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Dynamic Effects of Monetary Policy Shocks on Macroeconomic Volatility*

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Abstract

We develop a VAR that allows the estimation of the impact of monetary policy shocks on volatility. Estimates for the US suggest that an increase in the policy rate by 1% is associated with a rise in unemployment and inflation volatility of about 15%. Using a New Keynesian model, with search and matching labour frictions and Epstein-Zin preferences we show that these volatility effects are driven by the coexistence of agents' fears of unemployment and concerns about the (in) ability of the monetary authority to reverse deviations from the policy rule with the impact magnified by the agents' preferences.

Keywords: DSGE, Non-Linear SVAR, New Keynesian, Search and Matching Frictions, Epstein-Zin preferences, Stochastic Volatility

JEL Classification: E30, E40, E52, C11, C13, C15, C50

1 Introduction

Monetary authorities around the industrialised world are responsible for country's price, growth, employment and financial stability. They aim to meet these objectives via their instruments. This suggests that their actions have a first order effect on agents' economic behaviour. For instance, monetary authorities are able to alter households' and firms' consumption, investment and pricing plans by adjusting the policy rate. The transmission of monetary policy actions to the economy and their first order effects have been extensively studied by the monetary economics literature. However, the volatility implications – second order effects – induced by these actions have received less attention.

The question that arises naturally is: “do we need to be concerned about the volatility effects of monetary policy shocks?”. A number of recent events suggest that the answer to this question is in the affirmative. For example, the “taper tantrum” in June 2013 was associated with an increase in speculation that the Federal Reserve would start on a tapered end to quantitative easing (QE) in 2014. This discussion increased financial market volatility, pushed up long-term interest rates and induced heavy losses on bond investors. These anecdotal observations are further supported by empirical results present below. Using a structural VAR with stochastic volatility (extended to allow

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for feedback from the endogenous variables to the volatility), we show that monetary policy shocks increase macroeconomic volatility and the results are robust across identification schemes. It is also shown here that the monetary policy shock is responsible for about 40% to 50% of the forecast error variance contribution of all level shocks to the volatility of the endogenous variables. These volatility contributions are substantially higher than monetary policy shock's shares of explaining the level series (as it is commonly found in the literature and illustrated again here).

To understand how volatility is affected when monetary authorities decide to deviate unexpectedly from their Taylor type reaction function, we employ a stylised New Keynesian DSGE model, with search & matching labour frictions and Epstein-Zin preferences. The model is estimated using limited information impulse response matching techniques. Although the literature has questioned the ability of "simple" search and matching New Keynesian models to jointly replicate the dynamics of both unemployment and inflation (Krause and Lubik (2007) and [Gertler et al.](#) (2008)), our estimated model reproduces VAR responses remarkably well.

Simulations from the theoretical model suggest that the transmission of the policy shock to volatility depends on three modelling features: (i) the presence of labour market real frictions, (ii) the monetary authorities' desire for gradual policy adjustments and (iii) the existence of Epstein-Zin preferences. It is the coexistence of agents' fears about being prolonged unemployment and policymakers' preference for interest rate smoothing that causes volatility to increase significantly. It is only in this scenario that Epstein-Zin preferences have a quantitatively meaningful role. From an economic point of view, households acknowledge the real risk of becoming unemployed and the fact that during the unemployment spells additional adverse shock may occur. However, it is the combination of these risks together with the policy-rate smoothing parameter that causes monetary policy to have significant volatility effects and not the shock per-se. In other words, agents are not overly concerned that authorities are able to deviate unexpectedly from their objective function but they significantly price the fact that the central bank cannot fully undo such actions resulting in prolonged unemployment spells where they are vulnerable to further adverse shocks and future uncertainty rises.

The analysis presented here goes some way in explaining why authorities have been extremely cautious about when to initiate monetary policy normalisation as they may not be able to undo their actions quickly. More importantly, the simulations below give full support to FED's 'unconventional' policy actions undertaken during the Great Recession aiming to eliminate the 'vicious circle' of higher uncertainty, higher unemployment, lower inflation and support the recovery.

As in the empirical and theoretical literature on the impact of uncertainty shocks (Bloom (2009) and Fernandez-Villaverde et al. (2015)), our paper highlights the importance of these type of disturbances. However, the focus and results of our analysis are novel in one key respect. Unlike the bulk of the uncertainty literature, this paper attempts to model the transmission of monetary policy shocks to economic volatility and thus takes a step towards treating economic volatility as endogenous.

Regarding our empirical contribution, the study of Ludvigson et al. (2015) is the closely related. Ludvigson et al. (2015) develop a procedure that separates movements in volatility caused by primitive (first order) shocks and by uncertainty shocks. However, crucially, they do not identify the source of the primitive shocks. In contrast, our focus is on the impact of monetary policy shocks on volatility.

The papers closest to our theoretical work are the studies of Rudebusch and Swanson (2012) and Swanson ([2015](#)), who use a similar theoretical setup to the one employed here to understand the asset

pricing implications of volatility effects caused by level shocks. [Cacciatore and Ravenna \(2016\)](#) develop a real business cycle model, with labour search and matching frictions and an occasionally binding constraint on downward wage adjustment to understand the effect from a negative productivity shock on volatility.¹ Our paper is also related to the work of [Bikbov and Chernov \(2013\)](#) and [Campbell et al. \(2014\)](#), who uses macro-finance models to understand the relationships between monetary policy and bond risk premia. [Petrosky-Nadeau et al. \(2018\)](#) illustrate that when real business cycle models with search & matching friction are calibrated and solved carefully, then they can generate endogenous disasters. Our finding support fully their analysis, we illustrate below that only the version of the model with these labour frictions can give rise to endogenous disasters.²

Finally, our work is related to the heterogeneous agents (HA) literature that introduces unemployment into these incomplete markets models either to understand how different fiscal policies are transmitted to the economy ([McKay and Reis \(2016\)](#)) or to develop models that can account for extreme economic phenomena such as the Great Recession without employing large and persistent exogenous shocks ([Ravn and Sterk \(2017\)](#), [Den Haan et al. \(2018\)](#)). Agents in these models cannot fully insure against idiosyncratic unemployment risk and, therefore, they are concerned about their consumption level if they become unemployed. So when an adverse shock takes place, they act in a precautionary manner and increase savings. These concerns are more elevated in bad times as unemployment spells last longer making the agents' responses state dependent. In our setting, the specification of the utility function leads to state-dependence of responses. However this feature is now driven by the difference between current and steady-state consumption with agents responding by more in states where consumption is below the steady state.

The paper is organised as follows, Section 2 presents the empirical model and discusses the data and empirical results. Section 3 reviews the theoretical model, its calibration and presents the impulse response analysis. The final section concludes.

2 Empirical results

In order to estimate the impact of monetary policy shocks on second moments of key macroeconomic variables, we estimate an extended structural VAR model with stochastic volatility. The observation equation of the model is given by:

$$Z_t = c + \sum_{j=1}^P \beta_j Z_{t-j} + \sum_{k=1}^K b_k \tilde{h}_{t-k} + \Omega_t^{1/2} e_t, e_t \sim N(0, I_N) \quad (1)$$

In equation (1) Z_t is $N \times 1$ vector of endogenous variables and \tilde{h}_t denotes the $N \times 1$ vector of log stochastic volatilities. The coefficients are denoted by the $N \times N$ matrices β_j and b_k while I_N is a $N \times N$ identity matrix. The covariance matrix of the VAR residuals is time-varying and factored as:

$$\Omega_t = A^{-1} H_t A^{-1'} \quad (2)$$

$$H_t = \text{diag}(\exp(\tilde{h}_t)) \quad (3)$$

¹[Cacciatore and Ravenna \(2016\)](#) also use their framework to understand the state dependent amplification mechanism of exogenous uncertainty shocks.

²In our framework the endogenous disasters are caused by monetary policy shocks and not by productivity shocks as it is the case in [Petrosky-Nadeau et al. \(2018\)](#). Endogenous disasters in our model illustrate that the monetary policy could cause highly adverse economic conditions and this is why agents in the model: (i) are concerned and (ii) try to insure against these outcomes.

and the $N \times N$ diagonal matrix H_t holds the stochastic volatility of the orthogonalised shocks on the main diagonal ($\tilde{h}_t = [h_{1,t}, h_{2,t}, \dots, h_{N,t}]$). The structure of the A matrix is chosen by the econometrician to model the contemporaneous relationship amongst the reduced-form shocks. We discuss our choice of the structure of the A matrix in section 2.3 below.

The transition equation for the stochastic volatilities is given by the following VAR model:

$$\tilde{h}_t = \alpha + \theta \tilde{h}_{t-1} + \sum_{j=1}^K d_j Z_{t-j} + \eta_t, \eta_t \sim N(0, Q), E(e_t, \eta_t) = 0 \quad (4)$$

The constants and coefficients on lags are denoted by the $N \times 1$ and $N \times N$ matrices α and θ , respectively. Following standard practice in the literature on stochastic volatility models (see for e.g. Kim et al. (1998)), we allow \tilde{h}_t to depend on its first lag. However, the $N \times N$ coefficient matrices d_j also allow lagged endogenous variables to affect the log variances. If these coefficients are non-zero, then shocks to equation 1 have an impact on \tilde{h}_t and consequently on Ω_t and measures of the unconditional variance of Z_t . Note also that the stochastic volatility in mean formulation of equation 1 allows feedback from lagged volatilities to the endogenous variables.

The model in equations 1 and 4 contains two innovations relative to the standard BVAR with stochastic volatility (see Clark (2011)). First, it allows the elements of \tilde{h}_t to co-move while most of the previous literature assumes an independent AR or random walk process for each log variance. The specification used here thus captures the possibility that volatility of shocks to macroeconomic and financial variables may move together – a phenomenon that may be important during periods of recession and financial stress. Secondly, unlike previous applications of this model (see Mumtaz and Theodoridis (2015)), the terms $\sum_{k=1}^K b_k \tilde{h}_{t-k}$ and $\sum_{j=1}^K d_j Z_{t-j}$ in equations 1 and 4 allow a dynamic relationship between the level and volatility of the endogenous variables.³ One way to see this is to re-write the observation and transition equations jointly as an expanded VAR system:

$$\underbrace{\begin{pmatrix} Z_t \\ \tilde{h}_t \end{pmatrix}}_{(2N \times 1)} = \underbrace{\begin{pmatrix} c \\ \alpha \end{pmatrix}}_{(2N \times 1)} + \begin{pmatrix} \underbrace{\beta(L)}_{(N \times NP)} & \underbrace{b(L)}_{(N \times NK)} \\ \underbrace{d(L)}_{(N \times NK)} & \underbrace{\theta L}_{(N \times N)} \end{pmatrix} \begin{pmatrix} Z_t \\ \tilde{h}_t \end{pmatrix} + \underbrace{\begin{pmatrix} u_t \\ \eta_t \end{pmatrix}}_{(2N \times 1)} \quad (5)$$

$$\text{var} \left(\begin{pmatrix} u_t \\ \eta_t \end{pmatrix} \right) = \begin{pmatrix} A^{-1} H_t A^{-1'} & 0 \\ 0 & Q \end{pmatrix} \quad (6)$$

where $\beta(L)$, $b(L)$ and $d(L)$ denote lag polynomials of order P , K and K respectively. As discussed above, our interest lies in investigating the possible impact of monetary policy shocks on the second moments of the endogenous variables. The specification above enables us to calculate the impulse response of \tilde{h}_t and thus $\text{var}(Z_t)$ to a monetary policy shock identified via an appropriate structure for A .

Equation 5 reveals two restrictive features of the benchmark model. First, the coefficient matrices are time-invariant and the model does not directly account for structural change. Second, the error covariance matrix (equation 6) is assumed to be block diagonal and level (volatility) shocks have a lagged impact on volatility (levels). In the sensitivity analysis below (see Section 3.1.1), we relax these assumptions and show that our main results are qualitatively similar in the extended versions of the benchmark model.

³An exception is the univariate stochastic volatility in mean model of Chan (2017) that allows lagged effects from the data in the transition equation.

2.1 Data

The model is estimated using US data on the civilian unemployment rate, annual CPI inflation, an interest rate representing the policy instrument and the spread of 10 year government bonds over the three month T-bill rate. The data is monthly and, in the benchmark case, runs from 1947m1 to 2007m12, with the last few years dropped as they represent the period of unconventional monetary policy. The first ten years are used as a training sample with estimation carried out over the period 1957m1 to 2007m12. In the benchmark model, we use the three month T-bill rate as a proxy for the policy instrument. In an additional model specification we identify the monetary policy shock using an external instrument approach. As explained in section 2.3 below, this version of the model uses a one year government bond yield as the policy instrument. The data on the unemployment rate, CPI and the three month T-Bill rate is obtained from FRED, while the 1 and 10 year bond yield is obtained from Global Financial Data.

2.2 Estimation and impulse responses

The model is estimated using Bayesian methods. In the on-line appendix we state in detail the Gibbs sampling algorithm used to approximate the posterior distribution. In short, the algorithm is an extension of the MCMC methods used to estimate Bayesian VARs with stochastic volatility, presented for example in Cogley and Sargent (2005).⁴ The prior distribution for the VAR coefficients in equation 1 are based on existing studies and ‘shrink’ the VAR coefficient matrix towards an AR specification for each endogenous variable. We employ a similar prior for the transition equation and thus assume apriori that each log stochastic volatility follows an AR process and that there is no feedback from Z_{t-j} .

The impulse responses of $\log \text{var}(Z_t)$ to a monetary policy shock are calculated via Monte-Carlo integration. In particular, the impulse responses are defined as the difference between the following conditional expectations

$$IRF_t = E(\ln \text{var}(Z_{t+k}) \setminus \Psi_t, Z_{t-1}, \mu) - E(\ln \text{var}(Z_{t+k}) \setminus \Psi_t, Z_{t-1}) \quad (7)$$

where Ψ_t denotes the parameters and state variables of the model and μ is the monetary policy shock. The first term in equation 7 denotes a forecast of the log volatility conditioned on one of the structural shocks μ . Note that, the volatility of the endogenous variables depends on the structural shocks through equation 4 above. The second term is the baseline forecast of the log variance, i.e. conditioned on the scenario where the shock equals zero. Koop et al. (1996) describe how to approximate these conditional expectations via a stochastic simulation of the non-linear VAR model. We use 100 simulations to calculate IRF_t repeating this for 500 retained Gibbs draws. In order to account for history dependence of the non-linear responses, the calculation is done for $t = 1, 12, \dots, T$ i.e. every 12th month in the sample and the mean across time is reported in the figures below.

2.3 Model specification and identification

We set the lag length in the VAR model to 12 and use 3 lags of the endogenous variables in the transition equation 4 and 3 lags of the stochastic volatilities in the observation equation 1. As shown

⁴We use a particle Gibbs sampler (see Andrieu et al. (2010)). This is described in the technical appendix.

in the sensitivity analysis, the main results are very similar for longer lag lengths.

We consider three schemes to identify the monetary policy shock. The schemes are implemented by placing restrictions on the column of the A^{-1} matrix corresponding to the equation for the policy instrument. The remaining columns of the matrix correspond to a triangular structure.

The benchmark identification scheme uses contemporaneous sign restrictions to identify the monetary policy shock. We assume that a contractionary policy shock increases the short-term interest rate on impact and leads to a rise in unemployment and a fall in CPI inflation. The second scheme assumes a recursive structure and implies that monetary policy shocks have no contemporaneous impact on unemployment and inflation but can affect the term spread immediately. Finally, we follow [Gertler and Karadi \(2015\)](#) and identify the monetary policy shock using an external instrument. This version of the model uses the 1 year government bond yield as the measure of the policy rate and the estimation sample runs to 2012 m6. [Gertler and Karadi \(2015\)](#) argue that the use of the 1 year rate accounts for unconventional policy such as forward guidance. We use the benchmark instrument employed in [Gertler and Karadi \(2015\)](#) – i.e. surprise changes in three month ahead fed funds futures rate on FOMC dates. As discussed in [Mertens and Ravn \(2013\)](#), under the assumption that the instrument is relevant and uncorrelated with other structural shocks, the impulse vector to a unit shock can be recovered by a regression of the reduced form residuals on the instrument.⁵

3 Results

3.1 Impulse response to a monetary policy shock

Figure [1](#) presents the impulse response to a contractionary monetary policy shock normalised to increase the T-Bill rate by 100 basis points. The unemployment rate rises by about 0.2 percentage points at the two year horizon. Inflation displays a persistent decline of about 0.3 percentage points. Finally, the term spread falls by about 70 basis points on impact.

The last three rows of the figure present the response of the unconditional volatility to this shock. It is clear from the figure that the volatility of all endogenous variables rises in response to this shock. This is reflected in the measure of overall volatility, the log determinant of the covariance matrix of the endogenous variables which shows a persistent increase. The response of volatility is persistent lasting for about 2 years with the magnitude of the response of interest rate and inflation volatility slightly larger than the remaining variables.

Figure [2](#) presents the response of the volatility of the endogenous variables estimated using the three identification schemes discussed above. The second row of the figure shows the recursive identification schemes produces results very similar to the benchmark case. Similarly, when the external instrument is used to identify the monetary policy shock the impulse responses still suggest that volatility rises after a monetary contraction.

⁵[Gertler and Karadi \(2015\)](#) present a detailed evidence that suggests that three month ahead fed funds futures rate innovations provide a strong instrument to identify monetary policy shocks.

3.1.1 Robustness checks

Time variation As noted above, the benchmark model restricts the VAR coefficients to be fixed over time. To check the structural stability of the estimated impulse responses we extend the benchmark model to allow the coefficients to be time-varying. In particular, we estimate the following version of the model:

$$Z_t = c_t + \sum_{j=1}^P \beta_{t,j} Z_{t-j} + \sum_{k=1}^K b_{t,k} \tilde{h}_{t-k} + \Omega_t^{1/2} e_t \quad (8)$$

$$\Omega_t = A^{-1} H_t A^{-1'} \quad (9)$$

$$\tilde{h}_t = \alpha_t + \theta_t \tilde{h}_{t-1} + \sum_{j=1}^K d_{t,j} Z_{t-j} + \eta_t \quad (10)$$

where $\text{var}(\eta_t) = Q$. Letting $\underbrace{\Theta_t}_{N(NP+NK+1) \times 1} = \text{vec}([c_t, \beta_{t,1}, \dots, \beta_{t,P}, b_{t,1}, \dots, b_{t,K}])$ and $\underbrace{\Psi_t}_{N(N+NK+1) \times 1} = \text{vec}([\alpha_t, \theta_t, d_{t,1}, \dots, d_{t,K}])$ the evolution of the coefficients is determined by the additional transition equations:

$$\Theta_t = \Theta_{t-1} + \tilde{Q}_1^{1/2} v_{1t} \quad (11)$$

$$\Psi_t = \Psi_{t-1} + \tilde{Q}_2^{1/2} v_{2t} \quad (12)$$

where $(e_t', v_{1t}', v_{2t}')' \sim N(0, I_{\bar{N}})$ with $\bar{N} = N(NP + NK + 1) + N(N + NK + 1) + N$. The model can be estimated using an extended version of the Gibbs algorithm summarised above. The extension is described in the technical appendix.

The time-varying impulse responses of volatility to a 1 unit monetary contraction are shown in Figure 3. As in the benchmark case, sign restrictions are used to identify the policy shock. As noted in previous studies, there is some evidence suggesting that the impact of monetary policy on the real economy has declined over time (see Boivin and Giannoni (2006)). As in Boivin and Giannoni (2006) the inflation response becomes positive at medium horizons in the earlier part of the sample. While there is some weak evidence to suggest that the response of volatility may have been slightly larger during the 1970s and the first half of the 1980s, the impact on volatility remains positive and persistent throughout the sample period.

Further sensitivity checks The technical appendix provides a range of further checks. The benchmark model in equations 5 and 6 does not allow a contemporaneous relationship between level and volatility shocks. Following Alessandri and Mumtaz (2018), we extend the model and relax this assumption. Impulse responses from this version of the model (see Section 4.1, pages 14-17 in the appendix) support the conclusion that volatility rises after a monetary contraction.⁶ In addition, we show that the results survive if a longer lag length is used in the benchmark model. Similarly, versions of the model that include the Federal Funds rate, industrial production or stock returns produce results similar to the benchmark case. Finally, positioning the short-term interest rate first in the recursive order or using the Romer and Romer (2004) measure of monetary policy shocks as an

⁶In the technical appendix we present results using the proxy VAR of Gertler and Karadi (2015) extended to include the measure of uncertainty developed by Jurado et al. (2015). Results from this model, which allows for a contemporaneous impact of monetary policy shock on uncertainty, support our key conclusions.

instrument produces responses of volatility that support the results depicted in the second and third rows of Figure 2.

3.2 Variance decomposition

To investigate the importance of the monetary policy shock we construct the forecast error variance (FEV) decomposition for the benchmark model using the method described in Lanne and Nyberg (2016) for non-linear models. Table 1 presents the contribution of the monetary policy shock and compares it with the contribution of all 4 level shocks in the VAR model. The third and fourth columns of the table display the contribution to the FEV of volatility of the variables while the final two columns display the contribution to the FEV of the level. The final column of the table shows that, as highlighted by several previous studies, the monetary policy shock makes a modest contribution to future movements in the unemployment rate and inflation. As in Bernanke et al. (2005), the contribution to the FEV of the unemployment rate is about 10 percent while the contribution to inflation FEV does not exceed 5 percent. The contribution to the interest rates is higher, especially at shorter horizons. When compared to the contribution of all level shocks jointly (column 5 of the table), the monetary policy shock does not appear to be the most important component.

Column four of table 1 shows that the contribution of the monetary policy shock to the volatility of the variables is also modest in absolute terms and ranges from about 5 to 7 percent. However, in relative terms, the monetary policy shock appears to be important, especially at the one year horizon. For example, the total contribution of the level shocks to the FEV of unemployment volatility at this horizon is 16 percent. Almost half of this contribution comes from the monetary policy shock. Similarly, the monetary policy component in the contribution of level shocks to the FEV of inflation, interest rate and spread volatility accounts for 40 to 50 percent at the one year horizon. However, the relative importance of this shock declines at the 60 month horizon suggesting that other level or second moment shocks may play a role in the long run.

To investigate the economic importance of monetary policy transmission via volatility, we estimate a version of the benchmark model that restricts the effects of level shocks on second moments to be equal to zero (by setting $d_j = 0$ in equation 4).⁷ In Figure 4 we compare the impulse response to a monetary policy shock in the restricted and the benchmark model. It is clear from the figure that the response of unemployment and inflation is less persistent in the restricted case. This implies that the cumulated change in these variables is estimated be much smaller if the effect of policy shocks on volatility is assumed away.

4 Theoretical Analysis

In order to investigate the transmission of monetary policy shocks to volatility, we build and estimate a New Keynesian DSGE model. We first describe the building blocks of the key sectors of the DSGE model and consider how time-varying volatility arises in this set-up. We then use an estimated version

⁷Based on the deviance information criterion (DIC) of Spiegelhalter et al. (2002) the benchmark model is preferred to this restricted model. The DIC for the restricted model is -4526.68, while the corresponding estimate for the benchmark model is smaller (-4560.82) indicating an improved fit.

of the model to calculate the response of the key variables and their volatility to monetary policy shocks.

4.1 DSGE Model

Households: The economy is populated by a continuum of households ($h \in [0, 1]$) that attain utility from consumption $\tilde{C}_t(h)$ and leisure $1 - L_t(h)$, where $L_t(h)$ denotes the fraction of the household that is employed. Household's preferences are separable

$$u\left(\tilde{C}_t(h), \tilde{Z}_t, L_t(h)\right) = \frac{\left(\tilde{C}_t(h) - b\tilde{C}_{t-1}\right)^{1-\sigma_C}}{1-\sigma_C} - \chi_0 \tilde{Z}_t^{1-\sigma_C} \frac{L_t(h)^{1+\sigma_L}}{1+\sigma_L} \quad (13)$$

where σ_L is the inverse of the Frisch elasticity, σ_C stands for the inverse of intertemporal elasticity of substitution and $\tilde{Z}_t = Z\tilde{Z}_{t-1} \left(\frac{\tilde{Z}_{t-1}}{\tilde{Z}_{t-2}}\right)^{\rho_z} e^{\sigma_z \omega_{z,t}}$ denotes the non-stationary productivity process (the tilde indicates that the variable is non-stationary) where Z is the steady-state value of the productivity growth, ρ_z indicates the degree of persistence and σ_z is the standard deviation of the productivity growth process.

The empirical analysis above illustrates convincingly that the changes in the policy instrument have an impact on the level and volatility of endogenous variables. This evidence points to the existence of important non-linearities in the data that give rise to these effects. Given our stylised facts are related to volatility, it seems a natural starting point to investigate whether these non-linearities are due to agents' preferences. The analysis of [Rudebusch and Swanson \(2012\)](#) and [Swanson \(2015\)](#) suggest that when agents form recursive preferences ([Epstein and Zin \(1989\)](#)) then a productivity level shock induces the stochastic volatility of the series in the model to vary. We proceed, therefore, by assuming that agents have preferences of this form:

$$V_t(h) = u\left(\tilde{C}_t(h), \tilde{Z}_t, L_t(h)\right) + \beta \left(E_t V_{t+1}(h)^{1-\gamma}\right)^{\frac{1}{1-\gamma}} \quad (14)$$

The attractive feature of Epstein-Zin preferences is that the coefficient of relative risk aversion decouples from the intertemporal elasticity parameter. The parameter γ illustrates the degree of agents' desire for an early resolution of uncertainty over future consumption. Household maximises its utility function subject to its budget constraint which is:

$$\tilde{P}_t \tilde{C}_t(h) + \frac{\tilde{D}_t(h)}{R_t} + T_t(h) = \tilde{P}_t \tilde{W}_t L_t(h) + (1 - L_t(h)) \tilde{P}_t \tilde{B}_t + \tilde{D}_{t-1}(h) + \tilde{\Xi}_t(h) \quad (15)$$

where \tilde{P}_t is the price index, $\tilde{D}_t(h)$ is the one period risk free government debt, R_t is the return on investing on the government debt, \tilde{W}_t stands for the real wage, $T_t(h)$ is the lump sum taxes, \tilde{B}_t is the unemployment benefit and $\tilde{\Xi}_t(h)$ denotes firms' profits.

The budget constraint reveals the existence of real labour market frictions that lead some members of the household to become unemployed. However, they enjoy the same consumption levels as the employed members due to our complete markets assumption. The structure of the labour market is discussed below. Here we mention two pieces of evidence supporting the argument that search and matching frictions could be a vital part of the mechanism relating monetary policy shocks and second moments. Firstly, the empirical exercise undertaken in [Section 3.2](#) ([Figure 4](#)) reveals that the impact of monetary policy on unemployment is enhanced by the impact of this shock on volatility. This is indicative of the existence of frictions in the labour market that re-enforce the effects of volatility.

Secondly, there are a growing number of studies employing heterogeneous agents models that argue

in favour of including unemployment into these incomplete market models in order to better understand extreme economic episodes such as the Great Recession (Ravn and Sterk (2017), Den Haan et al. (2018)). The outcome of these studies is further supported by the work of Petrosky-Nadeau et al. (2018) that illustrates search and matching frictions in a real business cycle could give rise to endogenous disasters when the model is calibrated and solved carefully. These two different type of approaches seems to indicate that these labour frictions may have very rich nonlinear implications. Finally, during the ZLB period and the introduction of the forward guidance policy by the FED and Bank of England, unemployment became the primary policy variable in terms of monetary authorities communicating the end date of the excess stimulus in the economy.⁸

Labour Market: The existence of a real – search and matching – friction in the labour market (Mortensen and Pissarides (1994)) prevents all job-seekers ($U_t = 1 - (1 - \delta_N) L_{t-1}$) from being matched with vacancies (Υ_t) posted by firms and they end up unemployed ($u_t = 1 - L_t$). The matching technology is described by the following Cobb-Douglas (expression 16)

$$\mathcal{M}_t = \bar{\mu} U_t^\mu \Upsilon_t^{1-\mu} \quad (16)$$

$$L_t = (1 - \delta_N) L_{t-1} + Q_t^\Upsilon \Upsilon_t \quad (17)$$

$$\Psi_t = \kappa \tilde{Z}_t \Upsilon_t \quad (18)$$

While employment evolves according to equation 17, where δ_N is the separation probability. This formulation incorporates the assumption that new hires start working in the same period they are hired (Blanchard and Gali (2010)). Furthermore, firms in order to be able to hire a worker they need to post a vacancy and this incurs a cost (expression 18, see Mortensen and Pissarides (1994)). In other words, the cost is a linear function of the vacancies posted. This is different set-up than the cost of hiring function used in Gertler et al. (2008). However, this particular formulation implies that the cost is paid after the vacancy is filled and it reflects internal costs of adjusting the number of employees (such as training). This specification, thus, minimises the exposure of entrepreneur’s profits unsuccessful matches and, consequently, to uncertainty, since the cost is only paid after the vacancy is filled. This feature makes this formulation less suitable in our setting.

Final Good Producer: The next two paragraphs discuss the price Phillips curve. The set-up is quite standard and the nominal rigidities are introduced as a simple way to make output demand driven in the short run and to allow monetary policy to be able to affect the economic cycles. The final good is produced via the following production function

$$\tilde{Y}_t = \left[\int_0^1 \tilde{Y}_t(f)^{\frac{\varepsilon-1}{\varepsilon}} df \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad (19)$$

$$\tilde{Y}_t(f) = \left(\frac{P_t(f)}{P_t} \right)^{-\varepsilon} Y_t \quad (20)$$

where ε denotes the elasticity of substitution between differentiated intermediate goods ($f \in [0, 1]$). The demand for intermediate goods (expression 20) results from profit maximisation and the assumption that the final good producer operates under perfect competition.

⁸In August 2013, the Bank of England augmented its policy toolkit with (state dependent) forward guidance. Unemployment became a forward guidance threshold variable and the Bank of England started publishing its fan chart in order to better communicate with public its projection about real economy.

Intermediate Good Producers: Similar to [Krause and Lubik \(2007\)](#) and [Krause et al. \(2008\)](#) we assume that there is a continuum of firms ($f \in [0, 1]$) that post vacancies, combine employment, fixed capital and employ the following technology:

$$\tilde{Y}_t(f) = \tilde{Z}_t(L_t(f))^{1-\phi} K^\phi \quad (21)$$

to produce the intermediate good, where ϕ is the capital share in the production function. These producers solve a two-stage problem. In the first stage, taking the wage and the cost of filling a vacancy as given they decide how many vacancies to post and people to employ, these choices result from the maximisation of their profit function:

$$E_t \sum_{j=0}^{\infty} M_{t+j} \beta^j \left\{ \begin{aligned} & MC_{t+j}(f) \tilde{Y}_{t+j}(f) - \tilde{W}_{t+j} L_{t+j}(f) - R^K K - \kappa \tilde{Z}_{t+j} \Upsilon_{t+j}(f) \\ & + \tilde{\Theta}_{t+j}(f) \left((1 - \delta_N) L_{t+j-1}(f) + Q_{t+j}^Y(f) \Upsilon_{t+j}(f) - L_{t+j}(f) \right) \end{aligned} \right\} \quad (22)$$

where MC_t (the marginal cost), $\tilde{\Theta}_t$ (the shadow value of hiring an additional worker) are the Lagrange multipliers associated with the goods' production function and the employment's law of motion, respectively. Finally, M_t denotes the stochastic discount factor

$$M_{t+1} = \left[\frac{V_{t+1}}{(E_t V_{t+1}^{1-\gamma})^{\frac{1}{1-\gamma}}} \right]^{-\gamma} \left(\frac{\tilde{C}_t(h) - b\tilde{C}_{t-1}}{\tilde{C}_{t+1}(h) - b\tilde{C}_t} \right)^{\sigma_C}$$

In the second stage, producers set the price of the intermediate good that maximises their profits. The optimisation problem in this case reflects that prices are set in a staggered manner. This means that every period a fraction $(1 - \xi)$ of firms receive a random signal and set prices optimally ($\check{P}_t(f)$), while those firms who miss the signal set prices based on a rule of thumb backward looking indexation scheme ($\check{P}_t(f) = \Pi_{t-1}^\ell \check{P}_{t-1}(f)$). As explained in [Christiano et al. \(2005\)](#), this pricing setup allows us to replicate the hump shaped response of inflation to the monetary policy observed in the empirical section. The pricing problem is summarised by the following profit maximisation

$$\max_{\check{P}_t(f)} E_t \sum_{j=0}^{\infty} M_{t,t+j} (\beta\xi)^j \left[\left\{ \frac{\check{P}_t(f) \Pi_{t+j-1}^\ell}{\check{P}_{t+j}} - MC_{t+j}(f) \right\} \tilde{Y}_{t+j}(f) \right] \quad (23)$$

subject to

$$\tilde{Y}_{t+j}(f) = \left(\prod_{s=0}^j \Pi_{t+s-1}^\ell \frac{P_t(f)}{P_{t+j}} \right)^{-\varepsilon} Y_{t+j}$$

Wage Determination: The wage is determined by solving a Nash bargaining problem between workers and firms that takes place in order to decide how to split the surplus produced by a match (see [Mortensen and Pissarides \(1994\)](#) and [Krause et al. \(2008\)](#) amongst others). This simple framework is commonly used in the literature and assumes that newly hired workers get the existing wage, an assumption that is also supported by some empirical evidence ([Galuscak et al. \(2012\)](#)). To set the problem we need to define the value of the firm:

$$\tilde{J}_t^F = MC_t \frac{(1-\phi)\tilde{Y}_t}{L_t} - \tilde{W}_t + \beta E_t M_{t+1} (1 - \delta_N) \tilde{J}_{t+1}^F \quad (24)$$

On the other hand, the value of an employed and unemployed worker is given by:

$$\begin{aligned}\tilde{J}_t^W &= \tilde{W}_t - \chi_0 \tilde{Z}_t^{1-\sigma_C} L_t^{\sigma_L} \left(\tilde{C}_t - b\tilde{C}_{t-1} \right)^{\sigma_C} \\ &\quad + \beta E_t M_{t+1} \left\{ [1 - \delta_N (1 - Q_{t+1}^U)] \tilde{J}_{t+1}^W + \delta_N (1 - Q_{t+1}^U) \tilde{J}_{t+1}^U \right\}\end{aligned}\quad (25)$$

$$\tilde{J}_t^U = \tilde{B}_t + \beta E_t M_{t+1} \left\{ Q_{t+1}^U \tilde{J}_{t+1}^W + (1 - Q_{t+1}^U) \tilde{J}_{t+1}^U \right\}\quad (26)$$

and the wage results from the following bargaining problem:

$$\tilde{W}_t^{Nash} = \arg \max_{\tilde{W}_t} \left(\tilde{J}_t^W - \tilde{J}_t^U \right)^\eta \left(\tilde{J}_t^F \right)^{1-\eta}\quad (27)$$

Similarly to [Krause and Lubik \(2007\)](#) and [Leduc and Liu \(2016\)](#), we allow for real wage rigidity via the following norm:

$$\tilde{W}_t = \tilde{W}_{t-1}^{\iota_w} \left(\tilde{W}_t^{Nash} \right)^{1-\iota_w}\quad (28)$$

Government and Aggregation: The government in this economy runs a balanced budget:

$$\tilde{P}_t \tilde{G}_t + D_{t-1} + (1 - L_t) \tilde{P}_t \tilde{B}_t = T_t + \frac{D_t}{R_t}\quad (29)$$

where $\tilde{G}_t = g_t \tilde{Y}_t$ is government consumption and $g_t = g \left(\frac{g_{t-1}}{g} \right)^{\rho_g} e^{\sigma_g \omega_{g,t}}$ is the of the government share in the economy. Monetary policy is set based on Taylor Type rule:

$$\log(R_t) = r_t = \rho_R r_{t-1} + (1 - \rho_R) \left\{ \zeta_\Pi \log \left(\frac{\Pi_t}{\Pi} \right) + \zeta_u \log \left(\frac{u_t}{u} \right) \right\} + \omega_{R,t}\quad (30)$$

where Π is the inflation target, ρ_R is the interest rate smoothing parameter, ζ_Π and ζ_{Y^d} are the policy reaction coefficients to inflation and demand growth, respectively, and $\omega_{R,t} = \rho_{\epsilon_R} \omega_{R,t-1} + \sigma_{R\epsilon} \epsilon_{R,t}$ is the monetary policy shock. Finally, the market clearing condition is derived after a number of simple substitutions (see the Section 5.1, pages 21-22):

$$\frac{\tilde{Z}_t L_t^{1-\phi} K^\phi}{\Delta_t} = \tilde{C}_t + \tilde{G}_t + \tilde{\Psi}_t\quad (31)$$

The de-trended and steady-state calculations are discussed in the technical appendix (Section 5.1, pages 21-22).

4.2 Heteroscedasticity

The novel part of our analysis is that we focus on the volatility implications of the monetary policy shock. With the term volatility or measured uncertainty we refer to the heteroscedastic response of a variable, say x_t , defined as in [Basu and Bundick \(2017\)](#) and [Swanson \(2015\)](#)

$$\hat{\sigma}_{x,t} = 100 \ln \left(\frac{\sigma_{x,t}}{\sigma_x} \right)\quad (32)$$

where

$$\sigma_{x,t} = \text{var}(x_t) = E_t(x_{t+1} - E_t x_{t+1})^2\quad (33)$$

and σ_x is the stochastic steady-state standard deviation of the variable x_t .⁹ It is perhaps important to highlight that equation (33) coincides with the definition of volatility studied by [Jurado et al. \(2015\)](#). As explained in [Rudebusch and Swanson \(2012\)](#) and [Swanson \(2015\)](#), the higher moments of economy's endogenous state vector are time-varying ($\sigma_{x,t}$) due to (i) the additive separability of consumption in the period utility function and (ii) the Epstein-Zin preferences. According to these

⁹In the text below, we use the term uncertainty to refer to 'measured uncertainty'.

authors the additive separability property of consumption makes the model non-homogeneous and this is what induces a small degree of heteroscedasticity, which is further enhanced by the risk aversion parameter (γ).

The economic intuition behind these two technical conditions is actually quite simple. The additive separability property of consumption makes agents' responses to economic shocks depend on the current level of on the state of the economy. For instance, when the current level of consumption is low (or the marginal utility of consumption is high) then consumption uncertainty is higher (relative to the case where the initial level of consumption/output is high) and this reflects agents' elevated concerns about future shocks. As an adverse shock that lowers output further is going to induce a proportionally a larger reduction in utility relative to the case where the initial level of consumption/output was high. This channel is the further enhanced by Epstein-Zin preferences as the risk parameter reflects how much agents dislike elevated uncertainty (Rudebusch and Swanson (2012)).

