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## Estimation Procedure for "Price Formation in Call Auctions with Insider Information"

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**LEAF Working Paper Series**

No. 21-01

**Estimation Procedure for “Price Formation in Call Auctions with  
Insider Information”**

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# Estimation Procedure for “Price Formation in Call Auctions with Insider Information”

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This note provides the details of the estimation procedure in Brüner (2019). In Section 2 we derive the posterior distribution. Section 3 describes the MCMC algorithm used to obtain draws from the posterior distribution and in Section 4 we present the method for our model check. We conclude with a presentation of the estimation results.

JEL Classification: D82, G14, C11

Keywords: transaction cost, call market, asymmetric information, Bayesian econometrics, Monte Carlo Markov Chain algorithm

## 1 Introduction

Brüner (2019) considers an asset market organized as a sequence of call auctions. The asset’s true value at time  $t$  follows a random walk with drift parameter  $\mu$  and news shock  $\varepsilon_t$ , which is normally distributed with variance  $\sigma^2$  and is uncorrelated over time. There is a continuum of buyers, a continuum of sellers, and one insider. With probability  $\lambda$ , the insider learns the sign of  $\varepsilon_t$  prior to call auction  $t$ . With probability  $1 - \lambda$ , the insider does not receive a signal and she trades based on her (random) liquidity needs. Let  $q_t$  denote a trade indicator for call auction  $t$ , such

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that  $q_t = 1$ , if there are more buy orders than sell orders,  $q_t = -1$ , if there are more sell orders than buy orders, and  $q_t = 0$ , if there are equal numbers of buy and sell orders. In such a call auction market, asset price returns  $r_t$  follow the mixture model

$$r_t = \mu + \frac{S}{2} \cdot (q_t - q_{t-1}) + \zeta_{t-1} \cdot q_{t-1} \cdot \varepsilon_{t-1} | (\varepsilon_{t-1} > 0) + (1 - \zeta_{t-1}) \cdot \varepsilon_{t-1}, \quad (1)$$

where, for each  $t$ , the latent variable  $\zeta_t$  equals 1, if the insider received a signal about the direction of  $\varepsilon_t$ , and zero otherwise. In this mixture model specification,  $\lambda$  is a hyperparameter that represents the probability that  $\zeta_t = 1$ . The parameter  $S$  can be interpreted as a measure of transaction costs.

The prior distributions of the unknown parameters  $\mu$ ,  $S$ ,  $\lambda$  and  $\sigma$  are chosen to be noninformative. The joint prior of  $\mu$ ,  $S$  and  $\sigma$  is  $p(\mu, S, \sigma) \propto \sigma^{-2}$  and  $\lambda \sim \text{beta}(1, 1)$ .

Since the posterior distribution is not analytically tractable, we use numerical techniques to draw inferences.

## 2 Derivation of the Posterior Distribution

### Likelihood

Let  $\zeta = (\zeta_1, \zeta_2, \dots, \zeta_T)$  denote the vector of latent variables that indicate whether the potential insider received a signal in auction  $t$  or not. The likelihood of observing return  $r_t$  given the parameters  $\mu$ ,  $S$ ,  $\sigma$ ,  $\zeta$ ,  $\lambda$  is

$$\begin{aligned} p(r_t | \mu, S, \sigma, \zeta, \lambda) &= \left( \frac{2}{\sqrt{2\pi} \sigma} \exp\left\{-\frac{(q_{t-1}(r_t - \mu - \frac{S}{2}(q_t - q_{t-1})))^2}{2\sigma^2}\right\} \right)^{\zeta_{t-1}} \\ &\quad \times \left( \frac{1}{\sqrt{2\pi} \sigma} \exp\left\{-\frac{(r_t - \mu - \frac{S}{2}(q_t - q_{t-1}))^2}{2\sigma^2}\right\} \right)^{(1-\zeta_{t-1})} \\ &= 2^{\zeta_{t-1}} \frac{1}{\sqrt{2\pi} \sigma} \exp\left\{-\frac{(r_t - \mu - \frac{S}{2}(q_t - q_{t-1}))^2}{2\sigma^2}\right\}. \end{aligned}$$

The joint likelihood of the return vector  $r = (r_2, r_3, \dots, r_T)$  is then given by

$$p(r | \mu, S, \sigma, \zeta, \lambda) = \prod_{t=2}^T 2^{\zeta_{t-1}} \frac{1}{\sqrt{2\pi} \sigma} \exp\left\{-\frac{(r_t - \mu - \frac{S}{2}(q_t - q_{t-1}))^2}{2\sigma^2}\right\}.$$

