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# **Uniform Sharing**

# IFS Working Paper W19/30

Alex Wolf



# Uniform Sharing

### Alex Wolf\*

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#### Abstract

Collective models have become the go-to framework for intra-household allocations. Available empirical collective models are built for fixed sets of household members and accommodate diversity in household structures with difficulty. Individual-level data on food consumption from Bangladesh provides an opportunity to build a parsimonious model that applies naturally to households of all shapes and sizes. An intuitive assumption about how allocations change with household composition makes this possible. It also replaces previous models' identifying restrictions on individual demands across members or household structures and removes the need to classify members into types such as children or men. The resulting estimates allow a detailed and precise description of the age profiles of resource allocation using a new measure that summarizes an individual's expected consumption relative to others as a function of her characteristics. Estimates suggest that nearly a third of all variation in individual consumption is found within, rather than between households.

JEL Codes: D13

Keywords: collective model, intra-household allocation, resource share, inequality, poverty

### 1 Introduction

Governments, aid groups and international organizations expend considerable funds and effort to improve the lot of the worst off and to alleviate extreme poverty. To this end, an ability to reliably identify the poorest is of great importance. At the same time, there is increasing awareness that living standards can vary within households as well as between them (e.g. World Bank, 2018) and that such intra-household inequalities in consumption can be targeted by policy interventions.

But the measurement of consumption at the individual level is not straightforward. One reason is that members of households consume a great many things jointly. A common example of this is housing, where considerable savings can be achieved by living with others. In fact, such joint consumption is sometimes considered to be one of the reasons we observe multi-person households at all, others being the raising of children and, of course, love. But the existence of such *public goods* is not the only difficulty to overcome in order to apportion household consumption among individual members.

Next on the list is data availability. Even for *private goods* such as food or clothing, households form purchasing units. They buy jointly and then divide up quantities informally. Since available consumption data tends to effectively be data on purchases, it is not known which members consume how much of such goods.

<sup>\*</sup>The Institute for Fiscal Studies, 7 Ridgmount Street, London WC1E 7AE (email: alex.wolf@ifs.org.uk). I wish to thank Krishna Pendakur, Michel Azulai, Bram De Rock, Valérie Lechene, Caterina Mauri, Ryan Tierney, Jacob Penglase, Rachel Cassidy and Antonio Dalla Zuanna for extremely valuable input and advice.

The exception to this rule comes in the form of exclusive goods, which can be assumed to not be consumed by at least one member, and *assignable goods*, for which the allocation to the various members is known to some degree. Alcohol is an example of an exclusive good because children can be assumed not to consume any and clothing is commonly assignable in survey data. This assignability has its limits, though. Typically it is known how much clothing is consumed by men, women or children, but especially in developing countries, the vast majority of households have multiple men, women or children. This paper takes advantage of relatively recent data from Bangladesh, where respondents were asked to provide detailed information on individual food consumption, yielding an individually assignable good. Still, the presence of public, semi-public and unassigned private consumption means that important aspects are unobserved, making this a job for a *structural model* of household decision making.

Such models face a difficult task: To disentangle resource allocation from the effects of differences in preferences and joint consumption, where the latter in particular can have a range of effects on decisions. This is because jointly consumed goods effectively become cheaper for household members to consume, resulting in price effects, while the overall savings induce an income effect.

*Collective models* of the household (Chiappori, 1988, 1992) have become the standard approach to this problem. Their foundational assumption is that outcomes of the household decision process are Pareto efficient, meaning that any member can be made better off only at the expense of at least one other member. The household can then be thought of as maximizing a weighted sum of the members' utility functions. The weights, called *bargaining weights* may notably be functions of prices and incomes, resulting in an objective function that is not itself a well-behaved utility function.

Key to the success of these models is the elegant simplicity of this assumption. Previous models had either been unitary, treating the household as a maximizer of a single utility function, or they directly specified the decision process. The former did not allow much insight into the household (see Becker, 2009, for an exploration of what can be done using this framework), while the latter remained highly dependent on the process in question. At the same time, collective models have testable implications beyond a rejection of unitary behavior (Browning and Chiappori, 1998) and have generally fared well in such tests.

To obtain a convincing measure of overall resource allocation, a way has to be found to divide up public and semi-public consumption. This can be achieved by specifying a *consumption technology*, which converts such goods into private good equivalents. Using this setup, Browning et al. (2013) can identify *resource shares*, that is, the fractions of household resources allocated to each member. Their success spurned a number of high-profile innovations built on this strategy and made these structural models attractive to researchers investigating intra-household welfare and poverty.

These include Lewbel and Pendakur (2008); Bargain and Donni (2012), where individual demands are specified as Engel curves in a context of no price variation. As in Browning et al. (2013), identification is achieved by restricting preferences to be invariant to the formation of a couple's household and thus relies on the presence of singles and on their comparability to individuals in larger households. This limitation can be circumvented by relying on the abovementioned assignable goods. Dunbar et al. (2013) use such goods and achieve identification by restricting individual demands either across different *household structures*, in their case distinguished by the number of children, or across members of the same household. The relative simplicity of their model, coupled with the fact that singles are not needed has made it a popular choice for applications in developing country contexts (e.g. Calvi (2017); Brown et al. (2018)). Though these models are much more tractable and less onerous in data requirement than Browning et al. (2013), implementation continues to be difficult in practice. Authors have to contend with large standard errors, unstable estimates and difficult optimization procedures.<sup>1</sup>

Currently, all parametric collective models are estimated as systems of equations where separate equations are needed for each type t of household member. For instance in Bargain et al. (2014), these are men, women and children. These types have their origin in the names of members in theoretical models. They also make different households comparable when bringing a model to the data, an essential point since authors have to make do with cross-sections or, at best, short panels. In many cases, the motivation for types is also driven by data availability. Assumptions and definitions on which goods are exclusive or assignable normally run along the same lines.

But in the presence of data with true individual assignability, the system of types can be an obstacle. Such data is indeed available from the Bangladeshi BIHS, where respondents were asked to detail the meals and snacks eaten by household members over a 24 hour period, along with the ingredients used and how, by weight, each meal or snack was distributed. This permits the computation of individually assignable food expenditures.

The individually assignable good provides an opportunity to estimate more finely grained resource shares. Extending the standard approach to more types however, comes with complications. Most immediately, an equation must be added to the system for each new type, increasing the dimensionality of the model. In addition, the model quickly becomes even more complex when accounting for a range of household structures. Especially in a developing country such as Bangladesh, households come in all shapes and sizes. This means that multiple members will typically be of the same type, and one has no choice but to to assume that they share equally among themselves. The alternative is to discard any households that do not have exactly one member of each type. A system with four types using the former approach is set up by Brown et al. (2018), who also use the BIHS data on individual food expenditures and allow up to five households members of a given type.

