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The Effectiveness of a Predictor-corrector Technique in European Currency Option Valuation

Sang Woo Heo[a], Jinsuk Yang[b], SeungCheol Lim[c], Peter Cashel-Cordo[d†]

[a]*Professor of Mathematics, University of Southern Indiana, USA*
[b]*Assistant Professor of Finance, University of Southern Indiana, USA*
[c]*Professor of IT Convergence, Woosong University, Daejeon, Republic of Korea*
[d]*Professor of Economics, University of Southern Indiana, USA*

A B S T R A C T

In this work, we adopt a predictor-corrector technique to examine the accuracy of the Fractional Black-Scholes (FBS) model. Compared to the standard Black-Scholes (B-S) model, FBS model involves one additional parameter, a Hurst value (*H*) providing information whether the time series exhibits persistent or anti-persistent behavior. The FBS model, as a result, has been shown to provide more accurate predictions of option price [Heo et al. (2017) and reference therein]. Estimation accuracy of volatility and *H* values are key to better option price estimates. However, volatility and Hurst values are unknown prior to the closing time; consequently, the estimation of option prices relies heavily on the accuracy of volatilities and Hurst parameter estimation. In this study we compare option price estimation accuracy using three variations of calculating *H* values, and two volatility measures. We estimate two *H* values using historic data using one-month data (21 trading days) and three-month data (63 trading days), respectively, and by using predicted volatility estimates obtained using a binomial method, as a predictor and then used them to estimate implied *H* values. We subsequently correct the predicted volatility measure using the implied *H* value, the predictor-corrector technique. We investigate the accuracy of these FBS models and examine effectiveness of this predictor-corrector technique using Euro currency option (XDE) data traded in NASDAQ from November 2007 to June 2016.

Keywords: European Options, Fractional Black-Scholes model, Predictor-Corrector Technique

Ⅰ. Introduction

In developing option pricing models, a mathematically closed form is desirable because it is easy to implement for calculations and applications. One well-known model is the Black-Scholes model (B-S) (Black & Scholes, 1973). Because of its simplicity and reasonable accuracy, the B-S model remains popular in the financial world even though it is almost a half century old. This model assumes asset prices follow a standard Brownian motion [Gaussian and Markovian process] resulting in systematic pricing bias based on moneyness and time to maturity (MacBeth & Merville, 1979; Geske & Roll, 1984; Rubinstein, 1985). Since the standard Brownian motion has independent increments, the B-S model misses long- range dependence in financial markets (Willinger et al., 1999). In an effort to incorporate long-range dependence, a non-Markovian

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† Peter Cashel-Cordo
E-mail: cashel@usi.edu

Fractional Black Scholes model (FBS) (Elliott & Van der Hoek, 2003; Hu & Oksendal, 2003) was developed. The FBS model is mathematically closed and adopts a fractional Brownian motion so that it captures long-range dependence. Meng & Wang (2010) and Heo et al. (2009, 2017) demonstrate that the FBS model reduces option pricing error as compared to the B-S model. Yet, there are not many empirical studies using the FBS model with actual option data. Since the FBS model involves one additional parameter, Hurst value, which provides whether time series exhibits persistent or anti-persistent behavior, the model yields better results than the B-S model, but the FBS model still carries similar pricing bias depending on moneyness, the time to maturity, and other underlining parameters as the B-S model (Heo et al., 2009 & 2017).

Unlike other parameters involved in these models, the implied volatility and Hurst values are unknown prior to the trading day's closing time so that the estimation accuracy of option prices is sensitive to volatilities and Hurst values. It has been demonstrated that models using implied volatility produce more accurate estimates than using historical volatility (Katz & McCormick, 2005; Heo et al., 2017). Furthermore, recent studies show that implied volatility is more powerful predictor of future volatility than other types of historical volatility (Kim, 2016; Kim & Poonvoralak, 2019). There are still unanswered topics utilizing FBS models such as methods for obtaining better Hurst values and volatilities.

