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Reallocation and Productivity during Commodity Cycles

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Abstract

I study the firm-level dynamic response of a commodity-exporting economy to global cycles in commodity prices. To do so, I develop a heterogeneous-firms model that endogenizes declines in aggregate productivity through reallocation towards less productive firms. Within a given sector, commodity booms reallocate market share away from exporters because of currency appreciation and away from capital-intensive firms because of the increase in capital cost. I provide empirical evidence for these channels using microdata for Chile, the world's largest copper producer. When fed with the commodity super-cycle of 2003-2012, the calibrated model generates about 50% of the observed productivity decline.

Keywords: Productivity, Resource booms, Open economy macroeconomics

JEL Codes: D24, Q33, F41

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1 Introduction

Commodity-dependent economies are exposed to low-frequency cycles in international commodity prices (Reinhart et al., 2016). During a commodity price boom, productive inputs progressively reallocate away from tradable industries into nontradable services and the booming commodity sector (Corden and Neary, 1982). Because tradable industries are often associated with larger productivity gains in the long-run, commodity booms may backfire in the form of persistent currency appreciation and permanent productivity losses (Krugman, 1987; Alberola and Benigno, 2017).

Figure 1 illustrates these regularities using data for Chile, the largest copper producer in the world, during the commodity price super-cycle that started around 2003. Panel (a) shows the time paths of the real price of copper -by far the country's main produced and exported commodity- and the share of manufacturing in total output. The figure reveals large crowding-out effects of the commodity boom on the manufacturing share, which falls from 18% to 12% of GDP between 2003 and 2012.

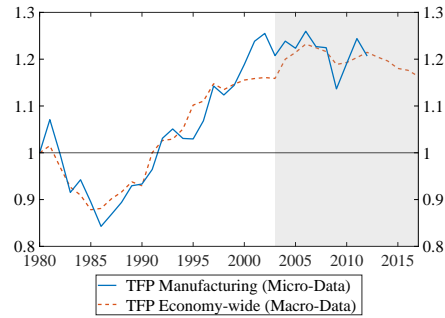
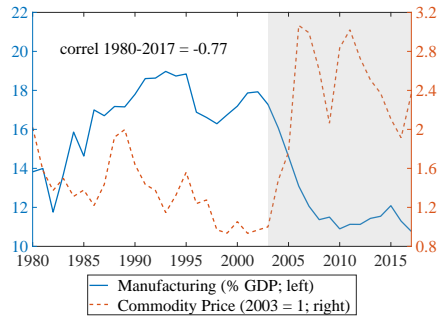
Traditional narratives in the literature argue that such reallocation is harmful to the economy by hindering positive externalities/spillovers from tradable sectors to the rest of the economy. Panel (b) of Figure 1 displays two aggregate measures of the country's total factor productivity during the most recent commodity price cycle. The solid line is the economy-wide productivity estimated using macroeconomic aggregates and standard Solow residual techniques. The dotted line corresponds to aggregate productivity in the manufacturing sector, obtained as the value-added weighted average of firm-level productivity estimates. Both measures deliver the same message: the commodity price super-cycle coincides with an unprecedented medium-run decline in the country's total factor productivity.

Theoretically, inefficient reallocation and low productivity are built into representative-firm models through spillovers or "learning-by-doing" externalities, assumed to be concentrated in the (non-commodity) tradable sector of the economy. In that environ-

Figure 1: Commodity Cycle, Sectoral Allocations and Productivity

(a) Manufacturing Share and P^{Co}

(b) Total Factor Productivity, 1980 = 1



Notes: Author’s calculations based on aggregated data from the Central Bank of Chile and micro data from ENIA (more details below). The gray area indicates the commodity boom period. Panel (a) reports the nominal manufacturing share in total output and the PPI-deflated copper price. Panel (b) shows two aggregated measures of the country’s total factor productivity (TFP). “TFP Economy-wide” refers to productivity estimated using macroeconomic data and standard Solow residual techniques, adjusting for input utilization rates and quality of the labor force. “TFP Manufacturing” is obtained using microdata from the country’s manufacturing Census. I compute firm-level TFP utilizing the method of [Wooldridge \(2009\)](#), and then I aggregate up using firm value-added weights.

ment, commodity booms crowd out the engine of the economy, which translates into low productivity economy-wide (e.g. [Alberola and Benigno, 2017](#)). In this paper, I develop a heterogeneous-firms model that endogenizes declines in aggregate productivity through reallocation dynamics, that is, a composition effect. Within a given sector, commodity booms reallocate market share away from exporter firms, because of currency appreciation. Because exporters in the data are (on average) more productive than non-exporters, the exchange rate channel induces a composition effect consistent with a decline in the average measured efficiency of operating firms. Similarly, resource booms reallocate market share away from capital-intensive firms in the (non-commodity) tradable sectors, because of the increase in the relative cost of capital, which is a consequence of the high capital-intensity in the booming commodity sector itself. Because capital-intensive firms are (on average) more productive than labor-intensive firms, the cost of capital channel interacts with the exchange rate channel to reinforce the overall decline in average productivity.

My main contribution is to link empirically and theoretically firm-level reallocation with sectoral shares and aggregate productivity dynamics during a commodity boom. On the empirical dimension, the extent of firm-level heterogeneity in capital intensities and export intensities determine the relative importance of each channel. For instance, if all firms use the same capital intensity, within-firm substitution between labor and capital plays no role as all firms face proportional increases in their unit costs. Likewise, if all firms were equally export-intensive, exchange rate dynamics would affect all plants symmetrically, and there would be no room for Dutch disease-like reallocation dynamics within sectors. Using manufacturing firm-level data for Chile, the largest raw copper producer globally, I document considerable within-sector variation of capital intensity in the cross-section of plants, suggesting that heterogeneous technologies with different exposures to changes in the cost of capital coexist even within manufacturing industries. Moreover, only about 22% of manufacturing firms engage in exporting activities, thereby being exposed to the exchange rate channel.¹

Motivated by this evidence, I empirically investigate the differential effects of commodity price booms on the relative performance of exporters versus non-exporters and capital-intensive versus labor-intensive firms within Chilean manufacturing industries during the period 1995-2012. The sample period analyzed includes the commodity price super-cycle that started around 2003, which provides a unique quasi-natural experiment to test the predictions of the theory proposed in this paper. First, I find that pre-boom exporters and capital-intensive firms lose market share relative to their non-exporter and labor-intensive counterparts during the commodity boom period 2003-2012. Second, the probability of exit from exporting increases significantly during periods of high commodity prices. Third, firms with relatively high capital-labor ratios in the pre-boom period significantly downsize their capital intensities during the boom. Overall, as pro-

¹Chilean copper mine production accounts for 27% of worldwide production in 2017 ([Comisión Chilena del Cobre, 2018](#), p. 141). The country's mining sector accounts for roughly 10% of GDP, 50% of total exports, and 21% of the economy-wide stock of physical capital, but less than 5% of the labor force.

ductive exporters and capital-intensive firms shrink, exit from exporting activities, and lessen their reliance on capital, the weighted average productivity in the cross-section of firms falls.

To formalize my empirical findings and quantify the relevance of the proposed channels in determining allocations and average productivity, I build a three-sector (commodity, exportable and nontradable) model of a small and financially open economy. The commodity and nontradable sectors are modeled as representative firms that combine labor and capital to produce. The main actors in this economy are a continuum of firms with heterogeneous productivity, aiming to represent the exportable/manufacturing sector. To deal with the differential effects that commodity shocks have on profits of exporters vs. non-exporters (within manufacturing industries), I borrow the framework introduced by [Melitz \(2003\)](#), in which firms trade off a fixed exporting cost against the possibility of serving the foreign market. In turn, to account for the significant cross-sectional heterogeneity and time variation observed in capital intensities across firms within manufacturing industries, I introduce a technology choice that allows firms to adjust their capital intensity in response to changes in relative input prices, along the lines of [Bustos \(2011\)](#). When choosing their technology, firms trade off higher fixed costs against a reduction in variable costs (or equivalently, a productivity boost).

As is well known, this type of framework leads to self-selection, in the sense that only the most productive firms find it profitable to pay the exporting and technology fixed costs. Intuitively, the profitability of becoming an exporter or adopting capital-intensive technology increases with firms' productivity type, while the costs of those choices are fixed and type-independent. This setup ensures there are always threshold productivity levels above which exporting and upgrading technology are worthwhile for the most productive firms in the economy.

The model is calibrated to reproduce selected macro and micro-level features of the Chilean economy, and it is used to study the country's dynamic response to an actual

commodity price cycle. In particular, to calibrate the parameters related to the exporting and capital intensity choices, I use the observed cross-sectional variation in export and capital intensities across firms within (3-digit) manufacturing industries.

When fed with an exogenously given commodity price boom-bust cycle, the calibrated model generates reallocation dynamics reminiscent of traditional Dutch disease narratives, but in a context in which reallocation is efficient. First, the resource sector crowds out labor and especially capital from manufacturing, consistent with the fact that mining production in Chile is substantially more capital-intensive than the typical manufacturing industry. Second, within the manufacturing sector, reallocation is shaped by firms' initial export and capital intensities. More specifically, using a model-simulated panel of firms, I show that exporters contract significantly relative to non-exporters during the boom, while the profits of capital-intensive firms fall disproportionately, findings consistent with the microdata. Third, entry/exit and upgrade/downgrade dynamics induce a composition effect that explains about half of the decline in measured manufacturing productivity between the pre-boom period 1995-2002 and the so-called super-cycle of 2003-2013. Fourth, the amplification effect generated by the cost of capital channel via the technology decision is quantitatively relevant. I find that the baseline model generates a productivity decline two times larger relative to a counterfactual economy with no capital intensity decision.

Related literature. While traditional work on commodity-dependence and the Dutch disease has focused on sector-level reallocation, this paper's main contribution is to link empirically and theoretically firm-level reallocation dynamics with aggregate outcomes. Two related articles using microdata to document the transmission channels from resource booms in the macroeconomy are [Allcott and Keniston \(2017\)](#) and [Benguria, Saffie, and Urzúa \(2018\)](#). [Allcott and Keniston \(2017\)](#) combine U.S. data on oil endowments at the county level with the Census of Manufactures to estimate how oil booms affect local manufacturing firms. They find that manufacturing as a whole is

not crowded out during oil booms, because the adverse effects on some tradable firms are offset by the positive impact on upstream and locally-traded subsectors. My paper differs in several important dimensions. First, I focus on commodity price shocks while they study oil discoveries. Second, and more important, they abstract from the two fundamental mechanisms emphasized in the present paper, as they propose a “within-country” model of Dutch disease reallocation, thereby eliminating the differential effects of exchange rate fluctuations on exporters’ relative performance versus non-exporters.

Second, [Benguria, Saffie, and Urzúa \(2018\)](#) exploit Brazilian regional variation in exposure to commodity price shocks and administrative firm-level data to disentangle similar channels like the ones studied here. While their emphasis is on labor market rigidities and the role of changes in the skill premium in shaping sectoral reallocation, I focus on the substitution between labor and capital. More importantly, by introducing a technology choice, I allow for an additional margin of adjustment that takes place within establishments, as I study how firms react to input price fluctuations by adjusting their optimal mix of labor and capital. My paper also provides an alternative explanation for persistent downturns in productivity during commodity price booms without relying on frictions or externalities.

By studying the effects of resource booms on sectoral allocations and productivity, the paper is tightly linked to the long-standing literature on Dutch disease or the “Resource Curse” (see [Corden and Neary, 1982](#); [Krugman, 1987](#); [Sachs and Warner, 2000](#); [Van der Ploeg, 2011](#); [Frankel, 2012](#); [Rodrik, 2012](#); [García-Cicco and Kawamura, 2015](#)). [Alberola and Benigno \(2017\)](#) propose a representative-firm three-sector commodity-exporter economy model to study the effects of commodity booms on long-run growth. In their model, when dynamic productivity spillovers are concentrated in the non-resource tradable sector, the commodity boom delays convergence to the world technology frontier and may even lead to a growth trap. While I do not consider spillover effects or any form of endogenous growth, I extend the environment

in other critical dimensions. First, I emphasize reallocation at the firm level within the manufacturing sector, which requires a framework with firm heterogeneity and an explicit distinction between exporters and non-exporters. Second, given the importance of relative input intensities in shaping reallocation, I allow for labor and capital in the production function and discipline their shares using firm-level data. I am not aware of other articles studying the capital intensity dimension in shaping firm-level reallocation dynamics during a commodity boom episode.

The remainder of the paper is organized as follows. Section 2 presents the firm-level data and summary descriptive statistics. Section 3 describes the quantitative model. Section 4 tests the ability of the model to replicate the empirical facts, and studies the most recent commodity super-cycle. Section 5 concludes.

2 Empirical Analysis

2.1 Data and Measurement

I use the ENIA (Encuesta Nacional Industrial Anual) longitudinal dataset of manufacturing firms provided by the Chilean statistical office INE (Instituto Nacional de Estadísticas). The baseline analysis uses a panel covering the period 1995-2012. The survey contains yearly information on establishments with more than ten employees, including an average of 5,000 observations per year. The data includes firm-industry, value-added, domestic sales, export sales, employment, intermediate inputs spending, and (self-reported) capital stock (book) value.²

Aggregating the micro-level data, ENIA accounts for 86% of aggregate manufacturing value-added and 50% of total manufacturing labor as recorded by the country's statistical office.

²Most firms in Chile are single-establishment. To ease exposition, in the main text, I use the words firm and establishment indistinctly.

Firm-level revenue total factor productivity (TFP) is estimated using the method of [Wooldridge \(2009\)](#). The export intensity of firm f is simply defined as the ratio of exports sales (x_{ft}) to total sales (s_{ft}), $X_{ft}^{\text{int}} = \frac{x_{ft}}{s_{ft}}$. In turn, the capital intensity of firm f is defined as $K_{ft}^{\text{int}} = \frac{k_{ft}/l_{ft}}{\sum_i k_{it}/\sum_i l_{it}}$, where k_{ft} and l_{ft} are capital and labor used by firm f in year t and the summations are done (3-digit) industry-wise.

The theory proposed below is designed to generate the critical groups of firms that are differentially affected by the exchange rate and cost of capital channels during a commodity boom: (a) non-exporters with low capital intensity, (b) non-exporters with high capital intensity, (c) exporters with low capital intensity, and (d) exporters with high capital intensity.