Resource shares are typically specified for each type either as linear indexes in observables or as logistic transformations of such indexes. In order to allow them to vary with household structure, the resource shares have to be made dependent on this structure in a flexible way, while also ensuring adding-up. Consider starting from a household that has one of each type of member and simply adding one woman. Clearly the women's resource share has to account for this, most likely by increasing collectively and decreasing on a per-woman basis. One may consider adding a dummy to the specification for each possible number of women. Since shares must sum to one, the other members' shares must also adapt. It may therefore seem sensible to include a fixed effect for each possible household structure in each type's resource share. But such an approach is both very costly and still not quite adequate because any covariates that are also included would remain fixed to a uniform effect size irrespectively of household structure. Lastly, these covariate effects need to be switched off entirely for households that do not contain any members of a given type, a case that becomes more common the more types are used.

To avoid these problems and to achieve considerable gains in parsimony, this paper takes a different approach. Rather than dividing household members into types, the model is written at the level of the individual. This is made possible by a new feature called uniform sharing, by which the resource share  $\eta_i$  of individual *i* is specified directly as a function of *household* composition, i.e. the set of members of the household and their attributes<sup>2</sup>. Individual demands for assignable goods are written as Engel curves as in Dunbar et al. (2013).

The new structure yields several advantages. First, identification is achieved across members of different but identically structured households. It is not necessary to restrict individual

<sup>&</sup>lt;sup>1</sup>See Tommasi and Wolf (2018); Wolf (2016) for alternative approaches to redressing this problem.

<sup>&</sup>lt;sup>2</sup>Throughout, I distinguish between a household's composition and its structure, which is implied by the composition but contains less information, such as a list of member types or simply the household size.

demands either across household members or across household structures, as was the case in previous models. Second, the model collapses to a single, individual-level equation rather than a system of multiple equations with cross-equation restrictions.<sup>3</sup> This makes estimation much more tractable with considerable gains in precision. Third, it naturally allows the inclusion of individuals from arbitrarily composed households, wherein each individual can still have a different resource share. The robust structure even allows the inclusion of households where individual assignable goods are observed for only a random subset of household members.

## 2 The Model

To introduce the model, this section starts by summarizing the model due to Browning et al. (2013) (hereafter BCL) on which the present paper is built. While BCL consider only twomember households, classified into husbands and wives, the version of their model sketched here is modified to include an arbitrary list of members who do not need to be cast into types. Notation also centers around the individual i, preparing the ground for an empirical model at the level of the individual.

The role of households in the model is threefold: First, they act as purchasing units, buying the goods that household members consume. This has an important consequence for the econometrician because, for the most part, consumption data consists of information on these joint purchases rather than on individual consumption. Second, households create economies of scale for their members through jointness of consumption. This will be modeled using a consumption technology by which purchased quantities are transformed into larger, shadow quantities for the members' consumption. Third, households determine the intra-household allocation through an unknown bargaining process which is assumed to be Pareto-efficient.

Denote by h the household of which individual i is a member and let  $H_i$  be the set of indices of the members of h, such that in particular  $i \in H_i$  and the household size is  $n = |H_i|$ . For ease of exposition, individual as well as household characteristics are suppressed in the notation, though they will become relevant and be explicitly introduced in the empirical model. For the moment, the subscript h will also be omitted, as only one household is considered.

The household buys K goods on the market at prices  $p = (p^1, \ldots, p^K)'$ . Let  $q = (q^1, \ldots, q^K)'$ be the vector of quantities purchased by the household. Total household expenditure is y = q'p. At the individual level,  $x_i = (x_i^1, \ldots, x_i^K)'$  denotes the vector of quantities consumed by the individual *i*. These quantities are important because they determine individual utility but they are not generally observable. Each individual has a utility function in own consumption  $U_i(x_i)$ . It is assumed that this function is the same for all individuals up to observables such as age, education, sex and the like.

A consumption technology describes how jointness of consumption leads to economies of scale in the household, meaning that there are savings from living in a household composed of multiple people, rather than alone. The technology is linear Barten (1964) scaling which transforms purchased quantities q into weakly larger private good equivalent quantities:

$$x = \sum_{j \in H_i} x_j = A^{-1}q \tag{1}$$

The matrix A is diagonal and contains the Barten scales  $a_k \leq 1$  for each good k. Values of  $a_k$  account for goods that are consumed, at least in part, jointly by the household members. A good can be anywhere between a purely private good, such as food, which is not consumed jointly and would have an associated Barten scale of  $a_k = 1$  to a good that is entirely public.

<sup>&</sup>lt;sup>3</sup>A single assignable good is used here, for K assignable goods there will be K equations.

Though such goods are perhaps hard to find in reality, rent and heating are often mentioned as examples. Such a good would have a Barten scale as small as  $a_k = \frac{1}{n}$ .

An important ingredient to understanding the effect of the consumption technology as well as the ways in which it is restrictive, is the *shadow price* of a good. The shadow price vector Ap gives the prices relevant for individuals within the household. Goods that are consumed jointly are modeled as having a small Barten scale, meaning that a private equivalent unit of such a good can be had for (relatively) cheap within the household. Private goods by contrast, remain as costly within the household as they are at market. For this reason, the consumption technology has two effects: It makes household members feel richer because their resources grow in real terms and it reorders the relative prices of goods, making private goods more expensive relative to ones with jointness in consumption.

This shadow price also offers a way to contrast the consumption technology formulation with an alternative way of modeling jointness in consumption where goods (or portions of goods) are divided a priori into public and private (see e.g. Donni, 2009). In such models, the shadow prices are individual Lindahl (1919) prices, which add up to market prices across individuals. These individual prices are a measure of the extent to which each individual benefits from a public good and they depend on preferences. In contrast, the consumption technology described here implies a single set of shadow prices for all members of the same household, producing private equivalent quantities for them to arbitrarily allocate among themselves. Though it models sharing, it is not actually linked in the model to this allocation, meaning that one member could in principle consume everything in the household and still benefit from economies of scale.<sup>4</sup>

To define the household problem, BCL assume Pareto efficiency in outcomes. This is the fundamental assumption of the collective model first made in Chiappori (1988) and Apps and Rees (1988). Efficiency yields the following problem:

$$\max_{q,\{x_j\forall j\in H_i\}} \sum_{j\in H_i} \lambda_j\left(\frac{p}{y}\right) U_j(x_j)$$
  
s.t.: 
$$\sum_{j\in H_i} x_j = A^{-1}q \text{ and } y = q'p$$
 (2)

The scalar functions  $\lambda_j \begin{pmatrix} p \\ y \end{pmatrix}$  are the *bargaining weights*, which may notably depend on relative market prices. For this reason maximand in Equation (2) is not, in general, a well-behaved utility function. The weights may incorporate the effects of other-regarding preferences through *caring*, whereby an individual derives utility from other household members'  $U_j(x_j)$  in addition to her own and the two are additively separable in her true welfare function. They may also depend on household or individual characteristics as well as on the consumption technology. Dependence on variables that can be plausibly assumed unrelated to preferences or the consumption technology plays an important role in some models. Such so-called *distribution factors* can improve estimates and even aid identification (e.g. in Browning et al., 1994; Dunbar et al., 2017). Under these and some additional technical conditions, BCL show that household demand for a good k can be written in budget share form as:

$$w^{k}(p, y, A) = \sum_{j \in H_{i}} \eta_{j}(p, y, A) w_{j}^{k} (Ap, y\eta_{j}(p, y, A))$$
(3)

where  $w^k$  is the household-level demand for good k expressed as a share of the household budget y and  $w_i^k$  is a corresponding *desired budget share* function for individual i. These

 $<sup>{}^{4}</sup>$ It is unclear how much can be gained by restricting this behavior in practice. Bargain et al. (2014) take this route through a reparametrization.

individual budget share functions should be thought of as the shares of own resources individuals would like to spend on the good at shadow prices. They take two arguments in this formulation, though possible dependence on characteristics is suppressed: First, they are functions of the shadow price vector Ap rather than market prices p. These shadow prices are the ones that are relevant for individual decision making. Second, the functions depend on *individual resources*  $y\eta_i(p, y, A)$ , the part of total household expenditure y which is allocated to i.