In this study, we examine the accuracy of two versions of Black-Scholes European Call and Put option pricing model using Euro currency option data (XDE) traded on the NASDAQ market from November 2007 to June 2016. For the B-S model, we follow the traditional method of using the implied volatility retrieved from a binomial tree model with 100 steps. In evaluating the FBS model, we use the implied *H* values obtained from FBS models, and the Hurst values from the built-in program in Mathematica® based on the Method of Moments with one-month data (21 trading days) and three-month data (63 trading days). Then we use the implied volatility obtained by the binomial method as a predictor and recover the corrected implied volatilities from the FBS model using three Hurst values. A model's accuracy is determined by measuring the mean absolute percentage error with respect to the actual option price (MAPE), also by the mean percent error with respect to option price (MPE), and the root mean squared error (RMSE). In Section II, we describe the B-S and FBS models and Section III discusses our research methodology and data. Estimation results are reported in Section IV. Section V concludes the study with suggestions for future research.

Ⅱ. Model

Fractional Brownian motion (fBm) with Hurst parameter $H \in (0,1)$ is the centered Gaussian process on a probability space defined by

$$B_0^H = 0 \quad \text{and} \quad E[(B_t^H - B_s^H)^2] = |t-s|^{2H}.$$

That is, the increment $B_t^H - B_s^H$ is normally distributed with zero mean and variance $|t-s|^{2H}$ for each $s,t \geq 0$. When $H = 1/2$, it is the standard Brownian motion. In fBm, the increments are positively correlated if $H > 1/2$ and they are negatively correlated if $H < 1/2$. When increments are positively correlated, the long-range dependence property makes fBm a better source of noise in modeling the stochastic evolution of asset prices. Therefore, the Brownian motion is replaced by fBm in the Black-Scholes model, the Hurst parameter provides an additional instrument of capturing market predictability (Qian & Rasheed, 2004). Heo et al. (2009) provides a brief history of fBm and the FBS model for pricing European options.

Using the time variable t, $0 \leq t \leq T$, where $t = 0$ corresponds to the issue date of the option and $t = T$ corresponds to its expiration date, we define the following variables:

$S = S(t)$: stock price at time t
X: strike price of option
r: current risk-free interest rate
σ: stock price volatility
$\tau = (T-t)$: time to expiration
H: Hurst parameter

Based on the results of Elliott and Van der Hoek (2003), and Hu and Øksendal (2003), the Fractional Black-Scholes (**FBS**) European Call option model is described below [Daye (2003)]:

$$C(S,t) = S(t) * N(d_1) - X * e^{-r\tau} N(d_2),$$

where

$$d_1 = \frac{\ln(\frac{S}{X}) + r\tau + \frac{\sigma^2}{2}(T^{2H} - t^{2H})}{\sigma\sqrt{T^{2H} - t^{2H}}},$$

$$d_2 = \frac{\ln(\frac{S}{X}) + r\tau - \frac{\sigma^2}{2}(T^{2H} - t^{2H})}{\sigma\sqrt{T^{2H} - t^{2H}}},$$

and

$$N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-u^2/2} du.$$

Using Put-Call parity, we can obtain the following version of the FBS European put option model

$$\begin{aligned} P(S,t) &= Xe^{-r\tau} + C(S,t) - S \\ &= Xe^{-r\tau} N(-d_2) - S(t) * N(-d_1). \end{aligned}$$

We note that if $H = 1/2$, the **FBS** formula is identical to the **B-S** formula.

Ⅲ. Methodology and Data

The calculation of the FBS model is very sensitive to two parameters, volatility (σ) and Hurst Parameter (H) because they are unknown prior to the trading day's closing time. In this study, we incorporate the implied volatility obtained by the binomial method with 100 steps. Then, because the FBS model has a mathematically closed form, we recover the implied Hurst values from the FBS model. For comparison, we compute Hurst values using historical one- and three-month (21 and 63 trading days, respectively) data, which are independent of the model, where we adopt the Method of Moments (Fractional Brownian Motion Process [h] in Mathematica® software). Then, we employ the Predictor-Corrector method to recover implied volatility values using the FBS model. This process is described as below:

(Step 1) An Implied Hurst Parameter (IH) is recovered from the FBS model. This IH depends on the predicted implied Volatility (IV_p) obtained from a binomial tree model with 100 steps.

