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Deliberation and Voting Rules¹

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Abstract

We analyse a formal model of decision-making by a deliberative committee. There is a given binary agenda. Individuals evaluate the two alternatives on both private and common interest grounds. Each individual has two sorts of private information going into committee: (a) perfect information about their personal bias and (b) noisy information about which alternative is best with respect to a (commonly held) normative criterion. Prior to a committee vote to choose an alternative, committee members engage in deliberation, modeled as a simultaneous cheap-talk game. We explore and compare equilibrium properties under majority and unanimity voting rules, paying particular attention to the character of debate (who influences who and how) and quality of the decision in each instance. On balance, majority rule induces more information sharing and fewer decision-making errors than unanimity. Furthermore, the influence and character of deliberation *per se* can vary more under majority rule than under unanimity.

1 Introduction

This paper concerns the relationship between deliberation and voting in committees. From at least one perspective, the issue is moot. In contrast to Condorcet, who saw the role of deliberation and debate largely in positive terms,¹ the recent political theory literature on “deliberative democracy” is more expressly normative, being concerned with questions of legitimacy and achieving a consensus sufficient to make voting irrelevant (see, for example, the essays in Bohman and Rehg, 1997 and in Elster, 2000). Thus, when summarizing the most optimistic view of deliberation, Elster (1986:112) writes that “there would not be any need for any aggregating mechanism, since a rational discussion would tend to produce unanimous preferences”.² And there is some support for this view in a very recent contribution to the formal mechanism design literature. Exploiting the revelation principle for Bayesian communication games, Gerardi and Yariv (2002) prove an equivalence result for all binary voting rules of the form, “choose x over y if at least r of n individuals, $1 < r < n$, vote for x against y ”. Specifically, suppose a non-unanimous binary voting rule r of the sort described is used to choose one of two fixed alternatives and suppose that, prior to voting, all individuals have the opportunity to make cheap talk statements about any decision-relevant private information they may have regarding the relative value of these alternatives;³ an *outcome* is then a probability of

¹Deliberation for Condorcet is necessary both to clarify individual interests and to formulate coherent agendas over which to vote: see Marquis de Condorcet, 1793; translated by Iain McLean and Fiona Hewitt, 1994:193.

²Not all normative theorists writing on deliberative democracy are enthusiastic about the value of the process. Particularly coherent critiques are offered by Christiano (1997), Johnson and Knight (1997) and Sanders (1997). Moreover, “tending to produce” unanimity is not the same as producing unanimity: “Even under ideal conditions there is no promise that consensual reasons will be forthcoming. If they are not, then deliberation concludes with voting, subject to some form of majority rule.” (Cohen, 1989:23). In a recent contribution, Dryzek and List (2002) argue that deliberation can (but need not) induce conditions under which democratic aggregation procedures are well-behaved and free of opportunities for strategic manipulation.

³Formally, a statement is “cheap talk” if the statement itself does not affect the speaker’s payoffs from outcomes (although, of course, cheap talk statements can influence, and are intended to influence, which outcome

winning for each alternative as a function of the underlying distribution of private information about these alternatives. The theorem Gerardi and Yariv prove is the following: under very general conditions on preferences and information, the set of sequential equilibrium outcomes achievable by augmenting a voting game with a prior communication stage is constant in the voting rule, r (Gerardi and Yariv, 2002: Proposition 2). In other words, although there is no assurance that all of the private information held by voters is necessarily realized in debate (the cheap talk communication stage), it is the case that the voting rule governing the final decision is immaterial, so apparently justifying the deliberative democracy thesis that voting becomes insignificant once talk is permitted.

The central observation used to prove the Gerardi-Yariv result is that all individuals voting unanimously can be a component of a sequential equilibrium. Because the voting rule r is presumed non-unanimous, it immediately follows that if $n - 1$ individuals are voting for the same alternative, the n^{th} individual can do no better than to vote for this alternative as well. And because this is true for *any* non-unanimous voting rule r , unanimous voting at any given profile of information effectively renders all such rules equivalent: if, at some rule r , an individual is willing to reveal information under the expectation that voting will be unanimous following such revelation, then that individual must also be willing to reveal exactly the same information at a rule r' conditional on the same expectation over voting. Specifying unanimous voting under any non-unanimous voting rule r is equivalent to saying the voting stage is irrelevant. It follows that, conditional on deliberation, any voting rule is in principle as good as any other.

On the other hand, if individuals behave as if voting mattered then the Gerardi-Yariv theorem does not hold in general. To see this, consider an example with seven voters in which the only private information is the individuals' prior preference for x over y ; in particular, suppose any additional information regarding the relative value of the alternatives is decision-irrelevant (is realized).

in that the ordinal preference profile over alternatives is independent of such information, pooled or otherwise. Suppose further that each individual conditions on the event that their vote is pivotal in the collective decision. Then whatever speeches individuals hear in debate, every individual must vote ‘sincerely’ for their most preferred alternative. Consequently, if four of the seven voters have a prior preference for x over y and the remaining three individuals have the opposite preference then, under majority rule $r = 4$, the unique outcome is x with probability one but, under a supramajority rule $r = 5$, the unique outcome is y with probability one: the voting rule matters.

Because we are concerned in what follows with understanding how details of the voting rule affect the character of deliberation, insisting only on sequential equilibria is inappropriate and, therefore, we assume throughout that individuals condition voting decisions on events at which their vote matters, that is, on being pivotal. In such a world, little is yet understood about optimal committee voting rules conditional on deliberation. Although we are not in a position to offer any sort of definitive analysis of the problem here, we do address two subsidiary questions with respect to majority rule and unanimity rule: first, “How does deliberation affect collective decisions under different voting rules?”; and second, “Can deliberation result in worse collective choices than those made in its absence?”.

It is useful to distinguish committee decision-making with *deliberation* from that with *debate*, more broadly considered. As understood in this paper, in deliberation at least two privately informed individuals engage in unmediated cheap talk over a collective decision to be made by these same individuals under a given voting rule; in debate at least two privately informed individuals engage in unmediated cheap talk prior to a decision being made. So deliberation here is understood as a subset of debate: in debate there is no requirement that those engaged in deliberation have any direct responsibility for the final decision, which might be made unilaterally by some quite distinct individual; in deliberation, however, the decision is necessarily by voting and those eligible to vote are precisely those eligible to deliberate. Although the existing formal strategic literature on decision-making with debate is now quite

considerable⁴, the corresponding literature on decision-making with deliberation is as yet very limited. In addition to Gerardi and Yariv (2002), to the best of our knowledge the only contributions are Doraszelski, Gerardi and Squintani (2001) who study a two-person unanimity problem in which both agents are known to share identical preferences conditional on full information; Coughlan (2000) who explicitly includes a cheap talk communication stage to the standard strategic model of voting in juries; Calvert and Johnson (1998) who explore a coordination role for cheap talk; and Austen-Smith (1990a, 1990b) who analyses a model of deliberative committee decision making with endogenous agenda-setting.

For Condorcet at least, perhaps the most important role of deliberation is in agenda-setting; nevertheless, we assume there is an exogenously fixed agenda. Although it is fairly natural to begin by asking what happens with fixed alternatives and then back up to ask how the alternatives for consideration might be chosen, the Condorcet Jury Theorem along with recent results on information aggregation through voting over fixed binary agendas (e.g. Ladha, 1992; McLennan, 1998; Duggan and Martinelli, 2001; Feddersen and Pesendorfer, 1996, 1997) raise a more concrete question about whether deliberation over given agendas is a salient issue. At least asymptotically as the electorate gets large, any majority or supermajority voting rule short of unanimity almost surely selects the alternative that would be chosen under the given rule were all voters fully informed and surely voted. However, committees in which deliberation is feasible are typically too small for asymptotic results to be useful.⁵ Thus there remains room for decision-relevant information sharing and argument in committees.

We consider a three person committee that must choose between two alternatives. Com-

⁴Examples include Battaglini (2002), Glazer and Rubinstein (2001), Ottaviani and Sorensen (2001), Krishna and Morgan (2001), Diermeier and Feddersen (2000), Lipman and Seppi (1995), Matthews and Postlewaite (1995), and Austen-Smith (1993a, 1993b).

⁵Condorcet (1793) surely felt this to be significant when claiming that the second form of debate he identifies (in which general questions are refined into “a number of clear and simple questions”) “could not take place outside an assembly without becoming very time-consuming” and “is of use only to men who are required to prepare or pronounce a joint decision” (*trans.* 1994:193).

mittee members' preferences have a *private interest* and a *common interest* component. An individual's private interest is her bias toward one or other of the alternatives and is private information to the individual; net of private interests, individuals' have a common interest in choosing the correct alternative for the realized state of nature, presumed unknown at the time of the decision. Each committee member observes a noisy private signal about the true state. We compare majority rule with unanimity rule both with and without debate.

There are two kinds of mistakes a committee can make given the information of its members. First, when the information available to the committee is sufficient to convince all members regardless of bias to support an alternative, the committee may choose the other alternative. We call this an *error in common interest*. Second, when the information available to the committee is insufficient to convince members to vote against their bias, the committee can choose an alternative that is not preferred by a majority. We call this second error an *error in bias*. Finally, when the committee never makes either kind of error in equilibrium we say that the equilibrium satisfies *full information equivalence*.

Overall, the results point to majority rule being superior both with respect to the expected quality of committee decisions and to the quality of debate it induces. Among other things, we show that with respect to pure strategy equilibria:

- (1) Without debate there are no equilibria under either rule that satisfy full information equivalence. However, there are equilibria under majority rule with debate that satisfy full information equivalence but there are no such equilibria under unanimity rule with debate.
- (2) Under majority rule with debate there are no equilibria that result in errors in common interest that would not also occur without debate. In contrast, there are equilibria under unanimity rule with debate that result in errors in common interest that would not occur without debate.
- (3) The only circumstances under which an equilibrium in the game with debate

under majority rule produces an error in bias are those in which an equilibrium satisfying full information equivalence is possible but not played. And although there exist circumstances under which debate weakly improves on no-debate under unanimity rule, we do not yet know whether this is a general property of debate equilibria under unanimity.

In the next three sections we describe our model and results for majority and unanimity rule. We conclude with a brief discussion of paths for future research.

2 A deliberative committee

Consider a three person committee, $N = \{1, 2, 3\}$, that has to choose an alternative $z \in \{x, y\}$; let x be the *status quo* policy. Individual preferences over the feasible alternatives can be decomposed into two parts, one reflecting purely private interests and one reflecting a notion of common good or fairness. Specifically, for any $i \in N$, i 's private interests are given by a utility

$$u_i(x) = 1 - u_i(y) \in \{0, 1\};$$

let $b_i \in \{x, y\}$ be i 's *bias*, where $b_i = z$ if and only $u_i(z) = 1$. The common good value of an alternative $z \in \{x, y\}$ is $f(z|\omega) \in \{0, 1\}$, describing which alternative is fair in state $\omega \in \{X, Y\}$. Then for any $z \in \{x, y\}$, $b_i \in \{x, y\}$ and $\lambda \in [0, 1]$, assume i 's preferences can be represented by⁶

$$U(z; b_i) = \lambda u_i(z) + (1 - \lambda) f(z|\omega).$$

In general, different individuals can be expected to have different moral systems or senses of what constitutes the common good. For example, suppose individuals are either Benthamite

⁶See Karni and Safra (2002) for an axiomatic justification of separable preferences for individuals with both private interests and a preference for fairness.

Utilitarians or Rawlsian Maximinimizers. Then reasons for choosing one alternative over another that are germane to the former can be utterly irrelevant to the latter and conversely. In this setting, productive debate might proceed either by a discussion of principles along, say, axiomatic grounds, or by seeking out reasons and arguments that are decision-relevant to both conceptions of how to evaluate the common good. Although such issues are, we think, quite important and worth thinking about more deeply, for now it is convenient simply to ignore such differences. So assume the evaluation function f is the same for everyone and satisfies $f(z|\omega) = 1$ if and only if $\omega = Z$. Similarly, without suggesting the assumption describes reality, it is convenient to suppose individuals value the common good in the same way, so λ is common across committee members.

There are two substantive sources of incomplete information. First, individual i 's bias $b_i \in \{x, y\}$ is known only to i : for all $i \in N$, assume the probability that $b_i = x$ is $1/2$. The second informational incompleteness concerns which of the two alternatives is most in the common interest, modeled as uncertainty over the realized state $\omega \in \{X, Y\}$... The common prior belief over $\{X, Y\}$ is assumed uniform. With probability $(1 - q) \in (0, 1)$ an individual $i \in N$ is either *uninformed*, observing no further information denoted $s_i = 0$, or, with probability q is *informed* and observes a noisy signal $s_i \in \{-1, 1\}$ from a common state-dependent distribution. Whether or not any $i \in N$ has observed any signal and, if so, which signal he or she received, is private information to i . Conditional on observing a signal, let

$$p = \Pr[s_i = 1|X, s_i \neq 0] = \Pr[s_i = -1|Y, s_i \neq 0]$$

and assume $p \in (\frac{1}{2}, 1)$. A pair $(p, q) \in (\frac{1}{2}, 1) \times (0, 1)$ is called an *information structure*.

In sum, therefore, an individual's type going into the committee decision-making process is a pair (b, s) where $b \in \{x, y\}$ is the alternative most in the individual's private interests and $s \in \{-1, 0, 1\}$ is the individual's signal regarding which alternative is fair. Hence, for each alternative $z \in \{x, y\}$, $i \in N$ has induced preferences going into committee given by

$$E[U(z; b_i)|s] = \lambda u_i(z) + (1 - \lambda) \Pr[Z|s] \in [0, 1].$$

Clearly, if $\lambda > 1/2$ then no type ever cares sufficiently about the common good for it to be decision relevant; therefore assume hereon that $\lambda \in (0, 1/2)$. Let $\pi \equiv \Pr[\omega = X]$ and define

$$\begin{aligned}\pi_x(\lambda) &= \min \{ \pi : \lambda + (1 - \lambda)\pi \geq (1 - \lambda)(1 - \pi) \}; \\ \pi_y(\lambda) &= \min \{ \pi : (1 - \lambda)\pi \geq \lambda + (1 - \lambda)(1 - \pi) \}.\end{aligned}$$

Then $\pi_x(\lambda)$ [respectively, $\pi_y(\lambda)$] is the decreasing [respectively, increasing] curve in Figure 1 below, illustrating induced preferences in (π, λ) -space. If $\pi < \pi_x(\lambda)$, an x -biased individual (i.e. i such that $b_i = x$) nevertheless strictly prefers y to x on grounds of expected fairness and, similarly, if $\pi > \pi_y(\lambda)$ then a y -biased individual (i.e. i such that $b_i = y$) strictly prefers x to y . The more individuals focus on their private interests (the higher is λ), the more evidence on the relative fairness of the two alternatives they require for such interests to be dominated.