This makes  $\eta_i(p, y, A)$  the all-important resource share. It gives the share of household resources allocated to individual *i*. The collection of all  $\eta_j$  in a household describes inequality within it. The  $\eta_j$  act as weights of the individual desired budget shares in household demand.

# **Assumption 1** (BCL). Assume that the BCL model of the household holds as outlined above so that in particular, Equation (3) holds.

Identification of this model by BCL relies on the observation of single households. These allow for the recovery of the functions  $w_i^k(p, y)$ , which directly determine the singles' demand. With this information, the remainder of the model can be recovered on couples' data. There are two important objections to this use of singles' demand: It supposes that members of couples have the same preferences given observables as singles<sup>5</sup> and, perhaps more importantly, it extrapolates their demands from market to intra-household prices. For goods with large economies of scale, this requires a considerable amount of (market-) price variation to be credible.

Instead, the approach chosen here is to abstract from prices as is done e.g. in Lewbel and Pendakur (2008); Bargain and Donni (2012). This puts the burden of identification on the expansion paths (or Engel Curves) of the now price-independent  $w_i^k(y)$ . In order to accomplish this, prices need to be assumed away in two places: The resource share and the desired budget share function. The former is the more straightforward.

**Assumption 2.** Resource shares  $\eta_i$  are independent of total household expenditure y and all demands are observed at constant relative prices  $\hat{p} = \frac{p}{|p|}$ .

Restricting  $\eta_i$  to be independent of y may seem strong, but it doesn't need to be. Correlates of y such as social status or even wealth may still affect allocations. Additionally, there is some evidence that the condition is not too far from reality. Both Menon et al. (2012) and Bargain et al. (2018) find that they cannot reject it on data from Italy and Bangladesh respectively.

But in order to write the desired budget share function  $w_i^k(p, y)$  without prices, an obstacle remains. Even with constant market prices, individuals still differ in the prices they face if they live in different households, especially in households of different sizes. This is because larger households can share some common expenses among more members, thereby reducing the shadow price by way of the Barten scales in A.

To accommodate these economies of scale and still be able to abstract from market prices, two approaches exist in the literature. One is to make assumptions that allow the explicit modeling of savings from scale economies without relying on price data by scaling individual resources, the remaining argument in  $w_i^k$ . This has the key advantage that it allows the estimation of scale economies, which are key to welfare comparisons across differently structured households. The other approach is to make  $w_i^k$  dependent directly on household structure and to achieve identification across households of the same structure. The advantage here is a simpler model that is not reliant on the observation of singles. Both approaches are introduced below and applied separately to the data in Section 4.

<sup>&</sup>lt;sup>5</sup>Using a nonparametric test, Hubner (2018) finds this not to be the case in both Russian and Spanish data.

#### 2.1 Including Scale Economies

In order to explicitly model scale economies assume, as in Lewbel and Pendakur (2008), that they are independent of the base expenditure (IB). The assumption is captured in the following statement about individual indirect utility.

Assumption 3 (Independence of the base - IB). There exists a function  $D_i(p, A)$ , such that

$$V_i(p, y, A) = V_i\left(p, \frac{y}{D_i(p, A)}\right) \tag{4}$$

Individual *i*'s utility under shadow prices Ap is equivalent to her utility under market prices but with her resources y scaled by the *scale economy index*  $D_i(p, A)$ . This index is a measure of the economies of scale *i* enjoys by living in a household with Barten scales A compared to living alone and facing market prices. Values smaller than one indicate economies of scale. Applying Roy's Identity in logs yields a relationship between demands in budget shares under market and intra-household prices:

$$w_i^k(Ap, y) = w_i^k\left(p, \frac{y}{D_i(p, A)}\right) + d_i^k(p, A)$$
(5)

where the term  $d_i^k(p, A) = \frac{\partial \ln D_i(p, A)}{\partial \ln p^k}$  is the elasticity of the scale economy index with respect to the price of good k. This elasticity depends on the nature of the good, as well as its budget share: If the price of a private good such as clothing rises, it will make both singles and members of larger households worse off, but it will also shift resources to a good that cannot be shared, reducing scale economies and thus increasing  $D_i(p, A)$ .

Applying Assumptions 2 and 3, Equation (3) can now be rewritten without prices. Since the consumption technology captured by A is at the level of the household, dependence on a given household is indicated by h in Equation (6) below.

$$w_{h}^{k}(y_{h}) = \sum_{j \in H_{i}} \eta_{j,h} w_{j,h}^{k} (y_{h}\eta_{j,h})$$

$$= \sum_{j \in H_{i}} \eta_{j,h} \left( w_{j}^{k} \left( \frac{y_{h}\eta_{j,h}}{D_{j,h}} \right) + d_{j,h}^{k} \right)$$
(6)

Further simplification is achieved by focussing on *private assignable goods* such as food. Though the associated Barten scales are equal to one, the consumption technology still matters in the demand for such goods: Household members experience savings on other goods that make them wealthier in real terms and raise the price of private goods relative to other goods.

Suppose for this exposition that one assignable good is available and let the household level budget share for *i*'s consumption of this good be  $w_{i,h}(y_h)$  and let  $w_i(y)$  be *i*'s individual desired budget share, a function of individual resources. Due to assignability, the desired budget shares will be zero for all members of the household but *i*. Then for *i*'s portion of the assignable good, Equation (6) can be written more simply as:

$$w_{i,h}(y_h) = \eta_{i,h} \left( w_i \left( \frac{y_h \eta_{i,h}}{D_{i,h}} \right) + d_{i,h} \right)$$
(7)

The budget share function  $w_i$  still needs a parametric specification. This job falls to a quadratic Engel curve in log individual resources as proposed by Banks et al. (1997). To allow for individual heterogeneity in tastes, individual characteristics will later be allowed to enter this specification in two ways: By modifying the intercept  $\alpha$  and a shifter  $\tau$ . The latter acts on

the budget share function the way a tax would, by shifting it up or down along the direction of individual resources.

