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**Metropolitan structures**

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Working Paper

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## Metropolitan Structures

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# Metropolitan Structures

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*February 17, 2020*

**Abstract:** How does the structure of metropolitan areas affect urbanization and welfare? Using the development of urban sub-centers in France in the 1970s, I study the short- and long-term effects of urban structural changes. To retrieve within metropolitan area effects, I exploit the fact that out of 11 planned sub-centers, only 9 have actually been developed. Using local population data between 1926 and 2015, I observe a polarization of growth in favor of sub-centers. As local gains might be offset by losses elsewhere, I develop a general equilibrium model to investigate global effects. Overall, the observed shift towards polycentric metropolitan structures leads to an increased urbanization (by about 900,000 inhabitants in 2015) and a 0.7% national welfare growth.

**Keywords:** Metropolitan areas, urbanization, structure, growth, mobility.

**JEL classification:** R11, R13, R58.

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*Cities should be built in the countryside, the air is healthier.*

— J.-L.-A. Commerson (1860)<sup>1</sup>

## 1 Introduction

The share of population living in urban centers is increasing globally, from 16% in 1900, and 50% in 2000, to about 68% in 2050 (Goldewijk, Beusen, and Janssen, 2010; United Nations, 2014). To accommodate the new urban population, many countries have planned and developed urban sub-centers within existing metropolitan areas; and hence, encouraging more polycentric metropolitan structures.<sup>2</sup> *New towns* or *satellite cities* are typical examples of such development. This global shift towards polycentricity has been recognized in the literature, as Pfister, Freestone, and Murphy (2000) put it: "Much recent literature in urban studies, geography, and planning portrays an inexorable evolution toward polycentricity as a new "postmodern" metropolitan form".

As shown in Figure 1a, the development of 772 such sub-centers has been registered worldwide since 1945.<sup>3</sup> Urban sub-centers have emerged on all continents, with important clusters in northern Europe, the eastern coast of the United States, India and north-east China. Furthermore, Figure 1b reveals that sub-centers in developing countries are fairly recent developments. Since 2000, 249 sub-centers have been developed in China, India, the Arabian Peninsula and across the African continent. Despite clear national differences, the long-run analysis of metropolitan structures in the developed world can provide insights on the future of metropolitan areas in the developing world.

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<sup>1</sup>Authors' own translation.

<sup>2</sup>Polycentric refers to a metropolitan area with multiple centers, as opposed to a monocentric metropolitan area where density is a monotonically decreasing function of distance to the metropolitan center in all directions. See Mori (2016) for a detailed discussion of the literature on these metropolitan structures.

<sup>3</sup>Count based on the International New Town Institute registry. Also, see Merlin (1991) and Fouchier (1999) for a discussion of this development in France and England, Shaw (1995) in India, Sorensen (2001) in Japan, Mason and Nigmatullina (2011) in Russia, among others national examples. Consult Delouvrier (1965) for a typical urban plan planning urban sub-centers.

Figure 1: Locations of urban sub-centers worldwide



Source: International New Town Institute. Accessed on: Oct 9<sup>th</sup>, 2019.

Link: <http://www.newtowninstitute.org/newtowndata/>.

Urbanization is a key driver of labor productivity, and ultimately, of economic growth. Approximately, 80% of global gross domestic product (GDP) is generated in cities (United Nations, 2014; Grubler and Fisk, 2012). The 2020 Global Economic Prospects report by the World Bank<sup>4</sup> lists urbanization as one of the main source of economic growth worldwide. Beyond mere labor productivity, urbanization is also likely to influence policies tackling climate change as it affects both commuting and density patterns. Hence, investigating whether (and if, how) various metropolitan structures affect urbanization in the short- and long-term is of great importance for both researchers and policy makers.

Despite the fact that the emergence of urban sub-centers constitutes possibly the most striking development of urban structures worldwide since World War II, the effect of such policies remains unclear. On the one hand, polycentric metropolitan structures offer the key advantage of restraining density growth, and subsequent congestion costs, in existing urban centers. Local urban plans, e.g., Washington (1961), Stockholm (version of 1958), Paris (1965), Strategy for the South East (1967) in England, Moscou (1973), among many others, have highlighted the reduction in central density as a main motivation for the development of sub-centers (Merlin and O’Callaghan, 1990). On the other hand, the effect of polycentric metropolitan structures on location-to-location distances, and hence, on commutes and agglomeration economies, is ambiguous. If we ignore possible sorting effects, the average commute length and duration is likely to increase if the metropolitan area stretches between several centers. However, employment sorting

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<sup>4</sup>World Bank (2020)

across different sub-centers can improve the spatial match between residents and workers. This is known as the "co-location hypothesis" (see, among others, [Gordon, Kumar, and Richardson, 1989](#)). Hence, polycentric metropolitan structures could, in theory, shorten the average location-to-location distance. Consequently, the effect of polycentric structures will (partly) depend on the share of individuals enjoying a better spatial match ([Cervero and Wu, 1998](#); [Aguilera, 2005](#)). In a nutshell, the attractiveness of polycentric metropolitan structures is then likely to depend on which force dominates between *lower congestion costs* following lower population density and better spatial matches, and *lower agglomeration economies* due to possible longer commutes and smaller productive spillovers. For this reason, this paper draws particular attention to the impact of urban sub-centers on sorting and spillover effects, both locally and globally.

Previous studies that have analyzed the effects of polycentric metropolitan structures on urbanization suffer from important limitations. First, at the local level, the existing evaluations suffer from the absence of proper control groups. I tackle this issue by exploiting the fact that not all planned sub-centers in France were eventually built. This allows me to retrieve local effects using an event study approach. Second, the scope of existing evaluations is narrow in two dimensions: (i) the focus is *solely* on local effects, when local gains might clearly be offset by losses elsewhere; (ii) only parts of the equilibrium components are analyzed (e.g., transport networks, land use, ...), when interaction effects might dominate in equilibrium. I mend these limitations by developing a structural model that I calibrate to fit French urban areas in order to analyze the global effects of urban sub-centers.

The analysis proceeds sequentially. I first investigate local effects, i.e. within metropolitan areas. I exploit the fact that, out of 11 planned sub-centers, only 9 have actually been developed, to construct a suitable control group. I argue that the decision to abandon the construction of 2 sub-centers was motivated solely by exogenous overall financial considerations, and that the selection of the 2 was as good as random. Before the reform, all initially selected sites display similar population trends and similar urban characteristics. I compare the demographic evolution of municipalities in a planned and built sub-center to those in sub-centers that were *only* planned. As outcome, I investigate population growth, as well the



demographic and economic composition of the sub-centers. The availability of local population data between 1926 and 2015 allows us to estimate local effects every 7 to 10 years.

In a second step, I turn to the global effects of developing polycentric structures on urbanization and welfare. I develop and apply a spatial quantifiable general equilibrium framework of the French economy with heterogeneous individuals, multiple economic sectors, amenity and productivity spillovers, as well as commuting and migration costs. Both productivity and amenity spillovers are individual type specific. Productivity consists of three parts: one that is endogenous and a function of past productivity; one that is endogenous and a function of spillovers from other places; and one that is exogenous and captures the local fundamentals to productivity, e.g., geography or productivity-related public investments. Similarly, local amenities are determined by three components: one that is endogenous and a function of past local amenities; one that is endogenous and a function of spillovers from other places; and one that is exogenous and captures the local fundamentals to amenities, e.g., as geography or amenity-related public investments.

Consistent with the literature, I find important local effects. Municipalities in developed sub-centers experience a very large growth (about 9 times larger growth on average by 2015), and are more likely to attract families and workers from the manufacturing sector. However, comparing other municipalities in the same metropolitan areas to their counterpart in metropolitan areas without sub-centers, I find that population growth in the first ones was about twice smaller by 2015. Thus, it indicates a displacement *within metropolitan areas* of the population growth towards sub-centers. Given the observed local effects, the overall effect of sub-centers on urbanization is then ambiguous.

Turning to a more structure approach to lift the ambiguity, I compare the observed urban strategy in the 9 largest French metropolitan areas between 1968 and 2015 to the urban strategy that would have taken place without intervention (i.e., the extrapolation of the pre-1968 trends in land use). I find that the construction of urban sub-centers across France's largest metropolitan areas increased the national urbanization share. The population opportunity gain is particularly large for the metropolitan areas of Paris, Lyon and Rouen. Overall, I estimate

that an additional 1 million individuals moved to the targeted metropolitan areas by 2015 due to the development of polycentric structures. I also show that these individuals came mostly from rural areas, and not from non-targeted metropolitan areas. These population patterns are associated to a 0.7% growth in national welfare by 2015. In equilibrium, positive spillovers effects (both local amenity and local productivity) dominate the increase in residential-to-workplace commuting costs.

The paper proceeds as follows. Before introducing the institutional and historical context around the development of urban sub-centers in France in Section 3, I briefly discuss the related literature in Section 2. In Section 4, I present reduced-form evidence of the effect of sub-centers on the urban population growth and composition. I present the spatial general equilibrium framework, calibrate it and present counterfactual analyses in Section 5. Section 6 concludes.

## 2 Literature review

This paper contributes to several strands of the literature. It directly relates to the urban literature on the internal structure of cities. The theoretical research on the endogenous within city structure dates back to Fujita and Ogawa (1980), Fujita and Ogawa (1982) and Imai (1982). These papers endogenise the location of both firms and workers throughout the city. Land use patterns emerge endogenously following the trade-off between productive spillovers and commutes. Lucas and Rossi-Hansberg (2002) extend these models to allow for substitutability between land and labor for firms and between housing and other goods for consumers. More recently, empirical research on metropolitan structures has emerge (see, among others, Muniz, Garcia-Lopez, and Galindo, 2008; Muniz and Garcia-Lopez, 2010; Garcia-Lopez, 2012). Ahlfeldt, Redding, Sturm, and Wolf (2015) analyzes in both a reduced-form and a general equilibrium framework how diffusion processes (of amenity and productivity) affects the internal structure of a city. Adopting a broader look, Monte, Redding, and Rossi-Hansberg (2018) recently proposed an approach which analyzes jointly the within and between metropolitan area effects, as also done in this paper. Michaels, Rauch, and Redding (2012) discuss the link between urbanization and structural transformation, which underly the

mechanisms modeled in the present paper.

