

Beccuti, Juan

Working Paper

On the optimality of price-posting in rental markets

Discussion Papers, No. 20-07

Provided in Cooperation with:

Department of Economics, University of Bern

Suggested Citation: Beccuti, Juan (2020) : On the optimality of price-posting in rental markets, Discussion Papers, No. 20-07, University of Bern, Department of Economics, Bern

This Version is available at:

<https://hdl.handle.net/10419/228895>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.



^b
**UNIVERSITÄT
BERN**

Faculty of Business, Economics
and Social Sciences

Department of Economics

**On the Optimality of Price-posting
in Rental Markets**

Juan Beccuti

20-07

March 2020

DISCUSSION PAPERS

Schanzeneckstrasse 1
CH-3012 Bern, Switzerland
<http://www.vwi.unibe.ch>

On the Optimality of Price-posting in Rental Markets*

Juan Beccuti[†]

Abstract

This paper considers a multi-period setting where a monopolist, with short-term commitment, rents one unit of a durable good to a single consumer in every period. The consumer's valuation constitutes his private information and remains constant over time. By using a mechanism design approach, the paper shows that the optimal renting strategy is to offer a simple price in every period. Although sophisticated mechanisms can make separation feasible when price-posting cannot achieve it, this happens precisely when separation is dominated by pooling. Moreover, the monopolist's choice of whether to discriminate or not depends on a simple and apparently myopic rule, reminiscent of its static equivalent.

Keywords: Durable good, renting, dynamic adverse selection, mechanism design, short-term commitment, price-posting.

JEL: D82, D86, D42.

*I thank Antonio Cabrales, Marco Celentani, Mikhail Drugov, María Angeles de Frutos, Daniel Garcia, Angel Hernando-Veciana, Miguel Kelly, Antoine Loeper, and Marc Möller for valuable discussions and suggestions. Financial support from Swiss National Science Foundation grant no. 178869 “Dynamic Screening without Commitment” is gratefully acknowledged.

[†]Department of Economics, University of Bern. Email: juan.beccuti@vwi.unibe.ch

1. Introduction

This paper considers a monopolist (*she*) renting a durable good to a single non-anonymous consumer (*he*) with unitary demand and private information about his valuation. While the dynamic pricing problem, when the monopolist has short-term commitment, is well understood, the optimality of simple price-posting remains an open issue.¹ The main contribution of this paper is to show that the monopolist cannot gain from offering a more sophisticated contract than a simple price in every period.

The literature on dynamic mechanism design has shown that, when the monopolist *sells* the durable good, posting a price in each period maximizes the monopolist's revenue (Skreta (2006); Doval & Skreta (2019*b*)). Renting differs from selling in that there exist new trade opportunities even after a positive transaction. A two-period analysis of the renting problem can be found in Bolton & Dewatripont (2005), where price-posting turns out to be the optimal mechanism. However, the optimality of price-posting in Bolton & Dewatripont (2005) is an artifact of the assumption that the monopolist and the consumer weigh future payoffs equally (Beccuti & Möller (2018)). In particular, price-posting turns to be suboptimal when the monopolist is more patient than the consumer. This bears the question of whether the optimality of price-posting in Bolton & Dewatripont (2005) is a consequence of restricting the analysis to a two-periods setting. This paper investigates whether price-posting remains the optimal renting mechanism in a multi-period environment.

The increment in the number of periods provides a setting in which the monopolist can engage in a *gradual learning* strategy. Hart & Tirole (1988) show that, when the number of periods is large, a monopolist renting a durable good is unable to price discriminate until the very end of the game. However, their setting restricts the monopolist to deterministic mechanisms: either the monopolist delivers the product after the consumer pays the required price or she does not deliver it at all.² Thus the monopolist's trade-off is simply between pooling (i.e., rent to any consumer at a low price) or price discrimination (i.e., rent to high types only at a high price).

The present paper extends the Hart & Tirole (1988) model by using a mechanism design approach to look for the optimal menu of contracts for each period. In particular, the monopolist is not restricted to set a sequence of prices and, as a consequence, the

¹See, for example, Hart & Tirole (1988). For a renting environment with anonymous consumers see, e.g., Bulow (1982).

²This restriction is present only in their short-term commitment case, while in the rest of the paper, they allow for general mechanisms.

monopolist may achieve separation through random mechanisms. While with deterministic mechanisms, the monopolist separates by refraining from trade with one type, with random mechanisms, the monopolist achieves separation by trading with both types with differing probabilities.

The paper’s main result is that a monopolistic renter cannot gain from proposing mechanisms more sophisticated than simple price-posting so that a restriction to price-posting comes without loss of generality. Consequently, gradual learning does not take place and the monopolist faces a similar trade-off as in the static case. The monopolist’s optimal strategy is to separate types when her belief about facing a high-type consumer is larger than a particular threshold and to pool otherwise.

Related Literature. The paper connects to the literature on dynamic adverse selection, which shows that the *ratchet effect* harms the mechanism designer’s market power due to her lack of long-term commitment (e.g., see the two-period incentive frameworks in Freixas et al. (1985), Laffont & Tirole (1987), and Laffont & Tirole (1988), or Hart & Tirole (1988) for the dynamic rental pricing analysis. Bikhchandani & McCardle (2012) study the dynamic pricing strategy when the monopolist and the consumer are not equally patient, while Devanur et al. (2019) study such Hart & Tirole (1988) environment for different levels of monopolist’s commitment.) The addition of an additional contractual dimension besides time may facilitate discrimination. Indeed, in Wang (1998), a principal facing an infinite time horizon achieves immediate separation of agents by specifying a menu of contracts with different qualities. Similarly, in Gerardi & Maestri (2018), immediate separation results from allowing the informed party to abandon the relationship in every period to claim an outside option. The main result of this paper supports the common view that time, as the only screening dimension, is not sufficient to discriminate types.

