  is in  $[0, 1]$  and is a solution of:

$$\delta^4 + 2\delta^3 - 4\delta^2 - 8\delta + 8 = 0. \quad (7)$$

(6) and (7) imply that  $\delta^+ < \delta^*$ ; so player 1 prefers to offer  $y^*$  only if both players would make acceptable offers in period 2 following rejection of  $y^*$ .

The Theorem will therefore be proved by showing that player 1 must offer  $y < y^*$  to secure acceptance in period 1 whenever  $\delta > \delta^*$ . This fact follows immediately from the observations that player 1 would always make an acceptable offer in period 2 following an offer of  $y^*$  in period 1; and that  $Y_2(0, y^*)$  must then exceed  $1 - y^*$ , since player 2 then prefers to delay.

□

We end this section by showing that the Rubinstein proposal (of  $y = 1 - \frac{\delta}{2}$ ) is accepted immediately in equilibrium if there is no deadline to bargaining.

Consider the following strategies for players 1 and 2. At any period  $t$  player 1 offers

$$y_t = \min(y_{t-1}, 1 - \frac{\delta}{2}),$$

rejects any offer of  $x_t < \frac{\delta}{2}$ , and accepts any offer of  $x_t \geq \frac{\delta}{2}$ ; while player 2 offers

$$x_t = \max(x_{t-1}, \frac{\delta}{2}),$$

rejects any offer of  $y_t > 1 - \frac{\delta}{2}$ , and accepts any offer of  $y_t \leq 1 - \frac{\delta}{2}$ .

If player 1 deviates by offering  $y_t > 1 - \frac{\delta}{2}$  then it is in player 2's interests to reject, since its equilibrium return is then  $\frac{\delta}{2}$ ; and if player 2 rejects then player 1 earns  $\frac{\delta}{2}$  in equilibrium. If player 1 offers  $y_t < 1 - \frac{\delta}{2}$  then player 2 will offer  $x_s \leq \frac{\delta}{2} < 1 - \frac{\delta}{2}$  for all  $s > t$ ; so player 1 is better off raising  $y_t$  to  $1 - \frac{\delta}{2}$ . This confirms that the stated strategies constitute an equilibrium, in which agreement is immediate whenever  $\delta < 1$ .

#### 4. Compromise Agreements with Incomplete Information

In Sections 2 and 3 we studied a model in which payoffs at terminal nodes are common knowledge. We have demonstrated that there is a deadline effect, with agreements only occurring in the first and last available periods; and that, in the latter case, the ultimate proposer takes all of the surplus. Both properties may be counterfactual in some applications: for example, negotiators tend not to make extreme proposals in final-offer arbitration (cf. Farber (1981)). In this

section we show how private information can modify these properties. We focus on a particularly simple framework in which the only private information concerns one player's discount factor.

We will study two versions of this story: in the first version there are only two available bargaining periods and the uninformed player makes an initial offer (if chosen as initial proposer) which is immediately accepted by a low discount-factor type, the other type rejecting and reaching a compromise agreement in the last period. A deadline effect is therefore consistent with compromise agreements. In the second version there are three available bargaining periods and an initial offer by the uninformed player is again accepted by the low discount-factor type, while the other type reaches agreement before the deadline.

We assume throughout that the prior probability that player 1 is of type  $t_1$  ( $t_2$ ) is  $p$  ( $1-p$ ). Type  $t_1$ 's discount factor is denoted by  $\delta$ , while type  $t_2$ 's discount factor is unity. Player 2 is known to have a discount factor of  $\theta$ . We will focus on (sequential) equilibrium play after Nature has chosen 2 to make the offer in period 1.

Suppose that 2 offers  $x$  in the first period of a game with a period 2 deadline. Type  $t_2$  rejects any  $x < 1$  while  $t_1$  accepts any  $x$  that is no more than

$$x^+ \equiv \delta / (2 - \delta).$$

Thus, if  $x = 1$  then both types accept; if  $x^+ \leq x < 1$  then only  $t_1$  accepts; while if  $x < x^+$  then neither type accepts. Hence 2's choice in the first period is between offering  $x^+$  (and sorting 1's types) and delaying w.p.1 by offering no concession. In the latter case 2 always earns  $\partial/2$ , while in the former case its offer of  $x^+$  is accepted with probability  $p$  in period 1; so that 2's expected return from offering  $x^+$  is  $x^+p + \partial x^+(1-p)/2$ . 2 clearly prefers to separate the types if, for example,  $\delta$  is small enough, for then the concession ( $x^+$ ) required to induce  $t_1$  to accept is relatively small, and would prove relatively costless were 1 to be of type  $t_2$ . In the latter event the game would continue till the deadline, but 2 would be unable to claim the entire cake if called on to propose in period 2, having offered a concession in period 1. It is easy to check that equilibria are unique for all parameter ( $\partial, p, \delta$ ) values.

This example can obviously be generalized to a game with more than two available periods in which each player's discount factor is private information. We conclude that deadline settlements may involve compromise agreements if some information is private.

In the second version of our model we suppose that there are three available periods. We will construct a sequential equilibrium in which 2's initial offer is accepted by  $t_1$  and rejected by  $t_2$ ; and if 1 is chosen to propose in period 2 then its offer is accepted by 2.