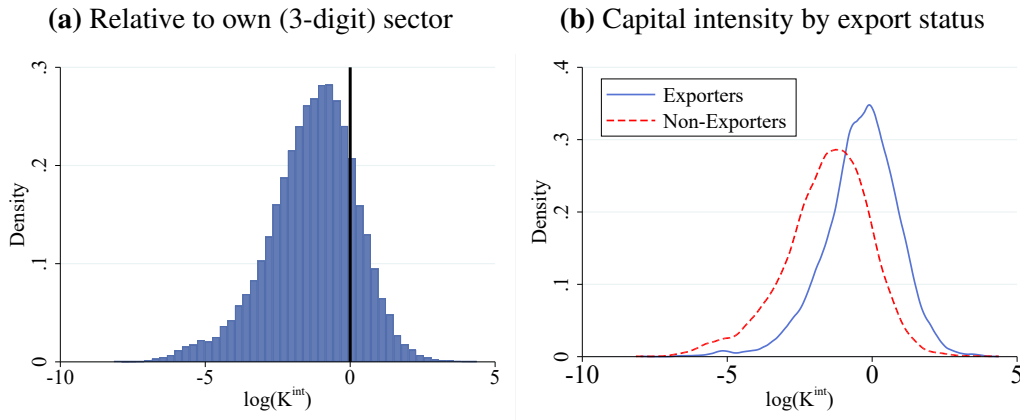
2.2 Firm Characteristics and Total Factor Productivity

In this subsection, I document several empirical regularities that are relevant for the analysis. First, I show significant heterogeneity in capital intensity across firms within 3-digit manufacturing industries. Capital-intensive firms are bigger and more productive than their labor-intensive counterparts. Second, I show that exporters are larger and more capital-intensive than non-exporter firms, findings that are consistent with the literature (see [Bernard and Jensen, 1999](#)).

Fact 1: *There is substantial cross-sectional heterogeneity in capital intensities and export-intensities within manufacturing industries.*

Panel (a) of [Figure 2](#) displays the distribution of (log) capital intensities K_{ft}^{int} across manufacturing firms pooling all years in the sample. The vertical line at zero indicates the frequency of firms with the same capital intensity of their own 3-digit industry ($K_f^{\text{int}} = 1$). I classify firms to the right (left) of the vertical line as capital-intensive or high-K (labor-intensive or low-K). It is clear from this figure that the distribution is very skewed, with relatively few firms located to the right of the vertical line. In turn, panel (b) shows that exporters are significantly more capital-intensive than non-exporters,

Figure 2: Capital Intensity Moments



Notes: Capital intensity of firm f is computed as $K_f^{\text{int}} = \frac{K_f/L_f}{\sum_i K_i/\sum_i L_i}$, where the summation is done (3-digit) industry-wise. The figures pool all years in the sample 1995-2012.

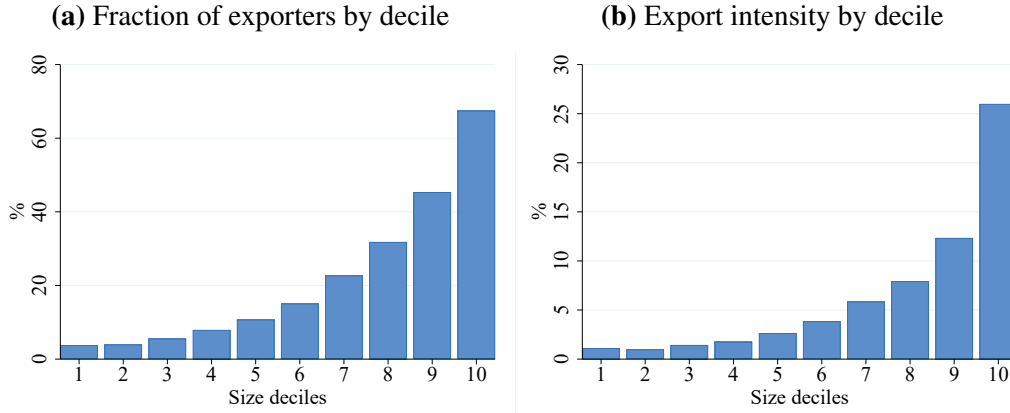
which means both channels will reinforce themselves.

In turn, Figure 3 displays export intensity moments across size deciles, with size measured as the number of employees. Panel (a) shows that larger firms are more likely to be exporters, while panel (b) shows that, conditional on exporting, larger firms are much more export-intensive than smaller firms. The next empirical fact documents how these dimensions of cross-sectional heterogeneity correlate with firm-level multi-factor productivity.

Fact 2: *Exporters and capital-intensive firms outperform non-exporters and labor-intensive firms, in terms of productivity and market shares.*

Exporters and capital-intensive firms outperform non-exporters and labor-intensive firms along many performance measures. Figure 4 illustrates this point using two relevant metrics for the analysis: productivity and market shares. Panel (a) of Figure 4 documents the estimated (revenue) total factor productivity distribution across firm-year pairs, grouped according to exporting and capital intensity status. Panel (b) replicates the exercise using firm-level sales-based market shares (firm sales over total sales). On average, exporters outperform non-exporters, and high-K firms outperform low-K firms.

Figure 3: Export Intensity Moments

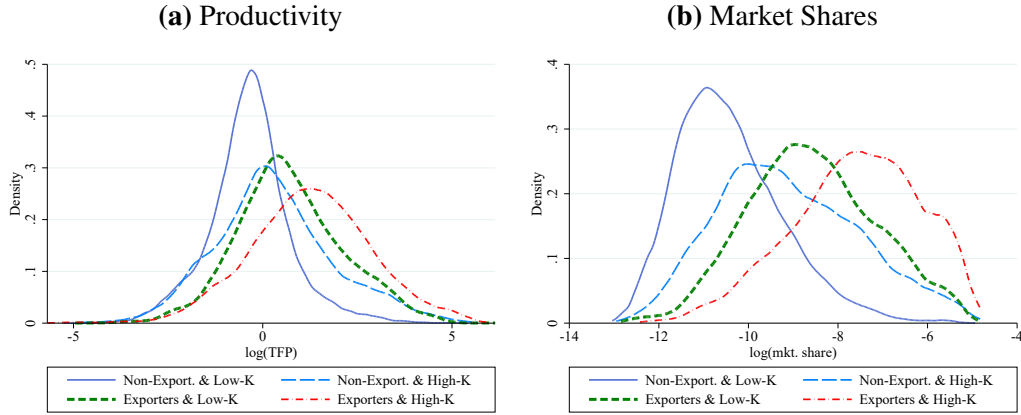


Notes: Panel (a) reports the fraction of exporters by size deciles. Firm-level size is proxied using the number of employees. Export intensity of firm f is simply defined as the ratio of exports sales (x_{ft}) to total sales (s_{ft}), $X_{ft}^{\text{int}} = \frac{x_{ft}}{s_{ft}}$. Panel (b) plots the average export intensity within each decile.

Naturally, the very selective group of high-K exporters (8% of the sample) is substantially more productive than the remaining groups, especially relative to the most numerous group of low-K non-exporters (71% of the sample). A similar sorting pattern holds when estimating productivity using pre-boom years only, or when using labor productivity. Online Appendix A presents panel regressions documenting systematically how exporters and capital-intensive firms display significantly higher revenue TFP relative to other groups in the economy, even after controlling by sector-year fixed-effects. The quantitative model developed in the next section is calibrated to approximately replicate the average productivity levels implied by the distributions in Figure 4.

In sum, I have documented that there is significant heterogeneity in export and capital intensities across firms within narrowly-defined sectors in Chile. These dimensions of cross-sectional variation matter, as they correlate positively with performance measures such as productivity, sales and profit measures. In the next subsection, I illustrate how these firm-level characteristics shape reallocation and average productivity during a commodity boom.

Figure 4: Firm-Level Distributions by Groups



Notes: Kernel density estimation. Firm-level revenue TFP is estimated using [Wooldridge \(2009\)](#).

2.3 Firm-Level Implications of a Commodity Boom

In this subsection, I present evidence that commodity price booms disproportionately affect the profitability and market share of export-oriented and capital-intensive firms (Fact 3). Second, along the extensive margin, I report significant declines in the probability of being an exporter and the likelihood of using more capital-intensive technologies during periods of high commodity prices. These facts suggest that firms react to changes in relative prices, redirecting production to the (small) domestic market and substituting towards more labor-intensive production functions.

Fact 3: *Exporters and capital-intensive firms lose market share during a commodity boom (intensive margin).*

To document how firm characteristics shape the intensive margin of adjustment during commodity booms, I estimate the following specification:

$$y_{ft} = \alpha X_{f0} Z_t + \beta K_{f0} Z_t + \gamma X_{f0} K_{f0} Z_t + \varphi_f + \varphi_{st} + \varepsilon_{ft} \quad (2.1)$$

where $y_{ft} = \ln(Y_{ft})$ denotes an outcome variable (such as real value-added or profits) for firm f in year t , X_{f0} is a dummy variable that takes the value 1 if firm f exports in its first period $t = 0$ in the sample, K_{f0} measures the capital intensity of firm f in period

$t = 0$, Z_t is a commodity cycle measure, while φ_{st} and φ_f represent sector-year and firm fixed effects, respectively. Because in this regression I am interested in the intensive margin of adjustment, I impose a tenure restriction and include only firms that operate at some point before and after the commodity price shock of 2004. The coefficient α in (2.1) measures the effect of commodity price fluctuations on the subsample of exporting firms with labor-intensive technologies, $\alpha = \frac{\partial y_{ft}}{\partial Z_t}(X_{f0} = 1; K_{f0} = 0)$. Similarly, β is the relative effect of commodity price fluctuations on the subsample of non-exporters with capital-intensive technologies, $\beta = \frac{\partial y_{ft}}{\partial Z_t}(X_{f0} = 0; K_{f0} = 1)$. Finally, γ is the incremental effect on firms that are simultaneously capital-intensive and exporters, so that $\alpha + \beta + \gamma = \frac{\partial y_{ft}}{\partial Z_t}(X_{f0} = 1; K_{f0} = 1)$. Note that the baseline impact of commodity shocks on the group of non-exporters with labor-intensive technologies ($X_{f0} = 0; K_{f0} = 0$) is absorbed by the sector-year fixed effects, so that all coefficients are interpreted relative to that base group.

Table 1 presents the results for regression (2.1). Columns (1) and (2) use real value-added as dependent variable, while columns (3) and (4) use real profits. Columns (1) and (3) present results using a continuous commodity cycle measure $Z_t = \tilde{P}_t^{Co} = \ln(P_{t-1}^{Co}/\bar{P}^{Co})$, while columns (2) and (4) use a commodity boom indicator $Z_t = \{0, 1\}$ that takes the value 1 in all the commodity boom years 2004-2012 and zero otherwise (See panel b of Figure 1). Table 1 shows that exporters and capital-intensive firms shrink significantly during periods of high commodity prices. The double interaction coefficient is also negative and significant, suggesting that capital-intensive exporter firms, the most efficient units in the economy, suffer a double hit: lower revenues due to currency appreciation and disproportionately higher variable costs through the cost of capital channel. Overall, using column (4) as benchmark, capital-intensive exporters face a $11.4\% = 100(0.0571 + 0.0166 + 0.0404)$ larger *decrease* in their real profits relative to firms in the base group of labor-intensive non-exporter firms.

Potential concerns with specification (2.1) include the possibility of confounding

Table 1: Panel Regressions: Commodity Booms and Outcome Variables

	Y_{ft} = Real value-added		Y_{ft} = Real Profits	
	$Z_t = \tilde{P}_t^{Co}$	$Z_t = \{0, 1\}$	$Z_t = \tilde{P}_t^{Co}$	$Z_t = \{0, 1\}$
	(1)	(2)	(3)	(4)
α	-0.0801	-0.0382	-0.0755	-0.0571
($X_f = 1; K_f = 0$)	(0.0265)	(0.0184)	(0.0265)	(0.0187)
β	-0.0273	-0.0149	-0.0283	-0.0166
($X_f = 0; K_f = 1$)	(0.0070)	(0.0047)	(0.0071)	(0.0050)
γ	-0.0348	-0.0390	-0.0360	-0.0404
	(0.0163)	(0.0151)	(0.0161)	(0.0148)
$\alpha + \beta + \gamma$	-0.1422	-0.0921	-0.1398	-0.1141
($X_f = 1; K_f = 1$)				
Firm FE	yes	yes	yes	yes
Sector \times Year FE	yes	yes	yes	yes
Adj. R^2	0.176	0.175	0.154	0.154
N. obs.	60,629	60,629	59,976	59,976

Notes: Results for regression (2.1):

$$y_{ft} = \alpha X_{f0} Z_t + \beta K_{f0} Z_t + \gamma X_{f0} K_{f0} Z_t + \varphi_f + \varphi_{st} + \varepsilon_{ft}$$

where $y_{ft} = \ln(Y_{ft})$ is an outcome variable for firm f in year t , X_{f0} is a dummy variable that takes the value 1 if firm f exports in its first period $t = 0$ in the sample, K_{f0} denotes the capital intensity of firm f in period $t = 0$, Z_t is a commodity price cycle measure, and φ_{st} and φ_f represent sector-year and firm fixed effects. Columns (1) and (2) use real value-added as dependent variable, while columns (3) and (4) use real profits. Columns (1) and (3) present results using $Z_t = \tilde{P}_t^{Co} = \ln(P_{t-1}^{Co}/\bar{P}^{Co})$ as the commodity cycle measure, while columns (2) and (4) use a commodity boom indicator Z_t that equals 1 in all the years 2004-2012 and zero otherwise (See panel b of Figure 1). Standard errors in parentheses.

factors such as financial frictions and the inclusion of the Great Recession in the estimation sample 1995-2012. Online Appendix B presents robustness analysis showing that my main results survive after controlling for potential confounding factors as well as trimming the sample to 1995-2007.