(Step 2) Using the implied Hurst Parameter from Step 1, we recover the corrected implied volatility (IV_c) from the FBS model.

(Step 3) Compute historical Hurst values H_1 and H_3 using the Method of Moments built in Mathematica® with one-month and three-month data, respectively.

(Step 4) Using the historical Hurst values H_1 and H_3 from Step 3, we recover corresponding corrected implied volatilities (IV_c), IV_{c1} and IV_{c3}, respectively from the FBS model.

(Step 5) Compute both B-S and FBS call and put option values with the combinations of

IV_p and IH, IV_c and IH
IV_p and H_1, IV_{c1} and H_1
IV_p and H_3, IV_{c3} and H_3.

We use Euro currency option data (XDE) traded on the NASDAQ from November 2, 2007 to June 30, 2016[1)] consisting of 6,366 different call and put options. The data set consists 672,804 observations. For a variety of reasons described below, it is necessary to screen the data. First, to eliminate transaction costs, observations with option prices less than or equal to $0.40 were deleted. We also delete outliers defined

1) The XDE data set was purchased from www.ivolatility.com. This set includes the volatility recovered from binomial tree model with 100 steps and LIBOR is used for the risk-free interest rates.

Table 1. Call Options: Predicted Implied Volatility vs. Corrected Implied Volatility

Call	Predicted Implied Volatility (IV_p)				Corrected Implied Volatility (IV_c)		
	B-S	FBS(IH)	FBS(H_1)	FBS(H_3)	FBS(IH)	FBS(H_1)	FBS(H_3)
MPE	10.2976	0.6991	8.4195	8.9503	0.6821	0.6746	0.6779
MAPE	11.1155	4.1673	17.7914	14.0642	4.1535	4.1589	4.1542
RMSE	0.7424	0.3336	0.7953	0.7644	0.3281	0.328	0.328

Table 2. Put Options: Predicted Implied Volatility vs. Corrected Implied Volatility

Put	Predicted Implied Volatility (IV_p)				Corrected Implied Volatility (IV_c)		
	B-S	FBS(IH)	FBS(H_1)	FBS(H_3)	FBS(IH)	FBS(H_1)	FBS(H_3)
MPE	-4.053	-1.4265	-7.658	-5.1624	0.1624	0.1327	0.1544
MAPE	5.7293	3.9902	12.4781	8.9956	2.8584	2.8655	2.8566
RMSE	0.4373	0.3652	0.5409	0.4785	0.135	0.1352	0.1349

as estimated B-S and FBS option values that are outside of Mean ± 3*STDEV. Option prices must satisfy the no-arbitrage boundary conditions:

$$P_{exact} \leq 0.98(X-S),\ C_{exact} \leq 0.98(S-X).$$

Otherwise, they were also deleted. Input variables IV_c and IH were recovered from the previous day's data, thus the first observation of each option was lost. The last filter required deleting options whose Hurst values are not in the range of 0 to 1. Consequently, the final dataset consists of 146,947 call option and 173,903 put option usable observations.

We also use two-percent filter around the exercise price (X) to define the moneyness. For call options, if $S > 1.02X$, then it is in-the-money (ITM), if $0.98X \leq S \leq 1.02X$, it is at-the-money (ATM). Otherwise, it is out-of-the-money (OTM). For put options, the definition of ITM and OTM are the opposite of those of call options.

To examine the accuracy of each model, we employ three different measures - mean absolute percent error (MAPE) for accuracy, mean percentage error (MPE) for bias, and root mean squared error (RMSE) for variation given by the following equations respectively:

$$MPE = \frac{1}{N}\sum \frac{Estimation - Exact\ Value}{Exact\ Value} \times 100(\%) \quad (1)$$

$$MAPE = \frac{1}{N}\sum \frac{|Estimation - Exact\ Value|}{Exact\ Value} \times 100(\%) \quad (2)$$

$$RMSE = \sqrt{\frac{1}{N}\sum (Estimation - Exact\ Value)^2}\ (\$) \quad (3)$$

Ⅳ. Empirical Results

The results of the accuracy tests for B-S and FBS models of all usable XDE option data are presented in the following tables. Tables 1 and 2 show the overall estimation errors for call and put options, respectively. Both tables demonstrate that the FBS model with the predicted volatility (IV_p) and the implied Hurst value (IH) outperforms the B-S model across all three accuracy measures.