Loosely speaking, agents in this economy price adverse shocks more heavily in 'bad times' when compared to 'good times'. This behaviour induces a wedge between the mode of the distribution x_t and its mean as the latter captures these elevated concerns. Figures 5 illustrates this phenomenon. We use the model developed in this study (and the estimates discussed below) to simulate the data. Panel A of Figure 5 shows the probability density function of unemployment rate, unemployment expected duration, labour income, GDP and annual inflation as deviations from their stochastic steady states when the monetary policy shock is drawn from its estimated distribution.¹⁰ It is apparent that even under one standard deviation monetary policy shocks the probability density functions displays a 'downward risk', meaning that the average unemployment rate and the average expected duration of being unemployed are higher than their modes. Similarly, the average labour income, GDP and inflation fall to left of their modes. We repeat the same exercise in Panel B of Figure 5, however, we apply larger shocks this time (two times their standard deviation). As expected, the asymmetry becomes more pronounced indicating that agents economic behaviour is also a function of the state of the economy.

The above simulations illustrate that under certain conditions monetary policy actions can have quite dramatic implications for the economy; for instance, unemployment rate could rise 6 percentage points (pps) above the stochastic steady state, while inflation, GDP and labour income could fall 10% below the stochastic steady state, respectively. These adverse economic conditions are taken into account by agents when they form their decisions optimally and try to minimise their exposure to these downward risks.

Table 3 compares a set of data estimated disaster statistics reported by Petrosky-Nadeau et al. (2018), with those predicted by the model that is subject to a monetary policy shock. The aim of this second exercise is to use data evidence to quantify the risks to which the agents in this economy are exposed and, consequently, to understand why they want to hedge against them. As in Petrosky-Nadeau et al., we apply the peak-to-trough method discussed in Barro and Ursua (2008) to to identify rare disasters. The disasters are defined as cumulative fractional declines in per capita output of at least 10%.

¹⁰Both the non-stationary productivity and government spending shocks are switched off for the rest of the analysis. To be precise, although the values of the latter two shocks are set to zero, their standard deviations are not. This affects peoples average behaviour as their expectations are based on the distribution of the two shocks and not just on their realisation.

When the economy is exposed to one standard deviation policy shocks, then the disaster probability is significantly less than 1%. However, if this highly unlikely event ever takes place then it lasts for almost 7 years and growth reduces by almost 12%.¹¹ Not surprisingly, when the economy is perturbed with larger policy shocks, then the disaster probability and the size of output collapse increase significantly and non-linearly, while the duration of disaster state decreases: The probability of disaster and its size rise to 5% and 14% respectively, while the duration of the disaster shortens to 4 years. Unlike in the case of one standard deviation shocks these estimates are significant closer to those obtained using actual data. Finally, the comparison of the second and third columns of Table 3 illustrates again that agents' responses to monetary policy shocks are different at different stages of the cycle.

4.3 Calibration

The model is estimated using limited information impulse response matching techniques (Christiano et al. (2005), Christiano et al. (2010)). However, the value of a small number of parameters is decided prior to the estimation. To be precise, the share of capital in the production (ϕ) and its depreciation rate have been calibrated to 0.36 and 0.025, numbers typically used in the literature (Christiano et al. (2005)). The steady-state unemployment (u) is set equal to 5.8% (the sample mean), while the steady-state value of output (y) to 1. The time discount factor (β) equals 0.995, while the both the steady-state value of inflation and productivity growth have been set to 2%. The last three parameters imply that the non-stochastic steady-state of the annual policy rate is 6%. Similar to Smets and Wouters (2007), the government spending and investment to GDP ratios are calibrated to 0.18 and 0.2, respectively. Finally, the steady-state value of the probability filling the vacancy is 70% (Hagedorn and Manovskii (2008)).

The parameters $\sigma_C, \sigma_L, b, \gamma, \xi, \iota, \zeta_{II}, \zeta_u, \zeta_R, \varepsilon, \mu, \eta, \delta_N, \bar{B} = \frac{B}{WL}, \iota_w, \frac{\Phi}{Y}, \rho_{\varepsilon_Z}, \rho_{\varepsilon_G}, \rho_{\varepsilon_R}, \sigma_{\varepsilon_Z}, \sigma_{\varepsilon_G}$ and σ_{ε_R} are selected to match the nonlinear VAR responses to an identified monetary policy shock using the benchmark identification scheme. The model is solved using third-order perturbation methods and the impulse responses are calculated relative to the stochastic steady state (Cacciatore and Ravenna (2016)).¹²

The objective of the estimation is for the model to be able to replicate not only the empirical responses to a monetary policy shock illustrated in Figure 1 but the also the contribution of the 'uncertainty channel' to these responses. This is defined as the difference between the unrestricted and restricted responses plotted in Figure 4 and discussed in Section 3.2. The latter set of targets ensures that the parameters that control the size of the uncertainty channel in the model are calibrated carefully and the predictions of the model about the importance of that channel are in line with those observed in the data.¹³ As it will become apparent later in our analysis, the last set of moments acts also a natural metric that allows us to assess which part of the transmission mechanism is responsible for the existence of the endogenous uncertainty channel. The process used to estimate the DSGE

¹¹In the online Appendix (Section 5.3) we develop a version of the model without search and matching real labour frictions but with sticky nominal wages. Although, this version of the model replicates the cyclical dynamics remarkably well, it fails to produce an endogenous disaster even when the economy is hit with 2 standard deviation policy shocks.

¹²No pruning is applied in our calculations, which have been implemented using Dynare 4.4.3. The model and replication files can be downloaded from authors' webpages. We also check if our results are robust if we apply Koop et al. (1996) methodology to calculate the IRFs and we find that our results are almost identical. We choose to employ the first method to calculate the IRFs as it dramatically reduces the estimation time.

¹³We would like to thank an anonymous referee for proposing this exercise.

contributions of uncertainty follows closely the steps employed for the empirical models. To be precise, the uncertainty contribution is defined as the difference between the responses obtained using the third order solution model minus those by using only the first order component of the solution.

4.4 Estimation Results

Figure 6 illustrates the ability of the theoretical model to replicate the identified empirical responses (Panel A) as well as the contribution of the uncertainty channel to these responses (Panel B), respectively. Even though the literature has questioned ability of a “simple” search and matching New Keynesian model to jointly replicate the dynamics of both unemployment and inflation (see the discussion in Krause and Lubik (2007) and Gertler et al. (2008)), our estimated model seems to be robust to this criticism as it reproduces the data dynamics remarkably well.

In this study, however, our results go further. The model is capable of replicating: (i) the VAR based stochastic volatility responses to a level monetary policy shock and, (ii) the empirically identified contribution of the uncertainty channel¹⁴. This is a new set of results that further supports the analysis of Petrosky-Nadeau et al. regarding the aptitude of DSGE models with search and matching labour market frictions of reproducing highly non-linear dynamics seen in the data.

Returning to the discussion of the estimated parameters, Table 2 illustrates that the model can replicate the data features mentioned in the previous paragraph by relying on a set of parameter values that have been extensively used in the literature. Perhaps the less common parameter estimate is that for the Epstein-Zin risk coefficient ($\gamma = 115.57$) with the estimated value indicating that agents dislike future uncertainty. This value is similar to the one used in the Rudebusch and Swanson (2012) and Swanson (2015). As explained in Swanson (2015) – and it is further illustrated in the next section – this parameter only has a small effect on the stochastic volatility of macroeconomic variables and therefore the large magnitude is innocuous¹⁵.

In terms of the utility kernel, the model demands some curvature ($\sigma_C = 2.15$, Chen et al. (2012)) and ($\sigma_L = 4.00$, Christiano et al. (2010)) in order to reconcile the predicted dynamics with those in the data. The degree of consumption smoothing ($b = 0.57$), is close to the estimates reported by Christiano et al. (2005).

The parameters that govern the labour market block of the model, the Cobb-Douglas matching parameter ($\mu = 0.40$), the job separation rate ($\delta_N = 0.18$), the income replacement ratio ($100\bar{B} = \frac{B}{WL} = 54\%$) and the steady-state, the bargaining power for workers ($\eta = 0.50$) search and matching friction ($100\frac{\Phi}{Y} = 1\%$) are again consistent with those in the literature (see for instance, Krause and Lubik (2007) and Krause et al. (2008), Gertler et al. (2008) and Hagedorn and Manovskii (2008) among others).

Consistent with the analysis Krause and Lubik (2007) and Leduc and Liu (2016) (among others), the model requires a high degree of real wage rigidity ($\iota_w = 0.89$) in order to be able to match the

¹⁴While the VAR-based volatility responses are all hump shaped, this feature is less evident in the model. This is likely to be because the model incorporates simplifying assumptions and does not capture all aspects of the data.

¹⁵This is not the case for asset price variables as they are functions of the stochastic discount factor. The risk parameter has a great influence on the second term of the stochastic discount factor that captures how agents ‘trade’ uncertainty across time. As it is discussed briefly in the next section, we discovered that the risk parameter variations have perhaps a larger impact on the stochastic steady-state than the cyclical dynamics regarding the macro variables.

volatility of unemployment in the data (Figure 5, Panel A, page 30, online appendix). As explained by these authors and it can be seen from equation (28), under this calibration paid wages decouple from productivity and this leads to high surplus for firms (relative to the situation of no real wage rigidity) stimulating vacancy creation.

Similar to Christiano et al. (2005) a high degree of price indexation ($\iota = 0.72$) and a small probability of resetting prices optimally ($1 - \xi = 0.03$) are needed for the model to match the response of inflation after a monetary policy shock. Furthermore, the steady-state value of firms' markup is 50% ($\varepsilon = 3.0$) a value similar to Smets and Wouters (2007) and Gertler et al. (2008). Finally, the estimates of the policy reaction coefficients are similar to those reported in the literature (Krause et al. (2008), Gertler et al. (2008), Leduc and Liu (2016)).

4.5 Impulse Response Analysis

The aim of this section is to discuss the transmission mechanism of monetary policy shocks to macroeconomic volatility. We illustrate this via impulse response analysis. We first describe the results in the benchmark model and then investigate the features of the model that drive the transmission of the shock.

4.5.1 Benchmark results

The blue solid line in Panel A of Figure 7 shows the agents' responses to a monetary policy shock estimated using the benchmark version of the model.

Households: Starting from the household side, as consumption moves away from its steady-state level due to the adverse policy shock, agents start becoming concerned about the fact that another adverse economic shock is going to take their current consumption even further away from its steady-state. For the agents, this is more costly in utility terms than if consumption had been above its steady-state prior to the adverse shock (see Rudebusch and Swanson (2012) and Swanson (2015)). To insure themselves against this downward risk they act in a precautionary manner and reduce current consumption by a larger amount (Basu and Bundick (2017) and Fernandez-Villaverde et al. (2015)).

Furthermore, in an economy with search and matching frictions, the agents face additional risks: i.e. the risk of job separation and unemployment. This enhances households' concerns about their expected consumption plan and intensifies their desire to hedge against this uncertainty. An adverse shock that reduces consumption when the latter is already below its steady state would have a larger detrimental impact when agents are unemployed. We consider the importance of these risks below.

Firms: Firms are owned by households and so they use the same stochastic discount factor to weight expected profits. This means that a profit reduction is more costly when the previous period profits had been below rather than above their steady-state. As it is explained carefully in Swanson (2015), these preferences induce entrepreneurs to devote more attention to generating profits in bad times. However, firms who face adverse economic conditions are exposed to: (i) paying a wage higher than

worker's productivity (taking into account future forgone costs of hiring) due to high wage rigidity and to, (ii) low expected demand for their output.

To mitigate these exposures firms pause hiring. As explained in [Bloom \(2009\)](#) and [Leduc and Liu \(2016\)](#) filling a vacancy is an irreversible decision that has long-term implications. Therefore, entrepreneurs act more cautiously and post even less vacancies pushing up unemployment.

Moreover, the firm recognises that a lower price could lead to a higher demand for its output and, consequently, more profits. Despite the fall in the marginal cost (caused by the fact that supply exceeds labour demand), the firm has an incentive to set an even lower price during bad times in order to secure more demand and, consequently, hedge itself against future more adverse economic outcomes.

As inflation falls and monetary authorities reduce the policy rate only gradually (due to their preferences of avoiding injecting too much interest rate volatility) the real interest rate remains persistently positive and this enhances the desire for saving.

Impact on volatility: As shown in the last row of Panel A of Figure [7](#), volatility increases after a monetary policy shock. As discussed in Section [4.2](#) above, the volatility of a variable can be viewed as a wedge between the mean and the mode of the distribution. When the adverse monetary policy shock occurs, agents in the economy form expectations about future events. Households take into account the probability of longer unemployment spells during which they may be exposed to additional negative shocks. Similarly, firms' expectations about profits are skewed downwards. However, as the time evolves and no further shocks are realised this wedge between expectations and what actually happens eases, and so does the volatility in the economy.

Contribution of uncertainty: The blue circle line in Panel B of Figure [7](#) isolates the contributions of uncertainty in the transmission of monetary policy. The uncertainty contribution is defined as the difference between the response derived by using the full third order solution of the model minus the responses produced using only the first term of that solution. For instance, unemployment increases 0.3 pps after one standard deviation monetary policy shock (Panel A of Figure [7](#), blue solid line/left y-axis) and 1/3 (30%) of this increase is due to the uncertainty (Panel B of Figure [7](#), blue circle line/left y-axis). Furthermore, uncertainty seems to account for almost 50% of the output, labour income and inflation fall. This exercise reveals that: (i) the endogenous uncertainty channel plays a sizeable role in the transmission of the monetary policy shock and (ii) it manifests itself as a demand type shock ([Leduc and Liu \(2016\)](#)).

4.5.2 Key Features of the model

It is interesting to consider the features of the model that drive the impact of monetary policy on second moments. In particular, the model contains three ingredients that play a role: (1) Search and Matching labour market frictions, (2) interest rate smoothing by the monetary authorities and (3) Epstein-Zin preferences. In order to gauge the role of these features of the model, we derive the impulse responses under the counterfactual scenario where these channels are switched off one by one.

Labour Market Frictions We pursue two exercises: (i) we consider what happens when non-stochastic steady-state probability of finding a job (Q^U) is (almost) equal to one (or the non-stochastic steady-state of unemployment is almost equal to zero) and (ii) what happens when the friction is removed completely but nominal wages adjust only gradual (sticky nominal wages, [Christiano et al. \(2005\)](#), [Smets and Wouters \(2007\)](#)). Due to the space required for the development of the second model, the latter exercise is conducted in the online Appendix (Section 5.3), but the intuition of the results coincides with the first experiment and it is, therefore, discussed here briefly.

Panel A of Figure [7](#) compares the agents' responses to a monetary policy shock derived by the benchmark version of the model (blue solid line) against the responses when implicitly there is no risk that agents will ever become unemployed ($Q^U = 1$, red dashed line).¹⁶ Panel B of Figure [7](#) on the other hand identifies the contribution of the uncertainty channel for both versions of the model. Without the search and matching labour market real friction the contribution of the endogenous uncertainty channel to the economy is substantially smaller (red cross line/right y-axis). This evidence seems to indicate that households fear that they can remain unemployed for an extended period of time (the expected average duration is given by $\frac{1}{1-(1-Q^U)^\sigma}$). Their fear is magnified by the possibility that, during this period, another adverse shock that moves their consumption away from its steady-state might arrive. To insure themselves against this uncertainty, they reduce consumption by more (relative to the situation where they move from unemployment to employment almost instantaneously) when the monetary shock takes place.

We develop in the online appendix (Section 5.3) a version of the model without search and matching frictions but with sticky nominal wages ([Christiano et al. \(2005\)](#), [Smets and Wouters \(2007\)](#)) and Epstein-Zin preferences (SW). The estimated version of the SW model again replicates the cyclical responses remarkably well. However, the steady-state value of the output stochastic volatility is almost zero and the model fails to produce distribution skewness and endogenous disasters.

These two quite different exercises seems to converge to the same conclusion that search and matching frictions are important. This is a result that seems to go hand in hand with the finding of the heterogenous agents literature. To be precise, [Ravn and Sterk \(2017\)](#) and [Den Haan et al. \(2018\)](#) (amongst others) argue convincingly about the necessity of incorporating search and matching friction (along with nominal price or/and wage rigidities) into these incomplete markets model in order to produce quantitative sizeable results without relying on large and very persistent shocks. It also coincides with the analysis of [Petrosky-Nadeau et al. \(2018\)](#), who argue that DSGE models with search and matching frictions can generate endogenous disasters.

Interest Rate Smoothing Taking the real labour frictions as given, we consider how discretionary monetary actions could cause uncertainty to increase endogenously and significantly. The systematic part of the monetary policy consists of the two parts: (i) the response to deviation from FED's inflation and unemployment gap objectives and (ii) the interest rate smoothing. Our investigation seems to suggest that it is the policymaker's desire to 'smooth' changes in the policy rate that actually causes uncertainty to rise after a monetary policy shock.¹⁷ To be precise, when the interest rate smoothing

¹⁶This is implemented in the model by lowering the non-stochastic steady-state unemployment rate to 0.01%

¹⁷In the online Appendix (Section 5.1) we illustrate that if the reaction coefficient to unemployment gap is increased beyond empirically plausible values (such as greater than one) then the uncertainty channel diminishes significantly. Although, this simulation lacks empirical support (as we do not observed such high values for α_u in the literature) the results further enforce the message of this paragraph. The higher policy response to unemployment countervails the

coefficient is set to zero ($\rho_R = 0$), the uncertainty channel disappears and the impact of monetary policy on second moments is close to zero. (Figure 8). This happens as the lack of preference for interest rate smoothing allows authorities to loosen policy very quickly in order to restore both the inflation and unemployment targets. In other words, agents do not seem to be particularly concerned about the ability of monetary authorities to discretionary deviate from their objective function as long as they can reverse their actions and restore economy's steady-state.

Epstein-Zin Risk Coefficient Figure 9 compares the responses (Panel A) and the uncertainty contributions (Panel B) in the benchmark case and the counterfactual case where the risk coefficient is set to zero ($\gamma = 0$). With $\gamma = 0$ the importance of the endogenous uncertainty channel declines and the uncertainty contributions are substantially smaller. Moreover, the response of volatility in the counterfactual case is smaller than the benchmark case.

However, these results also suggest that in relative terms the dramatic reduction in γ from 115 to 0 does not lead to effects that are extreme. As the economies' agents have a high desire of early resolution of future uncertainty, the central bank is expected to keep policy expansionary for longer to meet its objectives. As a result, the long-term interest rate falls by more than 30 bps and stays below its stochastic steady-state for more than a year. Finally, this exercise is an additional evidence in favour of endogenous uncertainty acts as a demand channel (shock).¹⁸

5 Conclusion

This study investigates the response of macroeconomic volatility to an unexpected increase in the policy rate. For this purpose we develop an empirical model that allows us to estimate the response of macroeconomic volatility to a monetary policy shock. To investigate the transmission channel of the shock, we build a simple New Keynesian model, with search and matching labour frictions and Epstein-Zin preferences.

The empirical model suggests that a 100 basis points increase in the policy rate causes unemployment and inflation volatility to rise by around 10% above its unconditional value. The theoretical model has been calibrated to match the SVAR responses. Simulations from the theoretical model suggest that it is the coexistence of agents' fears about being prolonged unemployment and monetary authorities' desire for gradual policy adjustments that causes volatility to increase to levels observed empirically. In other words, households understand the risks of becoming unemployed and the fact that during the unemployment spells additional adverse shock may occur. However, it is the combination of these risks together with the policy-rate smoothing parameter that causes monetary policy to have significant volatility effects and not the shock per-se. When these two conditions pre-exist, only then the Epstein-Zin preferences play a significant role.

smoothing parameter and the policy rate is decreased faster in order to support the recovery of the economy.

¹⁸In the online appendix (Section 5.1) we investigate what happens to the economy when the inflation target increases from 2% to 4%. We find that the uncertainty effects from this policy change are small.

References

- ALESSANDRI, P. AND H. MUMTAZ (2018): “Financial regimes and uncertainty shocks.” *Journal of Monetary Economics*.
- ANDRIEU, C., A. DOUCET, AND R. HOLENSTEIN (2010): “Particle Markov chain Monte Carlo methods,” *Journal of the Royal Statistical Society Series B*, 72, 269–342.
- BARRO, R. J. AND J. F. URSUA (2008): “Macroeconomic Crises since 1870,” *Brookings Papers on Economic Activity*, 39, 255–350.
- BASU, S. AND B. BUNDICK (2017): “Uncertainty Shocks in a Model of Effective Demand,” *Econometrica*, 85, 937–958.
- BERNANKE, B. S., J. BOIVIN, AND P. ELIASZ (2005): “Measuring the Effects of Monetary Policy: A Factor-Augmented Vector Autoregressive (FAVAR) Approach,” *The Quarterly Journal of Economics*, 120, 387–422.
- BIKBOV, R. AND M. CHERNOV (2013): “Monetary policy regimes and the term structure of interest rates,” *Journal of Econometrics*, 174, 27–43.
- BLANCHARD, O. AND J. GALI (2010): “Labor Markets and Monetary Policy: A New Keynesian Model with Unemployment,” *American Economic Journal: Macroeconomics*, 2, 1–30.
- BLOOM, N. (2009): “The Impact of Uncertainty Shocks,” *Econometrica*, 77, 623–85.
- BOIVIN, J. AND M. P. GIANNONI (2006): “Has Monetary Policy Become More Effective?” *The Review of Economics and Statistics*, 88, 445–462.
- CACCIATORE, M. AND F. RAVENNA (2016): “Uncertainty, Wages, and the Business Cycle,” mimeo.
- CAMPBELL, J. Y., C. PFLUEGER, AND L. M. VICEIRA (2014): “Monetary Policy Drivers of Bond and Equity Risks,” NBER Working Papers 20070, National Bureau of Economic Research, Inc.
- CHAN, J. C. C. (2017): “The Stochastic Volatility in Mean Model With Time-Varying Parameters: An Application to Inflation Modeling,” *Journal of Business & Economic Statistics*, 35, 17–28.
- CHEN, H., V. CURDIA, AND A. FERRERO (2012): “The Macroeconomic Effects of Large-scale Asset Purchase Programmes,” *Economic Journal*, 122, F289–F315.
- CHRISTIANO, L., M. EICHENBAUM, AND C. EVANS (2005): “Nominal Rigidities and the Dynamic Effects of a shock to Monetary Policy,” *Journal of Political Economy*, 113, 1–45.
- CHRISTIANO, L. J., M. TRABANDT, AND K. WALENTIN (2010): “DSGE Models for Monetary Policy Analysis,” in *Handbook of Monetary Economics*, ed. by B. M. Friedman and M. Woodford, vol. 3, 285–367.
- CLARK, T. E. (2011): “Real-Time Density Forecasts From Bayesian Vector Autoregressions With Stochastic Volatility,” *Journal of Business & Economic Statistics*, 29, 327–341.
- COGLEY, T. AND T. J. SARGENT (2005): “Drift and Volatilities: Monetary Policies and Outcomes in the Post WWII U.S,” *Review of Economic Dynamics*, 8, 262–302.

- DEN HAAN, W. J., P. RENDAHL, AND M. RIEGLER (2018): “Unemployment (Fears) and Deflationary Spirals,” *Journal of the European Economic Association*, forthcoming.
- EPSTEIN, L. G. AND S. E. ZIN (1989): “Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework,” *Econometrica*, 57, 937–69.
- FERNANDEZ-VILLAYERDE, J., P. A. GUERRON-QUINTANA, K. KUESTER, AND J. RUBIO-RAMIREZ (2015): “Fiscal Volatility Shocks and Economic Activity,” *American Economic Review*, 105, 3352–84.
- GALUSCAK, K., M. KEENEY, D. NICOLITSAS, F. SMETS, P. STRZELECKI, AND M. VODOPIVEC (2012): “The determination of wages of newly hired employees: Survey evidence on internal versus external factors,” *Labour Economics*, 19, 802–812.
- GERTLER, M. AND P. KARADI (2015): “Monetary Policy Surprises, Credit Costs, and Economic Activity,” *American Economic Journal: Macroeconomics*, 7, 44–76.
- GERTLER, M., L. SALA, AND A. TRIGARI (2008): “An Estimated Monetary DSGE Model with Unemployment and Staggered Nominal Wage Bargaining,” *Journal of Money, Credit and Banking*, 40, 1713–1764.
- HAGEDORN, M. AND I. MANOVSKII (2008): “The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited,” *American Economic Review*, 98, 1692–1706.
- JURADO, K., S. C. LUDVIGSON, AND S. NG (2015): “Measuring Uncertainty,” *American Economic Review*, 105, 1177–1216.
- KIM, S., N. SHEPHARD, AND S. CHIB (1998): “Stochastic Volatility: Likelihood Inference and Comparison with ARCH Models,” *Review of Economic Studies*, 65, 361–93.
- KOOP, G., H. PESARAN, AND S. POTTER (1996): “Impulse response analysis in nonlinear multivariate models,” *Journal of Econometrics*, 74, 119–147.
- KRAUSE, M. U., D. LOPEZ-SALIDO, AND T. A. LUBIK (2008): “Inflation dynamics with search frictions: A structural econometric analysis,” *Journal of Monetary Economics*, 55, 892–916.
- KRAUSE, M. U. AND T. A. LUBIK (2007): “The (ir)relevance of real wage rigidity in the New Keynesian model with search frictions,” *Journal of Monetary Economics*, 54, 706–727.
- LANNE, M. AND H. NYBERG (2016): “Generalized Forecast Error Variance Decomposition for Linear and Nonlinear Multivariate Models,” *Oxford Bulletin of Economics and Statistics*, 78, 595–603.
- LEDUC, S. AND Z. LIU (2016): “Uncertainty shocks are aggregate demand shocks,” *Journal of Monetary Economics*, 82, 20–35.
- LUDVIGSON, S. C., S. MA, AND S. NG (2015): “Uncertainty and Business Cycles: Exogenous Impulse or Endogenous Response?” NBER Working Papers 21803, National Bureau of Economic Research, Inc.
- MCKAY, A. AND R. REIS (2016): “The Role of Automatic Stabilizers in the U.S. Business Cycle,” *Econometrica*, 84, 141–194.

- MERTENS, K. AND M. O. RAVN (2013): “The Dynamic Effects of Personal and Corporate Income Tax Changes in the United States,” *American Economic Review*, 103, 1212–47.
- MORTENSEN, D. T. AND C. A. PISSARIDES (1994): “Job Creation and Job Destruction in the Theory of Unemployment,” *Review of Economic Studies*, 61, 397–415.
- MUMTAZ, H. AND K. THEODORIDIS (2015): “The International Transmission of Volatility Shocks: An Empirical Analysis,” *Journal of the European Economic Association*, 13, 512–533.
- PETROSKY-NADEAU, N., L. ZHANG, AND L.-A. KUEHN (2018): “Endogenous Disasters,” *American Economic Review*, 108, 2212–2245.
- RAVN, M. O. AND V. STERK (2017): “Job Uncertainty and Deep Recessions,” *Journal of Monetary Economics*, 90, 125–141.
- ROMER, C. D. AND D. H. ROMER (2004): “A New Measure of Monetary Shocks: Derivation and Implications,” *American Economic Review*, 94, 1055–1084.
- RUDEBUSCH, G. D. AND E. T. SWANSON (2012): “The Bond Premium in a DSGE Model with Long-Run Real and Nominal Risks,” *American Economic Journal: Macroeconomics*, 4, 105–43.
- SMETS, F. AND R. WOUTERS (2007): “Shocks and Frictions in US Business Cycles: a Bayesian DSGE Approach,” *American Economic Review*, 97, 586–606.
- SPIEGELHALTER, D. J., N. G. BEST, B. P. CARLIN, AND A. VAN DER LINDE (2002): “Bayesian measures of model complexity and fit,” *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 64, 583–639.
- SWANSON, E. (2015): “A Macroeconomic Model of Equities and Real, Nominal, and Defaultable Debt,” Mimeo, University of California Irvine.

A Tables

Variable	Horizon	All Level shocks		Monetary Policy shock		All Level shocks		Monetary Policy shock	
		Decomposition of volatility FEV		Decomposition of level FEV		Decomposition of level FEV		Decomposition of level FEV	
T-bill	12 mths	14.9 (7.88, 24.1)	6.45 (2.53, 11.8)	27.1 (20, 36.5)	15.7 (11.7, 20)	18 (11.3, 30.5)	5.93 (2.6, 11)	24.1 (16.6, 35.1)	7.98 (5.41, 11.1)
	24 mths	28.8 (18.2, 39.8)	5.8 (2.88, 11.2)	36.7 (23.5, 50.3)	6.6 (4.09, 10.8)	28.8 (18.2, 39.8)	5.8 (2.88, 11.2)	36.7 (23.5, 50.3)	6.6 (4.09, 10.8)
	60 mths	15.6 (8.87, 24.3)	7.14 (3.35, 12.6)	57 (46.4, 67.3)	10.2 (6.04, 15.7)	15.6 (8.87, 24.3)	7.14 (3.35, 12.6)	57 (46.4, 67.3)	10.2 (6.04, 15.7)
Unemployment	12 mths	20 (12.6, 31.2)	6.59 (3.29, 11.8)	48.6 (35.9, 62.1)	10.9 (5.44, 17.4)	20 (12.6, 31.2)	6.59 (3.29, 11.8)	48.6 (35.9, 62.1)	10.9 (5.44, 17.4)
	24 mths	30.4 (19.1, 41.7)	6.05 (3.43, 11.1)	48.4 (35.4, 60.4)	8.69 (4.59, 15.2)	30.4 (19.1, 41.7)	6.05 (3.43, 11.1)	48.4 (35.4, 60.4)	8.69 (4.59, 15.2)
	60 mths	17.8 (9.48, 28.9)	6.62 (2.76, 11.9)	72.2 (63.8, 80.6)	0.93 (0.27, 3.2)	17.8 (9.48, 28.9)	6.62 (2.76, 11.9)	72.2 (63.8, 80.6)	0.93 (0.27, 3.2)
Inflation	12 mths	22.7 (12.4, 34.7)	6.51 (3, 11.6)	57.6 (45.9, 69.6)	1.89 (0.41, 6.04)	22.7 (12.4, 34.7)	6.51 (3, 11.6)	57.6 (45.9, 69.6)	1.89 (0.41, 6.04)
	24 mths	31 (19.6, 40.2)	6.93 (3.85, 12.1)	56.7 (41.3, 69.2)	4.04 (1.24, 11.4)	31 (19.6, 40.2)	6.93 (3.85, 12.1)	56.7 (41.3, 69.2)	4.04 (1.24, 11.4)
	60 mths	12.3 (7.05, 19.6)	5.83 (2.05, 10.8)	52.3 (44.5, 61.3)	18.6 (13.7, 24)	12.3 (7.05, 19.6)	5.83 (2.05, 10.8)	52.3 (44.5, 61.3)	18.6 (13.7, 24)
Term-Spread	12 mths	15.7 (9.69, 25.9)	4.7 (1.9, 9.61)	39.1 (31.6, 48)	13.3 (9.85, 18.1)	15.7 (9.69, 25.9)	4.7 (1.9, 9.61)	39.1 (31.6, 48)	13.3 (9.85, 18.1)
	24 mths	26.6 (16.5, 39.6)	4.77 (2.15, 9.43)	40.3 (33, 48.2)	12 (8.78, 16.8)	26.6 (16.5, 39.6)	4.77 (2.15, 9.43)	40.3 (33, 48.2)	12 (8.78, 16.8)
	60 mths								

Table 1: Contribution to FEV of volatility and levels of endogenous variables. 68 percent error bands in parenthesis

Table 2: Structural Parameters

Parameters	Description	Values
<i>Calibrated Parameters</i>		
u	Steady State Unemployment Rate	0.06
$\frac{G}{\bar{Y}}$	Government Spending to GDP Ratio	0.18
$\frac{I}{\bar{Y}}$	Investment to GDP Ratio	0.20
$400\ln(\Pi)$	Inflation Target	2.00
$400\ln(Z)$	Steady State Value of Productivity Growth	2.00
Q^γ	Steady State Job Filling Probability	0.70
ϕ	Capital Production Share	0.36
δ_K	Capital Depreciation Rate	0.03
β	Time Discount Factor	0.995
<i>Estimated Parameters</i>		
$100\frac{\Phi}{\bar{Y}}$	Post Vacancy Resource Cost relative to GDP	1.00
$\bar{B} = \frac{B}{WL}$	Income Replacement Ratio	0.54
γ	Epstein-Zin Preference Risk	115.57
σ_C	Inverse of Intertemporal Elasticity of Substitution	2.15
b	Habit Smoothing	0.57
σ_L	Inverse of Frisch Elasticity	4.00
ε	Intermediate Goods Elasticity of Substitution	3.00
ξ	Calvo Probability	0.97
ι	Price Indexation	0.72
δ_N	Job Separation Rate	0.18
μ	Cobb-Douglas Matching	0.40
η	Bargain Parameter	0.50
ι_w	Real Wage Rigidity	0.89
ζ_Π	Inflation Monetary Policy Response	1.05
ζ_u	Unemployment Monetary Policy Response	0.40
ρ_R	Interest Rate Smoothing	0.71
ρ_{ϵ_R}	Policy Shock Persistence	0.93
ρ_{ϵ_Z}	Productivity Growth Shock Persistence	0.53
ρ_{ϵ_G}	Government Spending Shock Persistence	0.59
$100\sigma_{\epsilon_R}$	Policy Shock Standard Deviation	0.53
$100\sigma_{\epsilon_Z}$	Productivity Growth Shock Standard Deviation	1.58
$100\sigma_{\epsilon_G}$	Government Spending Shock Standard Deviation	0.01

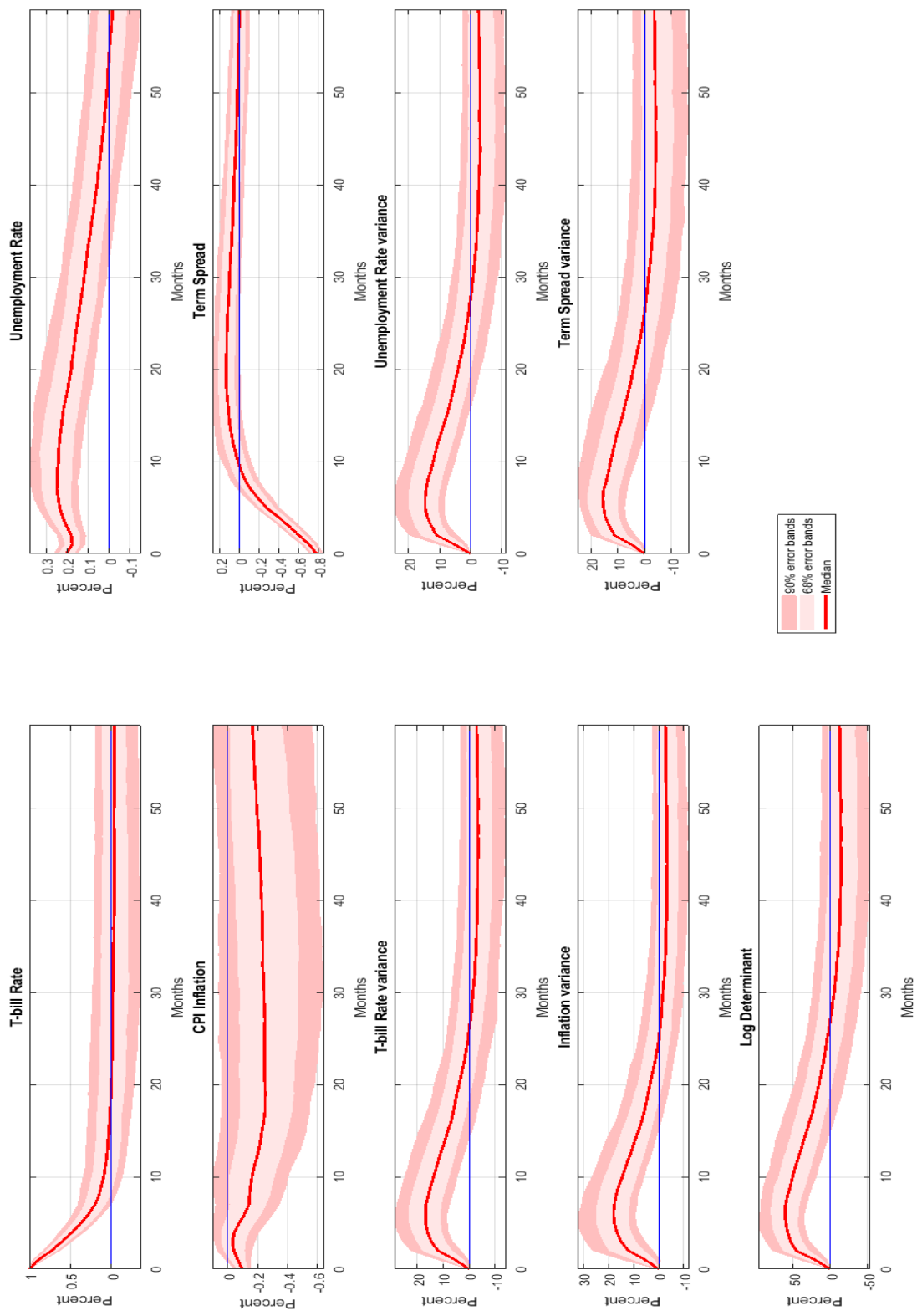
Table 3: Disaster Statistics

Metrics	Data	1 Standard Deviation	2 Standard Deviation
Probability	7.83	0.30	4.55
Size	21.99	11.84	14.46
Duration	3.72	6.35	3.76

Notes: The data disaster statistic estimates are those reported by [Petrosky-Nadeau et al. \(2018\)](#) (Table 4, pp. 2227). Starting from the stochastic steady state, the model is simulated for 50000 periods. Similar to [Petrosky-Nadeau et al. \(2018\)](#), we time-aggregate output into annual observations, and apply the peak-to-trough method to identify disasters as cumulative fractional declines in output of at least 10%. The disaster probabilities and average size are in percent, and the average duration is in terms of years.

