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## Optimal Monetary Policy in the Presence of Pricing-to-Market

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Optimal Monetary Policy  
in the Presence of Pricing-to-Market

von

Jochen Michaelis

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**Volkswirtschaftliche Diskussionsbeiträge**

# Optimal Monetary Policy in the Presence of Pricing-to-Market

Jochen Michaelis\*

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## Abstract

This paper presents a general-equilibrium framework to revisit the issues of optimal monetary policies and international policy coordination in a two-country model, focusing on the role of a pricing-to-market (PTM) policy by firms. Both countries may be different with respect to PTM. Using the set-up developed by Corsetti and Pesenti (2001a) and Betts and Devereux (2000a,b), we show that (i) for a given Foreign monetary stance, a Home monetary expansion is beneficial for both countries only if Home PTM is at an intermediate range; (ii) in a world Nash equilibrium Home and Foreign welfare are bell-shaped in the degrees of PTM; (iii) relative welfare crucially depends on the degrees of PTM; (iv) there is a welfare gain from cooperation even in the cases of no and full PTM.

Keywords: pricing to market, terms of trade, international coordination of monetary policies

JEL-classification: E 40, F 41, F 42

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## 1. Introduction

It is a truism that in a world with increasingly integrated national economies, monetary policy in each country affects economic welfare both at home and abroad. Due to the presence of beggar-thy-neighbor and beggar-thyself effects, however, the welfare effects are difficult to sign. The exploration of the international spillovers and the design of an optimal monetary policy in closed and open economies has become a cottage industry for that reason (see, e.g., Corsetti and Pesenti, 2001a,b; Obstfeld and Rogoff, 2002; Devereux and Engel, 2003; Sutherland, 2004). Given that wages and/or prices are predetermined, the core of the problem is a simple trade-off: monetary policy is useful for closing the output gap arising from monopolistic competition but may have an adverse terms-of-trade effect. The purpose of this paper is to ask whether monetary policy is beggar-thy-neighbor or beggar-thyself and to compare non-cooperative and cooperative optimal monetary policies.

To address the issues of interest, we set up a non-stochastic two-country general equilibrium model with imperfect competition on goods and labor markets and nominal wage and price rigidities. Some firms segment markets by country, they can charge different prices in domestic and foreign markets. In a similar framework Betts and Devereux (2000a,b) have shown that the sign of the terms-of-trade effect very much depends on the pricing policy of firms. If firms pre-set their export prices in the currency of the producer (consumer), the terms of trade of the expanding country will worsen (improve). In their model, the fraction of exporters who set prices in local currency of sale (pricing to market PTM) is symmetric across countries. Our framework instead allows the fraction of PTM firms to differ across the home and the foreign country, so that any change in the terms of trade can be separated in a change in export prices depending on home PTM and a change in import prices depending on PTM abroad. This distinction is crucial, since the increase in world aggregate demand is a function of the difference of the degrees of PTM, and since a given movement of the terms of trade is now compatible with various consumption and output (employment) allocations.<sup>1</sup>

Optimal policies are derived using as objective criterion welfare of the representative agent defined over the discounted flow of consumption, the utility of real balances and the disutility of work effort. Somewhat surprisingly, this most natural criterion is not very common in the related literature, where many contributions assume away the real balance term (see, e.g., Corsetti and Pesenti, 2001b; Sutherland, 2004). If, however, real balances are important for determining allocations of agents and monetary authorities are maximizing the welfare of agents then it should be included in the policymakers' problem. But this comes at a price. To get an exact solution for the welfare term we have to choose specific functional forms. In particular, utility is logarithmic in consumption and the elasticity of substitution between home and foreign goods is restricted to unity. The latter precludes current account imbalances and thus shuts off any long-run effects of money. But since the unitary value is within the range of empirical estimates of this parameter and no approximations of welfare are needed in order to characterize the optimal policy functions, the gain of this assumption, at least from our point of view, outweigh the costs in form of a loss in generality.<sup>2</sup>

In our two-country (Home and Foreign) set-up we show the following: (i) Home's terms of trade improve (worsen), if the sum of PTM-degrees in Home and Foreign is greater (less) than unity. (ii) For a given Foreign monetary stance, a Home monetary expansion is beggar-thyself (beggar-thy-neighbor), if Home PTM is "low" ("high"). Only if Home PTM is at an intermediate range, welfare will raise in both countries. This range vanishes in the case of perfect competition of labor and goods markets. (iii) In a world Nash equilibrium Home and Foreign welfare are bell-shaped in the degrees of PTM. (iv) The country which exhibits a higher degree of PTM than its neighbor will come up with a higher (or at least the same) welfare level. (v) If there is no terms-of-trade effect at the aggregate level, the Nash optimal monetary stance is identical for both countries; it does not matter which country experiences a higher degree of PTM in its economy. (vi) There is always a welfare gain from cooperation independently of the degrees of PTM.

The superiority of cooperation contrasts to parts of the literature. Assuming a world of no PTM Rogoff (1985) finds that the worsening of the terms of trade puts a brake on the policymakers' incentive to inflate in order to fill the output gap. Cooperation removes this brake, so that the equilibrium inflation rate rises implying a decline in welfare compared to the case of policy competition. Our result is different since we leave wages and prices as predetermined, not forward looking variables. We consider policies under commitment, which are not, in general, time-consistent in the Barro-Gordon sense. Betts and Devereux (2000a) find that the degree of PTM determines whether cooperation is good or bad. This result, however, hinges on an arbitrary assumed cost of inflation, which is absent in the welfare of private agents but part of the objective function of the policymaker.<sup>3</sup> Corsetti and Pesenti (2001b) analyze a stochastic two-country model and show that there are gains from cooperation when the degrees of PTM are strictly between zero and unity. In the polar cases of no and full PTM the movement in the terms of trade has no impact on relative consumption and output, there is no incentive to use the terms of trade strategically and thus no gain from cooperation (see also Benigno and Benigno, 2003, and Devereux and Engel, 2003). Obstfeld and Rogoff (2002) argue that even in the case where there are some gains from cooperation, these are likely to be very small. Benigno (2002), on the other hand, shows that these gains will be non-trivial if utility is not logarithmic in consumption. Sutherland (2004) makes a similar point emphasizing the case where the cross-country elasticity exceeds unity (for a discussion of the importance of this parameter for the sign of the welfare spill-over see also Tille, 2001, and Michaelis, 2004). Pappa (2002) provides a most general model and discusses how sensitive welfare responds to changes in key parameters like the intertemporal elasticity of substitution, the labor supply elasticity, the degree of openness etc.

The paper is structured as follows. The model is presented in Section 2. Section 3 presents the solution of the model and a discussion of the transmission mechanism of

monetary shocks. Section 4 discusses the design of an optimal monetary policy distinguishing between a world Nash equilibrium and a cooperative equilibrium. Section 5 concludes.

## 2. The Model

We consider a world of two countries of identical size, Home and Foreign, each populated by a continuum of identical households with population size normalized to unity and each specialized in the production of one type of goods. The national types of goods consists of a number of brands defined over a continuum of unit mass. Each brand, indexed by  $i$ , is produced by a single firm and sold world-wide. Home firms produce brands on the interval  $i \in [0,1]$ , whereas Foreign firms produce brands on the interval  $i \in [1,2]$ .

Following Betts and Devereux (2000a,b), we assume that a fraction of firms in each country can segment their markets by country, i.e., they can charge different prices for their pricing-to-market (PTM) goods in domestic and foreign markets. Before the exchange rate is known these firms set prices in the currency of the buyer. For the remaining firms the law of one price (LOP) holds. Prices for their LOP goods are assumed to be set in the currency of the seller. We extend the Betts/Devereux-scenario by allowing for asymmetries between countries with respect to the degree of PTM. The fraction of PTM-firms in the Home country,  $s$ , need not be equal to that in the Foreign country,  $s^*$ .<sup>4</sup>

### 2.1 Households

A representative Home household<sup>5</sup> maximizes its lifetime utility

$$U_t = \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[ (1-\delta) \ln C_{\tau} + \delta \ln \frac{M_{\tau}}{P_{\tau}} - \frac{\sigma}{2} (l_{\tau})^2 \right], \quad (1)$$

where  $M_t$  are nominal balances,  $P_t$  is Home's consumer price index,  $l_t$  are total hours worked by the household,  $\beta$  is the discount factor, and  $C_t$  is a consumption index defined as

$$C_t = \left( \frac{C_{H,t}}{\alpha} \right)^\alpha \left( \frac{C_{F,t}}{1-\alpha} \right)^{1-\alpha}. \quad (2)$$

Here,  $C_{H,t}$  and  $C_{F,t}$  are the consumption baskets of Home and Foreign goods, respectively.