### Priors

We assume an noninformative (and improper) joint prior for  $\mu$ ,  $S$  and  $\sigma$  :

$$p(\mu, S, \sigma) \propto \sigma^{-2}.$$

We further assume that *a priori*  $\lambda$  follows a beta distribution. The density of the beta distribution is given by

$$p(x; \alpha, \beta) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1}.$$

For the prior of  $\lambda$  we choose  $\alpha = \beta = 1$ , such that the beta distribution equals a uniform distribution on  $[0, 1]$ . Given  $\lambda$  the prior for  $\zeta$  is

$$p(\zeta|\lambda) = \prod_{t=2}^T \lambda^{\zeta_{t-1}} (1-\lambda)^{1-\zeta_{t-1}} = \lambda^h (1-\lambda)^{T-1-g-h},$$

where  $h = \sum_{t=2}^T \zeta_{t-1}$  and  $g = \sum_{t=2}^T q_{t-1}^2$ .

## Posterior

Multiplying the likelihood of the return vector with the prior densities we get the posterior density up to a normalizing constant:

$$\begin{aligned} p(\mu, S, \sigma, \zeta, \lambda|r) &\propto \sigma^{-2} \lambda^h (1-\lambda)^{T-1-g-h} \prod_{t=2}^T 2^{\zeta_{t-1}} \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{(r_t - \mu - \frac{S}{2}(q_t - q_{t-1}))^2}{2\sigma^2}\right\} \\ &\propto \sigma^{-(T+1)} \lambda^h (1-\lambda)^{T-1-g-h} 2^h \exp\left\{-\frac{\sum_{t=2}^T (r_t - \mu - \frac{S}{2}(q_t - q_{t-1}))^2}{2\sigma^2}\right\}. \end{aligned}$$

This posterior density is not analytically tractable and, therefore, we need to use numerical tools to draw inference about the parameters of interest.

## 3 Obtaining Draws from the Posterior Distribution

We simulate draws from the posterior distribution using a Gibbs sampler. As the parameters  $\zeta_1, \zeta_2, \dots, \zeta_T$ , and  $S$  are correlated, they are drawn in one batch (along with  $\mu$ ) with the Metropolis-Hastings (M-H) algorithm (Gelman et al., 2003). A single iteration of the Gibbs sampler proceeds as follows:

1. Draw  $\mu, S, \zeta|\lambda, \sigma, r$  with a M-H algorithm. The proposal density of  $\tilde{\mu}, \tilde{S}$  at the  $j$ -th step of the M-H algorithm is  $N((\mu^{j-1}, S^{j-1}), V)$ , where  $V$  is the inverse of the negative of the Hessian. The conditional posterior probability of  $\zeta_{t-1} = 1$  given  $\lambda$  is

$$P(\zeta_{t-1} = 1|\lambda, r) = \frac{p(\zeta_{t-1} = 1|\lambda, r)}{p(\zeta_{t-1} = 1|\lambda, r) + p(\zeta_{t-1} = 0|\lambda, r)} = \frac{2\lambda}{2\lambda + (1-\lambda)}.$$

The joint proposal density for a move from  $\{\mu, S, \zeta\}$  to  $\{\tilde{\mu}, \tilde{S}, \tilde{\zeta}\}$  is

$$\begin{aligned} q(\{\mu, S, \zeta\}, \{\tilde{\mu}, \tilde{S}, \tilde{\zeta}\} | \lambda, \sigma, r) &= (2\pi)^{-1} |V|^{-1/2} \\ &\times \exp\left\{\frac{-1}{2}((\tilde{\mu}, \tilde{S}) - (\mu, S))V^{-1}((\tilde{\mu}, \tilde{S}) - (\mu, S))'\right\} \\ &\times \left(\frac{2\lambda}{2\lambda + (1-\lambda)}\right)^{\tilde{h}} \left(\frac{1-\lambda}{2\lambda + (1-\lambda)}\right)^{T-1-g-\tilde{h}}. \end{aligned}$$

The proposal is accepted with probability  $\min\{\alpha(\{\mu, S, \zeta\}, \{\tilde{\mu}, \tilde{S}, \tilde{\zeta}\}), 1\}$ , where  $\alpha(\{\mu, S, \zeta\}, \{\tilde{\mu}, \tilde{S}, \tilde{\zeta}\})$