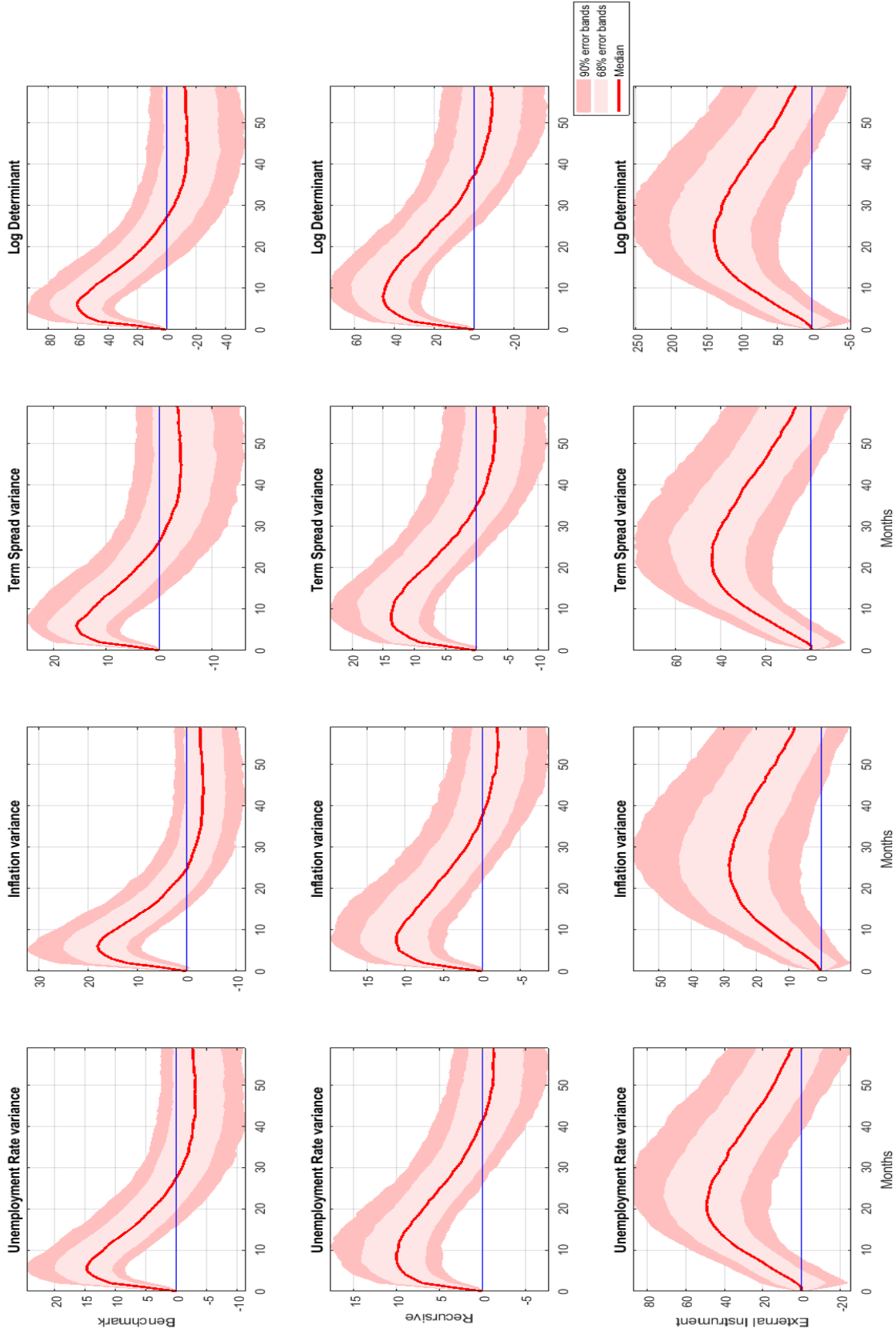
B Figures

Figure 1: Impulse response to a monetary policy shock



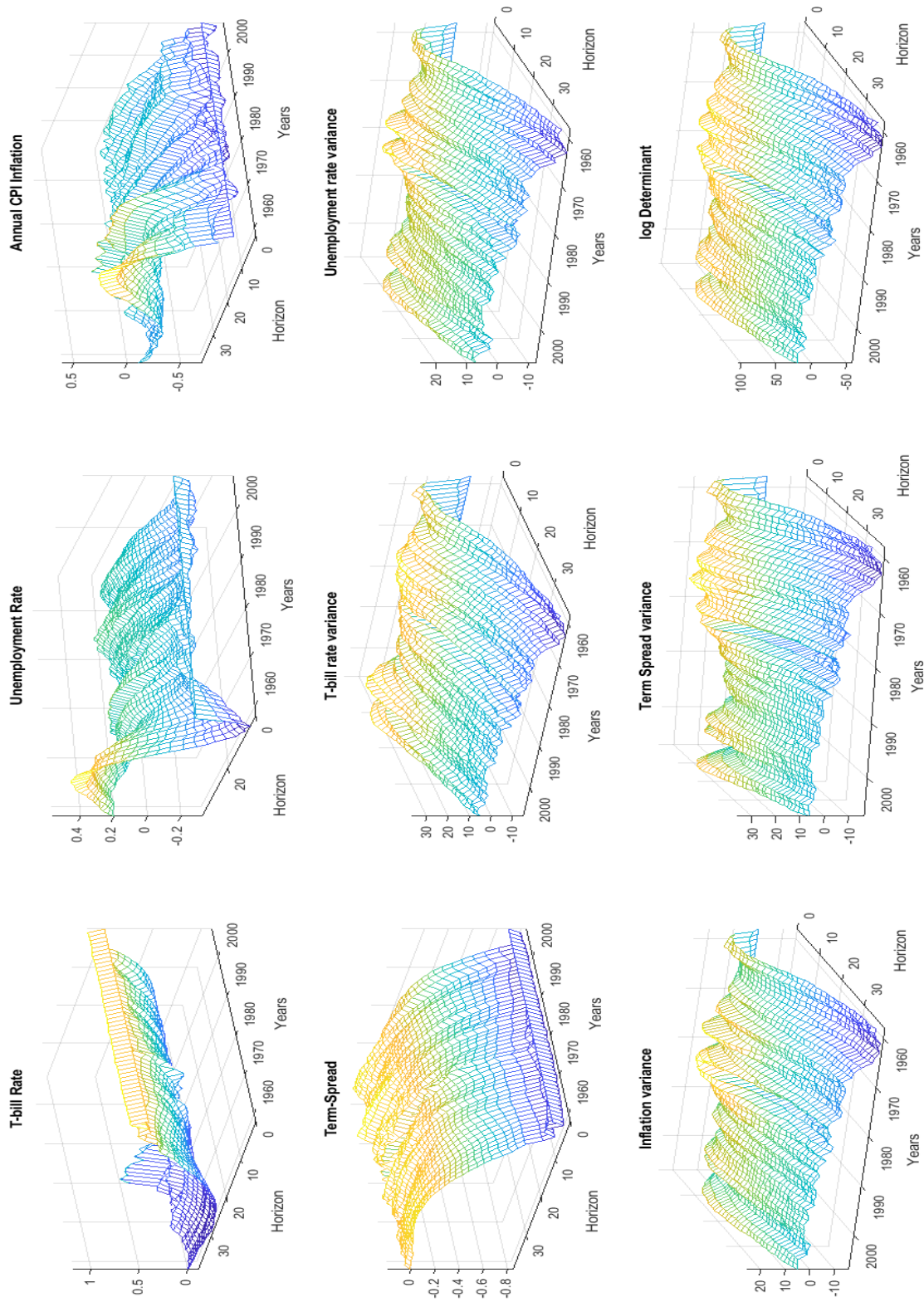
Notes: The solid line is the median. The light shaded area is the 68% error band while the dark shaded area is the 90% error band.

Figure 2: Impulse response to a monetary policy shock



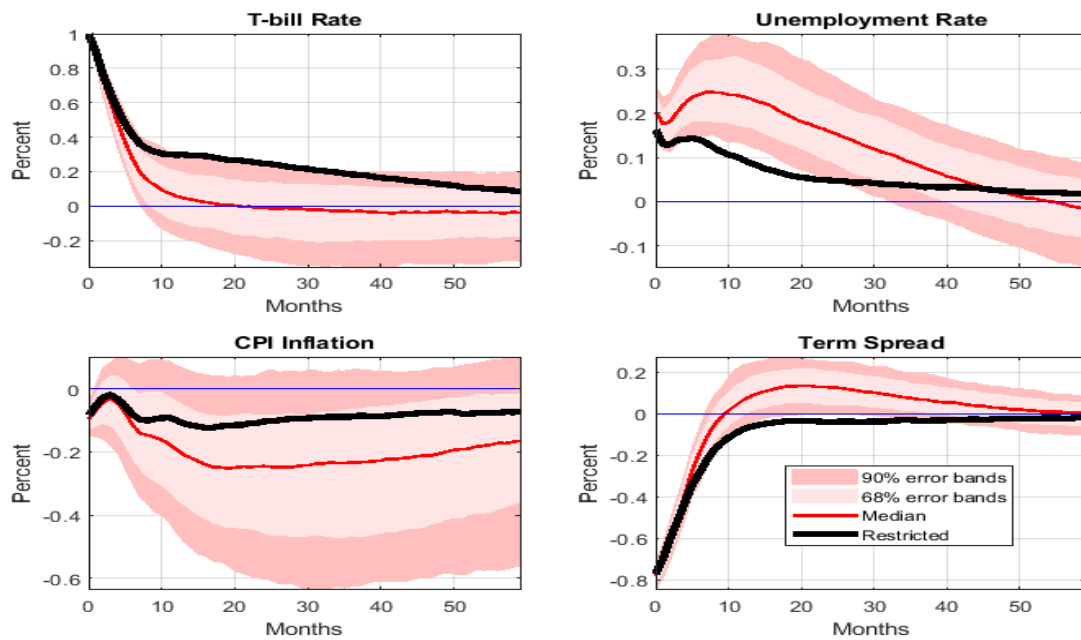
Notes: The solid line is the median. The light shaded area is the 68% error band while the dark shaded area is the 90% error band. The top row presents the responses in the benchmark model that uses sign restrictions. The second row presents results from a model that uses a Cholesky decomposition while the third row presents results from the model that uses fed fund futures rate surprises as an instrument to identify the shock.

Figure 3: Posterior median impulse response to a monetary policy shock using the model with time-varying coefficients



Notes: The impulse response is calculated every 12th month in the sample.

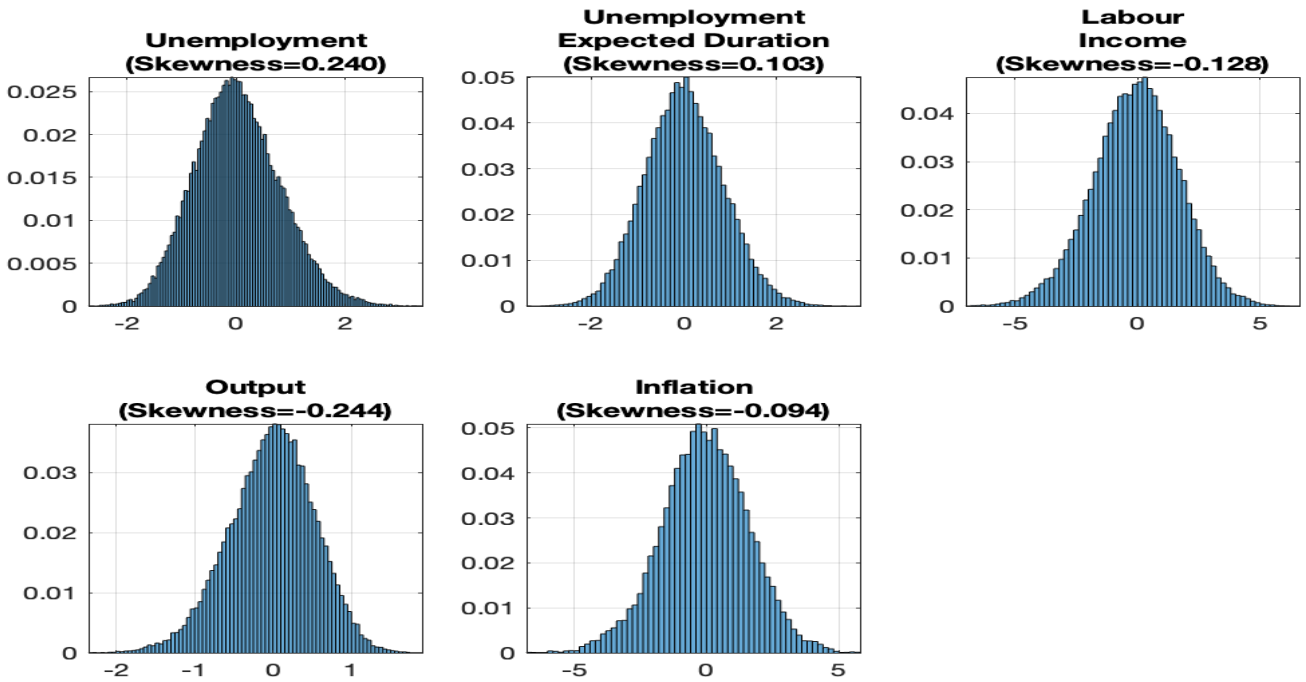
Figure 4: Impulse response to a monetary policy shock from the benchmark and restricted models



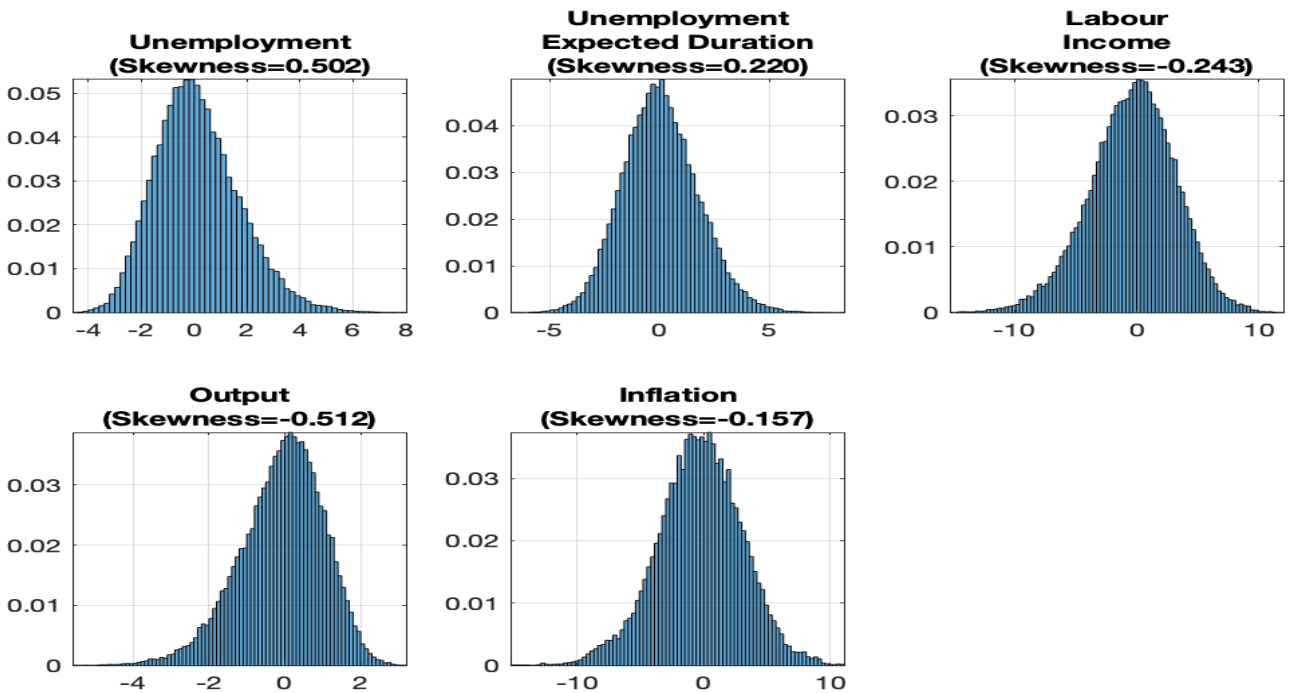
Notes: The light shaded area is the 68% error band while the dark shaded area is the 90% error band from the benchmark model. The thick black line shows the median response from the restricted model.

Figure 5: Downward Risk

Panel A: 1 Standard Deviation Shocks



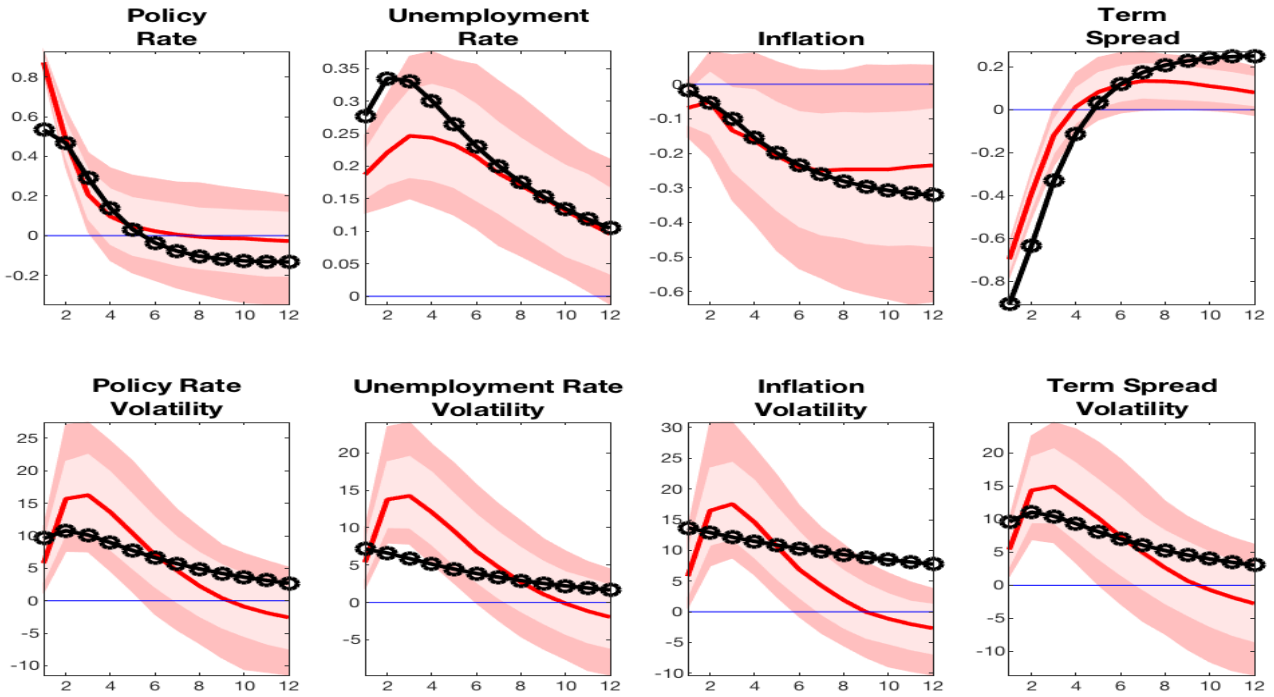
Panel B: 2 Standard Deviation Shocks



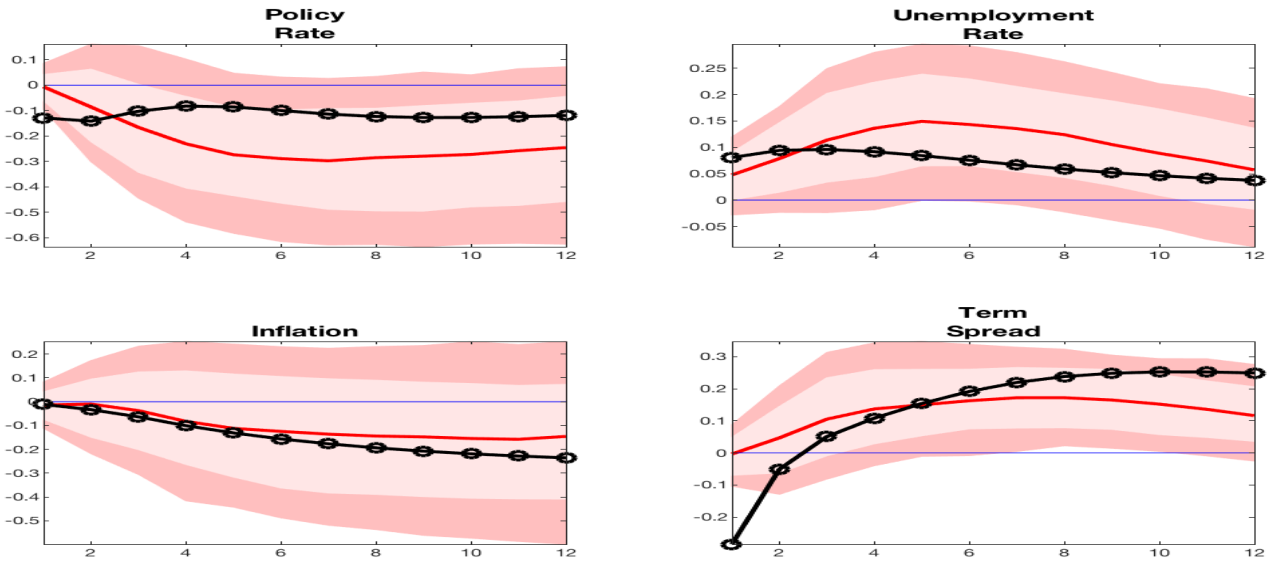
Notes: Starting from the stochastic steady state, the model is simulated for 50000 periods. The histogram illustrates the distribution of x_t as deviation from its stochastic steady-state. Unemployment and inflation are expressed in percentage points (x-axis), expected unemployment duration is measured in quarters, while the GDP and labour income are defined as percentage deviations.

Figure 6: The Fit of the DSGE Model

Panel A: Impulse Responses



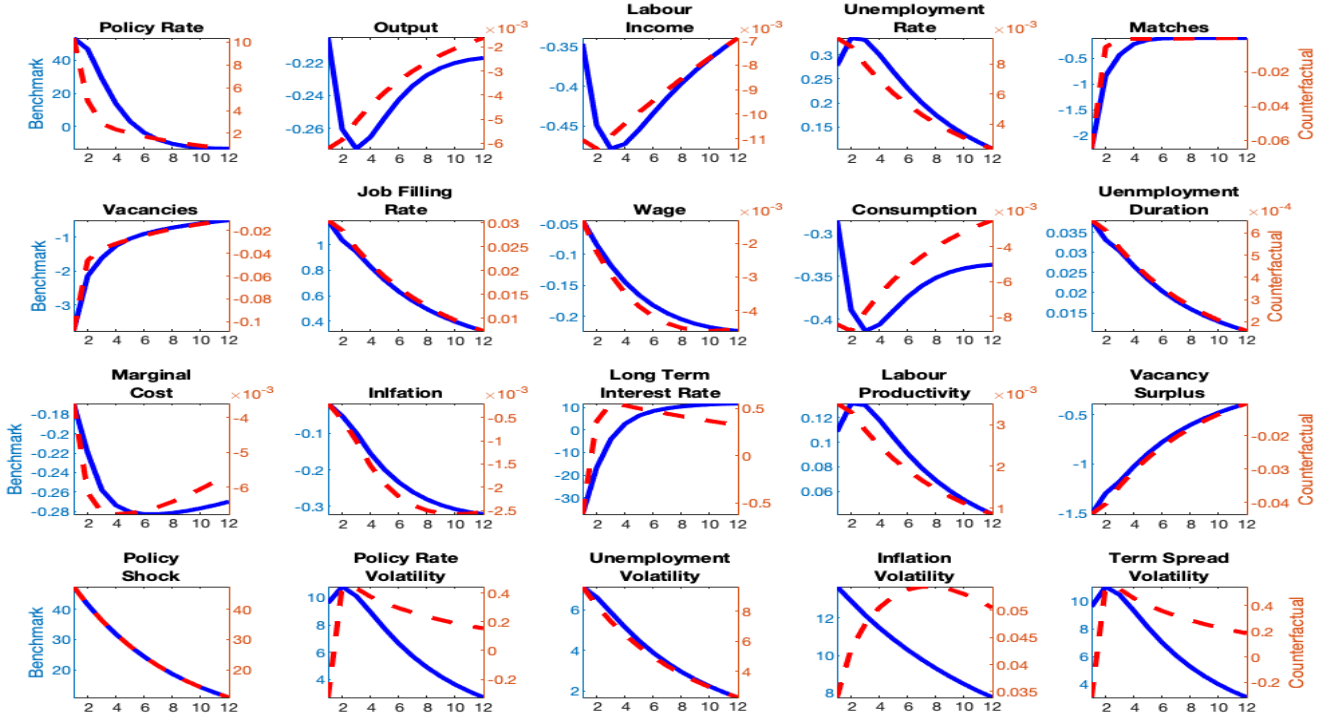
Panel B: Uncertainty Contribution



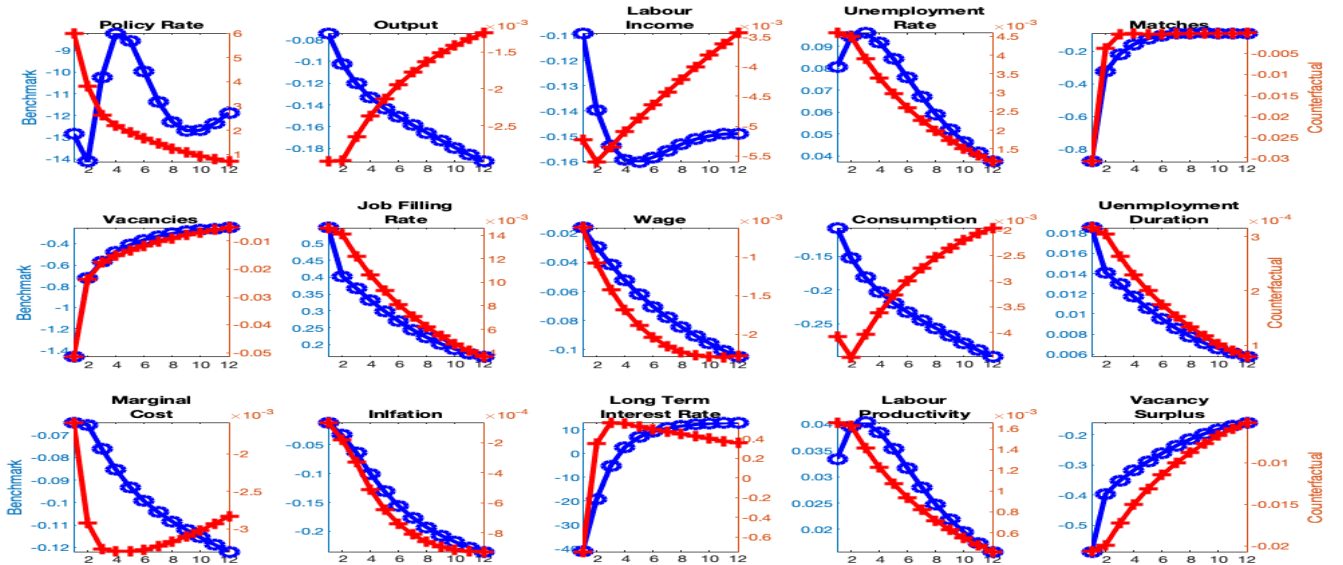
Notes: The responses from the empirical model have been adjusted to match the quarterly frequency of the DSGE model. The posterior distribution of the uncertainty contribution (shadow area) is obtained by subtracting the posterior distribution between the unrestricted and restricted empirical responses. The back dashed line illustrates the DSGE response to the policy shock. The policy rate and inflation are expressed in annual terms. The DSGE responses are calculated relative to the stochastic steady state. **Panel B:** The DSGE uncertainty contribution is calculated as the difference between the response derived by using the third order solution of the model minus the responses produced using first order solution.

Figure 7: Search and Matching Frictions

Panel A: Impulse Responses



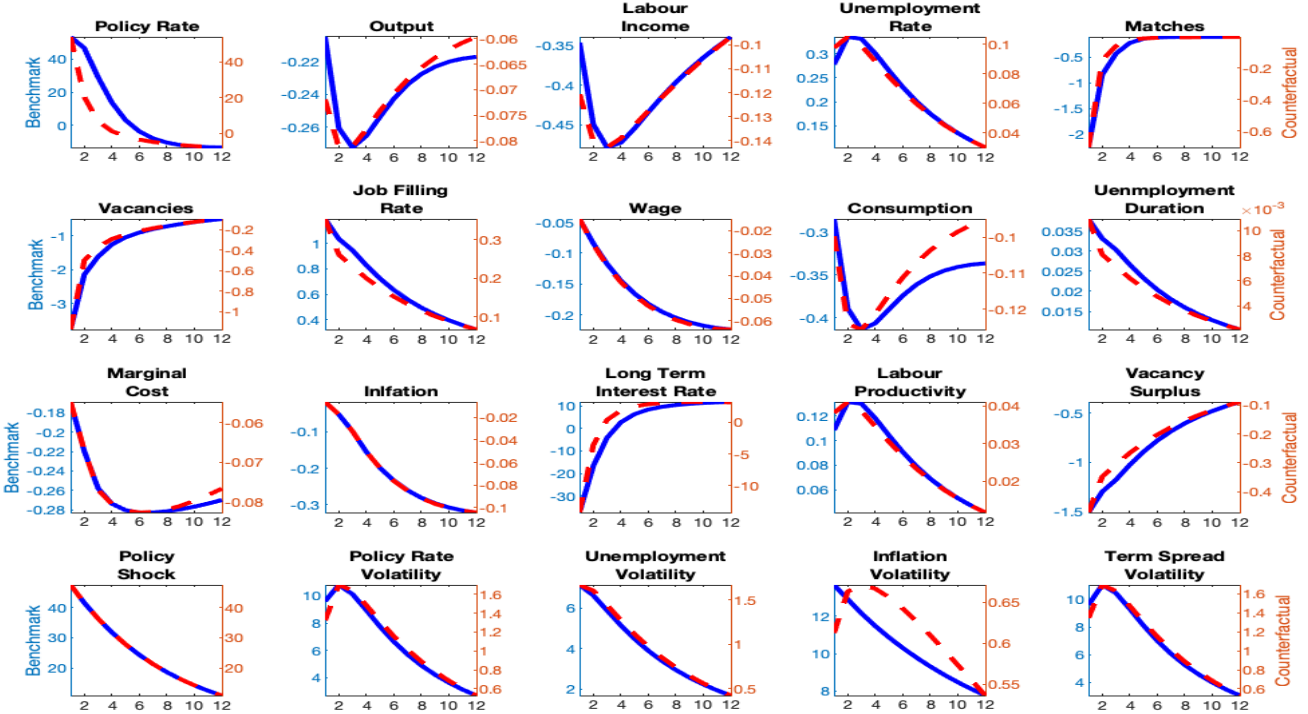
Panel B: Uncertainty Contribution



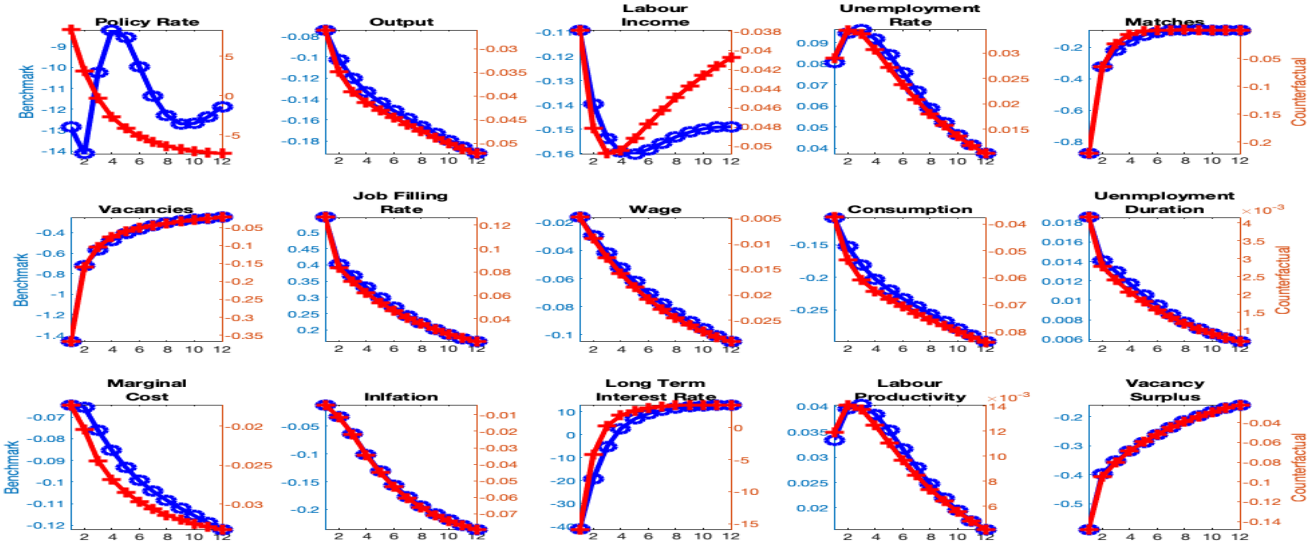
Notes: The blue solid (Panel A) and blue circle (Panel B) line (left y-axis) represents the benchmark version of the model ($u = 5.8\%$ or $Q^U = 63\%$), while the red dashed (Panel A) and red cross (Panel B) line (right y-axis) is the responses of the model when the unemployment rate is set to (almost) zero or the probability of finding a job is (almost) one ($u = 0$ or $Q^U = 100\%$). Rates are reported in annual basis points, inflation in annual percentage rates, the job filling probability in percentage points, unemployment duration in quarters. The responses are calculated relative to the stochastic steady state. **Panel B:** The uncertainty contribution is defined as the difference between the response derived by using the third minus the responses produced using only the first order solution of the model.

Figure 8: Policy Rate Smoothing Preferences

Panel A: Impulse Responses



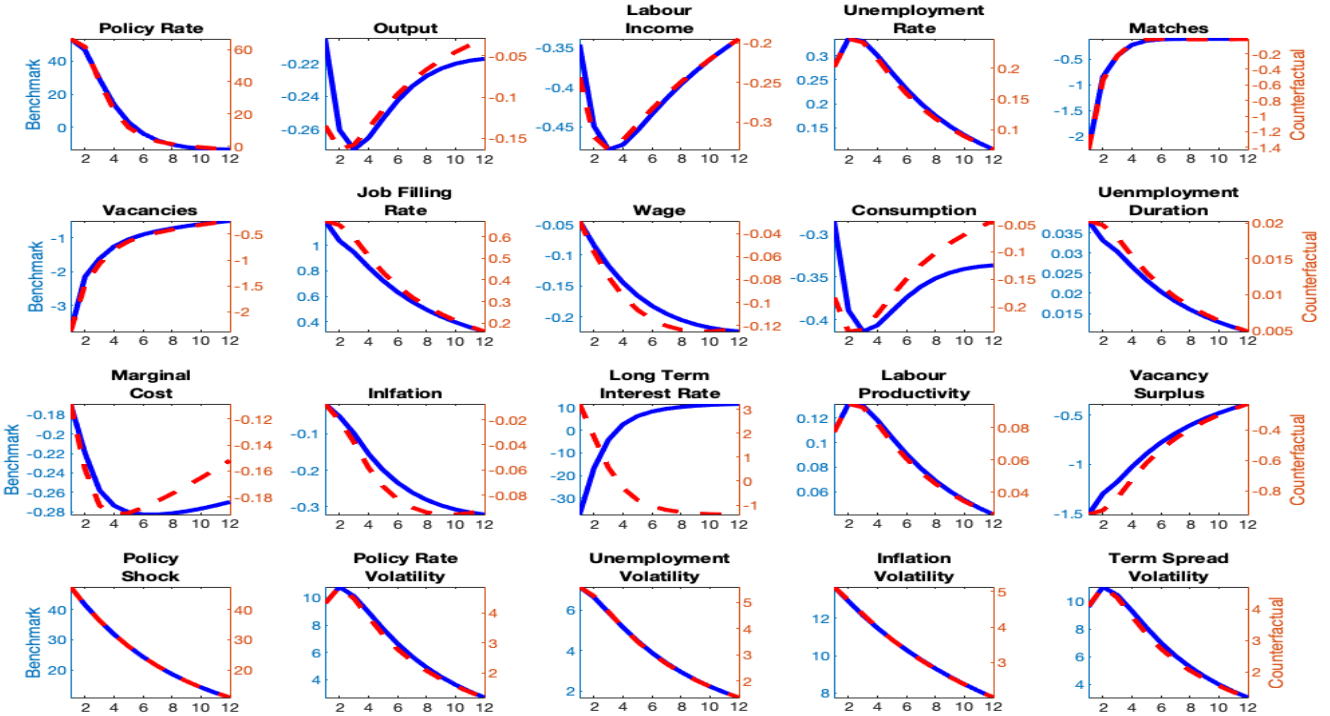
Panel B: Uncertainty Contribution



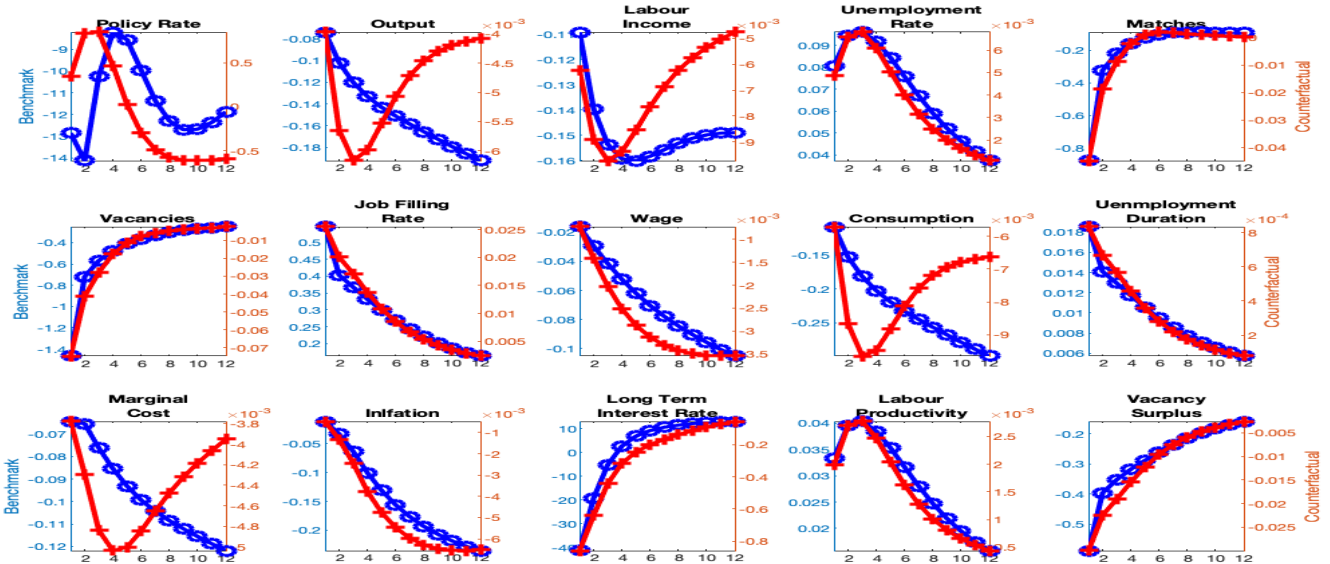
Notes: The blue solid (Panel A) and blue circle (Panel B) line (left y-axis) represents the benchmark version of the model ($\rho_R = 0.71$), while the red dashed (Panel A) and red cross (Panel B) line (right y-axis) denotes the responses of the model when the interest rate smoothing parameter is set to zero ($\rho_R = 0$). Rates are reported in annual basis points, inflation in annual percentage rates, the job filling probability in percentage points, unemployment duration in quarters. The responses are calculated relative to the stochastic steady state. **Panel B:** The uncertainty contribution is defined as the difference between the response derived by using the third minus the responses produced using only the first order solution of the model.