The parameter  $\alpha$  is the share of the Home good in the overall consumption basket. Due to the Cobb-Douglas specification of (2), the elasticity of substitution between the two available types of goods is restricted to unity. This assumption allows for a closed-form solution, but of course the tractability comes at the price of generality. In particular, because of a unitary terms-of-trade elasticity of exports and imports this assumption shuts off any current account imbalances and thus any long-run effects of monetary policies. Empirically, however, the assumption of unit elasticity seems warranted.<sup>6</sup> Moreover, the model is restricted to a unitary intertemporal elasticity of substitution and a marginal disutility of labor equal to two. For the implications of these parameters for the welfare effects of monetary policy see Pappa (2002).

In Eq. (2),  $C_{H,t}$  and  $C_{F,t}$  are CES aggregates across Home and Foreign brands,

$$C_{H,t} = \left( \int_0^1 (C_{H,t}(i))^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}; \quad C_{F,t} = \left( \int_1^{1+s^*} (C_{F,t}^{PTM}(i))^{\frac{\varepsilon-1}{\varepsilon}} di + \int_{1+s^*}^2 (C_{F,t}^{LOP}(i))^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}, \quad (3)$$

where  $C_{H,t}(i)$ ,  $C_{F,t}^{PTM}(i)$  and  $C_{F,t}^{LOP}(i)$  denote consumption of the Home variety  $i \in [0,1]$ , consumption of the Foreign PTM variety  $i \in [1,1+s^*]$  and consumption of the Foreign LOP variety  $i \in [1+s^*,2]$ , respectively. The parameter  $\varepsilon > 1$  represents the elasticity of substitution between any two goods produced in the same country.

The lifetime utility of Foreign households is analogously defined. In particular, Home and Foreign households are assumed to have identical discount factors and identical



preferences towards liquidity services and labor. However, we take into account the growing evidence of a significant degree of home bias in international trade (see Mc Callum, 1995; Engel and Rogers, 1996) by assuming that both Home and Foreign households have an equal bias for their own domestically produced good. This approach follows Warnock (2003). The consumption index of a Foreign household is given by

$$C_t^* = \left( \frac{C_{F,t}^*}{\alpha} \right)^\alpha \left( \frac{C_{H,t}^*}{1-\alpha} \right)^{1-\alpha}, \quad (4)$$

where  $C_{F,t}^*$  and  $C_{H,t}^*$  are respectively consumption of the Foreign and consumption of the Home good by a Foreign household. These baskets are in turn CES aggregates across brands as in (3). For  $\alpha > 1/2$  we have a home bias, that is, at given relative prices the ratio of Home goods to Foreign goods consumed in Home is higher than in Foreign. If  $\alpha = 1/2$ , Home and Foreign households have identical preferences.

The consumption-based price index, defined as the minimal expenditure required to purchase one unit of consumption  $C_t$  ( $C_t^*$ ), is given by

$$P_t = (P_{H,t})^\alpha (P_{F,t})^{1-\alpha}, \quad P_t^* = (P_{F,t}^*)^\alpha (P_{H,t}^*)^{1-\alpha} \quad (5)$$

where

$$P_{F,t} = \left( s^* (P_{F,t}^{PTM})^{1-\varepsilon} + (1-s^*) (P_{F,t}^{LOP})^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}} \quad (6)$$

$$P_{H,t}^* = \left( s (P_{H,t}^{*PTM})^{1-\varepsilon} + (1-s) (P_{H,t}^{*LOP})^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}$$

and

$$P_{H,t} = \left( \int_0^1 (P_{H,t}(i))^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}, \quad P_{F,t}^* = \left( \int_1^2 (P_{F,t}^*(i))^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}$$

$$\begin{aligned}
P_{F,t}^{PTM} &= \left( (s^*)^{-1} \int_1^{1+s^*} (P_{F,t}^{PTM}(i))^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}, & P_{H,t}^{*PTM} &= \left( s^{-1} \int_0^s (P_{H,t}^{*PTM}(i))^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}} \\
P_{F,t}^{LOP} &= \left( (1-s^*)^{-1} \int_{1+s^*}^2 (P_{F,t}^{LOP}(i))^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}, & P_{H,t}^{*LOP} &= \left( (1-s)^{-1} \int_s^1 (P_{H,t}^{*LOP}(i))^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}
\end{aligned} \tag{7}$$

In equation (5),  $P_{H,t}$  and  $P_{F,t}$  are the prices of Home and Foreign goods in Home currency, and  $P_{H,t}^*$  and  $P_{F,t}^*$  are the prices of Home and Foreign goods in Foreign currency. The import price indexes of Home and Foreign,  $P_{F,t}$  and  $P_{H,t}^*$ , are defined in equation (6), where  $P_{F,t}^{PTM}$  ( $P_{F,t}^{LOP}$ ) is the Home currency price of Foreign PTM (LOP) goods, and  $P_{H,t}^{*PTM}$  ( $P_{H,t}^{*LOP}$ ) is the Foreign currency price of Home PTM (LOP) goods, respectively. For the LOP goods we have

$$P_{H,t} = e_t P_{H,t}^{*LOP}, \quad P_{F,t}^{LOP} = e_t P_{F,t}^*, \tag{8}$$

where  $e_t$  is the nominal exchange rate (units of Home currency per unit of Foreign currency).

With a nominal wage  $W_t$ , the labor income of Home households is  $W_t l_t$ . Home households also receive profits on the ownership of domestic firms,  $\pi_t$ , and lump-sum transfers from the government,  $\tau_t$ . Their budget constraint is given by

$$W_t l_t + \pi_t + \tau_t + M_{t-1} + (1+i_t)P_{t-1}B_t = P_{H,t}C_{H,t} + P_{F,t}C_{F,t} + M_t + P_t B_{t+1}, \tag{9}$$

where  $M_{t-1}$  are nominal money holdings at the beginning of period  $t$ , and  $B_t$  are bonds accumulated during period  $t-1$  and carried over to period  $t$ . These bonds, denominated in Home currency, have a nominal yield  $i_t$ , their real rate of return is  $r_t$  with  $1+r_t = (1+i_t)P_{t-1}/P_t$ . Foreign households budget constraint reads

$$W_t^* l_t^* + \pi_t^* + \tau_t^* + M_{t-1}^* + (1+i_t) \frac{P_{t-1}}{e_t} B_t^* = P_{F,t}^* C_{F,t}^* + P_{H,t}^* C_{H,t}^* + M_t^* + \frac{P_t}{e_t} B_{t+1}^*, \tag{10}$$

where  $B_t^*$  are bond holdings by Foreign households.

Home and Foreign households make consumption decisions, choose nominal balances and set a nominal wage so as to maximize lifetime utility subject to their budget constraint.