Figure 1 here

It is analytically useful to define critical values for λ , $l_1(p), l_2(p) \in (0, 1/2)$, by

$$l_1(p) \equiv \frac{2p - 1}{2p} \text{ and } l_2(p) \equiv \frac{1}{p} l_1(p)$$

where there is no ambiguity, write $l_j = l_j(p)$. To interpret $l_1(p)$, let $S \equiv \sum_{i \in N} s_i$ be the sum of all individuals' signals. If this sum were common knowledge and if $\lambda < l_1$ then, irrespective of the distribution of signals across committee members, all individuals strictly prefer x [respectively, y] when $S \geq 1$ [respectively, $S \leq -1$]. And for $\lambda \in (l_1, l_2)$, all individuals strictly prefer x [respectively, y] only when $S \geq 2$ [respectively, $S \leq -2$]. When λ is below the threshold l_1 , individuals' induced preferences and behaviour in committee are in principle most sensitive to the opportunities offered by deliberation. From this perspective increases in p at a given λ are analogous to reductions in λ at a given p . Hence it suffices for the most part to focus on $\lambda < l_1$.

Once types are fixed, the committee decision-making process has two stages: the final stage is a vote (with no abstention) to choose between the two alternatives; this may be preceded

by a “debate” in which committee members simultaneously send a cheap talk message about the committee choice.

2.1 Strategies and equilibrium

The solution concept is a refinement of Perfect Bayesian Equilibrium in undominated (anonymous) strategies; although details of the refinement are discussed later, any subsequent reference to “equilibrium” or “equilibrium behaviour” refers to this solution concept. Anonymous strategies are imposed by definition and assumed throughout; anonymous strategies do not depend on the names of the agents.

A message strategy is a map

$$\mu : \{x, y\} \times \{-1, 0, 1\} \rightarrow \mathcal{M}$$

where \mathcal{M} is an arbitrary uncountable list of messages or speeches. Thus, any individual $i \in N$ with bias b_i and signal s_i makes a speech (sends message) $\mu(b_i, s_i) \in \mathcal{M}$. Let

$$\mathcal{M}_\mu \equiv [\cup_{\{x,y\} \times \{-1,0,1\}} \mu(b, s)] \subseteq \mathcal{M}$$

denote the range of μ . A *debate* is a list of messages $\mathbf{m} = (m_i, m_j, m_k) \in \mathcal{M}_\mu^3$; for any $i \in N$ and debate \mathbf{m} , let $M_{-i} = \sum_{j \neq i} m_j$.

A (pure) voting strategy is a map

$$v : \{x, y\} \times \{-1, 0, 1\} \times \mathcal{M}^3 \rightarrow \{x, y\}$$

So for any individual with bias b_i and signal s_i , $v(b_i, s_i, \mathbf{m}) \in \{x, y\}$ describes the individual’s vote conditional on b_i , s_i and the debate $\mathbf{m} = (m_i, m_j, m_k) \in \mathcal{M}^3$; by convention, the message listed first is invariably that of the individual i .

There are two sorts of constraint that any equilibrium strategy pair (μ, v) must satisfy. In any equilibrium, rational individuals vote for that alternative they most prefer conditional on their type, on the equilibrium messages heard in debate and on the event that they are pivotal

in the vote under the committee decision rule in effect. That is, for any $i \in N$ and signal s_i , given the voting behaviour v_{-i} of committee members other than i , given the voting rule and given a message strategy μ yielding debate $\mathbf{m} \in \mathcal{M}^3$, v must satisfy:

$$E[U(z; b_i)|s_i, \mathbf{m}, \mu, z, v_{-i}, \mathbf{vpiv}] > E[U(z'; b_i)|s_i, \mathbf{m}, \mu, z', v_{-i}, \mathbf{vpiv}]$$

implies i surely votes $v_i = z$ rather than $v'_i = z'$, where \mathbf{vpiv} denotes the event that i 's vote is pivotal with respect to x and y at the voting stage. Consequently, the first set of constraints, the *pivotal voting constraints*, insure that all individuals' voting behaviour is optimal conditional on their vote being pivotal at every information set. In some cases, such voting recommends voting with an individual's private interests and it is useful to have a term for this: say an individual *votes her bias* if $b_i = z$ implies i surely votes z . On the other hand, say that i *votes her signal* if, irrespective of bias, $s_i = 1$ [respectively, $s_i = -1$] implies i votes for x [respectively, y].

The second set of constraints, the *pivotal message constraints*, insure that every individual's message is optimal conditional on that message being pivotal for the final committee outcome, given individuals' voting strategies. Specifically, for any $i \in N$ and signal s_i , given the voting strategy v , the voting rule, and message strategies μ_{-i} for individuals other than i , μ must satisfy:

$$E[U(z; b_i)|s_i, m_i, \mu_{-i}, v, \mathbf{mpiv}(m_i, m'_i)] > E[U(z'; b_i)|s_i, m'_i, \mu_{-i}, v, \mathbf{mpiv}(m_i, m'_i)]$$

implies i surely makes the speech m_i rather than the speech m'_i , where $\mathbf{mpiv}(m_i, m'_i)$ denotes the event that the messages m_i and m'_i produce different outcomes at the voting stage.

Satisfying both sets of constraints gives rise to a variety of equilibria. To simplify the analysis and develop some intuition for how deliberation and voting interact, we focus on equilibria involving three important forms of debate. A (pure) message strategy μ is:

Separating in common interest if, for all $b \in \{x, y\}$ and any distinct $s, s' \in \{-1, 0, 1\}$,

$$\mu(b, s) \neq \mu(b, s');$$

Semi-pooling in common interest if, for all $b \in \{x, y\}$,

$$\mu(x, 0) = \mu(b, 1) \neq \mu(b, -1) = \mu(y, 0);$$

and *Pooling in common interest* if, for all $s, s' \in \{-1, 0, 1\}$,

$$\mu(x, s) = \mu(x, s') \text{ and } \mu(y, s) = \mu(y, s').$$

Further, say that μ is *separating in private interests* if, for all $s \in \{-1, 0, 1\}$, $\mu(x, s) \neq \mu(y, s)$.

An equilibrium (μ, v) is said to be a *separating debate equilibrium* if μ is separating in common interest; analogously, define semi-pooling and pooling debate equilibria.

Because debate is cheap talk there is always a pooling equilibrium in which no information is revealed in debate (Farrell, 1993). Further, private interest information (bias) matters in debate only insofar as it influences the audience's interpretation of any information offered regarding the common good. With respect to separating equilibria we need only consider equilibria that are separating in common interests and pooling in private interests. This is without loss of generality because if there exists an equilibrium with μ separating in common interests, then speakers are able to include a credible statement of their private interests too: information on bias does not matter under separation in common interest since all of the decision-relevant information is shared. The restriction to semi-pooling equilibria in which agents reveal how they would vote without deliberation is entirely *ad hoc*, adopted to reduce complexity and provide a method for comparing voting rules.⁷

We wish to understand how deliberation influences subsequent voting behaviour and thereby the quality of committee decisions. It is not enough simply to look for equilibria exhibiting more or less informative message strategies: any given informative message strategy can in principle be consistent in equilibrium with many voting strategies. Although some of the variation in voting behaviour at any given parameterization is eliminated through refinement, some

⁷In particular, we make no claim that these are the only sorts of equilibria; exploring alternatives is left for future work.

remains and is substantively interesting. Perhaps not surprisingly, given any attitude toward fairness (λ), the influence of deliberative argument on voting depends on the distribution of private bias in the committee and the relative likelihoods of any individual being informed (q) and the quality of the information conditional on being informed (p). Exactly how these features of the environment interact and the character of deliberative influence that they support, however, is not immediately apparent and turns out to be quite subtle. More detailed discussion of the particular sorts of influential equilibrium behaviour that can arise is deferred until the analysis, and we conclude this section with a brief description of the refinement (the formal definition is given in the Appendix).

The possibility, at any given parameterization of the model, of out-of-equilibrium messages and of undominated equilibrium voting profiles under which no individual is pivotal, motivates using a refinement to sharpen predictions. The refinement is essentially technical and has two components. First, individual vote decisions are subjected to individual-invariant trembles and we report behaviour in the limit as the trembles become vanishingly small. This insures that all individuals' equilibrium strategies are the limit of a sequence of best response strategies chosen conditional on being pivotal with strictly positive probability. The second component is a restriction on out-of-equilibrium beliefs at the debate stage. The issue here arises only for semi-pooling debate equilibria in which uninformed individuals are supposed to speak in support of their bias. There is little guidance on how best to proceed here and we simply assume that listeners hearing an out-of-equilibrium speech "I am uninformed" treat it as equivalent to hearing the speech "I believe y is likely the best choice" (where choosing y is without loss of generality).⁸

Although the comparative equilibrium properties of deliberative committee decision making with majority and unanimity voting are a main concern, it is necessary (and of independent

⁸We also considered an alternative specification: that out-of-equilibrium speeches claiming no information were believed surely. Although there turn out to be some differences, they are inconsequential; the derivations required, however, are considerably more tedious.

interest) to analyse behaviour under the two rules separately. Begin with majority rule.

3 Majority rule

Under majority rule, the alternative receiving at least two votes at the voting stage is the committee decision. This rule is inherently symmetric and, therefore, we consider symmetric strategies here. For any bias $b \in \{x, y\}$ let $-b \equiv \{x, y\} \setminus \{b\}$; recall $s \in \{-1, 0, 1\}$ and $\mathbf{m} \in \{-1, 0, 1\}^3$.

Definition 1 (1) A message strategy μ is symmetric if and only if, for all (b, s) , $\mu(b, s) = -\mu(-b, -s)$.

(2) A vote strategy v is symmetric if and only if, for all (b, s, \mathbf{m}) ,

$$v(b, s, \mathbf{m}) = x \Leftrightarrow v(-b, -s, -\mathbf{m}) = y.$$

Imposing symmetry on μ clearly adds little: by definition, if μ is separating, semi-pooling or pooling in common interest then μ is symmetric. Requiring symmetric voting strategies, however, although a mild restriction for the present model, has more bite.

Suppose there is no debate or that the message strategy is pooling in common interest (which, given debate is cheap talk here, always constitutes an equilibrium strategy). Then it is easy to check the symmetry assumptions imply that, for all $\lambda < 1/2$, the unique equilibrium voting profile is for all individuals to vote for their most preferred alternative conditional on their private signal and on being pivotal for the decision.

Proposition 1 Suppose there is no debate or that μ is pooling in common interests. Then, up to behaviour on the boundary $\lambda = l_1$, there is a unique symmetric voting equilibrium under majority rule; further, for any $z \in \{x, y\}$, if:

(1) $\lambda > l_1$ then, for all $s \in \{-1, 0, 1\}$, $v(z, s, \emptyset) = z$;

(2) $\lambda < l_1$ then $v(z, 1, \emptyset) = x$, $v(z, -1, \emptyset) = y$ and $v(z, 0, \emptyset) = z$.

Proofs for Proposition 1 and all subsequent results (save Proposition 5, where the proof is by example) are collected in the Appendix.

Say that a committee decision for $z \in \{x, y\}$ is *right* if z is chosen under the voting rule conditional on $\mathbf{s} = (s_1, s_2, s_3)$ being common knowledge at the time of the vote.⁹ Then, for $\lambda < l_1$ and majority rule, the only event in which the committee decision is not right, is when there are two uninformed individuals with identical bias for z and an informed agent with a signal supporting $z' \neq z$. In this case, all individuals vote for z' under full information but, in equilibrium, a majority votes under private information for z . Doing the calculation, the probability of “error” in the committee decision for $\lambda < l_1$ is no bigger than $3q(1-q)^2/8 \leq 1/18$. When $\lambda > l_1$, however, the likelihood of error jumps to $1/2$. Now suppose individuals have an opportunity for debate prior to voting.

Because information on common interests is intrinsically imperfect, we abuse language somewhat and say there is “full information” at the voting stage if the realized list of signals $\mathbf{s} \in \{-1, 0, 1\}^3$ is common knowledge. An equilibrium (μ, v) exhibits *full information equivalence* if and only if, for all \mathbf{s} , the equilibrium committee decision is always right. It is worth noting that full information equivalence does not imply all information is revealed in debate but only that, along the equilibrium path, committee decisions are those that would be made under common knowledge that $\mathbf{s} = \mathbf{m}$.

3.1 Separating debate equilibria

It is evidently possible for there to exist separating debate equilibria in which deliberation has no impact at all on individual voting behaviour. For example, suppose λ is sufficiently high relative to the quality of private information p ; then no feasible private signal or deliberative

⁹An alternative definition of the “right” decision is the alternative most likely in the common interest, conditional on the realized list of signals. When $\lambda < l_1$ and voting is majority rule, the two definitions recommend the same alternative but, in general, they are distinct because the definition in terms of common interest alone is insensitive to private bias.

argument can outweigh any individual bias and, therefore, fully revealing private information in debate can be an equilibrium strategy precisely because it is inconsequential. More interesting are those separating debate equilibria in which voting behaviour is responsive to deliberation.

In separating debate equilibria, speeches regarding the relative merits of the two alternatives are completely untainted by private interest and deliberation can generate consensus in individuals' induced preferences over $\{x, y\}$ whenever warranted by the realized list of signals. Moreover, if a separating debate equilibrium (μ, ν) exhibits full information equivalence, then the right committee decision is guaranteed because ν is uniquely defined by all individuals voting sincerely conditional on \mathbf{s} being common knowledge. Perhaps unfortunately, therefore, existence of such equilibria cannot always be assured.

Proposition 2 *Fix an information structure (p, q) . There is a unique value $\lambda(p, q) < l_1(p)$ such that there exists a full information equivalent symmetric separating debate equilibrium if and only if $\lambda \leq \lambda(p, q)$. Moreover,*

(1) *for all $p \in (1/2, 1)$, $\lambda(p, q)$ is strictly single-peaked in q on $(0, 1)$ with peak $\lambda^*(p) < l_1(p)$ such that $\lim_{p \rightarrow 1} \lambda^*(p) = \lim_{p \rightarrow 1} l_1(p) = 1/2$;*

(2) *for all $q \in (0, 1)$, $\lambda(p, q)$ is strictly increasing in p on $(\frac{1}{2}, 1)$ with maximum $\lambda^*(q) < 1/2$ such that $\lim_{q \rightarrow 1} \lambda^*(q) = 0$.*

At first glance, statements (1) and (2) of the proposition, taken together, may appear contradictory. However, they simply indicate that the order of limits is consequential: Figure 2 illustrates the function $\lambda(p, q)$ for three values of q .