2. Model

A monopolist and a consumer interact repeatedly during a finite number of periods $T \geq 2$. In each period t , the monopolist rents one unit of a durable good produced at zero cost.³ The consumer demands one unit of the good per period and is privately informed about his per-period valuation, denoted as his *type*. The consumer’s type can be low ($\theta_L > 0$) or high ($\theta_H > \theta_L$) and is constant across periods. Define $\Delta\theta = \theta_H - \theta_L$ and let $\beta \in (0, 1)$ denote the monopolist’s (prior) belief about the consumer’s probability of having a high type. Both players discount the future with the same discount factor $\delta \in (0, 1)$.

³The environment considered in the present paper is equivalent to the repeated sale of a perishable good.

The monopolist has short-term commitment, i.e., she can commit to the renting mechanism only for the current but not for future periods. In a setting in which there is direct communication between the consumer and the monopolist, as the one considered here, Bester & Strausz (2001) show that a modified revelation principle applies:⁴ It allows the monopolist to restrict to direct mechanisms and requires the consumer to reveal his true type only with strictly positive probability but not with certainty as in the static setting (Myerson (1981)).

Thus, at the beginning of every period, the monopolist offers a direct mechanism that specifies a payment from the consumer to the monopolist $w_m \in \mathfrak{R}$ and a likelihood of product-delivery $x_m \in [0, 1]$ conditional on the consumer's message $m \in \{l, h\}$. Let $q_L \in [0, 1)$ and $q_H \in (0, 1]$ denote the probability with which the low- and the high-type consumer reports $m = h$ respectively. After observing a message the monopolist updates her belief about the consumer's type following Bayes' rule: $\beta_h \equiv \frac{\beta q_H}{Q}$ and $\beta_l \equiv \frac{\beta(1-q_H)}{1-Q}$, where $Q \equiv \beta q_H + (1-\beta)q_L$ is the ex-ante likelihood that the consumer reports a high type. Without loss of generality, the analysis focuses on the case where $q_H \geq q_L$ (equivalently $\beta_h \geq \beta \geq \beta_l$). When $q_H < q_L$, it is possible to rename messages and to interchange their roles.

When the consumer reports m his (instantaneous) surplus is given by $x_m\theta - w_m$ while the monopolist's (instantaneous) payoff is w_m . In the next period, the monopolist uses her updated belief β_m to propose a new mechanism. In what follows, $V_{t+1}(\beta_m)$ denotes the monopolist's continuation value for period t . Similarly, $U_{t+1}^H(\beta_m)$ and $U_{t+1}^L(\beta_m)$ denote the high- and the low-type consumer's continuation values respectively.

Using Perfect Bayesian Equilibrium as the solution concept, the monopolist's proposal at any period must be sequentially optimal given the reporting history up to that period. This is, the monopolist looks for the mechanism that maximizes her expected payoff taking into account the consumer's strategic behavior and her (potentially updated) belief about the consumer's type. Therefore, in period t , the monopolist solves

$$\max_{x_l, x_h, w_l, w_h, q_L < 1, q_H > 0} Q[w_h + \delta V_{t+1}(\beta_h)] + (1-Q)[w_l + \delta V_{t+1}(\beta_l)],$$

subject to

$$x_h\theta_H - w_h + \delta U_{t+1}^H(\beta_h) \geq x_l\theta_H - w_l + \delta U_{t+1}^H(\beta_l), \quad (\text{with equality when } q_H < 1) \quad (IC_H)$$

$$x_l\theta_L - w_l + \delta U_{t+1}^L(\beta_l) \geq x_h\theta_L - w_h + \delta U_{t+1}^L(\beta_h), \quad (\text{with equality when } q_L > 0) \quad (IC_L)$$

$$x_h\theta_H - w_h + \delta U_{t+1}^H(\beta_h) \geq 0, \quad (PC_H)$$

$$x_l\theta_L - w_l + \delta U_{t+1}^L(\beta_l) \geq 0. \quad (PC_L)$$

⁴See Doval & Skreta (2019a) for a setting with general *communication devices*.

At any period, the consumer could choose not to participate and wait for the next period. In this case, the consumer not only gets zero instantaneous surplus, it is also assumed that he gets zero continuation values.⁵ Note also that, at some particular beliefs, the monopolist is indifferent among several mechanisms. To avoid more than one continuation equilibrium, the analysis assumes that the monopolist can credibly “promise” to offer the mechanism that reduces the consumer’s reluctance to report his type.⁶ For future references, a mechanism is a price-posting mechanism when there exist a $x_m \in \{0, 1\}$ and a price $p \in \Re$ such that $w_m = px_m$ for all $m \in \{l, h\}$.

The monopolist’s problem can be simplified using standard techniques.⁷ In particular, in any period the monopolist chooses x_h, x_l, q_L, q_H to solve the *reduced program*:

$$\begin{aligned} \max_{x_h, x_l, q_L < 1, q_H > 0} \quad & x_l \theta_L + Q \theta_H (x_h - x_l) + \\ & + (1 - Q) \delta V_{t+1}(\beta_l) + Q \delta \{V_{t+1}(\beta_h) - [U_{t+1}^H(\beta_l) - U_{t+1}^H(\beta_h)]\}, \end{aligned} \quad (1)$$

subject to

$$x_h - \frac{\delta}{\Delta \theta} [U_{t+1}^H(\beta_l) - U_{t+1}^H(\beta_h)] \geq x_l, \quad \text{with equality if } q_L > 0. \quad (DMC)$$