The solutions to these maximization problems are:

$$C_{t+1} = \beta(1+r_{t+1})C_t, \quad C_{t+1}^* = \beta \frac{(1+r_{t+1})q_t}{q_{t+1}} C_t^* \quad (11)$$

$$\frac{M_t}{P_t} = \frac{\delta}{1-\delta} \frac{1+i_{t+1}}{i_{t+1}} C_t, \quad \frac{M_t^*}{P_t^*} = \frac{\delta}{1-\delta} \frac{1+i_{t+1}}{1+i_{t+1}-e_{t+1}/e_t} C_t^* \quad (12)$$

$$C_{H,t}(i \in [0,1]) = \alpha \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} \left( \frac{P_{H,t}}{P_t} \right)^{-1} C_t \quad (13)$$

$$C_{F,t}^{PTM}(i \in [1, 1+s^*]) = s^*(1-\alpha) \left( \frac{P_{F,t}^{PTM}(i)}{P_{F,t}^{PTM}} \right)^{-\varepsilon} \left( \frac{P_{F,t}^{PTM}}{P_{F,t}} \right)^{-\varepsilon} \left( \frac{P_{F,t}}{P_t} \right)^{-1} C_t$$

$$C_{F,t}^{LOP}(i \in [1+s^*, 2]) = (1-s^*)(1-\alpha) \left( \frac{P_{F,t}^{LOP}(i)}{P_{F,t}^{LOP}} \right)^{-\varepsilon} \left( \frac{P_{F,t}^{LOP}}{P_{F,t}} \right)^{-\varepsilon} \left( \frac{P_{F,t}}{P_t} \right)^{-1} C_t$$

$$C_{F,t}^*(i \in [1,2]) = \alpha \left( \frac{P_{F,t}^*(i)}{P_{F,t}^*} \right)^{-\varepsilon} \left( \frac{P_{F,t}^*}{P_t^*} \right)^{-1} C_t^*$$

$$C_{H,t}^{*PTM}(i \in [0,s]) = s(1-\alpha) \left( \frac{P_{H,t}^{*PTM}(i)}{P_{H,t}^{*PTM}} \right)^{-\varepsilon} \left( \frac{P_{H,t}^{*PTM}}{P_{H,t}^*} \right)^{-\varepsilon} \left( \frac{P_{H,t}^*}{P_t^*} \right)^{-1} C_t^*$$

$$C_{H,t}^{*LOP}(i \in [s,1]) = (1-s)(1-\alpha) \left( \frac{P_{H,t}^{*LOP}(i)}{P_{H,t}^{*LOP}} \right)^{-\varepsilon} \left( \frac{P_{H,t}^{*LOP}}{P_{H,t}^*} \right)^{-\varepsilon} \left( \frac{P_{H,t}^*}{P_t^*} \right)^{-1} C_t^*$$

$$\frac{W_t}{P_t} = \frac{\phi}{\phi-1} \frac{\sigma}{1-\delta} C_t \cdot l_t, \quad \frac{W_t^*}{P_t^*} = \frac{\phi}{\phi-1} \frac{\sigma}{1-\delta} C_t^* \cdot l_t^* \quad (14)$$

Equations (11) are standard Euler equations describing the optimal intertemporal allocation of consumption. Note that the Foreign real rate of return,  $r_{t+1}^*$ , with  $1+r_{t+1}^* = (1+r_{t+1})q_t/q_{t+1}$  differs from Home real interest rate, if the real exchange rate, defined as  $q_t = e_t P_t^*/P_t$ , moves

from period  $t$  to  $t+1$ . The real interest rate parity holds. The money market equilibrium conditions (12) state that in Home and Foreign real money demand is increasing in consumption and decreasing in the nominal interest rate. Equations (13) describe the consumption demand of Home and Foreign households for all six categories of goods. Equations (14) are the optimal labor supply decisions. It is assumed that each household is monopolistic supplier of a specific type of labor and that all Home (Foreign) firms hire all Home (Foreign) types of labor. Given the negatively-sloped labor demand schedule for its type of labor, each household chooses the utility maximizing point on the labor demand curve. As (14) indicates, the optimal real wage is a mark-up,  $\psi^{-1}$ , on the marginal rate of substitution between leisure and consumption. The parameter  $\psi \equiv 1 - 1/\phi$  measures the degree of labor market competitiveness, and  $\phi (> 1)$  denotes the elasticity of input substitution (see Section 2.2 below).

Aggregating consumption demand over all Home varieties  $i \in [0,1]$  and all Foreign varieties  $i \in [1,2]$  gives world demand for Home and Foreign goods. By assuming symmetry across firms within each type of goods (but, of course, not across different types of goods) all relative prices within each type must be equal to unity. For  $P_{H,t}(i) = P_{H,t}$ ,  $P_{F,t}^{PTM}(i) = P_{F,t}^{PTM}$  etc., Eqs. (13) give the aggregate consumption demand for each type of goods.

## 2.2 Firms

Production of the Home (Foreign) good requires a continuum of differentiated labor inputs which are monopolistically supplied by Home (Foreign) households. The technology is given by  $Y_t^{(\phi-1)/\phi} = \int_0^1 (l_t(j))^{(\phi-1)/\phi} dj$  and  $(Y_t^*)^{(\phi-1)/\phi} = \int_0^1 (l_t^*(j^*))^{(\phi-1)/\phi} dj^*$ , where  $j$  and  $j^*$  indexes Home and Foreign households, respectively. In a symmetric equilibrium, output turns out to be linear in labor:

$$Y_t = l_t, \quad Y_t^* = l_t^* \quad (15)$$

Prices are set in advance and cannot be adjusted to a monetary shock within the period. Firms set prices at a level so as to achieve the optimal mark-up in the absence of such a shock. Let us have a look at Home PTM firms. Since they can segment markets, they have two parameters to maximize profits,  $P_{H,t}(i)$  and  $P_{H,t}^{*PTM}(i)$ . Profits are given by

$$\pi_t(i) = P_{H,t}(i) \cdot C_{H,t}(i) + e_t P_{H,t}^{*PTM}(i) \cdot C_{H,t}^{*PTM}(i) - W_t (C_{H,t}(i) + C_{H,t}^{*PTM}(i)).$$

Focusing on a symmetric equilibrium, the optimal price charged by Home PTM firms to Home residents is a constant mark-up,  $\kappa^{-1}$ , over marginal costs:

$$P_{H,t} = \frac{1}{\kappa} W_t \quad (16)$$

The parameter  $\kappa \equiv 1 - 1/\varepsilon$  serves as an index for product-market competitiveness. The price charged to Foreign residents,  $P_{H,t}^{*PTM}$ , will be set so that its expected Home currency value is a fixed mark-up over marginal costs too. Provided that the price elasticities of demand are the same in each country, the optimal mark-ups will be the same, so that even PTM firms do not price-discriminate across countries. Once prices are set, firms are ready to meet product demand. If there is an unexpected depreciation (appreciation) of the Home currency, profits of Home PTM firms will adjust, i.e. Home exporters of PTM goods get a higher (lower) revenue in domestic currency. It is straightforward to show that the optimal pricing of Home LOP firms also yields  $P_{H,t}$  given by (16). Foreign firms' optimal prices can be derived (and interpreted) in a similar way.

### 2.3 Governments and the Current Account

Home and Foreign government are assumed to use their seignorage income to finance a lump-sum transfer to their residents:  $M_t - M_{t-1} = P_t \tau_t$  and  $M_t^* - M_{t-1}^* = P_t^* \tau_t^*$ . Observing the bond market equilibrium,  $B_t + B_t^* = 0$  for all  $t$ , Home's current account may be written as

$$P_t C_t + P_t B_{t+1} = P_{H,t} C_{H,t} + e_t (1 - \alpha) P_t^* C_t^* + (1 + r_t) P_t B_t. \quad (17)$$

The left-hand side of (17) is period  $t$  expenditure for consumption and bonds, the right-hand side is income arising from product demand, repaid loans and interest payments. Foreign's current account is analogously defined.

### 3. The Solution of the Model

Our focus will be on the impact of permanent unanticipated changes in Home and Foreign money supply. We distinguish between three periods. In the initial period, the economy is in a steady state. In period  $t$ , the monetary shocks occur and we observe the short-run equilibrium which assumes that nominal wages are fixed before the shocks can be observed. In the long run (from period  $t+1$  onward), nominal wages adjust and all variables reach their new steady-state values. To save notation, we hereafter drop time indexes. Variables in the initial equilibrium are denoted by a zero subscript, variables in the new long-run equilibrium are indexed by an upperbar, short-run variables are not indexed.