Figure 9: Epstein-Zin Risk Coefficient

Panel A: Impulse Responses



Panel B: Uncertainty Contribution



Notes: The blue solid (Panel A) and blue circle (Panel B) line (left y-axis) represents the benchmark version of the model ($\gamma = 115.6$), while the red dashed (Panel A) and red cross (Panel B) line (right y-axis) denotes the responses of the model when the Epstein-Zin Risk Coefficient is set to zero ($\gamma = 0$). Rates are reported in annual basis points, inflation in annual percentage rates, the job filling probability in percentage points, unemployment duration in quarters. The responses are calculated relative to the stochastic steady state. **Panel B:** The uncertainty contribution is defined as the difference between the response derived by using the third minus the responses produced using only the first order solution of the model.

Online Appendix: Dynamic Effects of Monetary Policy Shocks on Macroeconomic Volatility

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1 Model Estimation

Consider the VAR model

$$Z_t = c + \sum_{j=1}^P \beta_j Z_{t-j} + \sum_{k=1}^K b_k \tilde{h}_{t-k} + \frac{1}{t} e_t, e_t \sim N(0, 1) \quad (1)$$

$$\tilde{h}_t = A^{-1} H_t A^{-1'} \quad (2)$$

$$\tilde{h}_t = \alpha + \theta \tilde{h}_{t-1} + \sum_{j=1}^K d_j Z_{t-j} + \eta_t, \eta_t \sim N(0, Q), E(e_t, \eta_t) = 0 \quad (3)$$

where $\tilde{h}_t = [h_{1t}, h_{2t}, \dots, h_{Nt}]$, $H_t = \text{diag} \exp \tilde{h}_t$.

1.1 Prior distributions and starting values

1.1.1 VAR coefficients

Let $\theta = \text{vec} [c; \beta_j; b_k]$. Following [Banbura et al. \(2007\)](#), we employ a Normal prior implemented via dummy observations. The priors are implemented by the dummy observations y_D and x_D that are defined as:

$$y_D = \begin{pmatrix} \frac{\text{diag}(\gamma_1 s_1 \dots \gamma_n s_n)}{\tau} \\ 0_{N \times (P-1) \times N} \\ \dots \\ 0_{EX \times N} \end{pmatrix}, \quad x_D = \begin{pmatrix} \frac{J_P \otimes \text{diag}(s_1 \dots s_n)}{\tau} 0_{NP \times EX} \\ 0_{N \times (NP) + EX} \\ \dots \\ 0_{EX \times NP} \quad I_{EX} \times 1/c \end{pmatrix} \quad (4)$$

where γ_1 to γ_n denote the prior mean for the parameters on the first lag obtained by estimating individual AR(1) regressions, τ measures the tightness of the prior on the VAR coefficients, and c is the tightness of the prior on the exogenous and pre-determined regressors. EX denotes the number of exogenous and pre-determined regressors in each equation. N denotes the total number of endogenous

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variables and P is the lag length. We set $\tau = 0.1$. We use a different value of c for the coefficients on the lagged volatilities and For the coefficients on the lagged volatilities c is set equal to 0.1. A flat prior is used for the intercept terms and the corresponding tightness is set equal to $c = 1000$. Note that these dummies do not directly implement a prior belief on the VAR error covariance matrix which is time-varying in our setting.

The priors for the coefficients are thus: $N(\mu_0, P_0)$ where $\mu_0 = (x_D' x_D)^{-1} x_D' y_D$ and $P_0 = S (x_D' x_D)^{-1}$ where S is a diagonal matrix with an estimate of the variance of Z_t (obtained using the training sample described below) on the main diagonal.

1.1.2 Elements of H_t

Following Cogley and Sargent (2005) we use a training sample (of 120 pre-sample observations from 1947M1 to 1957M12) to set the prior for the elements of the transition equation of the model. Let \hat{v}^{ols} denote the OLS estimate of the VAR covariance matrix estimated on the pre-sample data. The prior for \tilde{h}_t at $t = 0$ is defined as $\ln h_0 \sim N(\ln \mu_0, I_4)$ where μ_0 are the diagonal elements of the Cholesky decomposition of \hat{v}^{ols} .

1.1.3 Elements of A

The prior for the off-diagonal elements A is $A_0 \sim N(\hat{a}^{ols}, V \hat{a}^{ols})$ where \hat{a}^{ols} are the off-diagonal elements of \hat{v}^{ols} , with each row scaled by the corresponding element on the diagonal. $V \hat{a}^{ols}$ is assumed to be diagonal with the elements set equal to 100. We employ a tighter prior for elements where sign restrictions are imposed.

1.1.4 Parameters of the transition equation

The prior on the coefficients and error covariance of the transition equation (3) is implemented via dummy variables (see Banbura et al. (2007)), shrinking each equation towards an AR process. This set of artificial data also includes dummy variables to implement the inverse Wishart prior on Q and the coefficients on the predetermined regressors. The prior tightness parameter controlling the strength of the prior on the coefficients on the lagged volatilities is set equal to 0.05. The parameter that controls the prior tightness on the lagged pre-determined variables is also set to 0.05.

1.2 Simulating the posterior distributions

1.2.1 VAR coefficients

The distribution of the VAR coefficients conditional on all other parameters and the stochastic volatility \tilde{h}_t is linear and Gaussian: $\{Z_t, \tilde{h}_t\} \sim N_{T \setminus T, P_{T \setminus T}}$ where $T \setminus T = E_{T \setminus T} \{Z_t, \tilde{h}_t\}$, $P_{T \setminus T} = Cov_{T \setminus T} \{Z_t, \tilde{h}_t\}$. Following Carter and Kohn (2004) we use the Kalman filter to estimate $E_{T \setminus T}$ and $P_{T \setminus T}$ where we account for the fact that the covariance matrix of the VAR residuals changes through time. Note that since we condition on \tilde{h}_t and A , the form of the heteroscedasticity is known. To use

the Kalman filter we write the VAR in state space form as

$$\begin{aligned} y_t &= x_t \alpha + A^{-1} H_t A^{-1'}^{1/2} e_t \\ x_t &= x_{t-1} \end{aligned}$$

The Kalman filter is initialised at x_0 and P_0 and the recursions are given by the following equations for $t = 1, 2, \dots, T$

$$\begin{aligned} x_{t|t-1} &= x_{t-1|t-1} \\ P_{t|t-1} &= P_{t-1|t-1} \\ \eta_{t|t-1} &= y_t - x_{t|t-1} \alpha \\ f_{t|t-1} &= x_t P_{t|t-1} x_t' + A^{-1} H_t A^{-1'} \\ K_t &= P_{t|t-1} x_t' f_{t|t-1}^{-1} \\ x_{t|t} &= x_{t|t-1} + K_t \eta_{t|t-1} \\ P_{t|t} &= P_{t|t-1} - K_t x_t P_{t|t-1} \end{aligned}$$

The final iteration of the Kalman filter at time T delivers $x_{T|T}$ and $P_{T|T}$. This application of the [Carter and Kohn \(2004\)](#) algorithm to this heteroscedastic VAR model is equivalent to a GLS transformation of the model.

1.2.2 Element of A_t

Given a draw for α and \tilde{h}_t the VAR model can be written as $A' \tilde{Z}_t = e_t$ where $\tilde{Z}_t = Z_t - c + \sum_{j=1}^p \beta_j Z_{it-j} = v_t$ and $\text{VAR}(e_t) = H_t$. This is a system of linear equations with a known form of heteroscedasticity. The conditional distributions for a linear regression apply to each equation of this system after a simple GLS transformation to make the errors homoscedastic. The i th equation of this system is given as $v_{it} = -\alpha v_{-it} + e_{it}$ where the subscript i denotes the i th column while $-i$ denotes columns 1 to $i-1$. Note that the variance of e_{it} is time-varying and given by $\exp \tilde{h}_{it}$. A GLS transformation involves dividing both sides of the equation by $\exp \tilde{h}_{it}$ to produce $v_{it}^* = -\alpha v_{-it}^* + e_{it}^*$ where $*$ denotes the transformed variables and $\text{var}(e_{it}^*) = 1$. The conditional posterior for α is normal with mean and variance given by M^* and V^* :

$$\begin{aligned} M^* &= \left(V \hat{\alpha}^{ols} \right)^{-1} + v_{-it}^* v_{-it}^{*'} \left(V \hat{\alpha}^{ols} \right)^{-1} \hat{\alpha}^{ols} + v_{-it}^* v_{it}^* \\ V^* &= \left(V \hat{\alpha}^{ols} \right)^{-1} + v_{-it}^* v_{-it}^{*'} \end{aligned}$$

1.2.3 Elements of H_t

Conditional on the VAR coefficients and the parameters of the transition equation, the model has a multivariate non-linear state-space representation. Following recent developments in the seminal paper by [Andrieu et al. \(2010\)](#), we employ a particle Gibbs step to sample from the conditional posterior of

\tilde{h}_t . [Andrieu et al. \(2010\)](#) show how a version of the particle filter, conditioned on a fixed trajectory for one of the particles can be used to produce draws that result in a Markov Kernel with a target distribution that is invariant. However, the usual problem of path degeneracy in the particle filter can result in poor mixing in the original version of particle Gibbs. Recent development, however, suggest that small modifications of this algorithm can largely alleviate this problem. In particular, [Lindsten et al. \(2014\)](#) propose the addition of a step that involves sampling the ‘ancestors’ or indices associated with the particle that is being conditioned on. They show that this results in a substantial improvement in the mixing of the algorithm even with a few particles.¹ As explained in [Lindsten et al. \(2014\)](#), ancestor sampling breaks the reference path into pieces and this causes the particle system to collapse towards something different than the reference path. In the absence of this step, the particle system tends to collapse to the conditioning path. We employ particle Gibbs with ancestor sampling in this step.

Let $\tilde{h}_t^{(i-1)}$ denote the fixed the fixed trajectory, for $t = 1, 2, \dots, T$ obtained in the previous draw of the Gibbs algorithm. We denote the parameters of the model by θ , and $j = 1, 2, \dots, M$ represents the particles. The conditional particle filter with ancestor sampling proceeds in the following steps:

1. For $t = 1$

- (a) Draw $\tilde{h}_1^{(j)} \setminus \tilde{h}_0^{(j)}$, for $j = 1, 2, \dots, M - 1$. Fix $\tilde{h}_1^{(M)} = \tilde{h}_1^{(i-1)}$
- (b) Compute the normalised weights $p_1^{(j)} = \frac{w_1^{(j)}}{\sum_{j=1}^M w_1^{(j)}}$ where $w_1^{(j)}$ denotes the conditional likelihood: $w_1^{(j)} = \int \tilde{h}_1^{(j)} \exp \left\{ -0.5 \tilde{e}_1^{(j)2} - \tilde{e}_1^{(j)} \left(Z_1 - c + \sum_{j=1}^P \beta_j Z_{1-j} + \sum_{k=1}^K b_k \tilde{h}_{1,[-k]}^{(j)} \right) \right\}$ and $\tilde{h}_1^{(j)} = A^{-1} H_1^{(j)} A^{-1'}$ with $H_1^{(j)} = \text{diag} \exp \tilde{h}_{1,[0]}^{(j)}$. The subscript [0] denotes the contemporaneous value in the state vector while $[-k]$ denote the k lagged states.

2. For $t = 2$ to T

- (a) Resample $\tilde{h}_{t-1}^{(j)}$ for $j = 1, 2, \dots, M - 1$ using indices $a_t^{(j)}$ with $\Pr a_t^{(j)} = j = p_{t-1}^{(j)}$
- (b) Draw $\tilde{h}_t^{(j)} \setminus \tilde{h}_{t-1}^{(a_t^{(j)})}$, for $j = 1, 2, \dots, M - 1$ using the transition equation of the model (equation 3). Note that $\tilde{h}_{t-1}^{(a_t^{(j)})}$ denotes the resampled particles in step (a) above.
- (c) Fix $\tilde{h}_t^{(M)} = \tilde{h}_t^{(i-1)}$
- (d) Sample $a_t^{(M)}$ with $\Pr a_t^{(M)} = j = p_{t-1}^{(j)} \Pr \tilde{h}_t^{(i-1)} \setminus \tilde{h}_{t-1}^{(j)}, \alpha, \theta, d, Q$ where the density $\Pr \tilde{h}_t^{(i-1)} \setminus \tilde{h}_{t-1}^{(j)}, \alpha$ is computed as $\int /Q/^{-0.5} \exp \left\{ -0.5 \tilde{\eta}_t^{(j)2} - \tilde{\eta}_t^{(j)} \left(Z_t - c + \sum_{j=1}^P \beta_j Z_{t-j} + \sum_{k=1}^K d_k \tilde{h}_{t,[-k]}^{(j)} \right) \right\}$. This constitutes the ancestor sampling step. If $a_t^{(M)} = M$ then the algorithm collapses to the simple particle Gibbs.
- (e) Update the weights $p_t^{(j)} = \frac{w_t^{(j)}}{\sum_{j=1}^M w_t^{(j)}}$ where $w_t^{(j)}$ denotes the conditional likelihood: $w_t^{(j)} = \int \tilde{h}_t^{(j)} \exp \left\{ -0.5 \tilde{e}_t^{(j)2} - \tilde{e}_t^{(j)} \left(Z_t - c + \sum_{j=1}^P \beta_j Z_{t-j} + \sum_{k=1}^K b_k \tilde{h}_{t,[-k]}^{(j)} \right) \right\}$ and $\tilde{h}_t^{(j)} = A^{-1} H_t^{(j)} A^{-1'}$ with $H_t^{(j)} = \text{diag} \exp \tilde{h}_{t,[0]}^{(j)}$.

3. End

¹See [Nonejad \(2015\)](#) for a recent application of this algorithm.

4. Sample $\tilde{h}_t^{(i)}$ with $\Pr \tilde{h}_t^{(i)} = \tilde{h}_t^{(j)} = p_T^{(j)}$ to obtain a draw from the conditional posterior distribution

We use $M = 50$ particles in our application. The initial values μ_0 defined above are used to initialise step 1 of the filter.

1.2.4 Parameters of the transition equation

Conditional on the draw for the volatilities, the conditional posterior for $\bar{B} = \text{vec}([\alpha, \theta, d_1, \dots, d_j])$ the parameters of the VAR in equation 2 is Normal. Letting y and x denote the left and the right hand side of the VAR in 2 the conditional posterior of the coefficients is defined as

$$G(\bar{B} | \cdot) \sim N(B^*, Q^{-1} x^* x^{*'}^{-1})$$

where $B^* = (x^* x^{*'})^{-1} x^* y^*$ and x^* and y^* denote x and y appended with dummy observations.

The conditional posterior for Q is inverse Wishart and is given by

$$G(Q | \cdot) \sim IW(S^*, T^*)$$

where T^* denote the number of actual observations plus the number of dummy observations and $S^* = (y^* - x^* b^*)' (y^* - x^* b^*)$

The MCMC algorithm is applied using 100,000 iterations discarding the first 95,000 as burn-in.

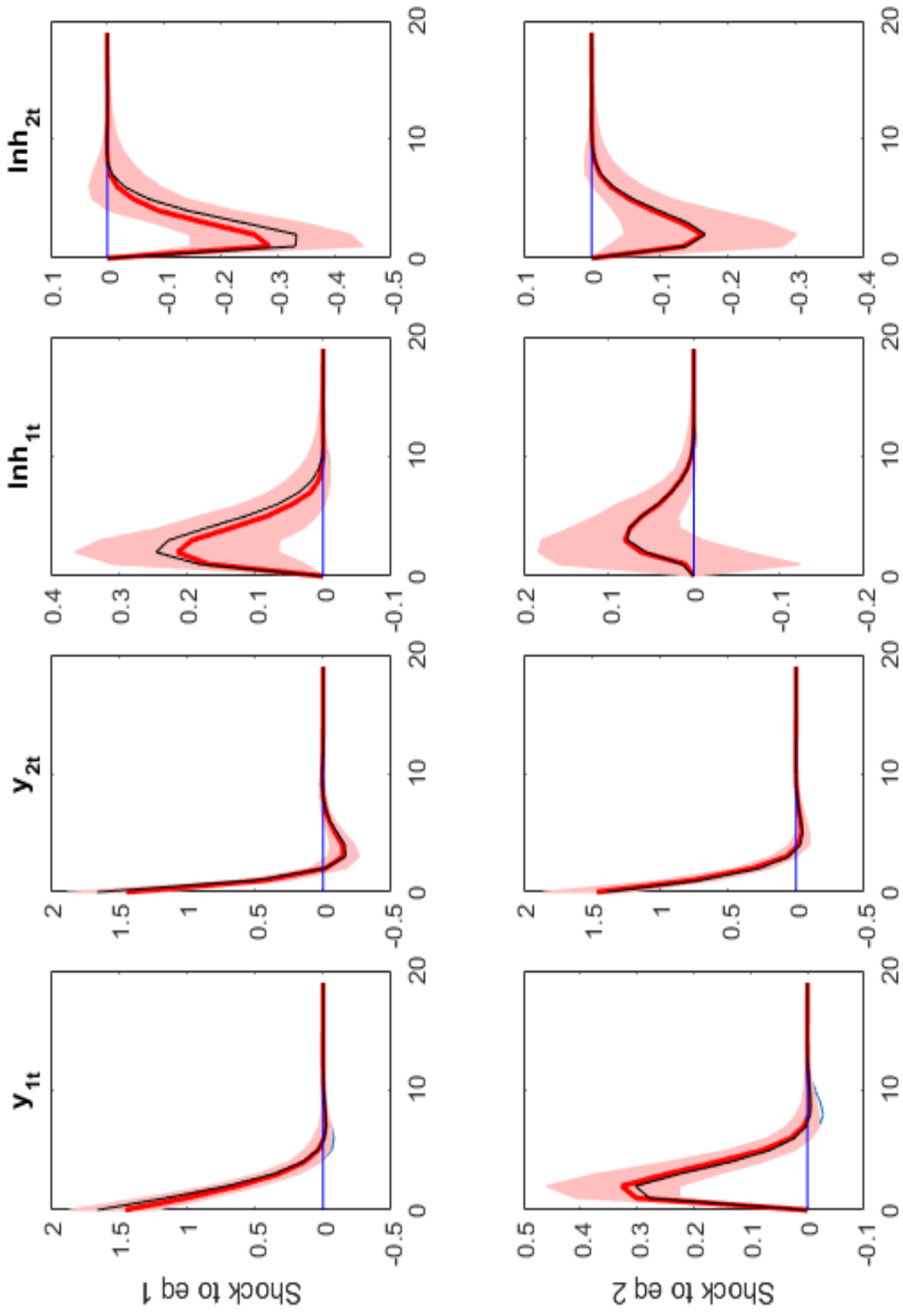
We conduct a small Monte-Carlo experiment to test the algorithm. We generate data from the following DGP

$$\begin{aligned} \begin{matrix} y_{1t} \\ y_{2t} \end{matrix} &= \begin{matrix} 0.5 & 0.2 \\ -0.2 & 0.5 \end{matrix} \begin{matrix} y_{1t-1} \\ y_{2t-1} \end{matrix} + \begin{matrix} -0.2 & -0.2 \\ 0.2 & 0.2 \end{matrix} \begin{matrix} \ln h_{1t-1} \\ \ln h_{2t-1} \end{matrix} + \begin{matrix} 1 & 0 \\ -1 & 1 \end{matrix} \begin{matrix} e_{1t} \\ e_{2t} \end{matrix} \\ \begin{matrix} \ln h_{1t} \\ \ln h_{2t} \end{matrix} &= \begin{matrix} 0.5 & -0.1 \\ -0.01 & 0.5 \end{matrix} \begin{matrix} \ln h_{1t-1} \\ \ln h_{2t-1} \end{matrix} + \begin{matrix} 0.1 & 0.01 \\ -0.1 & -0.1 \end{matrix} \begin{matrix} y_{1t-1} \\ y_{2t-1} \end{matrix} + \begin{matrix} 0.6 & -0.5 \\ -0.5 & 0.6 \end{matrix} \begin{matrix} v_{1t} \\ v_{2t} \end{matrix} \end{aligned}$$

where $e_{it}, v_{it} \sim N(0, 1)$ for $i = 1, 2$. We generate 500 observations and discard the first 100 to remove the effect of initial conditions. The experiment is repeated 100 times. At each iteration we calculate the impulse response to the two level shocks using an MCMC run of 5000 iterations.

A comparison of the true and estimated responses in figure 1 shows that the true response lies within the 95th percentile obtained from the simulation. This shows that the algorithm provides a satisfactory performance. Note that a previous version of the paper (see Mumtaz and Theodoridis (2015)) employs a single-move algorithm to draw from the conditional posterior distribution of the stochastic volatilities. The bias in the impulse responses in this previous version of the algorithm is larger and for this DGP the use of the particle Gibbs leads to a substantial improvement in estimation accuracy.

Figure 1: Monte Carlo Exercise



Notes: The black lines are true responses. The red line is the median across the Monte-Carlo replications. The shaded area is the 95th percentile

2 Model with time-varying coefficients

We also estimate the following version of the model:

$$Z_t = c_t + \sum_{j=1}^P \beta_{t,j} Z_{t-j} + \sum_{k=1}^K b_{t,k} \tilde{h}_{t-k} + \frac{1}{t} e_t \quad (5)$$

$$\tilde{h}_t = A^{-1} H_t A^{-1'} \quad (6)$$

$$\tilde{h}_t = \alpha_t + \theta_t \tilde{h}_{t-1} + \sum_{j=1}^K d_{t,j} Z_{t-j} + \eta_t \quad (7)$$

Letting $\tilde{h}_t = \text{vec} \begin{bmatrix} c_t, \beta_{t,1}, \dots, \beta_{t,P}, b_{t,1}, \dots, b_{t,K} \end{bmatrix}$ and $\tilde{h}_t = \text{vec}([\alpha_t, \theta_t, d_{t,1}, \dots, d_{t,K}])$ the evolution of the coefficients is determined by the additional transition equations:

$$\tilde{h}_t = \tilde{h}_{t-1} + \tilde{Q}_1^{1/2} v_{1t} \quad (8)$$

$$\tilde{h}_t = \tilde{h}_{t-1} + \tilde{Q}_2^{1/2} v_{2t} \quad (9)$$

The prior for \tilde{Q}_1 and \tilde{Q}_2 is inverse Wishart and is set following studies that employ VARs with time-varying parameters (see for e.g. [Benati and Mumtaz \(2007\)](#)).

$$p(\tilde{Q}_1) \sim IW(\tilde{Q}_{1,0}, \tilde{T}_{1,0})$$

$$p(\tilde{Q}_2) \sim IW(\tilde{Q}_{2,0}, \tilde{T}_{2,0})$$

where $\tilde{T}_{1,0} = \dim \tilde{Q}_{1,0} + 1$, $\tilde{T}_{2,0} = \dim \tilde{Q}_{2,0} + 1$. The scale matrices are calculated as $\tilde{Q}_{1,0} = V_{1,0} \times T_0 \times \kappa$ and $\tilde{Q}_{2,0} = V_{2,0} \times T_0 \times \kappa$ where $V_{1,0}$ and $V_{2,0}$ denote the covariance matrices of the coefficients and obtained by estimating a time-invariant VAR via OLS using the training sample $T_0 = 120$ and the initial value of \tilde{h}_t . We set the scaling parameter $\kappa = 0.0001$.

The Gibbs algorithm requires a change to draw the VAR coefficients and the coefficients of the transition equations. Conditional on the remaining parameters, the model has a linear state-space representation and the [Carter and Kohn \(2004\)](#) algorithm can be used to draw \tilde{h}_t and \tilde{h}_{t-1} . Given \tilde{h}_t and \tilde{h}_{t-1} , the covariances \tilde{Q}_1 and \tilde{Q}_2 can be drawn from the inverse Wishart distributions: $IW(\tilde{h}_t - \tilde{h}_{t-1})(\tilde{h}_t - \tilde{h}_{t-1})', \tilde{T}_{1,0} + 1$ and $IW(\tilde{h}_t - \tilde{h}_{t-1})(\tilde{h}_t - \tilde{h}_{t-1}), \tilde{T}_{2,0} + 1$.

3 Model that allows for correlation between the residuals of observation and transition equation

The Gibbs algorithm is taken from [Mumtaz \(2018\)](#). Consider the VAR model

$$\tilde{h}_t = \alpha + \theta \tilde{h}_{t-1} + \sum_{j=1}^Q d_j Z_{t-j} + S^{1/2} \eta_t \quad (10)$$

$$Z_t = c + \sum_{j=1}^P \beta_j Z_{t-j} + \sum_{k=1}^K b_k \tilde{h}_{t-k} + H_t^{1/2} e_t \quad (11)$$

where Z_t is a matrix of endogenous variables, $\tilde{h}_t = [h_{1t}, h_{2t}, \dots, h_{N,t}]$, $H_t = \text{diag} \exp \tilde{h}_t$ and $\tilde{s} = [s_1, s_2, \dots, s_N]$, $S = \text{diag}(\tilde{s})$. The $M = 2N$ disturbances $\varepsilon_t = \begin{pmatrix} \eta_t \\ e_t \end{pmatrix}$ are distributed normally

$N(0, \Sigma)$, where the diagonal elements of Σ are restricted to equal 1. For example for $M = 4, N = 2$:

$$\Sigma = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

3.1 Prior distributions and starting values

3.1.1 VAR coefficients

Let $\theta = \text{vec} [c; \beta_j; b_k]$. Following Banbura et al. (2007), we employ a Normal prior. The priors are implemented by the dummy observations y_D and x_D that are defined as:

$$y_D = \begin{pmatrix} \frac{\text{diag}(\gamma_1 s_1 \dots \gamma_n s_n)}{\tau} \\ 0_{N \times (P-1) \times N} \\ \dots \\ 0_{EX \times N} \end{pmatrix}, \quad x_D = \begin{pmatrix} \frac{J_P \otimes \text{diag}(s_1 \dots s_n)}{\tau} 0_{NP \times EX} \\ 0_{N \times (NP) + EX} \\ \dots \\ 0_{EX \times NP} \quad I_{EX} \times 1/c \end{pmatrix} \quad (12)$$

where γ_1 to γ_n denote the prior mean for the parameters on the first lag obtained by estimating individual AR(1) regressions, τ measures the tightness of the prior on the VAR coefficients, and c is the tightness of the prior on the exogenous and pre-determined regressors. EX denotes the number of exogenous and pre-determined regressors in each equation. N denotes the total number of endogenous variables and P is the lag length. We set $\tau = 0.1$. We use a different value of c for the coefficients on the lagged volatilities and the remaining pre-determined regressors. For the coefficients on the lagged volatilities c is set equal to 0.1. A flat prior is used for the intercept terms and the corresponding tightness is set equal to $c = 1000$. Note that these dummies do not directly implement a prior belief on the VAR error covariance matrix which is time-varying in our setting.

The priors for the coefficients are thus: $N(\theta_0, P_0)$ where $\theta_0 = (x_D' x_D)^{-1} x_D' y_D$ and $P_0 = S (x_D' x_D)^{-1}$ where S is a diagonal matrix with an estimate of the variance of Z_t (obtained using the training sample described below) on the main diagonal.

3.1.2 Elements of H_t

Following Cogley and Sargent (2005) we use a training sample (of 120 pre-sample observations) to set the prior for the elements of the transition equation of the model. Let \hat{v}^{ols} denote the OLS estimate of the VAR covariance matrix estimated on the pre-sample data. The initial value \tilde{h}_0 is set to μ_0 where μ_0 denotes the diagonal elements of the Cholesky decomposition of \hat{v}^{ols} .

3.1.3 Elements of

? decompose as $= L^{-1}DL^{-1}$ with the diagonal elements of D denoted by λ_k and a_{kj} denoting the lower triangular elements of L^{-1} . The prior for a_{kj} is assumed to be $N(0, 1)$ while the prior on D is implicit via the restriction that have diagonal elements that equal 1.

3.1.4 Parameters of the transition equation

The prior for VAR coefficients $\tilde{\alpha} = vec([\alpha; \theta; d_j])$ is set as above for . We assume an inverse Gamma prior for $\tilde{s} : IG(v_0, T_0), v_0 = 0.001, T_0 = 1$.

3.2 Simulating the posterior distributions

3.2.1 Coefficients

Conditional on S, H_t and , the model can be written as a SUR system with heteroscedasticity

$$\begin{aligned} Y_t &= X_t \beta_t + E_t \\ var(E_t) &= G_t G_t' \end{aligned} \tag{13}$$

where $G_t = diag[\tilde{s}^{-1/2}, \exp \tilde{h}_t^{-1/2}]$, $Y_t = \begin{bmatrix} h_{1t} \\ h_{2t} \\ \cdot \\ Z_t \end{bmatrix}$, $X_t = \begin{bmatrix} x_{1t} & 0 & 0 & 0 \\ 0 & x_{2t} & 0 & 0 \\ 0 & 0 & \cdot & 0 \\ 0 & 0 & 0 & \cdot \end{bmatrix}$, $E_t = \begin{bmatrix} \eta_{1t} \\ \eta_{2t} \\ \cdot \\ e_t \end{bmatrix}$

where x_{it} denotes the regressors in the ith equation of the system. Given a draw for the time-varying error covariance matrix, the coefficients have a conditional posterior that is normal: $N_{T \setminus T, P_{T \setminus T}}$. Following ? we use the Kalman filter to estimate the mean and variance of the conditional posterior where we account for the fact that the covariance matrix of the VAR residuals changes through time. To use the Kalman filter we define the transition equation as

$$t = t-1$$

The Kalman filter is initialised at β_0 and $P_{0|0}$ which are based on the priors for the coefficients introduced above and the recursions are given by the following equations for $t = 1, 2..T$

$$\begin{aligned}
\hat{y}_{t|t-1} &= \hat{y}_{t-1|t-1} \\
P_{t|t-1} &= P_{t-1|t-1} \\
\eta_{t|t-1} &= Y_t - X_t \hat{y}_{t|t-1} \\
f_{t|t-1} &= X_t P_{t|t-1} X_t' + G_t G_t' \\
K_t &= P_{t|t-1} X_t' f_{t|t-1}^{-1} \\
\hat{y}_{t|t} &= \hat{y}_{t|t-1} + K_t \eta_{t|t-1} \\
P_{t|t} &= P_{t|t-1} - K_t X_t P_{t|t-1}
\end{aligned}$$

The final iteration of the Kalman filter at time T delivers $\hat{y}_{T|T}$ and $P_{T|T}$.

3.3 Element of S

Given the residuals e_t , and the transition equations can be written as a VAR:

$$\begin{aligned}
\tilde{h}_t - S^{1/2} \mu_{\eta_t|e_t} &= \alpha + \theta \tilde{h}_{t-1} + \sum_{j=1}^Q d_j Z_{t-j} + \eta_t^* \\
\text{var}(\eta_t^*) &= S^{1/2} \Sigma_{\eta_t|e_t} S^{1/2}
\end{aligned} \tag{14}$$

$$\begin{aligned}
\mu_{\eta_t|e_t} &= e_t e'^{-1} \eta_e \\
\Sigma_{\eta_t|e_t} &= \Sigma_{\eta} - \Sigma_{\eta} e' e^{-1} e \Sigma_{\eta}
\end{aligned}$$

Note that η_t^* is uncorrelated with the residuals of the observation equation. The proposal density $q(\cdot)$ is defined as:

$$S_j = \kappa IG(v_1, T_1) + (1 - \kappa) IG(v(S_{j-1}), T, \bar{V})$$

where the parameter v_1 is set to $\tilde{\eta}'_{it} \tilde{\eta}_{it} + v_0$ and $T_1 = T_0 + T$ where T is the sample size and $\tilde{\eta}_{it}$ denotes the residuals of the i th transition equation. Letting the mean of the IG distribution equal S_{j-1} and standard deviation \bar{V} , the implied parameters of the second component of $q(\cdot)$ are defined as:

$$\begin{aligned}
v(S_{j-1}) &= 2S_{j-1} \left(1 + \frac{S_{j-1}^2}{\bar{V}^2} \right) \\
T \bar{V} &= 2 \left(2 + \frac{S_{j-1}^2}{\bar{V}^2} \right)
\end{aligned}$$

In the benchmark model, we set $\kappa = 0.5$ and \bar{V} is chosen to obtain a satisfactory acceptance rate. The draws accepted with probability:

$$\alpha = \frac{g(E_t/S_j) q(S_{j-1})}{g(E_t/S_{j-1}) q(S_j)}$$

where $g(E_t/S_j)$ denotes the posterior distribution evaluated at the j th draw of S given all other parameters at their values drawn in previous steps. With the model in the form of a VAR (equation [14](#)), the likelihood can be evaluated easily.

3.3.1 Elements of

? describe how to sample covariance matrices with restrictions on some of the elements and we follow their method in implementing the draw from this conditional posterior. ? decompose as $\Sigma = L^{-1}DL^{-1'}$. They show that when the diagonal elements of Σ are restricted to equal 1, then the diagonal elements of D (denoted by λ_k) satisfy:

$$\begin{aligned} \lambda_1 &= 1 \\ \lambda_k &= 1 - \prod_{j=1}^{k-1} a^{kj} \lambda_j, k = 2, 3, \dots, N \end{aligned} \tag{15}$$

where a^{kj} are lower diagonal elements of L^{-1} . They propose an independence Metropolis step to sample a^{kj} with a proposal density of the form:

$$f(a^{kj} | \varepsilon_t) = N(\mu_k, \tau V_k)$$

where $V = A_0^{-1} + \sum_{t=1}^T U_t \hat{D}^{-1} U_t$ and $\mu = V A_0^{-1} a_0 + \sum_{t=1}^T U_t \hat{D}^{-1} \varepsilon_t$. Here U_t is defined as the matrix:

$$U_t = \begin{bmatrix} 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ \varepsilon_{t,1} & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \varepsilon_{t,1} & \varepsilon_{t,2} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \varepsilon_{t,1} & \varepsilon_{t,2} & \varepsilon_{t,3} & 0 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & \cdot & \cdot & \cdot & \cdot & 0 & 0 & \varepsilon_{t,1} & \cdot & \varepsilon_{t,N} & \cdot \end{bmatrix}$$

and the diagonal elements of \hat{D} can be obtained by iterating between the equation for μ and equation 15. The draw is accepted with probability:

$$\alpha = \frac{g(\varepsilon_t | \text{new}) f(a_{\text{old}} | \varepsilon_t)}{g(\varepsilon_t | \text{old}) f(a_{\text{new}} | \varepsilon_t)}$$

with λ_k restricted to be greater than zero to ensure that Σ is positive definite. The expression for the likelihood function used to construct the posterior $g(\varepsilon_t | \cdot)$ is given in equation 2.7 in ?.

3.3.2 Elements of H_t

Conditional on the VAR coefficients and the parameters of the transition equation, the model has a multivariate non-linear state-space representation. It is convenient to express the state-space as:

$$\begin{aligned} F_t &= C + F_{t-1} + N_t \\ Z_t - H_t^{1/2} \mu_{e_t | \eta_t} &= c + \sum_{j=1}^P \beta_j Z_{t-j} + \sum_{k=1}^K b_k \tilde{h}_{t-k} + \tilde{e}_t \\ \text{var}(\tilde{e}_t) &= \Sigma_t = H_t^{1/2} \Sigma_{e_t | \eta_t} H_t^{1/2'} \end{aligned} \tag{16}$$

where:

$$F_t = \begin{pmatrix} \eta_{t+1} \\ \eta_t \\ \tilde{h}_t \\ \cdot \\ \tilde{h}_{t-k} \end{pmatrix} \quad (17)$$

$$C = \begin{pmatrix} 0 \\ 0 \\ \alpha + \sum_{j=1}^Q d_j Z_{t-j} \\ \cdot \\ 0 \end{pmatrix} \quad (18)$$

$$= S^{1/2} \begin{pmatrix} 0 & 0 & \cdot & \cdot & 0 \\ 1 & 0 & \cdot & \cdot & \cdot \\ 0 & \theta & \cdot & \cdot & 0 \\ 0 & 1 & \cdot & \cdot & 0 \\ \cdot & \cdot & 1 & \cdot & 0 \end{pmatrix} \quad (19)$$

$$N_t = \begin{pmatrix} \eta_{t+1} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$\mu_{e_t|\eta_t}$ denotes the conditional mean of e_t while $\Sigma_{e_t|\eta_t}$ is the conditional variance. These can be easily calculated using results for multi-variate normal distributions. Partitioning as:

$$= \begin{pmatrix} \eta & \eta_e' \\ \eta_e & e \end{pmatrix}$$

the conditional mean and variance are given by:

$$\begin{aligned} \mu_{e_t|\eta_t} &= \eta - \eta_e^{-1} \eta_e' \eta \\ \Sigma_{e_t|\eta_t} &= e - \eta_e^{-1} \eta_e' \eta_e \end{aligned}$$

Moreover:

$$\text{var}(N_t) = \tilde{Q} = \begin{pmatrix} \eta & \cdot & \cdot & \cdot & 0 \\ 0 & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & 0 & 0 & 0 \end{pmatrix}$$

Following recent developments in the seminal paper by [Andrieu et al. \(2010\)](#), we employ a particle Gibbs step to sample from the conditional posterior of F_t . [Andrieu et al. \(2010\)](#) show how a version of the particle filter, conditioned on a fixed trajectory for one of the particles can be used to produce draws that result in a Markov Kernel with a target distribution that is invariant. However, the usual problem of path degeneracy in the particle filter can result in poor mixing in the original version of particle Gibbs. Recent development, however, suggest that small modifications of this algorithm can largely alleviate this problem. In particular, [Lindsten et al. \(2014\)](#) propose the addition of a step that involves sampling the ‘ancestors’ or indices associated with the particle that is being conditioned on. They show that this results in a substantial improvement in the mixing of the algorithm even with a few particles.² As explained in [Lindsten et al. \(2014\)](#), ancestor sampling breaks the reference path into pieces and this causes the particle system to collapse towards something different than the reference path. In the absence of this step, the particle system tends to collapse to the conditioning path. We employ particle Gibbs with ancestor sampling in this step.

Let $F_t^{(i-1)}$ denote the fixed the fixed trajectory, for $t = 1, 2, \dots, T$ obtained in the previous draw of the Gibbs algorithm. We denote all the parameters of the model by θ , and $j = 1, 2, \dots, \tilde{M}$ indexes the particles. The conditional particle filter with ancestor sampling proceeds in the following steps:

1. For $t = 1$

(a) Draw $F_1^{(j)} \setminus F_0^{(j)}$, for $j = 1, 2, \dots, \tilde{M} - 1$. Fix $F_1^{(\tilde{M})} = F_1^{(i-1)}$

(b) Compute the normalised weights $p_1^{(j)} = \frac{w_1^{(j)}}{\sum_{j=1}^{\tilde{M}} w_1^{(j)}}$ where $w_1^{(j)}$ denotes the conditional likelihood: $\binom{j}{1}^{-0.5} - 0.5 \exp \tilde{e}_1 \binom{j}{1}^{-1} \tilde{e}'_1$ where $\binom{j}{1} = H_1^{(j)} \exp_{e_t \eta_t} H_1^{(j)'} \text{ with } H_1^{(j)} = \text{diag} \exp \tilde{h}_{1,[0]}^{(j)}$ and $\tilde{e}_1 = Z_1 - (H_1^{(j)})^{1/2} \mu_{e_t \eta_t} + c + \sum_{j=1}^P \beta_j Z_{t-j} + \sum_{k=1}^K b_k \tilde{h}_{1,[-k]}^{(j)}$ The subscript [0] denotes the contemporaneous value in the state vector while $[-k]$ denote the k lagged states.