The proof uses the fact that, as in the static case, the participation constraint of the high-type (PC_H) is redundant, and the participation of the low-type (PC_L) is binding at the optimum (implying $U_{t+1}^L = 0$ for any t). However, in contrast to the one-period setting, it is not clear which incentive constraint is binding at the optimum. Suppose (IC_L) is binding while (IC_H) is slack. Since (PC_L) is also binding, the price w_h is then equal to $x_h \theta_L$. Alternatively, if (IC_H) is the binding constraint, and since $w_l = x_l \theta_L$ (from (PC_L) binding), then

$$\begin{aligned} w_h &= x_l \theta_L + (x_h - x_l) \theta_H - \delta [U_{t+1}^H(\beta_l) - U_{t+1}^H(\beta_h)], \\ &= x_l \theta_L + (x_h - x_l) \theta_H - \delta [U_{t+1}^H(\beta_l) - U_{t+1}^H(\beta_h)] + x_h \theta_L - x_h \theta_L, \\ &= (x_h - x_l) \Delta \theta - \delta [U_{t+1}^H(\beta_l) - U_{t+1}^H(\beta_h)] + x_h \theta_L, \\ &\geq x_h \theta_L, \end{aligned}$$

⁵Since Bayes’ rule does not apply after non-participation, it can be assumed that, in such a case, the monopolist assigns probability one to face a high-type consumer. Hence, the monopolist sets a high price in the next period and the consumer’s continuation value is zero.

⁶For instance, in the last period T , the monopolist is indifferent between pooling and separating when $\beta = \theta_L / \theta_H$. Thus, when the monopolist’s prior belief is higher than θ_L / θ_H in $T - 1$, she promises to offer separation if her posterior belief becomes θ_L / θ_H after observing message l . Using the (IC_H), it can be seen that the incentives to truthfully report for the high-type consumer is larger when $U_T^H(\beta_l) = 0$ (due to future separation) than when $U_T^H(\beta_l) = \Delta \theta$ (due to future pooling). For the low-type, continuation values are zero under both mechanisms.

⁷A formal proof can be found in Beccuti & Möller (2019).

where the last inequality is due to the (DMC) . Thus, by making (IC_H) binding, the monopolist cannot be worse off.

The Dynamic Monotonicity Constraint (DMC) results from substituting the binding constraint (IC_H) into (IC_L) . It not only requires the allocation to be increasing in the reported type but also imposes a wedge between the high- and the low-type's allocation. This wedge is a consequence of the *ratchet effect*: If the high-type consumer makes his type public, he will lose his future information rents (i.e., $U_{t+1}^H(\beta_l) - U_{t+1}^H(\beta_h)$). Therefore, to induce the consumer to reveal his type, the monopolist has to compensate him. In particular, when the monopolist does not want to refrain from trade with the low-type consumer, the (DMC) determines that the compensation comes in form of a reduction in x_l . Notice that $\delta = 0$ recovers the static case and its standard monotonicity constraint, when the ratchet effect does not play any role and there is no need for such compensation.

The degree of separation that the monopolist can achieve is restricted by the (DMC) . Note, first, that an increment in x_h improves the monopolist's objective while it relaxes the constraint, implying $x_h^* = 1$. It follows that, separation by price-posting, i.e., with $x_l = 0$, may not be feasible if the number of periods remaining is sufficiently large. For instance, fully separating types via price-posting (with both types reporting truthfully) is not feasible when δ is sufficiently high since $\delta[U_{t+1}^H(0) - U_{t+1}^H(1)] = \delta[\sum_{i=0}^{T-(t+1)} \delta^i \Delta\theta] > \Delta\theta$. As an alternative, the monopolist may offer a semi-separating price-posting mechanism inducing $0 < \beta_l < \beta_h = 1$. Hart & Tirole (1988) show that, if the number of periods is large enough, even such semi-separating price-posting is not feasible (i.e., $\delta[U_{t+1}^H(\beta_l) - U_{t+1}^H(1)] > \Delta\theta$) and hence, when the monopolist is restricted to price-posting mechanism, she can only offer pooling.

However, for the same number of periods and monopolist's prior belief, the monopolist may still achieve some separation by offering a random delivery contract (i.e., $x_l \in (0, 1)$) and making (DMC) binding with $q_L > 0$, i.e., she can choose another pair $q_L > 0, q_H < 1$ such that $1 - \frac{\delta}{\Delta\theta}[U_{t+1}^H(\beta_l) - U_{t+1}^H(\beta_h)] = x_l > 0$. The choice between pooling and this type of semi-separation is absent in Hart & Tirole (1988), due to their restriction to simple price-posting mechanisms.

3. Optimality of price-posting

The next proposition presents the main result of the paper.

Proposition 1. *Independently of the number of remaining periods, the profit-maximizing renting mechanism is deterministic, $x_l, x_h \in \{0, 1\}$, i.e., it can be implemented by simple*

price-posting.

All formal proofs can be found in the Appendix. In the following, we explain the intuition for the result of Proposition 1.

In any period, the monopolist has two alternatives. She may pool types (i.e., $x_l = x_h^* = 1$), giving up to the possibility of learning about the consumer's valuation. Alternatively, she may induce separation either by refraining from trade with the low-type altogether or by offering him a random allocation. Note that the objective in the reduced program (1) is linear in x_l and decreases in x_l if $Q \geq \theta_L/\theta_H$. Therefore, if the monopolist chooses to separate types by inducing $Q \geq \theta_L/\theta_H$, then she does it optimally by setting $x_l^* = 0$. On the other hand, when the monopolist separates with $Q < \theta_L/\theta_H$, then her payoff increases with the likelihood of trading with the low-type. Thus, the monopolist would like x_l to be different but as close as possible to x_h^* . However, the (*DMC*) imposes a restriction for the maximum value of x_l . This restriction arises as a cost of separation and, since the monopolist expects to receive a message l with high probability, she is better off by not reducing the likelihood of trading with the low-type, i.e., by pooling types.