$$w_i(y) = \alpha_i + \beta(\ln y - \tau_i) + \gamma(\ln y - \tau_i)^2 \tag{8}$$

Since this function takes the logarithm of individual resources as argument, the scale economy index  $D_{i,h}$  is specified directly in logs. It may in principle depend on both individual and household characteristics. But recovery of a complex specification is difficult especially since effects of individual characteristics on  $\tau$  and  $\ln D$  can only be disentangled by relying on singles' data. For this reason, the specification give in Equation 9 focusses only on household size n, the most important factor in scale economies. The following functional form is chosen for the log of  $D_{i,h}$ :

$$\ln D_h = -\ln(\delta)(n^\theta - 1) \tag{9}$$

This specification is non-linear and differs from more common functional forms for equivalence scales, a related concept. The principal reason for this choice is the need to accommodate large households. Equivalence scales, which roughly correspond to  $1/D_h$ , are typically specified as functions that tend to infinity for arbitrarily large n. Instead, for  $\theta < 0$ , the expression in Equation 9 converges to  $\ln \delta$ , implying a limit of  $\delta$  for  $D_h$ . This reflects the idea that life in large households does not grow arbitrarily cheap. The parameter  $\delta$  therefore summarizes the degree of savings that is possible in the economic environment, while  $\theta$  regulates the speed at which efficiencies of scale are achieved as n increases. In this formulation, the index is normalized to one for n = 1, meaning singles. These are not in principle necessary for identification, which could be done by relying on the functional form, but they are useful and will be included below.

#### 2.2 Not Including Scale Economies

Alternatively to Assumption 3 and to the quadratic specification of Equation (8) it is possible to make the desired budget share function directly dependent on household structure as is done in Dunbar et al. (2013). This approach removes the explicit mention of the consumption technology, meaning that household scale economies cannot be estimated.

Starting from the basic BCL demand system in Equation 3, the idea is to replace the desired budget share functions  $w_j^k$ , which depend on Barten scales A, with household-specific versions  $w_{j,h}^k$ , which are assumed not to. Assuming that all households are observed at constant relative prices, the resulting system is shown in Equation (10).

$$w_{h}^{k}(y_{h}) = \sum_{j \in H_{i}} \eta_{j,h} w_{j,h}^{k} \left( y_{h} \eta_{j,h} \right)$$
(10)

Again suppose that one individually assignable good is available and let the household level budget share for this good be  $w_{i,h}$ . Then for *i*'s portion of the assignable good, Equation (10) becomes

$$w_{i,h}(y_h) = \eta_{i,h} w_{i,h} \left( \eta_{i,h} y_h \right) \tag{11}$$

The desired budget share function  $w_{i,h}^k$  is specified as a linear Engel curve in log individual resources where both the intercept and slope parameter may depend on the household in question:

$$w_{i,h}(y) = \alpha_{i,h} + \beta_{i,h}y \tag{12}$$

Because identification in this case will be across households of a given structure (see Section 2.4), it will be convenient to limit the extent to which demands may depend on the household of which i is a member.

Assumption 4 (Dependence on Household Structure). The highest order term of individual i's desired budget share function is invariant to i's household except through its structure s = s(h):

$$\beta_{i,h} = \beta_{i,s} \tag{13}$$

where the household structure s is an object that contains a limited amount of information about the household, most importantly the number of members n. In the application below, this will be a vector containing the number of i's co-members who are adults and the number who are children. In contrast, subscript h continues to denote objects that may depend on household composition.

Equation (11) resembles those of the Engel curve system by Dunbar et al. (2013), though theirs is a system at the level of the household h based on types, with one equation per type and an adjustment for cases where multiple members of the household are of the same type. As is the case with all other empirical collective models, a cross-equation restriction is added that the resource shares  $\eta_{t,h}$  must sum to one, where t stands for the member type. Identification is achieved there by restricting the slope parameters  $\beta_{t,h}$ . These are either required to be the same for all types in a given household (the more powerful 'SAP' assumption) or they are allowed to differ by type but cannot vary with the household's structure, meaning the number of children (the 'SAT' assumption).

Analogously, Equation (7) mirrors those of systems by Lewbel and Pendakur (2008); Bargain and Donni (2012). There too, cross-equation restrictions apply for assignable goods but identification relies on observing adults as singles and assuming stability of preferences between singles and members of couples conditional on scaling. These papers restrict their analysis to a short list of household compositions, avoiding the need for complex adjustments to the number of any type present in a household.

Because individual, rather than type-level, assignable goods are available in the data used in the present paper, there is a way to avoid the slope restrictions in the case without scale economies and the necessity to limit analyses to a small set of household structures. It involves a new assumption linking individuals across different households.

#### 2.3 Uniform Sharing

In order to estimate Equations (6) or (10), which are at the level of the individual i, a substitute needs to be found for the cross-equation restriction that resource shares add up to one. This job falls to Assumption 5, which specifies resource shares directly as a function of a household's composition.

Assumption 5 (Uniform Sharing). Individual *i*'s resource share  $\eta_{i,h}$  in household *h* can be expressed as the following function of *i*'s **resource weight**  $\rho_i$  and those of all members  $j \in H_i$  of the household:

$$\eta_{i,h} = \frac{\rho_i}{\sum_{j \in H_i} \rho_j} \tag{14}$$

The resource weight  $\rho_i$  is a function of individual characteristics, not of the household of which *i* is a member. By construction, it is directly related to the individuals' resource share  $\eta_{i,h}$  in the household. For a given set of other household members,  $\rho_i$  is approximately proportional to  $\eta_{i,h}$ . Because members of different characteristics are found with greater probability in certain

kinds of households rather than in others, the mapping is not one-to-one in population average terms, even within households of a given structure. Additionally, like the resource share itself,  $\rho_i$  does not depend on household resources  $y_h$ . Because it is independent of the household of which *i* is a member,  $\rho_i$  is arguably a better indicator than the resource share if one wishes to summarize the unequal distribution of resources across a large number of households that differ considerably in incomes as well as in composition.

Assumption 5 removes the need to keep track of (types of) household members and their frequencies in order to assure adding up of resource shares, a task that is particularly cumbersome if some households have no members of a given type. With this assumption, the model can be estimated as a single equation at the level of the individual, getting rid of the concept of types altogether.

The new specification for the resource share directly accounts for household composition. Let  $\sigma_h = \sum_{i \in H_i} \rho_i$ , then Equation (7) becomes:

$$w_{i,h}(y_h) = \frac{\rho_i}{\sigma_h} \left( w_i \left( \frac{\rho_i y_h}{\sigma_h D_h} \right) + d_h \right)$$
  
=  $\frac{\rho_i}{\sigma_h} \left( \alpha_i + \beta \left( \ln \left( \frac{\rho_i y_h}{\sigma_h D_h} \right) - \tau_i \right) + \gamma \left( \ln \left( \frac{\rho_i y_h}{\sigma_h D_h} \right) - \tau_i \right)^2 + d_h \right)$  (15)

And for the case without scale economies, Equation (11) becomes:

$$w_{i,h}(y_h) = \frac{\rho_i}{\sigma_h} w_{i,h} \left(\frac{\rho_i}{\sigma_h} y_h\right)$$
  
=  $\frac{\rho_i}{\sigma_h} \left(\alpha_{i,h} + \beta_{i,h} \left(\ln \rho_i - \ln \sigma_h + \ln y_h\right)\right)$  (16)

Assumption 5 is not entirely harmless. It fixes the relative resource shares of any two types of household members. For example, if  $\rho$  where allowed to vary only with age and sex, two women aged 20 who both live in households with men aged 36 will have the same relative resource shares w.r.t. those men, regardless of who else may be present in the household. Stated another way, the addition of a new member to a household does not modify the relative resources of the old members<sup>6</sup>. This rules out situations where for instance the addition of children may change the relative distribution of resources between their parents, although closely related mechanisms are possible, as  $\rho_i$  may be made dependent on whether *i* is a father or mother.