2. For $t = 2$ to T

(a) Resample $F_{t-1}^{(j)}$ for $j = 1, 2, \dots, \tilde{M} - 1$ using indices $a_t^{(j)}$ with $\Pr a_t^{(j)} = j = p_{t-1}^{(j)}$

(b) Draw $F_t^{(j)} \setminus F_{t-1}^{(a_t^{(j)})}$, for $j = 1, 2, \dots, \tilde{M} - 1$ using the transition equation of the model. Note that $F_{t-1}^{(a_t^{(j)})}$ denotes the resampled particles in step (a) above.

(c) Fix $F_t^{(\tilde{M})} = F_t^{(i-1)}$

(d) Sample $a_t^{(\tilde{M})}$ with $\Pr a_t^{(\tilde{M})} = j = p_{t-1}^{(j)} \Pr F_t^{(i-1)} \setminus F_{t-1}^{(j)}, C, \theta, \tilde{Q}$ where the density $\Pr F_t^{(i-1)} \setminus F_{t-1}^{(j)}, C$ is computed as $\tilde{Q}^{-0.5} - 0.5 \exp N_t^{(j)} \tilde{Q}^{-1} N_t^{(j)}$. This constitutes the ancestor sampling step. If $a_t^{(\tilde{M})} = \tilde{M}$ then the algorithm collapses to the simple particle Gibbs.

(e) Update the weights $p_t^{(j)} = \frac{w_t^{(j)}}{\sum_{j=1}^{\tilde{M}} w_t^{(j)}}$ where $w_t^{(j)}$ denotes the conditional likelihood: $\binom{j}{t}^{-0.5} - 0.5 \exp \tilde{e}_t \binom{j}{t}^{-1} \tilde{e}'_t$

3. End

²See [Nonejad \(2015\)](#) for a recent application of this algorithm.

4. Sample $F_t^{(i)}$ with $\Pr F_t^{(i)} = F_t^{(j)} = p_T^{(j)}$ to obtain a draw from the conditional posterior distribution

We use $\tilde{M} = 20$ particles in our application. The initial values μ_0 defined above are used to initialise step 1 of the filter.

4 Sensitivity Analysis

Figure 2 presents some additional robustness analysis. Each row of the figure presents the response of the volatilities in the alternate models. The top row of the figure shows results from a version of the model when the lag length K in transition equation 3 is increased to 12, the key results remain similar to the benchmark case. The second row assumes that the transition equation takes the form:

$$\tilde{h}_t = \alpha + \sum_{j=1}^K \theta_j \tilde{h}_{t-1} + \sum_{j=1}^K d_j Z_{t-j} + \eta_t, K = 3$$

Again, the key results survive. The third row of the figure shows that the main results are preserved when S&P 500 stock returns are added to the model in place of the term spread. When the T-bill rate is replaced by the Federal Funds rate in the benchmark model, the key results survive (fourth row). The fifth row presents results from a version of the model that replaces unemployment with industrial production growth. The monetary policy shock is identified by assuming that a contractionary shock that raises rates also leads to a fall in industrial production and a decrease in inflation on impact. The policy shock leads to an increase in the volatility of the endogenous variables. The second last row considers the version of the model identified using an external instrument. While in the text we use the shock based on fed funds futures, the results in the figure are based on the Romer and Romer (2004) index. These responses support the main conclusions. When the monetary policy shock is identified recursively, ordering the short-term rate first produces responses of volatility that are close to the benchmark case (final row of Figure 2).

Figure 3 reports a check on the main results using an alternative methodology. We augment the proxy VAR used in Gertler and Karadi (2015) to include the measure of economic uncertainty built by Jurado et al. (2015). This model suggests that a monetary contraction is accompanied by an increase in economic uncertainty. This supports the results obtained using our non-linear VAR model.

4.1 Correlation between level and volatility shocks

The benchmark model in equations 5 and 6 does not allow a contemporaneous relationship between level and volatility shocks. In this section we follow Mumtaz (2018) and Alessandri and Mumtaz (2018) and relax this assumption. This extended model is defined as:

$$Z_t = c + \sum_{j=1}^P \beta_j Z_{t-j} + \sum_{k=1}^K b_k \tilde{h}_{t-k} + H_t^{1/2} e_t \quad (20)$$

Figure 2: Sensitivity Analysis

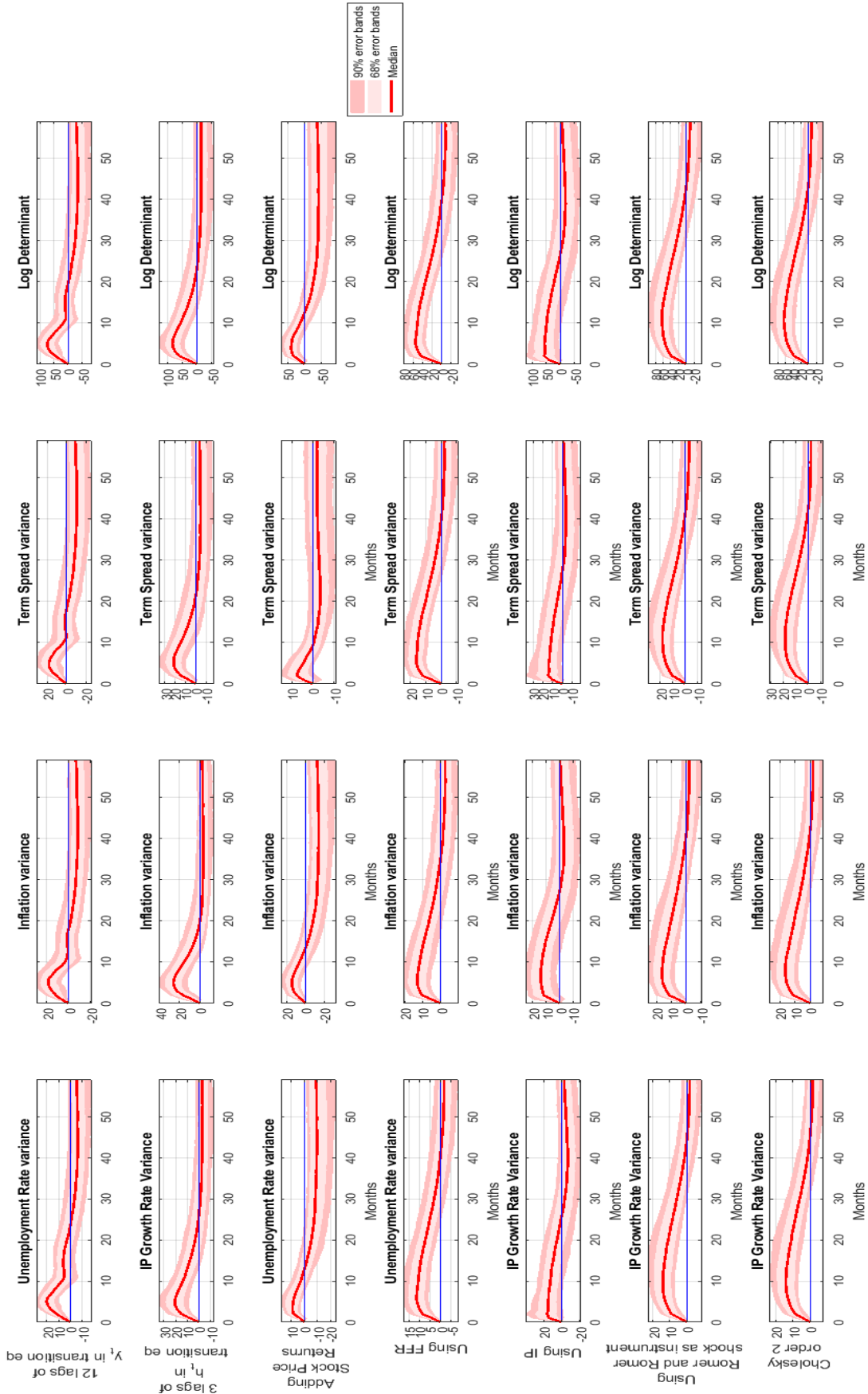
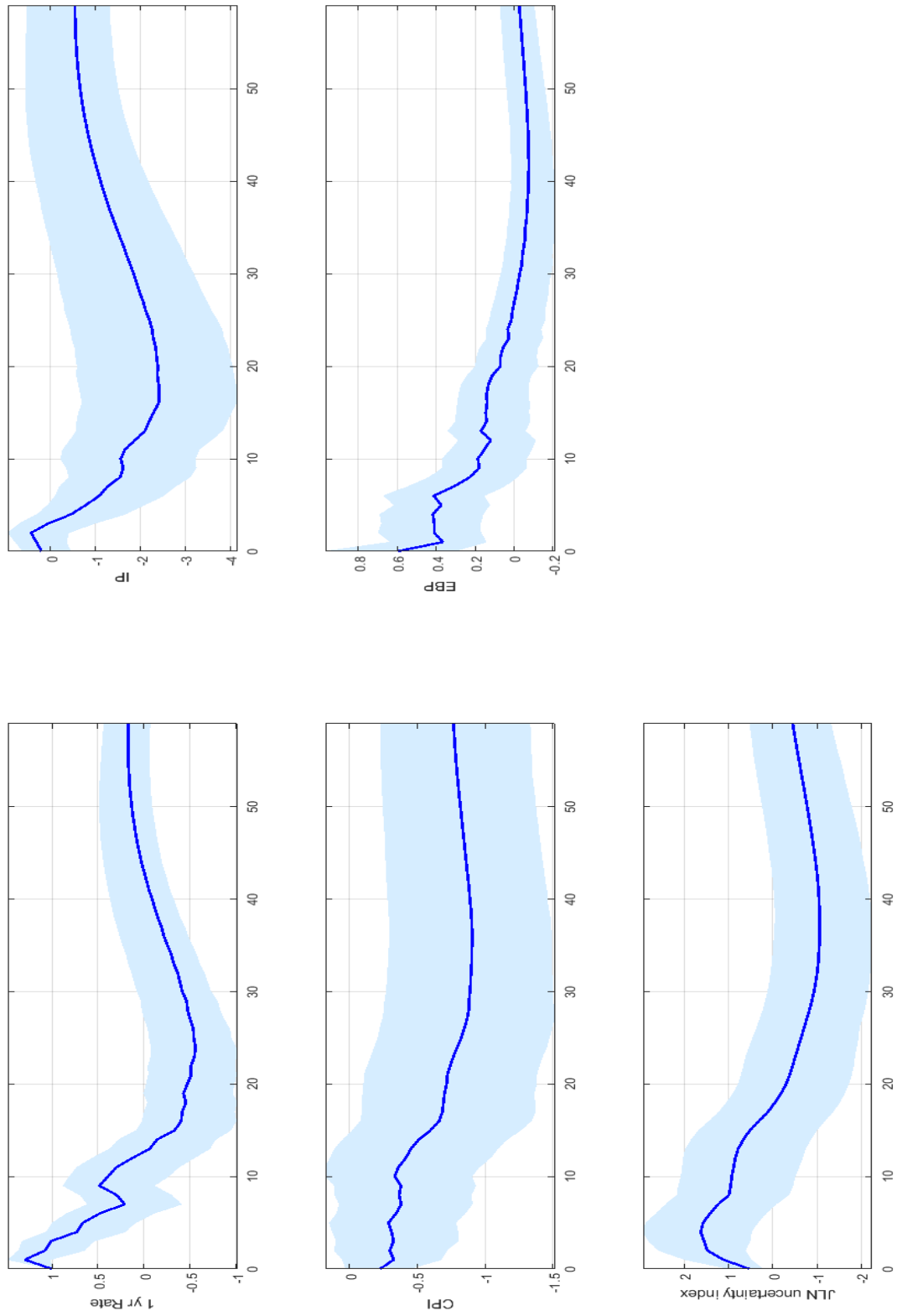


Figure 3: Results using the Gertler and Karadi (2015) VAR model augmented to include the Jurado et al. (2015) uncertainty index



$$\tilde{h}_t = \alpha + \theta \tilde{h}_{t-1} + \sum_{j=1}^K d_j Z_{t-j} + Q^{1/2} \eta_t \quad (21)$$

where $H_t = \text{diag}(\exp(-\tilde{h}_t))$ and $\begin{pmatrix} e_t \\ \eta_t \end{pmatrix} \sim N(0, \Sigma)$ with:

$$\Sigma = \begin{pmatrix} e_t & e_t \eta_t \\ e_t \eta_t & \eta_t \end{pmatrix} \quad (22)$$

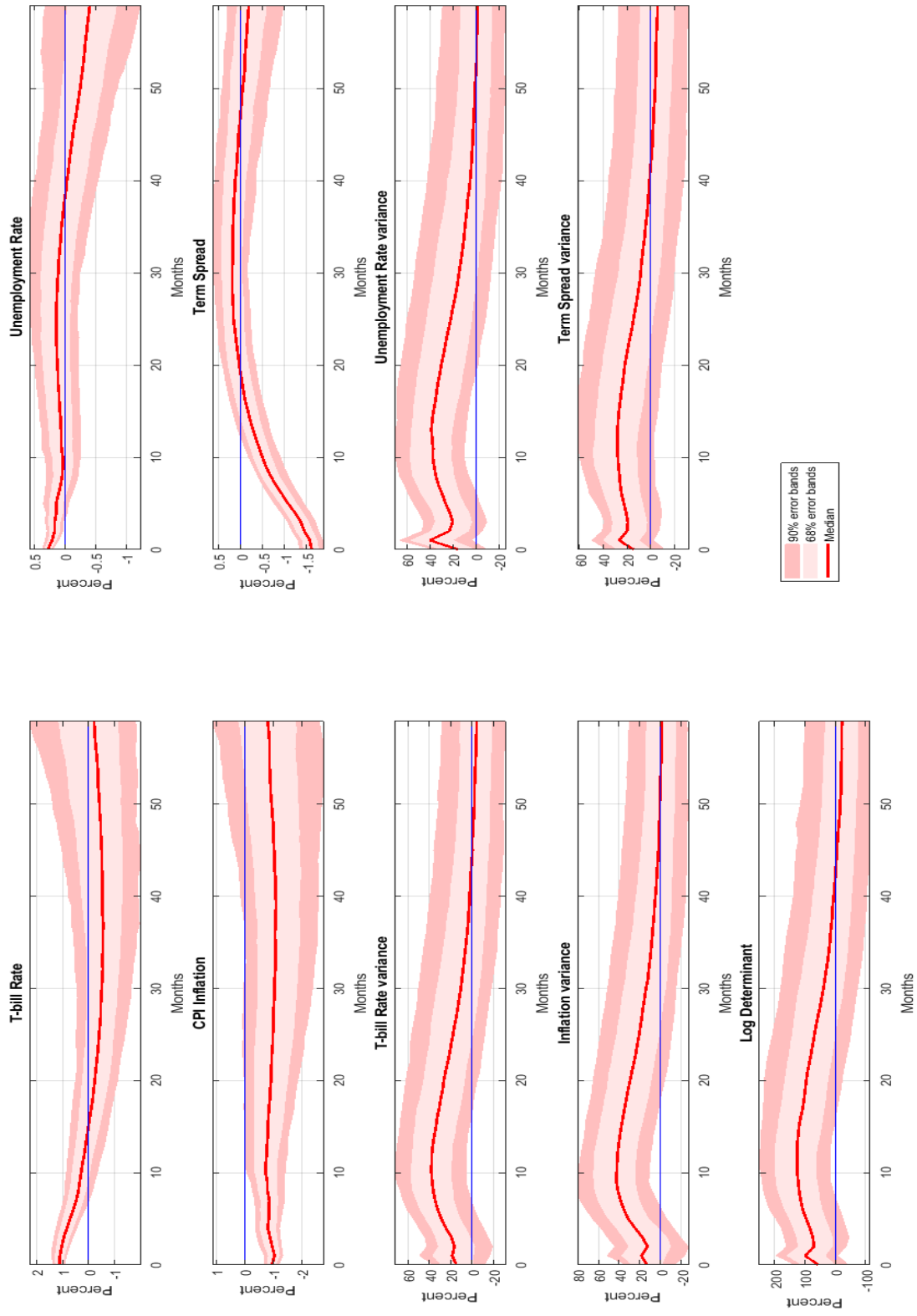
The diagonal elements of the covariance matrix are restricted to equal 1. With this specification, the covariance matrix between the residuals of equations 20 and 21 is given by:

$$\Sigma_t = \begin{pmatrix} H_t^{1/2} & 0 \\ 0 & Q^{1/2} \end{pmatrix} \begin{pmatrix} e_t & e_t \eta_t \\ e_t \eta_t & \eta_t \end{pmatrix} \begin{pmatrix} H_t^{1/2} & 0 \\ 0 & Q^{1/2} \end{pmatrix}' \quad (23)$$

We use the MCMC algorithm introduced in Mumtaz (2018) to approximate the posterior distribution and identify the monetary policy shock using contemporaneous sign restrictions used in the benchmark model. The impulse responses to the shock shown in Figure 4 suggest that, as in the benchmark case, volatility rises after a monetary contraction. However, Figure 4 also shows that the impulse responses are less precisely estimated. This is not surprising as several conditional posterior distributions in the extended model are non-standard and estimation is more complex and demanding. Nevertheless, based on the 68 percent highest posterior density intervals, the volatility responses are different from zero over the first year of the horizon. This provides moderately strong support to the conclusions reached using the benchmark analysis.³

³In the technical appendix we present results using the proxy VAR of Gertler and Karadi (2015) extended to include the measure of uncertainty developed by Jurado et al. (2015). Results from this model, which allows for a contemporaneous impact of monetary policy shock on uncertainty, support our key conclusions.

Figure 4: Impulse response to a monetary policy shock using the extended model in equations [20](#) and [21](#)



Notes: The solid line is the median. The light shaded area is the 68% error band while the dark shaded area is the 90% error band.

5 DSGE Model Analysis

5.1 A Model with Epstein-Zin Preferences and Search & Matching Frictions (Benchmark)

In this section we describe the building blocks of the key sectors of the DSGE model.

Households: The economy is populated by a continuum of households ($h \in [0, 1]$) that attain utility from consumption $\tilde{C}_t(h)$ and leisure $1 - L_t(h)$, where $L_t(h)$ denotes the fraction of the household that is employed. Household's preferences are separable

$$u(\tilde{C}_t(h), \tilde{Z}_t, L_t(h)) = \frac{\tilde{C}_t(h) - b\tilde{C}_{t-1}}{1 - \sigma_C} - \chi_0 \tilde{Z}_t^{1-\sigma_C} \frac{L_t(h)^{1+\sigma_L}}{1 + \sigma_L} \quad (24)$$

where σ_L is the inverse of the Frisch elasticity, σ_C stands for the inverse of intertemporal elasticity of substitution and \tilde{Z}_t denotes the non-stationary productivity process (the tilde indicates that the variable is non-stationary)

$$Z_t = \frac{\tilde{Z}_t}{\tilde{Z}_{t-1}} = Z (Z_{t-1})^{\rho_z} e^{\sigma_z \omega_{z,t}} \quad (25)$$

where Z is the steady-state value of the productivity growth, ρ_z indicates the degree of persistence and σ_z is the standard deviation of the productivity growth process. Furthermore, households have recursive preferences (Epstein and Zin (1989), Weil (1989, 1990))

$$V_t(h) = u(\tilde{C}_t(h), \tilde{Z}_t, L_t(h)) + \beta E_t V_{t+1}(h)^{1-\gamma} \frac{1}{1-\gamma} \quad (26)$$

The attractive feature of Epstein-Zin preferences is that the coefficient of relative risk aversion decouples from the intertemporal elasticity parameter. The parameter γ illustrates the degree of agents' desire for an early resolution of uncertainty over future consumption. Household maximises its utility function subject to its budget constraint which is:

$$\tilde{P}_t \tilde{C}_t(h) + \frac{\tilde{D}_t(h)}{R_t} + T_t(h) = \tilde{P}_t \tilde{W}_t L_t(h) + (1 - L_t(h)) \tilde{P}_t \tilde{B}_t + \tilde{D}_{t-1}(h) + \tilde{\pi}_t(h) \quad (27)$$

where \tilde{P}_t is the price index, $\tilde{D}_t(h)$ is the one period risk free government debt, R_t is the return on investing on the government debt, \tilde{W}_t stands for the real wage, $T_t(h)$ is the lump sum taxes, \tilde{B}_t is the unemployment benefit and $\tilde{\pi}_t(h)$ denotes firms' profits. This optimisation problem delivers (after some substitutions) the labour supply

$$\chi_0 \tilde{Z}_t^{1-\sigma_C} (L_t(h))^{\sigma_L} (\tilde{C}_t(h) - b\tilde{C}_{t-1})^{\sigma_C} + \tilde{B}_t = \tilde{W}_t \quad (28)$$

and the consumption Euler equation

$$\frac{1}{R_t} = \beta E_t \frac{\tilde{C}_t(h) - b\tilde{C}_{t-1}}{\tilde{C}_{t+1}(h) - b\tilde{C}_t} \frac{\tilde{P}_t}{\tilde{P}_{t+1}} E_t V_{t+1}^{1-\gamma} \frac{\gamma}{1-\gamma} V_{t+1}^{-\gamma} \quad (29)$$

If the stochastic discount factor is defined as:

$$M_{t+1} = \frac{V_{t+1}}{E_t V_{t+1}^{1-\gamma}} \frac{\tilde{C}_t(h) - b\tilde{C}_{t-1}}{\tilde{C}_{t+1}(h) - b\tilde{C}_t}^{-\gamma}, \quad (30)$$

then the former equation becomes:

$$\frac{1}{R_t} = \beta E_t \frac{M_{t+1}}{M_t} \quad (31)$$

Labour Market: The existence of a real – search and matching – friction in the labour market (Mortensen and Pissarides (1994)) prevents all job-seekers ($U_t = 1 - (1 - \delta_N) L_{t-1}$) from being matched with vacancies (v_t) posted by firms and they end up unemployed ($u_t = 1 - L_t$). The matching technology is described by the following Cobb-Douglas function:

$$M_t = \bar{\mu} U_t^\mu v_t^{1-\mu} \quad (32)$$

while the probability of filling the vacancy is given by

$$Q_t^r = \frac{M_t}{v_t} \quad (33)$$

and the probability of finding a job by

$$Q_t^u = \frac{M_t}{U_t} \quad (34)$$

employment evolves according to

$$L_t = (1 - \delta_N) L_{t-1} + Q_t^r v_t \quad (35)$$

where δ_N is the separation probability. Furthermore, firms in order to be able to hire a worker they need to post a vacancy and this incurs a cost

$$v_t = \kappa \tilde{Z}_t \quad (36)$$

Similar to Krause et al. (2008) and Di Pace and Villa (2016) we assume that the cost of posting a vacancy is non-linear, however, for $\sigma_\gamma = 1$ collapses to the one commonly used by the literature (see Mortensen and Pissarides (1994)).

Final Good Producer: The final good is produced via the following production function

$$\tilde{Y}_t = \int_0^1 \tilde{Y}_t(f)^{\frac{\varepsilon-1}{\varepsilon}} df^{\frac{\varepsilon}{\varepsilon-1}} \quad (37)$$

where ε denotes the elasticity of substitution between differentiated intermediate goods ($f \in [0, 1]$). The demand for intermediate goods

$$\tilde{Y}_t(f) = \frac{P_t(f)}{P_t}^{-\varepsilon} Y_t \quad (38)$$

results from profit maximisation and the assumption that the final good producer operates under perfect competition.

Intermediate Good Producers: Similar to [Krause and Lubik \(2007\)](#) and [Krause et al. \(2008\)](#) we assume that there is a continuum of firms ($f \in [0, 1]$) that post vacancies, combine employment, fixed capital and employ the following technology:

$$\tilde{Y}_t(f) = \tilde{Z}_t(L_t(f))^{1-\phi} K^\phi \quad (39)$$

to produce the intermediate good, where ϕ is the capital share in the production function. These producers solve a two-stages problem. In the first stage, taken the wage and the cost of filling a vacancy as given they decide how many vacancies to post and people to employ, these choices result from the maximisation of their profit function:

$$E_t \sum_{j=0}^{\infty} M_{t+j} \beta^j \left[MC_{t+j}(f) \tilde{Y}_{t+j}(f) - \tilde{W}_{t+j} L_{t+j}(f) - R^K K - \kappa \tilde{Z}_{t+j}(f) - \tilde{t}_{t+j}(f) (1 - \delta_N) L_{t+j-1}(f) + Q_{t+j}^Y(f) \tilde{t}_{t+j}(f) - L_{t+j}(f) \right] \quad (40)$$

with respect to employment delivers the relative price of good $\tilde{Y}_t(f)$ summarised by the following equations:

$$MC_t(f) = \frac{L_t(f)}{(1-\phi)\tilde{Y}_{t+j}(f)} \tilde{W}_t + \tilde{t}_t(f) - \beta E_t M_{t+1} \tilde{t}_{t+1}(f) (1 - \delta_N) \quad (41)$$

and

$$\tilde{t}_t(f) = \kappa \tilde{Z}_t \frac{(L_t(f))^{\sigma_Y - 1}}{Q_t^Y} \quad (42)$$

In the second stage, producers set the price of the intermediate good that maximises their profits. The optimisation problem in this case reflects that prices are set in a staggered manner. This means that every period a fraction $(1 - \xi)$ of firms receive a random signal and set prices optimally $\tilde{P}_t(f)$, while those firms who miss the signal set prices based on a rule of thumb backward looking indexation scheme $\tilde{P}_t(f) = \xi_{t-1} \tilde{P}_{t-1}(f)$. The pricing problem is summarised by the following profit maximisation

$$\max_{\tilde{P}_t(f)} E_t \sum_{j=0}^{\infty} M_{t,t+j} (\beta \xi)^j \left[\frac{\tilde{P}_t(f) \xi_{t+j-1}}{\tilde{P}_{t+j}} - MC_{t+j}(f) \tilde{Y}_{t+j}(f) \right] \quad (43)$$

subject to

$$\tilde{Y}_{t+j}(f) = \prod_{s=0}^j \frac{P_t(f)}{P_{t+s}(f)}^{-\varepsilon} Y_{t+j}$$

and this results in a system of equations that describe firms' optimal pricing decisions

$$\tilde{K}_{P,t}(f) = MC_t(f) \tilde{Y}_t(f) + \beta \xi E_t M_{t+1} \frac{t+1}{t}^\varepsilon \tilde{K}_{P,t+1}(f) \quad (44)$$

$$\tilde{F}_{P,t}(f) = \tilde{Y}_t(f) + \beta \xi E_t M_{t+1} \frac{t+1}{t}^{\varepsilon-1} \tilde{F}_{P,t+1}(f) \quad (45)$$

$$\tilde{P}_{P,t}(f) = \frac{\varepsilon}{\varepsilon - 1} \frac{\tilde{K}_{P,t}(f)}{\tilde{F}_{P,t}(f)} \quad (46)$$

$$1 = \xi \frac{t}{t-1}^{\varepsilon-1} + (1 - \xi) \tilde{P}_{P,t}(f)^{1-\varepsilon} \quad (47)$$

Wage Determination: The wage is determined by solving a Nash bargaining problem between workers and firms that takes place in order to decide how to split the surplus produced by a match (see [Mortensen and Pissarides \(1994\)](#) and [Krause et al. \(2008\)](#) among others). To set the problem we need to define the value of the firm, which can be derived from [\(41\)](#) after we use the value of an open vacancy and the free entry condition:

$$\tilde{J}_t^F = MC_t \frac{(1 - \phi) \tilde{Y}_t^d}{L_t} - \tilde{W}_t + \beta E_t M_{t+1} (1 - \delta_N) \tilde{J}_{t+1}^F \quad (48)$$

where $\tilde{Y}_t^d = \frac{\tilde{Y}_t}{\Delta_t}$ and \tilde{t} is defined below. On the other hand the value of an employed and unemployed worker is given by:

$$\begin{aligned} \tilde{J}_t^W &= \tilde{W}_t - \chi_0 \tilde{Z}_t^{1-\sigma_C} L_t^{\sigma_L} \tilde{C}_t - b \tilde{C}_{t-1}^{\sigma_C} \\ &+ \beta E_t M_{t+1} [1 - \delta_N] [1 - Q_{t+1}^U] \tilde{J}_{t+1}^W + \delta_N [1 - Q_{t+1}^U] \tilde{J}_{t+1}^U \end{aligned} \quad (49)$$

$$\tilde{J}_t^U = \tilde{B}_t + \beta E_t M_{t+1} [Q_{t+1}^U \tilde{J}_{t+1}^W + [1 - Q_{t+1}^U] \tilde{J}_{t+1}^U] \quad (50)$$

and the bargaining problem is described by:

$$\max_{\tilde{W}_t} \tilde{J}_t^W - \tilde{J}_t^U \quad \eta \quad \tilde{J}_t^F \quad 1-\eta \quad (51)$$

The total surplus is defined as:

$$\tilde{S}_t = \tilde{J}_t^F + \tilde{J}_t^W - \tilde{J}_t^U \quad (52)$$

and the solution to the problem is given by:

$$\tilde{J}_t^F = (1 - \eta) \tilde{S}_t \quad (53)$$

$$\bar{J}_t^W - \bar{J}_t^U = \eta \tilde{S}_t \quad (54)$$

Which can be used to derive the Nash wage:

$$\begin{aligned} \tilde{W}_t^{Nash} = & (1 - \eta) \chi_0 \tilde{Z}_t^{1-\sigma_C} L_t^{\sigma_L} \tilde{C}_t - b \tilde{C}_{t-1}^{\sigma_C} + \tilde{B}_t \\ & + \eta MC_t \frac{(1 - \phi) \tilde{Y}_t^d}{L_t} + \beta E_t M_{t+1} \tilde{w}_{t+1} (1 - \delta_N) \end{aligned} \quad (55)$$

Similarly to [Hall \(2005\)](#) and [Krause and Lubik \(2007\)](#), we allow for real wage rigidity via the following norm:

$$\tilde{W}_t = \tilde{W}_{t-1}^{\iota_w} \tilde{W}_t^{Nash} {}^{1-\iota_w} \quad (56)$$

Government: The government in this economy runs a balanced budget:

$$\tilde{P}_t \tilde{G}_t + \tilde{D}_{t-1} + (1 - L_t) \tilde{P}_t \tilde{B}_t = \tilde{T}_t + \frac{\tilde{D}_t}{R_t} \quad (57)$$

where G_t is government consumption and g_t is the of the government share in the economy:

$$\tilde{G}_t = g_t \tilde{Y}_t \quad (58)$$

that evolves in a stochastic manner:

$$\hat{g}_t = g + \rho_g (\hat{g}_{t-1} - g) + \sigma_g \omega_{g,t} \quad (59)$$

Monetary policy is set based on Taylor Type rule:

$$\log(R_t) = r_t = \rho_R r_{t-1} + (1 - \rho_R) \zeta_\Pi \log \frac{\pi_t}{\pi} + \zeta_u \log \frac{u_t}{u} + \omega_{R,t} \quad (60)$$

$$\omega_{R,t} = \rho_{\epsilon_R} \omega_{R,t-1} + \sigma_{R\epsilon} \epsilon_{R,t} \quad (61)$$

where π is the inflation target, ρ_R is the interest rate smoothing parameter, ζ_Π and ζ_{Y^d} are the policy reaction coefficients to inflation and demand growth, respectively.

Aggregation: The aggregated labour quantity demanded by firms is:

$$L_t = \int_0^1 L_t(f) df \quad (62)$$

and it is equal to the quantity supplied by household due the flexibility of wages. Using the latter equation and the intermediate good producer's production function, we obtain:

$$\tilde{Y}_t = \frac{\tilde{Z}_t L_t^{1-\phi} K^\phi}{\int_0^1 \frac{\tilde{P}_t(h)}{\tilde{P}_t}^{-\frac{\epsilon}{1-\phi}} dh} {}^{1-\phi} = \frac{\tilde{Z}_t L_t^{1-\phi} K^\phi}{t} \quad (63)$$

where \tilde{t} denotes the price dispersion and it evolves over time as follows:

$$\tilde{t}^{\frac{1}{1-\phi}} = (1 - \xi) \tilde{t}^{-\frac{\xi}{1-\phi}} + \xi \frac{\tilde{t}}{\tilde{t}-1} \tilde{t}^{\frac{\xi}{1-\phi}} \tilde{t}^{\frac{1}{1-\phi}} \quad (64)$$

Finally, using the expression about the aggregate profits:

$$\begin{aligned} \tilde{t} &= \tilde{P}_t \tilde{Y}_t - \int_0^1 \tilde{P}_t(h) \tilde{Y}_t(h) + \int_0^1 \tilde{P}_t(h) \tilde{Y}_t(h) - \tilde{P}_t \tilde{W}_t \int_0^1 L_t(h) \\ &\quad + \tilde{P}_t \tilde{W}_t \int_0^1 \tilde{Y}_t(h) - \tilde{W}_t \tilde{P}_t \int_0^1 N_t(h) - P_t \int_0^1 \tilde{t}(h) \end{aligned} \quad (65)$$

$$\tilde{t} = \tilde{P}_t \tilde{Y}_t - \tilde{P}_t \tilde{W}_t L_t - P_t \tilde{t} \quad (66)$$

and households budget constraint we derive the market clearing condition:

$$\frac{\tilde{Z}_t L_t^{1-\phi} K^\phi}{t} = \tilde{C}_t + \tilde{G}_t + \tilde{t} \quad (67)$$

Long-Term Interest Rate: As in [Rudebusch and Swanson \(2012\)](#) and [Swanson \(2015\)](#) the nominal consol price is given by:

$$P_t^L = 1 + E_t \beta \frac{M_{t+1}}{t+1} \kappa^L P_{t+1}^L \quad (68)$$

where κ^B denotes the rate of decay of the coupon on the consol and also controls the maturity of the asset. The long-term interest rate is then defined as:

$$r_t^L = \log \frac{\kappa^L P_t^L}{P_t^L - 1} \quad (69)$$

5.1.1 Stationary Equations

$$u(C_t, L_t) = \frac{C_t - h \frac{C_{t-1}}{\Gamma_t}}{1 - \sigma_C} - \chi_0 \frac{L_t^{1+\sigma_L}}{1 + \sigma_L} \quad (70)$$

$$\check{V}_t = E_t V_{t+1} \frac{1-\sigma_C}{t+1} \frac{1-\gamma}{t+1}$$

$$\ddot{V}_t = \check{V}_t^{\frac{1}{1-\gamma}}$$

$$V_t = u(C_t, L_t) + \beta \ddot{V}_t \quad (71)$$

$$\chi_0 N_t^{\sigma_L} C_t - h \frac{C_{t-1}}{t} \frac{\sigma_C}{t} + B = W_t^E \quad (72)$$

$$\frac{E_t V_{t+1} \frac{\sigma_C}{t+1}}{R_t^A} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \quad (73)$$

$$\frac{1}{R_t^A} = E_t \frac{C_t - h \frac{C_{t-1}}{\Gamma_t}}{C_{t+1} - h C_t} \stackrel{\sigma_C}{=} \frac{1}{t+1} E_t V_{t+1}^{1-\gamma} \frac{\gamma}{1-\gamma} V_{t+1}^{-\gamma}$$

$$M_{t+1} = E_t \frac{C_t - h \frac{C_{t-1}}{\Gamma_t}}{C_{t+1} - h C_t} \stackrel{\sigma_C}{=} \frac{1}{t+1} E_t V_{t+1}^{1-\gamma} \frac{\gamma}{1-\gamma} V_{t+1}^{-\gamma} \quad (74)$$

$$\frac{1}{R_t^A} = \beta E_t \frac{M_{t+1}}{t+1}$$

$$Y_t = \bar{K}^\phi L_t^{1-\phi}$$

$$MC_t = \frac{N_t(h)}{(1-\phi) \bar{Y}_t(h)} [W_t + \frac{t}{t+1} - \beta E_t M_{t+1} \frac{t}{t+1} (1-\delta_N)] \quad (75)$$

$$t = \frac{\kappa}{Q_t^Y} \quad (76)$$

$$W_t = \eta MC_t \frac{(1-\phi) Y_t}{L_t} + \beta E_t M_{t+1} \frac{t}{t+1} (1-\delta_N)$$

$$+ (1-\eta) \chi_0 L_t^{\sigma_L} C_t - h \frac{C_{t-1}}{t} \stackrel{\sigma_C}{=} + B \quad (77)$$

$$K_{P,t} = MC_t Y_t^d + \beta \xi E_t M_{t+1} \frac{t+1}{t} \stackrel{\varepsilon}{=} K_{P,t+1} \frac{t+1}{t+1} \quad (78)$$

$$F_{P,t} = Y_t^d + \beta \xi E_t M_{t+1} \frac{t+1}{t} \stackrel{\varepsilon-1}{=} F_{P,t+1} \frac{t+1}{t+1} \quad (79)$$

$$\frac{K_{P,t}}{F_{P,t}} = \frac{\varepsilon}{\varepsilon-1} \frac{K_{P,t}}{F_{P,t}} \quad (80)$$

$$1 = \xi \frac{t}{t-1} \stackrel{\varepsilon-1}{=} + (1-\xi) \frac{K_{P,t}}{F_{P,t}} \stackrel{-1-\varepsilon}{=} \quad (81)$$

$$\frac{1}{t} \stackrel{1-\phi}{=} = (1-\xi) \frac{K_{P,t}}{F_{P,t}} \stackrel{-\frac{\varepsilon}{1-\phi}}{=} + \xi \frac{t}{t-1} \stackrel{\frac{\varepsilon}{1-\phi}}{=} \frac{1}{t-1} \quad (82)$$

$$Y_t^d = C_{H,t} + G_t + I + \kappa \frac{t}{t} \quad (83)$$

$$Y_t = \frac{t}{t} Y_t^d \quad (84)$$

5.1.2 Steady States

We set $L = 1 - 0.058$ and we use the capital accumulation equation to

$$\delta_K K = I \quad (85)$$

$$\frac{\delta_K K}{Y} = \frac{I}{Y} \quad (86)$$

We also set

$$\frac{I}{Y} = 0.2 \quad (87)$$

The from the production function we derive

$$\begin{aligned} \frac{Y}{\bar{K}} &= \bar{K}^{\phi-1} L^{1-\phi} \\ \frac{Y}{\delta_K \bar{K}} &= \bar{K}^{\phi-1} \frac{L^{1-\phi}}{\delta_K} \\ \bar{K} &= \frac{L^{1-\phi} I}{\delta_K Y} \end{aligned} \quad (88)$$

From the stochastic discount factor we obtain

$$M = -\sigma_C$$

From the Philips curve equations we obtain

$$\begin{aligned} K_P &= \frac{MCY^d}{1 - \beta\xi^{(1-\iota)\varepsilon} 1 - \sigma_C} \\ F_P &= \frac{Y^d}{1 - \beta\xi^{(1-\iota)(\varepsilon-1)} 1 - \sigma_C} \\ -P &= \frac{1 - \xi^{(1-\iota)(\varepsilon-1)} \frac{1}{1-\varepsilon}}{1 - \xi} \\ -P &= \frac{\varepsilon}{\varepsilon - 1} \frac{K}{F} = \frac{\varepsilon}{\varepsilon - 1} \frac{\frac{MCY^d}{1 - \beta\xi M \Pi^{(1-\iota)\varepsilon} \Gamma}}{\frac{Y^d}{1 - \beta\xi M \Pi^{(1-\iota)(\varepsilon-1)} \Gamma}} \end{aligned} \quad (89)$$

$$-P = \frac{\varepsilon}{\varepsilon - 1} \frac{1 - \beta\xi^{(1-\iota)(\varepsilon-1)} 1 - \sigma_C}{1 - \beta\xi^{(1-\iota)\varepsilon} 1 - \sigma_C} MC \quad (90)$$

$$MC = -P \frac{\varepsilon - 1}{\varepsilon} \frac{1 - \beta\xi^{(1-\iota)\varepsilon} 1 - \sigma_C}{1 - \beta\xi^{(1-\iota)(\varepsilon-1)} 1 - \sigma_C} \quad (91)$$

$$MC = \frac{\varepsilon - 1}{\varepsilon} \frac{1 - \xi^{(1-\iota)(\varepsilon-1)} \frac{1}{1-\varepsilon}}{1 - \xi} \frac{1 - \beta\xi^{(1-\iota)\varepsilon} 1 - \sigma_C}{1 - \beta\xi^{(1-\iota)(\varepsilon-1)} 1 - \sigma_C} \quad (92)$$

Then from the price dispersion equation

$$= \frac{(1 - \xi)^{-\frac{\varepsilon}{1-\phi}} 1 - \phi}{1 - \xi \frac{(1-\iota)\varepsilon}{1-\phi}} \quad (93)$$

From the marketing clearing condition we also know that

$$Y^d = \frac{Y}{\kappa} \quad (94)$$

We use κ and the following expression to match the steady-state search and matching output loss

$$\kappa = \frac{1}{Y^d} \quad (95)$$

Similarly

$$C = (1 - g)Y^d - I - G + \kappa \sigma_T \quad (96)$$

$$= \frac{\kappa \sigma_T^{-1}}{Q^T}$$

From the demand for labour

$$W = MC \frac{(1 - \phi)Y}{N} - (1 - \beta M (1 - \delta_N)) \quad (97)$$

From the labour supply

$$\chi_0 = \frac{W^{-\eta} \left\{ \frac{MC^w \frac{(1-\phi)Y}{N} + \beta M \Theta \Gamma (1-\delta_N)}{1-\eta} \right\} - B}{N^{\sigma_L} \left(1 - \frac{b}{\Gamma} \right) C^{\sigma_C}}$$

5.2 Calibration

The model is estimated using limited information impulse response matching techniques (Christiano et al. (2005), Christiano et al. (2010)). However, the value of a small number of parameters is decided prior to the estimation. To be precise, the share of capital in the production (ϕ) and its depreciation rate have been calibrated to 0.36 and 0.025, numbers typically used in the literature (Christiano et al. (2005)). The steady-state unemployment (u) is set equal to 5.8% (the sample mean), while the steady-state value of output (y) to 1. The time discount factor (β) equals 0.995, while the both the steady-state value of inflation and productivity growth have been set to 2%. The last three parameters imply that the non-stochastic steady-state of the annual policy rate is 6%. Similar to Smets and Wouters (2007), the government spending and investment to GDP ratios are calibrated to 0.18 and 0.2, respectively. Finally, the steady-state value of the probability filling the vacancy is 70% (Hagedorn and Manovskii (2008)).