Figure 2 here

An implication of the proposition is that, for any signal quality $p < 1$, there exists a full information equivalent separating debate equilibrium only if the probability of individuals being uninformed, q , is neither too high nor too low. To see the intuition here, let (μ, ν) be a full information equivalent separating equilibrium and fix $p < 1$; then $\lambda < l_1(p)$. Because

$\lambda < l_1(p)$ and μ is separating in common interest, there is no difficulty satisfying the pivotal voting constraints; it is the pivotal message constraints that bind. Specifically, from the proof to Proposition 2, it is the pivotal message constraint on the uninformed individuals that defines those (λ, p, q) for which the full information equivalent separating debate equilibria exist. So consider an uninformed, y -biased individual i and suppose i is considering $m_i \in \{0, -1\}$ conditional on all others following the prescribed separating equilibrium strategies. Then i 's speech in debate is message pivotal in three events:

event	$s_j = m_j$	$s_k = m_k$	b_j	b_k	$\Pr[(\cdot) s_i = 0]$
(a)	0	0	x	x	$\frac{1}{4}(1 - q)^2$
(b)	1	-1	x	x	$\frac{1}{2}q^2p(1 - p)$
(c)	0	1	y	y	$\frac{1}{4}(1 - q)q$

If either event (a) or (b) occurs, i 's preferred outcome is y and, given equilibrium voting v , this is the committee decision if and only if i sends the message $m'_i = -1$ rather than the truthful message $m_i = 0$. On the other hand, i 's most preferred outcome at event (c) is x and this is the committee decision if and only if i sends message $m_i = 0$. The critical pivotal event as q goes to one is clearly (b), the case in which both of the other committee members are almost surely informed but with opposing signals. As the probability of being uninformed becomes negligible, the likelihood of event (b) being true conditional on i being pivotal increases in relative importance to the point that i chooses to deviate from reporting her lack of information (inducing all individuals to vote their bias for x) in favour of influencing the committee to support y .¹⁰ Similarly, when q goes to zero the most likely message pivot event is (a) with both j and k being uninformed; in this case i believes that the committee decision depends almost surely on the distribution of bias in the case i reports $m_i = s_i = 0$ but is (conditional on the event (a)) surely y if she sends message $m'_i = -1$.

¹⁰Of course, at p and q sufficiently large, the overall probability of an uninformed committee member being signal pivotal is negligible.

An alternative perspective on the separating equilibria identified in Proposition 2 is useful. Fixing $\lambda = 1/10$, Figure 3 identifies the set of parameter values (p, q) for which there is a full information equivalent separating debate equilibrium. As $\lambda \rightarrow 1/2$, this region shrinks toward a neighbourhood of the point $(1, 1)$ and, as $\lambda \rightarrow 0$, the region expands to fill the space of all information structures. Loosely speaking, given any probability of being informed, q , high p is equivalent to low λ .

Figure 3 here

For $\lambda \leq l_1$ and message strategy separating in common interest, the pivotal vote constraints are surely satisfied by individuals' voting on the basis of their full information induced preferences. Therefore the boundary of the region in Figure 3 for which the relevant equilibria exist is the set of informational structures at which uninformed individuals are just indifferent between revealing their lack of information and making a speech in support of their private interests. Of course, because making the latter speech is designed to encourage others to vote for the speaker's bias by influencing beliefs, the speech itself is not in terms of the speaker's bias *per se* but rather in terms of the common interest.

Proposition 2 claims that when individuals value the common good sufficiently highly and only a minimal amount of evidence in favour of an alternative being more in the common good is required to induce an individual to support that alternative, all private information on the common good can be credibly revealed in equilibrium and the subsequent voting behaviour results in full information equivalence. None of these properties, however, necessarily hold for semi-pooling debate equilibria.

3.2 Semi-pooling debate equilibria

By definition, in semi-pooling (SP) debate equilibria relatively informed individuals – that is, those for whom $s_i \neq 0$ – continue to make speeches advocating the alternative supported by their information, irrespective of their private interests, but uninformed individuals – those for

whom $s_i = 0$ – now make speeches advocating the alternative they favour on private interests alone. In view of Proposition 1, therefore, speeches in SP debate equilibria involve everyone effectively announcing how they would have voted without debate.

Beliefs regarding a speaker’s private bias are not the only thing that distinguish interpretation of speech under SP from that under separating message strategies. In a separating debate equilibrium, the particular values of the parameters q and p play an important part in defining when full information equivalent voting constitutes equilibrium behaviour, but have nothing to do with the interpretation of debate speeches *per se*.¹¹ In SP debate equilibria, however, this is no longer true: the likelihood of being informed and the quality of any information in fact received bear both on the interpretation of speech and on subsequent voting decisions. Not surprisingly therefore, there can be a variety of SP debate equilibria that, at least observationally, differ exclusively in voting behaviour. Depending on the information structure, identical debates (that is, any $\mathbf{m} \in \mathcal{M}_\mu^3$ or permutation thereof) can influence different individuals in different ways and lead to various profiles of voting decisions.

The different sorts of symmetric SP debate equilibria identified reflect different degrees to which individuals can be influenced by debate and signals. Some more language is useful. Fix a semi-pooling debate equilibrium (μ, v) . An uninformed individual i is *influenced in debate* if i ’s vote under v is not constant in the speeches of others; an informed individual i is *marginally influenced in debate* if either (1), (2) or (3) are true:

(1) i ’s signal is *against* i ’s bias, both of the others argue in favour of i ’s bias, and i votes *for* her bias

(2) i ’s signal is *for* i ’s bias, both of the others argue against i ’s bias, and i votes *against* her bias

¹¹Of course, $\lambda \leq l_1(p)$ is necessary for full information equivalent separating debate equilibria to exist at all. Nevertheless, any interpretation of (equilibrium) speech conditional on this constraint not binding is independent of p .

(3) i 's signal and both of the others' speeches are *against* i 's bias, and i votes *against* her bias;

an informed individual i is *influenced in debate* if all of (1), (2) and (3) obtain. The set of pure voting strategies, v , for the (symmetric) SP debate equilibria that exist is described in the Appendix, where their respective location in (p, q) -space is also illustrated for $\lambda = 1/10$. And it is worth remarking here that, whereas the binding constraint for the separating debate equilibrium is at the debate stage through the pivotal message constraints, the binding constraints on the SP equilibria are at the voting stage, through the pivotal voting constraints. Insofar as debate in SP equilibria is constant and only the voting responses to debate changes with the information structure, this shift in which constraints bite has some intuition.

Broadly speaking the distribution of semi-pooling debate equilibria on the space of information structures exhibits the following the pattern. Conditional on the quality of information, p , not being too low, in which case semi-pooling debate cannot be influential at all, if the probability of being informed,

q is low, at most the uninformed are influenced in debate;

q is moderate, informed individuals can be marginally influenced in debate at all save the lowest values of p , but uninformed individuals are influenced in debate only if p is moderate to high;

q is high, the informed and uninformed individuals are influenced in debate at all save the lowest values of p .

When the probability of being informed is high and all individuals are influenced in semi-pooling debate, the equilibrium voting strategy v is consistent with all individuals treating all speeches as true. Thus, all voting behaviour is "sincere" in that it reflects the balance of bias, signal and debate for each committee member; nevertheless, individual debate behaviour is not so sincere since the uninformed misrepresent their knowledge in debate, arguing for their bias

by adopting the speech of those informed in favour of that bias. Consequently, not all of the decision-relevant information is offered in semi-pooling debate and full information equivalence is not assured. For example, let (μ, v) be a semi-pooling debate equilibrium in which both informed and uninformed individuals are influenced; suppose $\mathbf{s} = (1, 0, 0)$ and individuals 2 and 3 are y -biased. Then by definition of μ , the debate is $\mathbf{m} = (1, -1, -1)$ and the equilibrium voting strategy v yields a unanimous vote for y . But (as shown in the Appendix, Figure 6) this sort of equilibrium only exists for $\lambda \leq l_1(p)$ which implies the right committee decision at \mathbf{s} is x . On the other hand, by Proposition 1, the outcome in this case without debate is also x . In other words, despite some speeches in semi-pooling debate occasionally failing to reflect any private information, such strategic speech-making turns out not to lead to any worse an outcome than the one under no debate.

The preceding example is not an artifact. Before making this assertion precise, it is useful to check some intuitive properties of voting behaviour in any (anonymous although not necessarily symmetric) SP debate equilibrium under majority rule.

Lemma 1 *In any SP debate equilibrium under majority rule,*

$$v(y, 0, -1, -1, -1) = y.$$

Assuming the committee makes decisions under majority rule, therefore, an uninformed y -biased agent surely votes her bias following any (semi-pooling) debate in which everyone argues for choosing y over x and, evidently, a completely symmetric argument applies for x -biased individuals; that is, $v(x, 0, 1, 1, 1) = x$ also. In other words, the symmetry of majority rule coupled with that of the semi-pooling message strategy μ implies a considerable degree of symmetry in SP equilibrium voting.

The next result, Lemma 2, says that SP debate equilibrium vote decisions are signal (claim 1) and bias (claim 2) monotonic.

Lemma 2 *Let (μ, v) be any SP debate equilibrium and $\mathbf{m} \in \mathcal{M}_\mu^3$. Then, under majority rule, for all signals $s > s'$ and each bias b :*

$$(1) \begin{cases} [v(b, s, \mathbf{m}) = y \Rightarrow v(b, s', \mathbf{m}) = y], \\ [v(b, s', \mathbf{m}) = x \Rightarrow v(b, s, \mathbf{m}) = x]; \end{cases}$$

$$(2) \begin{cases} [v(x, s, \mathbf{m}) = y \Rightarrow v(y, s, \mathbf{m}) = y], \\ [v(y, s, \mathbf{m}) = x \Rightarrow v(x, s, \mathbf{m}) = x]. \end{cases}$$

It seems sensible that in addition to bias and signal monotonicity, debate equilibria should also exhibit some sort of monotonicity in messages: if, given a distribution of bias and information in the committee, the only difference between two debates (m_i, \mathbf{m}_{-i}) , (m_i, \mathbf{m}'_{-i}) from i 's perspective is that the speeches of others \mathbf{m}'_{-i} are both more favourable to the individual's bias than are \mathbf{m}_{-i} , then i should vote his bias following (m_i, \mathbf{m}'_{-i}) if i votes his bias following (m_i, \mathbf{m}_{-i}) . This sort of monotonicity is satisfied by all of the debate equilibria considered so far and, as will become apparent shortly, all of those discussed in the next section on unanimity rule. But messages and votes are strategic decisions and, at least as far as we know at present, this form of monotonicity is not implied by the current assumptions on equilibrium behaviour.

Definition 2 *A voting strategy v satisfies debate monotonicity if and only if, for all (b_i, s_i, m_i) , $v(b_i, s_i, m_i, m_j, m_k) = b_i$, $b_i m'_j \geq b_i m_j$ and $b_i m'_k \geq b_i m_k$ imply $v_i(b_i, s_i, m_i, m'_j, m'_k) = b_i$.*

In words, if an agent is voting consistent with his bias after observing a signal and some debate then he must also be voting for his bias if he observes the same signal (and therefore sends the same message) and a debate that is more favourable for his bias. Note that debate monotonicity requires holding constant the agent's bias, signal and message. Moreover, debate monotonicity does not imply, for instance, that an uninformed y -biased individual who sends a message $m = -1$ and hears a split debate $\mathbf{m}_{-i} = (1, -1)$ surely votes his bias. Requiring debate monotonicity, then, is a substantively weak restriction; it is nevertheless very useful analytically.

Lemma 3 *In any symmetric SP debate equilibrium in which voting is debate monotonic, either there is a positive probability the vote of agent i is pivotal given debate $(m_i, m_j, m_k) = (1, -1, -1)$ or agents j and k are both voting for y .*

Recall the definition of a “right” committee decision as being the decision reached under decision making with full information on \mathbf{s} ; a “wrong” committee decision is any decision that is not “right”. There are, therefore, two sorts of error in committee decision making, depending on the distribution of induced preferences at any realized list of signals \mathbf{s} .

Definition 3 *The equilibrium (μ, ν) induces an*

- error in common interest at \mathbf{s} if all individuals’ induced preferences over $\{x, y\}$ are unanimous at \mathbf{s} , but the equilibrium committee decision is for the less preferred alternative;*
- error in bias at \mathbf{s} if each individual’s induced preference over $\{x, y\}$ at \mathbf{s} is completely defined by his or her bias, but the equilibrium committee decision is for the minority’s preferred alternative.*

Say that committee decision-making subject to a set of debate and voting rules α can make an error in common interest (bias) at (λ, p, q) if, under α , there exists an equilibrium (μ, ν) at (λ, p, q) that induces an error in common interest (bias).

Definition 4 *With respect to common interest (bias), committee decision-making subject to a set of rules α weakly dominates committee decision-making subject to a set of rules β at (λ, p, q) if (1) α can make an error in common interest (bias) at (λ, p, q) implies β can make an error in common interest (bias) at (λ, p, q) and (2) the converse is false.*

In words, an institutional arrangement α weakly dominates another, β , with respect to making a particular sort of error if, first, whenever there exists an equilibrium under α at which the committee makes the wrong decision for some \mathbf{s} , there also exists an equilibrium under β at which the committee makes the same mistake; and second, there exist circumstances under which β yields an error but every equilibrium under α results in the right decision.

Proposition 3 *Assume only separating, semi-pooling and pooling debate equilibria are played and that committee decisions are made by majority rule. If equilibrium voting is debate monotonic then,*

(1) *with respect to common interest, committee decision making with debate weakly dominates committee decision making without debate at almost all (λ, p, q) ;*

(2) *if separating equilibria are played whenever available, then (1) holds also with respect to bias.*

Two things are worth emphasizing about Proposition 3, the main result of this section. First, the result applies quite generally to all symmetric pure strategy SP debate equilibria exhibiting debate monotonicity (both with and without the technical refinement); and second, Proposition 3(1) does not say that for every feasible (λ, p, q) there exists a debate equilibrium that is, with respect to yielding “right” decisions at (λ, p, q) , weakly better with respect to errors in common interests than the no-debate equilibrium, but rather that *every* separating, semi-pooling and pooling equilibrium has this property at any feasible (λ, p, q) . But as indicated in the statement of Proposition 3(2), debate monotonicity is not enough to extend the result for errors in common interest to errors in bias. Errors in bias can only occur if (up to permutations)

$$\mathbf{s} \in \{(0, 0, 0), (0, 1, -1)\},$$

implying the probability of Y being the true state is $1/2$. *A priori*, there seems little reason to think that such errors in bias are any less important than errors in common interest. Although majority rule without debate is not immune to errors in bias (an example is given below), it is possible for debate to yield such an error where none would be made in its absence.