Identification of the resource weight  $\rho_i$  is up to a factor  $\lambda \in \mathbb{R}$ . This does not affect the resource share  $\eta_{i,h}$ , for which  $\lambda$  is irrelevant. Equation (15) requires the observation of households of different sizes for identification, such that the parameter  $D_h$  can be recovered. In contrast, for the case without scale economies in Equation (16), identification is achieved separately for households of a given *household structure*, meaning that restrictive identifying assumptions such as those used in DLP are not needed here.

#### 2.4 Identification

In Section 2.5  $\rho_i$  will be specified as a function of individual characteristics. Since it is identified only up to a factor  $\lambda \in \mathbb{R}$ , a normalization is needed. This is done there, and in the proofs below, by setting  $\rho_i$  to one for an individual with mean characteristics.

Identification makes use of the functional forms given in Equation (8) and (12). Since individual Engel curves are specified as polynomials in log resources, their second and first

 $<sup>^{6}</sup>$ This idea is analogous to the independence of irrelevant alternatives (IIA) property in multinomial logit models

derivatives, respectively, are constant. This means that the corresponding derivatives of household Engel curves in Equations (15) and (16) become relatively simple functions involving the resource weight  $\rho_i$ :

**Proposition 1** (Identification with scale economies). Let  $\rho_i = \rho(z_i) \in \mathcal{C}^0$  be the resource weight and slope term associated with individual *i* in a household of structure *s* with *n* members, with  $z_i \in \mathbb{R}^k$ ,  $k \in \mathbb{N}$ . Assume that the data allow recovery of the budget share function  $w_s(z_1, ..., z_n, \ln y_h) \in \mathcal{C}^2$  which has the form given in (15). Then the function  $\rho(z_i)$  is identified.

Proof of Proposition 1. Using the budget share function  $w_s$ , define the second derivatives with respect to  $y_h$  as

$$l_{s}(z_{1},..,z_{n}) = \frac{\partial^{2}}{\partial^{2} \ln y_{h}} w_{s}(z_{1},..,z_{n},\ln y_{h})$$

$$= \frac{2\rho(z_{1})}{\sum_{j\in 1,..,n}\rho(z_{j})}\gamma$$
(17)

To normalize  $\rho(z)$ , let  $\rho(\bar{z}) = 1$ . Evaluating  $l_s$  at  $\bar{z}$  for every member's argument obtains

$$l_s(\bar{z},..,\bar{z}) = \frac{2}{n}\gamma , \qquad (18)$$

identifying the curvature parameter  $\gamma$ . Varying an argument other than the first gives

$$l_s(\bar{z}, .., \bar{z}, z_i) = \frac{1}{n - 1 + \rho(z_i)}\gamma$$
(19)

which identifies  $\rho(z_i)$  and thereby the sum of resource weights  $\sigma_h$ .

The remaining parameters, fall into place. In particular, given the curvature of demands,  $D_h$  is identified as a left-right shifter through household size and  $/tau_i$  as a left-right shifter through individual characteristics.

For the case without scale economies, the proof is very similar. But this time, no comparison across household structures is needed.

**Proposition 2** (Identification without scale economies). Let  $\rho_i = \rho(z_i) \in \mathcal{C}^0$  and  $\beta_{s,i} = \beta_s(z_i) \in \mathcal{C}^0$  be, respectively, the resource weight and slope term associated with individual *i* in a household of structure *s* with *n* members, with  $z_i \in \mathbb{R}^k$ ,  $k \in \mathbb{N}$ . Assume that the data allow recovery of the budget share function  $w_s(z_1, ..., z_n, \ln y_h) \in \mathcal{C}^1$  which has the form given in (16). Then the function  $\rho(z_i)$  is identified.

Proof of Proposition 2. Using the budget share function  $w_s$ , define the slopes as

$$l_s(z_1, ..., z_n) = \frac{\partial}{\partial \ln y_h} w_s(z_1, ..., z_n, \ln y_h)$$
  
$$= \frac{\rho(z_1)}{\sum_{j \in 1, ..., n} \rho(z_j)} \beta_s(z_1)$$
(20)

To normalize  $\rho(z)$ , let  $\rho(\bar{z}) = 1$ . Evaluating  $l_s$  at  $\bar{z}$  for every member's argument obtains

$$l_s(\bar{z},..,\bar{z}) = \frac{1}{n} \beta_s(\bar{z}) , \qquad (21)$$

identifying the slope  $\beta_s$  at  $\bar{z}$ . Varying an argument other than the first gives

$$l_s(\bar{z}, .., \bar{z}, z_i) = \frac{1}{n - 1 + \rho(z_i)} \beta_s(\bar{z})$$
(22)

which identifies  $\rho(z_i)$ .

#### 2.5 Empirical Implementation

In Equations 15 and 16, no specification is given for the objects  $\rho_i$ ,  $\tau_i$ ,  $d_h$ ,  $\alpha_{i,h}$ ,  $\beta_{i,s}$ . They will be parametrized as linear indexes in characteristics z, including information about household composition in the case of  $d_h$  and  $\alpha_{i,h}$  and about household structure s = s(h) for  $\beta_{i,s}$ . This considerably simplifies the functional form for  $\sigma_h$  and thus for the resource share  $\eta_i$ :

$$\eta_i = \frac{\rho' z_i}{\sum_{j \in H_i} \rho' z_j} = \frac{\rho' z_i}{\rho' \sum_{j \in H_i} z_j}$$
(23)

This way, each individual's household budget share  $W_{i,h}$  becomes a known function of household income  $y_h$ , individual characteristics  $z_i$ , the sum of those same characteristics over all other household members  $\sum_{j \in H_i} z_j$  and by way of this sum, of household structure.

The structure of the household also enters both systems through parameters of the desired budget share functions  $\alpha_{i,h}$  and  $\beta_{i,s}$  as well as, in the case with scale economies,  $d_h$  and  $D_h$ . The latter of these is of special importance, summarizing the overall extent of the household's scale economies. It is also notoriously difficult to estimate (Lewbel and Pendakur, 2008, see ). For this reason, the choice of functional form for  $D_h$  is of some importance. It is specified here as a function only of the household size n:

$$\ln D_h = -\ln\phi(n^{-\theta} - 1) \tag{24}$$

There are two important motivations for this choice. First, a low dimensional functional form is needed to obtain precise estimates. Second, the function converges to  $\phi \in (0, 1)$  for large n, while the speed of this convergence is regulated by  $\theta > 0$ . This allows for a rapid accumulation of scale efficiencies when going from very small to medium households without implying extreme savings in the very large households that are also present in the data. In contrast a linear approach would imply no limit to the savings that can be obtained by forming ever larger households.