Beyond economics, the pros and cons of various metropolitan structures have been the topic of an important body of research in urban geography and urban planning, with the limitations discussed in the introduction. Among many others, let us highlight the contributions of [Gordon and Richardson \(1996\)](#) and [Pfister et al. \(2000\)](#) which question the apparent consensus in these literatures about the "inexorable evolution toward polycentricity" ([Pfister et al., 2000](#)). Even though very different from this paper in their approach and methodology, they share the starting point of this paper, i.e. the benefits of polycentric metropolitan structures might not be as straightforward as advertised.

Concerning the methodology, this paper follows a strategy similar to [Kline and Moretti \(2014\)](#). Sub-center planning is similar to regional development programs in the sense that they both have important local effects, but that they both require employing a more structure approach to understand their effects on economic activity as a whole. More generally, this paper relates to both the literature on event study designs ([Fuest, Peichl, and Siegloch, 2018](#); [Schmidheiny and Siegloch, 2019](#)) and the literature on quantifiable spatial general equilibrium models in an urban context ([Ahlfeldt et al., 2015](#)).

The discussion about metropolitan structures also relates to the literature on city size and natural city boundaries. On top of administrative boundaries and metropolitan statistical areas (US definition)/functional urban areas (EU definition), different strategies to delineate cities have been proposed ([Holmes and Lee, 2010](#); [Rozenfeld, Rybski, Andrade, Batty, Stanley, and Makse, 2008](#)). This paper delineates agglomerations using the City Clustering Algorithm (CCA) as in [Rozenfeld, Rybski, Gabaix, and Makse \(2011\)](#) for reasons detailed in Section 3. Furthermore, the possible benefits of building urban sub-centers relates closely to the question of the optimal city size ([Au and Henderson, 2006](#); [Desmet and Rossi-Hansberg, 2013](#); [Albouy, Behrens, Robert-Nicoud, and Seeger, 2019](#)).

Finally, metropolitan structures and transport networks are naturally connected. [Baum-Snow \(2007\)](#), [Garcia-Lopez, Hemet, and Viladecans-Marsal \(2017b\)](#) and [Garcia-Lopez, Hemet, and Viladecans-Marsal \(2017a\)](#), among others, show how transport connections (public or private) shape the structure of an urban area. [Garcia-Lopez et al. \(2017a\)](#), in particular, analyze the consequences of the

RER on the polycentricity of Paris’ metropolitan area. Overall, this strand of the literature finds that transport nodes attracts density and that more transport connections lead to a more decentralized urban structure.

### 3 Institutional and historical background

**Delineating metropolitan areas** — To analyze urban structures, it is essential to move beyond administrative boundaries and focus on metropolitan areas. The recent urban literature offers mainly two alternative methods to delineate such areas. First, statistical areas (e.g., Metropolitan Statistical Areas in the United-States, Functional Urban Areas in the European Union, ...) are commonly defined by the various national statistical offices using both population counts and commutes. A central location reaching a minimum population threshold is used as starting points. All other locations (e.g., counties or municipalities) with a share of residence-workplace commutes to the central location above a fixed threshold are then included in the metropolitan area. Unfortunately, this method is often ill-suited to analyze urban sub-centers. Imagine the case of a relatively large sub-center. This sub-center could qualify to become the central location of a new metropolitan area; and therefore, not be included in the original metropolitan area.

In this paper, I adopt the second alternative to delineate such areas which does not suffer for the same potential problem. Namely, I employ the City Clustering Algorithm (CCA) approach which defines metropolitan area as a maximally connected cluster of populated sites, following [Rozenfeld et al. \(2008\)](#) and [Rozenfeld et al. \(2011\)](#). As cells, I use municipal centroids. Note that data at a lower level in 2015 would be available, hence, permitting a finer definition of metropolitan borders. However, the historical data used in this paper for both the reduced form and structural approaches is at the municipal level. Hence, metropolitan borders finer than the municipalities would not add any information to the analysis. Using 2015 data (i.e., end of the period analyzed), I use as starting point for the algorithm the centroid of the municipality giving it’s name to the metropolitan area (e.g., municipality of Paris for the Paris Metropolitan Area). Then, I recursively grow the cluster by adding all municipalities within a radius  $r$  with a population

density,  $D$ , larger than a threshold  $D^*$ . The cluster stops growing when there is no municipality within  $r$ km of the cluster with population density  $D > D^*$ . In the analysis, I take  $r = 10$ km as this is the minimum distance needed to "reach" another municipality out of Marseille; and  $D^* = 5$  hab/km<sup>2</sup> as this is the average density at 100km from the 9 urban centers considered. I do so for the 9 largest metropolitan areas in France (i.e., Paris, Marseille, Lyon, Lille, Toulouse, Bordeaux, Nantes, Strasbourg and Rouen).

**The emergence of sub-centers around Paris** — In the early 1960s, the French government and central planners expected important growth in the French metropolitan areas. Paris' population was expected to be multiplied by 1.65 (from 8.4 to 14 millions), the sum of commutes would at least triple and the number of housing units would rise from 3.2 to 6 millions (Delouvrier, 1965). Even without population growth, the spatial extent of Paris was expect to increase as the average number of rooms available per capita in the region was a third lower than in other similar sized foreign metropolitan areas (e.g., London or New-York).<sup>5</sup>

To answer this expected growth, the 1965 urban plan for the metropolitan area of Paris (S.D.A.U.R.P., Delouvrier, 1965) introduced the first urban sub-centers in France in the modern era. Precisely, Delouvrier (1965) advocates that "to answer the long-term demographic growth [...], 150.000 ha of new building land are necessary"<sup>6</sup> concentrated in specific locations (i.e., sub-centers) to avoid large urban sprawl and preserve natural spaces. These sub-centers have taken the name of "villes nouvelles" (i.e., direct translation of the English "New Towns"). In total, 7 sub-centers were planned in Paris' metropolitan area.

Intuitively, the central planners thought of these sub-centers as new neighborhoods outside of the city, in the sense that they were neither specialized productive clusters nor pure residential centers. Table 1 reveals how the first 63ha were allocated in 1969. A quarter of the new building land was to be used for residential purposes, 11% for commercial uses and 14% for public services. Whereas a significant share of local residents were expected to work in the sub-centers, the majority of workers still commuted outside of the sub-center towards Paris' center. In 1991,

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<sup>5</sup>As Delouvrier (1965) puts it: "Even if the metropolitan area of Paris did not increase by a single inhabitant, its spatial expansion was unavoidable".

<sup>6</sup>Authors' own translation.

even after the completion of important investments in public and private transport infrastructure which I detail below, only 15% of regional employment was within 30min by car of the sub-centers, and only 4% within 30min by public transport.<sup>7</sup>

Table 1: Planned land use in Paris' sub-centers (1969)

Usage	Land used	Share of total land used
Residential area	16,036 ha	25.5%
Commercial area	6,654 ha	10.6%
Public services	8,535 ha	13.6%
Planned amenities	12,878 ha	20.5%
Green spaces	4,575 ha	7.3%
Transport land use	6,596 ha	10.5%
Rest	7,542 ha	12.0%

Figure 2(a) displays the location of these sub-centers around Paris city center. Red (yellow) municipalities are located within a (non-)realized sub-center. White municipalities are other municipalities in the metropolitan area but outside any planned sub-center. Moreover, to avoid large increase in commuting times and distances, the construction of sub-centers is often combined with major transport infrastructure investments. In France, the development of sub-centers in the metropolitan area of Paris was coupled with the construction of the first French suburban rapid transit system ("Réseau Express Régional (RER)" in French). Figure 2(b) reveals the location of these RER lines. The general equilibrium framework in this paper accounts for the availability of such public transit system when modeling commuting costs.

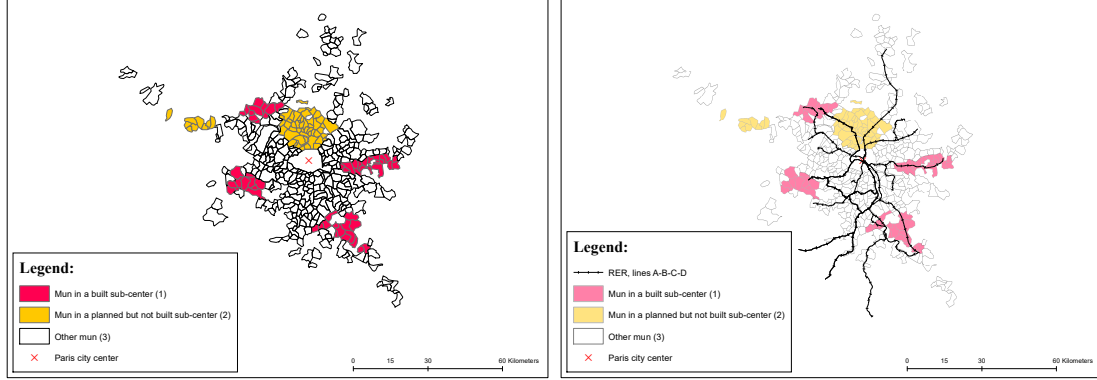
When construction of the sub-centers started in 1969, the costs incurred by the State were revealed larger than expected. As an example, the public investments for the sole sub-center of Evry amounted to 4.6 billion Francs (1985 value) by 1986, which is equivalent to about 1 billion Euros nowadays. Even though these spending attracted by then about 4.75 times more private investments in housing, retail, offices, ..., a similar initial investments could not be supported for 7 different sub-centers. Hence, it was decided in 1969 to abandon the construction of 2 out of the 7 sub-centers. These are the planned sub-centers of Mantes-la-Jolie (West of Paris) and of the Vallée de Montmorency (North of Paris). To the best of my knowledge, the choice to abandon these two is as good as random. All 7 planned sub-centers circled Paris with a centroid at about 20km of the center.

<sup>7</sup>Fouchier (1999), consult: [link here](#).

Figure 2: Sub-centers and suburban train in Paris

(a) Sub-centers

(b) RER



Notes: "Mun" refers to municipalities.