As a result, the monopolist has only to decide whether to pool types or to separate them by incurring the cost of not trading with the low-type. As in the two-period setting, when the monopolist offers a separating price-posting, she does it by inducing true telling from the low-type consumer. With such a mechanism, she is certain of facing a high-type after observing the message h . Therefore, for $q_L = 0$, it must hold that $1 - \frac{\delta}{\Delta\theta} U_{t+1}^H(\beta_l) > 0$ since the monopolist can only offer a mechanism that satisfies the (*DMC*). As $U_{t+1}^H(\beta)$ is decreasing in β , we can define $\hat{\beta}_l$ as the smallest β_l for which this inequality is still satisfied. In correspondence to $\hat{\beta}_l$, let \hat{q}_H denote the likelihood with which the high-type is required to tell the truth to induce the posterior $\hat{\beta}_l$. With the help of \hat{q}_H , the next corollary states a rule governing the monopolist's choice between pooling and separation.

Corollary 1. *The monopolist finds it optimal to separate types when $\beta\hat{q}_H\theta_H \geq \theta_L$, and to pool them otherwise.*

Whether it is optimal to induce information revelation or not depends on a simple and apparently myopic rule. To choose between separation and pooling, in any given period the monopolist only needs to compare the payoffs of that period, disregarding all future payoffs. In particular, separation is optimal if the maximum feasible present period payoff from inducing the low-type to report truthfully is larger than the payoff from pooling both types, i.e., if $\beta \cdot \hat{q}_H \cdot \theta_H \geq \theta_L$.⁸

⁸Note, however, that the monopolist needs to take into account continuation values to decide whether to fully or to semi-separate.

Continuation values matter only in that they determine how much separation the monopolist can possibly induce. This limitation goes unnoticed in a setting with two periods because, with only one period to go, future information rents can never exceed the information rents of the current period. Only by considering the case with more than two periods, it becomes clear that the degree of separation the monopolist can achieve can become restricted, making her choice between pooling and separation non-trivial. The following example illustrates these points.

Example. *The rule states that the monopolist's ability to learn is limited according to*

$$\frac{\Delta\theta}{\delta} > U_{t+1}^H(\beta_l), \quad (2)$$

with $\beta_h = 1$ (i.e., $q_L = 0$), and $\beta_l = \frac{\beta(1-q_H)}{1-\beta q_H}$ from Bayes' rule.

In a two-period settings, the consumer's continuation values are (see, e.g., Bolton & Dewatripont (2005))

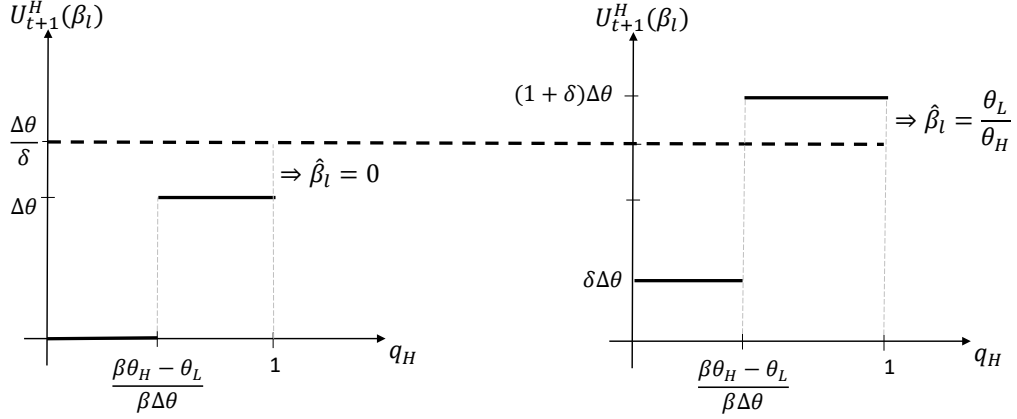
$$U_{t+1}^H(\beta) = \begin{cases} 0 & \text{if } \beta \geq \frac{\theta_L}{\theta_H}, \\ \Delta\theta & \text{if } \beta < \frac{\theta_L}{\theta_H}, \end{cases}$$

while in the three-period case,

$$U_{t+1}^H(\beta) = \begin{cases} 0 & \text{if } \beta > \frac{\theta_L}{\theta_H} \frac{\theta_H + \delta\Delta\theta}{\theta_L + \delta\Delta\theta} \\ \delta\Delta\theta & \text{if } \beta \in \left[\frac{\theta_L}{\theta_H}, \frac{\theta_L}{\theta_H} \frac{\theta_H + \delta\Delta\theta}{\theta_L + \delta\Delta\theta} \right] \\ (1 + \delta)\Delta\theta & \text{if } \beta < \frac{\theta_L}{\theta_H} \end{cases}.$$

Note that, in the former case, the condition (2) holds for any β_l , while in the latter case, the monopolist cannot induce $\beta_l < \theta_L/\theta_H$ when δ is sufficiently large.

The following picture illustrates the rule for the two-period setting (at the left), and for the three-period one (at the right) when the monopolist's prior belief $\beta \in \left[\frac{\theta_L}{\theta_H}, \frac{\theta_L}{\theta_H} \frac{\theta_H + \delta\Delta\theta}{\theta_L + \delta\Delta\theta} \right]$.