To see why the restriction to playing separating debate equilibria where available is necessary for Proposition 3(2), assume y is the right decision at the situation $\mathbf{s} = (0, 0, 0)$. Since y is the right decision by assumption, at least two of the committee are y -biased (say, $i = 1, 2$), so y is surely the no-debate equilibrium decision (Proposition 1). Now let (μ, v) be a symmetric

semi-pooling debate equilibrium satisfying debate monotonicity. There are two debates possible in equilibrium, depending on individual 3's bias. If $b_3 = y$ then $\mathbf{m} = (-1, -1, -1)$ and Lemma 1 implies there is no error; so suppose $b_3 = x$, yielding $\mathbf{m} = (-1, -1, 1)$. By anonymity, if (μ, v) results in an error at this debate, each uninformed y -biased individual $i \in \{1, 2\}$ must vote for x conditional on sending a message $m_i = -1$ and hearing a split debate $\mathbf{m}_{-i} = (-1, 1)$; that is an error in bias implies

$$v(y, 0, -1, -1, 1) = x.$$

And it turns out that indeed there exist SP debate equilibria in which the uninformed adopt such voting decisions (see the Appendix, Lemma 4); that is, uninformed individuals speak in favour of their bias in debate but vote against that bias if they hear a split debate among the other committee members. Although perhaps unusual, such voting behaviour by uninformed individuals may not be absurd. Given that an uninformed individual i delivers the speech $m_i = -1$, when i hears a split debate $(m_j, m_k) = (1, -1)$, she might reason that if her vote is pivotal, it is most likely to be the speaker who presented the minority opinion, $m_j = 1$, who is voting against the majority position, $m_i = m_k = -1$, advocated in debate. In turn, this suggests that j is relatively more likely to be informed in which case, conditional on being pivotal, i voting for the minority deliberative opinion is the best thing to do.

Similar reasoning applies to the remaining possibility, $\mathbf{s} = (0, 1, -1)$: if the bias distribution is $\mathbf{b} = (y, x, y)$, then the debate has to be $\mathbf{m} = (-1, 1, -1)$ and, in the SP debate equilibrium in which uninformed individuals vote against their bias conditional on hearing a split debate, the first two individuals vote for x to produce an error. In this case, however, there is no guarantee that the no-debate equilibrium outcome is right: if $\mathbf{b}' = (x, y, y)$, the right decision is y but the no-debate outcome is x .

It is not hard to see from the preceding discussion that a necessary condition for an error in bias (not involving errors in common interest) to result from debate is that uninformed individuals vote against their bias conditional on observing a split debate. In the Appendix,

we show that all of the symmetric semi-pooling debate equilibria exhibiting such behaviour exist only if the separating debate equilibrium also exists. Proposition 3(2) then follows.

4 Unanimity rule

In this section we ask how variations in the voting rule influence the character and extent of deliberation. Further, much of the more normative literature on deliberation suggests that requiring unanimity at the voting stage promotes more information sharing and argument in debate, since some form of consensus is now essential for pro-active committee decision.¹² In this section we demonstrate that such reasoning is mistaken.

Unlike with majority rule, the *status quo* policy is consequential under unanimity rule: the *status quo* x can be rejected in favour of y only if all three committee members vote for y against x .¹³ And since unanimity rule is evidently not symmetric, there is no good reason to insist, or even focus, on symmetric equilibria; in fact, quite the contrary is true. Consequently, we no longer look for symmetric voting strategies, although we maintain the presumption of anonymity. Moreover, there are (as discussed momentarily) multiple voting equilibria without debate under unanimity rule. It follows that any sort of comparative statement across majority and unanimity rules is necessarily tempered by these fundamental differences between the two decision schemes.

¹²Particularly direct examples include: “It should be remembered that veto power or unanimity represents a constraint that induces deliberation: when parties can block outcomes, actors have incentives to find reasons that are convincing to all, not just to the majority” (Eriksen, 2001:15/16); “Hence the unanimity requirement in jury verdicts, which is intended to encourage through deliberation as necessary for a conviction” (Shapiro, 2000:12); “The necessity of a consensus of all jurors which flows from the requirement of unanimity, promotes deliberation and provides some insurance that the opinions of each of the jurors will be heard and discussed” (South Australian High Court, 1993; quoted in Walker and Lane, 1994:2)

¹³A seemingly plausible alternative assumption here, is to take a fair lottery over $\{x, y\}$ as the status quo and require a unanimous vote to insure either alternative surely. But then decision making is over a three, rather than two, alternatives, a quite different scenario.

Suppose there is no debate stage and note that an individual is pivotal in voting only if both of the other committee members are voting for y . Then it cannot be the case that all types surely vote for y in any equilibrium, irrespective of their bias or signal. It is easy to see why: suppose the claim false and consider an individual i with signal $s_i = 1$ and bias for x . Then the event that i is vote pivotal under unanimity contains no additional decision-relevant information for i , in which case, given signal $s_i = 1$, voting for x surely is the best decision. Depending on the parameters (λ, p, q) there can exist distinct no-debate equilibria with unanimity rule and, for some (λ, p, q) , the only no-debate equilibria are in mixed strategies. The possible pure voting strategies are described in the table below, where the entries in each cell are the vote pairs $[v(y, s_i, \emptyset), v(x, s_i, \emptyset)]$; all no-debate equilibrium mixed strategies have support in this set of pure strategies.¹⁴

		No-debate equilibria, $[v(y, s_i, \emptyset), v(x, s_i, \emptyset)]$				
s_i	1	2	3	4	5	
-1	y,y	y,y	y,y	y,y	y,x	
0	y,y	y,y	y,x	y,x	y,x	
1	x,x	y,x	x,x	y,x	y,x	

Table 1: Pure strategy no-debate voting equilibria

In common with the story for SP debate equilibrium voting under majority rule, broadly speaking, the better the quality of the information the more willing are uninformed individuals to vote for y , effectively delegating the committee decision to the informed committee members, who likewise are more willing to vote their signal irrespective of bias (**1**). And again, the presence of uninformed individuals is important here. As is apparent from Figure 7 in the

¹⁴A description of the mixed strategy equilibria and of the sets of information structures for which the various no-debate equilibria exist can be found in the Appendix.

Appendix, as signal quality declines individuals with an x -bias become increasingly unwilling to vote against their bias.

It is immediate that no-debate equilibrium under unanimity can yield errors in bias: suppose the equilibrium is **1** and all individuals are both uninformed and x -biased; then there is a unanimous vote for y where in fact x is the right decision.

The multiplicity of no-debate equilibria under unanimity rule makes an unequivocal statement about the likelihood of an equilibrium committee decision being “right” contingent on the particular equilibrium played. However, the bounds are clear. The smallest likelihood of error is when voting strategy **1** is equilibrium behaviour. Here, an error occurs only in state Y when all individuals are informed but one sees an incorrect signal; this occurs with probability $q^3 p^2 (1 - p) / 2$. At the other extreme for $\lambda \leq l_1$, when **4** is equilibrium behaviour there are multiple events at which error can occur; doing the (tedious) calculation gives the likelihood of error as $[4pq(1 - 2q + pq) - pq^3(p + 2p^2 - 4)] / 16$. And for $\lambda > l_1$, under **5**, the likelihood of error is $1/2$. More interesting, is what happens when deliberation precedes any committee vote.

Proposition 4 *There exists no separating debate equilibrium under unanimity.*

Comparing this result with Proposition 2 undermines any general claim that requiring unanimity to make policy changes induces more deliberation in committee than requiring only a majority. Depending on individual attitudes toward the common interests and on the information structure, deliberation can be fully informative under majority rule but not under unanimity.¹⁵

¹⁵A similar impossibility result is proved by Doraszelski, Gerardi and Squintani (2001), the only other model that, to the best of our knowledge, considers deliberation under unanimity rule. DGS study a two-person committee that is choosing between a status quo and a given alternative policy; rejection of the status quo requires unanimous approval. There are two states of the world from the common interest perspective, say X and Y , and both individuals strictly prefer x (respectively, y) in state X (respectively, Y). Where they differ

Although full separation, and thereby full information equivalence, is impossible under unanimity, deliberation can nevertheless be informative under some circumstances. We prove the claim by example. Under unanimity rule, any individual who so chooses can guarantee a committee decision for x with her vote alone. This suggests a debate strategy in which a y -biased individual argues for y in debate irrespective of any private information: if such an individual is persuaded, either by her private information or by deliberation, that x is most in her interests then her own vote insures this outcome whatever she says in debate; but if she is left preferring y over x then her deliberative argument can be pivotal. Similarly, an individual for whom x is most in his private interests has nothing to lose by sharing his information on the relative common good properties of the two alternatives. Formally, the suggested pattern of deliberation is described by the asymmetric message strategy, $\bar{\mu}$: for all distinct $s, s' \in \{-1, 0, 1\}$,

$$\bar{\mu}(x, s) \neq \bar{\mu}(x, s') \text{ and } \bar{\mu}(y, s) = \bar{\mu}(x, -1).$$

Thus, under $\bar{\mu}$, all y -biased individuals pool in common interests and all x -biased individuals separate in common interest. For want of a better term, then, call any debate equilibrium $(\bar{\mu}, v)$ a *bias-driven debate equilibrium*.

In any bias-driven debate equilibrium, those who are most likely to want change (y -biased individuals) argue consistently for this alternative irrespective of their signal, so suppressing any information they might have in support of the *status quo* x ; against this obscurantism, those

is in the attitudes about making errors and these attitudes (parametrized by some real number from the unit interval) are private information. In addition to learning their particular attitude to error, each individual also observes a noisy binary signal regarding the true state of the world. Inter alia, DGS study what happens when both individuals can give cheap-talk signals about their signals prior to voting. Their main results are that there is no separation in debate and deliberation is influential only in the case when an individual's signal conflicts with her disposition and prior belief: "When there is a conflict between a player's preferences and her private information about the state, she votes in accordance with her private information only if it is confirmed by the message she receives from her opponent" (p.2).

most likely to resist change (x -biased individuals) are willing to reveal all of their information in debate, whether or not it suggests that in fact y is the better alternative on common interest grounds. But despite this willingness on the part of x -biased committee members to make a case for y when appropriate, the only credible arguments are those who argue (at least weakly) on behalf of x ; any effort by an x -bias individual to argue for y is confounded by the incentives for those with a private interest for y also arguing that case. So there is small hope here of achieving any sort of consensus through deliberation alone. But such a lack of deliberative consensus need not imply that deliberation cannot yield unanimous voting in committee.

Equilibria involving such asymmetric deliberation do exist; Figure 4 illustrates an example for $\lambda = 1/10$. As indicated in the diagram, a necessary but not sufficient condition on the information structure (p, q) for the bias-driven equilibrium to exist is that the no-debate voting equilibrium **1** also exists at (p, q) .

Figure 4 here

The identified bias-driven equilibrium is the pair $(\bar{\mu}, \bar{v})$. The message strategy $\bar{\mu}$ is defined above and the voting strategy \bar{v} is described in Table 2, where the pairs in the two “ m_i ”-columns are the votes, $(\bar{v}(y, \cdot), \bar{v}(x, \cdot))$.

s_i	M_{-i}	$m_i \in \{-1, 0\}$	$m_i = 1$
-1	≤ 0	y, y	y, y
	1	y, x	y, x
	2	x, x	x, x
0	≤ -1	y, y	y, y
	$[-1+1]$	y, x	x, x
	$[0+0]$	y, x	y, x
	≥ 1	x, x	x, x
1	≥ -2	x, x	x, x

Table 2: Voting strategy \bar{v}

The binding constraints on $(\bar{\mu}, \bar{v})$ are two pivotal voting constraints: the lower boundary illustrated in Figure 4 describes the locus of information structures at which an x -biased individual with signal against his bias is indifferent at \bar{v} between voting for y (as required) or x conditional on hearing a split debate, $(m_j, m_k) = (-1, 1)$; the upper boundary describes the locus of information structures at which an y -biased individual with signal against her bias is indifferent at \bar{v} between voting for x (as required) or y conditional on hearing a uniform debate, $(m_j, m_k) = (-1, -1)$.

The most interesting thing to note about the strategy \bar{v} is that an uninformed y -biased individual i votes for y against x if i makes any speech $m_i \in \{-1, 0\}$ (weakly) in support of choosing y , but votes for x against y if, for some reason, i advocates choosing x , $m_i = 1$, and the others are divided in debate, $(m_j, m_k) = (-1, 1)$. In other words, under \bar{v} , an individual with a given signal, hearing given speeches by others in debate, nevertheless votes differently depending on the particular cheap talk speech she delivers; in this case, the individual “talks herself into voting against her bias”. Such behaviour is not, it turns out, unreasonable: because the subsequent votes of others depend in part on the arguments they hear in debate, the pivotal voting constraint facing an individual following one speech does not necessarily coincide with that following a different speech. In fact, although, in the equilibrium $(\bar{\mu}, \bar{v})$, this particular behaviour is off-equilibrium-path, it proves essential to support existence of $(\bar{\mu}, \bar{v})$ as equilibrium behaviour at all. If, as seems intuitive, the uninformed y -biased individual’s vote is independent of her own message at any debate (in particular, at the debate $(m_j, m_k) = (-1, 1)$), then not all of the message pivotal constraints can be satisfied along the equilibrium path.

Similar considerations apply, although less evidently, elsewhere in the equilibrium voting profile. From Table 2, an x -biased individual with a signal against her bias ($s_i = -1$) is required to vote for y conditional on $M_{-i} = -1$ whatever speech she makes. However, if the probability of others being informed, q , is sufficiently low, then such an individual strictly prefers to vote for x in the event she sends the off-path message $m'_i = 1$ supporting her private

bias rather than her signal, $\bar{\mu}(x, -1) = -1$ (or a speech $m_i = 0$) but not otherwise. Moreover, if the individual is presumed to vote for x conditional on sending $m'_i = 1$, then $(\bar{\mu}, \bar{v})$ cannot describe equilibrium behaviour at any information structure.

Recall that the probability of the committee choosing the wrong alternative in the no-debate equilibrium **1** is $q^3 p^2 (1 - p)/2$. Under the bias-driven debate equilibrium $(\bar{\mu}, \bar{v})$, the probability of the committee making an error in common interest falls to zero but that of making an error in bias remains strictly positive: if $b_1 = y$, $b_2 = b_3 = x$ and all individuals are uninformed, the debate under $(\bar{\mu}, \bar{v})$ is $\mathbf{m} = (-1, 0, 0)$ and, given \bar{v} , all individuals vote for the wrong outcome, y , exactly as in the no-debate equilibrium **1**. Nevertheless, it is clear by inspection that the debate equilibrium $(\bar{\mu}, \bar{v})$ also weakly dominates the no debate equilibrium **1** with respect to bias $(\bar{\mu}, \bar{v})$, that is, when $\mathbf{s} \in \{(0, 0, 0), (0, -1, 1)\}$. Whenever there is an error in bias alone under $(\bar{\mu}, \bar{v})$ there is also an error under **1** without debate, but the converse is false: let $\mathbf{s} = (0, 0, 0)$ and $b_i = x$ all i ; then without debate the wrong decision y is made but with debate the decision is x . It follows that the bias-driven debate equilibrium $(\bar{\mu}, \bar{v})$ weakly dominates the no debate equilibrium **1**. This is perhaps to be expected: debates supported by $\bar{\mu}$ necessarily make committee members strictly more informed at the voting stage than they are without debate.¹⁶ In general, however, the weak dominance result for $(\bar{\mu}, \bar{v})$ does not extend to all bias-driven debate equilibria.