The abandoned ones did not significantly differ in long term population trends, in previous transport accessibility or geographical location. The event study analysis in Section 4 formally tests for difference in population trends up to 40 years before the reform. No significant differences are observed.

**The emergence of sub-centers in the rest of France** — The planning and subsequent construction of urban sub-centers in metropolitan areas other than Paris followed shortly (between 1966-1969 for planning and 1969-1972 for construction).<sup>8</sup> In total, four additional sub-centers were constructed in four different metropolitan areas other than Paris. Table 2 summarizes the emergence of sub-centers across France. Note that sub-centers were built over several municipalities (up to 21 in Marne-la-Vallée).

**Measuring the polycentricity of French metropolitan areas** — The concept of polycentricity is multi-dimensional. The number of sub-centers, their size relative to the city core but also relative to each other, the location of the sub-centers, ... affect the degree of polycentricity of a metropolitan area. Hence, measuring polycentricity with a single index is (at least) a difficult task. That being said, the urban and planning literature often resorts to the use of the rank-size coefficient. Denote the size of city  $i$  by  $S_i$ , and its rank by  $R_i$ . Coefficient  $\beta$

<sup>8</sup>See the timeline of the sub-center developments in France in Figure 11.

Table 2: Urban sub-centers in France

Sub-center name	MA	Nb. of mun.	Pop. growth	Housing growth	Realized
Cergy-Pontoise	Paris	11	706.63	477.96	Yes
Evry	Paris	4	3797.72	902.42	Yes
Marne-la-Vallée	Paris	21	1429.49	1057.76	Yes
Melun-Sénart	Paris	10	817.39	590.58	Yes
St-Quentin-en-Yvelines	Paris	7	1213.76	696.57	Yes
Rives de l'Etang de Berre	Marseille	5	447.86	257.97	Yes
L'Isle-d'Abeau	Lyon	5	1005.40	477.26	Yes
Le Vaudreuil	Rouen	1	160.47	10.22	Yes
Lille-Est	Lille	1	152.94	15.81	Yes
Mantes-la-Jolie	Paris	10	286.26	217.87	No
Vallée de Montmorency	Paris	20	162.30	38.81	No

*Notes:* Housing growth is measured based on housing units. Growth in population and housing units is measured between 1968 and 2015 in percent.

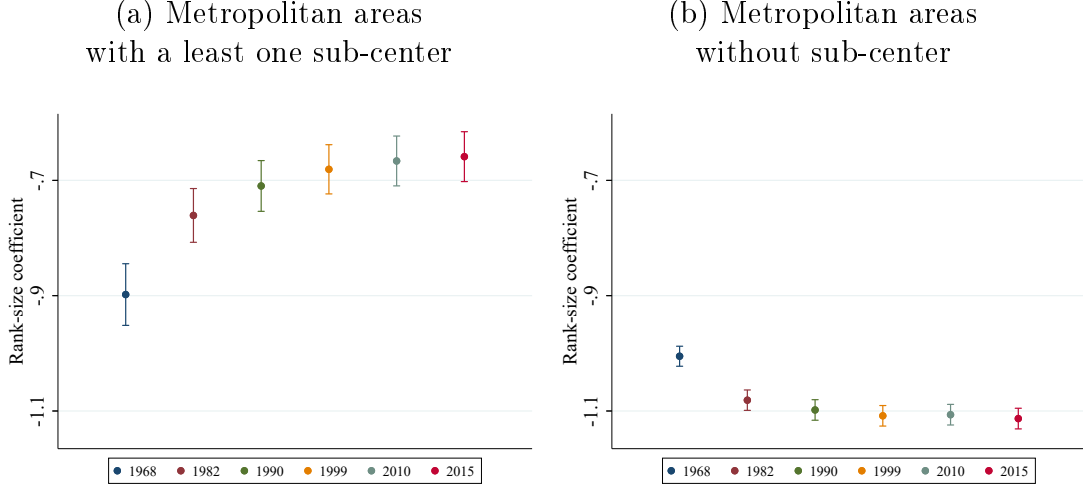
in (1) is the rank-size coefficient. A larger value for  $\beta$  indicates a more polycentric metropolitan structure, as it implies that the size of smaller sub-centers is relatively large compared to the size of larger sub-centers.

$$\log(S_i) = \beta \log(R_i) + u_i \quad (1)$$

Figure 3 reveals the evolution of the rank-size coefficient in metropolitan areas with at least one sub-center (3a) and in metropolitan areas without (3b) between 1968 and 2015. Following the construction of sub-centers, I observe an increase of the rank-size coefficient for the first types of metropolitan areas between 1968 and 1990. The coefficient is stable afterwards. The picture is reversed for metropolitan areas without sub-center. They appear to have become slightly but significantly more monocentric between 1968 and 1990. This might result from spillover effects across metropolitan areas. I investigate this question in Section 5.



Figure 3: Polycentrity as measured by the rank-size coefficient



Notes: 95% confidence intervals are displayed.

**Data source** — The data used in this paper mostly comes from the French National Institute for Statistics and Economic Studies (INSEE). This includes, population count at the municipal level since 1926, housing unit construction count since 1968, municipality-to-municipality commuting data and département-to-département (NUTS3-NUTS3) migratory data. Historical wage data is provided by the Institute for Research and Information in Health Economics (IRDES). Finally, [Chapelle and Eyméoud \(2017\)](#) provided us with housing rents (in euros per m<sup>2</sup>) at the local level.

## 4 Reduced-form evidence

**Event study design** — I analyze the effects of building a sub-center on population growth and composition using an event study design following [Schmidheiny and Siegloch \(2019\)](#). Consider the set  $\mathbb{T}$  of treated units. All units  $i \in \mathbb{T}$  receives a single treatment at time  $t$ .<sup>9</sup> I observe the dependent variable  $y_{it}$  at different time periods  $t$ . As the treatment effect is allowed to vary over time, I am interested

<sup>9</sup>I use municipal level data recorded between 1926 and 2015 at varying intervals of 7 to 10 years. Even though all sub-centers were not created on the same year (see Figure 11), all creations took place in the time window 1968-1975. Hence, it appears that, from a practical perspective, all treated municipalities are treated at the same time (i.e., they are not yet treated in 1968 and all treated in 1975).

in studying its dynamics over a window ranging from  $\underline{t} < 0$  periods prior to the event to  $\bar{t} > 0$  after the event.  $\beta_t$  is then the coefficient of interest. Formally, I identify local effects using the following model:

$$y_{it} = \sum_{t=\underline{t}}^{\bar{t}} \beta_t D_{it} + \mu_i + \lambda_t + u_{it}, \quad (2)$$

$y_{it}$  is the outcome of interest.  $D_{it}$  is the interaction of a year dummy and the treatment indicator equal to 1  $\forall i \in \mathbb{T}$ .  $\mu_i$  and  $\lambda_t$  are i- and t-specific fixed effects.  $u_{it}$  is the error term.

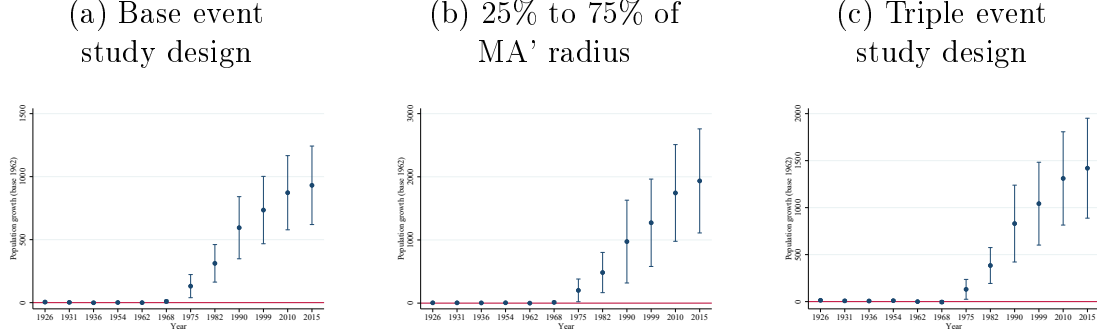
In the preferred specification (labeled "base event study design"), the control group includes municipalities in a planned, *but not developed*, sub-center. As they were selected to be part of a sub-center at the same time and under the same goal than the actually treated ones, they likely constitute the most fitting control group. However, to test the robustness of the results, I estimate two alternative event study specifications. First, I consider in the control group all municipalities located at a similar distance of the center than the treated ones (i.e., within 25% to 75% of a metropolitan areas' radius). Second, I estimate a triple interaction event study model in which the effect on treated sub-centers is compared to planned sub-centers relative to all other municipalities in the metropolitan area.

The reminder of this Section is organized in three parts. I first look at the population growth in sub-centers, before turning to the population composition in sub-centers. Finally, I focus on the population growth in municipalities outside the sub-centers to investigate possible polarized growth effects towards sub-centers.

**Population growth in sub-centers** — Figure 4 investigates the local population growth in sub-centers. Figure 4a reveals the evolution of the treatment effect using the base specification following (2). Treatment effects are estimated relative to 1962. As expected, population grows very similarly across the two groups between 1926 and 1968. Consistent with the urban plans, population grows significantly more in the treated municipalities from 1975 onwards. By 2015, they grew about 10 times more than the control municipalities. The two robustness specifications reveal very similar results (Figure 4b-c). Overall, the baseline specification appears rather conservative relative to the robustness specifications (by

2015, the point estimates in Figure 4b (4c) imply a 20 (15) times larger growth).

Figure 4: Population growth in sub-centers

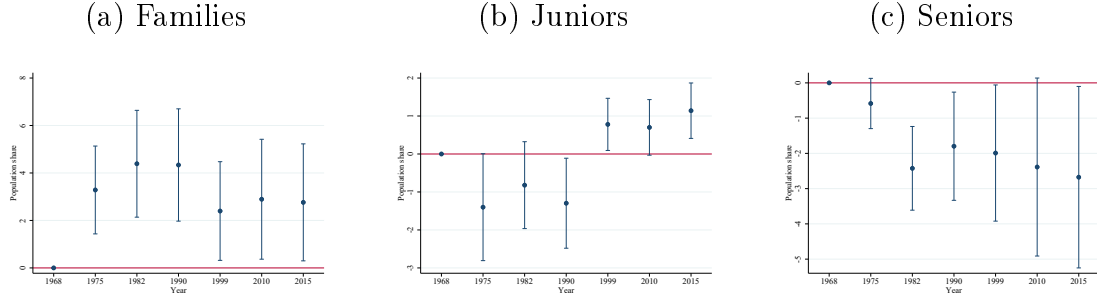


Notes: "MA" stands for Metropolitan Area.