This was not the case using the predicted volatility (IV_p) and historical Hurst values estimated with one-month (H_1) and three-month (H_3) data sets. The FBS model, using H_1 or H_3, was not any better than B-S model for both call and put options. In the case of call options, the MAPE's of both FBS call and put options are noticeably worse than that of B-S model. In the case of put options, all three measures show that the FBS model yields higher errors than the B-S models.

When the corrected volatility (IV_c) is used, regardless

Table 3. Call Options by Moneyness[2)]

Call		Predicted Implied Volatility (IV_p)				Corrected Implied Volatility (IV_c)		
		B-S	FBS(IH)	FBS(H_1)	FBS(H_3)	FBS(IH)	FBS(H_1)	FBS(H_3)
ITM	MPE	6.0513	2.057	5.3279	5.8881	2.0387	2.0429	2.0431
	MAPE	6.3509	3.0077	6.8261	6.761	2.9859	2.986	2.986
	RMSE	0.9848	0.542	0.96	0.9859	0.5331	0.5331	0.5331
ATM	MPE	10.7263	0.2787	7.2086	9.4188	0.2596	0.2082	0.2275
	MAPE	11.6191	3.7741	17.372	14.0326	3.7675	3.7785	3.7733
	RMSE	0.7138	0.1531	0.7978	0.7366	0.1496	0.1497	0.1496
OTM	MPE	14.0969	-0.2409	12.6629	11.5358	-0.2548	-0.2313	-0.2403
	MAPE	15.3569	5.6996	29.083	21.3444	5.6869	5.692	5.6833
	RMSE	0.4193	0.1234	0.5845	0.4897	0.1225	0.1221	0.122

Table 4. Put Options by Moneyness[3)]

PUT		Predicted Implied Volatility (IV_p)				Corrected Implied Volatility (IV_c)		
		B-S	FBS(IH)	FBS(H_1)	FBS(H_3)	FBS(IH)	FBS(H_1)	FBS(H_3)
ITM	MPE	-2.7039	-1.4219	-3.4611	-3.171	-0.0129	-0.016	-0.0169
	MAPE	3.2007	2.397	4.1374	3.801	1.304	1.3036	1.3013
	RMSE	0.5868	0.5034	0.6303	0.612	0.1736	0.1736	0.1734
ATM	MPE	-4.1724	-1.822	-7.9813	-5.559	0.2809	0.2484	0.2806
	MAPE	6.6119	4.8016	13.1637	9.8932	3.2413	3.2592	3.2426
	RMSE	0.3932	0.3194	0.5626	0.4518	0.1184	0.1186	0.1183
OTM	MPE	-5.5245	-1.042	-12.2829	-7.1171	0.252	0.1937	0.2316
	MAPE	7.8375	5.0669	21.6268	14.2293	4.3119	4.3171	4.3081
	RMSE	0.2168	0.151	0.3818	0.2865	0.0916	0.0917	0.0915

of call and put options, the FBS model with any Hurst value outperforms the B-S model across the board. All three FBS(IH), FBS(H_1) and FBS(H_3) results show that their pricing errors have been improved and they have very similar pricing errors. In fact, the FBS model with IV_c produces less errors than the corresponding FBS model with IV_p. Particularly, we notice significant improvement of FBS(H_1) and FBS(H_3).

This evidence shows that if this predictor - corrector method is adopted, the estimation accuracy of FBS models is very sensitive to the choice of volatility estimate, but insensitive to Hurst parameter whether it is implied (IH) or historical (H_1, H_3). There are different methods to obtain Hurst parameter as summarized in Biagini et al. (2008) but research does not provide clear evidence that one method is better choice over the others. By incorporating the predictor - corrector method, the method obtaining Hurst parameters is less critical.