#### 3 Data

The data come from two waves of the Bangladesh Integrated Household Survey (International Food Policy Research Institute, 2016)<sup>7</sup>. The data was collected in 2012 and 2015 by the International Food Policy Research Institute (IFPRI) and is intended to be nationally representative. For the main results, a combined sample is used, adjusting for inflation in PPP terms.

In addition to a thorough list of food and nonfood expenditures at household level, the survey contains a section on individual food consumption. In it, the household member responsible for food preparation is asked about what was eaten and who ate how much for the most recent normal day. A first portion is concerned with the list of 'menus' that were consumed by any household members. Such menus may include simple snacks or tea all the way to family dinners. For each menu a list of ingredients is given, which can be priced thanks to the extensive food expenditure section.

The second portion of the individual food section asks the same respondent for an estimate of how many grams of each menu was eaten by which household member as well as whether a household member skipped any menus. For curry, respondents are also asked to separately indicate how many grams of the protein portion was consumed by each household member.

 $<sup>^{7}</sup>$ This same data is also used by Brown et al. (2018), who also compute individual food expenditure but aggregate it by type.



Figure 1: Fit for food budget shares as a function of age - restricted cubic splines.

To estimate the two models in (15) and (16), household budget shares for individual assignable goods are needed. This role is filled here by *individual food budget shares*  $w_i$ , the shares of expenditure on food for individual *i* in total household expenditure. These were constructed as follows. First, total household expenditure  $y_h$  was constructed using all available expenditure information and total household food expenditure was constructed separately. Then, individual food consumption on the most recent normal day was priced using the above mentioned information along with prices which were carefully constructed using information from the household food expenditure section. Extrapolating these individual expenses over the whole year yields values which exceed household food expenditure by about a factor of two and which frequently exceed total household expenditure. Instead, an individual's share in the most recent normal day's food expenditure was multiplied by the household's food budget share to obtain individual food budget shares.

A fit to the resulting household budget shares for individual food consumption is shown in Figure 1. Two separate curves are shown for males and females, which were obtained using a regression on restricted cubic splines with four knots at equally spaced quantiles of the age distribution. The pattern shown may be unsurprising, with adults consuming about three times the value of food as infants and males consuming more food by value than females at all ages.

The estimation sample differs from the full sample by the following selection. Households were dropped if some individuals were recorded as being permanently absent from the household or if there was no information on their food consumption during the most recent normal day. Likewise, households were dropped if their total expenditure was in the top or bottom 0.25% of the sample. Also missing from the estimation sample are any households which had either no head, multiple heads or missing information for any members on sex, age or relationship to head. This leaves a total of 44167 individuals across 10838 households, roughly half from the 2012 round and half from the 2015 round.

As Table 1 shows, households in the estimation sample are highly varied in composition. Only 39% of households are nuclear in the sense that they consist of one man, one woman and children, which are defined as those under 16 years of age. These households contain 38% of all individuals in the sample, meaning that a solid majority live in non-standard household structures. Household sizes also vary widely. While 5.7% of individuals live in households of

	Number of Adults				
Number of Children	1	2	3	4 +	Total
0	216	1,970	1,932	1,587	5,705
1	524	4,752	$3,\!256$	$3,\!399$	$11,\!931$
2	$1,\!158$	$7,\!396$	$3,\!330$	$2,\!671$	$14,\!555$
3	664	$3,\!970$	1,764	1,411	$7,\!809$
4+	344	2,035	817	971	$4,\!167$
Total	2,906	20,123	11,099	10,039	44,167

Table 1: Numbers of individuals by household structure - adults and children

just two people, a similar share of 5.9% live in households composed of eight or more members.

Variable	Specification	Distribution
HH expenditure $y_h$	Logarithmic, demeaned	Mean: 5874 USD PPP per year
		Std: 4249 USD
Protein budget share $w_i^p$	$\%$ of $y_h$	Mean: $5.7\%$
		Std: 7.1%
Non-protein budget share $w_i^n$	$\%$ of $y_h$	Mean: $7.7\%$
		Std: $7.6\%$
Age	In years, demeaned, splines	Mean: 27.0 years
		Median: 23 years
Sex	Male dummy	47.7% male
Education	In years of schooling,	Adult median: 4 years
	demeaned by age, splines	Median of positive: 5 years
HH Location	Dhaka dummy	29.6% in Dhaka
HH Assets	Logarithmic, demeaned, splines	Mean 1963 USD PPP
		Std: 3506 USD
N Adults	Number excluding self	Mean: 2.8
N Children	Number excluding self	Mean: 1.9

 Table 2: Summary statistics of key variables

Key variables used in estimation are summarized in Table 2. Total annual household expenditure  $y_h$  and individual food budget shares  $w_i$  were already described above. The remaining variables in the list enter the model as shifters of the resource weight  $\rho_i$  and of the individual desired budget share parameters. Both nutritional requirements and intra-household allocations are suspected to depend highly on an individual's age, which therefore plays an important role. Bangladesh is a young country with a median age of 25.6 years in 2015, not far from the median in the data.

Education is cast into years of schooling. A large share (40.8%) of adults report not having completed the equivalent of a single year of schooling and only 1.3% report any education past high school. Nearly a third of individuals in the data live in Dhaka, the capital. The variable is demeaned by age, giving a deviation in years from the age-conditional mean for use in estimation.

## 4 Results

To focus on the development of individual resources over age and sex, these variables enter the empirical specification in a highly flexible way: As separate sets of restricted cubic splines with four knots for males and females. This allows for highly flexible and entirely separate developments of the resource weight  $\rho_i$  as well as two desired budget share parameters in each of the two models,  $\alpha_{i,h}$  and  $\beta_{i,s}$  in Model (16) and  $\alpha_{i,h}$  and  $\tau_i$  in Model (15). An age-conditional deviation from mean education also enters the same parameters in the same highly flexible way. Demeaned log household assets are interacted with the age splines and enter the resource weight but are assumed to be a distribution factor, meaning they do not enter preferences. In addition, the resource weight  $\rho_i$  may depend on dummies for head of household or spouse of the head as well as a dummy for a male residing in the capital Dhaka.

This selection of distribution factors for the resource weight is informed by its nature. They should be shifters of the relative bargaining position of a member i and therefore need to vary within households. Good choices for covariates fall into two categories, those with intrahousehold variation such as age and sex and interaction terms of these with household-level variables. For suspected household-level shifters of allocations, this provides a particularly flexible environment: They can be interacted with whichever individual observable they are believed to act on. For instance, an urban location dummy may be thought to differentially affect highly educated and less highly educated members. In that case, it should be interacted with educational attainment. This flexibility is not present in type-based models, where any effects on allocations must along type lines.

Household structure enters the models differently. For Model (15), where scale economies are explicitly accounted for, the number of adults in h other than i and the number of children in h other than i enter the parameter  $d_h$ , while the total household size enters  $D_h$  as per Equation (24). Individual characteristics are not included in  $d_h$  without loss of generality, since such effects cannot be disentangled from those in  $\alpha_i$  (Wolf, 2016, see ).