**Demographic composition in sub-centers** — I now turn to the demographic and economic composition of sub-centers. Based on individual age, I define three demographic categories: *families* regroup individuals aged between 0 and 20, as well as individuals between 30 and 50; *juniors* refer to individuals between 20 and 30; and *seniors* to individuals over 50 years. Figure 5 reports the treatment effect of being part of a sub-center using the base specification (2) on the share of each demographic group in the total municipal population.

Relative to municipalities in only planned sub-centers, the share of families in constructed sub-centers is about 3 percentage point larger between 1975 and 2015. Reversely, the share of seniors in constructed sub-centers is about 2 percentage points lower. Hence, I observe a specialization of sub-centers into hosting families at the expense of seniors. Importantly, this is not an effect of housing unit size, as I compare municipalities with similar housing characteristics. Instead, it is more likely to be due to the important public investment (e.g., schools, green spaces, ...) that were made in constructed sub-centers which are more likely to attract families. No clear pattern for students appears. This is possibly due to the fact that few sub-centers hosted a new university (e.g., Cergy-Pontoise, near Paris or Lille-Est, near Lille), but most did not. Hence, obtaining no significant average effect is not surprising.

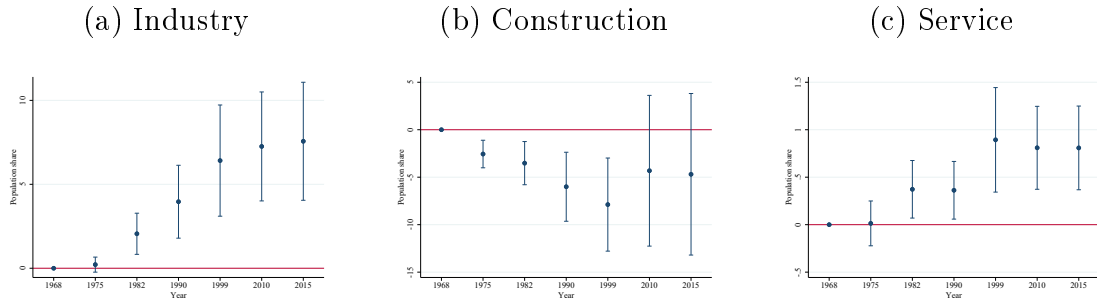
Figure 5: Demographic composition in sub-centers



*Notes:* Families regroup individuals aged between 0 and 20, plus between 30 and 50; students refer to individuals between 20 and 30; and seniors to individuals over 50 years.

**Sectoral composition in sub-centers** — Based on individual occupation at the place of residence, I compute the share of workers in each sector in the total residential workforce. As agriculture is very residual within the metropolitan area, I look at the *industry*, *construction* and *service* sectors using the base specification (2). Figure 6 displays the treatment effects over time. I observe a strong increase in the share of industry workers residing in sub-centers (by about 7 percentage points in 2015), as well as a smaller but significant increase in the share of service workers (by about 0.75 percentage points in 2015). This increase appear to have come at the cost of a decrease in the share of construction workers residing in a sub-center. I observe a significant treatment effect until 1999 of about -8 percentage points. The treatment effect is of similar magnitude afterwards even though less precisely estimated.

Figure 6: Sectoral composition in sub-centers



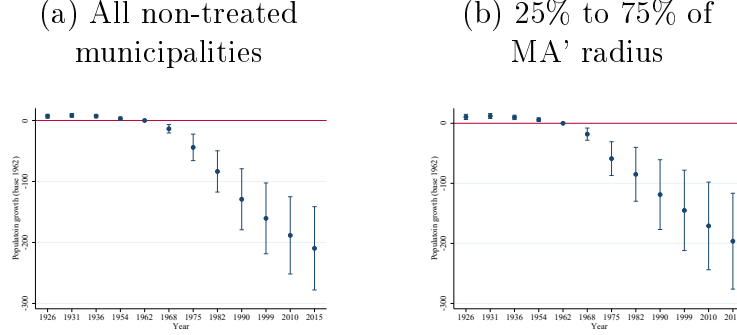
**Population growth outside the sub-centers** — Let me label municipalities within metropolitan areas with at least one sub-center, but outside of it as

*remaining municipalities*. Representative of these municipalities have long argued that the construction of sub-centers has lead the urban development to concentrate in these centers at their expense. For the 1970s French urban planners, this would appear less as a critic and more as a confirmation that the general guidelines specified by the urban plans have been followed. After all, developing polycentric metropolitan structure were motivated as a tool to restrain population growth in existing centers without extensive urban sprawl. However, even though the limited growth in remaining municipalities was targeted by the urban plans, it remains vital to evaluate its magnitude if one wants to understand the overall effect of polycentric structures on urbanization.

To evaluate the growth slowing effect of sub-centers on remain municipalities, I compare these municipalities to municipalities in metropolitan areas without sub-center. Precisely, I compare remaining municipalities in the metropolitan area of Paris, Marseille, Lyon, Lille and Rouen (i.e., those with at least one sub-center) to the municipalities in the metropolitan area of Toulouse, Bordeaux, Nantes and Strasbourg (i.e., those without sub-center). The choice of these last four metropolitan area is directed by the fact that they are the four largest metropolitan area in France after Paris, Marseille, Lyon, Lille and Rouen. I first do so considering all municipalities in the metropolitan areas without sub-center as control group (Figure 7a), then I restrict the control group to municipalities within 25% and 75% of the metropolitan areas' radius (Figure 7b).

Figure 7 reveals that remaining municipalities grew significantly less from 1975 onwards than their counterpart in metropolitan areas without sub-center. The effect reaches its lower point in 2015 with a relative growth twice smaller. The picture is very similar with both control groups. In combination with the growth effect in sub-centers seen in Figure 4, this result naturally raises the question of the net effect of polycentric metropolitan structures on urbanization. Given possible spillover effects, it appears useful to investigate this question in a more structured approach. I to so in the next Section.

Figure 7: Population growth outside the sub-centers



Notes: "MA" stands for Metropolitan Area.

## 5 General equilibrium effects of sub-centers

I develop a standard economic geography model to evaluate the effects of polycentricity on urbanization and welfare. I start by outlining the model (Section 5.1), before calibrating the model to fit the French economic geography (Section 5.2). I then present the results (Section 5.3).

### 5.1 Model

Consider a country composed of a finite number of locations  $i \in \{1, \dots, I\}$  on a lattice. Time is discrete and indexed by  $t \in \{0, 1, \dots\}$ . Further consider  $\theta \in \Theta$  individual types as a combination of age groups and economic sectors. At time  $t$ , the national economy is populated by a total of  $H_t$  individuals who are endowed with one unit of labor each, which they supply inelastically. I further assume that individuals are not forward looking.

In the framework, the underlying increase in urbanization, which initially motivates the development of polycentric metropolitan structures, arises from two sources exogenously. First, in any period, I consider the total number of workers in each sector as exogenously given; hence, exogenously accounting for the structural transformation away from agriculture. This is in line with insights from the macroeconomics literature which explains structural transformation by a more rapid productivity growth in agriculture than in non-agriculture (Baumol, 1967; Ngai and Pissarides, 2007; Rogerson, 2008). Second, the urban plans which anticipates an increase in urbanization on a 50 years horizon naturally act as an

urbanization force. In the framework, the urban plans are exogenously given. The goal is then to see how urbanization and welfare evolve given different urban plans.

### 5.1.1 Individual utility and location choice

Individuals have preferences linear in a consumption index. For individual  $o$ , of type  $\theta$ , residing in  $i \in I$ , originating from  $j \in I$ , working in  $k \in I$ , at time  $t$ , the consumption index ( $C_{ijkot}$ ) depends on consumption of a single final good ( $c_{ijkot}$ ), consumption of housing units ( $l_{ijkot}$ ), increases with locational amenities ( $b_{it}$ ), decreases with distance to place of origin ( $d_{ij}$ ), decreases with commuting distance to workplace ( $d_{ik}$ ) and is affected by an idiosyncratic shock ( $z_{ijkot}$ ). The aggregate consumption index is assumed to take the following Cobb–Douglas form:

$$C_{ijkot}^\theta = \frac{b_{it}^\theta}{d_{ij}d_{ik}} \left( \frac{c_{ijkot}^\theta}{\beta} \right)^\beta \left( \frac{l_{ijkot}^\theta}{1-\beta} \right)^{1-\beta} z_{ijkot}^\theta, \quad 0 < \beta < 1. \quad (3)$$

The idiosyncratic shock ( $z_{ijkot}^\theta$ ) describes the heterogeneity in utility that individuals derive from living in  $i$ , working in  $k$  and originating from  $j$  at time  $t$ . For each individual, this idiosyncratic component is drawn from an independent Fréchet distribution following [Eaton and Kortum \(2002\)](#),  $F(z_{ijkot}^\theta) = e^{-(z_{ijkot}^\theta)^{-\epsilon}}$ .