The parameters $\sigma_C, \sigma_L, b, \gamma, \xi, \iota, \zeta_{II}, \zeta_u, \zeta_R, \varepsilon, \mu, \eta, \delta_N, \bar{B} = \frac{B}{WL}, \iota_w, \frac{\Phi}{Y}, \rho_{\varepsilon_Z}, \rho_{\varepsilon_G}, \rho_{\varepsilon_R}, \sigma_{\varepsilon_Z}, \sigma_{\varepsilon_G}$ and σ_{ε_R} are selected to match the nonlinear VAR responses to an identified monetary policy shock using the benchmark identification scheme. The model is solved using third-order perturbation methods and the impulse responses are calculated relative to the stochastic steady state (Cacciatore and Ravenna (2016)).⁴

The objective of the estimation is for the model to be able to replicate not only the empirical responses to a monetary policy shock illustrated in Figure 1 (page 35 in the main text) but the also the contribution of the ‘uncertainty channel’ to these responses. This is defined as the difference between the unrestricted and restricted responses plotted in Figure 4 (page 38 in the main text) and discussed in Section 3.2 (in the main text). The latter set of targets ensures that the parameters that control the size of the uncertainty channel in the model are calibrated carefully and the predictions of the model about the importance of that channel are in line with those observed in the data.⁵ As it will become

⁴No pruning is applied in our calculations, which have been implemented using Dynare 4.4.3. The model and replication files can be downloaded from authors’ webpages. We also check if our results are robust if we apply Koop et al. (1996) methodology to calculate the IRFs and we find that our results are almost identical. We choose to employ the first method to calculate the IRFs as it dramatically reduces the estimation time.

⁵We would like to thank an anonymous referee for proposing this exercise.

apparent later in our analysis, the last set of moments acts also a natural metric that allows us to assess which part of the transmission mechanism is responsible for the existence of the endogenous uncertainty channel. The process used to estimate the DSGE contributions of uncertainty follows closely the steps employed for the empirical models. To be precise, the uncertainty contribution is defined as the difference between the responses obtained using the third order solution model minus those by using only the first order component of the solution.

5.3 Estimation Results

Figure 5 illustrates the ability of the theoretical model to replicate the identified empirical responses (Panel A) as well as the contribution of the uncertainty channel to these responses (Panel B), respectively. Even though the literature has questioned ability of a “simple” search and matching New Keynesian model to jointly replicate the dynamics of both unemployment and inflation (see the discussion in Krause and Lubik (2007) and Gertler et al. (2008)), our estimated model seems to be robust to this criticism as it reproduces the data dynamics remarkably well.

In this study, however, our results go further. The model is capable of replicating: (i) the VAR based stochastic volatility responses to a level monetary policy shock and, (ii) the empirically identified contribution of the uncertainty channel.⁶ This is a new set of results that further supports the analysis of Petrosky-Nadeau et al. regarding the aptitude of DSGE models with search and matching labour market frictions of reproducing highly non-linear dynamics seen in the data.

Returning to the discussion of the estimated parameters, Table 6 shows that the model can replicate the data features mentioned in the previous paragraph by relying on a set of parameter values that have been extensively used in the literature. Perhaps the less common parameter estimate is that for the Epstein-Zin risk coefficient ($\gamma = 115.57$) with the estimated value indicating that agents dislike future uncertainty. This value is similar to the one used in the Rudebusch and Swanson (2012) and Swanson (2015). As explained in Swanson (2015) – and it is further illustrated in the next section – this parameter only has a small effect on the stochastic volatility of macroeconomic variables and therefore the large magnitude is innocuous.⁷

In terms of the utility kernel, the model demands some curvature ($\sigma_C = 2.15$, Chen et al. (2012)) and ($\sigma_L = 4.00$, Christiano et al. (2010)) in order to reconcile the predicted dynamics with those in the data. The degree of consumption smoothing ($b = 0.57$), is close to the estimates reported by Christiano et al. (2005).

The parameters that govern the labour market block of the model, the Cobb-Douglas matching parameter ($\mu = 0.40$), the job separation rate ($\delta_N = 0.18$), the income replacement ratio ($100\bar{B} = \frac{B}{WL} = 54\%$) and the steady-state, the bargaining power for workers ($\eta = 0.50$) search and matching friction ($100\frac{\Phi}{Y} = 1\%$) are again consistent with those in the literature (see for instance, Krause and Lubik (2007) and Krause et al. (2008), Gertler et al. (2008) and Hagedorn and Manovskii (2008) among others).

⁶The empirical volatility responses are all hump shaped, while these hump is less pronounced in the theoretical responses. Due to the simplicity of the model, this feature is expected.

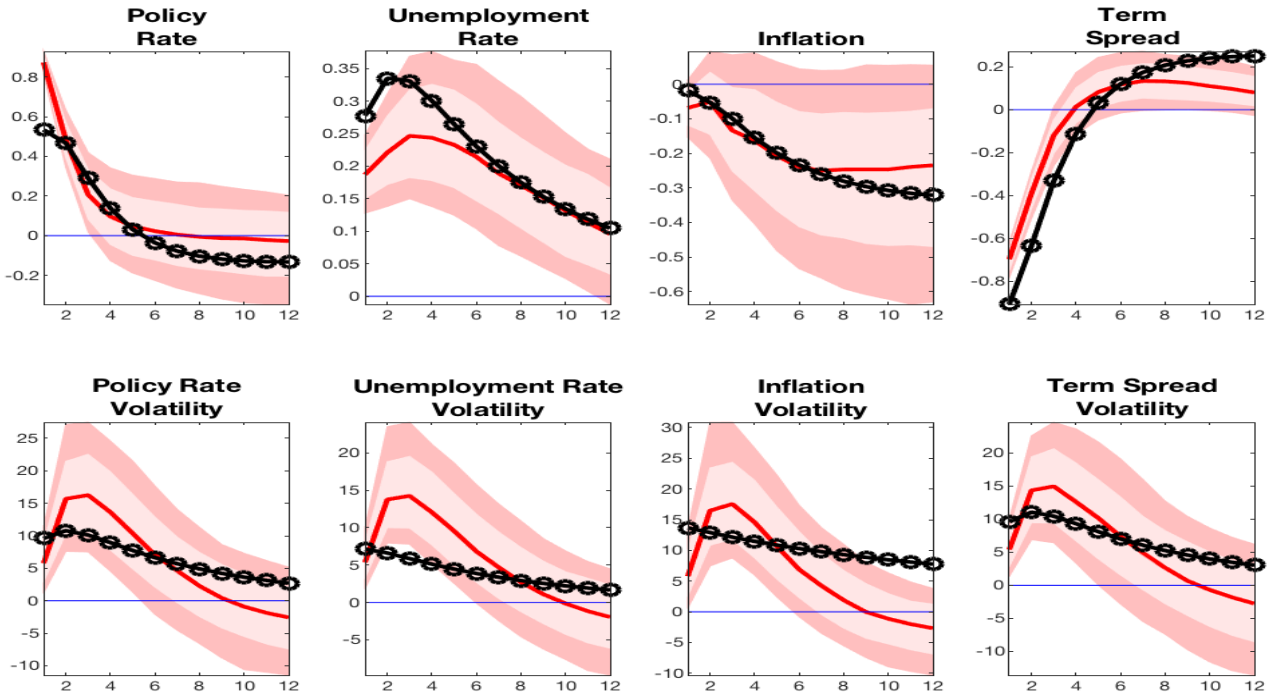
⁷This is not the case for asset price variables as they are functions of the stochastic discount factor. The risk parameter has a great influence on the second term of the stochastic discount factor that captures how agents ‘trade’ uncertainty across time. As it is discussed briefly in the next section, we discovered that the risk parameter variations have perhaps a larger impact on the stochastic steady-state than the cyclical dynamics regarding the macro variables.

Consistent with the analysis [Krause and Lubik \(2007\)](#) and [Leduc and Liu \(2016\)](#) (among others), the model requires a high degree of real wage rigidity ($\iota_w = 0.89$) in order to be able to match the volatility of unemployment in the data (Figure 5, Panel A). As explained by these authors and it can be seen from equation (28, in the main text), under this calibration paid wages decouple from productivity and this leads to high surplus for firms (relative to the situation of no real wage rigidity) stimulating vacancy creation.

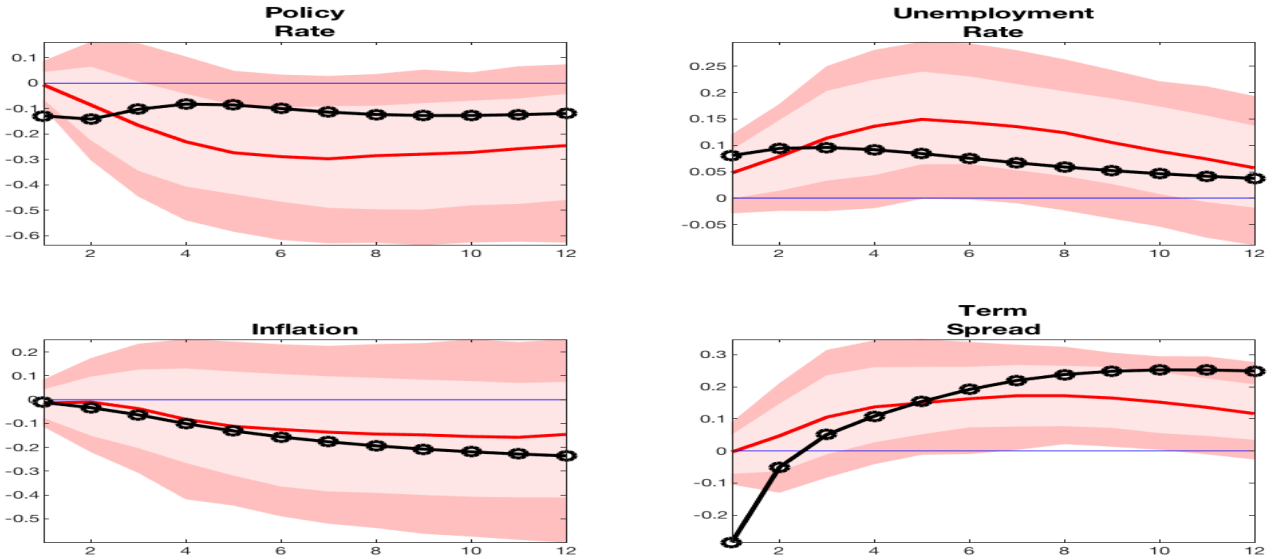
Similar to [Christiano et al. \(2005\)](#) a high degree of price indexation ($\iota = 0.72$) and a small probability of resetting prices optimally ($1 - \xi = 0.03$) are needed for the model to match the response of inflation after a monetary policy shock. Furthermore, the steady-state value of firms' markup is 50% ($\varepsilon = 3.0$) a value similar to [Smets and Wouters \(2007\)](#) and [Gertler et al. \(2008\)](#). Finally, the estimates of the policy reaction coefficients are similar to those reported in the literature ([Krause et al. \(2008\)](#), [Gertler et al. \(2008\)](#), [Leduc and Liu \(2016\)](#)).

Figure 5: The Fit of the DSGE Model

Panel A: Impulse Responses



Panel B: Uncertainty Contribution



Notes: The responses from the empirical model have been adjusted to match the quarterly frequency of the DSGE model. The posterior distribution of the uncertainty contribution (shadow area) is obtained by subtracting the posterior distribution between the unrestricted and restricted empirical responses. The back dashed line illustrates the DSGE response to the policy shock. The policy rate and inflation are expressed in annual terms. The DSGE responses are calculated relative to the stochastic steady state. **Panel B:** The DSGE uncertainty contribution is calculated as the difference between the response derived by using the third order solution of the model minus the responses produced using first order solution.

Table 1: Structural Parameters

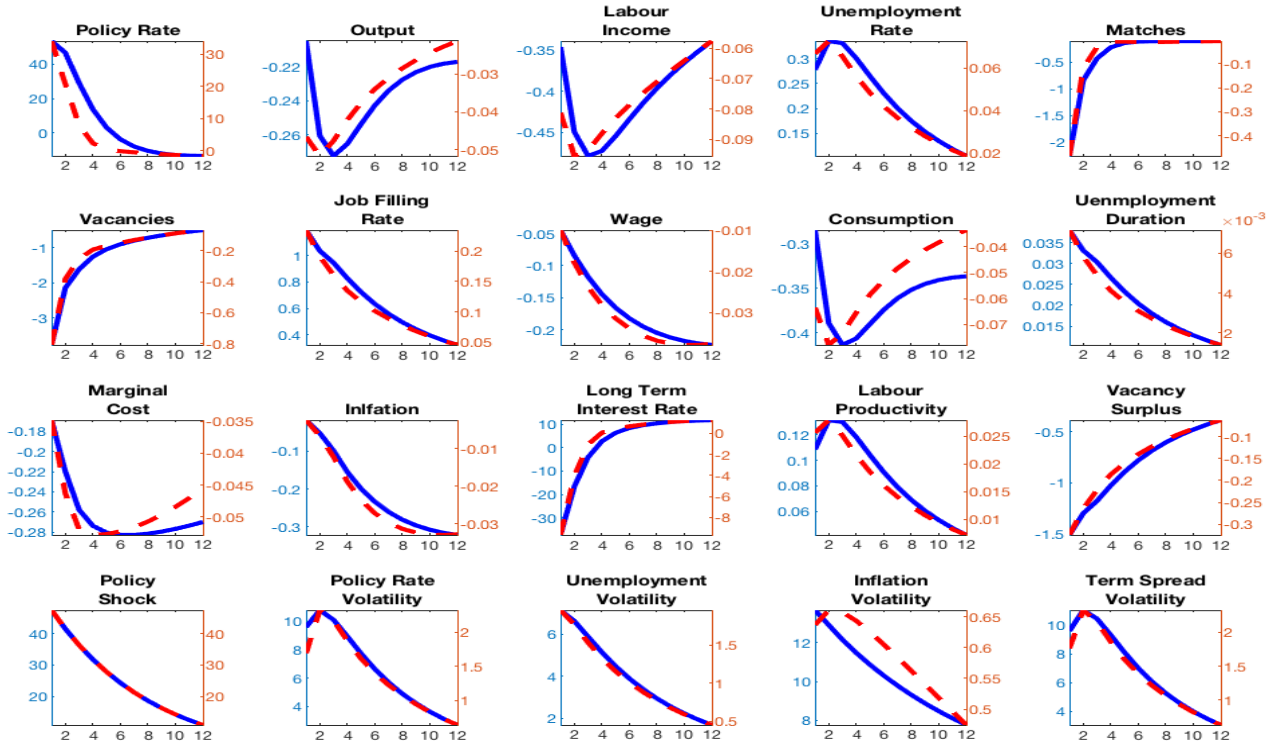
Parameters	Description	Values
<i>Calibrated Parameters</i>		
u	Steady State Unemployment Rate	0.06
$\frac{G}{\bar{Y}}$	Government Spending to GDP Ratio	0.18
$\frac{I}{\bar{Y}}$	Investment to GDP Ratio	0.20
$400\ln(\pi)$	Inflation Target	2.00
$400\ln(Z)$	Steady State Value of Productivity Growth	2.00
Q^x	Steady State Job Filling Probability	0.70
ϕ	Capital Production Share	0.36
δ_K	Capital Depreciation Rate	0.03
β	Time Discount Factor	0.995
<i>Estimated Parameters</i>		
$100\frac{\Phi}{\bar{Y}}$	Post Vacancy Resource Cost relative to GDP	1.00
$\bar{B} = \frac{B}{WL}$	Income Replacement Ratio	0.54
γ	Epstein-Zin Preference Risk	115.57
σ_C	Inverse of Intertemporal Elasticity of Substitution	2.15
b	Habit Smoothing	0.57
σ_L	Inverse of Frisch Elasticity	4.00
ε	Intermediate Goods Elasticity of Substitution	3.00
ξ	Calvo Probability	0.97
ι	Price Indexation	0.72
δ_N	Job Separation Rate	0.18
μ	Cobb-Douglas Matching	0.40
η	Bargain Parameter	0.50
ι_w	Real Wage Rigidity	0.89
ζ_Π	Inflation Monetary Policy Response	1.05
ζ_u	Unemployment Monetary Policy Response	0.40
ρ_R	Interest Rate Smoothing	0.71
ρ_{ϵ_R}	Policy Shock Persistence	0.93
ρ_{ϵ_Z}	Productivity Growth Shock Persistence	0.53
ρ_{ϵ_G}	Government Spending Shock Persistence	0.59
$100\sigma_{\epsilon_R}$	Policy Shock Standard Deviation	0.53
$100\sigma_{\epsilon_Z}$	Productivity Growth Shock Standard Deviation	1.58
$100\sigma_{\epsilon_G}$	Government Spending Shock Standard Deviation	0.01

5.3.1 Additional Simulations

Unemployment Gap Policy Reaction Coefficient In this paragraph we investigate how the monetary policy shock is transmitted when monetary authorities have elevated preferences to stabilise the output gap ($\zeta_u = 1.25$).

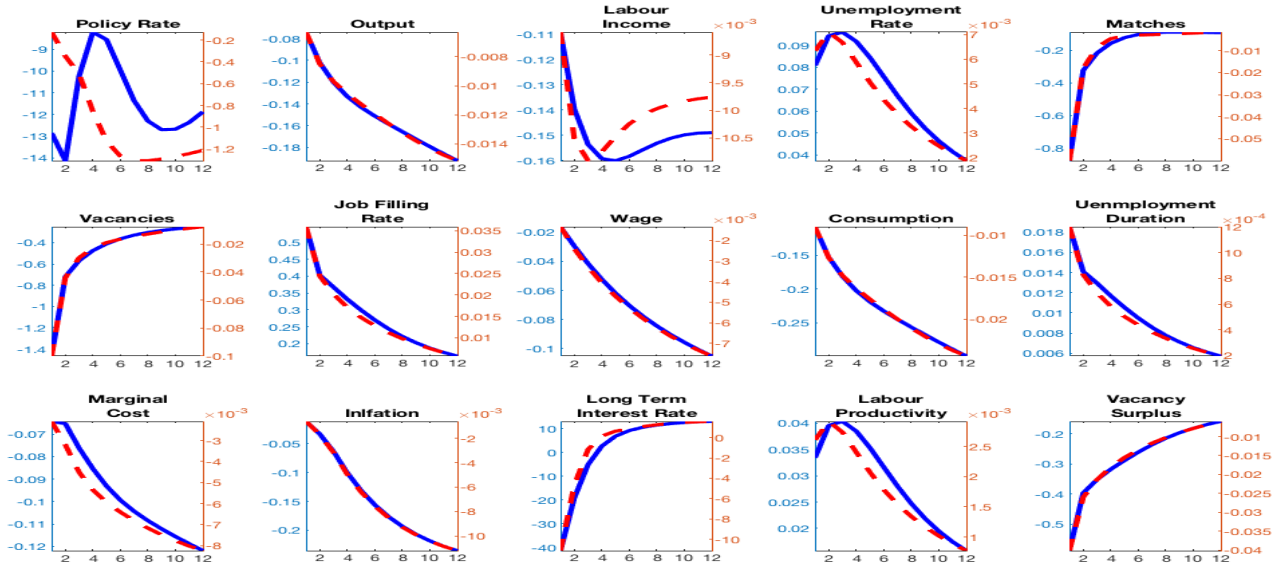
Inflation Target: It was mentioned in the main text that the ZLB could restrict policy maker's ability to generate loose monetary policy expectations and this would cause volatility to increase by more after a contractionary policy shock. One of the counter measures proposed to circumvent this constrain is to increase the inflation target. Figure 8 investigates further the link between the non-zero steady-state inflation and the macroeconomic volatility. The simulation seem to suggest that the effect from this policy change on the endogenous uncertainty is small; the channel is actually marginally reduced. As explained by Ascari and Sbordone (2014) the slope of the Phillips curve is a function of the steady-state inflation and when this is not zero then the Phillips curve becomes flatter. This reflects that agents become more forward looking since expected inflation is going to erode their average markup (King and Wolman (1999)). This makes inflation less responsive to the monetary policy, so it falls by less but it also rises by less to the changes to the policy rate. As a results, the long-term interest rate falls by less but it less lower for longer time than in the economy with a 2% inflation target.

Figure 6: Unemployment Gap Policy Reaction Coefficient



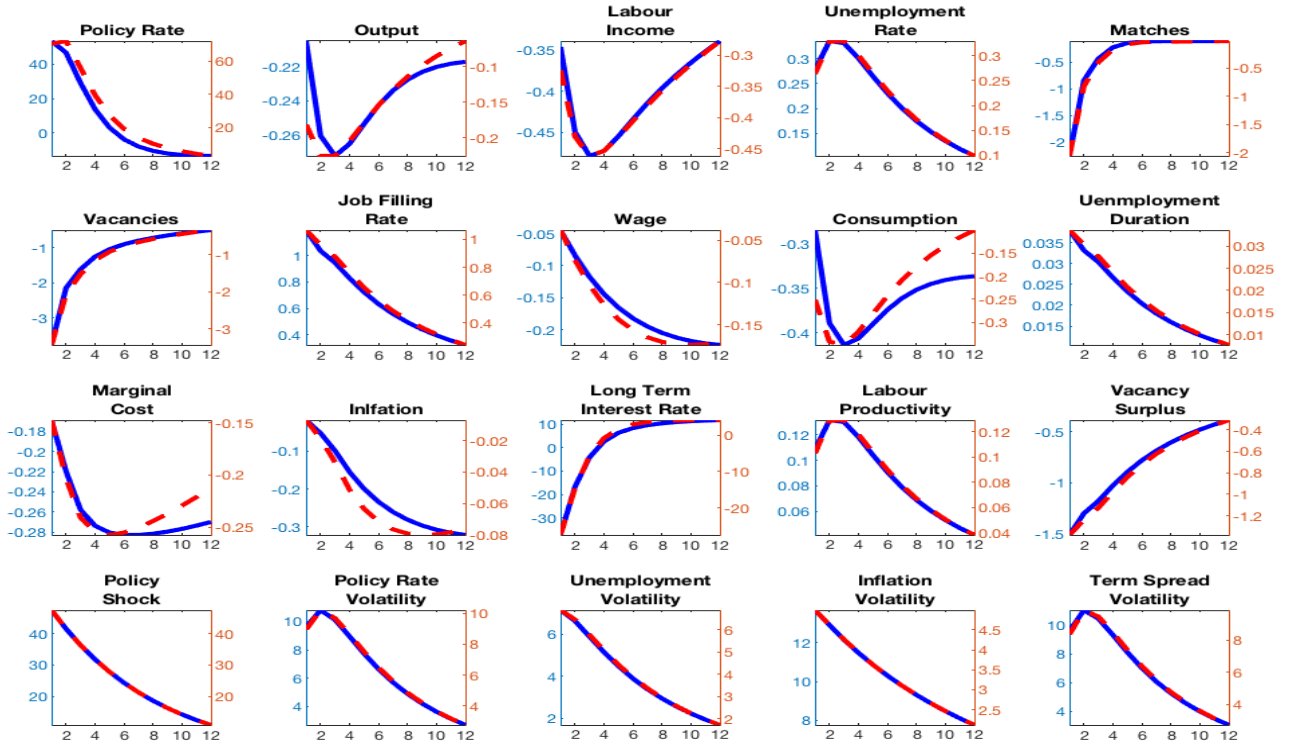
Notes: The blue solid line (left y-axis) represents the benchmark version of the model ($\zeta_u = 0.40$), while the red dashed line (right y-axis) denotes the responses of the model when the policy reaction coefficient to unemployment gap is set to 1.25 ($\rho_R = 1.25$). Rates are reported in annual basis points, inflation in annual percentage rates, the job filling probability in percentage points, unemployment duration in quarters. The responses are calculated relative to the stochastic steady state.

Figure 7: Unemployment Gap Policy Reaction Coefficient: Uncertainty Contribution



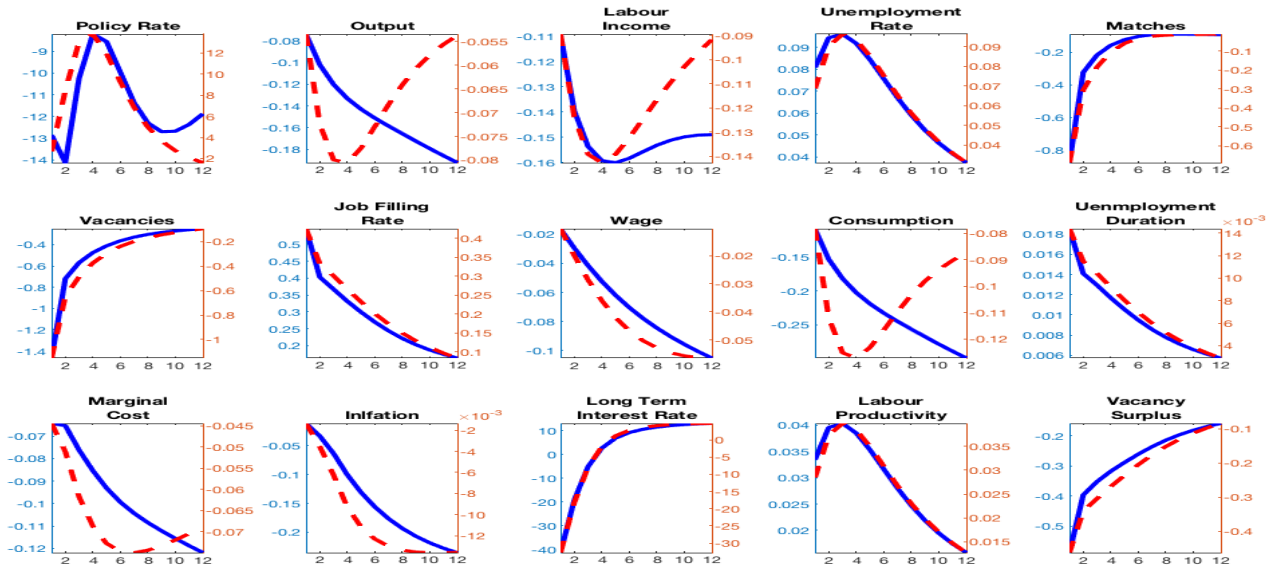
Notes: The blue solid line (left y-axis) represents the benchmark version of the model ($\zeta_u = 0.40$), while the red dashed line (right y-axis) denotes the responses of the model when the policy reaction coefficient to unemployment gap is set to 1.25 ($\zeta_u = 1.25$). Rates are reported in annual basis points, inflation in annual percentage rates, the job filling probability in percentage points, unemployment duration in quarters. The responses are calculated relative to the stochastic steady state. The uncertainty contribution is defined as the difference between the response derived by using the third minus the responses produced using only the first order solution of the model.

Figure 8: Inflation Target



Notes: The blue solid line (left y-axis) represents the benchmark version of the model ($400 * \log(\Pi) = 2$), while the red dashed line (right y-axis) denotes the responses of the model when the inflation target is set to four percent ($400 * \log(\Pi) = 4$). Rates are reported in annual basis points, inflation in annual percentage rates, the job filling probability in percentage points, unemployment duration in quarters. The responses are calculated relative to the stochastic steady state.

Figure 9: Inflation Target: Uncertainty Contribution



Notes: The blue solid line (left y-axis) represents the benchmark version of the model ($400 * \log(\Pi) = 2$), while the red dashed line (right y-axis) denotes the responses of the model when the inflation target is set to four percent ($400 * \log(\Pi) = 4$). Rates are reported in annual basis points, inflation in annual percentage rates, the job filling probability in percentage points, unemployment duration in quarters. The responses are calculated relative to the stochastic steady state. The uncertainty contribution is defined as the difference between the response derived by using the third minus the responses produced using only the first order solution of the model.

5.4 A Two Agents Model with Epstein-Zin Preferences and Search & Matching Frictions (Incomplete Markets)

We develop in this section a two agents version of the model with Epstein-Zin preferences that differ in terms of their labour skills. The separation rate for high labour skilled (HLS) jobs is lower than the separation rate of low labour skill (LLS) jobs ((Bruckner and Pappa (2012), Pappa et al. (2015))), while the HLS workers have a higher bargain power than LLS workers. Labour markets are segmented meaning that HLS employees cannot apply for LLS jobs and vice versa. Finally, the HLS workers own the majority of the government debt assets in the economy. We use this model to illustrate how the uncertainty induced by the monetary policy shock affects the labour income inequality across the two groups, as well as their consumption levels.

5.4.1 Model

Households: We assume that the economy is populated by two types of (high and low labour skilled) households have the same utility kernel. The job separation rate determines the type of skills as high skilled jobs get destroyed less frequently (Bruckner and Pappa (2012), Pappa et al. (2015)). For simplicity, we assume that the preference parameters are the same across both types.

$$u(\tilde{C}_{j,t}, \tilde{Z}_t, N_{j,t}) = \frac{\tilde{C}_{j,t}(h) - b\tilde{C}_{j,t-1}^{1-\sigma_C}}{1-\sigma_C} - \chi_{j,0} \tilde{Z}_t^{1-\sigma_C} \frac{L_{j,t}(h)^{1+\sigma_L}}{1+\sigma_L} \quad (98)$$

The budget constraint defined as in the benchmark version of the model, however, an adjustment portfolio cost has been added in order to ensure that the non-stochastic steady state of the model and the dynamics around it are well defined (see Schmitt-Grohe and Uribe (2003)).

$$\tilde{C}_{j,t} + \left(1 + \frac{\psi_D}{2}\right) \frac{\tilde{D}_{j,t}}{\tilde{C}_{j,t}} - \frac{D_j}{C_j} \frac{\tilde{D}_{j,t}}{R_t^A} + T_{j,t} = \tilde{W}_{j,t} L_{j,t} + (1 - N_{j,t}) \tilde{B}_t + \frac{\tilde{D}_{j,t-1}}{t} + j_t \quad (99)$$

Both types of households have recursive preferences

$$V_{j,t} = u(\tilde{C}_{j,t}, \tilde{Z}_t, N_{j,t}) + \beta E_t V_{j,t+1}^{1-\gamma} \frac{1}{1-\gamma} \quad (100)$$

Also both types of households decide optimally how much to consume, to work and save by solving the following problem

$$L_j \quad V_{j,0} - \sum_{i=0}^{\infty} \mu_{j,t+i} \left[V_{t+i} - u(\tilde{C}_{j,t+i}, \tilde{Z}_{j,t+i}, L_{j,t+i}) - \beta E_t V_{j,t+i+1}^{1-\gamma} \frac{1}{1-\gamma} \right. \\ \left. - \sum_{i=0}^{\infty} \beta^i \tilde{\lambda}_{j,t+i} \left[\tilde{W}_{j,t+i} L_{j,t+i} + \frac{\tilde{D}_{j,t+i-1}}{\Pi_{t+i}} - \tilde{C}_{j,t+i} \right. \right. \\ \left. \left. - \left(1 + \frac{\psi_D}{2}\right) \frac{\tilde{D}_{j,t+i}}{\tilde{C}_{j,t+i}} - \frac{D_j}{C_j} \frac{\tilde{D}_{j,t+i}}{R_{j,t+i}^A} \right] \right] \quad (101)$$

The marginal utility of consumption is give by:

$$\mu_{j,t} \bar{C}_{j,t}(h) - b\bar{C}_{j,t-1}^{-\sigma_C} = \tilde{\lambda}_{j,t} \left[1 - \psi_D \frac{1}{R_t^A} \frac{\bar{D}_{j,t}}{\bar{C}_{j,t}} - \frac{D_j}{C_j} \frac{\bar{D}_{j,t}}{\bar{C}_{j,t}} \right]^2 \quad (102)$$

The supply of labour is based on

$$\chi_{j,0} \bar{Z}_t^{1-\sigma_C} N_{j,t}^{\sigma_L} \bar{C}_{j,t}(h) - b\bar{C}_{j,t-1}^{-\sigma_C} \left[1 - \psi_D \frac{1}{R_t^A} \frac{\bar{D}_{j,t}}{\bar{C}_{j,t}} - \frac{D_j}{C_j} \frac{\bar{D}_{j,t}}{\bar{C}_{j,t}} \right]^2 = \tilde{W}_{j,t} - \tilde{B}_t \quad (103)$$

While the allocation of consumption across time is decided using

$$\frac{1}{R_t^A} = \beta E_t \frac{\tilde{\lambda}_{j,t+1}}{\tilde{\lambda}_{j,t}} \frac{1}{1 + \frac{\psi_D}{2} \frac{\bar{D}_{j,t}}{\bar{C}_{j,t}} - \frac{D_j}{C_j} \frac{\bar{D}_{j,t}}{\bar{C}_{j,t}} + \psi_D \frac{\bar{D}_{j,t}}{\bar{C}_{j,t}} - \frac{D_j}{C_j} \frac{\bar{D}_{j,t}}{\bar{C}_{j,t}}} \quad (104)$$

The first order condition with respect to preferences is

$$E_t \mu_{j,t+1} = \mu_{j,t} \beta E_t V_{j,t+1}^{1-\gamma} \frac{1}{1-\gamma} V_{j,t+1}^{-\gamma} \quad (105)$$

After some simple transformation we obtain the stochastic discount factor

$$M_{j,t+1} = \frac{V_{j,t+1}}{E_t V_{j,t+1}^{1-\gamma} \frac{1}{1-\gamma}} \frac{\bar{C}_{j,t} - b\bar{C}_{j,t-1}}{\bar{C}_{j,t+1} - b\bar{C}_{j,t}} \frac{1}{1-\gamma} \quad (106)$$

From the last two equation we obtain

$$\frac{1 - \psi_D \frac{1}{R_{t+1}^A} \frac{\bar{D}_{j,t+1}}{\bar{C}_{j,t+1}} - \frac{D_j}{C_j} \frac{\bar{D}_{j,t+1}}{\bar{C}_{j,t+1}}}{R_t^A} = \beta E_t \frac{M_{j,t+1}}{M_{j,t}} \quad (107)$$

Labour Market: Every firm posts two type of vacancies (high and low skilled type vacancies) and agents from each households can apply only to one type of jobs. As it was explained earlier, some households pose high and some low labour skills. This implies that low labour skilled agents cannot apply for high skilled jobs and vice versa (segmentation). The matching technology for both labour markets is the same

$$M_{j,t} = \bar{\mu}_j U_{j,t}^{\mu_j} \frac{1-\mu_j}{j,t} \quad (108)$$

As in the benchmark economy the probability of filling the vacancy and finding a job in each market are given respectively by:

$$Q_{j,t}^Y = \frac{M_{j,t}}{j,t} \quad (109)$$

$$Q_{j,t}^U = \frac{M_{j,t}}{U_{j,t}} \quad (110)$$

The number of unemployed workers in each market equals

$$U_{j,t} = 1 - (1 - \delta_{j,N}) N_{j,t-1} \quad (111)$$

and the degree of market tightness for each type skilled jobs is

$$X_{j,t} = \frac{j,t}{U_{j,t}} \quad (112)$$

Intermediate Good Producers: The technology employed for the production of the intermediate good is

$$\tilde{Y}_t = \tilde{Z}_t L_{H,t}^{\phi_S(1-\phi)} L_{L,t}^{(1-\phi_S)(1-\phi)} K^\phi \quad (113)$$