Let $\theta_L = 1$, $\theta_H = 2$, $\beta = 2/3$, and $\delta = 3/4$. In the two-period setting, there is no limit, $\beta \cdot 1 \cdot \theta_H = 4/3 > 1$, and the monopolist separates types. However, when $T = 3$, $\hat{\beta}_l = 0.5$ and, from Bayes' rule, $\hat{q}_H = \frac{\beta - \hat{\beta}_l}{\beta(1 - \hat{\beta}_l)} = 0.5$. It follows that $\beta \cdot \hat{q}_H \cdot \theta_H = 2/3 < 1$ and the monopolist finds it optimal to pool types. Alternatively, if $\beta > 3/4$, then $\beta \cdot \hat{q}_H \cdot \theta_H > 1$ and the monopolist offers separation.

4. Conclusion

The dynamic pricing problem when a monopolistic renter has short-term commitment has been broadly studied. In particular, Hart & Tirole (1988) show that such a monopolist cannot price discriminate consumers until the very end of the horizon. This paper uses a mechanism design approach to derive the optimal renting strategy and shows that a restriction to price-posting comes without loss of generality. The choice between separation or pooling follows a simple and apparently myopic rule that only considers current payoffs. In particular, in a two consumer-types setting, separation is optimal when the monopolist can induce types to reveal themselves in a way that makes the likelihood of renting at a high price larger than the ratio of types.

Appendix A - Proofs

Proof of Proposition 1: The discussion following the reduced program at (1) already shows that $x_h = 1$ in any period. It remains to look for the optimal x_l . The next lemma states some useful properties to proceed with the proof of the proposition.

Lemma 1. *In every period t , the monopolist's payoffs satisfy:*

- If $\beta \leq \theta_L/\theta_H$: $V_t(\beta) = \theta_L + \delta V_{t+1}(\beta)$.
- If $\beta \geq \theta_L/\theta_H$: $\theta_L + \delta V_{t+1}(\beta) \leq V_t(\beta) \leq \begin{cases} \beta \hat{q}_H \theta_H + \delta V_{t+1}(\beta), & \text{if } \hat{q}_H \geq \frac{\theta_L}{\theta_H \beta}, \\ \theta_L + \delta V_{t+1}(\beta), & \text{otherwise,} \end{cases}$,

where $\hat{q}_H \equiv \frac{\beta - \hat{\beta}_l}{\beta(1 - \hat{\beta}_l)}$ and $\hat{\beta}_l \equiv \inf\{\beta_l : U_{t+1}^H(\beta_l) \leq \frac{\Delta\theta}{\delta}\}$.

Proof of Lemma 1: If $\beta \leq \theta_L/\theta_H$, by renting to both types of consumers, the monopolist achieves the solution under commitment.

When $\beta \geq \theta_L/\theta_H$, the monopolist can always achieve what she would get by pooling types ($w_h = w_l = \theta_L$, $x_h = x_l = 1$), which implies (w.l.g.) that her posterior belief would be equal to her prior belief. For the upper-bound, the analysis proceed by induction. The last two periods are well known (see, e.g., Bolton & Dewatripont (2005), Chapter 9). Period T payoffs are

$$V_T(\beta) = \begin{cases} \beta q_H^* \theta_H \\ \theta_L \end{cases}, \quad U_T^H(\beta) = \begin{cases} 0 & \text{if } \beta \geq \frac{\theta_L}{\theta_H} \\ \Delta\theta & \text{if } \beta \leq \frac{\theta_L}{\theta_H} \end{cases}, \quad (3)$$

with the optimal $q_H^* = 1$, while period $T - 1$ payoffs are,

$$V_{T-1}(\beta) = \begin{cases} \beta q_H^* \theta_H + \delta \beta \theta_H \\ \beta q_H^* \theta_H + \delta \theta_L \\ \theta_L + \delta \theta_L \end{cases}, \quad U_{T-1}^H(\beta) = \begin{cases} 0 & \text{if } \beta \geq \frac{\theta_L}{\theta_H} \frac{\theta_H + \delta \Delta\theta}{\theta_L + \delta \Delta\theta} \\ \delta \Delta\theta & \text{if } \beta \in \left[\frac{\theta_L}{\theta_H}, \frac{\theta_L}{\theta_H} \frac{\theta_H + \delta \Delta\theta}{\theta_L + \delta \Delta\theta} \right] \\ (1 + \delta) \Delta\theta & \text{if } \beta \leq \frac{\theta_L}{\theta_H} \end{cases}, \quad (4)$$

with $q_H^* = \frac{\beta \theta_H - \theta_L}{\beta \Delta\theta}$ when $\beta \geq \frac{\theta_L}{\theta_H} \frac{\theta_H + \delta \Delta\theta}{\theta_L + \delta \Delta\theta}$ and $q_H^* = 1$ when $\beta \in \left[\frac{\theta_L}{\theta_H}, \frac{\theta_L}{\theta_H} \frac{\theta_H + \delta \Delta\theta}{\theta_L + \delta \Delta\theta} \right]$. It is straightforward to check that the lemma holds in $T - 1$ and T since $\frac{\theta_L}{\theta_H} < q_H^* \leq \hat{q}_H \leq 1$.

Now, let us assume the lemma also holds for $t + 1$.