Locational amenities in  $i$  at  $t$  are a function of two components. A density related one,  $\tilde{b}_{it}^\theta$ , which captures possible congestion. It is a function of  $i$ 's population density at  $t-1$  and the inverse distance weighted sum of population densities in  $i' \neq i$  at  $t-1$ .  $\mathbb{W}_{ii'}$  is max row sum normalized scalar where the  $(i, i')$  entry is defined as the inverse distance between location  $i$  and location  $i'$ . A time specific local residential amenity component indexed  $it$  which captures local natural amenities and amenity-related public investments. Formally, we have:

$$b_{it}^\theta = g_{it}^{b\theta} \times \tilde{b}_{it}^\theta, \quad \text{with} \quad \tilde{b}_{it}^\theta = \left( \frac{h_{i,t-1}^\theta}{l_i} \right)^{\omega_t^{1\theta}} \left( \int_I \mathbb{W}_{ii'} \frac{h_{i',t-1}^\theta}{l_{i'}} di' \right)^{\omega_t^{2\theta}}. \quad (4)$$

Using utility maximization and taking the final good as numeraire, denoting wages from an individual of type  $\theta$  working in  $k$  at  $t$  by  $w_{kt}^\theta$  and housing prices by  $Q_{it}$ , indirect utility of individual  $o$ , residing in  $i \in I$ , originating from  $j \in I$ , and

working in  $k \in I$  at time  $t$  is given by:

$$U_{ijkot}^\theta = \frac{b_{it}^\theta z_{ijkot}^\theta w_{kt}^\theta Q_{it}^{\beta-1}}{d_{ij} d_{ik}}. \quad (5)$$

Let us define the attractiveness of a combination of residence, origin and workplace for type  $\theta$ , as  $v_{ijkot}^\theta = (b_{it}^\theta w_{kt}^\theta)^\epsilon (d_{ij} d_{ik} Q_{it}^{1-\beta})^{-\epsilon}$ . I can then express the probability,  $\pi_{ijkot}^\theta$ , that an individual, originating from  $j$ , chooses to reside in  $i$  and work in  $k$ ; the probability,  $\pi_{it}^\theta$ , that an individual resides in  $i$ ; the probability,  $\pi_{kt}^\theta$ , that an individual works in  $k$  as:

$$\pi_{ijkot}^\theta = \frac{v_{ijkot}^\theta}{\sum_i \sum_j \sum_k v_{ijkot}^\theta}, \quad \text{and} \quad \pi_{it}^\theta = \frac{\sum_j \sum_k v_{ijkot}^\theta}{\sum_i \sum_j \sum_k v_{ijkot}^\theta}, \quad \text{and} \quad \pi_{kt}^\theta = \frac{\sum_i \sum_j v_{ijkot}^\theta}{\sum_i \sum_j \sum_k v_{ijkot}^\theta}. \quad (6)$$

Consequently, the expected utility for individuals of type  $\theta$  in the area is then:

$$\mathbb{E}[u]_t^\theta = \Gamma\left(\frac{\epsilon-1}{\epsilon}\right) \left[ \left( \sum_{i=1}^I \sum_{j=1}^I \sum_{k=1}^I v_{ijkot}^\theta \right)^{\frac{1}{\epsilon}} \right]. \quad (7)$$

### 5.1.2 Labor market

I assume a single final good is traded without costs within the country, and consider the following production function:

$$y_{kt}^\theta = T_{kt}^\theta h_{kt}^\theta, \quad (8)$$

where  $y_{kt}^\theta$  is the output of the final good in  $k$ ,  $h_{kt}^\theta$  is the local employment of type  $\theta$ , and  $T_{kt}^\theta$  is the locational productivity. The approach taken to model the labor market is guided by the availability of localized long-term data. The functional form allows the use of precise housing unit data on a long time period when analyzing individuals residential choices (i.e., 1968-2015 as detailed in the calibration).

Locational productivity are a function of both: a workplace density component,  $\tilde{T}_{kt}$ , which is a function of population density in  $k$  at  $t-1$  and the inverse distance weighted sum of population density in  $k' \neq k$  at  $t-1$ ; and a locational productivity component which captures local productive natural characteristics and production-related public investments.  $\mathbb{W}_{kk'}$  is max row sum normalized scalar



where the  $(k, k')$  entry is defined as the inverse distance between location  $k$  and location  $k'$ .

$$T_{kt}^\theta = g_{kt}^{T^\theta} \times \tilde{T}_{kt}^\theta, \quad \text{with} \quad \tilde{T}_{kt}^\theta = \left( \frac{h_{k,t-1}^\theta}{l_k} \right)^{\gamma_t^{1\theta}} \left( \int_I \mathbb{W}_{kk'} \frac{\tilde{h}_{k',t-1}^\theta}{l_{k'}} dk' \right)^{\gamma_t^{2\theta}}. \quad (9)$$

Therefore, in equilibrium, wages  $(w_{kt}^\theta)$  are directly determined by the importance of agglomeration economies in productivity  $(T_{kt}^\theta)$ .

### 5.1.3 Housing market clearing

Clearing the residential housing market requires that the demand for housing units equal its supply in each location:  $l_{it}$ . This condition can then be written as:

$$(1 - \beta) \frac{\sum_\theta \mathbb{E}[w_{kt|i}^\theta] h_{it}^\theta}{Q_{it}} = l_{it}. \quad (10)$$

### 5.1.4 Equilibrium

The general equilibrium of the model can be referenced by the following vectors  $\{\pi^1, \dots, \pi^\Theta, Q, w^1, \dots, w^\Theta\}$ .

---

**Proposition 1** *Assuming strictly positive, finite, and exogenous characteristics , there exist unique general equilibrium vectors  $\{\pi^1, \dots, \pi^\Theta, Q, w^1, \dots, w^\Theta\}$ .*

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PROOF: See the proof of Proposition 1 in the Appendix, Section B. *Q.E.D.*

## 5.2 Calibration

The calibration proceeds in 8 steps. In the first step, I define the metropolitan areas of Paris, Marseille, Lyon, Lille, Rouen, Toulouse, Bordeaux, Nantes, and Strasbourg using the CCA. In steps 2-3, I estimate the mobility parameters. I derive historical wages at the local level in step 4. In steps 5-6, I obtain the historical amenities and historical workplace employment; which I then use in steps 7-8 to determine the elasticities of amenities and productivity.

### 5.2.1 Set of locations (step 1)

The set of locations considered includes all municipalities in the metropolitan areas (defined using the CCA) that contain at least one sub-center: Paris, Marseille, Lyon, Lille, and Rouen. These are the five largest metropolitan areas in France.<sup>10</sup> Additionally, I include all municipalities in the next four largest metropolitan areas in France: Toulouse, Bordeaux, Nantes, and Strasbourg. Finally, the centroids of the departments (NUTS3) in the rest of France (RoF) is considered as additional locations. Table 3 summarizes the set of locations included in the analysis.

Table 3: Metropolitan areas considered

MA	Nb. of mun.	Nb. of obs.	Pop. 1968	Pop. 2015	Contains an SC
Paris	337	337	7,860,937	9,510,857	Yes
Marseille	42	42	1,509,626	1,861,632	Yes
Lyon	79	79	1,197,200	1,558,690	Yes
Lille	198	198	2,114,602	2,153,718	Yes
Rouen	158	158	1,427,857	2,172,400	Yes
Toulouse	43	43	450,157	839,620	No
Bordeaux	19	19	532,737	695,363	No
Nantes	15	15	384,670	560,691	No
Strasbourg	17	17	345,726	441,155	No
RoF (NUTS3)	34,150	91	34,769,332	46,068,871	No

Notes: Metropolitan areas are obtained using the CCA.

### 5.2.2 Mobility costs: migration (step 2) and commuting (step 3)

In this paper, mobility costs are of two types. First, migration costs refer to the costs of changing ones residential location. Second, the commuting costs are the costs associated with the distance from one's residential location to one's workplace.

As is customary in the quantitative economic geography literature, I model mobility frictions  $d_{ij}$  and  $d_{ik}$  as an exponential function of distance.  $\kappa_1$  and  $\kappa_2$  govern the translation of distance into mobility frictions. I further consider that commuting frictions are determined by the distance between  $i$  and  $k$ , but also by the availability of public transport between these two locations.  $\lambda_{ik}$  captures the distance by regional train (RER) between locations in Paris metropolitan area, which was developed jointly with the Parisian sub-centers. Formally, I assume:

$$d_{ij} = \exp(\kappa_1 \tau_{ij}), \quad \text{and} \quad d_{ik} = \exp(\kappa_2 \tau_{ik} + \mu \lambda_{ik}) \quad (11)$$

<sup>10</sup>Even though Rouen is actually not one of the largest cities in France, it's metropolitan area indeed quite large (see Table 3).

Accordingly, there are semi-log gravity equations for mobility flows between residence  $i$  and origin  $j$  (and between residence  $i$  and workplace  $k$ ) in terms of travel time between  $i$  and  $j$ , and  $i$  and  $k$ :

$$\ln \pi_{ij} = \kappa_1 \tau_{ij} + \omega_{1i} + \zeta_{1j} + \eta_{1k} + \varepsilon_{1ij}. \quad (12)$$

$$\ln \pi_{ik} = \kappa_2 \tau_{ik} + \mu \lambda_{ik} + \omega_{2i} + \zeta_{2j} + \eta_{2k} + \varepsilon_{2ik}. \quad (13)$$

where  $\omega_{\cdot i}$  are residence fixed effects capturing residence characteristics,  $\zeta_{\cdot j}$  are origin characteristics, and  $\eta_{\cdot k}$  captures workplace characteristics. The parameters  $\kappa_1$  and  $\kappa_2$  are the semi-elasticities of migration and commuting flows, respectively, with respect to the great circle distance.  $\mu$  is the semi-elasticity of commuting flows with respect to distance by public transport. They are defined as  $\kappa_1 = \xi_1 \times \epsilon$ ,  $\kappa_2 = \xi_2 \times \epsilon$  and  $\mu = \rho \times \epsilon$ , where  $\xi_1$ ,  $\xi_2$  and  $\rho$  are the travel-cost parameters and  $\epsilon$  is the heterogeneity parameter from the Fréchet distributed shock on individuals' utility. Table 4 displays the estimation of (12) and (13). I follow Ahlfeldt et al. (2015) and take  $\epsilon = 6.83$ . Then, I obtain  $\xi_1 = -0.0023$ ,  $\xi_2 = -0.0366$  and  $\rho = 0.0176$  based on these estimates. The positive coefficient obtain for  $\mu$  reflects the fact that, to commute, more people switch to public transport as distance between their residence and workplace locations increase.