Proposition 5 *Assume committee decisions are made by unanimity rule. There exist (λ, p, q) at which the committee makes the wrong decision under a bias-driven debate equilibrium $(\bar{\mu}, \hat{v})$, but makes the right decision under a no-debate equilibrium, v^0 .*

Proof We show by example that bias-driven debate can support errors in common interest in settings where the committee decision under the relevant (pure strategy) no-debate equilibrium is the right decision. Assume $\lambda = 1/10$ (this particular value is inessential). The message

¹⁶This is true even if the debate is $\mathbf{m} = (-1, -1, -1)$; in this case all individuals know there is no x -biased committee member with a signal $s \geq 0$.

strategy $\bar{\mu}$ is defined above; the vote strategy \hat{v} is described in Table 3 where, as usual, the “ m_i ”-columns are the votes, $(\hat{v}(y, \cdot), \hat{v}(x, \cdot))$.

s_i	M_{-i}	$m_i = -1$	$m_i = 0$	$m_i = 1$
-1	≤ -1	y,y	y,y	y,y
	$[-1+1]$	y,y	y,y	y,x
	$[0+0]$	y,y	y,y	y,y
	1	y,x	y,x	y,x
	2	x,x	x,x	x,x
0	-2	y,y	y,y	y,y
	-1	y,x	y,x	y,y
	$[-1+1]$	y,x	y,x	y,y
	$[0+0]$	y,x	y,x	x,x
	≥ 1	x,x	x,x	x,x
1	≥ -2	x,x	x,x	x,x

Table 3: Voting strategy \hat{v}

Insisting on the technical equilibrium refinement (individually independent trembles) leads to difficulties off the (postulated) equilibrium path here. In particular, the strategy pair $(\bar{\mu}, \hat{v})$ is an equilibrium and survives the refinement only on a line cutting through the set $(p, q)[\mathbf{1}] \subset (\frac{1}{2}, 1) \times (0, 1)$ on which the no-debate voting equilibrium **1** exists. However, the no-debate equilibrium **1** obviously exists without insisting on the refinement and lifting the refinement further results in $(\bar{\mu}, \hat{v})$ constituting equilibrium behaviour on a nonempty set of information structures having strictly positive measure: see Figure 5.¹⁷

Figure 5 here

¹⁷As indicated in the Appendix, establishing these claims formally is both tedious and computationally demanding, so we omit the details. All of the derivations supporting this example and the figures in the text, however, are available from the authors on request.

In the figure, the region below the two intersecting thick lines is the set of information structures for which $(\bar{\mu}, \hat{\nu})$ is an (unrefined) equilibrium; the downward sloping thin line is the lower boundary of $(p, q)[\mathbf{1}]$. If the technical equilibrium refinement is imposed, the set of information structures delineating those $(\bar{\mu}, \hat{\nu})$ debate equilibria surviving the refinement is precisely the downward sloping thick line, that is, the upper boundary of the unrefined set.

Consider any $(p', q') \in (p, q)[\mathbf{1}]$ for which $(\bar{\mu}, \hat{\nu})$ is a bias-driven debate equilibrium (the information structure $(0.68, 0.30864)$ works for the refined, non-generic, case). Assume the realized profile of signals is $\mathbf{s} = (-1, 0, 0)$ so the right committee decision is y . Assume that the two uninformed individuals, $i = 2, 3$, are both x -biased. Then under the no-debate voting equilibrium $\mathbf{1}$, the committee unanimously votes for the right alternative y . Under the debate equilibrium $(\bar{\mu}, \hat{\nu})$, however, the realized debate is $\mathbf{m} = \mathbf{s} = (-1, 0, 0)$ but subsequent equilibrium voting has both individuals 2 and 3 voting for x , thus vetoing y and leading to the wrong committee decision. \square

The reason for the error in the example establishing Proposition 5 is not hard to see. In the relevant information structure, the probability of any individual being informed is sufficiently low that a single noisy speech for y is insufficient to offset any private bias for x . When there is no debate, however, the uninformed x -biased individuals condition on being pivotal, that is, on the event that both of the other committee members are surely voting for y , in which event there is positive probability on both individuals observing signals for Y being the true state. On balance, the *ex ante* possibility of there being two signals in favour of Y conditional on being pivotal without debate, is stronger support for choosing y than knowing as a result of debate that there is at most one signal in favour of Y .

Proposition 5 implies that an analogous claim to Proposition 3 (which holds with or without the trembles refinement) is not available. The result does not imply that deliberation is on balance detrimental to the quality of committee decision making under unanimity rule and it seems unlikely that this is the case. What is true, is that, in comparison with majority rule,

requiring unanimous voting induces quite distinct sorts of deliberation and incentives to share information in debate. And on balance, majority rule offers more opportunity for credible deliberation and symmetric information sharing.

5 Discussion

Despite the fact that the role of deliberation in agenda-setting *per se* may likely prove the most important, there is still a great deal to be learned about deliberation over fixed agendas. Assuming a fixed agenda, the particular issue we address in this paper concerns the connection between the voting rule adopted by a committee for making a decision and the character of any deliberation preceding the vote. In this regard, the informal literature on deliberation and consensus claims (among other things) that unanimity rule promotes deliberation. The intuition underlying this argument is that because it is necessary to arrive at a consensus to reach a decision, more information is revealed by committee members in an effort to persuade the last individual. As our model demonstrates, however, such reasoning is fallacious. Although unanimity rule creates incentives for supporters of the *status quo* to reveal information, it likewise creates incentives for others to conceal information favouring that *status quo*. This in turn generates an externality rendering information from all members of the committee suspect. In contrast, majority rule balances the incentives of those biased for and against the *status quo* to the extent that, under at least some circumstances, everyone can truthfully reveal their private information in debate. Moreover, even when majority rule cannot support purely truthful communication we show that majority rule (for a reasonable subclass of semi-pooling strategies) does not result in the committee making decisions that everyone in the committee would oppose if all information were shared; this is not true of deliberation under unanimity rule where such mistakes can occur.

The analysis underlying our results depends on what is, at least from a standard game-theoretic perspective, a fairly natural conception of committee deliberation, specifically, delib-

eration as strategic information transmission. And within this framework, there are some fairly obvious extensions, including sequential speechmaking, consequential variation in the relative weights individuals' place on private interests, and so on. However, the usual apparatus of incomplete information games may in fact to be too restrictive to address some of the important questions considered in the normative political theory literature. And a key issue in this regard concerns whether or not all consequential deliberation is inherently informational. If it turns out that in fact arguments predicated on strategic information transmission models fail to capture the salient features of committee deliberation precisely because these features are not intrinsically informational, then the relevance of our discussion to the normative literature becomes moot.

There seem to be two principal ways in which deliberation might not be informational. Loosely speaking, the first involves equilibrium selection in coordination games (Farrell, 1987; Rabin, 1994; Calvert and Johnson, 1998) and the second involves argument through analogy and precedent (Aragones, Gilboa, Postlewaite and Schmeidler, 2001).

Although it is surely the case that coordination and argument through analogy do not concern information of the sort considered in the model here, they are both intrinsically concerned with some form of informational imperfection. This is most evident for coordination games; here, no new information regarding the state of the world is produced in debate but the extent to which speech is informative is the extent to which any *strategic* uncertainty is resolved. Thus speech can lead to *ex post* Pareto efficiency gains by facilitating coordination on a particular equilibrium and, in the typical case where the distribution of payoffs is not neutral across equilibria, any tension in deliberation involves the equilibrium on which to coordinate.

Aragones, Gilboa, Postlewaite and Schmeidler (AGPS) observe that not all persuasive arguments involve changes in beliefs through information sharing. Rather, many arguments are by analogy, whereby the speaker makes explicit to the listener relations between known facts that the listener may not have seen. As an example, they suggest an individual, initially predisposed against US intervention in the Iraqi invasion of Kuwait, may be induced to change

her mind after an analogy is drawn between Hussein's actions toward Kuwait and Hitler's actions toward Poland. It is, AGPS claim, perfectly reasonable to assume that while both individuals are fully aware of the cases involved, only one of them has made any connection between the two.

There is a strong intuition for analogies being important for debate and it seems apparent that the setting is not one usefully captured by orthodox Bayesian theory. Nevertheless, analogic arguments still seem to be fundamentally concerned with information transmission, albeit of qualitatively different sort to that in the standard framework: the speaker in the example is pointing out a connection of which the listener was previously unaware. From this perspective, information asymmetries remain critical to any notion of consequential debate and what AGPS, along with those looking at the role of debate in coordination games, make explicit is that we are going to have look for new tools if we hope to model all of the relevant forms such information asymmetries might take. On the other hand, if AGPS are correct in claiming both that information is not the issue and that it is the relations between known sets of facts, or "cases", that form the basis of much persuasive rhetoric, then models permitting failures of logical, as well as informational, omniscience are going to prove important. For it seems that logically omniscient individuals under complete and full information are going to know all possible connections between facts.

6 Appendix

6.1 Proofs

We first derive some important threshold inequalities exploited in some of the formal arguments below.

Given a message strategy μ and debate $\mathbf{m} \in \mathcal{M}_\mu^3$, any equilibrium vote strategy v has to satisfy the pivotal voting constraints: that is, conditional on being pivotal at v , a b -biased agent i who observes a signal $s \in \{-1, 0, 1\}$ weakly prefers to vote for z rather than z' under majority rule if and only if

$$E[U(z; b)|s, \mathbf{m}, \mu, z, v_{-i}, \mathbf{vpiv}] \geq E[U(z'; b)|s, \mathbf{m}, \mu, z', v_{-i}, \mathbf{vpiv}]$$

and, by definition of being pivotal, if the individual votes z in this event then z surely wins. With this in mind, let $b = z = y$, $z' = x$ and substitute for preferences $U(\cdot; y)$ into the inequality to yield

$$\begin{aligned} & E[U(y; y)|\cdot, \mathbf{vpiv}] - E[U(x; y)|\cdot, \mathbf{vpiv}] \\ = & \Pr[Y|s, \mathbf{m}, \mathbf{vpiv}] + \Pr[X|s, \mathbf{m}, \mathbf{vpiv}]\lambda \\ & - \Pr[X|s, \mathbf{m}, \mathbf{vpiv}](1 - \lambda) \\ = & \Pr[Y|s, \mathbf{m}, \mathbf{vpiv}] + (1 - \Pr[Y|s, \mathbf{m}, \mathbf{vpiv}])(2\lambda - 1), \end{aligned}$$

where the strategy pair (μ, v_{-i}) is understood and, in obvious notation, we write $\Pr[Z|\cdot] \equiv \Pr[\omega = Z|\cdot]$, $Z \in \{X, Y\}$.¹⁸ It follows that a y -biased individual votes for y rather than for x at v only if

$$\lambda \geq \frac{1}{2} \left(1 - \frac{\Pr[Y|s, \mathbf{m}, \mathbf{vpiv}]}{1 - \Pr[Y|s, \mathbf{m}, \mathbf{vpiv}]} \right).$$

¹⁸An analogous inequality can be derived for the pivotal signaling constraints in similar fashion (although it is important in this case to fix the vote strategy v across all individuals, including the one to whom a particular constraint applies).

By Bayes rule,

$$\begin{aligned}
& \Pr[Y|s, \mathbf{m}, \mathbf{vpiv}] \\
&= \frac{\Pr[Y|s] \Pr[\mathbf{vpiv}|Y, \mathbf{m}]}{\Pr[Y|s] \Pr[\mathbf{vpiv}|Y, \mathbf{m}] + \Pr[X|s] \Pr[\mathbf{vpiv}|X, \mathbf{m}]} \\
&= \frac{\Omega(s)}{\Omega(s) + \Phi(\mathbf{m})}
\end{aligned}$$

where

$$\Omega(s) \equiv \frac{\Pr[Y|s]}{\Pr[X|s]} \text{ and } \Phi(\mathbf{m}) \equiv \frac{\Pr[\mathbf{vpiv}|X, \mathbf{m}]}{\Pr[\mathbf{vpiv}|Y, \mathbf{m}]}.$$

So a y -biased individual votes for y rather than for x at v only if

$$\lambda \geq \frac{1}{2} \left(1 - \frac{\Omega(s)}{\Phi(\mathbf{m})} \right) \dots$$

Similarly, an x -biased ($b = x$) individual who observes signal s weakly prefers to vote for x at v only if

$$\lambda \geq \frac{1}{2} \left(1 - \frac{\Phi(\mathbf{m})}{\Omega(s)} \right) \dots$$

Further, if voting strategies are symmetric and committee decision making is by majority rule, the following are easily checked:

- i.* $\Phi(m, m', m'') = \Phi(m, m'', m')$.
- ii.* $\Phi(m, m', m'') = \frac{1}{\Phi(-m, -m', -m'')}$
- iii.* $\Phi(0, m, -m) = 1$

Proof of Proposition 1 Let v^0 be any symmetric pure strategy voting equilibrium and, without loss of generality, let $i \in N$ have y -bias ($b_i = y$) and signal $s_i \in \{-1, 0, 1\}$. For $j \neq i$ and $k \neq i$, (s_j, s_k) must satisfy exactly one of the following: (a) $s_j = s_k = 0$; (b) $s_j = -s_k \neq 0$; (c) $s_j = s_k \neq 0$; (d) $s_j + s_k = -1$; (e) $s_j + s_k = 1$. Then for each $\omega \in \{X, Y\}$,

$$\Pr[a|\omega] = (1 - q)^2, \quad \Pr[b|\omega] = q^2 p(1 - p), \quad \Pr[c|\omega] = \frac{1}{2} q^2 (2p - 1);$$

and

$$\Pr[d|X] = \Pr[e|Y] = \frac{1}{2} (1 - q) q (1 - p), \quad \Pr[d|Y] = \Pr[e|X] = \frac{1}{2} (1 - q) q p.$$

Now suppose i is vote pivotal. Then $j \neq i$ and $k \neq i$ must be voting for different alternatives. Furthermore, v^0 symmetric implies that, conditional on i being pivotal, (d) can be true of (s_j, s_k) if and only if (e) can be true of (s_j, s_k) . Hence, although not every possibility in $\{(a), \dots, (e)\}$ need have strictly positive probability conditional on i vote pivotal, v^0 symmetric implies

$$\Pr[\mathbf{vpiv}|v^0, Y] = \Pr[\mathbf{vpiv}|v^0, X].$$

By Bayes rule, therefore, $\Pr[Y|s_i, v^0, \mathbf{vpiv}]$ in this case is simply

$$\frac{\Pr[\mathbf{vpiv}|v^0, Y] \Pr[Y|s_i]}{\Pr[\mathbf{vpiv}|v^0]} = \Pr[Y|s_i].$$