#### 3.1 The Initial Steady State

In the initial steady state all markets clear in each country. To get a closed-form solution for the aggregate variables, one needs the assumption of zero net foreign assets. For  $B_0 = B_0^* = 0$ , it is straightforward to show that the steady-state levels of output, employment and consumption in Home and Foreign are given by

$$Y_0 = l_0 = C_0 = \sqrt{\frac{\kappa\psi}{\sigma}}, \quad Y_0^* = l_0^* = C_0^* = \sqrt{\frac{\kappa\psi}{\sigma}} \quad (18)$$

and that the equilibrium value for the nominal exchange rate is  $e_0 = M_0/M_0^*$ . The initial steady state is a flexible-price equilibrium, where money is neutral. The less competitive the labor and goods markets (low  $\phi, \varepsilon$ ), the higher are the price mark-ups and the lower are labor and product demand and hence aggregate output, employment and consumption. Note that the social optimum is characterized by perfect labor and goods markets ( $\kappa, \psi \rightarrow 1$ ), where goods prices are equal to marginal costs and where the real wage is equal to the marginal rate of substitution between consumption and leisure.

### 3.2 The Short-run and the Long-run Equilibrium

In period  $t$  a once-and-for-all unanticipated change in Home and/or Foreign money supply occurs. In the case of a monetary expansion we have  $M = \bar{M} \geq M_0$  and/or  $M^* = \bar{M}^* \geq M_0^*$ . For convenience and without loss of generality, we normalize the initial money supply in Home and Foreign to unity:  $M_0 = M_0^* = 1$ .

Table 1 about here

Table 1 presents the general solution of the model. Because of the absence of current account imbalances, money is neutral in the long run. All real variables, i.e., consumption, output, employment and real money supply return to their pre-shock level (see (24) and (25)).

Turning to the short-run equilibrium, Eq. (26) shows that the nominal exchange rate depreciates in proportion to the rise in relative Home money supply and instantaneously jumps to its new steady-state level. There is no short-run over- or undershooting, since due to

our specification of the utility function (1) both the interest rate elasticity and the consumption elasticity of money demand are equal to unity.

Equations (27) describe the import price indices. The change in Home import prices is determined by the Foreign degree of PTM,  $s^*$ . For  $s^* = 0$ , all Foreign exporters pre-set their prices in Foreign currency, and due to the law of one price a depreciation of the Home currency forces them to raise their prices in Home currency. As  $s^*$  takes on positive (and higher) values, the price pressure gets milder, since the prices of the imported PTM goods are pre-set in Home currency and cannot be adjusted to the variation of the exchange rate. When  $s^* = 1$ , the depreciation leaves the Home currency price of Home imports unchanged.

The increase in Home import prices worsens Home terms of trade, but this may be offset by an increase in its export prices. Depending on the Home degree of PTM,  $s$ , we observe two polar cases. When  $s = 0$ , prices are pre-set in Home currency, and the law of one price now forces Home exporters to lower their price in the Foreign currency one-to-one with the exchange rate. In this case, the Home currency price of Home exports does not change. If  $s = 1$ , Home exporters pre-set their prices in Foreign currency,  $P_H^*$  remains unchanged, and a depreciation raises the Home currency price of Home exports in proportion to the movement in the exchange rate. For intermediate values of  $s$ , the increase in the exchange rate exceeds the decrease in the Foreign currency price of Home exports, implying that the Home currency price of Home exports goes up.

The net effect on Home's terms of trade is given by Eq. (28). When  $s + s^* < 1$ , the terms of trade worsen, the increase in the Home currency export prices does not offset the increase in Home import prices. For  $s + s^* > 1$ , the terms of trade improve, and for  $s + s^* = 1$ , the change in export prices equals the rise in import prices, so that the terms of trade do not move. Eq. (28) replicates an important result already obtained by Betts and Devereux (2000a,b) and extends its application to the case of country specific degrees of PTM: the sign of the terms-



of-trade response to an increase in relative Home money supply crucially depends on the sum of Home's and Foreign's degree of PTM.

The increase in Home import prices contributes to domestic inflation. The smaller the share of Foreign goods in Home consumer price index,  $1 - \alpha$ , and the larger the fraction of Foreign PTM firms,  $s^*$ , the smaller is the increase in  $P$  (see Eq. (29)). Put different, the degree of Home price stickiness is increasing in the degree of Foreign PTM. In the case of full Foreign PTM,  $s^* = 1$ , the import prices and thus the consumer price index does not change at all (remember that  $P_H$  is assumed to be fix). Analogously, the depreciation of the Home currency constitutes a deflationary bias in Foreign, which is maximized at  $s = 0$ . As Home's degree of PTM,  $s$ , gets larger, the fraction of firms lowering their prices in Foreign currency and thus the decrease in  $P^*$  gets smaller see Eq. (29)). For  $s = 1$ , a Home monetary expansion does not alter Foreign's consumer price index. From these price effects we can immediately conclude that the depreciation of the short-run real exchange rate and thus the short-run deviation from PPP is increasing in the degrees of PTM (see Eq. (30)). With full PTM in both countries ( $s = s^* = 1$ ), the short-run real exchange rate moves one-to-one with the nominal exchange rate. If  $s$  and/or  $s^*$  is smaller than one, the correlation between real and nominal exchange rates is increasing in the home bias parameter, since with a home bias consumer prices are more insulated (see Warnock, 2003). Only in the polar case of no PTM ( $s = s^* = 0$ ) and identical preferences ( $\alpha = 1/2$ ), the real exchange rate does not go up, i.e. the increase in the nominal exchange rate is just offset by the decline in relative prices.

The increase in real balances in both Home and Foreign generates a boost in product demand. Since world product demand (world consumption) moves in proportion to world real money supply, the size of the demand shock very much depends on the pricing policy of firms. Defining the world real money supply,  $M_W$ , as geometric average of Home and Foreign real money supply, using  $1/2$  as weights, we get from (23):

$$M_W = kz \cdot (M)^{\frac{1+(1-\alpha)(s^*-s)}{2}} (M^*)^{\frac{1+(1-\alpha)(s-s^*)}{2}}. \quad (32)$$

The world money average increases in  $M$  with elasticity  $[1+(1-\alpha)(s^*-s)]/2$ , which is maximized when a constant Home price index  $P$  ( $s^*=1$ ) is coupled with a fall in the Foreign price index  $P^*$  ( $s=0$ ). On the other hand, the demand effect reaches a minimum for  $s^*=0$  and  $s=1$ , where now an increase in  $P$  is coupled with a constant  $P^*$ . In the Betts/Devereux-scenario of identical degrees of PTM in each country,  $s=s^*$ , the elasticity equals  $1/2$ , and PTM does not affect the increase in world product demand.

Concerning the splitting of the world demand effect between Home and Foreign consumption note first that, irrespective of  $s$  and  $s^*$ , Home nominal consumption expenditure,  $PC$ , moves in proportion with Home money supply and that Foreign consumption expenditure,  $P^*C^*$ , does not depend on  $M$ . Since Home households spend a constant fraction of total expenditures for Home goods,  $P_H C_H = \alpha PC$ , and  $P_H$  is assumed to be constant, Home consumption of domestic goods,  $C_H$ , rises. The impact on  $C_H$  does not depend on PTM, but is of course increasing in the home bias (see (19)). The remaining fraction  $1-\alpha$  of the increase in nominal consumption expenditure is spent for higher imports:  $P_F C_F = (1-\alpha)PC$ . As stressed above, when there is no Foreign PTM ( $s^*=0$ ), all Foreign firms are forced to raise their price in Home currency, so that the higher expenditure for Foreign goods is completely absorbed by higher Home import prices  $P_F$ . Home consumption of Foreign goods,  $C_F$ , does not alter (see (20)). As the fraction of Foreign LOP firms declines, i.e.  $s^*$  takes on positive values, the price effect gets milder leading to a positive  $C_F$ -effect. At full Foreign PTM, Home import prices are constant forcing up the consumption effect to a maximum. Summarizing, as can be seen from (21), the impact of a Home monetary expansion on overall Home consumption  $C$  is increasing in  $s^*$ , whereas  $s$  does not matter.