Fact 4: *Exporters are more likely to exit from foreign markets; capital-intensive firms are more likely to downsize their technology's capital-labor ratio during a commodity boom (extensive margin).*

When the commodity shock is persistent enough, the exchange rate's protracted appreciation induces some pre-existing established exporters to exit from foreign markets. Similarly, some pre-boom capital-intensive firms are not able to bear the increase in the cost of capital, and thus are forced to downsize to less capital-intensive technologies. To document the effects of commodity booms on the extensive margin, that is, firms' decisions to exit from exporting and downsize their capital-labor ratios, I follow the literature and specify a dynamic linear probability model.³ The following specification treats the global cycle of commodity prices as exogenous from the viewpoint of Chilean manufacturing firms. I estimate:

$$Y_{ft} = \alpha_1 Y_{ft-1} + \alpha_2 Y_{ft-2} + \beta_1 Y_{ft-1} Z_t + \beta_2 Y_{ft-2} Z_t + \varphi_{st} + \varphi_f + \varepsilon_{ft} \quad (2.2)$$

where Y_{ft} can take the form of an export dummy $Y_{ft} = X_{ft} = 1$ if firm f exports in year t or a capital intensity dummy $Y_{ft} = K_{ft} = 1$ if firm f is classified as capital-intensive in year t (according to the definition in Figure 2), Z_t is a commodity cycle measure, and φ_{st} and φ_f are sector-year and firm fixed effects. The regression also includes firm-level controls such as size and productivity (not shown in equation 2.2). As before, I use the same two alternative measures for the commodity cycle. The lagged dependent variable is included because of the well-known fact that up-front sunk entry costs induce state-dependence in the exporting and capital-intensity decisions.⁴ I interact lags of

³See Roberts and Tybout (1997), Bernard and Jensen (1999), and Lincoln et al. (2019).

⁴These assumptions are consistent with the theory proposed below.

the dependent variable with the commodity cycle measure in order to understand to what extent the probabilities of continuing to export and using the capital-intensive technology are affected by commodity price fluctuations. I introduce two lags to capture the idea that the negative effects of persistent commodity booms take some time to build up.

Table 2: Panel Analysis: Dynamic Linear Probability Model

	$Y_{ft} = X_{ft} = \{0, 1\}$		$Y_{ft} = K_{ft} = \{0, 1\}$	
	$Z_t = \{0, 1\}$	$Z_t = \tilde{P}_t^{Co}$	$Z_t = \{0, 1\}$	$Z_t = \tilde{P}_t^{Co}$
	(1)	(2)	(3)	(4)
α_1	0.3250 (0.0164)	0.3420 (0.0128)	0.2460 (0.0152)	0.2660 (0.0118)
α_2	0.0785 (0.0151)	0.0490 (0.0112)	0.0880 (0.0143)	0.0630 (0.01075)
β_1	0.0322 (0.0194)	0.0760 (0.0225)	0.0439 (0.0177)	0.0670 (0.0207)
β_2	-0.0541 (0.0196)	-0.0840 (0.0123)	-0.0530 (0.0178)	-0.0716 (0.0210)
$100(\alpha_1 + \alpha_2)$	40.4%	39.1%	33.4%	32.9%
$100(\beta_1 + \beta_2)$	-2.2%	-0.8%	-0.9%	-0.5%
Firm FE	yes	yes	yes	yes
Sector×Year FE	yes	yes	yes	yes
Adj. R^2	0.151	0.151	0.141	0.141
N. obs.	41,907	41,907	41,907	41,907

Notes: Results for regression (2.2):

$$Y_{ft} = \alpha_1 Y_{ft-1} + \alpha_2 Y_{ft-2} + \beta_1 Y_{ft-1} Z_t + \beta_2 Y_{ft-2} Z_t + \varphi_{st} + \varphi_f + \varepsilon_{ft}$$

where Y_{ft} can take the form of an export dummy $Y_{ft} = X_{ft} = 1$ if firm f exports in year t (columns 1 and 2) or a capital intensity dummy $Y_{ft} = K_{ft} = 1$ if firm f is above the median capital intensity of its own industry in year t (columns 3 and 4). As before, Z_t is a commodity cycle measure and φ_{st} and φ_f are sector-year and firm fixed effects. Columns (1) and (3) present results using $Z_t = \tilde{P}_t^{Co} = \ln(P_{t-1}^{Co}/\bar{P}^{Co})$ as the commodity cycle measure, while columns (2) and (4) use a commodity boom indicator $Z_t = 1$ that equals 1 in all the years 2004-2012 and zero otherwise (See panel b of Figure 1). Standard errors in parentheses.

Table 2 reports the results. The coefficient on Y_{ft-j} is the marginal increase in the probability of exporting in period t if firm f exported in $t - j$. The interaction terms are interpreted as the incremental/detrimental effect of the commodity boom on the probability of continuing to export. For instance, column (1) shows that an exporter

in $t - 1$ has a 30% higher probability of being an exporter in period t ; if the firm also exported in $t - 2$, the probability further increases by about five percentage points. Regarding the interactions, for firms that exported last year, the commodity boom has a positive (sometimes not significant) effect on the probability of exporting today. But the adverse effects are significant for firms that exported two years ago. Moreover, across specifications, the negative impact of $t - 2$ dominates the positive effect of $t - 1$ in absolute value and significance. Similar correlations hold in columns (3) and (4) for the probability of using capital-intensive technologies. The next section develops a quantitative general equilibrium model designed to quantify the roles of the exchange rate and cost of capital channels, distinguishing between the intensive margin and the extensive margin of adjustment. The model is calibrated to match facts 1 and 2 and tested in terms of its ability to reproduce facts 3 and 4.

3 Model Description

Consider a small and financially open commodity-exporting economy with four types of goods: nontradable (N), exportable (X), importable (M), and commodity (Co). Households consume N , X and M , while the economy only produces N , X , and Co . Commodity production is entirely sold abroad at international price p^{Co} , the only exogenous driving force in the model. The household supplies labor, accumulates capital, smooths consumption via foreign borrowing, and owns firms.

Exportable varieties are produced by a continuum of firms with heterogeneous productivity, while two representative firms produce the nontradable and the commodity good, respectively. Firms in all sectors “compete” for the economy-wide labor and capital, so that changes in relative prices shape the market shares across sectors X , N and Co , and across firms within sector X . Investment goods are produced using a technology that combines nontradable goods N and importable goods M . Sector

N aims to represent truly nontradable services (e.g., utilities, construction), and the imported M component of investment represents capital goods not produced by the small open economy.

3.1 Household

There is an infinitely-lived representative household that maximizes lifetime utility:

$$\sum_{t=0}^{\infty} \beta^t \frac{\left[C_t - \varphi \frac{L_t^\zeta}{\zeta} \right]^{1-v}}{1-v}, \quad (3.1)$$

where C and L are consumption and labor supply, while the parameters β , v , ζ , and φ govern time discounting, the intertemporal elasticity of substitution, the Frisch elasticity of labor supply, and the marginal rate of substitution between consumption and leisure. The consumption bundle C is defined as a nested CES aggregator of the three types of goods C^N , C^X , and C^M :

$$C_t = \left[\chi_N^{\frac{1}{\epsilon}} (C_t^N)^{\frac{\epsilon-1}{\epsilon}} + (1 - \chi_N)^{\frac{1}{\epsilon}} (C_t^T)^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}} \quad (3.2)$$

$$C_t^T = \left[\chi_X^{\frac{1}{\epsilon}} (C_t^X)^{\frac{\epsilon-1}{\epsilon}} + (1 - \chi_X)^{\frac{1}{\epsilon}} (C_t^M)^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}, \quad (3.3)$$

where χ_N and χ_X control the weights of each type of good, and ϵ and ε govern the elasticity of substitution between goods. In turn, exportable consumption is a bundle over a continuum of manufacturing varieties indexed by ω :

$$C_t^X = \left[\int_{\omega \in \Omega_t} (q_{dt}(\omega))^\rho d\omega \right]^{\frac{1}{\rho}}, \quad (3.4)$$

where $q_{dt}(\omega)$ is the domestic demand of variety ω , Ω_t is the set of varieties available in period t , and $\sigma = 1/(1 - \rho) > 1$ is the elasticity of substitution among varieties (ρ is the inverse markup). The household supplies labor, accumulates capital, smooths

consumption via foreign borrowing, and owns firms.

The budget constraint can be written as:

$$p_t C_t + p_t^I I_t + B_{t+1} = w_t L_t + r_t^k K_t + (1 + r^*) B_t + \Pi_t, \quad (3.5)$$

where p_t is the price of the consumption, which is also a model-based proxy for the real exchange rate (RER); p_t^I is the price of the investment good, B_t is the country's net foreign asset position that pays exogenous interest rate r^* , w_t is the wage, I_t and K_t are investment and capital with rental rate r_t^k , and Π_t collects nominal profits from the ownership of firms.⁵ The aggregate stock of capital evolves according to:

$$K_{t+1} = (1 - \delta^k) K_t + I_t, \quad (3.6)$$

where δ^k is the depreciation rate. The household chooses $\{C_t, I_t, L_t, K_{t+1}, B_{t+1}\}$ to maximize (3.1) subject to the budget constraint (3.5) and law of motion (3.6). Cost minimization yields static choices for $\{C_t^N, C_t^T, C_t^X, C_t^M\}$.⁶

3.2 Exportable Varieties

The exportable sector is the core of the analysis. It features heterogeneous firms à la [Melitz \(2003\)](#), augmented with physical capital and a technology choice. The market structure is monopolistic competition. There is an infinite pool of forward-looking potential entrants that consider making an initial investment, modeled as a one-time sunk entry cost f_e , to draw a permanent productivity type z from a distribution $g(z)$ with positive support over $(0, \infty)$ and continuous cumulative distribution $G(z)$.

⁵More specifically, $\Pi_t = \Pi_t^N + \Pi_t^X + \tau \Pi_t^{Co} - \frac{\phi}{2} \left(\frac{K_{t+1}}{K_t} - 1 \right)^2 K_t$, where it is assumed that a fraction $0 \leq \tau \leq 1$ of commodity wealth is owned by domestic residents, while the remaining $(1 - \tau)$ is owned by the rest of the world (foreign direct investment). To ease exposition, I set $\tau = 1$ for now, but relax this assumption later in the calibration section.

⁶Online Appendix C provides additional details.

After learning z , firms with a sufficiently low draw optimally decide to exit and never produce. In turn, successful entrants decide (i) between two technologies that combine labor and capital but differ in their capital share α , and (ii) whether to serve the foreign market and become an exporter. To fix notation, let $j \in \{l, h\}$ index whether a firm uses the technology with low ($j = l$) or high ($j = h$) capital intensity, and let $s \in \{d, x\}$ denote whether a firm is purely domestic ($s = d$) or an exporter ($s = x$).

Technology choice. The production technology is a Cobb-Douglas of the form: $y(z_{sj}) = z_{sj} k_{sj}^{\alpha_j} l_{sj}^{1-\alpha_j}$, where k_{sj} and l_{sj} are capital and labor, α_j is the capital share with $\alpha_h > \alpha_l$, and z_{sj} denotes effective productivity for the different groups, given by:

$$z_{sj} = \begin{cases} z_{dl} = z & \text{if non-exporter } (s = d), \text{ labor-intensive } (j = l) \\ z_{dh} = z \cdot \kappa_h & \text{if non-exporter } (s = d), \text{ capital-intensive } (j = h) \\ z_{xl} = z \cdot \kappa_x & \text{if exporter } (s = x), \text{ labor-intensive } (j = l) \\ z_{xh} = z \cdot \kappa_h \cdot \kappa_x & \text{if exporter } (s = x), \text{ capital-intensive } (j = h) \end{cases}$$

where $\kappa_h \geq 1$ and $\kappa_x \geq 1$ are calibration parameters that I discipline by matching the productivity gaps among the different groups of firms illustrated in Figure 4. κ_h can be interpreted as a fixed cost advantage derived from using the capital-intensive technology (as in Bustos, 2011). κ_x can be interpreted as a fixed productivity boost arising from being exposed to export markets (consistent with the evidence in Bernard and Jensen, 1999, and many others). The basic technology with low capital intensity (α_l) entails a (per-period) fixed operational cost f_d , while adopting the capital-intensive technology (α_h) requires a larger fixed cost $f_d + f_h$.

Exporting choice. Firms serving the domestic market face a demand schedule given by: $q_{dt}(z) = (p_t(z))^{-\sigma} (p_t^X)^{\sigma} C_t^X$. Also serving the foreign market entails paying a fixed (per-period) exporting cost f_x , which gives firms access to foreign demand given by $q_{xt}(z) = (p_t(z))^{-\sigma} \gamma$, where γ controls the size of the foreign market. I assume the same price elasticity for both domestic and foreign demands. In essence, firms trade off the benefits of lower variable costs (via κ_h and κ_x) and larger demand (via γ) against

larger fixed operation costs (via f_h and f_x).

Pricing rule. The unit cost function is of the form $\frac{\phi_{jt}}{z_{sj}}$, where $\phi_{jt} = \left(\frac{r_t^k}{\alpha_j}\right)^{\alpha_j} \left(\frac{w_t}{1-\alpha_j}\right)^{1-\alpha_j}$ is the weighted average price of the composite input. Each firm charges a constant markup ($1/\rho$) over unit cost. Then, for any pair $s \in \{d, x\}$ and $j \in \{l, h\}$, a firm with productivity z will set a price equal to $p_{sjt}(z) = \left(\frac{1}{\rho}\right) \left(\frac{\phi_{jt}}{z_{sj}}\right)$.