While the number of employed people in each market evolves according to

$$L_{j,t} = (1 - \delta_{j,N}) L_{j,t-1} + Q_{j,t}^U \quad (114)$$

The profit function is given by

$$E_t \sum_{i=0}^{\infty} M_{j,t+i} \beta^i \left[P_t^W \tilde{Y}_t(h) - \tilde{W}_{H,t} L_{H,t} - \tilde{W}_{L,t} L_{L,t} - R_t^K K_{t-1}(h) - \kappa_H \tilde{Z}_t L_{H,t} - \kappa_L \tilde{Z}_t L_{L,t} + (1 - \delta_{H,N}) N_{H,t-1} + Q_{H,t}^U L_{H,t} - N_{H,t} + (1 - \delta_{L,N}) N_{L,t-1} + Q_{L,t}^U L_{L,t} - N_{L,t} \right] \quad (115)$$

The maximisation of the above objective function determines the demand for low and high skilled workers

$$P_t^W = \frac{N_{H,t}}{\phi_S(1-\phi) \tilde{Y}_t} \tilde{W}_{H,t} + \tilde{w}_{H,t} - \beta E_t M_{H,t+1} \tilde{w}_{H,t+1} (1 - \delta_{H,N}) \quad (116)$$

$$P_t^W = \frac{N_{L,t}}{(1-\phi_S)(1-\phi) \tilde{Y}_t} \tilde{W}_{L,t} + \tilde{w}_{L,t} - \beta E_t M_{L,t+1} \tilde{w}_{L,t+1} (1 - \delta_{L,N})$$

and the number of posted vacancies

$$\tilde{j}_{j,t} = \frac{\kappa_j \tilde{Z}_t}{Q_{j,t}^U} \quad (117)$$

The intermediate good producers after they decided how many employer to recruit and new jobs to post, they set prices. Namely, prices are chosen to maximises profits. Only a fraction $(1 - \xi)$ of producers receive a random signal and set prices optimally. The remaining (ξ) producers miss this signal and set prices based on a backward looking indexation rule.

$$\max_{\tilde{P}_t(h)} E_t \sum_{j=0}^{\infty} M_{t,t+j} (\beta \xi)^j \left[\frac{\tilde{P}_t(h) \tilde{Y}_{t+j}(h)}{\tilde{P}_{t+j}} - P_{t+j}^W(h) \tilde{Y}_{t+j}(h) \right]$$

The price Philip curve is described by the following four equations, while the last equation describes

the evolution of price dispersion

$$\tilde{K}_{P,t} = P_t^W \tilde{Y}_t^d + \beta \xi E_t M_{t+1} \frac{t+1}{t} \frac{\varepsilon}{\varepsilon-1} \tilde{K}_{P,t+1} \quad (118)$$

$$\tilde{F}_{P,t} = \tilde{Y}_t^d + \beta \xi E_t M_{t+1} \frac{t+1}{t} \frac{\varepsilon-1}{\varepsilon} \tilde{F}_{P,t+1} \quad (119)$$

$$\frac{1}{t} = \frac{\varepsilon}{\varepsilon-1} \frac{K_{P,t}}{F_{P,t}} \quad (120)$$

$$1 = \xi \frac{t}{t-1} \frac{\varepsilon-1}{\varepsilon} + (1-\xi) \frac{1}{t} \frac{\varepsilon-1}{\varepsilon} \quad (121)$$

$$\frac{1}{t} \frac{\varepsilon-1}{\varepsilon} = (1-\xi) \frac{1}{t} \frac{\varepsilon-1}{\varepsilon} + \xi \frac{t}{t-1} \frac{\varepsilon-1}{\varepsilon} \frac{1}{t-1} \quad (122)$$

Labour Market and Wage Bargaining: As in the benchmark version of the model the surplus created by the match is split between the employee and the employer

$$\tilde{S}_{j,t}^N = \tilde{W}_{j,t} - \chi_{j,0} \tilde{Z}_t^{1-\sigma_C} L_{j,t}^{\sigma_L} \tilde{C}_{j,t}^{\sigma_C} + \beta E_t M_{j,t+1} [1 - \delta_{j,N} (1 - X_{j,t+1})] \tilde{S}_{j,t+1}^N + \delta_{j,N} (1 - X_{j,t+1}) \tilde{S}_{j,t+1}^U$$

This is done via a Nash bargain problem

$$\eta_j \tilde{w}_{j,t} = 1 - \eta_j \tilde{S}_{j,t}^W \quad (123)$$

and this process determines the wage paid to high and low skilled workers respectively

$$\begin{aligned} \tilde{W}_{H,t} = & \eta P_t^W \frac{\phi_S (1-\phi) \tilde{Y}_t}{N_{H,t}} + \beta E_t M_{H,t+1} \tilde{W}_{H,t+1} (1 - \delta_{H,N}) \\ & + (1-\eta) \chi_{H,0} \tilde{Z}_t^{1-\sigma_C} L_{H,t}^{\sigma_L} \tilde{C}_{H,t} - b \tilde{C}_{H,t-1}^{\sigma_C} \\ & 1 - \psi_D \frac{1}{R_t^A} \frac{\tilde{D}_{H,t}}{\tilde{C}_{H,t}} - \frac{D_H}{C_H} \frac{\tilde{D}_{H,t}}{\tilde{C}_{H,t}}^2 + \tilde{B}_t \end{aligned} \quad (124)$$

$$\begin{aligned} \tilde{W}_{L,t} = & \eta P_t^W \frac{(1-\phi_S) (1-\phi) \tilde{Y}_t}{N_{L,t}} + \beta E_t M_{L,t+1} \tilde{W}_{L,t+1} (1 - \delta_{L,N}) \\ & + (1-\eta) \chi_{L,0} \tilde{Z}_t^{1-\sigma_C} L_{L,t}^{\sigma_L} \tilde{C}_{L,t} - b \tilde{C}_{L,t-1}^{\sigma_C} \\ & 1 - \psi_D \frac{1}{R_t^A} \frac{\tilde{D}_{L,t}}{\tilde{C}_{L,t}} - \frac{D_H}{C_H} \frac{\tilde{D}_{L,t}}{\tilde{C}_{L,t}}^2 + \tilde{B}_t \end{aligned} \quad (125)$$

Retailers: The final good production function is CES aggregator

$$\tilde{Y}_t = \int_0^1 \tilde{Y}_t(h)^{\frac{\varepsilon-1}{\varepsilon}} dh \quad (126)$$

While the demand for intermediate good is give by

$$\tilde{Y}_t(h) = \frac{\tilde{P}_t(h) \frac{\varepsilon-1}{\varepsilon}}{\tilde{P}_t} \tilde{Y}_t \quad (127)$$

Market Clearing Conditions: The market clearing conditions are some simple substitutions

$$\tilde{C}_t + \tilde{G}_t + \tilde{I}_t + \kappa_H \tilde{Z}_{H,t} + \kappa_L \tilde{Z}_{L,t} = \frac{\tilde{Z}_t L_{H,t}^{\phi_S(1-\phi)} L_{L,t}^{(1-\phi_S)(1-\phi)} K^{\phi}}{t} \quad (128)$$

where

$$t = \int_0^1 \frac{\tilde{P}_t(h)^{-\frac{\epsilon}{1-\phi}}}{\tilde{P}_t} dh \quad (129)$$

denotes the price dispersion.

Government: Unlike the benchmark version, the government relies on both lump-sum taxes and debt to finance its consumptions and transfers. Government budget constraint is given by

$$\tilde{P}_t \tilde{G}_t + \tilde{D}_{t-1} + (1 - L_{H,t}) \tilde{P}_t \tilde{B}_{H,t} + (1 - L_{L,t}) \tilde{P}_t \tilde{B}_{L,t} = \tilde{T}_t + \frac{\tilde{D}_t}{R_t}$$

where the lump sum taxes are described by the following debt targeting rule

$$\tilde{T}_t = \tilde{T}_{H,t} + \tilde{T}_{L,t} = \tilde{T} \tilde{D}_{t-1}^{\tau_D} \quad (130)$$

The debt reaction coefficient has been calibrated $\tau_D = 0.025$ to ensure that debt is a slow moving variable. G_t is government consumption and g_t is the share of the government in the economy:

$$\tilde{G}_t = g_t \tilde{Y}_t \quad (131)$$

that evolves in a stochastic manner:

$$\hat{g}_t = g + \rho_g (\hat{g}_{t-1} - g) + \sigma_g \omega_{g,t} \quad (132)$$

5.4.2 Calibration

This version of the model gives rise to a number of new parameters that do not exist in benchmark version of the model. As we do not want this version of the model to move far away from the estimated benchmark model, its calibration is carried out as follows:

1. We use the same values for the calibrated parameters that overlap between the two versions. Furthermore:
 - (a) The total unemployment steady-state value is set equal to its data average value 5.8, however, the share of low and high labour skilled workers is estimated
 - (b) The steady-state government debt to GDP ratio is equal to 40% to match the average in the data
2. We use the benchmark estimates for the preference coefficients and the Phillip curve parameters
3. Similarly, we use the same estimates for the parameters that control the high labour skilled labour market block.

Table 2: Calibrated Parameters

Parameters	Description	Values
u	Steady State Unemployment Rate	0.06
u_L	Steady State Unemployment Rate High Labour Skilled Workers	$u - u_H$
$\frac{G}{Y}$	Government Spending to GDP Ratio	0.18
$\frac{I}{Y}$	Investment to GDP Ratio	0.20
$\frac{B}{Y}$	Government Debt to GDP Ratio	0.40
$400\ln(\pi)$	Inflation Target	2.00
$400\ln(Z)$	Steady State Value of Productivity Growth	2.00
Q^r	Steady State Job Filling Probability	0.70
ϕ	Capital Production Share	0.36
δ_K	Capital Depreciation Rate	0.03
β	Time Discount Factor	0.995
$\psi_{D,H}$	Portfolio Adjustment Cost High Labour Skilled Workers	0.00

4. A small set of parameters described in Table 4 is estimated

Figures 10 and 11 display the fit of the model. Again, it could be argued that the incomplete version of the model fits the empirical stylised facts very well, despite the restrictive calibration process undertaken here.

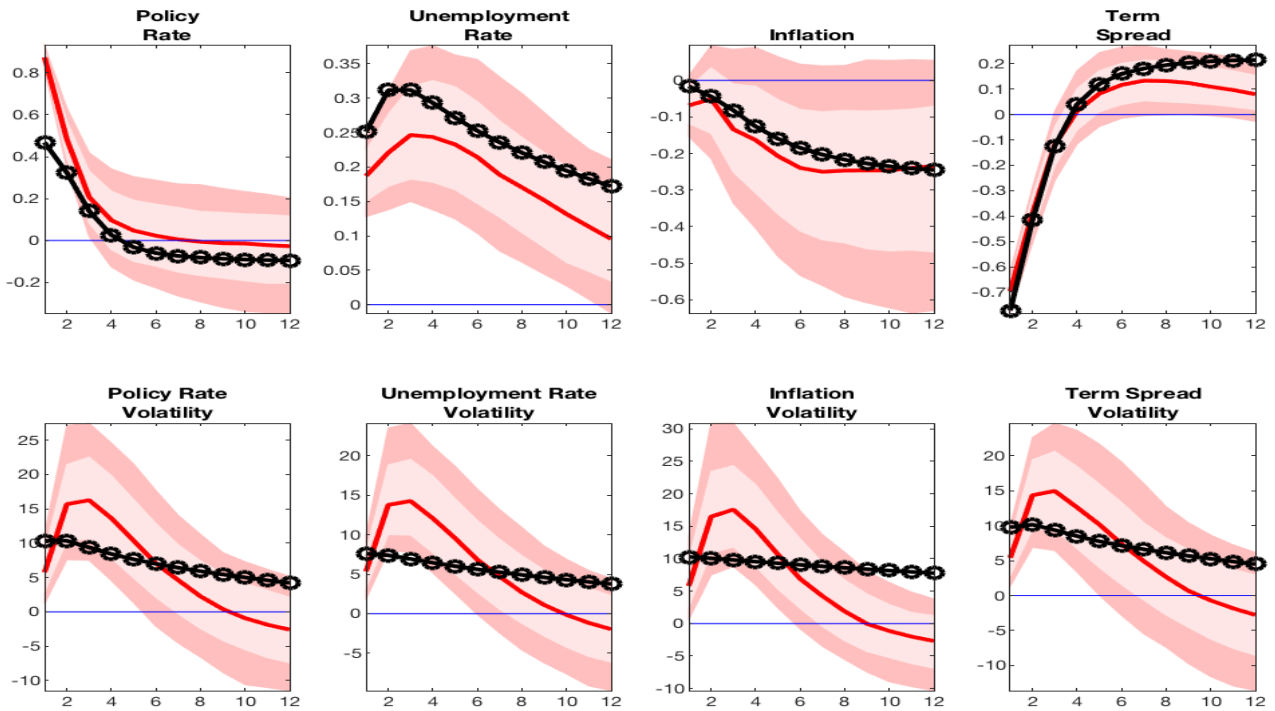
Table 3: Benchmark Model Parameter Estimates

Parameters	Description	Values
$100\frac{\Phi}{\bar{Y}}$	Post Vacancy Resource Cost relative to GDP	1.00
$\bar{B} = \frac{B}{WL}$	Income Replacement Ratio	0.54
γ	Epstein-Zin Preference Risk	115.29
σ_C	Inverse of Intertemporal Elasticity of Substitution	2.15
b	Habit Smoothing	0.57
σ_L	Inverse of Frisch Elasticity	4.00
ε	Intermediate Goods Elasticity of Substitution	3.00
ξ	Calvo Probability	0.97
ι	Price Indexation	0.72
$\delta_{N,H}$	Job Separation Rate High Labour Skilled Workers	0.05
μ_H	Cobb-Douglas Matching High Labour Skilled Workers	0.40
μ_L	Cobb-Douglas Matching Low Labour Skilled Workers	0.40
η_H	Bargain Parameter High Labour Skilled Workers	0.50
$\iota_{W,H}$	Real Wage Rigidity High Labour Skilled Workers	0.89
ζ_{Π}	Inflation Monetary Policy Response	1.05
ζ_u	Unemployment Monetary Policy Response	0.40
ρ_R	Interest Rate Smoothing	0.71

Table 4: Estimated Parameters

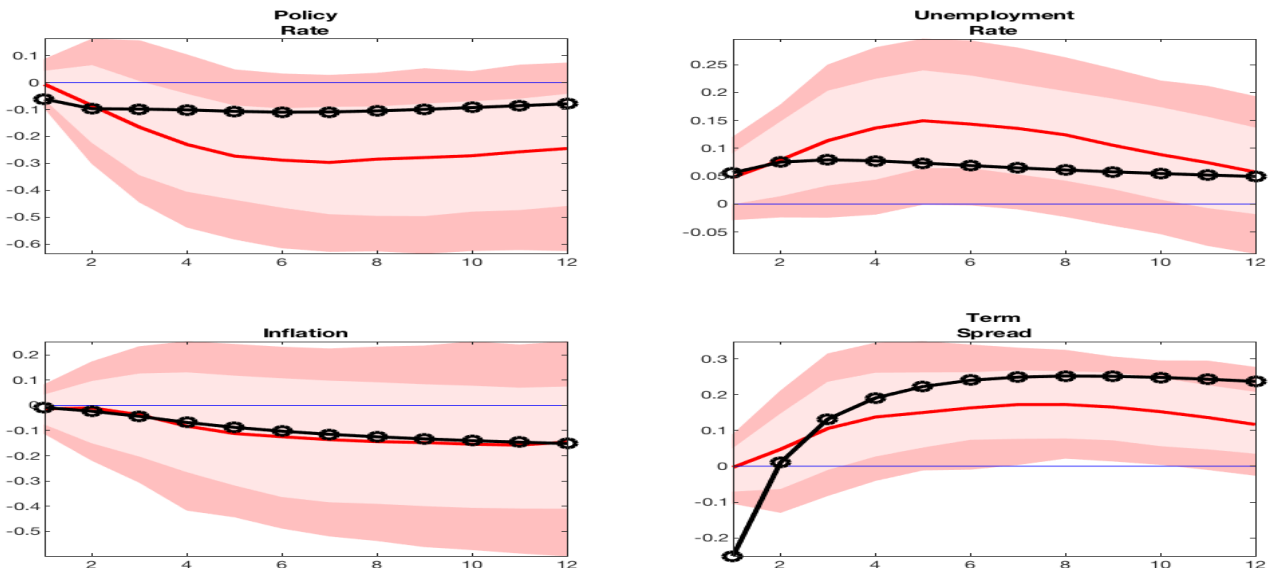
Parameters	Description	Values
ϕ_S	Share of High Labour Skilled Workers in the Production Function	0.28
$\psi_{D,L}$	Portfolio Adjustment Cost Low Labour Skilled Workers	0.02
$\frac{B_L}{B}$	Share of Government Debt Assets Owned by Low Labour Skilled Workers	0.10
$\delta_{N,L}$	Job Separation Rate Low Labour Skilled Workers	0.25
η_L	Bargain Parameter Low Labour Skilled Workers	0.09
$\iota_{W,L}$	Real Wage Rigidity Low Labour Skilled Workers	0.91
ρ_{ε_Z}	Productivity Growth Shock Persistence	0.57
ρ_{ε_G}	Government Spending Shock Persistence	0.54
ρ_{ε_R}	Policy Shock Persistence	0.94
$100\sigma_{\varepsilon_Z}$	Productivity Growth Shock Standard Deviation	1.44
$100\sigma_{\varepsilon_G}$	Government Spending Shock Standard Deviation	0.03
$100\sigma_{\varepsilon_R}$	Policy Shock Standard Deviation	0.58

Figure 10: The Fit of the DSGE Model with Incomplete Markets



Notes: The responses from the empirical model have been adjusted to match the quarterly frequency of the DSGE model. The back dashed line illustrates the DSGE response to the policy shock. The policy rate and inflation are expressed in annual terms. The DSGE responses are calculated relative to the stochastic steady state.

Figure 11: The Fit of the DSGE Model with Incomplete Markets: Uncertainty Effect



Notes: The responses from the empirical model have been adjusted to match the quarterly frequency of the DSGE model. The posterior distribution of the uncertainty contribution (shadow area) is obtained by subtracting the posterior distribution between the unrestricted and restricted empirical responses. The back dashed line illustrates the DSGE response to the policy shock. The policy rate and inflation are expressed in annual terms. The DSGE responses are calculated relative to the stochastic steady state. The DSGE uncertainty contribution is calculated as the difference between the response derived by using the third order solution of the model minus the responses produced using first order solution.

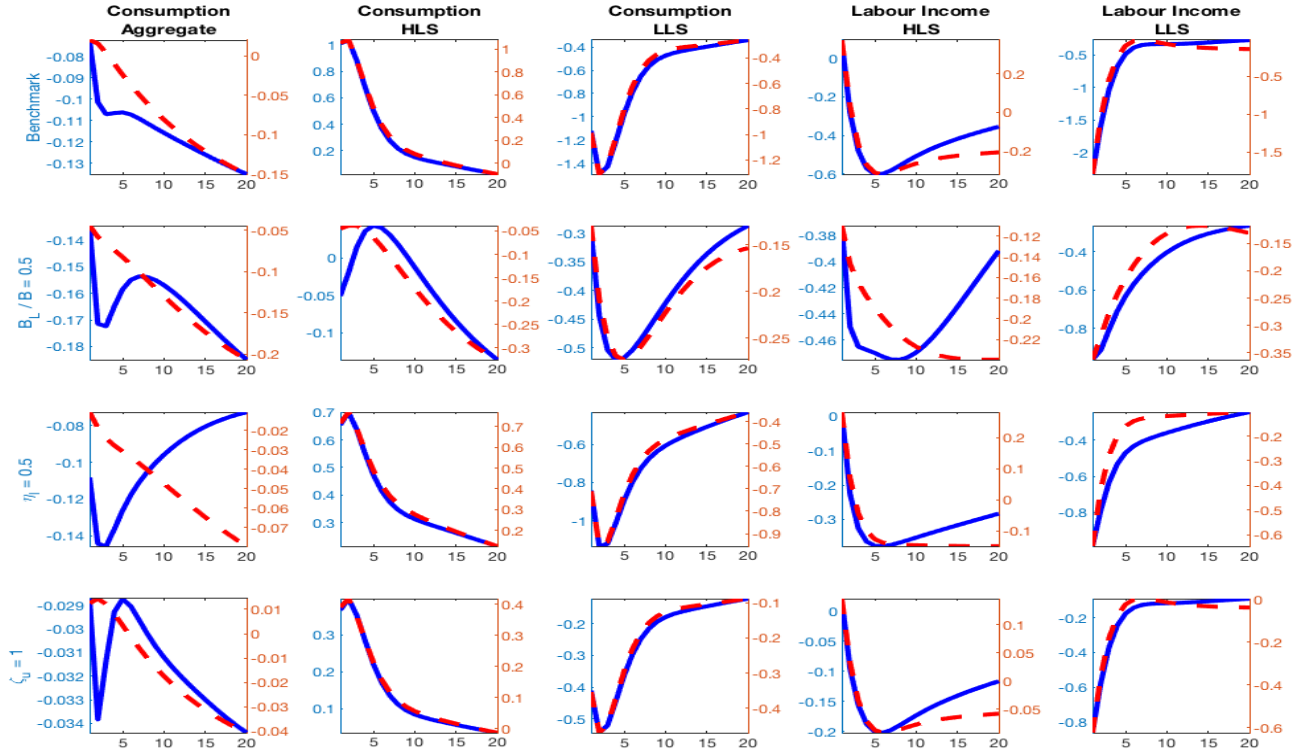
5.4.3 Simulations & IRFs

The first row of Figure 12 displays agents' responses to a monetary policy shock when the structural parameters are set equal to the values reported in Tables 2, 3 and 4. Not surprisingly, the labour income inequality (defined as the difference between the HLS and LLS labour income) increases by almost 2% and this is mostly driven by the fall of LLS labour income (the HLS consumption reduces only marginally). Uncertainty, drives more than 50% of this income inequality increase. When it comes to consumption, the results are even more interesting. Uncertainty, contributes about 1.2% to the fall of the LLS consumption and this explains almost 80% of the reduction of the consumption of this group. However, uncertainty turns out to have a positive effect on the HLS consumption. In this environment, the two type households not only can smooth consumption by allocating it across time but also trading it between the groups. The LLS group is more exposed after an adverse monetary policy and as a such it forms stronger precautionary motives. This implies that they reduce consumption and supply labour by a lot more than the HLS employees. For the HLS group the inflation fall (which also boosts the real value of their assets) together with their smaller exposure future risk (relative to the LLS workers) makes their current consumption cheaper. The increase in the HLS consumption only mitigates some of the adverse effect of uncertainty in the aggregate consumption (which fall by 0.15%).

In the second row, we illustrate how the responses are altered (relative to the first row) when the share of government debt assets owned by the LLS group increases to 50%. The income inequalities opens up by significantly less (relative to the first), while the contribution of the uncertainty to the LLS consumption has been reduced dramatically. Qualitatively, similar results are obtained when the bargain power of the LLS workers increases to 0.5 (third row). However, the results are significantly smaller quantitatively. In the final exercise (fourth row), it is investigate what happens when the policy reaction coefficient (ζ_u) increase to one ($\zeta_u = 1$). Again both the labour income inequality increase by less and the contribution of the uncertainty is smaller than in the first exercise.

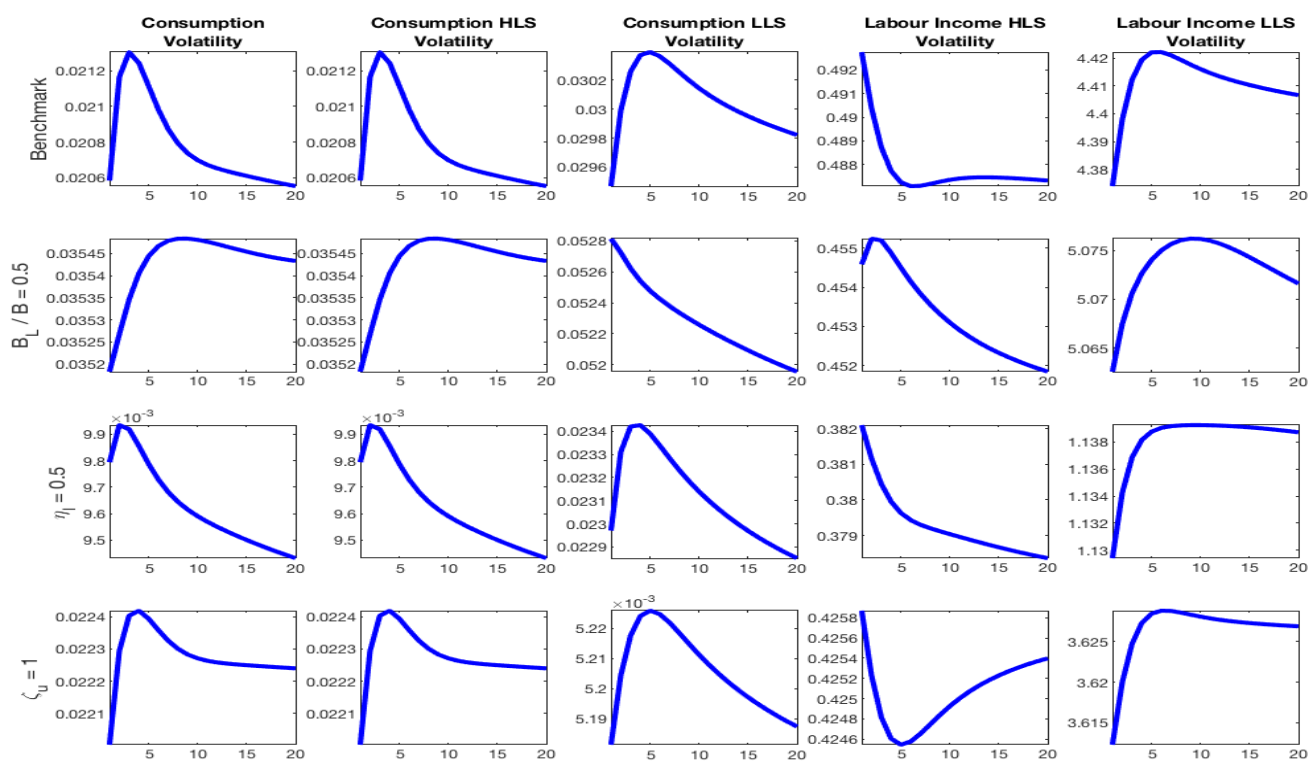
Finally, Figure 13 plots the volatility responses of all variables discussed in this exercise for all four exercises.

Figure 12: Incomplete Markets Simulations



Notes: The blue solid line (left y-axis) represents the responses of the state variables to the system, while the red dashed line (right y-axis) denotes the uncertainty contribution to these responses. The uncertainty contribution is defined as the difference between the response derived by using the third minus the responses produced using only the first order solution of the model. Rates are reported in annual basis points, inflation in annual percentage rates, the job filling probability in percentage points, unemployment duration in quarters. The responses are calculated relative to the stochastic steady state. HLS and LLS stand for high and low labour skilled workers. The first row displays the responses of the version of the model that uses the parameters reported in Tables 2, 3 and 4. The second row shows what happens when the non stochastic steady-state low skilled workers' share of assets increases to 50%. The third row illustrates how agents respond to a monetary policy when the low skilled workers increases to 0.5. The last row presents the transmission of the shock when the policy reaction coefficient increases to one ($\zeta_u = 1$).

Figure 13: Incomplete Markets Simulations: Volatility Responses



Notes: The volatility responses are presented in levels. HLS and LLS stand for high and low labour skilled workers. The first row displays the responses of the version of the model that uses the parameters reported in Tables 2, 3 and 4. The second row shows what happens when the non stochastic steady-state low skilled workers' share of assets increases to 50%. The third row illustrates how agents respond to a monetary policy when the low skilled workers increases to 0.5. The last row presents the transmission of the shock when the policy reaction coefficient increases to one ($\zeta_u = 1$).

5.5 A with Epstein-Zin Preferences and Sticky Nominal Wages (Sticky Wages)

Households: As in the benchmark version of the model the household receives utility from consumption while they dislike labour

$$u(\tilde{C}_t, \tilde{Z}_t, L_t) = \frac{\tilde{C}_t - b\tilde{C}_{t-1}^{1-\sigma_C}}{1-\sigma_C} - \chi_0 \tilde{Z}_t^{1-\sigma_C} \frac{L_t^{1+\sigma_L}}{1+\sigma_L} \quad (133)$$

Their budget constraint has been modified slightly as there are not unemployed agents in this economy

$$\tilde{P}_t \tilde{C}_t + \frac{\tilde{D}_t}{R_t} + T_t = \tilde{P}_t \tilde{W}_t L_t + \tilde{D}_{t-1} + \dots \quad (134)$$

Again households' preferences have a recursive format

$$V_t = u(\tilde{C}_t, \tilde{Z}_t, L_t) + \beta E_t V_{t+1}^{1-\gamma} \frac{1}{1-\gamma} \quad (135)$$

The following Lagrangian problem allows them to select optimally how much to consume and save

$$\begin{aligned} L = & V_0 - \sum_{i=0}^{\infty} \mu_{t+i} [V_{t+i} - u(\tilde{C}_{t+i}, \tilde{Z}_{t+i}, L_{t+i}) - \beta E_t V_{t+i+1}^{1-\gamma} \frac{1}{1-\gamma} \\ & - \sum_{i=0}^{\infty} \beta^i \tilde{\lambda}_{t+i} [\tilde{P}_{t+i} \tilde{W}_{t+i} L_{t+i} + \tilde{D}_{t+i-1} - \tilde{P}_{t+i} \tilde{C}_{t+i} - \frac{\tilde{D}_{t+i}}{R_{t+i}}] \end{aligned} \quad (136)$$

The marginal utility of consumption is given

$$\tilde{C}_t - b\tilde{C}_{t-1}^{1-\sigma_C} = \frac{\tilde{\lambda}_t \tilde{P}_t}{\mu_t} \quad (137)$$

The allocation of consumption across time is based on the following Euler equation

$$\tilde{\lambda}_t = \beta E_t \tilde{\lambda}_{t+1} R_t^A \quad (138)$$

While from the first order condition with respect to V_t

$$E_t \mu_{t+1} = \mu_t \beta E_t V_{t+1}^{1-\gamma} \frac{1}{1-\gamma} V_{t+1}^{-\gamma} \quad (139)$$

And some substitutions we obtain the stochastic discount factor

$$M_{t+1} = \frac{V_{t+1}}{E_t V_{t+1}^{1-\gamma} \frac{1}{1-\gamma}} \frac{\tilde{C}_t - b\tilde{C}_{t-1}^{1-\sigma_C}}{\tilde{C}_{t+1} - b\tilde{C}_t^{1-\sigma_C}} \sigma_C \quad (140)$$

And the Euler equation becomes

$$\frac{1}{R_t^A} = \beta E_t \frac{M_{t+1}}{t+1} \quad (141)$$

Wages: We follow [Erceg et al. \(2000\)](#) and assume that each monopolistically competitive household supplies a differentiated labour service to the production section. They set their nominal wage and supply any amount of labour demanded by the firms at that wage rate. For convenience, we assume that there exists a representative firm that combines households' labour inputs into a homogeneous input good - L_t^d - using a CES production function

$$L_t^d = \int_0^1 L_{\mathcal{Z},t}^{\frac{1}{\lambda_w}} d\mathcal{Z}^{\lambda_w} \quad (142)$$

where λ_w denotes the mark-up in the labour market. Taking \tilde{W}_t and $\tilde{W}_{\mathcal{Z},t}$ as given, the aggregator's demand for the labour hours of household \mathcal{Z} results from profit maximisation

$$\begin{aligned} \max_{h_{\mathcal{Z},t}} \quad & \tilde{W}_t \int_0^1 L_{\mathcal{Z},t}^{\frac{1}{\lambda_w}} d\mathcal{Z}^{\lambda_w} - \int_0^1 \tilde{W}_{\mathcal{Z},t} L_{\mathcal{Z},t} \\ L_{\mathcal{Z},t} = \quad & \frac{\tilde{W}_{\mathcal{Z},t}}{\tilde{W}_t}^{-\frac{\lambda_w}{\lambda_w-1}} L_t^d \end{aligned} \quad (143)$$

The aggregate wage arises from the profit condition and the demand curve

$$\tilde{W}_t = \int_0^1 \tilde{W}_{\mathcal{Z},t}^{1-\lambda_w} d\mathcal{Z}^{1-\lambda_w} \quad (144)$$

In each period, a fraction $1 - \xi_w$ of households receive a random signal and they are allowed to reset wages optimally - \tilde{W}_t^* . All other households can only partially index their wages by past inflation. The problem of setting wages can be described as follows

$$\max_{\tilde{W}_{\mathcal{Z},t}} L \quad V_0 - \sum_{i=0}^{\infty} \mu_{t+i} \left[-\chi_0 \tilde{Z}_t^{1-\sigma_C} \frac{L_{\mathcal{Z},t+i}^{1+\sigma_L}}{1+\sigma_L} - \sum_{s=1}^i \beta^s \tilde{\lambda}_{t+i} \frac{\iota_w}{t+s} \tilde{W}_{\mathcal{Z},t} L_{\mathcal{Z},t+i} \right]$$

subject to

$$L_{\mathcal{Z},t+i} = \frac{\sum_{s=1}^i \frac{\iota_w}{t+s-1} \tilde{W}_{\mathcal{Z},t}}{\frac{\iota_w}{t+i} \tilde{W}_{t+i}}^{-\frac{\lambda_w}{\lambda_w-1}} L_{t+i}^d$$

The labour supply is determined by the following four equations that describe the wage Phillips curve

$$f_t^1 = \frac{1}{\lambda_w} \tilde{W}_t^* \frac{\tilde{\lambda}_t^{-1}}{\mu_t} \tilde{W}_t^{\frac{\lambda_w}{\lambda_w-1}} N_t^d + \beta \xi_w E_t \frac{\iota_w}{t+1} \tilde{W}_{t+1}^* \frac{1}{\lambda_w-1} f_{t+1}^1 \quad (145)$$

and

$$f_t^2 = \chi_0 \tilde{Z}_{t+i}^{1-\sigma_C} \frac{\tilde{W}_t}{\tilde{W}_t^*} \frac{\lambda_w(1+\sigma_L)}{\lambda_w-1} L_{t+i}^d + \beta \xi_w E_t \frac{\iota_w}{t+1} \tilde{W}_{t+1}^* \frac{\lambda_w(1+\sigma_L)}{\lambda_w-1} f_{t+1}^2 \quad (146)$$

$$\tilde{W}_t^{\frac{1}{1-\lambda_w}} = \xi_w \frac{\iota_w}{t} \frac{1}{1-\lambda_w} \tilde{W}_{t-1}^{\frac{1}{1-\lambda_w}} + (1 - \xi_w) \tilde{W}_t^* \frac{1}{1-\lambda_w} \quad (147)$$

Wage dispersion evolves according to

$$w_t = \xi_w \frac{\iota_w}{t} \frac{\lambda_w}{1-\lambda_w} \frac{\tilde{W}_{t-1}}{\tilde{W}_t} \frac{\lambda_w}{1-\lambda_w} w_{t-1} + (1 - \xi_w) \frac{\tilde{W}_t^*}{\tilde{W}_t} \frac{\lambda_w}{1-\lambda_w}$$

Intermediate Good Producers: The technology given for the production of the intermediate good is given by

$$\tilde{Y}_t(h) = \tilde{Z}_t L_t^d(h)^{1-\phi} \tilde{K}_{t-1}^\phi \quad (148)$$

Firstly, intermediate good producers act as price takers and demand the amount of labour that maximises their profits

$$E_t \sum_{i=0}^{\infty} M_{t+i} \beta^i P_t^W \tilde{Y}_t(h) - \tilde{W}_t L_t^d(h) - R_t^K \tilde{K}_{t-1}(h) \quad (149)$$

Namely, the labour demand is

$$P_t^W = \frac{L_t^d(h)}{(1-\phi) \tilde{Y}_t(h)} \tilde{W}_t \quad (150)$$

Next they set prices in order to maximise profits. Only a fraction $(1 - \xi)$ of producers receive a random signal and set prices optimally. The remaining (ξ) producers miss this signal and set prices based on a backward looking indexation rule.

$$\max_{\tilde{P}_t(h)} E_t \sum_{j=0}^{\infty} M_{t,t+j} (\beta \xi)^j \frac{\tilde{P}_t(h)^{\frac{\iota_\pi}{t+j-1}}}{\tilde{P}_{t+j}} - P_{t+j}^W \tilde{Y}_{t+j}(h)$$

The price Philip curve is described by the following four equations, while the last equation describes the evolution of price dispersion

$$\tilde{K}_{P,t} = P_t^W \tilde{Y}_t^d + \beta \xi E_t M_{t+1} \frac{t+1}{t} \frac{\epsilon}{\iota_\pi} \tilde{K}_{P,t+1} \quad (151)$$

$$\tilde{F}_{P,t} = \tilde{Y}_t^d + \beta \xi E_t M_{t+1} \frac{t+1}{t} \frac{\epsilon-1}{\iota_\pi} \tilde{F}_{P,t+1} \quad (152)$$

$$\frac{\epsilon}{t} = \frac{\epsilon}{\epsilon-1} \frac{K_{P,t}}{F_{P,t}} \quad (153)$$

$$1 = \xi \frac{t}{\iota_\pi} \frac{\epsilon-1}{t-1} + (1 - \xi) \frac{\epsilon-1}{t} \quad (154)$$

$$\frac{1}{t} \frac{1}{1-\phi} = (1 - \xi) \frac{\epsilon}{t} \frac{1}{1-\phi} + \xi \frac{t}{\iota_\pi} \frac{1}{1-\phi} \frac{1}{t-1} \quad (155)$$

Final Good Producers: Use combine the output produced in the previous stage and the following CES technology

$$\tilde{Y}_t = \int_0^1 \tilde{Y}_t(h)^{\frac{\varepsilon-1}{\varepsilon}} dh^{\frac{\varepsilon-1}{\varepsilon}} \quad (156)$$

To produce the final good. Finally, the demand for intermediate good is given by

$$\tilde{Y}_t(h) = \frac{\tilde{P}_t(h)^{\frac{\nu\pi}{t-1}}}{\tilde{P}_t} \tilde{Y}_t^{-\varepsilon} \quad (157)$$

Market Clearing Conditions:

$$\tilde{C}_t + \tilde{G}_t + \tilde{I}_t = \frac{\tilde{Z}_t \frac{L_t}{\Delta_t^w} K^\phi}{t} \quad (158)$$

Government:

$$\tilde{P}_t \tilde{G}_t + \tilde{D}_{t-1} = T_t + \frac{\tilde{D}_{t-1}}{R_t} \quad (159)$$