Substituting for $U(\cdot; y)$ into the pivotal voting constraint (with debate ignored) and collecting terms, voting for y is a best response for i if and only if:

$$\begin{aligned} & E[U(y; y)|s, y, v_{-i}^0, \mathbf{vpiv}] - E[U(x; y)|s, x, v_{-i}^0, \mathbf{vpiv}] \\ &= \Pr[Y|s, \mathbf{vpiv}] + \Pr[X|s, \mathbf{vpiv}]\lambda - \Pr[X|s, \mathbf{vpiv}](1 - \lambda) \\ &= \Pr[Y|s, \mathbf{vpiv}] + (1 - \Pr[Y|s, \mathbf{vpiv}])(2\lambda - 1) \geq 0 \end{aligned}$$

where the dependency on v_{-i}^0 is understood. It follows that a y -biased individual votes for y rather than for x at v^0 only if

$$\begin{aligned} \lambda &\geq \frac{1 - 2\Pr[Y|s_i, \mathbf{vpiv}]}{2(1 - \Pr[Y|s_i, \mathbf{vpiv}])} \\ &= \frac{(1 - 2\Pr[Y|s_i])}{2(1 - \Pr[Y|s_i])}. \end{aligned}$$

If $s_i = 1$, $\Pr[Y|s_i] = (1-p)$ and the constraint for voting y is $\lambda \geq l_1(p)$; if $s_i \leq 0$, $\Pr[Y|s_i] \geq 1/2$ and the constraint for voting y is $\lambda \geq 0$. This proves the proposition. \square

Proof of Proposition 2 Let (μ, v) be a full information equivalent separating debate equilibrium at (p, q) . Given μ is separating in common interests, it is immediate from the definition of $l_1(p)$ that $\lambda \leq l_1(p)$ is necessary and sufficient for v to satisfy the pivotal voting constraints

and be full information equivalent voting. We therefore have to check the pivotal message constraints, given $\lambda \leq l_1(p)$.

Without loss of generality, consider a y -biased individual $i \in N$. It is straightforward to check that if $s_i = -1$ then $m_i = -1$ is the unique best response to μ_{-i} . Suppose i has signal $s_i = 0$. Given (μ_{-i}, v) and $s_i = 0$, it is clear that i never strictly prefers sending message $m_i'' = 1$ rather than sending $m_i = 0$; and i is willing to send the message $m_i = s_i = 0$ rather than deviate to a speech $m_i' = -1 < s_i$ if and only if

$$E[U(z; y)|0, 0, \boldsymbol{\mu}_{-i}, v, \mathbf{mpiv}(0, -1)] \geq E[U(z'; y)|0, -1, \boldsymbol{\mu}_{-i}, v, \mathbf{mpiv}(0, -1)].$$

Given (μ, v) , i is message pivotal at $s_i = 0$ between $m_i = 0$ and $m_i' = -1$ if either (a) both j and k are uninformed, have a bias for x , and send messages $m_j = m_k = 0$, or (b) both j and k are informed, have a bias for x , and send messages $m_j = -m_k = 1$, or (c) j is uninformed and sends $m_j = s_j = 0$, k is informed and sends message $m_k = s_k = 1$, and both j, k have a bias for y . Suppose i sends the truthful message $m_i = s_i = 0$. Then the committee decision is surely x . On the other hand, if i sends the message $m_i' = -1$, the committee decision is surely y . With these remarks in mind, compute

$$\Pr[Y|s_i, \mu_{-i}, v, \mathbf{mpiv}(m, m')] = \frac{\Pr[\mathbf{mpiv}(m, m')|\mu_{-i}, v, Y] \Pr[Y|s_i]}{\Pr[\mathbf{mpiv}(m, m')|\mu_{-i}, v, Y] \Pr[Y|s_i] + \Pr[\mathbf{mpiv}(m, m')|\mu_{-i}, v, X] \Pr[X|s_i]}$$

where

$$\begin{aligned} \Pr[\mathbf{mpiv}(0, -1)|\mu_{-i}, v, Y] &\equiv \left[\frac{1}{4}(1-q)^2 + \frac{1}{2}q^2p(1-p) + \frac{1}{2}q(1-p)(1-q) \right], \\ \Pr[\mathbf{mpiv}(0, -1)|\mu_{-i}, v, X] &\equiv \left[\frac{1}{4}(1-q)^2 + \frac{1}{2}q^2(1-p)p + \frac{1}{2}qp(1-q) \right]. \end{aligned}$$

Since $\Pr[Y|s_i = 0] = 1/2$, i is willing to send $m_i = 0$ rather than $m'_i = -1$ only if

$$\begin{aligned}\lambda &\leq \frac{1 - 2\Pr[Y|0, \mu_{-i}, v, \mathbf{mpiv}(0, -1)]}{2(1 - \Pr[Y|0, \mu_{-i}, v, \mathbf{mpiv}(0, -1)])} \\ &= \frac{q(1-q)(2p-1)}{[(1-q)^2 + 2q^2(1-p)p + 2qp(1-q)]} \\ &< l_1(p).\end{aligned}$$

Now suppose, $s_i = 1$. If ever i prefers to send a message $m''_i = 0$ rather than the message $m_i = 1$, then i surely prefers to send a message $m'_i = -1$ rather than the message $m_i = 1$. So it suffices to identify when sending $m_i = 1$ is a best response for i . Given (μ, v) , i is message pivotal between $m_i = 1$ and $m'_i = -1$ at events (a') both j and k are uninformed and send messages $m_j = m_k = 0$, or (b') both j and k are informed and send messages $m_j = -m_k = 1$, or (c') j is uninformed and sends $m_j = s_j = 0$, k is informed and sends message $m_k = s_k = 1$, and both j, k have a bias for y , or (d') where j is uninformed and sends $m_j = s_j = 0$, k is informed and sends message $m_k = s_k = -1$, and both j, k have a bias for x . Then whichever event obtains, if i sends the truthful message $m_i = s_i = 1$, the committee decision is surely x and, if i sends the message $m'_i = -1$, the committee decision is surely y . Thus

$$\begin{aligned}\Pr[\mathbf{mpiv}(1, -1)|\mu_{-i}, v, Y] &\equiv \\ &[(1-q)^2 + \frac{1}{2}q^2p(1-p) + \frac{1}{4}q(1-p)(1-q) + \frac{1}{4}qp(1-q)],\end{aligned}$$

and

$$\begin{aligned}\Pr[\mathbf{mpiv}(1, -1)|\mu_{-i}, v, X] &\equiv \\ &[(1-q)^2 + \frac{1}{2}q^2(1-p)p + \frac{1}{4}qp(1-q) + \frac{1}{4}q(1-p)(1-q)].\end{aligned}$$

Rehearsing the same argument as before, *mutatis mutandis*, yields that i is willing to send $m_i = 1$ rather than $m'_i = -1$ only if

$$\begin{aligned}\lambda &\leq \frac{1 - 2\Pr[Y|1, \mu_{-i}, v, \mathbf{mpiv}(1, -1)]}{2(1 - \Pr[Y|1, \mu_{-i}, v, \mathbf{mpiv}(1, -1)])} \\ &= \frac{(2p-1)}{2p} = l_1(p).\end{aligned}$$

Therefore the binding message pivot constraint is that on the uninformed individual, in which case there exists a full information equivalent separating debate equilibrium if and only if

$$\lambda \leq \frac{q(1-q)(2p-1)}{[(1-q)^2 + 2q^2(1-p)p + 2qp(1-q)]}$$

Maximizing the RHS of this inequality with respect to q and p in turn, substituting back and taking limits appropriately yields Proposition 2(1) and 2(2), completing the proof. \square

Proof of Lemma 1 Let (μ, v) be an SP debate equilibrium and suppose the lemma is false at (μ, v) . Assume individual i is uninformed ($s_i = 0$), has bias $b_i = y$ and that $(m_i, m_j, m_k) = (-1, -1, -1)$. Given μ is semi-pooling in common interests, a message $m = -1$ is sent in debate only if the sender has a signal $s = -1$ or is y -biased and uninformed. By supposition

$$v(y, 0, -1, -1, -1) = x.$$

By μ semi-pooling, it must be that for $j, k \neq i$,

$$(s_j, s_k) \in \{(0, 0), (0, -1), (-1, -1)\}.$$

If ever $s_j = 0$, then μ semi-pooling implies j is y -biased and the supposition requires j to vote surely for x . Hence, individual i cannot be vote pivotal if $(s_j, s_k) = (0, 0)$. And if i is vote pivotal under majority rule and $(s_j, s_k) = (0, -1)$, it must be that k votes for y ; and if $(s_j, s_k) = (-1, -1)$, j, k must (given majority rule) have opposite bias. In any case, the pivotal voting constraints imply that a y -biased individual is willing to vote for x at $\mathbf{m} = (-1, -1, -1)$ if and only if

$$\lambda \leq \frac{1}{2} \left(1 - \frac{\Omega(0)}{\Phi(-1, -1, -1)} \right)$$

where, for any signal s and debate $\mathbf{m} \in \mathcal{M}_\mu^3$,

$$\Omega(s) \equiv \frac{\Pr[Y|s]}{\Pr[X|s]} \text{ and } \Phi(\mathbf{m}) \equiv \frac{\Pr[\mathbf{vpiv}|X, \mu, v, \mathbf{m}]}{\Pr[\mathbf{vpiv}|Y, \mu, v, \mathbf{m}]}$$

Given $\Omega(0) = 1$, there exist $\lambda \in (0, 1/2)$ satisfying the inequality only if $\Phi(-1, -1, -1) > 1$. Because i can be pivotal at $\mathbf{m} = (-1, -1, -1)$ given $s_i = 0$ only at the events identified above, we have

$$\begin{aligned}\Phi(-1, -1, -1) &= \frac{[\frac{1}{2}(1-q)q(1-p) + \frac{1}{2}q^2(1-p)^2]}{[\frac{1}{2}(1-q)qp + \frac{1}{2}q^2p^2]} \\ &= \frac{(1-p)(1-qp)}{p(1-q+qp)}.\end{aligned}$$

But $p \geq 1/2$ implies $\Phi(-1, -1, -1) < 1$. This contradiction proves the lemma. \square

Proof of Lemma 2 The pivotal voting constraints imply a y -biased individual is willing to vote for y at $\mathbf{m} \in \mathcal{M}_\mu^3$ given a signal s if and only if

$$\lambda \geq \frac{1}{2} \left(1 - \frac{\Omega(s)}{\Phi(\mathbf{m})} \right) ..$$

Similarly, an x -biased individual who observes signal s weakly prefers to vote for x at $\mathbf{m} \in \mathcal{M}_\mu^3$ if and only if

$$\lambda \geq \frac{1}{2} \left(1 - \frac{\Phi(\mathbf{m})}{\Omega(s)} \right) ..$$

By assumption, $p > 1/2$; hence, Ω is strictly decreasing in s . The claims now follow directly. \square

Proof Suppose $(m_i, m_j, m_k) = (1, -1, -1)$ and assume individual i who sends message $m_i = 1$ cannot be pivotal and that both agents j and k always vote for x irrespective of their bias and signal. Then it must be the case that

$$v(y, -1, -1, 1, -1) = x. \tag{*}$$

Consider such a y -biased individual who has observed signal $s = -1$ and sent message $m = -1$ and observes a split debate $(1, -1)$ and who is supposed to vote for x . There can be no event such that this agent's vote is pivotal for this debate since otherwise he must vote for y . To see this note that the observed split debate and the assumption of the SP signalling strategy

implies at most one other agent has observed the signal 1 so it follows that if there is a positive probability the agent is pivotal he should vote for y . To ensure such an agent votes for x it must be the case that his vote cannot be pivotal. But then, since the other agent sending message $m_k = -1$ is always voting for x by assumption we get the requirement that

$$v(1, 1, 1, -1, -1) = x.$$

Symmetry and anonymity implies

$$v(-1, -1, -1, 1, 1) = y. \tag{**}$$

But equations (*) and (**) imply a violation of debate monotonicity. \square

Proof of Proposition 3(1) Fix any feasible information structure (p, q) . By Proposition 1, there is a unique equilibrium in pure strategies without debate: when $\lambda < l_1(p)$, all informed individuals surely vote their signal and all uninformed individuals vote their bias; when $\lambda > l_1(p)$ all individuals vote their bias. Let v^0 denote this no-debate voting strategy and let (μ, v) be any pure strategy debate equilibrium (in undominated strategies and subject to the maintained technical refinement). Then the proposition is trivial if μ is either separating or pooling in common interest. Suppose (μ, v) is a semi-pooling debate equilibrium.

Under a semi-pooling equilibrium, all individuals offer make speeches that reveal how they would have voted without debate. For a committee decision distinct to the no-debate decision, therefore, at least one person must change their vote as a consequence of the debate. As a consequence of debate, that is, either an informed individual votes against her signal or an uninformed individual votes against her bias. Moreover, if the outcome is going to be worse with debate than without, it must be that an individual who changes her vote switches to the worse outcome. Let y be the right outcome; then the committee can make an error in common interest by choosing x following debate only if y is defined by unanimous induced preferences at \mathbf{s} . So there can be an error in common interests only if (up to permutations)

$$\mathbf{s} \in \{(-1, -1, -1), (0, -1, -1), (1, -1, -1), (-1, 0, 0)\}.$$

We consider each case in turn. Throughout, the SP debate equilibrium (μ, v) is fixed and taken as understood. Let $\mathbf{v} = (v_1, v_2, v_3) \in \{x, y\}^3$ denote a list of votes.