Let \hat{q}_{Hm} denote \hat{q}_H when the monopolist's prior belief is β_m where $m = \{l, h\}$. After some algebraic manipulations, the solution of the reduced program 1 has an upper-bound determined by the solution of

$$\begin{aligned} & \max_{x_l, q_L < 1, q_H > 0} x_l \theta_L + Q \theta_H (1 - x_l) + \\ & \quad + \delta [(1 - Q) \beta_l \hat{q}_{Hl} \theta_H + Q \beta_h \hat{q}_{Hh} \theta_H] \\ & \quad + \delta^2 [(1 - Q) V_{t+2}(\beta_l) + Q \{V_{t+2}(\beta_h) - [U_{t+2}^H(\beta_l) - U_{t+2}^H(\beta_h)]\}] \quad (5) \\ \text{subj. to:} \quad & 1 - \frac{\delta^2}{\Delta\theta} [U_{t+2}^H(\beta_l) - U_{t+2}^H(\beta_h)] \geq x_l, \quad \text{with equality if } q_L > 0, \end{aligned}$$

since the objective is larger than in the reduced program and the constraint has been relaxed. To see the latter, note that the monopolist extracts a weakly larger surplus when her belief increases:

$$[U_{t+1}^H(\beta_l) - U_{t+1}^H(\beta_h)] \geq \delta[U_{t+2}^H(\beta_l) - U_{t+2}^H(\beta_h)] \geq 0, \quad (6)$$

and, therefore,

$$1 - \frac{\delta^2}{\Delta\theta}[U_{t+2}^H(\beta_l) - U_{t+2}^H(\beta_h)] \geq 1 - \frac{\delta}{\Delta\theta}[U_{t+1}^H(\beta_l) - U_{t+1}^H(\beta_h)] \geq x_l. \quad (7)$$

Recall that, by definition, \hat{q}_H is decreasing in the posterior belief induced by it. Suppose w.l.g. that \hat{q}_{Hl} yields a posterior (denoted by $\hat{\beta}_u$) lower than the one yielded by \hat{q}_{Hh} (denoted by $\hat{\beta}_h$).⁹ Then

$$\begin{aligned} (1-Q)\beta_l\hat{q}_{Hl} + Q\beta_h\hat{q}_{Hh} &= (1-Q)\beta_l\frac{\beta_l - \hat{\beta}_u}{\beta_l(1 - \hat{\beta}_u)} + Q\beta_h\frac{\beta_h - \hat{\beta}_h}{\beta_h(1 - \hat{\beta}_h)} \\ &\leq (1-Q)\beta_l\frac{\beta_l - \hat{\beta}_u}{\beta_l(1 - \hat{\beta}_u)} + Q\beta_h\frac{\beta_h - \hat{\beta}_u}{\beta_h(1 - \hat{\beta}_u)} \\ &= \beta\frac{\beta - \beta_l}{\beta(1 - \beta_l)} \\ &= \beta\tilde{q}_H. \end{aligned} \quad (8)$$

Since, by assumption, β_l is feasible at the reduced program, it is also feasible at program (5) due to the relaxed constraint. It follows that $\beta_l \geq \hat{\beta}_l$ and $\tilde{q}_H \leq \hat{q}_H$ by definition of $\hat{\beta}_l$ and \hat{q}_H respectively. Hence, adding and subtracting $\delta(x_l\theta_L + Q\theta_H(1 - x_l))$ to the previous objective, the solution of program (5) has an upper-bound given by

$$\begin{aligned} &\max_{x_l, q_L < 1, q_H > 0} (1 - \delta)[x_l\theta_L + Q\theta_H(1 - x_l)] + \delta\beta\hat{q}_H\theta_H + \\ &+ \delta[x_l\theta_L + Q\theta_H(1 - x_l) + \delta(1 - Q)V_{t+2}(\beta_l) + \delta Q\{V_{t+2}(\beta_h) - [U_{t+2}^H(\beta_l) - U_{t+2}^H(\beta_h)]\}], \\ \text{subj. to:} \quad &1 - \frac{\delta^2}{\Delta\theta}[U_{t+2}^H(\beta_l) - U_{t+2}^H(\beta_h)] \geq x_l, \quad \text{with equality if } q_L > 0. \end{aligned} \quad (9)$$

The expression in the second line of the objective function is the reduced program at $t + 1$ subject to a relaxed constraint.¹⁰ Thus, by the inductive argument, that expression is at

⁹Although out of the scope of this paper, it can be shown that this supposition is correct. Intuitively, an *optimistic* monopolist (i.e., $\beta \geq \theta_L/\theta_H$) wants to remain as optimistic as possible when she separate types.

¹⁰The allocation x_l corresponds to period t and not to $t + 1$. However, since the new constraint relaxes compare to the one at t , the solution of the expression in curly brackets contains the solution of the reduced program in next period.

least $V_{t+1}(\beta)$. Hence, the upper-bound (9) can be written as

$$\begin{aligned} & \max_{x_l, q_L < 1, q_H > 0} (1 - \delta)[x_l \theta_L + Q \theta_H (1 - x_l)] + \delta \beta \hat{q}_H \theta_H + \delta V_{t+1}(\beta) \\ \text{subj. to:} \quad & 1 - \frac{\delta^2}{\Delta \theta} [U_{t+2}^H(\beta_l) - U_{t+2}^H(\beta_h)] \geq x_l, \quad \text{with equality if } q_L > 0. \end{aligned}$$

Because $(1 - \delta)[x_l \theta_L + Q \theta_H (1 - x_l)] + \delta \beta \hat{q}_H \theta_H$ is linear in x_l , the allocation x_l takes one of the extreme values. Note that when $Q \leq \theta_L / \theta_H$, that expression increases with x_l . However, if $Q \leq \theta_L / \theta_H$, then $\hat{q}_H \leq \theta_L / \theta_H$ and the expression is lower or equal than θ_L even for $x_l = 1$. On the other hand, when $Q \geq \theta_L / \theta_H$, by reducing x_l the objective increases while the constraint relaxes, i.e., $x_l = 0$. As a consequence, the constraint holds with strict inequality since

$$1 - \frac{\delta^2}{\Delta \theta} [U_{t+2}^H(\beta_l) - U_{t+2}^H(\beta_h)] > 1 - \frac{\delta}{\Delta \theta} [U_{t+1}^H(\beta_l) - U_{t+1}^H(\beta_h)] = 0$$

when $U_{t+1}^H(\beta_l) - U_{t+1}^H(\beta_h) = \frac{\Delta \theta}{\delta}$, or

$$1 - \frac{\delta^2}{\Delta \theta} [U_{t+2}^H(\beta_l) - U_{t+2}^H(\beta_h)] \geq 1 - \frac{\delta}{\Delta \theta} [U_{t+1}^H(\beta_l) - U_{t+1}^H(\beta_h)] > 0$$

when $U_{t+1}^H(\beta_l) - U_{t+1}^H(\beta_h) < \frac{\Delta \theta}{\delta}$. Hence, $q_L = 0$ and $Q = \beta q_H \leq \beta \hat{q}_H$ by definition of \hat{q}_H .