What are the effects of a Home monetary shock on Foreign consumption? Such a shock has no impact on total Foreign consumption expenditure,  $P^*C^*$ , and because of  $P_F^*C_F^* = \alpha P^*C^*$  no impact on Foreign expenditure for their own goods,  $P_F^*C_F^*$ . From the assumption of a constant  $P_F^*$  follows a constant consumption level  $C_F^*$ , (see (19)). With respect to Foreign demand for Home goods, we know from  $P_H^*C_H^* = (1-\alpha)P^*C^*$  that the consumption of Home goods increases if and only if the price of Home goods in terms of Foreign currency,  $P_H^*$ , declines. This, in turn, needs a positive fraction of Home LOP firms, since at full Home PTM ( $s = 1$ ) all Home export prices are pre-set in Foreign currency fixing  $P_H^*$  and leading to a constant  $C_H^*$ . The lower Home PTM,  $s$ , the larger the fall in  $P_H^*$  and the larger the boost in Home's exports to Foreign (see (20)). As a result, the impact of a Home monetary expansion on overall Foreign consumption  $C^*$  is decreasing in  $s$ , whereas  $s^*$  does not matter (see (21)).

Short-run output and employment in the Home economy unambiguously boost because of higher Home demand for domestic goods and higher exports to Foreign. As just described, the latter are falling in  $s$ , so that the impact of a Home monetary innovation on domestic output and employment is decreasing in Home PTM (see (22)). Similarly, by pushing up Home demand for Foreign goods, the monetary expansion at Home increases Foreign output and employment, with the only exception of no Foreign PTM where the increase in import prices leaves no room for an increase in real demand for Foreign goods (see (22)).

#### 4. Welfare-maximizing monetary policy

In this section we ask for the design of an optimal monetary policy. Due to the distortions in the labor and the goods markets and due to terms-of-trade effects a surprise monetary expansion may raise welfare in the country where it takes place. The sign of the welfare spill-

over on its neighbor(s) is a priori unknown, all three cases are possible depending on the degrees of PTM. In Section 4.1 we assume policy makers in Home and Foreign who follow a policy of maximizing welfare of domestic households, taking as given the monetary stance abroad. The solution of the two reaction functions is the world Nash equilibrium discussed in Section 4.2. In Section 4.3 we derive the implications of our model for the coordination of international monetary policies by comparing the world Nash equilibrium with the cooperative equilibrium.

#### 4.1 Welfare Effects

The policy problem faced by Home monetary authorities is to maximize the intertemporal utility function (1),  $V$ :

$$V = (1 - \delta) \ln C + \delta \ln \frac{M}{P} - \frac{\sigma}{2} (l)^2 + \frac{\beta}{1 - \beta} \left[ (1 - \delta) \ln \bar{C} + \delta \ln \frac{\bar{M}}{\bar{P}} - \frac{\sigma}{2} (\bar{l})^2 \right], \quad (33)$$

where use has been made of the assumption that from period  $t + 1$  onwards the economy is in the new steady state. We assume that the monetary authorities are able to commit to pre-announced rules. For a more detailed discussion of this issue see Corsetti and Pesenti (2001b). Eq. (23) shows that real balances are in proportion to consumption,  $M / P = kC$ . Thus we can replace the term  $(1 - \delta) \ln C + \delta \ln(M / P)$  by  $\ln C + \delta \ln k$  in (33) in order to get the welfare effects of a Home monetary expansion as

$$\frac{\partial V}{\partial M} = \frac{1}{C} \frac{\partial C}{\partial M} - \sigma \frac{\partial l}{\partial M}, \quad \frac{\partial V^*}{\partial M} = \frac{1}{C^*} \frac{\partial C^*}{\partial M} - \sigma^* \frac{\partial l^*}{\partial M} \quad (34)$$

Observing the expressions given in Table 1, (34) takes the form

$$\frac{\partial V}{\partial M} \Big|_{M=M^*=1} = (1 - s(1 - \alpha)) [1 - \kappa\psi] + (1 - \alpha) [s + s^* - 1] \quad (35)$$

$$\left. \frac{\partial V^*}{\partial M} \right|_{M=M^*=1} = s^* (1 - \alpha) [1 - \kappa\psi] - (1 - \alpha) [s + s^* - 1], \quad (36)$$

where the derivatives have been evaluated at the initial steady state. The properties of (35) and (36) are summarized in

**Proposition 1:** (i) *There is a critical share  $\underline{s}$  of Home PTM firms, given by*

$$\underline{s} \equiv \frac{\kappa\psi - \alpha - s^* (1 - \alpha)}{\kappa\psi (1 - \alpha)}, \text{ such that for any } s > \underline{s} \text{ a Home monetary expansion raises Home}$$

*welfare. When } s < \underline{s}, \text{ a monetary expansion is beggar-thyself, Home welfare decreases.}*

(ii) *There is a critical share  $\bar{s}$  of Home PTM firms, given by  $\bar{s} \equiv 1 - s^* \kappa\psi$ , such that for any } s < \bar{s} \text{ (} s > \bar{s}) \text{ Foreign welfare is increasing (decreasing) in a Home monetary expansion.}*

(iii) *If and only if Home PTM is at an intermediate range, that is if } \underline{s} < s < \bar{s}, \text{ welfare will raise in both countries.}*

**Proof:** (i) The multiplier  $\partial V / \partial M$  is monotonically increasing in  $s$ , and the substitution of  $\underline{s}$  for  $s$  in (35) yields  $\partial V / \partial M = 0$ . (ii) Likewise, the multiplier  $\partial V^* / \partial M$  is monotonically decreasing in  $s$ , and the substitution of  $\bar{s}$  for  $s$  in (36) yields  $\partial V^* / \partial M = 0$ . (iii) From  $\kappa\psi \leq 1$  follows the relation  $\underline{s} \leq \bar{s}$  which, given the proofs of part (i) and (ii), is a necessary and sufficient condition for the existence of a range for  $s$ , where both countries benefit.

As indicated by (35) and (36), the overall welfare effects of a Home monetary impulse stem from two sources, namely monopolistic distortions in the goods and/or labor markets ( $1 - \kappa\psi > 0$ ) and a terms-of-trade effect ( $s + s^* \neq 1$ ). Notice that a home bias is no distortion in a welfare theoretic sense constituting an inefficiency. Concerning market imperfections, it holds that the less competitive goods and labor markets, the lower are consumption and output

in the initial steady state, and the higher is the welfare benefit of a given increase in aggregate product demand. The welfare benefit from higher consumption exceeds the disutility of more employment (less leisure). Turning to the terms-of-trade effect, we note - once again - that its sign depends on the sum of the PTM degrees. Furthermore, the terms-of-trade effect matters more in a more open economy (low home bias). For  $s + s^* > 1$ , Home observes an improvement in its terms of trade strengthening the welfare benefit stemming from higher product demand. Foreign, on the other hand, observes a worsening in its terms of trade mitigating the welfare benefit. The reverse is true for  $s + s^* < 1$ .

Concerning the overall welfare effect, the terms-of-trade effect may offset the aggregate demand effect. For full Home PTM ( $s = 1$ ), the positive demand effect is coupled with an improvement in Home terms of trade, Home welfare unambiguously rises. As  $s$  declines, the improvement in the terms of trade is getting smaller and turns into a worsening for  $s + s^* < 1$ . The increase in Home consumption does not depend on Home PTM, but as  $s$  declines, a larger fraction of Home firms is forced to cut their prices in Foreign currency, so that Foreign households demand more Home goods raising Home production (employment) and lowering Home's welfare benefit. If  $s$  falls short of the threshold  $\underline{s}$ , the (negative) terms-of-trade effect exceeds the demand effect, the Home monetary expansion is beggar-thyself.

The impact of a Home monetary expansion on Foreign welfare is given by Eq. (36). With full Home PTM, monetary policy is a beggar-thy-neighbor instrument. When  $s = 1$ , the Foreign import price index does not decline implying that Foreign consumption is unaffected by a monetary expansion in the Home economy. Foreign output and employment, however, go up because of a higher demand by Home households. Foreign faces a welfare loss. As  $s$  declines, the output and employment effect does not alter, but the strength of the terms-of-trade effect is diminishing, so that Foreign consumption increases. For  $s < \bar{s}$ , the sign of the welfare spill-over effect turns into positive.