(I) $(s_1, s_2, s_3) = (-1, -1, -1)$. Under v^0 all individuals vote for y and, given the signal profile and definition of μ , the debate must be $\mathbf{m} = (-1, -1, -1)$. Consequent on \mathbf{m} , therefore, there are essentially two possible voting outcomes $\mathbf{v} = (v_1, v_2, v_3)$ that result in a mistake:

(a) $\mathbf{v} = (x, x, x)$. In this case, all agents are supposed to vote for x . Then, by Lemma 1, $v(y, 0, -1, -1, -1) = y$ and so, by signal monotonicity (Lemma 2.1), $v(y, -1, -1, -1, -1) = y$. Hence, $b_i = x$ for all i , so we must have $v(x, -1, -1, -1, -1) = x$. Consider any x -biased agent who is supposed to vote for x here. For this debate, there is a positive probability of being pivotal and $\Phi(-1, -1, -1)$ is defined. Specifically,

$$\begin{aligned} \Phi(-1, -1, -1) &= \frac{\Pr[\mathbf{vpiv}|X, \mu, v, -1, -1, -1]}{\Pr[\mathbf{vpiv}|Y, \mu, v, -1, -1, -1]} \\ &= \frac{\frac{1}{2}q(1-q)(1-p) + \frac{1}{2}q^2(1-p)^2}{\frac{1}{2}q(1-q)p + \frac{1}{2}q^2p^2} \\ &= (1-p) \frac{1-qp}{p(1-q+qp)}. \end{aligned}$$

Now $\Omega(-1) = p/(1-p)$ so the agent is willing to vote for x only if

$$\begin{aligned} \lambda &\geq \frac{1}{2} \left(1 - \frac{(1-p)(1-p)(1-qp)}{p(1-q+qp)} \right) \\ &= \frac{1}{2} \frac{(2p-1)(1-qp(1-p))}{p^2(1-q+qp)} \\ &> 1 \end{aligned}$$

But then, by Proposition 1, the no-debate equilibrium v^0 requires all individuals to vote their bias which makes x the right outcome and contradicts the supposition of an error here.

(b) $\mathbf{v} = (x, x, y)$ or $\mathbf{v} = (y, x, x)$. For either of these possibilities to constitute equilibrium behaviour here requires $v(x, 0, -1, -1, -1) = x$. But then the same logic as for (a) applies and we obtain a contradiction.

(II) $(s_1, s_2, s_3) = (0, -1, -1)$. It must be the case that the uninformed agent is x -biased since otherwise all the messages are -1 and the argument in case I(a) applies. As indicated (and without loss of generality), assume $s_1 = 0$ and therefore, by μ semi-pooling, $(m_1, m_2, m_3) = (1, -1, -1)$. By Lemma 3, if 1 is not pivotal then the right decision must be made. So if there is an error, 1 must have positive probability of being pivotal here. And for 1's vote to be pivotal it must be the case that individuals 2 and 3 are different (if they have the same bias, send the same message and observe the same messages from others then they vote the same way). But since $m_2 = m_3 = -1$, μ semi-pooling implies $s_j \leq 0$, $j = 2, 3$, and moreover $m_j = -1$ and $s_j = 0$ imply $b_j = y$. There can be only one such agent $j \in \{2, 3\}$ in the pair if a vote is pivotal, so the other agent, k , must have observed $s_k = -1$. In this case the uninformed x -biased agent, $i = 1$, who sends message $m_1 = 1$ must believe that, conditional on being pivotal, exactly one other agent k has observed signal $s_k = -1$. In which case, by $l_1 > \lambda$, individual $i = 1$ prefers to vote for y .

(III) $(s_1, s_2, s_3) = (1, -1, -1)$. Then $(m_1, m_2, m_3) = (1, -1, -1)$. By the same argument as for (II), if a vote is pivotal it must be the case that agents 2 and 3 are different. Consequently, at least one of these agents must have observed the signal $s_j = -1$; let $j = 2$. It follows that agent 1 cannot be y -biased: for if $b_1 = y$, then he would vote for y conditional on being pivotal because he knows the third agent has not seen $s = 1$. Either $b_2 = x$ or $b_3 = x$; assume $b_2 = x$. Since 2 and 3 must be different, it must be that $b_3 = y$ and $s_3 \leq 0$. It follows that $k = 3$ votes for y , implying both individuals 1 and 2 are voting for x . Now individual 1 is x -biased and $s_1 = 1$; therefore 1 prefers to vote for x only if

$$\lambda \geq \frac{1}{2} \left(1 - \frac{\Phi(1, -1, -1)}{\Omega(1)} \right).$$

Because $\lambda < l_1$ the above inequality can be satisfied only if $\Phi(1, -1, -1) > 1$ but, given the voting strategies described above,

$$\Phi(1, -1, -1) = \frac{p(q^2(1-p)^2 + \frac{1}{2}q(1-q)(1-p))}{(1-p)(q^2p^2 + \frac{1}{2}q(1-q)p)} < 1$$

since $p > 1/2$.

(IV) $(s_1, s_2, s_3) = (0, 0, -1)$. By μ semi-pooling, if $b_1 = b_2 = y$ then $m_1 = m_2 = -1$ and, therefore, by Lemmas 1 and 2(1), both individuals surely vote y . On the other hand, because informed individuals vote their signal and uninformed individual vote their bias when $\lambda < l_1$ and there is no debate, if $b_1 = b_2 = x$ then the decision under no debate is x and evidently a debate equilibrium cannot do worse. To obtain a mistake therefore, it is necessary that $b_1 \neq b_2$; without loss of generality, assume $b_1 = x$ and $b_2 = y$. Then the debate is $(m_1, m_2, m_3) = (1, -1, -1)$. By Lemma 3, if there is an error there must be positive probability of $i = 1$ being pivotal at this debate. But then individuals 2 and 3 must be voting differently and therefore, by $m_2 = m_3$ and $M_{-2} = M_{-3}$, have different biases. By semi-pooling debate, $m_j = -1$ implies either $s_j = 0$ and $b_j = y$ or $s_j = -1$. Hence, individual 1 knows surely that $s_2 + s_3 \leq -1$ in which case, since $\lambda < l_1$ and $s_1 = 0$, 1 surely votes y .

Because (I) through (IV) exhaust the possibilities for errors in common interest, we are done. \square

The following lemma is useful for proving Proposition 3(2). Let μ be the semipooling message strategy and define the (SP equilibrium path) voting profile R by:

	s_i		
M_{-i}	-1	0	1
-2	y,y	y,y	y,x
0	y,y	x,y	x,x
2	y,x	x,x	x,x

where the entries in each cell describe the vote pair, $[v(y, s_i, \mathbf{m}), v(x, s_i, \mathbf{m})]$ and $\mathcal{M}_\mu = \{-1, 1\}$.

Lemma 4 *If (μ, v) and (μ, v') are both symmetric and debate monotonic semipooling debate equilibria under which uninformed individuals vote against their bias on hearing a split debate.*

Then, along the equilibrium path, $v = v'$ and equilibrium voting decisions are described by the profile R .

Proof. Consider equilibrium path voting behaviour. By hypothesis, along the equilibrium path uninformed individuals vote against their bias on hearing a split debate $(-1, 1)$; that is,

$$v(y, 0, -1, -1, 1) = x \text{ and } v(x, 0, 1, -1, 1) = y \quad (1)$$

By (1) and debate monotonicity,

$$v(y, 0, -1, 1, 1) = x \text{ and } v(x, 0, 1, -1, -1) = y \quad (2)$$

By (1) and Lemma 2 (signal monotonicity),

$$v(y, 1, 1, -1, 1) = x \text{ and } v(x, -1, -1, -1, 1) = y \quad (3)$$

By (3) and debate monotonicity,

$$v(y, 1, 1, 1, 1) = x \text{ and } v(x, -1, -1, -1, -1) = y \quad (4)$$

By (3) and Lemma 2 (bias monotonicity),

$$v(x, 1, 1, -1, 1) = x \text{ and } v(y, -1, -1, -1, 1) = y \quad (5)$$

Similarly, by (4) and Lemma 2 (bias monotonicity),

$$v(x, 1, 1, 1, 1) = x \text{ and } v(y, -1, -1, -1, -1) = y \quad (6)$$

And by Lemma 1,

$$v(y, 0, -1, -1, -1) = y \text{ and } v(x, 0, 1, 1, 1) = x \quad (7)$$

There remain two (equilibrium path) decisions to be determined; specifically, for each $z \in \{x, y\}$

$$v(z, -1, -1, 1, 1) \text{ and } v(z, 1, 1, -1, -1)$$

Suppose first that individual $i \in N$ has $v(x, -1, -1, 1, 1) = y$. Then both of the other two committee members observe a split debate. Hence, (1) through (7) imply there exists a unique event at which i 's vote is pivotal: there exists an uninformed ($s_j = 0$) x -biased individual j who has sent message $m_j = 1$, hears a split debate and votes for y ; and there exists an informed ($s_k = 1$) individual k who has sent message $m_k = 1$, hears a split debate and votes for x . But then i 's unique undominated vote decision is to vote for x . Therefore,

$$v(x, -1, -1, 1, 1) = x$$

in which case, by symmetry

$$v(y, 1, 1, -1, -1) = y$$

Now suppose that individual $i \in N$ has $v(y, -1, -1, 1, 1) = x$. Then i 's vote is pivotal in exactly the same case as above; but since i is now presumed y -biased, we conclude

$$v(y, -1, -1, 1, 1) = y$$

so by symmetry

$$v(x, 1, 1, -1, -1) = x.$$

And because there exist no further unspecified equilibrium path voting decisions, this proves the lemma. \square

Proof of Proposition 3(2) To prove the result, it suffices to show there exists a symmetric and debate monotonic semipooling debate equilibrium at (λ, p, q) in which uninformed individuals vote against their bias on hearing a split debate only if there exists a separating debate equilibrium at (λ, p, q) . A necessary condition for any such semipooling debate equilibrium to exist is for the pivotal constraints to hold along equilibrium path. So consider a y -biased individual who has signal $s = 0$, sends message $m = -1$ and observes a split debate $(-1, 1)$. By hypothesis, $v(y, 0, -1, -1, 1) = x$. By Lemma 4, the unique equilibrium voting path in any such semipooling debate equilibrium is described by the strategy R, defined above. Therefore,

there are three events at which the vote of an uninformed individual i , having sent message $m_i = -1$ and observed a split debate $(m_j, m_k) = (-1, 1)$, is pivotal:

Either both j and k are uninformed: j is y -biased, $M_{-j} = 0$ and votes x ; k is x -biased, $M_{-k} = -2$ and votes y ;

Or j is uninformed, y -biased and votes x given $M_{-j} = 0$; k is informed with $s_k = 1$, y -biased and votes y given $M_{-k} = -2$;

Or j is informed with $s_j = -1$, y -biased and votes y given $M_{-j} = 0$; k is informed with $s_k = 1$, x -biased and votes x given $M_{-k} = -2$.

Substituting into the pivotal voting constraint and rearranging yields $v(y, 0, -1, -1, 1) = x$ can be an undominated best response only if

$$\lambda \leq \lambda_R \equiv \frac{q(1-q)(2p-1)}{2[(1-q)^2 + qp(1-q) + 2q^2p(1-p)]}.$$

>From the proof to Proposition 2, the binding pivotal message constraint for the separating debate equilibrium requires

$$\lambda \leq \lambda_S \equiv \frac{q(1-q)(2p-1)}{[(1-q)^2 + 2qp(1-q) + 2q^2p(1-p)]}.$$

Hence, at any information structure $(p, q) \in (\frac{1}{2}, 1) \times (0, 1)$,

$$\lambda_R < \lambda_S \Leftrightarrow 0 < (1-q)^2 + 2q^2p(1-p)$$

which is obviously true. This fact proves the result. \square

Proof of Proposition 4 Suppose by way of contradiction that (μ, ν) is a separating debate equilibrium. Then no new information is revealed by the fact that a vote is pivotal and, therefore, the sincere voting strategy is weakly dominant; in particular, given $\mathbf{m} \in \mathcal{M}_\mu^3$, $\mathbf{m} = \mathbf{s}$ and $\lambda < l_1$ implies

$$v(y, s_i, \mathbf{m}) = \begin{cases} y & \text{if } s_i + M_{-i} \leq 0 \\ x & \text{otherwise} \end{cases} \quad \text{and} \quad v(x, s_i, \mathbf{m}) = \begin{cases} y & \text{if } s_i + M_{-i} < 0 \\ x & \text{otherwise} \end{cases}$$

with the sincere strategy being defined analogously for $\lambda > l_1$. We show that an uninformed ($s = 0$) individual with y -bias strictly prefers to send message $m = -1$ to message $m = 0$, thus violating the relevant pivotal message constraint for μ separating in common interests. Because the event that an individual is message or vote pivotal under unanimity implies that both the other committee members are making similar decisions, to prove the result it suffices to check the case $\lambda < l_1$. Given $\lambda < l_1$, μ separating and v sincere, an uninformed y -biased individual i is message pivotal between $m_i = 0$ and $m'_i = -1$ under unanimity rule with *status quo* x if and only if (a) $(m_j, m_k) = (s_j, s_k) = (0, 0)$ and at least one of j, k has an x -bias, or (b) $(m_j, m_k) = (s_j, s_k) = (-1, 1)$ and at least one of j, k has an x -bias. Therefore, recalling

$$\Pr[Y|s_i, \mu_{-i}, v, \mathbf{mpiv}(m, m')] = \frac{\Pr[\mathbf{mpiv}(m, m')|\mu_{-i}, v, Y] \Pr[Y|s_i]}{\Pr[\mathbf{mpiv}(m, m')|\mu_{-i}, v, Y] \Pr[Y|s_i] + \Pr[\mathbf{mpiv}(m, m')|\mu_{-i}, v, X] \Pr[X|s_i]}$$

and

$$\begin{aligned} \Pr[\mathbf{mpiv}(0, -1)|\mu_{-i}, v, Y] &\equiv \left[\frac{3}{4}(1-q)^2 + \frac{3}{2}q^2p(1-p)\right], \\ \Pr[\mathbf{mpiv}(0, -1)|\mu_{-i}, v, X] &\equiv \left[\frac{3}{4}(1-q)^2 + \frac{3}{2}q^2(1-p)p\right]. \end{aligned}$$

By v sincere, in either event (a) or (b), individual i votes her bias whatever message she delivers. On the other hand, both j and k vote surely for y in these events if $m'_i = -1$ and at least one of them votes for x otherwise. Therefore, since $\Pr[Y|s_i = 0] = 1/2$, substituting into the relevant message pivot constraint implies that i is willing to send $m_i = 0$ rather than $m'_i = -1$ only if

$$\lambda \leq \frac{1 - 2 \Pr[Y|0, \mu_{-i}, v, \mathbf{mpiv}(0, -1)]}{2(1 - \Pr[Y|0, \mu_{-i}, v, \mathbf{mpiv}(0, -1)])} = 0$$

which contradicts $\lambda > 0$. \square

6.2 Refinement and derivations

In this Appendix we define the technical (trembles) equilibrium refinement and describe the approach to identifying particular classes of equilibria discussed in the paper. With some abuse

to the notation in the text, it is useful to begin by redefining some variables. Fix an individual $i \in N$ and hereafter suppress any individual-specific subscripts. Assume also that committee decision making is by majority rule; similar constructions apply to the case of unanimity.