Value functions. Regardless of their productivity type, all operating firms are subject to a constant probability δ of a bad shock that forces them to exit the market. Firms can also exit endogenously when their present discounted value becomes negative. Type- z firm chooses technology $j \in \{l, h\}$ and export status $s \in \{d, x\}$ decisions yielding the largest present discounted value:

$$V_t(z) = \max\{V_{dlt}(z), V_{dht}(z), V_{xlt}(z), V_{xht}(z)\}, \quad (3.7)$$

where

$$V_{sjt}(z) = \max\left\{0, \pi_{sjt}(z) + \frac{(1-\delta)}{(1+r^*)} V_{t+1}(z)\right\}, \quad s \in \{d, x\} \quad j \in \{l, h\}. \quad (3.8)$$

As shown in Online Appendix C, profits can be written as follows:

$$\pi_{sjt}(z) = \begin{cases} \pi_{dlt}(z) = \frac{1}{\sigma} \left(\frac{1}{\rho} \cdot \frac{\phi_{lt}}{z}\right)^{1-\sigma} (p_t^X)^\sigma C_t^X - f_d \\ \pi_{dht}(z) = \frac{1}{\sigma} \left(\frac{1}{\rho} \cdot \frac{\phi_{ht}}{z\kappa_h}\right)^{1-\sigma} (p_t^X)^\sigma C_t^X - f_d - f_h \\ \pi_{xlt}(z) = \frac{1}{\sigma} \left(\frac{1}{\rho} \cdot \frac{\phi_{lt}}{z\kappa_x}\right)^{1-\sigma} ((p_t^X)^\sigma C_t^X + \gamma) - f_d - f_x \\ \pi_{xht}(z) = \frac{1}{\sigma} \left(\frac{1}{\rho} \cdot \frac{\phi_{ht}}{z\kappa_x\kappa_h}\right)^{1-\sigma} ((p_t^X)^\sigma C_t^X + \gamma) - f_d - f_x - f_h, \end{cases} \quad (3.9)$$

where all fixed costs are valued in units of the numeraire. This environment gives rise to productivity cutoff rules that determine firms' entry/exit into/from domestic and foreign markets (\bar{z}_{xt}), as well as the adoption of the capital-intensive technology (\bar{z}_{ht}). The

domestic entry cutoff, (\bar{z}_{dt}) , is given by the following zero-value condition:

$$V_{dtt}(\bar{z}_{dt}) = 0. \quad (3.10)$$

Firms with $z < \bar{z}_{dt}$ optimally exit. In turn, firms in the range $\bar{z}_{dt} \leq z < \bar{z}_{xt}$ serve the domestic market only, and decide to use the cheaper labor-intensive technology. Intuitively, these firms lack the efficiency to be profitable when paying f_x or f_h .

The next step is to pin down the marginal exporter type $z = \bar{z}_{xt}$ and the marginal adopter of the capital-intensive technology $z = \bar{z}_{ht}$. Given the fixed costs structure, we need to consider two possible cases regarding the ordering of these two cutoffs. If $\bar{z}_x < \bar{z}_h$ (case 1), the marginal exporter uses the low-K technology, while the marginal adopter is an exporter type. In case 1, the model does not generate non-exporters using the capital-intensive technology (8% of the sample); all adopters are exporters.

Conversely, if $\bar{z}_h < \bar{z}_x$ (case 2), the marginal type that chooses the high-K technology is a non-exporter, while the marginal exporter uses the high-K technology. In case 2, the model does not generate exporters using the low-K technology (13% of the sample); all exporters are capital-intensive. As in [Bustos \(2011\)](#), I calibrate the model as in case 1, because it is closer to the data. Thus, we have the following marginal conditions:

$$V_{dtt}(\bar{z}_{xt}) = V_{xlt}(\bar{z}_{xt}) \quad (3.11)$$

$$V_{xlt}(\bar{z}_{ht}) = V_{xht}(\bar{z}_{ht}). \quad (3.12)$$

Free Entry Condition. The industry equilibrium is characterized by a mass \mathcal{M}_t of firms, a mass \mathcal{M}_{et} of prospective entrants, and a distribution $\mu_t(z)$ of productivity types $z \in (0, \infty)$. The mass of entrants is implicitly pinned by the free entry condition:

$$\int_{\bar{z}_{dt}}^{\infty} V_t(z)g(z)dz - \bar{f}_e = 0, \quad (3.13)$$

where $\overline{f_e}$ is a sunk entry cost valued in units of the numeraire. Because firms learn z after paying $\overline{f_e}$, prospective entrants consider the expected present value of entering $(\int_{\overline{z}_{dt}}^{\infty} V_t(z)g(z)dz)$ net of the sunk cost. Free entry drives this wedge to zero.⁷

Distribution of Firms. The firm productivity distribution evolves as follows:

$$\mathcal{M}_t\mu_t(z) = \begin{cases} (1 - \delta)\mathcal{M}_{t-1}\mu_{t-1}(z) + \mathcal{M}_{et}g(z) & \text{if } z \geq \overline{z}_{dt} \\ 0 & \text{otherwise} \end{cases}. \quad (3.14)$$

In a nutshell, the measure of firms today equals the measure of firms that survive both, exogenous exit (via the iid exit shock δ) and endogenous exit (if $z < \overline{z}_{dt}$), plus the flow of new entrants. By the law of large numbers, the latter is given by the unconditional distribution $g(z)$ scaled by the mass of potential entrants \mathcal{M}_{et} .

3.3 Commodity and Nontradable Sectors

The emphasis here is on the transmission mechanisms from exogenous commodity *price* booms to the rest of the economy. As such, I keep commodity production as simple as possible, abstracting from optimal exhaustion issues.⁸ I assume there is a representative firm in the commodity sector with technology given by $Y_t^{Co} = A^{Co} \left[(K_t^{Co})^{\alpha^{Co}} (L_t^{Co})^{1-\alpha^{Co}} \right]^\eta$, where α^{Co} governs the capital intensity and $\eta < 1$ induces decreasing returns to scale. The constant A^{Co} is set to target the empirical share of commodity output in total GDP. Sectoral profits are then given by $\Pi_t^{Co} = p_t^{Co}Y_t^{Co} - w_tL_t^{Co} - r_t^kK_t^{Co}$, where p_t^{Co} is the exogenous price of the commodity good.

⁷In practice, I add an endogenous convex component to the entry cost, which captures congestion effects in firm creation. The assumed functional form is:

$$f_e(\mathcal{M}_{et}) = \overline{f_e} + \phi_e [\exp(\mathcal{M}_{et} - \overline{\mathcal{M}}_e) - 1],$$

where $\overline{\mathcal{M}}_e$ is the steady state mass of entrants, and ϕ_e controls the degree of congestion effects. This simple set-up helps to match the empirical entry rate and is computationally convenient to compute transitional dynamics.

⁸The implicit assumption is that commodity reserves are still large. See [Arezki et al. \(2017\)](#) for an in-depth analysis of optimal exhaustion and the macroeconomic effects of oil *discoveries*.

In the model, I also distinguish between pure nontradable goods N and potentially exportable varieties (sector X , described above). For similar reasons, I keep the nontradable production simple by assuming a representative firm that maximizes profits given by $\Pi_t^N = p_t^N Y_t^N - w_t L_t^N - r_t^k K_t^N$, where p_t^N is the relative price of the nontradable good. The technology is given by $Y_t^N = A^N (K_t^N)^{\alpha^N} (L_t^N)^{1-\alpha^N}$, where α^N denotes the capital share. The constant A^N is set to target the empirical share of nontradable services in total GDP. This sector helps the model capturing the differential performance of (non-commodity) tradable varieties (which are worse-off due to the exchange rate appreciation) versus pure nontradable services (favored by the wealth effect).

Finally, investment goods are produced by a set of competitive firms operating a CES technology that combines locally produced nontradables services (ID_t^N) and importable goods (ID_t^M), with shares χ_I and elasticity of substitution ξ . This simple set-up is aimed to account for the fact that emerging countries rely heavily on imported investment (capital and intermediate goods) to source the domestic economy.

3.4 Equilibrium

Suppose the economy is in a stationary equilibrium in $t = 0$, associated with given commodity price p_0^{Co} . Unexpectedly, in $t = 1$ all the agents learn the economy is buffeted by a commodity price boom-bust cycle $\{p_t^{Co}\}_{t=1}^T$, and re-optimize under perfect foresight.

Definition. Given initial conditions for the net foreign asset position (B_0), the economy-wide stock of capital (K_0) and the initial measure of firms ($\mathcal{M}_0 \mu_0(z)$), and given an exogenous sequence of commodity prices $\{p_t^{Co}\}_{t=1}^T$, a competitive equilibrium consists of sequences of the following objects: (i) value functions $\{V_t(z)\}_{t=1}^T$, (ii) mass of entrants $\{\mathcal{M}_{et}\}_{t=1}^T$, (iii) measure of firms $\{\mathcal{M}_t \mu_t(z)\}_{t=1}^T$, (iv) operational, exporting, and technology cut-offs $\{\bar{z}_{dt}, \bar{z}_{xt}, \bar{z}_{ht}\}_{t=1}^T$, (v) decision rules for firms in the X sector $\{y_{sjt}(z), l_{sjt}(z), k_{sjt}(z), p_{sjt}(z)\}_{t=1}^T$, (vi) decision rules for firms in the Co and N sec-

tors $\{L_t^{Co}, K_t^{Co}, Y_t^{Co}, L_t^N, K_t^N, Y_t^N\}$, (vii) decision rules for the representative household $\{C_t, L_t, K_{t+1}, B_{t+1}\}_{t=1}^T$, and (viii) aggregate prices $\{w_t, r_t^k, p_t, p_t^X, p_t^N, p_t^I\}_{t=1}^T$, such that, for all $t = 1, \dots, T$:

1. (Maximization of Firms, X varieties): Given prices $\{w_t, r_t^k, p_t^X\}_{t=1}^T$ and cutoffs $\{\bar{z}_{dt}, \bar{z}_{xt}, \bar{z}_{ht}\}_{t=1}^T$, the value function $\{V_t(z)\}_{t=1}^T$ solves the problem of firms in sector X , and the sequences $\{y_{sjt}(z), l_{sjt}(z), k_{sjt}(z), p_{sjt}(z)\}_{t=1}^T$ are the associated decision rules.
2. (Maximization of Household): Given prices $\{w_t, r_t^k, p_t, p_t^X, p_t^N, p_t^I\}_{t=1}^T$, the decision rules $\{C_t, L_t, K_{t+1}, B_{t+1}\}_{t=1}^T$ solve the problem of the household.
3. (Maximization of Firms, Co and N sectors): Given prices $\{w_t, r_t^k, p_t^N\}_{t=1}^T$, allocations $\{L_t^{Co}, K_t^{Co}, Y_t^{Co}, L_t^N, K_t^N, Y_t^N\}$ solve the firms problem.
4. (Inputs Markets Clear):

$$L_t = \mathcal{M}_t \int_{\bar{z}_{dt}}^{\infty} l_t(z) \mu_t(z) dz + L_t^N + L_t^{Co} \quad (3.15)$$

$$K_t = \mathcal{M}_t \int_{\bar{z}_{dt}}^{\infty} k_t(z) \mu_t(z) dz + K_t^N + K_t^{Co} \quad (3.16)$$

5. (Goods Markets Clear):

$$\left[\mathcal{M}_t \left(\int_{\bar{z}_{dt}}^{\infty} (q_t(z))^\rho \mu_t(z) dz \right) \right]^{\frac{1}{\rho}} = C_t^X \quad (3.17)$$

$$Y_t^N = C_t^N + ID_t^N \quad (3.18)$$

6. (Free Entry Condition (FEC)): The mass of entrants satisfy the FEC (3.13).
7. (Laws of Motion): The measure of firms $\mathcal{M}_t \mu_t(z)$ and the capital stock K_{t+1} evolve according to (3.14) and (3.6), respectively.
8. (Net Foreign Asset Position): The aggregate resource constraint induces a law of motion for the net foreign asset position given by:

$$B_{t+1} = (1 + r^*)B_t + TB_t, \quad (3.19)$$

where $TB_t \equiv X_t - M_t$ is the trade balance, while exports and imports are given by:

$$X_t \equiv p_t^{Co} Y_t^{Co} + \mathcal{M}_t \left[\int_{\bar{z}_{xt}}^{\infty} p_{xt}(z) q_{xt}(z) \mu_t(z) dz \right] \quad (3.20)$$

$$M_t \equiv C_t^M + ID_t^M + \Phi_t + \mathcal{F}_t \quad (3.21)$$

9. (Transversality Condition B_{T+1}): In period $t = T$ the economy has settled in the new steady state with a finite and stable net foreign asset position.⁹

Equations (3.15) and (3.16) illustrate how the model can account for reallocation dynamics *between* sectors Co , X and N , as well as reallocation *within* the exportable sector X . These input market clearing conditions pin down the equilibrium wage w_t and rental rate of capital r_t^k . Equation (3.17) links household demands with firms supply of varieties. The N sector market clearing condition (3.18) includes consumption C_t^N and the nontradable services used to produce the investment good ID_t^N . Similarly, total exports (3.20) include commodity exports and the aggregation of exportable varieties. In turn, total imports (3.21) include the term used for consumption C_t^M , investment ID_t^M , capital adjustment costs Φ_t and fixed costs \mathcal{F}_t .¹⁰ Online Appendices C and D contain detailed derivations, including the full set of dynamic equilibrium conditions as well as computational algorithms to compute transitional dynamics. Next, I proceed to quantitatively characterize the key channels of the model, and illustrate how commodity booms induce reallocation dynamics consistent with a decline in average productivity.

⁹Note that the small open economy has a unit root in the net foreign asset position. As such, even transitory shocks can have permanent effects (see [Schmitt-Grohé and Uribe, 2003](#) and [Alberola and Benigno, 2017](#)).

¹⁰Recall capital adjustment costs $\Phi_t = \frac{\phi}{2} \left(\frac{K_{t+1}}{K_t} - 1 \right)^2 K_t$ and fixed costs are given by $\mathcal{F}_t = \mathcal{M}_t f_d + \mathcal{M}_t p_{xt} f_x + \mathcal{M}_t p_{ht} f_h + \mathcal{M}_{et} f_e (\mathcal{M}_{et})$. The variables p_{xt} and p_{ht} denote the time-varying fractions of exporters and the fraction of capital-intensive firms, respectively.

4 Quantitative Analysis

This section illustrates the ability of the model to reproduce relevant firm-level and macroeconomic dynamics after a commodity price boom-bust cycle episode. The quantitative analysis delivers three main messages. First, in commodity-exporting economies, exogenous commodity price booms induce reallocation from very productive exporters and capital-intensive firms towards less productive (non-exporters and labor-intensive) firms, thereby rationalizing a fall in aggregate (average) productivity.

Second, the model does a good job at linking reallocation dynamics at the firm level with relevant macroeconomic outcomes. At the aggregate level, and depending on assumptions, the baseline model generates between 40% and 60% of the observed productivity fall in the commodity boom. Both the RER channel and the cost of capital channel are quantitatively important, explaining 1/3 and 2/3 of the overall decline, respectively.