So we have to distinguish between three scenarios. For  $s < \underline{s}$ , Home does not have an incentive to boost its money supply, since it would be beggar-thyself. The second scenario is  $\underline{s} < s < \bar{s}$ , where a Home monetary expansion is beneficial for both countries. In the extreme case of perfect competition on labor and product markets this scenario collapses ( $\underline{s} = \bar{s} = 1 - s^*$ ), since monetary policy does not fill any output gap, but is reduced to its distributional implications via the terms of trade effect. For  $s > \bar{s}$ , we have a positive impact on Home but a negative impact on Foreign welfare. This case emphasizes the strategically interdependence of monetary policy, it opens up a rationale for competitive depreciations.

In order to assess the empirical relevance of the critical shares  $\underline{s}$  and  $\bar{s}$ , it is appropriate to run a quantitative calibration exercise. We set the degree of product market and labor market distortions as to yield a labor supply, which in the initial steady state deviates from the distortion free level of employment by 7%. This number is slightly above the US-American and slightly below the unemployment rate in the Euro area. The required value is  $\kappa\psi = 0.865$ . Estimates of the Foreign degree of PTM can be derived from estimates of the degree of exchange rate pass-through into import prices, since import prices increase in the exchange rate with elasticity  $1 - s^*$  (see (27)). Campa and Goldberg (2002) provide such estimates across countries and report that the United States has relatively low pass through ( $s^* = 0.74$ ). The Euro area, approximated by a non-weighted average of Germany, France and Italy, has a medium degree of pass through ( $s^* = 0.4$ ), whereas Japan has high pass through ( $s^* = 0.12$ ). Similar results are also found by Brauer (2003).

Table 2 about here

As Table 2 indicates, in the US-American case of a high degree of Foreign PTM the critical  $\underline{s}$  is negative, a monetary expansion in the US would be prosper-thyself. This holds

for identical preferences ( $\alpha = 0.5$ ) and of course even stronger in case of a home bias. Moreover, we can expect that a US monetary expansion is prosper-thy-neighbor: the critical share  $\bar{s} = 0.36$  is far above the values found by, for instance, Marston (1990), Knetter (1993) and Brauer (2003), who consistently report low PTM by US exporters. If, however, 'Home' is assumed to be the Euro area, the results are less clear-cut. At least for a low home bias (large terms-of-trade effect), the critical share  $\underline{s}$  is in the range of estimated PTM by Euro exporters (for Germany, for instance, Knetter (1993) estimates  $s = 0.36$ ). So a monetary expansion in the Euro area is likely to be beneficial for the Euro area, but a beggar-thyself outcome can not be ruled out. Given the large value of  $\bar{s} = 0.65$ , a monetary impulse by the European Central Bank probably generates a positive welfare externality to its neighbors. Lastly, consider the Japanese case. Assuming a PTM of about 60 percent for Japanese exporters, which is in line with the studies by Marston (1990), Parsley (1993) and Gagnon and Knetter (1995)<sup>7</sup>, it is likely that a Japanese monetary expansion would be beggar-thyself and prosper-thy-neighbor. In summary, our calculations suggest that a win-win situation is the rule, but for a wide range of plausible parameter constellations we will observe a beggar-thyself outcome.

## 4.2 World Nash Equilibrium

In this section we derive the optimal monetary stances in a world Nash equilibrium. Both countries are assumed to behave optimally by maximizing welfare of domestic households, taking as given the monetary stance abroad. The optimal monetary stance is defined by the point where the marginal utility of the increase in short-run consumption equals the marginal

disutility of the increase in short-run employment:  $\frac{1}{C} \frac{\partial C}{\partial M} = \sigma l \frac{\partial l}{\partial M}$  for Home and

$\frac{1}{C^*} \frac{\partial C^*}{\partial M^*} = \sigma l^* \frac{\partial l^*}{\partial M^*}$  for Foreign. By observing the expressions in Table 1, these first-order

conditions can be rearranged to



$$\alpha + s^*(1-\alpha) = \kappa\psi \left[ \alpha M + (1-\alpha)(M^*)^s M^{1-s} \right] \left[ \alpha M + (1-\alpha)(1-s)(M^*)^s M^{1-s} \right] \quad (37)$$

$$\alpha + s(1-\alpha) = \kappa\psi \left[ \alpha M^* + (1-\alpha)M^s (M^*)^{1-s} \right] \left[ \alpha M^* + (1-\alpha)(1-s^*)M^{s^*} (M^*)^{1-s^*} \right]. \quad (38)$$

The world Nash equilibrium is the solution of the reaction functions, Eq. (37) for Home and Eq. (38) for Foreign. These functions indicate that Home and Foreign monetary stances are strategic substitutes, the optimal response to a monetary expansion in Foreign is a contraction in Home vice versa. In general, a Foreign expansion boosts both Home consumption and Home employment. This, however, affects the marginal utility and the marginal disutility of a change in Home's money supply asymmetrically, the former remains constant<sup>8</sup>, whereas the latter increases. To restore an optimum, a Home monetary contraction is required. The only case in which Home does not react to a change in  $M^*$  is when  $s = 0$ . In the case of no Home PTM, Home employment does not respond to Foreign policies, the first-order condition for the optimal monetary stance in Home is fulfilled at the same level of  $M$ . Analogously, Foreign does not respond to a change in  $M$  when there is no Foreign PTM ( $s^* = 0$ ).

A closed form solution for the world Nash equilibrium can not be derived for all parameter constellations, we are thus restricted to some special cases and numerical simulations. Three scenarios allow for a solution in closed form: firstly, the Betts/Devereux-scenario of identical degrees of PTM in each country, secondly, the degrees of PTM in Home and Foreign are different but sum up to unity, and thirdly, no PTM in at least one country. Let us start with the case of perfect symmetry,  $s = s^*$ . Not surprisingly, the first-order conditions are simultaneously fulfilled if and only if Home and Foreign take the same monetary stance:  $M = M^*$ . Plugging this result into (37) (or (38)) leads to the optimal monetary stance in a world Nash equilibrium:

$$M^N \Big|_{s=s^*} = (M^*)^N \Big|_{s=s^*} = \sqrt{\frac{1-\alpha+s(1-\alpha)}{\kappa\psi(1-s(1-\alpha))}}. \quad (39)$$

With an optimal policy in place, Home production (employment) and consumption are:

$$Y^N \Big|_{s=s^*} = C^N \Big|_{s=s^*} = \sqrt{\frac{1-\alpha+s(1-\alpha)}{\sigma(1-s(1-\alpha))}}. \quad (40)$$

Inserting (39) and (40) into (33) gives Home welfare as

$$V^N \Big|_{s=s^*} = \ln \sqrt{\frac{1-\alpha+s(1-\alpha)}{\sigma(1-s(1-\alpha))}} - \frac{1-\alpha+s(1-\alpha)}{2(1-s(1-\alpha))} + \frac{\delta}{1-\beta} \ln k + \frac{\beta}{1-\beta} \left[ \ln \bar{C} - \frac{\sigma}{2} (\bar{l})^2 \right]. \quad (41)$$

The same expressions hold for Foreign production, consumption and welfare. The properties of (39) - (41) are stated in

**Proposition 2:** *Suppose that Home and Foreign have identical degrees of PTM. Then*

(i) *in a world Nash equilibrium the optimal monetary stance,  $M^N$ , is increasing in the degree of PTM and decreasing in the competitiveness of the labor and product markets.*

(ii) *the Nash optimal monetary policy offsets the distortions due to monopolistic product and labor markets completely.*

(iii) *Home and Foreign welfare are bell-shaped in the degree of PTM with a maximum at  $s = 1/2$ .*

(iv) *For  $s = 1/2$ , the Nash optimal monetary policy supports the first-best allocation (social optimum)  $Y^{so} = C^{so} = \sqrt{1/\sigma}$ .*

**Proof:** (i) immediately follows from the inspection of (39), (ii) follows from the absence of  $\kappa$  and  $\psi$  in (40). For  $s = 1/2$ , we get  $\partial V^N / \partial s = 0$  from (41), which proofs part (iii). To proof part (iv), insert  $s = 1/2$  into (40).