Therefore, the solution of the reduced program at (1) has an upper-bound of $\beta \hat{q}_H \theta_H + \delta V_{t+1}(\beta)$ when $\hat{q}_H \geq \theta_L / \theta_H$, and of $\theta_L + \delta V_{t+1}(\beta)$ otherwise. This completes the inductive argument of Lemma 1.

Proof of Proposition 1: At period t the monopolist solves the reduced program (1). Because the objective is linear in x_l , then $x_l^* \in \{0, 1 - \frac{\delta}{\Delta \theta} [U_{t+1}^H(\beta_l) - U_{t+1}^H(\beta_h)], 1\}$, with the allocation $x_l = 1 - \frac{\delta}{\Delta \theta} [U_{t+1}^H(\beta_l) - U_{t+1}^H(\beta_h)] \in (0, 1)$ as the only one that is not a price-posting.

Assuming it is only feasible to induce $Q \leq \theta_L / \theta_H$, then it must be that $\beta \hat{q}_H \leq \theta_L / \theta_H$ and, by Lemma 1, the monopolist cannot get larger payoffs than those under pooling, i.e., $x_l = 1$.

Now, assume it is feasible to induce $Q \geq \theta_L / \theta_H$. As argued above, for such Q a lower x_l increases the monopolist's objective while relaxes the *(DMC)*. Thus, if the monopolist wants to offer a mechanism with such Q , her optimal strategy would be to separate with a price-posting that allocates $x_l^* = 0$.

On the other hand, if the monopolist wants to induce $Q < \theta_L / \theta_H$ (when a larger Q is feasible), the objective is increasing in x_l and the *(DMC)* becomes binding: $x_l =$

$\min\{1 - \frac{\delta}{\Delta\theta}[U_{t+1}^H(\beta_l) - U_{t+1}^H(\beta_h)], 1\}$. Hence, she may pool types (i.e., $x_l = x_h^* = 1$) or separate them by offering the non price-posting $x_l = 1 - \frac{\delta}{\Delta\theta}[U_{t+1}^H(\beta_l) - U_{t+1}^H(\beta_h)] \in (0, 1)$.

Next, we proceed by induction to show that the monopolist prefers pooling than separation with such non price-posting. From (3) and (4), it follows that the proposition holds for periods $T - 1$ and T . Suppose it also holds at $t + 1$ (i.e., setting a price is optimal) when the monopolist gets $V_{t+1}(\beta)$.

At period t , after plugging $x_l = 1 - \frac{\delta}{\Delta\theta}[U_{t+1}^H(\beta_l) - U_{t+1}^H(\beta_h)]$ into the reduced program (1) and after some algebraic operations, the monopolist solves:

$$\begin{aligned} \max_{q_L < 1, q_H > 0} \quad & \theta_L + (1 - Q)\delta V_{t+1}(\beta_l) + Q\delta V_{t+1}(\beta_h) \\ & - (1 - Q)\delta \frac{\theta_L}{\Delta\theta}[U_{t+1}^H(\beta_l) - U_{t+1}^H(\beta_h)], \quad (10) \\ \text{s.t.:} \quad & Q < \theta_L/\theta_H, \quad [U_{t+1}^H(\beta_l) - U_{t+1}^H(\beta_h)] \in (0, \frac{\Delta\theta}{\delta}). \end{aligned}$$

Together with Lemma 1, the inequalities (6), and the equation (8), the previous program has an upper-bound of

$$\begin{aligned} \max_{q_L < 1, q_H > 0} \quad & \theta_L + \delta\{\beta\tilde{q}_H\theta_H + (1 - Q)\delta V_{t+2}(\beta_l) + Q\delta V_{t+2}(\beta_h) \\ & - (1 - Q)\delta \frac{\theta_L}{\Delta\theta}[U_{t+2}^H(\beta_l) - U_{t+2}^H(\beta_h)]\}, \quad (11) \\ \text{s.t.:} \quad & Q < \theta_L/\theta_H, \quad [U_{t+2}^H(\beta_l) - U_{t+2}^H(\beta_h)] \in (0, \frac{\Delta\theta}{\delta^2}). \end{aligned}$$

Because $Q < \theta_L/\theta_H$, it follows that $\beta\tilde{q}_H < \theta_L/\theta_H$. Thus, previous objective is not larger than

$$\theta_L + \delta\{\theta_L + (1 - Q)\delta V_{t+2}(\beta_l) + Q\delta V_{t+2}(\beta_h) - (1 - Q)\delta \frac{\theta_L}{\Delta\theta}[U_{t+2}^H(\beta_l) - U_{t+2}^H(\beta_h)]\}.$$

Notice that the expression in curly brackets is the objective (10) when the monopolist offers the non price-posting at period $t + 1$ with $Q < \theta_L/\theta_H$. However, by the induction argument, at $t + 1$ the monopolist finds it optimal to offer a price-posting. In particular, with such low Q , the monopolist offers pooling. Thus, previous expression has an upper-bound of $\theta_L + \delta V_{t+2}(\beta)$ and hence of $V_{t+1}(\beta)$ due to Lemma 1. Therefore, when $x_l = 1 - \frac{\delta}{\Delta\theta}[U_{t+1}^H(\beta_l) - U_{t+1}^H(\beta_h)] \in (0, 1)$, in period t the monopolist cannot achieve a payoff larger than under pooling, i.e., $\theta_L + \delta V_{t+1}(\beta)$, concluding the induction argument. ■

Proof of Corollary 1:

The proof of Proposition 1 has shown that the monopolist offers pooling if she wants to induce $Q \leq \theta_L/\theta_H$. On the other hand, separation with $x_l^* = 0$ occurs if the monopolist wants $Q \geq \theta_L/\theta_H$. It remains to determine the level of Q to be induced by the monopolist.