On the other hand, Model (16) needs to account for household structure in a less formal fashion as laid out in Section 2. To this end, the parameters  $\alpha_{i,h}$  and  $\beta_{i,s}$  may depend on numbers of children and adults as described above for  $d_h$ . Both models are very well behaved in estimation and yield precise and robust estimates.

	Model $(16)$		Model $(15)$		
Parameter	Estimate	t-stat	Estimate	t-stat	
$ ho_{head}$	0.0931***	(7.28)	0.0877***	(7.63)	
$ ho_{spouse}$	$-0.0487^{***}$	(-3.92)	$-0.0538^{***}$	(-4.90)	
$ ho_{Dhaka\cdot male}$	$0.0361^{***}$	(4.24)	$0.0384^{***}$	(4.81)	
$ ho_{agesplines}$	Yes		Yes		
$ ho_{edusplines}$	Yes		Yes		
$\rho_{assetssplines}$	Yes		Yes		
$\phi$ .			$0.490^{***}$	(9.07)	
heta			$0.735^{***}$	(3.97)	
Individuals	42977		43234		
$R^2$ non-protein	0.685		0.687		
$R^2$ protein	0.485		0.492		

Table 3: Key parameter estimates from Models with (15) and without (16) scale economies.

Estimates of key parameters for both models are shown in Table 3, a more complete version

of which is available in Appendix A. Despite the considerable differences between the two structural models, they show considerable agreement in their estimate of the resource weight function. As shown in the table, being the head of a household goes along with a considerable increase in  $\rho$  while the household head's spouse has a smaller resource weight. Allocations are shifted towards males in Dhaka, which is home to about 30% of the sample, defying expectations on the part of the author that women would be relatively better off in the more modern city.

The parameters  $\phi$  and  $\theta$  relate to the scale economy function, and are estimated with surprising precision. The limiting scale economy index for very large households,  $\phi$ , is estimated at 0.49, meaning that the largest households in the data come very close to 50% savings compared to living alone. For the estimate of  $\theta$  shown here, much of these savings happen 'early', meaning when moving from singles to small multi-person households: A couple is already estimated to save 25% compared to singles.

The most striking results in terms of the resource weight are not visible in Table 3 because they concern the dependence of allocations on an individual's age. Figure 2 shows predicted resource weights as a function of age and sex for a non-head, non-spouse of head living outside of Dhaka, with education at the adult mean. Confidence bands have been added around the two prediction lines. Again, results are very similar across models and, as the graphs illustrate, estimates are extraordinarily precise.



Figure 2: Estimated resource weights  $\hat{\rho}$  as a function of the individual's age.

The graphs show that age is the most powerful driver of differences in intra-household allocations. Young children are allocated only small shares of household resources which grow quickly as they age. Figure 2 also speaks to gender inequality in intra-household allocations. While a positive male-female gender gap opens up as children age, it is reversed for working age adults, at least when they are neither the head nor the spouse. Note though that 82% of heads are male and 99% of spouses of heads are female, meaning that the gender imbalance is estimated to run the other way for much of this subpopulation.

The strongest result, though, is the age profile itself. It is worth noting that a graph such as this could not have been drawn based on previous empirical models of the household. For both sexes, the resource weight experiences a steep rise during childhood and peaks in early adulthood. The early rise is particularly important. It starts at birth and rises almost linearly at a steep angle. A worthwhile comparison is Figure 1, where a linear rise can also be observed, but is much less steep. The interpretation of these estimates is therefore that infants consume little more than food, while young adults have a much more varied consumption basket.

The early rise also means that a great deal of variation in intra-household resources is hidden

when all children are grouped together as a type of household member. In the results found here a division into boys and girls would also do very little to improve the situation. This point is of relevance not only for models of intra-household allocations but also for equivalence scales, which typically treat all children equally regardless of age.

Two other variables enter the resource weight: The effect of the age-conditional deviation from mean education is shown in Figures 3, with that of log household resources delegated to Appendix A. The former is shown here because it presents an intriguing pattern. While more education usually yields an advantage in intra-household allocations for adults, the reverse is true for children. This may be due to compensation by the household for children child labor in the home, which is not otherwise accounted for in the model.



Figure 3: Estimated effects on resource weights  $\hat{\rho}$  of the deviation from age-conditional mean education.

## 5 Discussion

Having estimated two models that return separate resource shares for all members of households, it is natural to ask how much additional variation in resources, or inequality, is revealed this way. This turns out to be a substantial amount and it raises the question of what to make of all this intra-household inequality. First, a key question is to what extent inequalities reflect differences in needs or preferences versus unfairness. This question is important when considering whether and how to implement targeted programs for poverty reduction. Second and relatedly, the large differences found between children of different ages should be of concern when comparing across households as well as within. Third, since child poverty is a fundamental concern, it raises the question of what can be learned about differences between children that are unlikely to be explained by differences in need.

#### 5.1 Inequality Decomposition

To quantify how much inequality is revealed by differentiating individuals of the same type within a household, I use a measure of inequality that can be easily decomposed in an additive fashion. This measure, mean log deviations (MLD), has been used for this reason in similar contexts (see Brown et al., 2018). Let  $r_i$  be an individual's estimated individual resources and denote the average individual resources in the dataset simply by r. Then the MLD inequality measure for the dataset is given by

	Model $(16)$		Model (15) (scaled)	
	MLD	Percent	MLD	Percent
Between households	0.171	67.1	0.162	70.7
Between types; within households	0.068	26.6	0.055	24.0
Between individuals; within households, types	0.016	6.4	0.012	5.3
Total	0.255	100.0	0.230	100.0

Table 4: Decomposition of mean log deviations in (scaled) individual resources

$$MLD_{tot} = \frac{1}{N} \sum_{i=1}^{N} \ln r - \ln r_i$$
(25)

Now using mean resources at the subgroup level, this sum can be easily expanded by telescoping. In this case, let  $r_h$  be the average resources in *i*'s household, also known as per capita total expenditure. Then the above sum can be decomposed into a between-household and a within-household portion by adding and subtracting the term  $\ln(r_h)$ . Similarly, let  $r_{t,h}$  be the average resources for *i*'s type *t* in *i*'s household *h*. Then the MLD of resources can be decomposed as

$$MLD_{tot} = \frac{1}{N} \sum_{i=1}^{N} \ln r - \ln r_i$$
  
=  $\frac{1}{N} \left( \sum_{i=1}^{N} \ln r - \ln r_h + \sum_{i=1}^{N} \ln r_h - \ln r_{t,h} + \sum_{i=1}^{N} \ln r_{t,h} - \ln r_i \right)$   
=  $MLD_h + MLD_t + MLD_i,$  (26)

where  $MLD_h$  is the part due to inequality between households,  $MLD_t$  the part due to inequality between types within households and  $MLD_i$  the part due to inequality between individuals of the same type within the same household. The types are men, women, girls and boys, where adults are defined those 16 years or older. The results of this decomposition for both models are shown in Table 4. In the case of Model (16),  $r_i$  is given by  $\hat{\eta}_{i,h}y_h$ , meaning that resources are not scaled. In contrast, since Model (15) provides estimates of household scale economies, these are included in the measure of now private equivalent individual resources as  $r_i = \hat{\eta}_{i,h}y_h/\hat{D}_h$ .