Theoretically, the FBS model outperforms B-S model but we have notice that it is not the case if we use wrong combinations of volatility IV_p and Hurst values (H_1, H_3). So, we closely examine the pricing errors by moneyness. Table 3 and Table 4 present the pricing errors by the moneyness for call and put options, respectively. We notice from Table 3 with MPE that for call options, all models with predicted volatility (IV_p) overestimate the pricing

2) Among 146,947 usable call options, 49,239 are ITM, 48,105 are ATM and 49,603 are OTM options.

3) Among 173,903 usable put options, 64,743 are ITM, 54,197 are ATM and 54,963 are OTM options.

values except the case of FBS(IH) with OTM. With the corrected volatility (IV_c), FBS model with any Hurst value underestimates only in the case of OTM. For put options, Table 2 reveals that all models with predicted volatility (IV_p) underestimate the option values. Even if we consider the moneyness, the same results are observed in Table 4. Contrary to call options, all models with the corrected volatility (IV_c) underestimate only in the case of ITM.

For both call and put options, FBS(H_1) and FBS (H_3) with predicted volatility (IV_p) yield significant pricing errors in the ATM and OTM cases, which contributes to the overall pricing errors in Table 1 and Table 2.

With the corrected volatility, FBS(IH), FBS(H_1) and FBS(H_3) yield almost the same pricing errors across the board, particularly, the pricing errors in the three ITM case. As far as RMSE is concerned, all FBS models with corrected implied volatility produce similar pricing errors, but RMSE decreases in the order of ITM, ATM, and OTM. Thus, it seems to be that the FBS model is the best fit in the OTM case. However, the result of MAPE has the opposite order. This pattern was also documented in Heo et al. (2009). One plausible explanation is the average option call (put) values for ITM, ATM, and OTM are \$9.94 (\$10.64), \$2.93 (\$3.01), and \$1.62 (\$1.64), respectively. So even if RMSE of the OTM case is smaller than that of the ITM case, the percentage error could be greater. For instance, when we examine the RMSE and MAPE of FBS(IH) with IV_c for call options, MAPE is 2.9859% and RMSE is \$0.5331 in the case of ITM and, MAPE is 5.6869% and RMSE is \$0.1225 in the case of OTM. So relative to the call option values (\$9.94 vs. \$1.62), RMSE \$0.5331 is relatively in percentage terms smaller than RMSE \$0.1225. Therefore, FBS is a better fit in the ITM case.

Our results are consistent with the systematic pricing bias documented in Geske & Roll (1984) and reference therein. In our case, all FBS models with the corrected volatility underestimate in the case of OTM for call options (less than -0.25%) and ITM for put options (less than -0.017%), which are very small. This bias is the opposite of that in Black (1975). This may be the case that we only considered the moneyness ignoring time to maturity.

Ⅴ. Conclusion

In this article, we adopt the predictor - corrector method to estimate the accuracy of the FBS model using Euro currency option data (XDE) traded on the NASDAQ from November 2, 2007 to June 30, 2016. The comparison results are presented by variance estimations (Predicted Implied Volatility vs. Corrected Implied Volatility), Hurst parameter, and by moneyness. Our results show that when corrected volatility (IV_c) was used, FBS(IH), FBS(H_1) and FBS (H_3) produce least pricing errors across the board and all three have almost same pricing errors. Hence, when the predictor - corrector technique is applied, FBS model is very sensitive to the choice of volatility but insensitive to Hurst parameter. The Hurst parameter is well studied but it is difficult to measure from real life data [Clegg (2006)]. Hence, our technique makes easy on this problem because our findings show that the accuracy does not heavily depend on the choice of the Hurst parameter if we adopt this technique. However, our study has limitations because we only tested XDE option and used FBS model driven implied Hurst parameter and the Hurst parameter based on the moment method using one and three- month data. As summarized in Biagini et al. (2008), there are several methods to obtain the Hurst parameter. It would be interesting to apply this technique using Hurst parameters derived from different method to other European options and expand this technique to American options.

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