$$\begin{aligned} &= \frac{\lambda^{\tilde{h}}(1-\lambda)^{T-1-g-\tilde{h}}2^{\tilde{h}} \exp\left\{-\frac{\sum_{t=2}^T (r_t - \tilde{\mu} - \tilde{S}/2(q_t - q_{t-1}))^2}{2\sigma^2}\right\}}{\lambda^h(1-\lambda)^{T-1-g-h}2^h \exp\left\{-\frac{\sum_{t=2}^T (r_t - \mu - \frac{S}{2}(q_t - q_{t-1}))^2}{2\sigma^2}\right\}} \\ &\times \frac{\exp\left\{-1/2((\mu, S) - (\tilde{\mu}, \tilde{S}))V^{-1}((\mu, S) - (\tilde{\mu}, \tilde{S}))'\right\}}{\exp\left\{-1/2((\tilde{\mu}, \tilde{S}) - (\mu, S))V^{-1}((\tilde{\mu}, \tilde{S}) - (\mu, S))'\right\}} \\ &\times \frac{\left(\frac{2\lambda}{2\lambda+(1-\lambda)}\right)^h \left(\frac{1-\lambda}{2\lambda+(1-\lambda)}\right)^{T-1-g-h}}{\left(\frac{2\lambda}{2\lambda+(1-\lambda)}\right)^{\tilde{h}} \left(\frac{1-\lambda}{2\lambda+(1-\lambda)}\right)^{T-1-g-\tilde{h}}} \\ &= \frac{\exp\left\{-\frac{\sum_{t=2}^T (r_t - \tilde{\mu} - \tilde{S}/2(q_t - q_{t-1}))^2}{2\sigma^2}\right\}}{\exp\left\{-\frac{\sum_{t=2}^T (r_t - \mu - \frac{S}{2}(q_t - q_{t-1}))^2}{2\sigma^2}\right\}}. \end{aligned}$$

2. Draw  $\lambda | \mu, S, \zeta, \sigma, r$  from a  $\text{beta}(h+1, T-1-g-h+1)$  distribution.
3. Draw  $\sigma | \mu, S, \zeta, \lambda, r$  with a M-H algorithm. The proposal density of a move from  $\sigma$  to  $\tilde{\sigma}$  is the scaled inverse  $\chi^2$ -distribution:

$$q(\sigma, \tilde{\sigma}) = \frac{((T-1)/2)^{(T-1)/2}}{\Gamma((T-1)/2)} \sigma^{T-1} \tilde{\sigma}^{-(T+1)} \exp\left\{-\frac{(T-1)\sigma^2}{2\tilde{\sigma}^2}\right\}.$$

The probability that proposal  $\tilde{\sigma}$  is accepted is then given by  $\min\{\alpha(\sigma, \tilde{\sigma}), 1\}$ , where

$$\begin{aligned} \alpha(\sigma, \tilde{\sigma}) &= \frac{\tilde{\sigma}^{-(T+1)} \exp\left\{-\frac{\sum_{t=2}^T (r_t - \mu - \frac{S}{2}(q_t - q_{t-1}))^2}{2\tilde{\sigma}^2}\right\}}{\sigma^{-(T+1)} \exp\left\{-\frac{\sum_{t=2}^T (r_t - \mu - \frac{S}{2}(q_t - q_{t-1}))^2}{2\sigma^2}\right\}} \\ &\times \frac{\tilde{\sigma}^{T-1} \sigma^{-(T+1)} \exp\left\{-\frac{(T-1)\tilde{\sigma}^2}{2\sigma^2}\right\}}{\sigma^{T-1} \tilde{\sigma}^{-(T+1)} \exp\left\{-\frac{(T-1)\sigma^2}{2\tilde{\sigma}^2}\right\}} \\ &= \frac{\exp\left\{-\frac{\sum_{t=2}^T (r_t - \mu - \frac{S}{2}(q_t - q_{t-1}))^2}{2\tilde{\sigma}^2}\right\} \tilde{\sigma}^{T-1} \exp\left\{-\frac{(T-1)\tilde{\sigma}^2}{2\sigma^2}\right\}}{\exp\left\{-\frac{\sum_{t=2}^T (r_t - \mu - \frac{S}{2}(q_t - q_{t-1}))^2}{2\sigma^2}\right\} \sigma^{T-1} \exp\left\{-\frac{(T-1)\sigma^2}{2\tilde{\sigma}^2}\right\}}. \end{aligned}$$

## 4 Model Choice

We test our model against the alternative that  $\lambda = 0$  and, consequently, all  $\zeta_{t-1}$ , for  $t = 2, \dots, T$ , are zero. Let model  $M_1$  denote the unrestricted model ( $\lambda > 0$ ) and model  $M_2$  denote the restricted ( $\lambda = 0$ ) model. All parameters of model  $M_i$ ,  $i \in \{1, 2\}$  are grouped into the set  $\theta_i$ . The posterior odds ratio of the two models is

$$\frac{Pr(M_1|y)}{Pr(M_2|y)} = \frac{Pr(M_1) \int p(r|\theta_1, M_1) p(\theta_1|M_1) d\theta_1}{Pr(M_2) \int p(r|\theta_2, M_2) p(\theta_2|M_2) d\theta_2}.$$

We assume equal prior probabilities for the two models. Thus, the posterior odds ratio is the ratio of the marginal likelihoods of the two models, also called the Bayes factor. Note that the marginal likelihood is the (inverse of) the normalizing constant and therefore, in order to find the model that best explains the data, we need to calculate the ratio of two normalizing constants. From the many methods available to tackle this problem, we choose iterative bridge sampling proposed by Meng and Wong (1996).