Third, the model with firm dynamics does not rely on spillover externalities (as in [Krugman, 1987](#) or [Alberola and Benigno, 2017](#)) to generate a productivity decline during a commodity boom. Instead, pure composition effects arising from efficient reallocation across firms can look like a ‘Dutch disease’ without really being a disease. As emphasized in the seminal [Corden and Neary \(1982\)](#), the commodity boom is welfare-improving for the commodity producer, as the economy can consume more using the proceeds from commodity exports during the boom, and interest payments on the accumulated net foreign assets after the boom ends ([Alberola and Benigno, 2017](#)).

4.1 Calibration

Table 3 reports a set of parameters set a priori either using standard values in the literature or based on direct firm-level data. All data moments used in the calibration are averages over the pre-commodity boom period 1995-2003 for which I have microdata.

The model period is one year. I set the time preference parameter $\beta = 0.96$ to target a long-run interest rate of 4%. I set the inverse of the intertemporal elasticity of substitution equal to $v = 1$ (log utility), and the Frisch elasticity similar to the baseline value documented by [Ríos-Rull et al. \(2012\)](#), which is 0.72 ($\zeta = 1 + 1/0.72 = 2.4$). The elasticities of substitution for the consumption and investment baskets ϵ , ε and ξ are set to standard values used in the literature. Capital depreciation is set at $\delta^k = 0.08$, while the exogenous exit shock probability is set to $\delta = 0.08$ so that the model's steady-state reproduces the average between entry and exit rates observed in the microdata.

Table 3: Externally Calibrated Parameters

Symbol	Value	Description	Source/Target
β	0.96	discount	$r = 4\%$
v	1	inverse IES	log utility
ζ	2.4	Frisch elasticity	literature
ϵ	1.1	substitution $C^N - C^T$	literature
ε	1.2	substitution $C^X - C^M$	literature
ξ	0.75	substitution $ID^N - ID^M$	literature
δ^k	0.08	depreciation of capital	macrodata
δ	0.08	exit shock	microdata
f_e	1	fixed entry cost	normalized

Table 4: Internally Calibrated Parameters and Model Fit

Symbol	Value	Description	Target	Data	Model
A^{Co}	0.29	scale pr. fc. N	Y^{Co}/Y	10%	10%
\bar{B}	0.50	SS NFA	TB/Y	1%	1%
τ	0.73	share Co SOE	REN/Y	-3%	-3%
φ	4.53	labor supply scale	$Y = 1$	1	1
κ_x	1.08	cost advantage α_h	$\ln(\tilde{z}_x/\tilde{z}_d)$	1.20	1.21
κ_h	1.45	cost advantage X	$\ln(\tilde{z}_h/\tilde{z}_l)$	1.90	1.74
γ	0.25	foreign size	$\ln(va50/va25)$	0.87	0.98
ρ	0.69	inverse markup	$\ln(va75/va50)$	1.16	1.18
μ_z	-2.10	$\ln z \sim N(\mu_z, \sigma_z)$	$\ln(va90/va10)$	4.07	3.32
σ_z	0.99	$\ln z \sim N(\mu_z, \sigma_z)$	$\ln(va95/va5)$	5.41	5.59
$100f_d$	0.34	operational cost	$\ln(va99/va1)$	7.64	6.48
f_x	0.16	exporting cost	fraction exporters	22%	22%
f_h	1.06	tech. adoption cost	fraction adopters	8%	8%
α_l	0.12	K share Low- K	cost share α_l	0.12	0.12
α_h	0.37	K share High- K	cost share α_h	0.37	0.37
χ^X	0.88	share C^X in C^T	$(Y^X + X^X)/Y$	0.48	0.49
χ^N	0.51	share C^N in C	Y^N/Y	0.41	0.40
A^N	0.80	scale pr. fc. N	L^N/L	0.41	0.46
α^N	0.30	K share Co sector	K^N/K	0.40	0.39
α^{Co}	0.60	K share Co sector	K^{Co}/K	0.20	0.14
χ^I	0.02	share ID^N in I	ID^M/C^M	4.40	4.36
η	0.45	DRS Co sector	$\Delta Y^{Co}/Y$	15p.p.	14p.p.
ϕ	50	K adjustment cost	$\Delta I/I$	26%	28%
ϕ_e	4	congestion cost	entry volatility	4%	4%

The remaining parameters, listed in Table 4, are chosen to match selected cross-sectional and macroeconomic data moments. Several macroeconomic targets, listed in the first block of Table 4 are exactly matched by construction. I normalize the initial state of the economy to have a zero net-foreign asset position, $\bar{B} = 0$. I set the share of commodity wealth in the hands of domestic residents $\tau = 0.73$ to get the -3% deficit in the net income accounts (rents) observed in the data. I set the fixed resource parameter \bar{R} in the commodity sector to match the share of mining in total output in Chile, $p^{Co}Y^C/Y = 0.1$. The scale parameter of the labor supply is chosen to normalize an initial steady-state nominal output of $Y = 1$. The middle block of Table 4, which includes 17 parameters $\{\kappa_x, \kappa_h, \gamma, \rho, \mu_z, \sigma_z, f_d, f_x, f_h, \alpha_l, \alpha_h, \chi^X, \chi^N, A^N, \alpha^N, \alpha^{Co}, \chi^I\}$,

is jointly calibrated by minimizing a loss function given by the sum of squared residuals associated with the following set of moments: (1) the productivity gap between the median exporter and the median non-exporter, (2) the productivity gap between the median high-K and the median low-K firm, (3) the value-added ratio between the 50th and 25th percentiles of the value-added distribution, $\ln(va_{50}/va_{25})$, (4) $\ln(va_{75}/va_{50})$, (5) $\ln(va_{90}/va_{10})$, (6) $\ln(va_{95}/va_{5})$, (7) $\ln(va_{99}/va_{1})$, (8) the fraction of exporters, (9) the fraction of capital-intensive firms, (10) the (average) capital cost share among low-K firms (11) the (average) capital cost share among capital-intensive firms (12) the value-added exportable share in total GDP, (13) the value-added nontradable share in GDP, (14) the share of labor in the N sector, (15) the share of capital in the N sector, (16) the share of capital in the Co sector, and (17) the ratio of imported investment (measured as capital goods imports plus intermediate goods imports) to imported consumption. To map the model-based sectors to the ISIC rev.3 trade classification data, I classify ‘Mining and Quarrying’ as commodity Co sector; ‘Manufacturing,’ ‘Trade,’ ‘Transportation and Storage,’ and ‘Financial Services’ as exportable X sector; and finally, ‘Utilities,’ ‘Construction,’ and ‘Government and Community Services’ as the nontradable N sector.

The last block of Table 4, composed of parameters $\{\eta, \phi, \phi_e\}$, is calibrated to match moments from the transitional dynamics. The level of decreasing returns in commodity production (η) is set to match the peak-to-trough change in the share of commodity output during the transition ($\Delta Y^C/Y$). Notice that the commodity cycle fed into the model is so large and persistent that under constant returns to scale (CRS), the commodity sector would take over the whole economy, effectively inducing full de-industrialization as in the hypothetical case studied in [Alberola and Benigno \(2017\)](#). To avoid this counterfactual result while keeping commodity production as simple as possible requires significant amounts of decreasing returns ($\eta = 0.45$). For similar reasons, the model requires significant degrees of capital adjustment costs ($\phi = 80$),

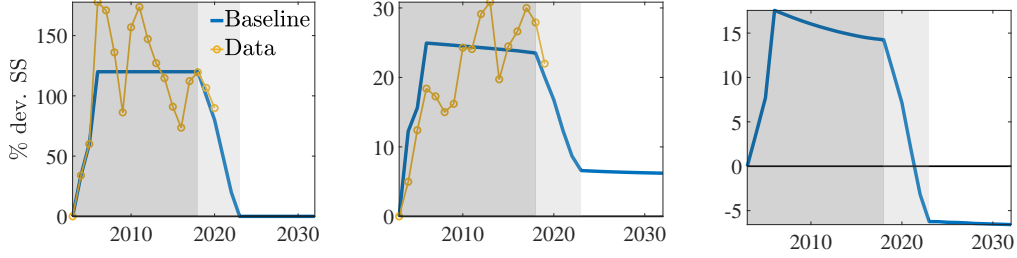
which avoids a too-rapid capital accumulation in the commodity sector (which occurs in the data, but with unmodeled time-to-build dynamics). The data moment used to discipline ϕ is the economy-wide investment boom in the data, measured as the ratio between average (real) investment in the pre-boom period 1995-2003 and the commodity boom period 2004-2012. Finally, the parameter governing congestion cost at entry ϕ_e is set to match the observed entry rate volatility in the manufacturing sector.

4.2 Transition Dynamics during a Commodity Cycle

In this section, I compute the macroeconomic and firm-level dynamic response of the small open commodity exporter economy to an unexpected commodity price boom-bust cycle. At time $t = 0$ (2003 in the data) the economy is in the initial stationary equilibrium associated with commodity price p_0^{Co} . At the beginning of time $t = 1$ (2004 in the data), the commodity price boom-bust cycle is revealed once and for all to all the agents and the transition equilibrium is solved under perfect foresight. I feed the model with a smoothed version of the actual commodity price observed in the data, illustrated in panel (a) of Figure 5. Because I am interested in the long-run transition to the new steady state, I abstract from the large but temporary commodity price swings observed in the data around the Global Financial Crisis of 2009. As can be seen in the figure, the overall commodity super-cycle lasted long after that recession.

Figure 5: Exogenous Trigger and Endogenous Price Response

(a) Commodity Price (p_t^{Co}) (b) Real Exchange Rate (p_t) (c) Input Price Ratio ($\frac{r_t^k}{w_t}$)



Notes: The solid lines depict the time series in the baseline model, while the circled lines are data counterparts when available. The dark and light gray shades represent the *exogenous* boom and bust cycle path fed to the model, illustrated in panel (a). Panels (b) and (c) are *endogenous* prices responses. The real exchange rate p_t is defined as the price of the domestic basket in terms of the foreign basket; an increase in p_t means a real appreciation.

Panels (b) and (c) of Figure 5 report the dynamic response of the endogenous prices directly related with the two channels emphasized in this paper: the real exchange rate (p_t) and the ratio of the rental rate of capital to the wage (r_t^k/w_t). Each panel displays the time paths for the baseline model (solid lines) and data counterparts when available. Panel (b) shows that the real exchange rate appreciates persistently during the boom phase, thereby reducing the revenue of exporters relative to non-exporter firms. The model is broadly consistent with the medium-run evolution of the real exchange rate (p_t) while at the same time capturing the peak appreciation of about 25% observed in the year 2007. Similarly, the cost of capital relative to the cost of labor r_t^k/w_t increases by about 30% (panel c), inducing a cost disadvantage to capital-intensive firms. That is, during the commodity boom, variable input costs $\phi_j(\alpha_j)$ increase relatively more in high-K firms ($j = h$) than in low-K firms ($j = l$).

4.3 Firm-Level Implications of Commodity Price Cycles

Intensive margin.— In this subsection, I momentarily abstract from entry/exit dynamics and focus on the intensive margin of adjustment. I thus focus on the question: How do pre-boom exporters and capital-intensive firms perform during the commodity boom,

relative to their non-exporter and labor-intensive counterparts? To isolate the intensive margin, I re-estimate the panel regressions reported in Section 2 on model-simulated data. Specifically, starting from the steady-state distribution of incumbents, I draw new firms from the unconditional distribution of entrants, and then I iterate the measure of firms forward imposing the exit shock δ . At each point in time, I use the aggregate general equilibrium prices (p_t and r_t^k/w_t) and thresholds (\bar{z}_{dt} , \bar{z}_{xt} and \bar{z}_{ht}) obtained from the transition equilibrium to compute firms' optimal decisions.

Table 5 reports the results comparing the model-based panel regressions against their data counterparts obtained in Section 2. The model does a good job of reproducing the untargeted elasticities between initial export status, capital intensity, and firm-level performance measures during the recent commodity price boom. In the model (data), the commodity price super-cycle induced a -9% (-8%) decline in real value-added (columns 1 and 2) for firms that are exporters and using the low-K technology ($X_f = 1; K_f = 0$), relative to the base group of non-exporters with low-K. When looking at the very selected group of exporters with high-K ($X_f = 1; K_f = 1$), the decline in real value-added reaches -15% (-14%). Roughly speaking, in a model without heterogeneity in capital intensities, we would underestimate the effect by about 40%. A similar pattern arises when focusing on real profits (columns 3 and 4); however, the model mildly underestimates the negative effects recorded in the data. Figure 6 breaks down total profits between profits from domestic and foreign sales for illustrative productivity types. The top row in the figure compares the performance of the average exporter (\tilde{z}_x) with the average non-exporter (\tilde{z}_d). In contrast, the bottom row displays the average exporter with low-K technology (\tilde{z}_{xl}) against the average exporter with high-K technology (\tilde{z}_{xh}).