Let $b = -1$ if the individual's bias is for y and let $b = 1$ if her bias is for x . Similarly, let $\omega = -1$ if the state of the world is Y and let $\omega = 1$ if the state is X . Let m, m' etc denote the relevant individual's message in any debate and let θ, ρ denote the messages of the other two committee members; by convention, when writing any debate $\mathbf{m} = (m, \theta, \rho) \in \mathcal{M}^3 = \{-1, 0, 1\}^3$, the relevant individual's message is always listed first.

6.2.1 Refinement

For any profile $(b, s, m, \theta, \rho) \in \{-1, 1\} \times \{-1, 0, 1\}^4$ and any $z \in \{x, y\}$, let

$$v(b, s, m, \theta, \rho) \in [0, 1]$$

be the probability that an individual with bias b and signal s , having sent debate message m and heard messages θ, ρ , votes for alternative y . Since there is no abstention, $1 - v(b, s, m, \theta, \rho)$ is the probability that the individual votes for x . Similarly, for any $(b, s) \in \{-1, 1\} \times \{-1, 0, 1\}$ and any $m \in \{-1, 0, 1\}$ let

$$\mu(b, s, m) \in [0, 1]$$

be the probability that an individual with bias b and signal s sends debate message m ; by assumption, $\sum_{m \in \{-1, 0, 1\}} \mu(b, s, m) = 1$.

With this notation, an anonymous message strategy is a triple

$$\mu = (\mu(b, s, -1), \mu(b, s, 0), \mu(b, s, 1))$$

and an anonymous voting strategy is simply a pair of vote-probabilities sufficiently described by

$$v = v(b, s, m, \theta, \rho).$$

Let (μ, v) be an equilibrium in pure strategies, i.e. the adding up constraints are satisfied and $\mu(b, s, m) \in \{0, 1\}$ for each message m and $v(b, s, m, \theta, \rho) \in \{0, 1\}$. It is irrelevant to apply any refinement to separating debate equilibria as there is no out-of-equilibrium behaviour to worry about. So assume for this discussion that μ is semi-pooling in common interests. Then the only messages supposed to be sent in equilibrium are $m = -1$ and $m' = 1$. However, if ever a message $m = 0$ is observed in a semi-pooling equilibrium, we assume all individuals surely identify the (out of equilibrium) message with the message $m = -1$. Now consider the voting strategy v .

Given v , define a perturbed voting strategy component-wise by

$$v(b, s, m, \theta, \rho; \varepsilon) = \begin{cases} 1 - \varepsilon & \text{if } v(b, s, m, \theta, \rho) = 1 \\ \varepsilon & \text{otherwise} \end{cases},$$

where $\varepsilon > 0$ and small. For each ε , let

$$v(\varepsilon) = ((v(b, s, m, \theta, \rho; \varepsilon)))$$

Then the pure strategy pair (μ, v) survives the technical refinement (individual-invariant trembles) if

$$\lim_{\varepsilon \rightarrow 0} (\mu, v(\varepsilon)) = (\mu, v).$$

6.2.2 Derivations

For each rule and any conjectured equilibrium strategy pair (μ, v) , we have to identify the message pivot and vote pivot constraints for each possible event. Typically, there are a great many such events to check. To see why, consider an individual with bias b and signal s who is supposed to send message m in debate; then there are two possible deviations from m and, for each deviation, there are multiple distinct pivot events. And given a realized debate, the vote pivot constraints for the individual have to be checked for each possible message he might have sent, both in and out of equilibrium, and for each possible debate that might be realized.

Finally, this family of constraints has to be checked for consistency. Not surprisingly, the algebra becomes very cumbersome and tedious very rapidly. We therefore wrote a program using the Maple V symbolic manipulation package in Scientific Workplace 4.1 to identify the relevant pivot events and do the algebra. This is available from the authors on request.

6.3 Semi-pooling equilibria for majority rule

Table 4 describes the voting strategies, v , for the (symmetric) SP debate equilibria that exist. As indicated, each column headed by a bold-faced letter is a particular SP debate equilibrium and the voting behaviour is described in terms of an individual's signal, s_i , and the sum of the others' debate messages, M_{-i} ; the y -biased individual's prescribed vote is listed first.

		SP debate equilibria, [$v(y, s_i, \mathbf{m}), v(x, s_i, \mathbf{m})$]							
s_i	M_{-i}	A	B	C1	C2	R	M1	M2	U
-1	-2	y,y	y,y	y,y	y,y	y,y	y,y	y,y	y,x
	0	y,y	y,y	y,y	y,y	y,y	y,x	y,x	y,x
	2	y,y	y,x	y,x	x,x	y,x	y,x	y,x	y,x
0	-2	y,x	y,x	y,y	y,y	y,y	y,y	y,x	y,x
	0	y,x	y,x	y,x	y,x	x,y	y,x	y,x	y,x
	2	y,x	y,x	x,x	x,x	x,x	x,x	y,x	y,x
1	-2	x,x	y,x	y,x	y,y	y,x	y,x	y,x	y,x
	0	x,x	x,x	x,x	x,x	x,x	y,x	y,x	y,x
	2	x,x	x,x	x,x	x,x	x,x	x,x	x,x	y,x

Table 4: SP debate equilibrium voting strategies under majority rule

o SP equilibrium A has all informed individuals surely vote their signals and the uninformed always vote their bias.

- Under B, uninformed individuals always vote their bias and only those informed individuals with a signal *against* their bias who are marginally influenced in debate.
- In SP equilibrium C1, the informed agents' voting behaviour is the same as under B but now the uninformed individuals can be influenced in debate.
- C2 is the 'most influential' of the SP equilibria available; along the equilibrium path, every individual's voting behaviour in C2 coincides with that in a separating debate equilibrium. Indeed, this last property is true of all uninformed voters in both C1 and C2 SP equilibria.
- In the equilibrium R, informed individuals are marginally influenced in debate exactly as in B and C1; the singular feature of R is the voting behaviour of the uninformed. Although they make speeches in support of their bias, they vote *against* their bias unless (like the informed individuals) they hear two speeches in debate that favour their bias.
- Debate in both equilibria M1 and M2 has very little impact. Under M1 the uninformed are influenced in debate but the informed are only marginally influenced in debate: they vote their bias unless they have a signal against their bias *and* both of the other speeches in debate support the alternative favoured by that signal. And although the voting behaviour of informed agents is the same in M2 as in M1, uninformed individuals under M2 surely vote their bias irrespective of the debate.
- Finally, debate in U is informative but utterly uninfluential. Despite the message strategy being semi-pooling in common interest, all individuals always vote their bias.

Figure 6 describes those information structures for which the various pure strategy SP equilibria exist; the dotted outline in the figure is the boundary for the full information equivalent separating equilibrium.

Figure 6 here

6.4 No-debate equilibria for unanimity rule

Recall the pure strategy no-debate equilibria under unanimity rule from Table 1 of the text:

No-debate equilibria, [$v(y, s_i, \emptyset), v(x, s_i, \emptyset)$]					
s_i	1	2	3	4	5
-1	y,y	y,y	y,y	y,y	y,x
0	y,y	y,y	y,x	y,x	y,x
1	x,x	y,x	x,x	y,x	y,x

For any pair of pure strategy no-debate equilibria, $\mathbf{a}, \mathbf{b} \in \{\mathbf{1}, \dots, \mathbf{5}\}$, let $\mathbf{a-b}$ denote a mixed voting strategy profile that involves individuals randomizing between their respective vote decisions under pure strategies \mathbf{a} and \mathbf{b} (in fact, only one type of person is ever required to use a non-degenerate lottery). Figure 7 describes the distribution of voting equilibria under unanimity rule with no debate, assuming a typical value, $\lambda = 1/10$.

Figure 7 here

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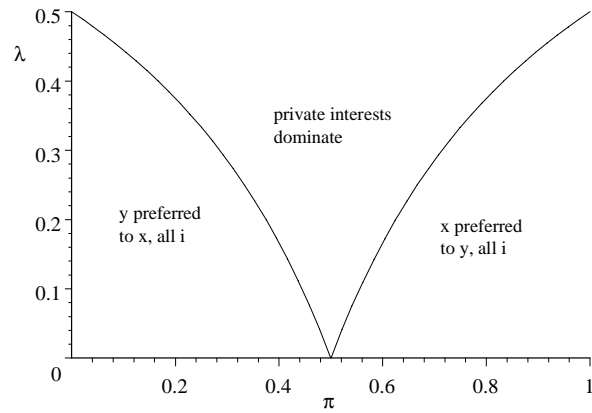


Figure 1: Induced preferences

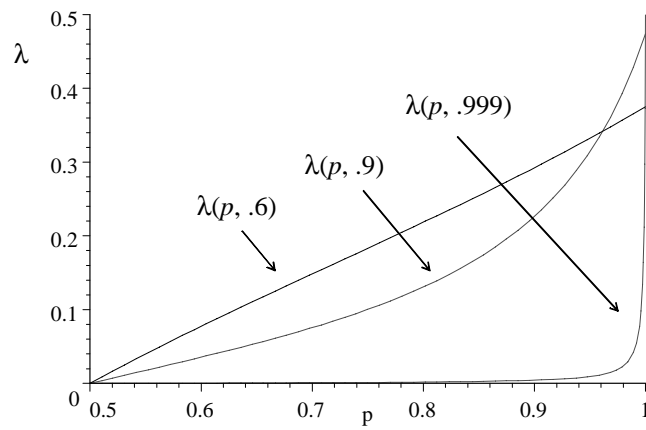


Figure 2: $\lambda(p, q)$ for $q \in \{.6, .9, .999\}$

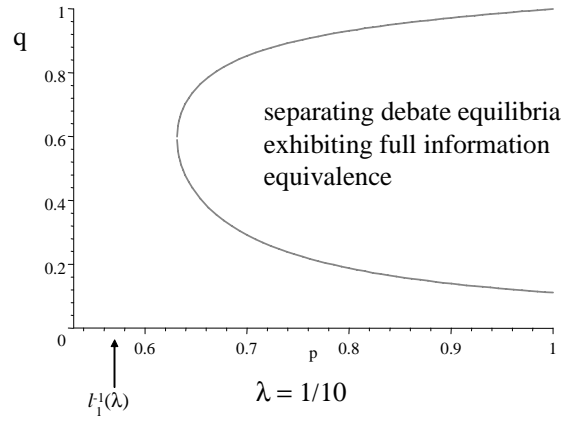


Figure 3: Separating debate equilibria

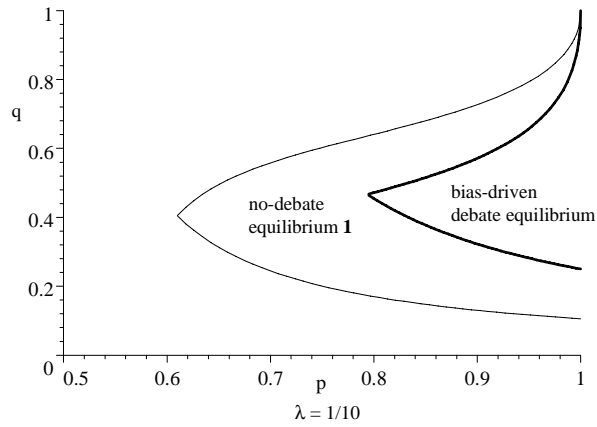


Figure 4: Bias-driven debate equilibrium $(\bar{\mu}, \bar{v})$

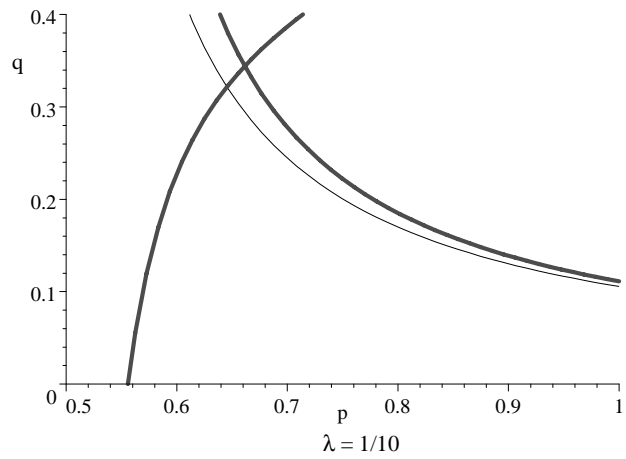


Figure 5: Bias-driven debate equilibrium $(\hat{\mu}, \hat{v})$

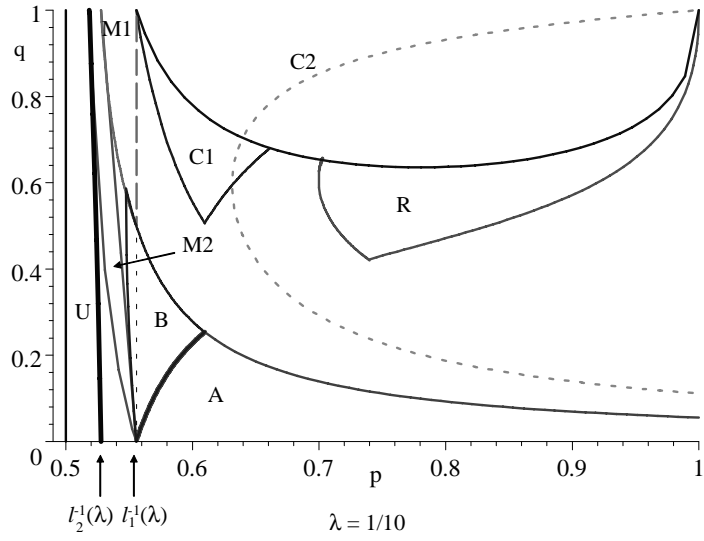


Figure 6: Semi-pooling debate equilibria

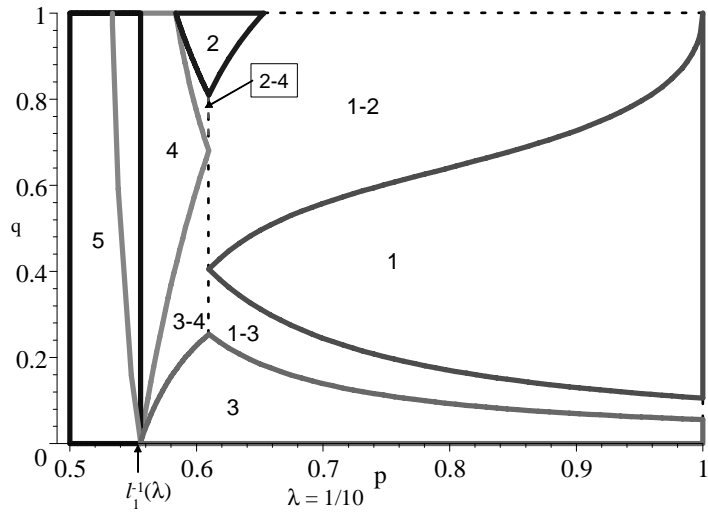


Figure 7: Voting equilibria under unanimity with no debate