As explained above, each country's incentive to expand its money supply is increasing in the degree of PTM. Thus, both countries optimally choose a higher money supply, the higher is  $s$ . For  $s > 1/2$ , each country generates a negative welfare spill-over by attempting to

improve its terms of trade at the expense of its neighbor. In a symmetric equilibrium, however, where both countries are assumed to be identical with respect to the degree of PTM, the equilibrium terms of trade are independent of the monetary stance (see (28) with  $M = M^*$ ), so that there will only be a too expansive policy (inflationary bias). When  $s < 1/2$ , a similar line of reasoning leads to the conclusion that the Nash optimal monetary stance will have a deflationary bias. This kind of spill-over reaches a maximum in the polar cases of no and full PTM. For  $s = 1/2$ , there is no terms-of-trade effect even at the national level and hence neither a deflationary nor an inflationary bias.

In a symmetric equilibrium it is not possible to use the terms-of-trade effect strategically. Thus, there is no trade off between the aims of closing the output gap and improving the terms of trade. This in turn opens up the possibility – and this is, of course, utility maximizing – to pursue a policy that closes the output gap completely (see Part (ii) of Proposition 2). In the case  $s = 1/2$ , the Nash optimal monetary stance leads to the first-best allocation because of the absence of any inflationary or deflationary bias.

Now consider the case where the PTM parameters in Home and Foreign are allowed to be different but sum up to unity:  $s + s^* = 1$ . In this case the reaction functions (37) and (38) deliver the world Nash optimal monetary stance in Home and Foreign as

$$M^N \Big|_{s+s^*=1} = (M^*)^N \Big|_{s+s^*=1} = \sqrt{\frac{1}{\kappa\psi}}. \quad (42)$$

With an optimal policy in place, Home consumption, output (employment) and welfare are:

$$C^N \Big|_{s+s^*=1} = Y^N \Big|_{s+s^*=1} = \sqrt{\frac{1}{\sigma}} \quad (43)$$

$$V^N \Big|_{s+s^*=1} = \ln \sqrt{\frac{1}{\sigma}} - \frac{1}{2} + \frac{\delta}{1-\beta} \ln k + \frac{\beta}{1-\beta} \left[ \ln \bar{C} - \frac{\sigma}{2} (\bar{l})^2 \right]. \quad (44)$$

These results establish

**Proposition 3:** *Suppose that the PTM parameters in Home and Foreign sum up to unity. Then the world Nash optimal monetary stance*

*(i) is identical for both countries, and*

*(ii) always supports the first-best allocation.*

Provided that there is no terms-of-trade effect at the aggregate level, i.e.  $s + s^* = 1$ , it does not matter which country experiences a higher degree of PTM in its economy. Once again, there is no trade off between the aims of closing the output gap and improving the terms of trade.

Next consider the case, where there is no Home PTM (and an arbitrary degree of Foreign PTM). For  $s = 0$ , the first-order condition (37) delivers the Nash optimal monetary stance in Home as

$$M^N|_{s=0} = \sqrt{\frac{\alpha + s^*(1-\alpha)}{\kappa\psi}}. \quad (45)$$

This leads to

**Proposition 4:** *Suppose that there is no PTM in Home. Then the Nash optimal monetary stance in Home is increasing in the degree of PTM in Foreign.*

**Proof:** From examination of (45).

Due to a lower increase in its import prices and thus a lower reduction in its terms of trade, the marginal utility of a higher Home money supply is increasing in the Foreign degree of PTM,  $s^*$  (see footnote 8). For  $s^* = 1$ , the terms-of-trade effect vanishes, and Home's optimal monetary stance implements the first-best allocation.

For the general case we have to resort to numerical simulations of the model. Based on experimentation with alternative parameter values we establish

- Proposition 5:** (i) *If Home exhibits a "low" or "high" degree of PTM, i.e. for  $s < s^*$  (if  $s + s^* < 1$ ) respectively  $s > s^*$  (if  $s + s^* > 1$ ) the optimal monetary stance in Home is higher than its Foreign counterpart. For medium degrees of Home PTM,  $M^N$  falls short of  $(M^*)^N$ .*
- (ii) *If  $M^N$  exceeds (is lower than)  $(M^*)^N$ , Home consumption and output exceeds (is lower than) Foreign consumption and output.*
- (iii) *If the degree of Home PTM is greater than or equal to the degree of Foreign PTM,  $s \geq s^*$ , Home welfare will be greater than or equal to Foreign welfare,  $V \geq V^*$ . If  $s \leq s^*$  then  $V \leq V^*$ .*

To gain some intuition let us start with  $s = s^*$ , where we know from Eqs. (39) - (41) that Home and Foreign are identical with respect to the optimal money supply, consumption, output and welfare. Moreover, the higher the degree of PTM, the higher consumption and output, and the lower is the marginal utility of consumption and the higher is the marginal disutility of employment. Now suppose that this equilibrium is distorted by an increase in the degree of Home PTM,  $s$ . How does such a distortion affect the first-order conditions for the optimal monetary stances in Home and Foreign? In Home, the marginal utility of a higher  $M$  remains constant (note from (21) that  $C$  does not depend on  $s$ ), whereas due to lower exports to Foreign and thus due to a lower output effect the marginal disutility of a higher  $M$  goes down. As a consequence, a higher  $s$  forces  $M^N$  up. In Foreign, the output effect of a higher Foreign money supply  $M^*$  is independent of  $s$  (see (22)), but because of higher imports from Home, the rise in  $C^*$  is increasing in  $s$  (see (21)). As a result,  $(M^*)^N$  goes up too. For  $s + s^* > 1$ , we have a high level of consumption and output (employment) in the initial

equilibrium, so that the drop in the marginal disutility of Home employment is quite large, whereas the increase in the marginal utility of money in Foreign is quite low. The increase in  $M^N$  dominates the increase in  $(M^*)^N$ , relative money supply shifts in favor of Home. For  $s + s^* < 1$ , however, the initial level of consumption and output is low implying a low reduction in the marginal disutility of Home employment and a high increase in the marginal utility of Foreign money. In this case, the increase in  $M^N$  falls short of the increase in  $(M^*)^N$ , relative money supply shifts in favor of Foreign.

Part (ii) of Proposition 5 immediately follows from (21) and (22): the economy with a higher relative money supply experiences both higher consumption and higher employment (output). In terms of welfare, we get the result that in a world Nash equilibrium, relative welfare is unambiguously determined by the degrees of PTM. A country which exhibits a higher degree of PTM than its neighbor will come up with a higher (or at least the same) welfare level.

### 4.3 Cooperative Equilibrium

In this section we discuss the issue of international monetary coordination: are there any welfare gains from cooperation, and if so, how are they related to the degrees of PTM? Suppose that the two monetary authorities sign a binding contract on a cooperative policy. Policy makers agree on a single social welfare function which they jointly maximize. Since Home and Foreign are assumed to be symmetric, it is most natural to maximize the average of the national welfare levels:

$$V^W = \frac{1}{2}(V + V^*). \quad (46)$$

Moreover, due to the symmetry assumption, Home and Foreign exert the same influence on world welfare, so that both countries will implement the same monetary stance:  $M = M^*$ . To

derive the optimal cooperative policy, insert the expressions from Table 1 and maximize (46) with respect to  $M$  (or, equivalently, with respect to  $M^*$ ). This delivers the optimal cooperative policy as

$$M^{coop} = (M^*)^{coop} = \sqrt{\frac{1}{\kappa\psi}}. \quad (47)$$

In a cooperative equilibrium Home consumption, output (employment) and welfare are:

$$Y^{coop} = C^{coop} = \sqrt{\frac{1}{\sigma}} \quad (48)$$

$$V^{coop} = \ln \sqrt{\frac{1}{\sigma} - \frac{1}{2}} + \frac{\delta}{1-\beta} \ln \frac{\delta}{(1-\delta)(1-\beta)} + \frac{\beta}{1-\beta} \left[ \ln \bar{C} - \frac{\sigma}{2} (\bar{l})^2 \right]. \quad (49)$$

The properties of (47) - (49) are stated in

**Proposition 6:** *In a cooperative equilibrium the optimal monetary policy*

(i) *is independent of the degrees of PTM, and*

(ii) *supports the first best allocation, so that*

(iii) *there is always a welfare gain from cooperation,  $V^{coop} > V^N$ , the only exception being  $s + s^* = 1$ , where  $V^{coop} = V^N$ .*

**Proofs:** Part (i) and part (ii) follow from (47) and (49) by inspection. Part (iii) is obtained by the comparison of (44) with (49).