Table 5: Panel Regressions: Commodity Booms and Outcome Variables

	Y_{ft} = Real value-added		Y_{ft} = Real Profits	
	Data	Model	Data	Model
	(1)	(2)	(3)	(4)
ψ_1	-0.0801	-0.0902	-0.0755	-0.0444
$(X_f = 1; K_f = 0)$	(0.0265)	(0.0048)	(0.0265)	(0.0022)
ψ_2	-0.1422	-0.1540	-0.1398	-0.1110
$(X_f = 1; K_f = 1)$	(0.0666)	(0.0064)	(0.0625)	(0.0041)
Firm FE	yes	yes	yes	yes
Sector \times Year FE	yes	yes	yes	yes
Adj. R^2	0.176	0.104	0.154	0.127
N. obs.	60,629	77,432	59,976	77,432

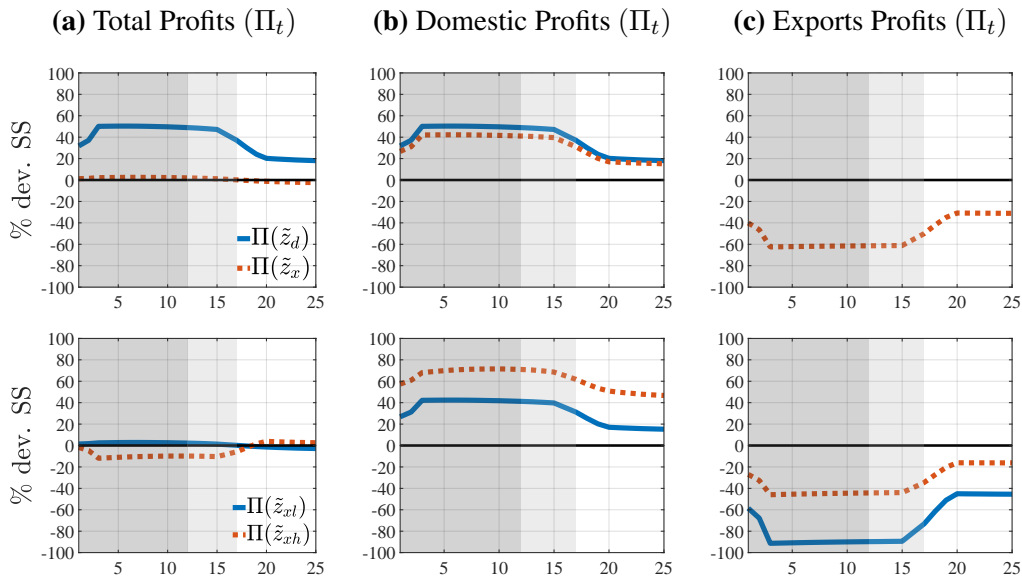
Notes: Results for regression (2.1). Because in the model all capital-intensive firms ($K_f = 1$) are also exporters ($X_f = 1$), the triple interaction is removed from the specification as follows:

$$y_{ft} = \psi_1 X_{f0} Z_t + \psi_2 K_{f0} Z_t + \varphi_f + \varphi_{st} + \varepsilon_{ft}$$

where the mapping from data-based coefficients to model-based coefficients is $\psi_1 = \alpha$ and $\psi_2 = \alpha + \beta + \gamma$. Recall that $y_{ft} = \ln(Y_{ft})$ is an outcome variable for firm f in year t , X_{f0} is a dummy variable that takes the value 1 if firm f exports in its first period $t = 0$ in the sample (conditional on $t = 0$ being in the pre-boom period 1995-2003), K_{f0} denotes the capital intensity of firm f in period $t = 0$, Z_t is a commodity price cycle measure, and φ_{st} and φ_f represent sector-year and firm fixed effects. Columns (1) and (2) use real value-added as dependent variable, while columns (3) and (4) use real profits. All columns present results using a commodity boom indicator Z_t that equals 1 in all the years 2004-2012 and zero otherwise (see Panel b of Figure 1). Standard errors in parentheses.

The first row of Figure 6 confirms that relatively low productivity firms (represented here by \tilde{z}_d) enjoy high domestic demand, which more than compensates them for the economy-wide increase in input costs during the boom. In turn, the average exporter (\tilde{z}_x) exhibits a similar increase in its profits from domestic markets; however, the value of their export sales plummet by more than 80% at peak (relative to the initial steady-state) as a consequence of the sharp appreciation of the real exchange rate. Overall, the average non-exporter increases profits by almost 40% at peak, while the average exporter peaks at only 10%. Note that the total effect of the commodity boom for the ‘typical’ exporter is positive, as the wealth effect in domestic markets dominates the negative impact of exchange rate appreciation on foreign sales.

Figure 6: Intensive Margin: Profits Responses to the Commodity Boom



Notes: The columns report total profits and its breakdown. The first row compares profits for the average exporter (\tilde{z}_x) versus the average purely-domestic type (\tilde{z}_d). The second row compares the average low-K firm (\tilde{z}_l) with the average high-K firm (\tilde{z}_h). The dark and light gray shades represent the boom and bust cycle path fed to the model. All series are in percent deviation from the initial steady state.

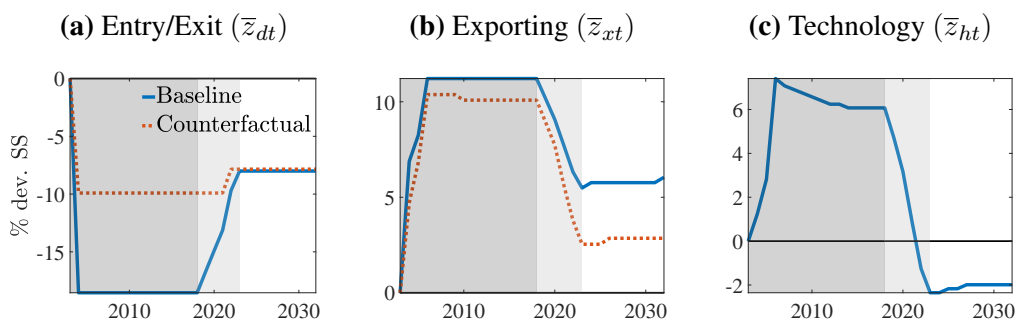
The second row of Figure 6 isolates the cost of capital channel. Within exporters, those using the capital-intensive technology are worst-off in terms of total profits during the commodity boom, with a net decline of about 10% at peak, due to the adverse effects of the real exchange rate channel on export sales. In this case, the evolution of domestic and foreign profits is quite similar for both types; however, the typical high-K exporter assigns more weight to their export sales than the typical low-K exporter. Overall, low-K exporters still experience an increase in total profits; that is, for these firms, on average, the positive wealth effect in domestic markets outweighs the adverse effects of the RER channel on export competitiveness. On the other hand, the average high-K exporter experiences an overall decline in total profits, as the RER channel, compounded with the rise in the cost of capital, dominates the benefits of higher domestic sales.

Extensive margin.— In this subsection, I study the reallocation of market shares due to changes in firm export and technology decisions, the extensive margin. As emphasized

above, the commodity boom also induces pervasive composition dynamics by shifting the productivity thresholds that determine firm selection.

Figure 7 displays the time paths for these cutoffs governing entry/exit as well as upgrading/downgrading operating technology. In order to isolate the channels, each panel displays the time paths for the baseline model (solid lines) and a counterfactual simulation with export decision but without technology choice (dotted lines). In particular, the cutoff that determines entry into the domestic market falls by 17% at peak (panel a of Figure 7), allowing some initially unprofitable low-productivity firms to start operations and serve booming local demand. In turn, both the exporting and adoption cutoffs respectively increase by 12% and 5% at peak (panels b and c), thereby inducing some pre-boom exporters to exit the foreign markets as well as some capital-intensive firms to downgrade their technology. It is clear from Figure 7 that the different channels interact and reinforce themselves, as both \bar{z}_d and \bar{z}_x react significantly more in the baseline model than in the ‘RER only’ counterfactual with no technology choice.

Figure 7: Selection: Cutoff Dynamics



Notes: The solid lines depict the time series in the baseline model, while the dotted lines correspond to a counterfactual without technology decision. The dark and light gray shades represent the exogenous boom and bust cycle path fed to the model, illustrated in panel (a). Figures are measured in percent deviation from the steady state.

While there are not clear data analogs for these cutoffs, I test their plausibility by comparing the entry/exit dynamics implied by the model’s transition against those

obtained from the data. Specifically, I run the linear probability model described in Section 2 on the artificial panel of firms. Table 6 shows the results. Regarding the export decision in columns (1) and (2), for firms that exported last year, the model (data) shows a significant decline of -3.3% (-2.2%) in the probability of exporting today. The commodity boom significantly hampers the ability of exporters to keep paying fixed costs. Regarding the technology choice (columns 3 and 4), the analogous figure is -2.3% (-0.9%), which confirms that many firms in the exportable sector downgrade to less capital-intensive technologies during the boom. Despite some timing disagreements between $t - 1$ and $t - 2$, the model does a reasonable job replicating the overall effects observed in the data as measured by the long-run effects $100(\beta_1 + \beta_2)$.

Table 6: Panel Analysis: Dynamic Linear Probability Model

	$Y_{ft} = X_{ft} = \{0, 1\}$		$Y_{ft} = K_{ft} = \{0, 1\}$	
	Data	Model	Data	Model
	(1)	(2)	(3)	(4)
α_1	0.3250 (0.0164)	0.5910 (0.0181)	0.2460 (0.0152)	0.297 (0.0462)
α_2	0.0785 (0.0151)	-0.230 (0.0216)	0.0880 (0.0143)	-0.0712 (0.0402)
β_1	0.0322 (0.0194)	-0.248 (0.0201)	0.0439 (0.0177)	-0.0703 (0.0401)
β_2	-0.0541 (0.0196)	0.233 (0.0206)	-0.0530 (0.0178)	0.0573 (0.0408)
$100(\alpha_1 + \alpha_2)$	40.4%	36.1%	33.4%	22.6%
$100(\beta_1 + \beta_2)$	-2.2%	-1.5%	-0.9%	-1.3%
Firm FE	yes	yes	yes	yes
Sector×Year FE	yes	yes	yes	yes
Adj. R^2	0.151	0.445	0.141	0.552
N. obs.	41,907	48,537	41,907	48,537

Notes: Results for regression (2.2).

$$Y_{ft} = \alpha_1 Y_{ft-1} + \alpha_2 Y_{ft-2} + \beta_1 Y_{ft-1} Z_t + \beta_2 Y_{ft-2} Z_t + \varphi_{st} + \varphi_f + \varepsilon_{ft}$$

where Y_{ft} is an export dummy $Y_{ft} = X_{ft} = 1$ if firm f exports in year t (columns 1 and 2) or a capital intensity dummy $Y_{ft} = K_{ft} = 1$ if firm f is above the median capital intensity of its own industry in year t (columns 3 and 4). All specifications use a commodity boom indicator that equals 1 in the period 2004-2012 and zero otherwise; φ_{st} and φ_f are sector-year and firm fixed effects. Standard errors in parentheses.

4.4 Aggregate Productivity and Macroeconomic Ratios

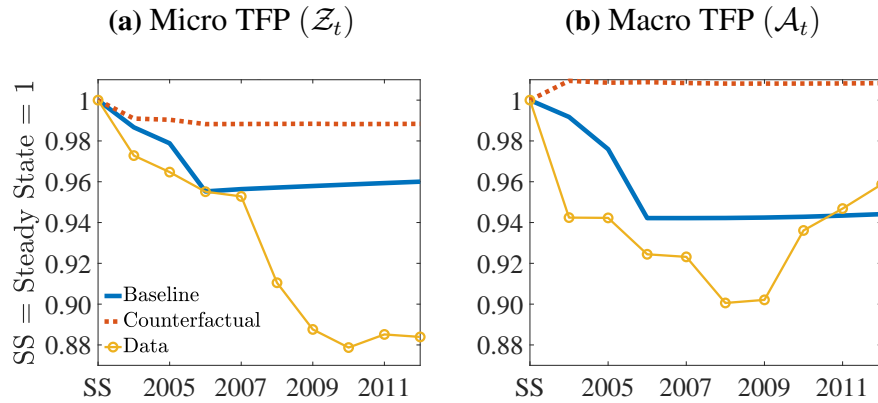
This section links the microeconomic reallocation dynamics described above with aggregate macroeconomic outcomes. First, I compute model-based measures of aggregate productivity and compare them against the data. Next, I assess the model's performance along other critical macroeconomic ratios. I follow the literature in computing model-based productivity measures as weighted average of individual productivity:

$$\mathcal{Z}_t = \sum_f \omega_{ft} \log z_f \quad (4.22)$$

where ω_{ft} is the time-varying (sales-based) weight for firm f in year t , and z_f is the model-based idiosyncratic productivity of firm f . Alternatively, I construct a Solow-based productivity measure using aggregated measures of the exportable sector:

$$\mathcal{A}_t = \frac{p_t^X Y_t^X + p_{xt}^X X_t^X}{(K_t^X)^{\alpha^X} (L_t^X)^{1-\alpha^X}}. \quad (4.23)$$

Figure 8: Productivity: Model vs Data



Notes: The solid lines depict the time series in the baseline model, while the dotted lines correspond to a counterfactual without technology decision. The circled lines are data counterparts. Figures are normalized to one in the steady state (the year 2003 in the data).

Figure 8 presents the time paths for these productivity measures during the commodity

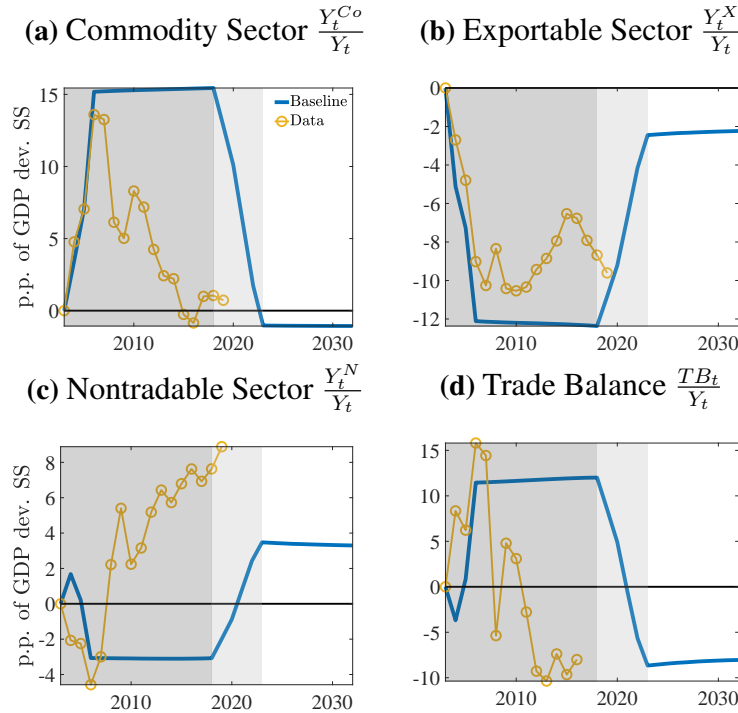
boom. The figures also display data counterparts and the ‘RER only’ counterfactual simulation introduced above. In the model, the economy is in the steady state until $t = 0$ (the year SS=2003), and then adjust to the commodity super-cycle from $t = 1$ onward. In the data, there is time variation from the beginning of the sample in 1995 until 2003 ($t = 0$). I define the average TFP level for the period 1995-2003 as the steady state in the data, and normalize this value to one. On average, in the period 2004-2012, the baseline (counterfactual) model generates 50% (30%) of the productivity fall observed in the microdata (Z_t). The analogous figures for the A_t metric are 40% (20%) .