According to this decomposition, a per-capita measure of inequality captures only around 70% of all inequality in consumption. Around 5% of mean log deviations are hidden if one assumes that members of the same type in the same household have the same resources. This is despite the fact that four types are considered such that individuals are often the only household members of their type in which case  $r_{t,h}$  will be equal to  $r_i$ .

#### 5.2 Poverty and Needs

It is important to note that the inequality described above is not synonymous with unfairness. This is especially clear when comparing adults to young children. Though they consume much less by the above estimates, they are likely to have very different needs. A less clear case is that of older adults who are also estimated to receive a smaller share of the various pies though this would seem to come from non-food consumption (see Figure 1). In both of these cases, a difference in needs or preferences does not exclude the existence of power imbalances

	Number of Adults			
Number of Children	1	2	3	4+
1	1862	1432	1648	1523
2	1935	1421	1613	1521
3	1577	1401	1404	1378
4+	1415	1262	1375	1421

Table 5: Children's mean estimated resources  $\hat{\eta}_{i,h}y_h/\hat{D}_h$  in 2011 PPP USD per year

that reinforce or partially offset any inequality that would exist in an 'equal bargaining power' scenario. Here, bargaining power is synonymous with the *bargaining weight*  $\lambda_i$  from Equation (2). Importantly for considerations involving children and the infirm, this weight can incorporate the effects of caring by other household members (see Section 2). Such a power imbalance would seem to be the more likely driver of the difference between older men and women.

The large differences in allocations to children of different ages are also an issue for comparisons between different households. This is typically done by statistical agencies using equivalence scales (see e.g. OECD, 2013). The intent of such scales is to adjust incomes (or expenditures) of differently structured households so that adjusted numbers reflect *household welfare*. This term is difficult to define in any context (Nelson, 1993) and becomes especially fraught in the intra-household context, when the theoretical framework is explicitly set up to avoid the concepts of household utility or household preferences. Nonetheless, equivalence scales have important applications in policy making, insurance and legal systems. They satisfy a need to make comparisons without which there could not be a single poverty rate or a meaningful median equivalent income measure.

Equivalence scales in use as policy tools are blunt tools that classify household members as adults or children, and prescribe a scaling factor to each household structure based on this classification. Doing this, they have to account for at least two factors: The difficult concept of needs and economies of scale in consumption. The latter are the key reason why equivalence scales for households composed only of adults are not simply equal to the number of adults. Larger households should be better off with the same per capita expenditure due to joint consumption. Both scale economies and needs are important for the relative impact on the scale of adding an adult versus a child.

But as Figure 2 shows, there are large differences in consumption, and presumably in needs, between children of different ages. Judging purely from these estimates, adjusting equivalence scales for children's ages should lead to large corrections for many households.

To compare the living conditions of children in differently structured households, Table 5 lists mean estimated annual resources  $\hat{\eta}_{i,h}y_h/\hat{D}_h$  in PPP US dollars for children. These are presented here without confidence intervals but show a clear pattern of fewer resources in larger households. The numbers take into account the estimated intra-household allocation and scale economies from Model (15) as well as observed household expenditure.

# 6 Conclusion

Using data on the individual consumption of food, a private assignable good, this paper has shown that resource allocations within households can be recovered using a parsimonious collective model. Rather than relying on types of members and either grouping some household members together or selecting only households of a given structure, a *resource weight* function is estimated for each individual, which is proportional to her share in household resources by the assumption of uniform sharing.

The resource weight also serves as a concise summary of how resources are allocated within a set of very differently composed households. By abstracting from types, the model permits the study of intra-household inequality along dimensions that do not involve distinctions between such types. For instance between highly and less highly educated household members.

Estimation on data from Bangladesh yields a detailed picture of intra-household consumption inequality between members of different characteristics and notably allows the construction of a detailed age-profile of allocations from birth to old age. A key result from the age profile is that the resources allocated to children depend greatly on their ages, with very young children consuming only a small fraction of what adults consume.

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# A Tables and Figures

	Model	Model (16)		Model (15)		
Parameter	Estimate	t-stat	Estimate	t-stat		
$\rho_{head}$	0.0931***	(7.28)	0.0877***	(7.63)		
$\rho_{spouse}$	$-0.0487^{***}$	(-3.92)	$-0.0538^{***}$	(-4.90)		
$\rho_{Dhaka \cdot male}$	$0.0361^{***}$	(4.24)	$0.0384^{***}$	(4.81)		
$\rho_{agesplines}$	Yes		Yes			
$\rho_{edusplines}$	Yes		Yes			
$\rho_{assets splines}$	Yes		Yes			
$\alpha_0^n$	$13.87^{***}$	(19.18)	$19.66^{***}$	(38.60)		
$\alpha^n_{agesplines}$	Yes		Yes			
$\alpha^n_{edusplines}$	Yes		Yes			
$\alpha_{Nkids}^{n}$	-0.0768	(-0.28)				
$\alpha_{Nadults}^{n}$	$-1.298^{***}$	(-4.61)				
$\alpha_0^p$	$32.13^{***}$	(39.62)	$26.65^{***}$	(53.94)		
$\alpha^p_{agesplines}$	Yes		Yes			
$\alpha^{p}_{edusplines}$	Yes		Yes			
$\alpha^p_{Nkids}$	$1.168^{***}$	(3.78)				
$\alpha_{Nadults}^{p}$	$1.273^{***}$	(4.12)				
$\beta_0^n$	$-15.59^{***}$	(-32.70)	$-16.69^{***}$	(-35.71)		
$\beta_{agesplines}^n$	Yes		No			
$\beta_{edusplines}^{n}$	Yes		No			
$\beta_{Nkids}^{n}$	0.210	(1.12)				
$\beta_{Nadults}^{n}$	$0.603^{***}$	(2.98)				
$\beta_0^p$	$6.486^{***}$	(12.51)	$-6.473^{***}$	(-9.83)		
$\beta_{agesplines}^{\check{p}}$	Yes	. ,	No	. ,		
$\beta_{adusplings}^{p}$	Yes		No			
$\beta_{Nhido}^{p}$	$0.929^{***}$	(4.55)				
$\beta_{Nadulto}^{p}$	-0.426*	(-1.93)				
$\gamma_0^n$			$-1.729^{***}$	(-7.00)		
$\tau_{aaesplines}$			Yes	( )		
$ au_{edusplines}$			Yes			
$d_{Nkids}^n$			$-1.417^{***}$	(-9.78)		
$d_{Nadults}^{n}$			$0.469^{***}$	(3.20)		
$d_{Nkids}^p$			$1.358^{***}$	(10.01)		
$d_{Nadults}^{p}$			-0.302**	(-2.15)		
$\phi$			$0.490^{***}$	(9.07)		
θ			0.735***	(3.97)		
Individuals	42977		43234			
$R^2$ non-protein	0.685		0.687			
$R^2$ protein	0.485		0.492			

Table 6: Parameter estimates from Models with (15) and without (16) scale economies.



Figure 4: Estimated effects on resource weights  $\hat{\rho}$  of log household assets.