Let

$$x^* \equiv \delta(2\delta - \partial\delta + 4 - 4\partial)/(8 - 4\partial - 2\delta^2 + \partial\delta^2 - 2\partial\delta).$$

We claim that the following strategies and beliefs constitute a sequential equilibrium if  $\partial$  is close enough to zero: 2 would offer  $x^*$  in period 1, which  $t_1$  would accept and  $t_2$  would reject, so that 2 would infer from rejection that 1 was of type  $t_2$ ; in period 2 player 2 would repeat its period 1 offer of  $x_1$  which either type would reject, while either type would offer

$$y^*(x_1) \equiv (2 - \partial + \partial x_1^*) / (2 - \partial)$$

which 2 would accept; the period 3 proposer would make a take-it-or-leave-it offer.

Our claim can be confirmed by noticing that, given its beliefs after period 1 rejection, 2 must repeat its offer in period 2 since  $t_2$  would reject it in any case; so  $t_1$  would also reject 2's period 2 offer as  $\delta \in (0,1)$ . On the other hand, if  $\partial < 2/3$  then the offer of  $y^*(x^*)$  is accepted by 2 and both types of 1 prefer acceptance of  $y^*(x^*)$  to delaying agreement until period 3. Using these observations, it is easy to check that  $x^*$  is the minimum share for 1 which  $t_1$  would accept in period 1 (rejection being a dominant strategy for  $t_2$ ). Thus, if  $\partial$  is low enough, 2 would prefer to offer  $x^*$  and separate the two types (at the cost of allowing  $t_2$  to commit) than to make any offer which pools the two types in accepting or rejecting. If Nature



were to select 2 and 1 to propose in periods 1 and 2 respectively, then in equilibrium 2 would reach a partially delayed compromise agreement with  $t_2$ .

## 5. Conclusions

Since Hicks (1963) economists have explained costly delay in bargaining by postulating that players are uncertain about each others' payoffs in the event of agreement and/or disagreement. This explanation has recently been formalized in the literature on bargaining with incomplete information.

We have presented an alternative account for delay in the empirically important case where bargaining is subject to a deadline when casual empiricism and experimental evidence both suggest that agreements tend to be concentrated at the outset of bargaining and close to the deadline. We explain this deadline effect by supposing that players endogenously commit during bargaining: by rejecting an offer a player commits not to subsequently accept any poorer offer.

Our results shed light on the question of whether agreements to refer unsettled issues to arbitration tends to 'chill' bargaining, reducing the likelihood of a negotiated settlement. The associated literature largely attributes negotiation failures to inconsistent beliefs about the arbitrated outcome; but Farber and Bazerman's (1989) evidence suggests that this explanation may be difficult to reconcile with the stylized fact that settlement probabilities are higher under

final-offer than under conventional arbitration. Farber and Bazerman suggest, rather, that their evidence might be explained by a reluctance to concede when all offers can be presented to the arbitrator (as in American public sector interest arbitration).<sup>8</sup> Farber and Bazerman do not model the precise way in which these concessions affect the arbitrator's award. We (in effect) provide a simple model in which the arbitrator ignores all but the most favorable concessions and randomizes between them; and demonstrate that this may result in (costly) arbitration even if all parameters are common knowledge.

## FOOTNOTES

<sup>1</sup> Kennan's (1986) exhaustive survey, for instance, does not mention the possibility of an efficiency explanation.

<sup>2</sup> It is easy to see that an alternative repeated demand game in which both players propose simultaneously each period and the game ends when demands are compatible has equilibria with delay (exploiting the multiplicity of solutions in a one-shot Nash demand game). However, these equilibria are Pareto-dominated by other solutions without delay; so this resolution of the Hicks's Paradox is unconvincing.

<sup>3</sup> See Gul, Sonnenschein and Wilson (1986) and Gul and Sonnenschein (1988). The Coase Conjecture may be false if there may (with positive probability) be no surplus to divide (Ausubel and Deneckere (1986)) or if both sides are privately informed (Cramton (1987)) or if the surplus shrinks over time (Hart (1989)).

<sup>4</sup> But cf. Hicks (1963), p.147: "Adequate knowledge will always make a settlement possible."

<sup>5</sup> Our account is compatible with the explanation for deadline effects that Roth et al. (1988) fn.2 attribute to labor negotiators: that it is difficult to sell an agreement to union members if there is still time to continue negotiations. See also Walton and McKersie (1965) on a labor negotiator's capacity to commit.

- <sup>6</sup> Readers should distinguish between our assumption that proposals must be monotonic from the equilibrium property of many incomplete information models: that proposals are strictly monotonic.
- <sup>7</sup> But notice that delay only occurs when it is not 'too' inefficient, i.e., when  $\delta$  is high enough.
- <sup>8</sup> By contrast, public sector interest arbitration in the U.K. (the Whitney Council system) ignores proposals rejected during negotiations, while offers to settle civil cases out of court are inadmissible as evidence, precisely for fear that their prospective use in court may deter negotiations from offering concessions. See McCormick (1958), p.158b and Wheeler (1977).

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