As noted above, the evolution of these measures reflects reallocation dynamics within the exportable sector from high to low-productivity firms, due to the combined effect of the RER channel and the increase in the cost of capital. The ‘RER only’ counterfactual simulation isolates the pure effect of the RER channel by assuming no heterogeneity in capital intensities; in essence, I drop the firm’s technology decision and re-run the analysis. This counterfactual reveals that both channels are quantitatively important and able to generate an economically significant productivity hangover during commodity booms. On average, the baseline model generates a decline in Z_t that is 2.5 times larger than the ‘counterfactual,’ revealing the strong amplification effects induced by the cost of capital channel. Notably, the ‘RER only’ counterfactual fails to deliver a decline in the Solow-based A_t measure, which is inconsistent with the data. When comparing the baseline simulation against the data counterparts, it is clear that the model has a better fit when considering the period immediately before the Global Financial Crisis of 2008-2009. Arguably, the financial crisis triggers additional unmodeled forces that would explain the ‘excess’ productivity hangover in the data, relative to the baseline model.

Finally, I briefly illustrate the performance of the model across other untargeted macroeconomic moments. I construct (nominal) sectoral value-added ratios as a share of total GDP and the trade balance-to-GDP ratio. Figure 9 reports the results comparing the

baseline model against the analogous time series constructed from macroeconomic data. The figure shows that the model does a reasonable job in reproducing the reallocation patterns *across* economic sectors (panels (a)-(c)), while at the same time capturing the large but transitory trade surplus (panel (d)).

Figure 9: Macroeconomic Ratios: Sectoral Shares and the Trade Balance



Notes: The solid lines depict the time series in the baseline model, while the circled lines are data counterparts. The dark and light gray shades represent the exogenous boom and bust cycle path fed to the model. Figures are measured in percentage points (p.p.) absolute deviation from the steady state.

However, the model also yields a larger and more persistent boom. On the one hand, this behavior is expected as the data series was heavily interrupted by the Global Financial Crisis of 2008, which is not captured in the model's stylized simulation. Second, in practice, it takes time for the commodity boom to be internalized by economic agents, while the simulation here is solved under perfect foresight. [Fornero and Kirchner \(2018\)](#) examine the role of learning about the persistence of commodity price shocks for saving-investment dynamics in a DSGE model estimated for Chile. They provide a

similar intuition for the short-lived trade surplus, as economic agents slowly internalize the positive wealth effect. A fraction of the increase in consumption goes to imported goods, especially under an appreciated real exchange rate. When the boom fades away, the economy consumes the interest payments of the accumulated net foreign assets, the so-called “financial channel” emphasized in [Alberola and Benigno \(2017\)](#).

5 Final Remarks

This paper investigates the firm-level dynamic response of a commodity-dependent economy to persistent global cycles in commodity prices. The article’s main contribution consists of linking firm-level reallocation dynamics with aggregate productivity while recovering classic insights from the so-called Dutch disease literature. As is well-known, commodity booms reallocate market share away from exporters because of currency appreciation and away from capital-intensive firms because of increased capital costs. As these firms are, on average, more productive, the exchange rate and the cost channels combine to reallocate market share from high to low-productivity firms.

I build a multi-sector general equilibrium model with heterogeneous firms able to rationalize the above channels through cross-sectional compositional dynamics while being consistent with important macroeconomic moments. I calibrate the model to microdata for Chile and show that the model is consistent with macroeconomic and cross-sectional moments during the recent commodity price boom of 2003-2012. To effectively focus on the novel role of firm-level reallocation dynamics during a commodity boom episode, the analysis abstract from possible learning-by-doing or learning-by-exporting externalities. I leave for future research the joint study of reallocation with learning spillovers, which opens the door to study the effects of commodity dependence on long-term growth as well as the role of government interventions.

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For Online Publication

Online Appendix for “Reallocation and Productivity during Commodity Cycles”

A Firm Characteristics and TFP

To document in a systematic fashion how exporters and capital-intensive firms outperform their non-exporters and labor-intensive counterparts, I run:

$$\ln(Y_{ft}) = \alpha X_{f0} + \beta K_{f0}^{int} + \delta X_{f0} K_{f0}^{int} + \gamma' Z_{ft} + \varphi_{st} + \varepsilon_{ft} \quad (\text{A.1})$$

where Y_{ft} denotes a productivity measure for firm f in year t , X_{f0} is a dummy variable that takes the value of 1 if firm f exports in its first period $t = 0$ (conditional on $t = 0$ being in the pre-boom period 1995-2003), K_{f0}^{int} denotes firm f period $t = 0$ capital intensity, Z_{ft} are firm-level controls, and φ_{st} represents sector-year fixed effects. Firm-level multi-factor productivity is estimated using the method of [Wooldridge \(2009\)](#) and, under the assumption of constant returns to scale, using cost shares as in [Foster, Haltiwanger, and Krizan \(2001\)](#).

Table 7 presents the results. Pre-boom exporters and capital-intensive firms are significantly more (revenue) productive than their non-exporters and labor-intensive analogs. Similar results emerge when using alternative firm-level outcome variables such as real value-added and real profits.

Table 7: Panel Regressions: Firm Characteristics and Productivity

	Dependent Variable: ln(Productivity)			
	CRS	WLP	CRS	WLP
	Sample: 1995-2007		Sample: 1995-2012	
X_{f0}	0.569 (0.0258)	0.691 (0.0281)	0.611 (0.0253)	0.657 (0.0281)
K_{f0}^{int}	0.084 (0.0061)	0.108 (0.0066)	0.098 (0.0060)	0.111 (0.0066)
$X_{f0}K_{f0}^{int}$	0.155 (0.0142)	0.159 (0.0155)	0.149 (0.0140)	0.172 (0.0157)
Firm FE	no	no	no	no
Sector×Year FE	yes	yes	yes	yes
Adj. R^2	0.080	0.030	0.116	0.066
N. obs.	52,138	52,138	63,687	63,687

Notes: Results for regression (A.1). Standard errors in parentheses. All specifications include a control for firm size (not reported). CRS: Elasticities obtained using cost shares (constant returns to scale). WLP: [Wooldridge \(2009\)](#) estimation (decreasing returns to scale).

B Robustness

Table 8 augments the baseline panel regressions presented in Section 2 with an interaction between firm-level size and the commodity price shock. The purpose of this interaction is to check the robustness of my main results to a financial friction channel that differentially affects firms of different sizes. Column (1) in Table 8 displays the baseline result. Columns (2)-(4) show that the results survive the introduction of these interactions.

Table 8: Panel Regressions: Commodity Booms and Outcome Variables

	Dependent Variable: ln (Real Profits)			
	(1)	(2)	(3)	(4)
$X_{f0}\tilde{P}_{t-1}^{Co}$	-0.079 (0.0292)	-0.082 (0.0291)	-0.059 (0.0302)	-0.062 (0.0302)
$K_{f0}^{int}\tilde{P}_{t-1}^{Co}$	-0.024 (0.0074)	-0.026 (0.0076)	-0.019 (0.0076)	-0.022 (0.0077)
$X_{f0}K_{f0}^{int}\tilde{P}_{t-1}^{Co}$	-0.032 (0.0152)	-0.031 (0.0152)	-0.033 (0.0152)	-0.033 (0.0152)
$TFP_{f0}\tilde{P}_{t-1}^{Co}$		-0.033 (0.0181)		-0.036 (0.0182)
$SIZE_{f0}\tilde{P}_{t-1}^{Co}$			-0.018 (0.0074)	-0.019 (0.0074)
Firm FE	yes	yes	yes	yes
Sector×Year FE	yes	yes	yes	yes
Adj. R^2	0.169	0.169	0.169	0.169
N. obs.	59,281	59,281	59,281	59,281

	Dependent Variable: ln (Real value-added)			
	(1)	(2)	(3)	(4)
$X_{f0}\tilde{P}_{t-1}^{Co}$	-0.092 (0.0291)	-0.095 (0.0292)	-0.077 (0.0300)	-0.079 (0.0300)
$K_{f0}^{int}\tilde{P}_{t-1}^{Co}$	-0.021 (0.0073)	-0.023 (0.0074)	-0.018 (0.0075)	-0.020 (0.0075)
$X_{f0}K_{f0}^{int}\tilde{P}_{t-1}^{Co}$	-0.032 (0.0157)	-0.031 (0.0156)	-0.033 (0.0157)	-0.032 (0.0157)
$TFP_{f0}\tilde{P}_{t-1}^{Co}$		-0.027 (0.0190)		-0.029 (0.0190)
$SIZE_{f0}\tilde{P}_{t-1}^{Co}$			-0.014 (0.0075)	-0.015 (0.0075)
Firm FE	yes	yes	yes	yes
Sector×Year FE	yes	yes	yes	yes
Adj. R^2	0.169	0.169	0.169	0.169
N. obs.	59,281	59,281	59,281	59,281

Notes: Results for regression (2.1) with additional controls. Standard errors in parentheses. The variables $SIZE_{f0}$ and TFP_{f0} are constructed as firm f quintile in the size and productivity distributions in its first period $t = 0$ in the sample. Size is measured as the number of workers, while firm-level productivity is estimated using the method of Wooldridge (2009).

C General Equilibrium System

The full set of general equilibrium conditions can be written as a system of the 44 equations listed in this Appendix, in the following 44 endogenous variables:

$$\begin{aligned}
 \text{Endogenous (44)} &= \{C^N, C^T, C^X, C^M, Y^X, Y^N, Y^{Co}, p^{X^X}\} = 8 \\
 &= \{L^N, L^X, L^{Co}, K^N, K^X, K^{Co}, ID^N, ID^M\} = 8 \\
 &= \{C, I, K, L, Y, X, M, TB, REN, B, r^k, w, p, p^N, p^T, p^X, p^I\} = 17 \\
 &= \{V(z), V_{sj}(z), \mu(z), \bar{z}_d, \bar{z}_h, \bar{z}_x, \mathcal{M}, \mathcal{M}_e, \phi_j, \Phi(\cdot), \mathcal{F}\} = 11
 \end{aligned}$$

C.1 Household

The household chooses $\{C_t, I_t, K_{t+1}, B_{t+1}\}$ to maximize (3.1) subject to the budget constraint (3.5) and law of motion for capital (3.6). The first-order conditions are given by:

$$\varphi L_t^{\zeta-1} = \frac{w_t}{p_t} \quad (\text{C.1})$$

$$\frac{1}{(1+r^*)} = \beta \left[\frac{C_t - \varphi \frac{L_t^\zeta}{\zeta}}{C_{t+1} - \varphi \frac{L_{t+1}^\zeta}{\zeta}} \right]^v \left(\frac{p_t}{p_{t+1}} \right) \quad (\text{C.2})$$

$$p_t^I + \phi \left(\frac{K_{t+1}}{K_t} - 1 \right) = \beta \left[r_{t+1}^k + (1 - \delta^k) p_{t+1}^I + adj_{t+1} \right] \quad (\text{C.3})$$

$$adj_t = \phi \left(\frac{K_{t+1}}{K_t} \right) \left(\frac{K_{t+1}}{K_t} - 1 \right) - \frac{\phi}{2} \left(\frac{K_{t+1}}{K_t} - 1 \right)^2$$

$$K_{t+1} = (1 - \delta^k) K_t + I_t \quad (\text{C.4})$$

The nested CES aggregators (3.2) and (3.3) yield the following demand system:

$$C_t^N = \chi_N \left(\frac{p_t}{p_t^N} \right)^\epsilon C_t \quad (\text{C.5})$$

$$C_t^T = (1 - \chi_N) \left(\frac{p_t}{p_t^T} \right)^\epsilon C_t \quad (\text{C.6})$$

$$C_t^X = \chi_X \left(\frac{p_t}{p_t^X} \right)^\epsilon C_t^T \quad (\text{C.7})$$

$$C_t^M = (1 - \chi_X) (p_t^T)^\epsilon C_t^T \quad (\text{C.8})$$

with prices given by (recall the price of importable goods is the numeraire $p_t^M = 1$):

$$p_t = \left[\chi_N (p_t^N)^{1-\epsilon} + (1 - \chi_N) (p_t^T)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} \quad (\text{C.9})$$

$$p_t^T = \left[\chi_X (p_t^X)^{1-\epsilon} + (1 - \chi_X) \right]^{\frac{1}{1-\epsilon}}. \quad (\text{C.10})$$

C.2 Exportable Varieties

The production function is of the form $y = z_{sj} k_{sj}^{\alpha_j} l_{sj}^{1-\alpha_j}$, where k and l denote capital and labor for a firm with status $s = \{d, x\}$ and $j = \{l, h\}$, and z_{sj} denotes effective idiosyncratic productivity given by:

$$z_{sj} = \begin{cases} z & \text{if } s = d \text{ and } j = l; \text{ pays } f_d & \text{(non-exporter; labor-intensive)} \\ z\kappa_h & \text{if } s = d \text{ and } j = h; \text{ pays } f_d + f_h & \text{(non-exporter; capital-intensive)} \\ z\kappa_x & \text{if } s = x \text{ and } j = l; \text{ pays } f_d + f_x & \text{(exporter; labor-intensive)} \\ z\kappa_x\kappa_h & \text{if } s = x \text{ and } j = h; \text{ pays } f_d + f_x + f_h & \text{(exporter; capital-intensive)} \end{cases}$$

This technology gives rise to a composite input price of the form:

$$\phi_{jt} = \left(\frac{r_t^k}{\alpha_j} \right)^{\alpha_j} \left(\frac{w_t}{1 - \alpha_j} \right)^{1 - \alpha_j}, \quad j = \{l, h\} \quad (\text{C.11})$$

Thus, the cost function and pricing rule for any $s = \{d, x\}$ and $j = \{l, h\}$ can be written as:

$$\begin{aligned} c_{sjt}(z) &= \frac{\phi_j}{z_{sjt}} \\ p_{sjt}(z) &= \frac{1}{\rho} \cdot c_{sjt}(z) \end{aligned}$$

The demand system for any triplet $S = \{D, X\}$ (sales destination), $s = \{d, x\}$ (exporter status), and $j = \{l, h\}$ (technology choice) is given by:

$$q_{sjt}^S(z) = \begin{cases} \left[\frac{p_{sjt}(z)}{p_t^X} \right]^{-\sigma} C_t & \text{if } S = D, s = d, x \\ \left[p_{sjt}(z) \right]^{-\sigma} \gamma & \text{if } S = X, s = x \end{cases}$$