Let  $w_{ij}$  for  $j = 1, \dots, n_i$  be the generated draws of model  $i$  and  $s_i = n_i/(n_1 + n_2)$ ,  $i \in \{1, 2\}$ . Define  $l_{ij} = q_1(w_{ij})/q_2(w_{ij})$ , where  $q_i(\cdot)$  is the (unnormalized) posterior density of model  $i$ . Meng and Wong (1996) show that the following iterative method converges to the desired ratio of normalizing constant, and hence, the posterior odds ratio:

$$\hat{r}_O^{(t+1)} = \frac{\frac{1}{n_2} \sum_{j=1}^{n_2} \left( \frac{l_{2j}}{s_1 l_{2j} + s_2 \hat{r}_O^{(t)}} \right)}{\frac{1}{n_1} \sum_{j=1}^{n_1} \left( \frac{1}{s_1 l_{1j} + s_2 \hat{r}_O^{(t)}} \right)} \quad (2)$$

## 5 Results

Table 1 shows the results using 20 stocks traded in pure call auction markets at Euronext Paris in 2006. The results are based on 3000 iterations of the Gibbs sampler, where the first 1000 iterations were discarded. The results for the Bayes factor (B) provide clear support for the unrestricted model.

Table 1: Estimation results. (Standard deviations in parentheses.)

Name	$S$	$\lambda$	$\sigma$	$\mu$	B	$P(\lambda > 0)$
AES Chemunex	3.036 (0.253)	0.129 (0.049)	3.676	0.307 (0.146)	613.89	1.00
Altarea	1.092 (0.048)	0.084 (0.048)	0.516	0.103 (0.0201)	66.04	0.99
Artois Nom.	2.217 (0.126)	0.05 (0.038)	1.383	0.306 (0.082)	17.81	0.95
Banque Tarneaud	0.837 (0.051)	0.104 (0.048)	0.711	0.06 (0.030)	340.74	1.00
Banque de la Reunion	1.076 (0.020)	0.639 (0.046)	0.434	0.025 (0.018)	> 1000	1.00
Cambodge Nom.	3.241 (0.358)	0.053 (0.043)	2.758	0.652 (0.224)	10.41	0.91
Credit Foncier Communal d'Alsace	1.052 (0.068)	0.041 (0.038)	0.972	0.013 (0.045)	23.02	0.96
Chauffage Urbain	5.447 (0.671)	0.068 (0.061)	2.898	0.08 (0.378)	5.50	0.85
Cofitem	0.994 (0.062)	0.149 (0.055)	0.772	0.042 (0.03)	464.99	1.00
Credit Agricole Centre Loire	0.403 (0.035)	0.101 (0.047)	0.464	-0.035 (0.019)	485.45	1.00
Credit Agricole Oise	0.93 (0.035)	0.446 (0.04)	0.638	-0.039 (0.023)	> 1000	1.00
Distriborg	4.703 (1.457)	0.182 (0.118)	6.402	0.938 (0.785)	14.71	0.94
Exacompta Clairefontaine	1.816 (0.099)	0.144 (0.067)	1.019	0.052 (0.041)	93.03	0.99
Finatis	2.133 (0.172)	0.092 (0.057)	1.79	0.129 (0.09)	35.78	0.97
Fonciere des Murs	1.182 (0.091)	0.13 (0.05)	1.197	-0.046 (0.033)	65.71	0.99
Fromageries Bel	1.444 (0.062)	0.198 (0.051)	0.916	0.066 (0.041)	> 1000	1.00
Grand Marnier	1.155 (0.118)	0.084 (0.053)	0.935	0.075 (0.042)	27.95	0.97
Hotels et Casino de Deauville	2.519 (0.318)	0.434 (0.161)	1.515	0.181 (0.131)	154.50	0.99
Icade Fonciere des Pimonts	0.761 (0.072)	0.079 (0.057)	0.728	0.09 (0.039)	52.59	0.98
Institut de Participations de l'Ouest	0.977 (0.069)	0.2 (0.066)	0.779	0.029 (0.019)	429.05	1.00
Average	1.8507	0.1704	1.5251	0.1513		



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