Table 4: Gravity estimations of migration and commuting

	Migration		Commuting	
	(I)	(II)	(III)	(IV)
Great circle distance (km)	-0.013 (0.0005)	-0.013 (0.0005)	-0.252 (0.0132)	-0.252 (0.0131)
Distance by public transport (km)			0.121 (0.0691)	0.121 (0.0687)
Estimation	PPML	NB	PPML	NB
Observations	8,836	8,836	6,116	6,116

*Notes:* Migration estimates based département (NUTS2) to département (NUTS2) mobility data. Commuting estimates based municipality to municipality mobility data. PPML stands for Pseudo Poisson Maximum Likelihood, and NB for Negative Binomial.

### 5.2.3 Historical wages by type (step 4)

Historical wages at the regional level are available for the period 1968-2015. However, the wage variation by type  $\theta$  is only recorded for the later years. To circumvent this data limitation, I assume a type-specific bonus-malus,  $\lambda^\theta$ , (based on 2015 wage data by type from the INSEE) which I then apply to the historical regional wages. Denoting the the median wage in year  $t$  in region  $i$  by  $\bar{w}_{kt}$ , I set  $w_{kt}^\theta = (1 + \lambda^\theta) \times \bar{w}_{kt}$ . Table 5 reports  $\lambda^\theta$  for each type.

Table 5: Type-specific bonus-malus

Econ. sector	Industry			Construction			Service		
Age group	Families	Juniors	Seniors	Families	Juniors	Seniors	Families	Juniors	Seniors
$\lambda^\theta$	-0.16	-0.44	0.13	0.19	-0.23	0.34	0.05	-0.37	0.20

### 5.2.4 Historical amenities (step 5) and workplace employments (step 6)

Using (6), I retrieve the overall locational amenities ( $b_{it}^\theta$ ) and the workplace employment ( $h_{kt}^\theta$ ) at each location and each time period.

---

**Proposition 2** *For each time period, given known values for the parameters  $\{\epsilon, \beta, \kappa_1, \kappa_2, \mu\}$  and the observed data  $\{\mathbf{H}, \boldsymbol{\pi}^1, \dots, \boldsymbol{\pi}^\Theta, \mathbf{w}^1, \dots, \mathbf{w}^\Theta, \boldsymbol{\tau}, \boldsymbol{\lambda}\}$ , there exist unique (up-to-scale) vectors  $\{\mathbf{b}^1, \dots, \mathbf{b}^\Theta, \mathbf{h}^1, \dots, \mathbf{h}^\Theta\}$  that close the model.*

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PROOF: See the proofs of Proposition 2 in the Appendix, Section C. *Q.E.D.*

### 5.2.5 Local and network-driven elasticities of amenities (step 7)

From (4) and taking logarithms, I can estimate the elasticity associated to one's own density,  $\omega_t^{1\theta}$ , the elasticity associated to the inverse distance weighted densities in other locations,  $\omega_t^{2\theta}$ , and retrieve the revealed amenity-related public investments,  $g_{it}^{b\theta}$  from (4) as follows:

$$\log(b_{it}^\theta) = \omega_t^{1\theta} \log\left(\frac{\tilde{h}_{i,t-1}^\theta}{l_i}\right) + \omega_t^{2\theta} \log\left(\int_I \mathbb{W}_{ii'} \frac{\tilde{h}_{i',t-1}^\theta}{l_{i'}} di'\right) + \log(g_{it}^{b\theta}). \quad (14)$$

As instruments for past population density ( $\log(\frac{h_{i,t-1}^\theta}{l_i})$ ) and past diffused population density ( $\log(\int_I \mathbb{W}_{ii'} \frac{h_{i',t-1}^\theta}{l_{i'}} di')$ ), I employ the latitude of a location ( $\log(x_i)$ ) and the sum of inverse distance weighted latitude ( $\log(\int_I \mathbb{W}_{ii'} x_{i'} di')$ ).  $g_{it}^{b\theta}$  are then obtain residually.

Over all individual types, a clear pattern emerges. Whereas the network-driven elasticity of density parameters are negative, but relatively small over time (at about -0.05), the own elasticity of density is larger in magnitude and stable over time at about -0.7 between 1968 and 2015. These coefficients imply that locational amenities strongly decrease with local population density, but also with congestion from nearby locations (even though to a lesser extent).

### 5.2.6 Local and network-driven elasticities of productivity (step 8)

Under profit maximization, agglomeration economies in productivity determine wages in  $i$ . Hence, the local elasticity of productivity parameter ( $\gamma_t^{1\theta}$ ), the network-driven elasticity of productivity parameter ( $\gamma_t^{2\theta}$ ), and the revealed productivity-related public investments,  $g_{it}^{T\theta}$  can be retrieve by estimating:

$$\log(w_{kt}) = \gamma_t^{1\theta} \log(\frac{\tilde{h}_{k,t-1}^\theta}{l_k}) + \gamma_t^{2\theta} \log(\int_I \mathbb{W}_{kk'} \frac{\tilde{h}_{k',t-1}^\theta}{l_{k'}} dk') + \log(g_{kt}^{T\theta}). \quad (15)$$

As instruments for past workplace population ( $\log(\frac{h_{k,t-1}^\theta}{l_k})$ ) and past diffused population count ( $\log(\int_I \mathbb{W}_{kk'} \frac{h_{k',t-1}^\theta}{l_{k'}} dk')$ ), I employ the latitude of a location ( $\log(x_k)$ ) and the sum of inverse distance weighted latitude ( $\log(\int_I \mathbb{W}_{kk'} x_{k'} dk')$ ).  $g_{kt}^{T\theta}$  are obtain residually. Similarly to residential amenities, the elasticity of productivity diffusion parameters remain stable at a small positive value for all years. The local elasticity of productivity parameters are stable at about 0.13 for all individual types.

## 5.3 Analysis

In this Section, I compare the observed urban developments in France between 1968 and 2015 to a counterfactual scenario under which housing units growth in each locations would have continue on their trends from before 1968, i.e., without

the central planner’s intervention. In what follows, I first formalize the counterfactual scenario, before comparing the difference between the two scenarios in terms of urbanization and welfare.

### 5.3.1 Baseline and counterfactual

The baseline scenario corresponds to the observed evolution of housing units in France’s largest metropolitan areas between 1968 and 2015, i.e., urban sub-centers were developed in Paris, Marseille, Lyon, Lille, and Rouen. In the counterfactual scenario, I redistribute the observed increase in the stock of housing units in sub-centers according to the distribution of housing units in 1968. The redistribution is only applied within metropolitan areas with at least a sub-center. The evolution of housing units in the other metropolitan areas remains unchanged. Formally, denoting the set of all municipalities in a metropolitan area with at least one sub-center as  $I^m$ , I set the counterfactual housing stock in  $i$  at  $t$ ,  $\tilde{l}_{it}$  as follows:<sup>11</sup>

$$\tilde{l}_{it} = \tilde{l}_{i,t-1} + \frac{l_{i,1968}}{\sum_{i \in I^m} l_{i,1968}} \sum_{i \in I^m} l_{it} - l_{i,t-1}, \forall i \in I^m. \quad (16)$$

I arbitrarily add another constraint to the counterfactual scenario to make it more realistic. I suppose that the municipality of Paris, i.e., center of Paris’ metropolitan area, does not grow in housing units relative to the baseline scenario. Hence, the redistribution only applies to other municipalities in the area. Due to a unique architecture and urban characteristics, housing unit growth (esp. building higher) is strongly restricted in Paris *intra muros*. Even though interesting from a theoretical perspective, a counterfactual scenario ignoring these restrictions would have little empirical relevance.

### 5.3.2 Results

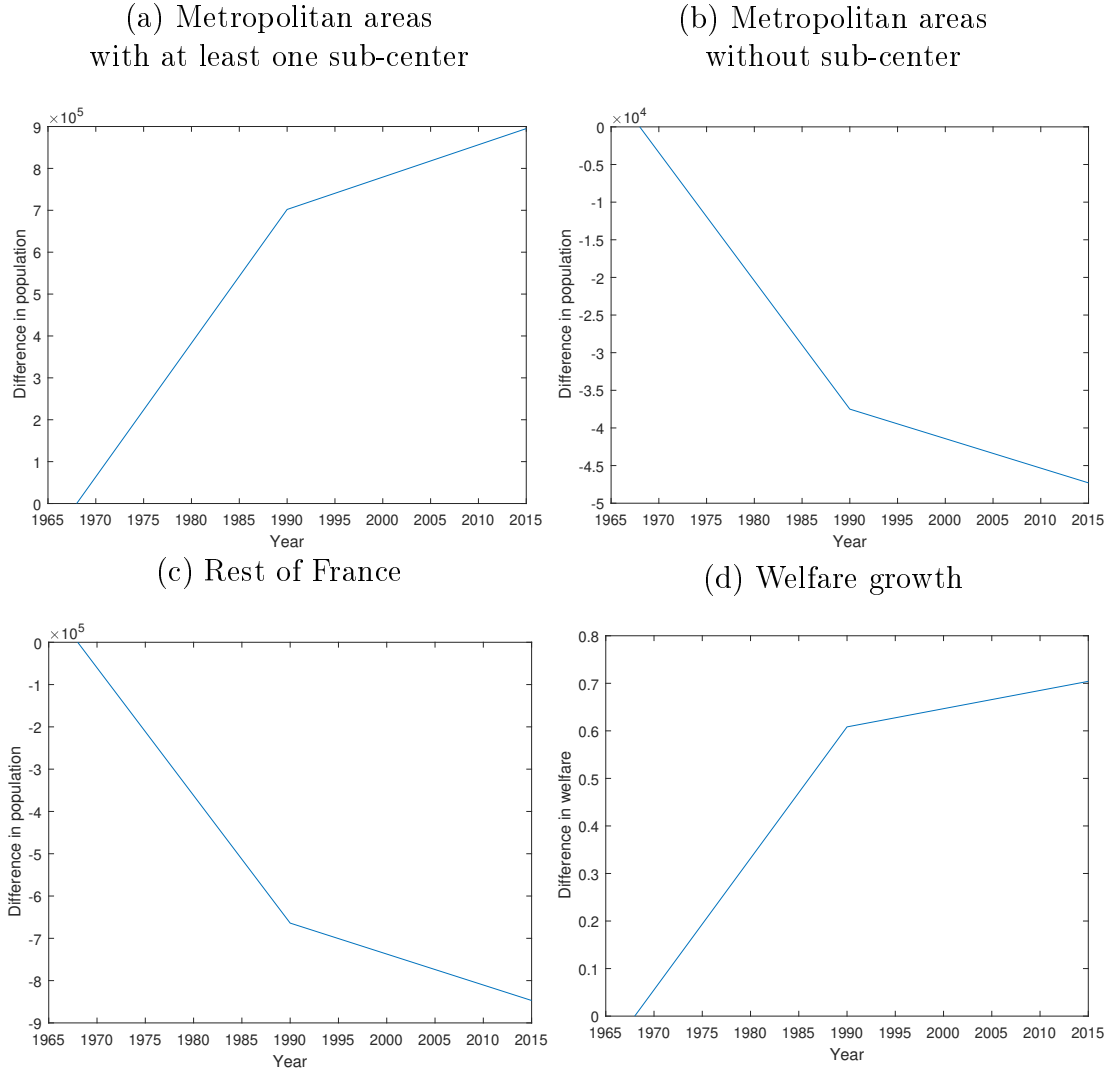
Figure 8 compares population and welfare between the baseline and the counterfactual scenarios at each period. Hence, *a positive value implies a larger value under the baseline than under the counterfactual*. Figures 8a-b-c focus on population difference in metropolitan areas with a sub-center (8a), without a sub-center