5.5.1 Stationary Equations

$$u(C_t, L_t) = \frac{C_t^{1-\sigma_C}}{1-\sigma_C} - \chi_0 \frac{L_t^{1+\sigma_L}}{1+\sigma_L} \quad (160)$$

$$\check{V}_t = E_t \left[V_{t+1} \frac{1-\sigma_C}{t+1} \right]^{1-\gamma}$$

$$\ddot{V}_t = \check{V}_t^{\frac{1}{1-\gamma}}$$

$$V_t = u(C_t, L_t) + \beta \ddot{V}_t \quad (161)$$

$$C_t - b \frac{C_{t-1}}{t}^{-\sigma_C} = \frac{\lambda_t}{\mu_t}$$

$$\frac{E_t \frac{t+1}{R_t^A} \frac{\sigma_C}{t+1}}{R_t^A} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \quad (162)$$

$$M_{t+1} = \frac{V_{t+1}}{\check{V}_t} \frac{1-\sigma_C}{t+1}^{-\gamma} \frac{C_t}{C_{t+1}}^{\sigma_C} \quad (163)$$

$$\frac{1}{R_t^A} = \beta E_t \frac{M_{t+1}}{t+1}$$

$$Y_t = \bar{K}^\phi L_t^{1-\phi}$$

$$P_t^W = \frac{\frac{L_t}{\Delta_t^w}}{(1-\phi)\tilde{Y}_t} W_t \quad (164)$$

$$K_{P,t} = P_t^W Y_t^d + \beta \xi E_t M_{t+1} \left(\frac{t+1}{t}\right)^{\frac{\varepsilon}{\iota_\pi}} K_{P,t+1} \quad (165)$$

$$F_{P,t} = Y_t^d + \beta \xi E_t M_{t+1} \left(\frac{t+1}{t}\right)^{\varepsilon-1} F_{P,t+1} \quad (166)$$

$$\frac{1}{t} = \frac{\varepsilon}{\varepsilon-1} \frac{K_{P,t}}{F_{P,t}} \quad (167)$$

$$1 = \xi \left(\frac{t}{t-1}\right)^{\varepsilon-1} + (1-\xi) \left(\frac{t}{t-1}\right)^{-1-\varepsilon} \quad (168)$$

$$\frac{1}{t^{1-\phi}} = (1-\xi) \left(\frac{t}{t-1}\right)^{-\frac{\varepsilon}{1-\phi}} + \xi \left(\frac{t}{t-1}\right)^{\frac{\varepsilon}{1-\phi}} \frac{1}{t-1} \quad (169)$$

$$Y_t^d = C_{H,t} + G_t + I \quad (170)$$

$$Y_t = {}_t Y_t^d \quad (171)$$

$$L_t = {}_t L_t^d \quad (172)$$

$$f_t = \frac{1}{\lambda_w} (W_t^*)^{-\frac{1}{\lambda_w-1}} \frac{\lambda_t}{\mu_t} (W_t)^{\frac{\lambda_w}{\lambda_w-1}} N_t^d + \beta \xi_w E_t \left(\frac{t}{t+1}\right)^{-\frac{1}{\lambda_w-1}} \frac{\tilde{W}_{t+1}^*}{W_t^*} \left(\frac{t}{t+1}\right)^{\frac{1}{\lambda_w-1}} f_{t+1} \frac{1-\sigma_C}{t+1} \quad (173)$$

$$f_t = \chi_0 \frac{W_t}{W_t^*} \left(\frac{t}{t+1}\right)^{\frac{\lambda_w(1+\sigma_L)}{\lambda_w-1}} L_{t+1}^d \frac{1+\sigma_L}{t+1} + \beta \xi_w E_t \left(\frac{t}{t+1}\right)^{-\frac{\lambda_w(1+\sigma_L)}{\lambda_w-1}} \frac{W_{t+1}^*}{W_t^*} \left(\frac{t}{t+1}\right)^{\frac{\lambda_w(1+\sigma_L)}{\lambda_w-1}} f_{t+1} \frac{1-\sigma_C}{t+1} \quad (174)$$

$$W_t^{\frac{1}{1-\lambda_w}} = \xi_w \left(\frac{t}{t-1}\right)^{\frac{\lambda_w}{1-\lambda_w}} \frac{W_{t-1}}{t} \frac{1}{1-\lambda_w} + (1-\xi_w) (W_t^*)^{\frac{1}{1-\lambda_w}} \quad (175)$$

$$\frac{w}{t} = \xi_w \left(\frac{t}{t-1}\right)^{\frac{\lambda_w}{1-\lambda_w}} \frac{W_{t-1}}{t W_t} \frac{\lambda_w \Delta w}{1-\lambda_w} + (1-\xi_w) \frac{W_t^*}{W_t} \frac{\lambda_w}{1-\lambda_w}$$

5.5.2 Steady-States

We assume that $L^d = 1/3$, we know

$$\delta_K K = I \quad (176)$$

$$\frac{\delta_K K}{Y} = \frac{I}{\bar{Y}} \quad (177)$$

We also assume that

$$\frac{I}{Y} = 0.2 \quad (178)$$

The from the Production function

$$\begin{aligned} \frac{Y}{K} &= \bar{K}^{\phi-1} L^{1-\phi} \\ \frac{Y}{\delta_K K} &= \bar{K}^{\phi-1} \frac{L^{1-\phi}}{\delta_K} \\ \bar{K} &= \frac{L^{1-\phi} I}{\delta_K Y}^{\frac{1}{1-\phi}} \end{aligned} \quad (179)$$

From the stochastic discount factor we obtain

$$M = -\sigma_C$$

From the PP equations we obtain

$$\begin{aligned} K_P &= \frac{P^W Y^d}{1 - \beta \xi^{1-\sigma_C}} \\ F_P &= \frac{Y^d}{1 - \beta \xi^{1-\sigma_C}} \\ - &= \frac{1 - \xi^{(1-\iota_\pi)(\varepsilon-1)} \frac{1}{1-\varepsilon}}{1 - \xi} \\ - &= \frac{\varepsilon}{\varepsilon - 1} \frac{K}{F} = \frac{\varepsilon}{\varepsilon - 1} \frac{\frac{P^W Y^d}{1 - \beta \xi \Pi^{(1-\iota_\pi)\varepsilon} M \Gamma}}{\frac{Y^d}{1 - \beta \xi \Pi^{(1-\iota_\pi)(\varepsilon-1)} M \Gamma}} \end{aligned} \quad (180)$$

$$- = \frac{\varepsilon}{\varepsilon - 1} P^W \quad (181)$$

$$P^W = \frac{\varepsilon - 1 - 1 - \beta \xi^{(1-\iota_\pi)(\varepsilon-1)} M}{\varepsilon \frac{1 - \beta \xi^{(1-\iota_\pi)\varepsilon} M}} \quad (182)$$

Then from the price dispersion equation

$$= \frac{(1 - \xi)^{-\frac{\varepsilon}{1-\phi}}}{1 - \xi^{\frac{(1-\iota_\pi)\varepsilon}{1-\phi}}} \frac{1-\phi}{1-\phi} \quad (183)$$

From the marketing clearing condition we also know that

$$Y^d = \frac{Y}{\lambda_w} \quad (184)$$

Similarly

$$C = (1 - g) Y^d - I \quad (185)$$

From the demand for labour

$$1 = \xi_w \lambda_w^{-1} \frac{1}{1-\lambda_w} \frac{1}{1-\lambda_w} + (1 - \xi_w) \frac{W^*}{W} \frac{1}{1-\lambda_w}$$

$$\frac{W^*}{W} = \frac{1 - \xi_w \iota_w^{-1} \frac{1}{1-\lambda_w} \frac{1}{\Gamma} \frac{1}{1-\lambda_w}^{1-\lambda_w}}{1 - \xi_w}$$

$$w = \frac{(1 - \xi_w) \frac{W^*}{W} \frac{\lambda_w}{1-\lambda_w}}{1 - \xi_w (\iota_w^{-1}) \frac{\lambda_w}{1-\lambda_w} \frac{1}{\Gamma} \frac{\lambda_w}{1-\lambda_w}} \quad (186)$$

$$N^d = \frac{L}{w} \quad (187)$$

$$W = P^W \frac{(1 - \phi) Y}{L^d} \quad (188)$$

$$W^* = \frac{W^*}{W} W \quad (189)$$

From the labour supply schedule

$$f = \frac{\frac{1}{\lambda_w} (W^*)^{-\frac{1}{\lambda_w-1}} \frac{(1-b)C}{\Gamma}^{-\sigma_C} (W)^{\frac{\lambda_w}{\lambda_w-1}} L^d}{1 - \beta \xi_w \frac{1-\iota_w}{\lambda_w-1} \left(\frac{1+(1-\sigma_C)(\lambda_w-1)}{\lambda_w-1} \right)} \quad (190)$$

$$\chi_0 = \frac{1 - \beta \xi_w E_t \frac{\lambda_w(1+\sigma_L)(1-\iota_w)}{\lambda_w-1} \frac{\lambda_w(1+\sigma_L)+(1-\sigma_C)(\lambda_w-1)}{\lambda_w-1} f}{\frac{W}{W^*} \frac{\lambda_w(1+\sigma_L)}{\lambda_w-1} (N^d)^{1+\sigma_L}} \quad (191)$$

5.5.3 Calibration

Table 5: Calibrated Parameters

Parameters	Description	Values
\bar{L}	Steady State of Average Hours	1/3
$\frac{G}{\bar{Y}}$	Government Spending to GDP Ratio	0.18
$\frac{I}{\bar{Y}}$	Investment to GDP Ratio	0.20
$400 \ln(\pi)$	Inflation Target	2.00
$400 \ln(Z)$	Steady State Value of Productivity Growth	2.00
ϕ	Capital Production Share	0.36
δ_K	Capital Depreciation Rate	0.03
β	Time Discount Factor	0.995

Table 6: Estimated Parameters

Parameters	Description	Values
γ	Epstein-Zin Preference Risk	24.73
σ_C	Inverse of Intertemporal Elasticity of Substitution	1.00
b	Habit Smoothing	0.93
σ_L	Inverse of Frisch Elasticity	1.15
ε	Intermediate Goods Elasticity of Substitution	12.94
ξ	Calvo Price Probability	0.82
ι	Price Indexation	0.00
ε_w	Wages Elasticity of Substitution	3.84
ξ_w	Calvo Wage Probability	0.53
ι_w	Wage Indexation	0.78
ζ_Π	Inflation Monetary Policy Response	2.29
ζ_Y	Output Monetary Policy Response	0.30
ρ_R	Interest Rate Smoothing	0.71
ρ_{ε_R}	Policy Shock Persistence	0.18
ρ_{ε_z}	Productivity Growth Shock Persistence	0.83
ρ_{ε_g}	Government Spending Shock Persistence	0.13
$100\sigma_R$	Policy Shock Standard Deviation	0.22
$100\sigma_z$	Policy Productivity Growth Standard Deviation	5.00
$100\sigma_g$	Government Spending Shock Standard Deviation	0.01

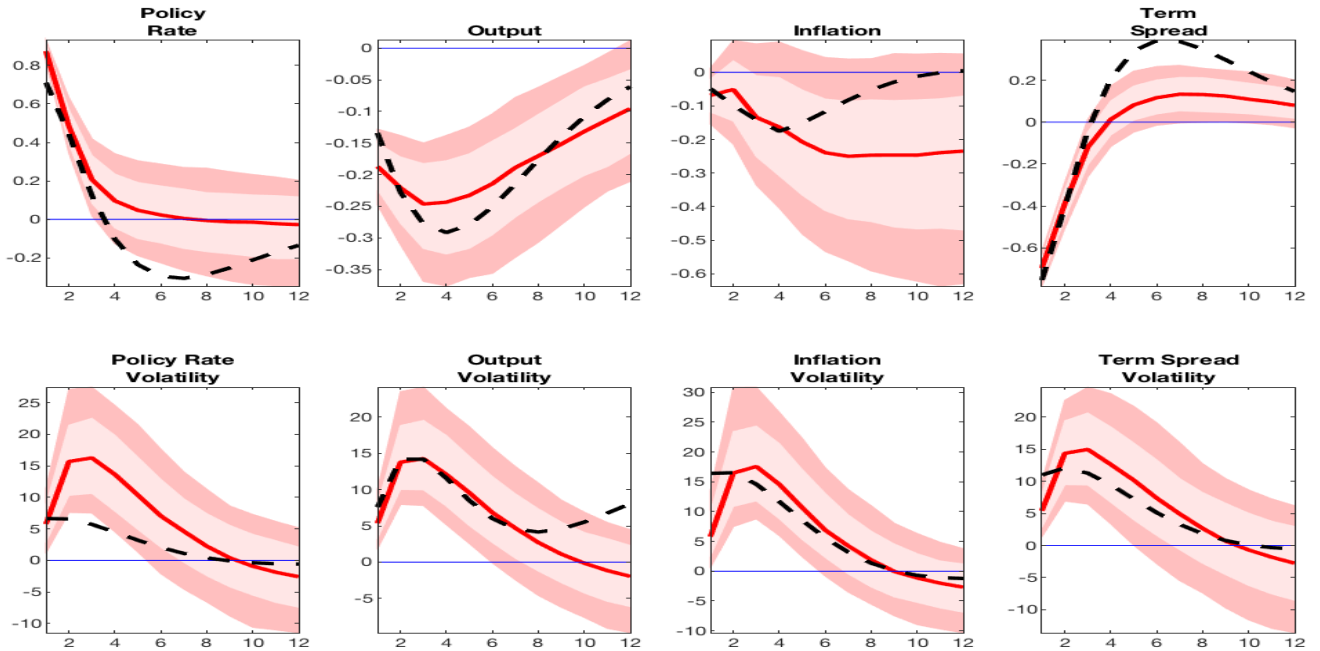
5.5.4 Simulations & IRFs

Table 7: Disaster Statistics

Metrics	Data	1 Standard Deviation	2 Standard Deviation
Probability	7.83	-	0.02
Size	21.99	-	10.85
Duration	3.72	-	3.67

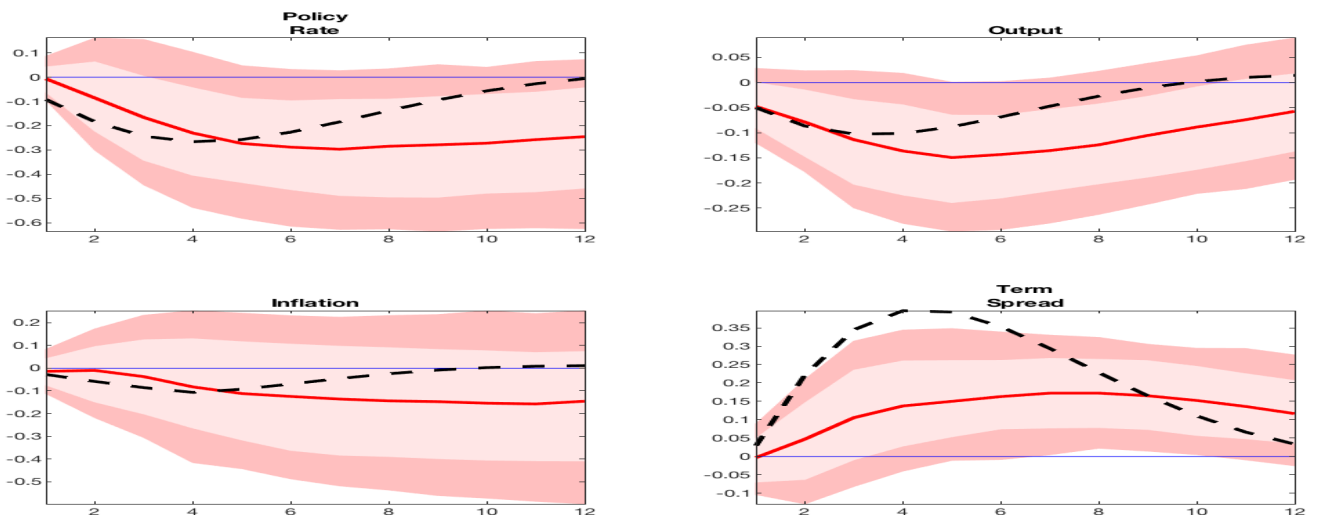
Notes: The data disaster statistic estimates are those reported by [Petrosky-Nadeau et al. \(2018\)](#) (Table 4, pp. 2227). Starting from the stochastic steady state, the model is simulated for 50000 periods. Similar to [Petrosky-Nadeau et al. \(2018\)](#), we time-aggregate output into annual observations, and apply the peak-to-trough method to identify disasters as cumulative fractional declines in output of at least 10%. The disaster probabilities and average size are in percent, and the average duration is in terms of years.

Figure 14: The fit of the DSGE model with Sticky Wages



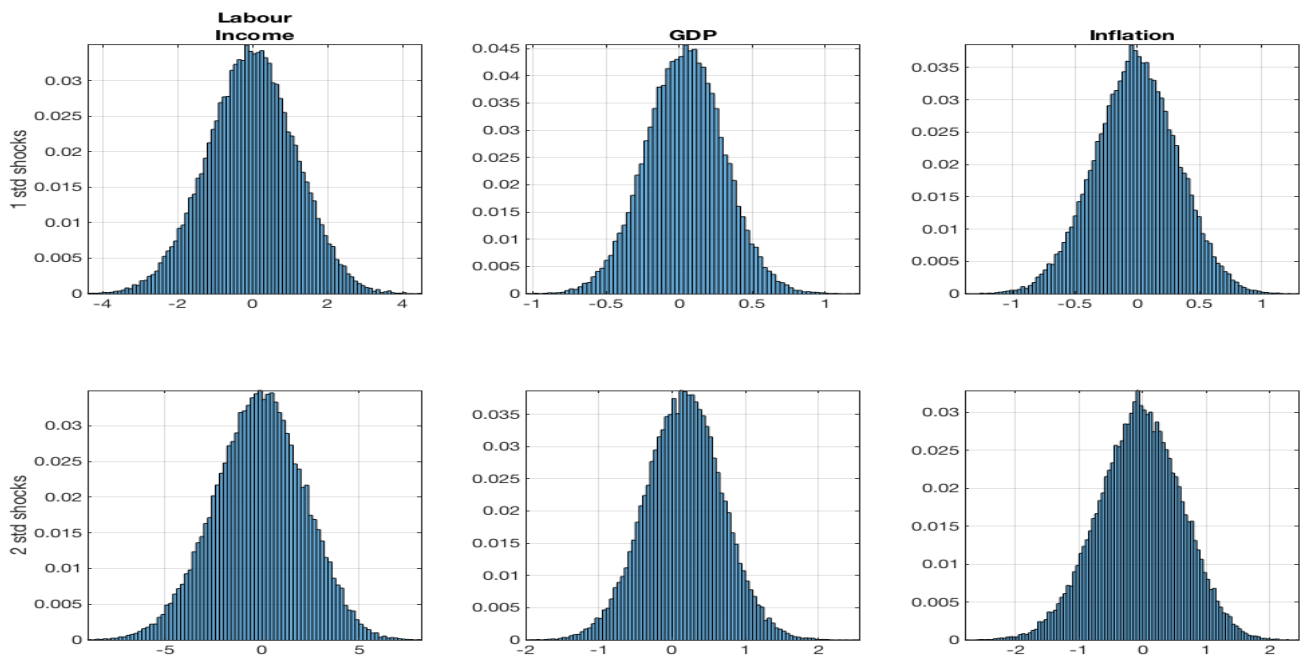
Notes: The responses from the empirical model have been adjusted to match the quarterly frequency of the DSGE model. The back dashed line illustrates the DSGE response to the policy shock. The policy rate and inflation are expressed in annual terms.

Figure 15: Uncertainty Effect DSGE model with Sticky Wages: Uncertainty Contribution



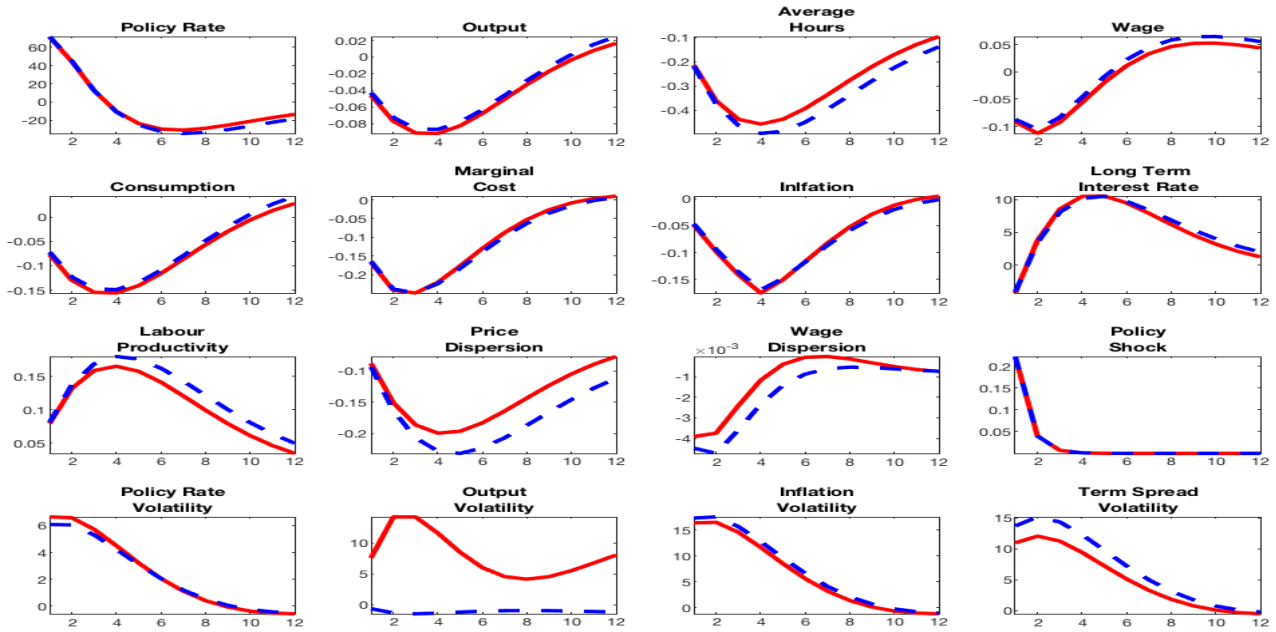
Notes: The responses from the empirical model have been adjusted to match the quarterly frequency of the DSGE model. The back dashed line illustrates the DSGE response to the policy shock. The policy rate and inflation are expressed in annual terms. The DSGE response is the difference between the response derived by using the third order solution of the model and the one obtained by using the first order solution.

Figure 16: Downward Risk



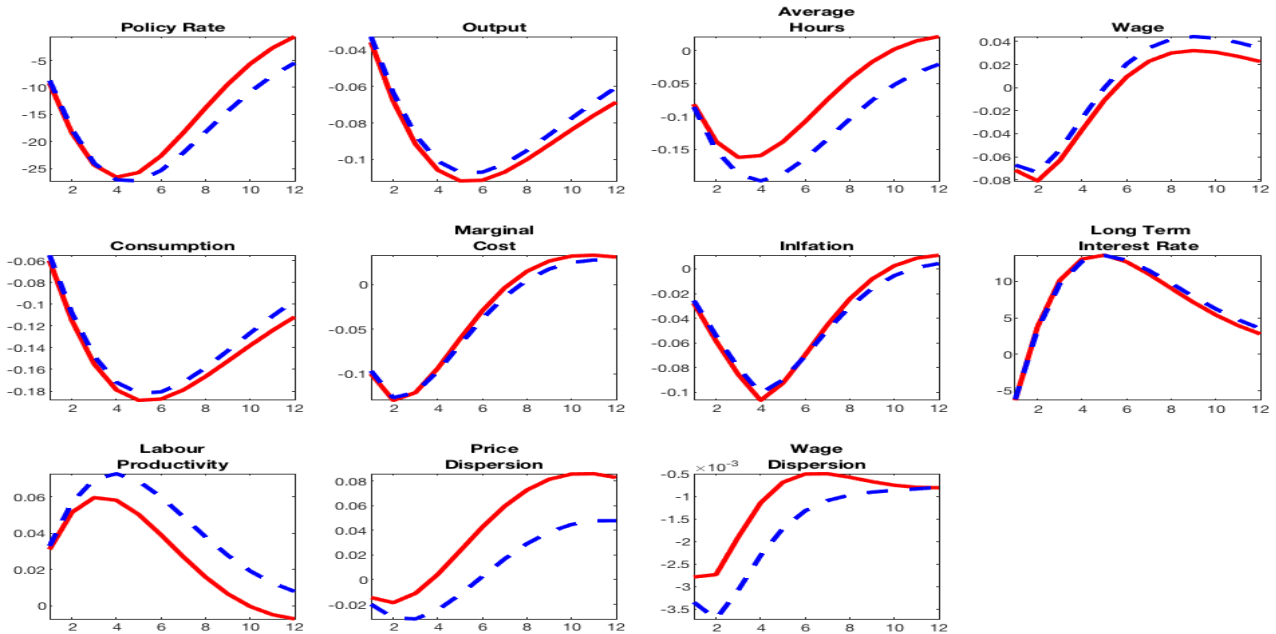
Notes: Starting from the stochastic steady state, the model is simulated from 50000 periods. The histogram illustrates the distribution of x_t as deviation from its stochastic steady-state. For the simulation in the first row we use the standard deviation value that results from the estimation, while we use two times this estimate for the second row.

Figure 17: Epstein-Zin Risk Coefficient



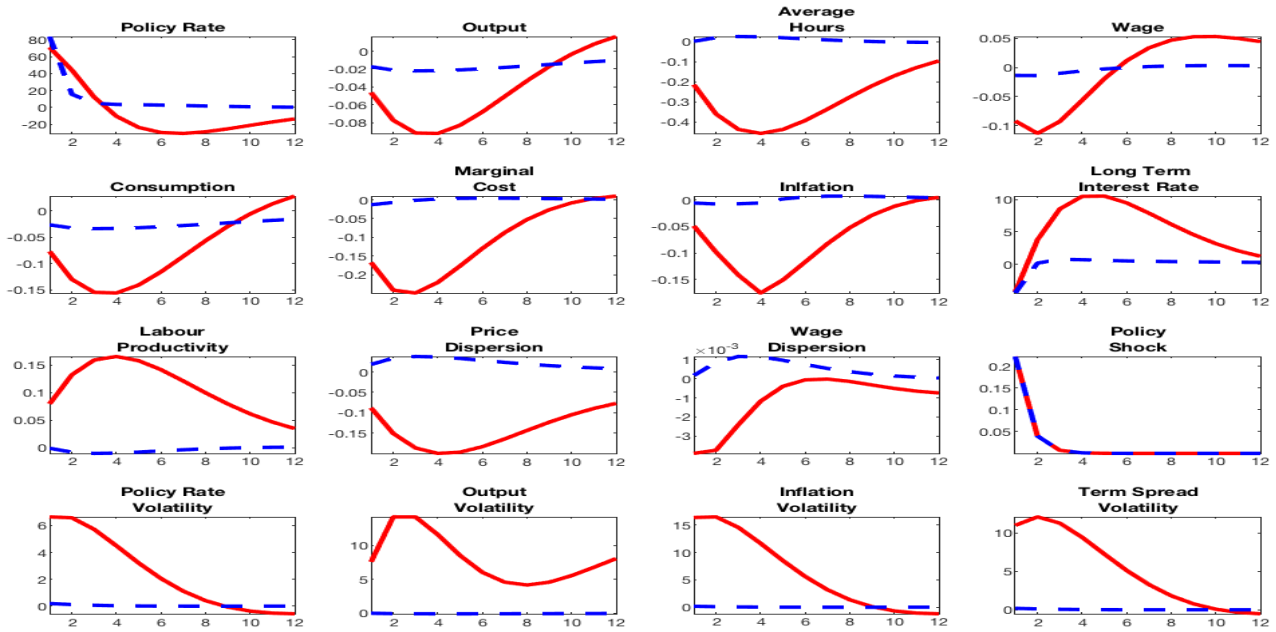
Notes: The red solid line represents the benchmark version of the model ($\gamma = 24.73$), while the blue dashed line is the responses of the model when agents in the economy form expected utility preferences ($\gamma = 0$). Rates are reported in annual basis points, inflation in annual percentage rates and Job finding and filling probability in percentage points. The responses are calculated relative to the stochastic steady state.

Figure 18: Epstein-Zin Risk Coefficient: Uncertainty Contribution



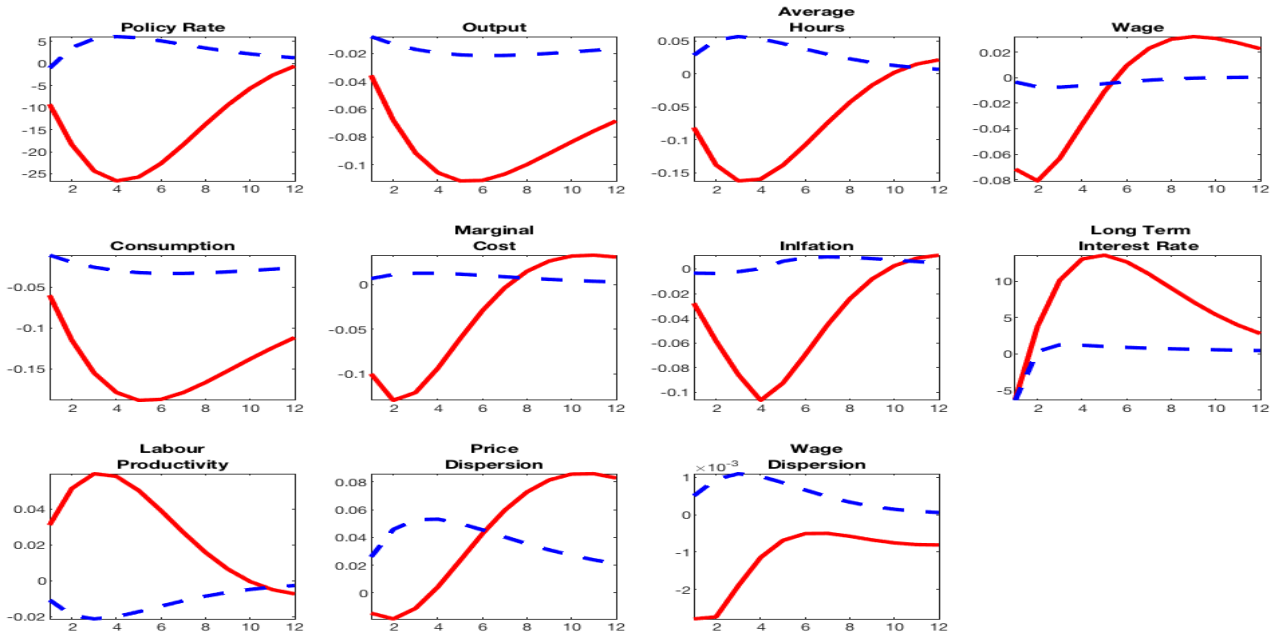
Notes: The red solid line represents the benchmark version of the model ($\gamma = 24.73$), while the blue dashed line is the responses of the model when agents in the economy form expected utility preferences ($\gamma = 0$). Rates are reported in annual basis points, inflation in annual percentage rates and Job finding and filling probability in percentage points. The responses are calculated relative to the stochastic steady state. The uncertainty contribution is calculated as the difference between the response derived by using the third order solution of the model minus the responses produced using first order solution.

Figure 19: Policy Rate Smoothing Preferences



Notes: The red solid line represents the benchmark version of the model ($\rho_R = 0.71$), while the blue dashed line is the responses of the model when authorities have no preferences for interest rate smoothing ($\rho_R = 0$). Rates are reported in annual basis points, inflation in annual percentage rates and Job finding and filling probability in percentage points. The responses are calculated relative to the stochastic steady state.

Figure 20: Policy Rate Smoothing Preferences: Uncertainty Contribution



Notes: The red solid line represents the benchmark version of the model ($\rho_R = 0.71$), while the blue dashed line is the responses of the model when authorities have no preferences for interest rate smoothing ($\rho_R = 0$). Rates are reported in annual basis points, inflation in annual percentage rates and Job finding and filling probability in percentage points. The responses are calculated relative to the stochastic steady state. The uncertainty contribution is calculated as the difference between the response derived by using the third order solution of the model minus the responses produced using first order solution.

References

- ALESSANDRI, P. AND H. MUMTAZ (2018): "Financial regimes and uncertainty shocks." *Journal of Monetary Economics*.
- ANDRIEU, C., A. DOUCET, AND R. HOLENSTEIN (2010): "Particle Markov chain Monte Carlo methods," *Journal of the Royal Statistical Society Series B*, 72, 269–342.
- ASCARI, G. AND A. M. SBORDONE (2014): "The Macroeconomics of Trend Inflation," *Journal of Economic Literature*, 52, 679–739.
- BANBURA, M., D. GIANNONE, AND L. REICHLIN (2007): "Bayesian VARs with Large Panels," CEPR Discussion Papers 6326, C.E.P.R. Discussion Papers.
- BENATI, L. AND H. MUMTAZ (2007): "U.S. evolving macroeconomic dynamics: a structural investigation," Working Paper Series 746, European Central Bank.
- BRUCKNER, M. AND E. PAPP (2012): "Fiscal Expansions, Unemployment, And Labor Force Participation: Theory And Evidence," *International Economic Review*, 53, 1205–1228.
- CACCIATORE, M. AND F. RAVENNA (2016): "Uncertainty, Wages, and the Business Cycle," mimeo.
- CARTER, C. AND P. KOHN (2004): "On Gibbs sampling for state space models," *Biometrika*, 81, 541–53.
- CHEN, H., V. CURDIA, AND A. FERRERO (2012): "The Macroeconomic Effects of Large-scale Asset Purchase Programmes," *Economic Journal*, 122, F289–F315.
- CHRISTIANO, L., M. EICHENBAUM, AND C. EVANS (2005): "Nominal Rigidities and the Dynamic Effects of a shock to Monetary Policy," *Journal of Political Economy*, 113, 1–45.
- CHRISTIANO, L. J., M. TRABANDT, AND K. WALENTIN (2010): "DSGE Models for Monetary Policy Analysis," in *Handbook of Monetary Economics*, ed. by B. M. Friedman and M. Woodford, vol. 3, 285–367.
- COGLEY, T. AND T. J. SARGENT (2005): "Drift and Volatilities: Monetary Policies and Outcomes in the Post WWII U.S.," *Review of Economic Dynamics*, 8, 262–302.
- DI PACE, F. AND S. VILLA (2016): "Factor complementarity and labour market dynamics," *European Economic Review*, 82, 70–112.
- EPSTEIN, L. G. AND S. E. ZIN (1989): "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework," *Econometrica*, 57, 937–69.
- ERCEG, C. J., D. W. HENDERSON, AND A. T. LEVIN (2000): "Optimal monetary policy with staggered wage and price contracts," *Journal of Monetary Economics*, 46, 281–313.
- GERTLER, M. AND P. KARADI (2015): "Monetary Policy Surprises, Credit Costs, and Economic Activity," *American Economic Journal: Macroeconomics*, 7, 44–76.
- GERTLER, M., L. SALA, AND A. TRIGARI (2008): "An Estimated Monetary DSGE Model with Unemployment and Staggered Nominal Wage Bargaining," *Journal of Money, Credit and Banking*, 40, 1713–1764.

- HAGEDORN, M. AND I. MANOVSKII (2008): "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited," *American Economic Review*, 98, 1692–1706.
- HALL, R. E. (2005): "Employment Fluctuations with Equilibrium Wage Stickiness," *American Economic Review*, 95, 50–65.
- JURADO, K., S. C. LUDVIGSON, AND S. NG (2015): "Measuring Uncertainty," *American Economic Review*, 105, 1177–1216.
- KING, R. AND A. L. WOLMAN (1999): "What Should the Monetary Authority Do When Prices Are Sticky?" in *Monetary Policy Rules*, National Bureau of Economic Research, Inc, NBER Chapters, 349–404.
- KOOP, G., H. PESARAN, AND S. POTTER (1996): "Impulse response analysis in nonlinear multivariate models," *Journal of Econometrics*, 74, 119–147.
- KRAUSE, M. U., D. LOPEZ-SALIDO, AND T. A. LUBIK (2008): "Inflation dynamics with search frictions: A structural econometric analysis," *Journal of Monetary Economics*, 55, 892–916.
- KRAUSE, M. U. AND T. A. LUBIK (2007): "The (ir)relevance of real wage rigidity in the New Keynesian model with search frictions," *Journal of Monetary Economics*, 54, 706–727.
- LEDUC, S. AND Z. LIU (2016): "Uncertainty shocks are aggregate demand shocks," *Journal of Monetary Economics*, 82, 20–35.
- LINDSTEN, F., M. I. JORDAN, AND T. B. SCHÖN (2014): "Particle Gibbs with Ancestor Sampling," *Journal of Machine Learning Research*, 15, 2145–2184.
- MORTENSEN, D. T. AND C. A. PISSARIDES (1994): "Job Creation and Job Destruction in the Theory of Unemployment," *Review of Economic Studies*, 61, 397–415.
- MUMTAZ, H. (2018): "A generalised stochastic volatility in mean VAR," Working Papers 855, Queen Mary University of London, School of Economics and Finance.
- MUMTAZ, H. AND K. THEODORIDIS (2015): "Dynamic Effects of Monetary Policy Shocks on Macroeconomic Volatility," Working Papers 760, Queen Mary University of London, School of Economics and Finance.
- NONEJAD, N. (2015): "REPLICATING THE RESULTS IN A NEW MODEL OF TREND INFLATION USING PARTICLE MARKOV CHAIN MONTE CARLO," *Journal of Applied Econometrics*, n/a–n/a.
- PAPPA, E., R. SAJEDI, AND E. VELLA (2015): "Fiscal consolidation with tax evasion and corruption," *Journal of International Economics*, 96, 56–75.
- PETROSKY-NADEAU, N., L. ZHANG, AND L.-A. KUEHN (2018): "Endogenous Disasters," *American Economic Review*, 108, 2212–2245.
- ROMER, C. D. AND D. H. ROMER (2004): "A New Measure of Monetary Shocks: Derivation and Implications," *American Economic Review*, 94, 1055–1084.

- RUDEBUSCH, G. D. AND E. T. SWANSON (2012): "The Bond Premium in a DSGE Model with Long-Run Real and Nominal Risks," *American Economic Journal: Macroeconomics*, 4, 105–43.
- SCHMITT-GROHE, S. AND M. URIBE (2003): "Closing small open economy models," *Journal of International Economics*, 61, 163–185.
- SMETS, F. AND R. WOUTERS (2007): "Shocks and Frictions in US Business Cycles: a Bayesian DSGE Approach," *American Economic Review*, 97, 586–606.
- SWANSON, E. (2015): "A Macroeconomic Model of Equities and Real, Nominal, and Defaultable Debt," Mimeo, University of California Irvine.
- WEIL, P. (1989): "The equity premium puzzle and the risk-free rate puzzle," *Journal of Monetary Economics*, 24, 401–421.
- (1990): "Nonexpected Utility in Macroeconomics," *The Quarterly Journal of Economics*, 105, 29–42.