Assume that the monopolist separates with $q_L = 0$ (i.e., $\beta_h = 1$). Then, $Q = \beta q_H \geq \theta_L/\theta_H$ must be feasible, i.e., q_H must be such that (DMC) holds: $U_{t+1}^H(\beta_l) - U_{t+1}^H(\beta_h) \leq \frac{\Delta\theta}{\delta}$. The monopolist solves:

$$\begin{aligned} \max_{q_H} \quad & \beta q_H \theta_H + (1 - \beta q_H) \delta V_{t+1}(\beta_l) + \beta q_H \delta [V_{t+1}(1) - U_{t+1}^H(\beta_l)], \\ \text{s.t.} \quad & q_H \geq \frac{\theta_L}{\theta_H \beta}. \end{aligned} \tag{12}$$

On the other hand, assume $Q \geq \theta_L/\theta_H$ with $q_L \neq 0$. In this case, q_L, q_H must be such that $U_{t+1}^H(\beta_l) - U_{t+1}^H(\beta_h) = \frac{\Delta\theta}{\delta}$ to have $x_l = 0$. Now, the monopolist solves

$$\begin{aligned} \max_{q_L, q_H} \quad & Q \theta_L + (1 - Q) \delta V_{t+1}(\beta_l) + Q \delta V_{t+1}(\beta_h), \\ \text{s.t.} \quad & U_{t+1}^H(\beta_l) - U_{t+1}^H(\beta_h) = \frac{\Delta\theta}{\delta} \end{aligned}$$

This objective has an upper bound of

$$\begin{aligned} & \theta_L + (1 - Q) \delta V_{t+1}(\beta_l) + Q \delta V_{t+1}(\beta_h), \\ & \leq \theta_L + \delta \beta \hat{q}_H + \delta^2 [(1 - Q) V_{t+2}(\beta_l) + Q V_{t+2}(\beta_h)], \end{aligned}$$

by application of Lemma 1, equation (8) and the discussion that followed. Note that the last line is the payoff that the monopolist would get after pooling in period t and postponing screening to the next period. However, pooling is dominated by immediate separation when $Q \geq \theta_L/\theta_H$.

Therefore, q_h^* solves problem (12) in which $\beta \hat{q}_H \geq \theta_L/\theta_H$ by assumption. Additionally, since $\hat{q}_H \leq 1$, it follows that $\beta \geq \theta_L/\theta_H$ and, by Lemma 1, the lower bound for monopolist's payoff comes from pooling. Hence, the monopolist is better off by separating types when $\beta \hat{q}_H \geq \theta_L/\theta_H$. ■

References

- Beccuti, J. & Möller, M. (2018), ‘Dynamic adverse selection with a patient seller’, *Journal of Economic Theory* **173**, 95–117.
- Beccuti, J. & Möller, M. (2019), ‘Screening by mode of trade’, *Unpublished Manuscript*.
- Bester, H. & Strausz, R. (2001), ‘Contracting with imperfect commitment and the revelation principle: The single agent case’, *Econometrica* **69**(4), 1077–1098.

- Bikhchandani, S. & McCardle, K. (2012), ‘Behavior-based price discrimination by a patient seller’, *The BE Journal of Theoretical Economics* **12**(1).
- Bolton, P. & Dewatripont, M. (2005), *Contract Theory*, MIT Press.
- Bulow, J. (1982), ‘Durable-goods monopolists’, *Journal of Political Economy* **90**(2), 314–332.
- Devanur, N., Peres, Y. & Sivan, B. (2019), ‘Perfect bayesian equilibria in repeated sales’, *Games and Economic Behavior* .
- Doval, L. & Skreta, V. (2019a), ‘Mechanism design with limited commitment’, *Unpublished Manuscript* .
- Doval, L. & Skreta, V. (2019b), ‘Optimal mechanism for the sale of a durable good’, *Unpublished Manuscript* .
- Freixas, X., Guesnerie, R. & Tirole, J. (1985), ‘Planning under incomplete information and the ratchet effect’, *Review of Economic Studies* **52**, 173–191.
- Gerardi, D. & Maestri, L. (2018), ‘Dynamic contracting with limited commitment and the ratchet effect’, *Unpublished Manuscript* .
- Hart, O. & Tirole, J. (1988), ‘Contract renegotiation and coasian dynamics’, *Review of Economic Studies* **55**(4), 509–540.
- Laffont, J. & Tirole, J. (1987), ‘Comparative statics of the optimal dynamic incentive contract’, *European Economic Review* **31**(4), 901–926.
- Laffont, J. & Tirole, J. (1988), ‘The dynamics of incentive contracts’, *Econometrica* **56**, 1153–1175.
- Myerson, R. (1981), ‘Optimal auction design’, *Mathematics of Operations Research* **6**(1), 58–73.
- Skreta, V. (2006), ‘Sequentially optimal mechanisms’, *Review of Economic Studies* **73**(4), 1085–1111.
- Wang, G. (1998), ‘Bargaining over a menu of wage contracts’, *Review of Economic Studies* **65**(2), 295–305.