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<sup>11</sup>For the first period after the initial period, we have:  $\tilde{l}_{i,t-1} = l_{i,t-1}$ .

(8b) and in the rest of France (8c). Figure 8d displays the national welfare growth. In equilibrium, welfare is by construction equalized across all locations.

Figure 8: Urbanization and welfare (1968-2015)

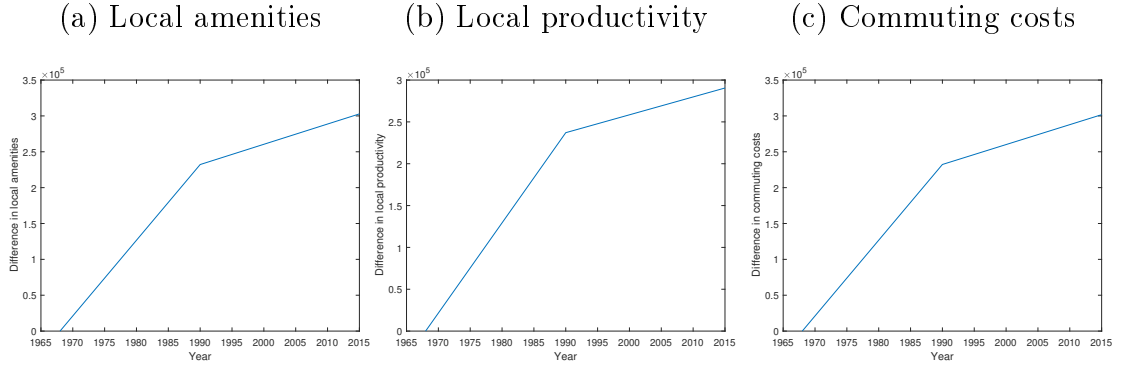


Overall, the effect of modifying the metropolitan structure of France's largest urban centers appears to have increased urbanization relative to the counterfactual. Targeted metropolitan areas appear to have gained almost 1 million inhabitant by 2015. Non-targeted metropolitan areas would have grown slightly more by 2015; overall, by 50,000 inhabitants. The majority of individuals that moved to targeted metropolitan areas came from the rest of France, which "lost" about 850,000 inhabitants. Furthermore, the development of polycentric structures lead to a 0.7% increase in the national welfare level by 2015. Concerning the dynamic of these effects, it appears that the largest share of the effects took place in the

first 20 years following the reform, and continued on a slightly flatter trend in the next 20 years. This is directly in line with the reduced form dynamic effects obtained in Section 4.

As detailed in the introduction and modeled in Section 5.1, the effects of metropolitan structures on urbanization and welfare are determined by a trade-off between spillovers and commuting costs. Figure 9 illustrates each component of this trade-off. Sub-centers are first and foremost motivated to avoid a large density increase in existing urban centers; and hence, avoid a reduction in residential amenities. Indeed, Figure 9a reveals that when sub-centers are developed the sum (across all residential locations) of local amenities enjoyed increases.

Figure 9: Local amenities and the co-location hypothesis



*Notes:* The sums across all locations of local amenities and local productivity are displayed in Panel (a) and (b), respectively. Panel (c) shows the sum of user weighted commuting costs across all bi-lateral commuting connections.

Furthermore, it is fruitful to look at Figures 9b and 9c in the context of the co-location hypothesis. This hypothesis stipulates that polycentric structures would improve the spatial match in residential and workplace locations. Hence, reducing commuting costs and increasing productivity spillovers. However, as we have seen, recent empirical studies have questioned the empirical relevance of the hypothesis (Aguilera, 2005; Cervero and Wu, 1998). Surprisingly, the results here agree with both sides. Polycentric structures lead to an increase in commuting costs (Figure 9c), but this increase does not translate in an decrease in local productivity. The sum of local productivity is actually increasing as the increase in commuting costs is out-weighted by the increase in urbanization (Figure 9b). Overall, this last results highlights the importance of studying metropolitan structures with a



general approach. Studying only parts of the local spatial market fails to capture the interactions of the effects, which would lead to biased conclusions.

Table 6 breaks down the population effects observed above by metropolitan area. Unsurprisingly, Paris is the largest winner of the reform. By 2015, Paris' population increased by 549,000 inhabitants more under a polycentric structure. As Paris is the metropolitan area that went through the strongest change, it is not surprising that the largest effects are observed there. Conversely, Lille, the metropolitan area in which the sub-center was localized very close to the city center ( $\approx 5\text{km}$ ), did not benefit from developing a sub-center. All metropolitan areas without sub-center experienced a slightly smaller growth given the construction of the sub-centers, but the effects remain minimal relative to their overall population.

Table 6: Net population change by Metropolitan Area

Year	MA with at least one sub-center					MA without sub-center			
	Paris	Marseille	Lyon	Lille	Rouen	Toulouse	Bordeaux	Nantes	Strasbourg
1990	425,600	7,700	85,900	-104,600	286,900	-12,530	-10,040	-8,240	-6,680
2015	549,000	-14,400	167,400	-134,200	326,500	-15,740	-12,680	-10,410	-8,470

*Notes:* Difference in population count between the counterfactual and the baseline is displayed for 1990 and 2013 for all metropolitan areas defined using the city clustering algorithm as detailed in 3.

Finally, I look at the population growth in metropolitan areas with a sub-center by economic sector and demographic group in Table 7. To ease the interpretation of the results, I report percentage change from the baseline to the counterfactual. Overall, polycentric structures have induced a population growth between 3% and 7% in all sectors and demographic groups. The growth appears particularly important when looking at juniors, i.e., individuals between 20 and 30 years old. Joining this result to the reduced form results in Section 4, I can conclude that polycentric structures have increased the level of sorting within the targeted metropolitan areas. Families have moved in sub-centers; thus, leaving space in more central locations to seniors and juniors. This result is in line with nowadays descriptive evidence of the demographic composition of the sub-centers and their metropolitan areas.

Table 7: Population growth in metropolitan areas with at least one sub-center, by economic sector and demographic group

Year	Economic sector			Demographic group		
	Industry	Construction	Service	Juniors	Families	Seniors
1990	3.727	3.746	3.725	6.402	3.105	4.812
2015	4.594	4.566	4.660	7.091	3.712	5.986

*Notes:* Population growth (%) in metropolitan areas with at least one sub-center, defined using the city clustering algorithm as detailed in [3](#), is displayed.

## 6 Conclusion

In this paper, I looked at the urbanization and welfare effects of metropolitan structures in both the short- and long-term. The analysis of within metropolitan areas effects revealed important polarization of growth towards sub-centers, as well as important demographic and sectoral sorting patterns.

These results were obtained using a event study design exploiting the fact that only 9 out of 11 planned sub-centers were actually developed to build a suitable control group. At the global level, comparing the observed urban plans to the a counterfactual without intervention (i.e., extrapolating the pre-reform trends), I observe that the shift towards polycentricity is associated with an increase in urbanization (about 900,000 inhabitants by 2015) and in welfare (0.7% growth in national welfare level).

Overall, the development of polycentric metropolitan structures appears to increase local amenity (through lower central density) and productivity spillovers (through higher urbanization share), which dominate the induced increase in residence-to-workplace commuting costs. Hence, the general shift towards polycentricity observed worldwide to host the increasing urban population is predicted to increase urbanization and welfare globally. Given the positive association between urbanization and labor productivity, these structural reforms are likely to be stable drivers of economics growth in the coming years.