When countries cooperate, they internalize the welfare spillovers. As aforementioned, for  $s + s^* > 1$ , each country generates a negative welfare spill-over by attempting to improve its terms of trade at the expense of its neighbor. But the equilibrium terms of trade are independent of the monetary stance, so that there will only be a too expansive policy (inflationary bias). When  $s + s^* < 1$ , the Nash optimal monetary stance will have a

deflationary bias. Monetary coordination improves welfare by eliminating these biases. This result is in stark contrast to Corsetti and Pesenti (2001b) and Devereux and Engel (2003), who argue that there will be no gain from cooperation in the cases of no and full PTM. For  $s + s^* = 1$ , there are no terms-of-trade effects and thus no welfare spill-overs. In such a case monetary coordination is of no consequence (Betts and Devereux, 2000a).

## 5. Conclusions

The key message of our paper is that both the magnitude of the degree of pricing to market and its asymmetry between countries is decisive for the transmission mechanism and the welfare effects of monetary policy. In particular, these parameters are decisive for the question whether monetary policy is beggar-thy-neighbor or beggar-thyself. By comparing non-cooperative and cooperative optimal monetary policies we find, firstly, that there is always a gain from cooperation, and secondly, that the gain reaches a maximum at the polar cases of no and full pricing to market since in these cases the movement in the terms of trade and thus the welfare spill-over is at a maximum in the non-cooperative setting.

Our framework can be extended in several ways, for instance by a less restrictive preference specification. Most prominent is the role of the cross-country elasticity of substitution. Assuming a value different from unity would imply long-term effects of monetary policy via current-account imbalances opening up interesting interactions with various distortions like monopolistic competition on goods and labor markets or imperfections concerning the financial market structure. Further research will show whether the second-order approximation technique put forward by Sutherland (2004) overcomes the problem of obtaining exact and explicit solutions when the cross-country elasticity of substitution differs from unity.



Another promising line of research is the joint determination of optimal fiscal and monetary policy. Steps in this direction have been taken by Schmitt-Grohé and Uribe (2004b). If policymakers were able to neutralize the monopolistic competition distortions by subsidies on wages and production, the flexible price equilibrium would be efficient, and monetary policy could be used for some other objectives. Stochastic open economy models à la Obstfeld and Rogoff (2000, 2002) take up this idea and analyze how alternative monetary policy *rules* perform in mitigating demand, supply and liquidity-preference shocks. If, however, the government has no access to non-distortionary instruments, we will be back in a second-best world where the design of optimal policies will be an exciting subject of further research.

Table 1: Solution of the model

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Determinants of Home and Foreign welfare

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Short-run consumption

$$C_H = z\alpha M \qquad C_F^* = z\alpha M^* \qquad (19)$$

$$C_F = z(1-\alpha)M^{s^*} (M^*)^{1-s^*} \qquad C_H^* = z(1-\alpha)M^{1-s} (M^*)^s \qquad (20)$$

$$C = zM^{1-(1-\alpha)(1-s^*)} (M^*)^{(1-\alpha)(1-s^*)} \qquad C^* = z(M^*)^{1-(1-\alpha)(1-s)} M^{(1-\alpha)(1-s)} \qquad (21)$$

Short-run output and employment

$$Y = l = z(\alpha M + (1-\alpha)M^{1-s} (M^*)^s) \qquad Y^* = l^* = z(\alpha M^* + (1-\alpha)M^{s^*} (M^*)^{1-s^*}) \qquad (22)$$

Short-run real money supply

$$\frac{M}{P} = kC \qquad \frac{M^*}{P^*} = kC^* \qquad (23)$$

Long-run consumption, output and employment

$$\bar{C} = \bar{Y} = \bar{l} = z \qquad \bar{C}^* = \bar{Y}^* = \bar{l}^* = z \qquad (24)$$

Long-run real money supply

$$\frac{\bar{M}}{\bar{P}} = kz \qquad \frac{\bar{M}^*}{\bar{P}^*} = kz \qquad (25)$$


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Prices

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$$e = \bar{e} = M / M^* \qquad \text{Nominal exchange rate} \qquad (26)$$

Home and Foreign import price index

$$P_F = \frac{1}{kz} \left( \frac{M}{M^*} \right)^{1-s^*} \qquad P_H^* = \frac{1}{kz} \left( \frac{M}{M^*} \right)^{s-1} \qquad (27)$$

$$tot \equiv \frac{eP_H^*}{P_F} = \left( \frac{M}{M^*} \right)^{s+s^*-1} \quad \text{Short-run terms of trade} \quad (28)$$

Home and Foreign consumer price index

$$P = \frac{1}{kz} \left( \frac{M}{M^*} \right)^{(1-\alpha)(1-s^*)} \quad P^* = \frac{1}{kz} \left( \frac{M}{M^*} \right)^{-(1-\alpha)(1-s)} \quad (29)$$

$$q \equiv \frac{eP^*}{P} = \left( \frac{M}{M^*} \right)^{1-(1-\alpha)(2-s-s^*)} \quad \text{Short-run real exchange rate} \quad (30)$$

$$\overline{tot} = \overline{q} = 1 \quad \text{Long-run tot and real exchange rate} \quad (31)$$

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where  $z \equiv \sqrt{\kappa\psi/\sigma}$  and  $k \equiv \frac{\delta}{(1-\delta)(1-\beta)}$

Table 2: Critical shares for Home degree of PTM

$\kappa\psi = 0.865$									
	$s^* = 0.74$			$s^* = 0.4$			$s^* = 0.12$		
$\alpha$	0.5	0.6	0.7	0.5	0.6	0.7	0.5	0.6	0.7
$\underline{s}$	-0.01	-0.09	-0.22	0.38	0.30	0.17	0.71	0.63	0.50
$\bar{s}$	0.36			0.65			0.90		

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<sup>1</sup> Corsetti and Pesenti (2001b) take up a similar approach by allowing the degrees of PTM to differ across countries. They consider the case in which exporters pre-set prices in foreign currency but are able to modify them after observing exchange-rate changes. The pass-through elasticity is assumed to be constant and exogenous to the model. This approach, however, appears inconsistent with working from first principles. If there are no menu costs or the like impeding a price change, the profit-maximizing response to an exchange-rate change is a complete pass-through. In other words, the assumption of incomplete pass-through contradicts with the assumption of profit-maximizing firms.

<sup>2</sup> In Schmitt-Grohé and Uribe (2004a) and Sutherland (2004) a second-order approximation of the welfare measure is derived in order to analyze the case of a cross-country elasticity different from unity.

<sup>3</sup> For a derivation of loss functions typically assumed in the literature on monetary policy evaluation that are grounded in the welfare of private agents see Woodford (2002).

<sup>4</sup> The empirical literature indicates that the degree of PTM differs between both industries and countries (see for instance Marston, 1990; Goldberg and Knetter, 1997; and Campa and Goldberg, 2002). We take the degrees of PTM as given, i.e. we do not endogenise the currency of price setting. This issue is discussed in Taylor (2000), Aizenman (2004), and Devereux et al. (2004).

<sup>5</sup> Since we will focus on symmetric equilibria, where all households are identical within a country, we omit any household index and interpret all variables in both per-capita and aggregate terms.

<sup>6</sup> The benchmark estimates are in the range [1,2]. Many studies, see for instance Chari et al. (2000) and Smets and Wouters (2002), set the elasticity at 1.5. More recent studies, however, suggest that the price elasticities of imports and exports have declined over time (Marquez, 1999). According to Hooper and Marquez (1995) the median value of the estimates for Germany, the United Kingdom and Japan is 0.6. Bergin (2004) gets the result that the elasticity of substitution between home and foreign goods is not significant different from unity. So a very plausible range for this elasticity is [0.5,1.5], and the unitary assumption falls right in the middle.

<sup>7</sup> For a critical review of this literature see Kikuchi and Sumner (2002).

<sup>8</sup> The marginal utility of a higher  $M$  is  $\frac{1}{C} \frac{\partial C}{\partial M} = (\alpha + s^* (1 - \alpha)) \frac{1}{M}$ , it does not depend on  $M^*$ .