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## A Supplemental tables and figures

Figure 10: The 1965 urban plan for the metropolitan area of Paris

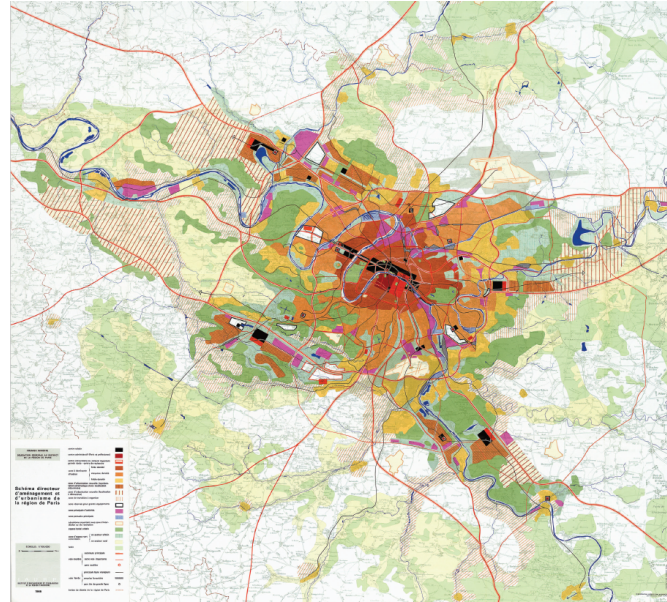
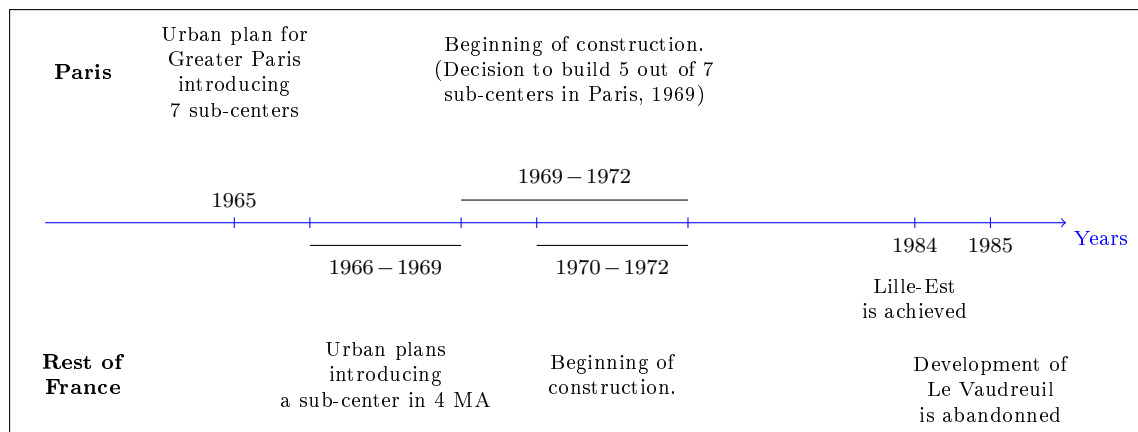


Figure 11: Timeline of the sub-center developments in France



## B Proof: Existence and uniqueness of equilibrium

Given that, in equilibrium, wages at the workplace are directly determined by the importance of agglomeration economies in productivity. Hence,  $w_{kt}^\theta$  is determined using:

$$w_{kt}^\theta = g_{kt}^{T\theta} \left( \frac{h_{k,t-1}^\theta}{l_k} \right)^{\gamma_t^{1\theta}} \left( \int_I \mathbb{W}_{kk'} \frac{\tilde{h}_{k',t-1}^\theta}{l_{k'}} dk' \right)^{\gamma_t^{2\theta}}. \quad (17)$$

I can then obtain the expected residential wages  $\mathbb{E}[w_i|k]^\theta$ .

$$\mathbb{E}[w_i|kt]^\theta = \sum_{k=1}^I \frac{(w_{kt}^\theta/d_{ik})^\epsilon}{\sum_{k=1}^I (w_{kt}^\theta/d_{ik})^\epsilon} w_{kt}^\theta; \quad (18)$$

I can then rewrite the attractiveness of a tri-lateral connection as

$$v_{ijkt}^\theta = (b_{it} \sum_{k=1}^I \frac{(w_{kt}^\theta/d_{ik})^\epsilon}{\sum_{k=1}^I (w_{kt}^\theta/d_{ik})^\epsilon} w_{kt}^\theta)^\epsilon (d_{ij} d_{ik} Q_{it}^{1-\beta})^{-\epsilon}, \quad (19)$$

and therefore the residential density as:

$$h_{it}^\theta = H^\theta \times \frac{\sum_j \sum_k (b_{it} \sum_{k=1}^I \frac{(w_{kt}^\theta/d_{ik})^\epsilon}{\sum_{k=1}^I (w_{kt}^\theta/d_{ik})^\epsilon} w_{kt}^\theta)^\epsilon (d_{ij} d_{ik} Q_{it}^{1-\beta})^{-\epsilon}}{\sum_i \sum_j \sum_k (b_{it} \sum_{k=1}^I \frac{(w_{kt}^\theta/d_{ik})^\epsilon}{\sum_{k=1}^I (w_{kt}^\theta/d_{ik})^\epsilon} w_{kt}^\theta)^\epsilon (d_{ij} d_{ik} Q_{it}^{1-\beta})^{-\epsilon}} \quad (20)$$

Assuming that the utility of the chosen units is such that  $(\frac{\gamma}{\mathbb{E}[u]})^\epsilon = 1$ , it follows that the residential land market clearing (10) provides the following system:

$$l_{it} = \sum_{\theta} \left[ \frac{\alpha(1-\beta) \sum_{k=1}^I \frac{(w_{kt}^\theta/d_{ik})^\epsilon}{\sum_{k=1}^I (w_{kt}^\theta/d_{ik})^\epsilon} w_{kt}^\theta}{Q_i} \times H \sum_j \sum_k \frac{(b_{it} \sum_{k=1}^I \frac{(w_{kt}^\theta/d_{ik})^\epsilon}{\sum_{k=1}^I (w_{kt}^\theta/d_{ik})^\epsilon} w_{kt}^\theta)^\epsilon}{(d_{ij} d_{ik} Q_{it}^{1-\beta})^\epsilon} \right] \quad (21)$$

Hence, there exists a unique vector  $Q_{it}$  which solves (21). Given housing prices, I can then derive  $\pi_{ijkt}^1, \dots, \pi_{ijkt}^\Theta$ .

Q.E.D.



## C Proof: Calibration

The derivation below is valid for every type  $\theta$  and time period  $t$  independently. Hence, for simplicity of the notation, I omit the  $\theta$  and  $t$  indexes. The share of individuals residing in  $i$  has been defined as follows:

$$\begin{aligned} H_i &= H\pi_i \\ &= H \frac{\sum_{j=1}^I \sum_{k=1}^I v_{ijk}^\theta}{\sum_{r=1}^I \sum_{j=1}^I \sum_{k=1}^I v_{ijk}^\theta} \end{aligned} \quad (22)$$

For every  $t$ , I can rewrite this condition as a system of  $I$  equations for the  $I$  unknown residential amenities  $B_i$  as follows:

$$D_i(B) = H_{Ri} - H \frac{\sum_{j=1}^I \sum_{k=1}^I v_{ijk}}{\sum_{r=1}^I \sum_{j=1}^I \sum_{k=1}^I v_{ijk}} = 0. \quad (23)$$

**Lemma 3:** Given the parameters  $\{\epsilon, \kappa_1, \kappa_2, \zeta, \beta\}$ , and observables  $\{\mathbf{Q}, \boldsymbol{\rho}_{ij}, \boldsymbol{\rho}_{ik}, \mathbf{w}_k, \mathbf{H}_i\}$ , the system in (23) exhibits the following properties:

**Property 1:**  $D(B)$  is continuous.

**Property 2:**  $D(B)$  is homogeneous of degree zero.

**Property 3:**  $\sum_{i=1}^I D_i(B) = 0$ .

**Property 4:**  $D(B)$  exhibits gross substitution:

$$\frac{\partial D_i(B)}{\partial B_j} > 0, \quad \forall i, j, i \neq j \quad (24)$$

$$\frac{\partial D_i(B)}{\partial B_i} < 0, \quad \forall i \quad (25)$$

**PROOF:** Properties 1 and 2 of Lemma 3 follow directly from an inspection of (23). Property 3 is satisfied by noting:

$$\sum_{i=1}^I D_i(B) = H - H = 0 \quad (26)$$

Property 4 can be established by noting:

$$\frac{\partial D_i(B)}{\partial B_j} = H \frac{\epsilon B_i^{2\epsilon-1} (\sum_{j=1}^I \sum_{k=1}^I (d_{ij} d_{ik} Q_i^{1-\beta})^{-\epsilon} (w_k)^\epsilon)^2}{[\sum_{i=1}^I \sum_{j=1}^I \sum_{k=1}^I B_i^\epsilon (d_{ij} d_{ik} Q_i^{1-\beta})^{-\epsilon} (w_k)^\epsilon]^2} > 0. \quad (27)$$

Using property 2, which implies  $\nabla D_i(B)B = 0$ , it follows that:

$$\frac{\partial D_i(B)}{\partial B_i} < 0, \quad \forall i. \quad (28)$$

Thus, gross substitution is established.

Q.E.D.

**Lemma 4:** There exists a unique vector  $B$  which solves (23).

**PROOF:** I proceed in two steps. First, I show that there exists at most one (normalized) vector  $B$  which solves (23). Second, I show a vector  $B$  that solves (23) exists.

Gross substitution requires that  $D(B) = D(B')$  cannot occur if  $B$  and  $B'$  non-collinear vectors. By homogeneity of degree zero, I can assume that  $B' \geq B$  and  $B_i = B'_i$  for some  $i$ . Now suppose that I lower (or keep constant)  $B'$  in all locations except in  $i$  one at a time. By gross substitution,  $B_i$  will increase in at least one step. Hence,  $D(B) > D(B')$  which is a contradiction.

By homogeneity of degree zero, the search for an equilibrium amenity vector can be restricted to the unit simplex  $\Delta = \{\sum_{i=1}^I B_i = 1\}$ . Define on  $\Delta$  the function  $D^+(\cdot)$  by  $D_i^+(B) = \max\{D_i(B), 0\}$ .  $D^+(\cdot)$  is continuous. Denote  $\alpha(B) = \sum_{i=1}^I [B_i + D_i^+(B)]$  with  $\alpha(B) \geq 1, \forall B$ . Then define the function  $f(\cdot)$  from the closed convex set  $\Delta$  into itself as:

$$f(B) = [1/\alpha(B)][B + D^+(B)]. \quad (29)$$

By Brouwer's Fixed Point Theorem, there exist a  $B^* \in \Delta$  such that  $B^* = f(B^*)$ . Since  $\sum_{i=1}^I D_i(B) = 0$ , it follows that at the fixed point for amenity,  $B^* = f(B^*)$  and  $D_i(B) = 0$  for all  $i$ .

Q.E.D.

Homogeneity of degree zero implies that the equilibrium amenity vector is unique up to a normalization. I impose the normalization that the geometric mean amenity is equal to 1, i.e.,  $\left[ \prod_{i=1}^I B_i \right]^{1/I} = 1$ .

Once  $B_i$  is obtained,  $H_k$  naturally follows from (6).

Q.E.D.