Firm-level profits from domestic sales ($S = D$) are given by:

$$\pi_{sjt}^D(z) = \begin{cases} \pi_{dlt}^D(z) = [p_{dlt}(z) - c_{dlt}(z)] q_{dlt}^D(z) - f_d \\ \pi_{dht}^D(z) = [p_{dht}(z) - c_{dht}(z)] q_{dht}^D(z) - f_d - f_h \\ \pi_{xlt}^D(z) = [p_{dlt}(z) - c_{dlt}(z)] q_{xlt}^D(z) - f_d \\ \pi_{xht}^D(z) = [p_{dht}(z) - c_{dht}(z)] q_{xht}^D(z) - f_d - f_h \end{cases}$$

Similarly, firm-level profits from export sales ($S = X$) are given by:

$$\pi_{sjt}^X(z) = \begin{cases} \pi_{dlt}^X(z) = 0 \\ \pi_{dht}^X(z) = 0 \\ \pi_{xlt}^X(z) = [p_{xht}(z) - c_{xht}(z)] q_{xlt}^X(z) - f_x \\ \pi_{xht}^X(z) = [p_{xht}(z) - c_{xht}(z)] q_{xht}^X(z) - f_x \end{cases}$$

Thus, total profits can be written as in equation (3.9) in the main text:

$$\pi_{sjt}(z) = \begin{cases} \pi_{dlt}^D(z) + \pi_{dlt}^X(z) = \frac{1}{\sigma} \left(\frac{1}{\rho} \cdot \frac{\phi_{lt}}{z} \right)^{1-\sigma} (p_t^X)^\sigma C_t - f_d \\ \pi_{dht}^D(z) + \pi_{dht}^X(z) = \frac{1}{\sigma} \left(\frac{1}{\rho} \cdot \frac{\phi_{ht}}{z\kappa_h} \right)^{1-\sigma} (p_t^X)^\sigma C_t - f_d - f_h \\ \pi_{xlt}^D(z) + \pi_{xlt}^X(z) = \frac{1}{\sigma} \left(\frac{1}{\rho} \cdot \frac{\phi_{lt}}{z\kappa_x} \right)^{1-\sigma} ((p_t^X)^\sigma C_t + \gamma) - f_d - f_x \\ \pi_{xht}^D(z) + \pi_{xht}^X(z) = \frac{1}{\sigma} \left(\frac{1}{\rho} \cdot \frac{\phi_{ht}}{z\kappa_x\kappa_h} \right)^{1-\sigma} ((p_t^X)^\sigma C_t + \gamma) - f_d - f_x - f_h \end{cases}$$

Given profits, we can evaluate the value functions of a firm with productivity level z :

$$V_t(z) = \max\{V_{dlt}(z), V_{dht}(z), V_{xlt}(z), V_{xht}(z)\} \quad (\text{C.12})$$

$$V_{sjt}(z) = \max\{0, \pi_{sjt}(z) + (1 - \delta)\beta V_{t+1}(z)\}; \quad s = \{d, x\}; \quad j = \{l, h\} \quad (\text{C.13})$$

Given the value functions, I pin down the productivity cutoffs from the marginal conditions:

$$V_{dlt}(\bar{z}_{dt}) = 0 \quad (\text{C.14})$$

$$V_{dlt}(\bar{z}_{xt}) = V_{xlt}(\bar{z}_{xt}) \quad (\text{C.15})$$

$$V_{xlt}(\bar{z}_{ht}) = V_{xht}(\bar{z}_{ht}) \quad (\text{C.16})$$

The free entry condition pins down the mass of entrants and is given by:

$$\int_{\bar{z}_{dt}}^{\infty} V_t(z)g(z)dz = \bar{f}_e + \phi_e [\exp(\mathcal{M}_{et} - \bar{\mathcal{M}}_e) - 1] \quad (\text{C.17})$$

while the law of motion for the distribution and the mass of active firms are:

$$\mathcal{M}_{t+1}\mu_{t+1}(z) = \begin{cases} (1 - \delta)\mathcal{M}_t\mu_t(z) + \mathcal{M}_{et+1}g(z), & \text{if } z \geq \bar{z}_{dt+1} \\ 0, & \text{otherwise} \end{cases} \quad (\text{C.18})$$

$$\mathcal{M}_{t+1} = (1 - \delta)\mathcal{M}_t \int_{\bar{z}_{dt+1}}^{\infty} \mu_t(z)dz + \mathcal{M}_{et+1} \int_{\bar{z}_{dt+1}}^{\infty} g(z)dz \quad (\text{C.19})$$

Next, I deal with aggregation. Integrating the production of individual varieties yields:

$$Y_t^X = \left[\mathcal{M}_t \left(\int_{\bar{z}_{dt}}^{\bar{z}_{xt}} (q_{dlt}^D(z))^\rho \mu_t(z)dz + \int_{\bar{z}_{xt}}^{\bar{z}_{ht}} (q_{xlt}^D(z))^\rho \mu_t(z)dz + \int_{\bar{z}_{ht}}^{\infty} (q_{xht}^D(z))^\rho \mu_t(z)dz \right) \right]^{\frac{1}{\rho}} \quad (\text{C.20})$$

Similarly, I aggregate the value of foreign sales in the X sector:

$$pX_t^X \equiv \mathcal{M}_t \left[\int_{\bar{z}_{xt}}^{\bar{z}_{ht}} p_{xlt}(z)q_{xlt}^X(z)\mu_t(z)dz + \int_{\bar{z}_{ht}}^{\infty} p_{xht}(z)q_{xht}^X(z)\mu_t(z)dz \right] \quad (\text{C.21})$$

Finally, we need to aggregate capital and labor used in the whole exportable sector. Given the cost structure outlined above, we can write the total cost function, which combines labor and

capital used both directly in production and to cover the fixed operational costs, as:

$$TC_{sjt}^S(z) = \frac{q_{sjt}^S(z)}{z_{sj}} \phi_{jt} + \mathbb{F}; \quad S = D, X; \quad s = d, x; \quad j = l, h;$$

where $\mathbb{F} = [f_d + f_h \mathbf{1}(j = h)] \mathbf{1}(s = d) + f_x \mathbf{1}(s = x)$ collects fixed costs for any pair $s = \{d, x\}$ and $j = \{l, h\}$. By Shepard's lemma, input demands by a type- z firm are:

$$l_{sjt}^S(z) = \begin{cases} \frac{(1-\alpha_j)\phi_{jt}}{w_t} \cdot \left[(p_t^X)^\sigma C_t^X \left(\frac{\rho}{\phi_j} \right)^\sigma (z_{sj})^{\sigma-1} \right] & \text{if } S = D \\ \frac{(1-\alpha_j)\phi_{jt}}{w_t} \cdot \left[\gamma \left(\frac{\rho}{\phi_j} \right)^\sigma (z_{sj})^{\sigma-1} \right] & \text{if } S = X \end{cases}$$

$$k_{sjt}^S(z) = \begin{cases} \frac{\alpha_j \phi_{jt}}{r_t^k} \cdot \left[(p_t^X)^\sigma C_t^X \left(\frac{\rho}{\phi_{jt}} \right)^\sigma (z_{sj})^{\sigma-1} \right] & \text{if } S = D \\ \frac{\alpha_j \phi_{jt}}{r_t^k} \cdot \left[\gamma \left(\frac{\rho}{\phi_{jt}} \right)^\sigma (z_{sj})^{\sigma-1} \right] & \text{if } S = X \end{cases}$$

where it is easy to see that $l_{dj}^X = k_{dj}^X = 0$ for any j (domestic firms $s = d$ do not sell abroad $S = X$). Finally, integrating over all active types, I compute aggregate input demands in the exportable sector:

$$L_t^X = \mathcal{M}_t \int_{\bar{z}_{dt}}^{\bar{z}_{xt}} l_{dlt}^D(z) \mu_t(z) dz + \mathcal{M}_t \int_{\bar{z}_{xt}}^{\bar{z}_{ht}} (l_{xlt}^D(z) + l_{xlt}^X(z)) \mu_t(z) dz$$

$$+ \mathcal{M}_t \int_{\bar{z}_{ht}}^{\infty} (l_{xht}^D(z) + l_{xht}^X(z)) \mu_t(z) dz \quad (\text{C.22})$$

$$K_t^X = \mathcal{M}_t \int_{\bar{z}_{dt}}^{\bar{z}_{xt}} k_{dlt}^D(z) \mu_t(z) dz + \mathcal{M}_t \int_{\bar{z}_{xt}}^{\bar{z}_{ht}} (k_{xlt}^D(z) + k_{xlt}^X(z)) \mu_t(z) dz$$

$$+ \mathcal{M}_t \int_{\bar{z}_{ht}}^{\infty} (k_{xht}^D(z) + k_{xht}^X(z)) \mu_t(z) dz \quad (\text{C.23})$$

C.3 Commodity Good

This sector features a representative firm that maximizes $\Pi_t^{Co} = p_t^{Co} Y_t^{Co} - w_t L_t^{Co} - r_t^k K_t^{Co}$ subject to the DRS technology:

$$Y_t^{Co} = A^{Co} \left[(K_t^{Co})^{\alpha^{Co}} (L_t^{Co})^{1-\alpha^{Co}} \right]^\eta.$$

The first order conditions are:

$$K_t^{Co} = \eta(\alpha^{Co}) \frac{p_t^{Co} Y_t^{Co}}{r_t^k} \quad (\text{C.24})$$

$$L_t^{Co} = \eta(1 - \alpha^{Co}) \frac{p_t^{Co} Y_t^{Co}}{w_t} \quad (\text{C.25})$$

Plugging these back into the DRS technology yields:

$$[Y_t^{Co}]^{\frac{1-\eta}{\eta}} = \frac{p_t^{Co}}{\left[\frac{1}{\eta} \left(\frac{r_t^k}{\alpha^{Co}} \right)^{\alpha^{Co}} \left(\frac{w_t}{1-\alpha^{Co}} \right)^{1-\alpha^{Co}} \right]} \quad (C.26)$$

C.4 Nontradable Good

There is a representative firm that maximizes $\Pi_t^N = p_t^N Y_t^N - w_t L_t^N - r_t^k K_t^N$ subject to the CRS technology:

$$Y_t^N = A^N (K_t^N)^{\alpha^N} (L_t^N)^{1-\alpha^N}.$$

The first-order conditions for the nontradable sector are also fairly standard:

$$K_t^N = p_t^N \alpha^N \frac{Y_t^N}{r_t^k} \quad (C.27)$$

$$L_t^N = p_t^N (1 - \alpha^N) \frac{Y_t^N}{w_t} \quad (C.28)$$

Plugging these back in the production function yields:

$$p_t^N = \left(\frac{1}{A^N} \right) \left(\frac{r_t^k}{\alpha^N} \right)^{\alpha^N} \left(\frac{w_t}{1 - \alpha^N} \right)^{1-\alpha^N} \quad (C.29)$$

C.5 Investment Good

The CES technology to produce investment goods is given by:

$$I_t = \left[\chi_I^{\frac{1}{\xi}} (ID_t^N)^{\frac{\xi-1}{\xi}} + (1 - \chi_I)^{\frac{1}{\xi}} (ID_t^M)^{\frac{\xi-1}{\xi}} \right]^{\frac{\xi}{\xi-1}}.$$

Cost minimization yields the following demands:

$$ID_t^N = \chi_I \left(\frac{p_t^I}{p_t^N} \right)^{\xi} I_t \quad (C.30)$$

$$ID_t^M = (1 - \chi_I) (p_t^I)^{\xi} I_t, \quad (C.31)$$

which plugged back into the CES technology yields the price of investment:

$$p_t^I = \left[\chi_I (p_t^N)^{1-\xi} + (1 - \chi_I) \right]^{\frac{1}{1-\xi}} \quad (C.32)$$

C.6 Market Clearing and Definitions

The model is closed with the following set of market clearing conditions:

$$Y_t^X = C_t^X \quad (\text{C.33})$$

$$Y_t^N = C_t^N + ID_t^N \quad (\text{C.34})$$

$$L_t = L_t^X + L_t^N + L_t^{Co} \quad (\text{C.35})$$

$$K_t = K_t^X + K_t^N + K_t^{Co} \quad (\text{C.36})$$

$$B_{t+1} = (1 + r^*)B_t + TB_t + REN_t \quad (\text{C.37})$$

in which the following definitions apply:

$$TB_t \equiv X_t - M_t \quad (\text{C.38})$$

$$REN_t \equiv -(1 - \tau)p_t^{Co}Y_t^{Co} \quad (\text{C.39})$$

$$X_t \equiv p_t^{Co}Y_t^{Co} + pX_t^X \quad (\text{C.40})$$

$$M_t \equiv C_t^M + ID_t^M + \Phi_t + \mathcal{F}_t \quad (\text{C.41})$$

$$\Phi_t \equiv \frac{\phi}{2} \left(\frac{K_{t+1}}{K_t} - 1 \right)^2 K_t \quad (\text{C.42})$$

$$\mathcal{F}_t \equiv \mathcal{M}_t f_d + \mathcal{M}_t p_{xt} f_x + \mathcal{M}_t p_{ht} f_h + \mathcal{M}_{et} f_e(\mathcal{M}_{et}) \quad (\text{C.43})$$

$$Y_t \equiv p_t^X Y_t^X + X_t^X + p_t^N Y_t^N + p_t^{Co} Y_t^{Co} - \Phi_t - \mathcal{F}_t \quad (\text{C.44})$$

where p_{xt} and p_{ht} in (C.43) denote the time-varying fractions of exporters and capital-intensive firms, given by:

$$p_{xt} = \frac{\int_{\bar{z}_{xt}}^{\infty} \mu_t(z) dz}{\int_{\bar{z}_{dt}}^{\infty} \mu_t(z) dz}$$

$$p_{ht} = \frac{\int_{\bar{z}_{ht}}^{\infty} \mu_t(z) dz}{\int_{\bar{z}_{dt}}^{\infty} \mu_t(z) dz}.$$

C.7 Transition Algorithm

- **Set up:** The economy is in the calibrated initial steady state up until $t = 0$. The commodity price boom-bust cycle $\{p_t^{Co}\}_{t=1}^T$ (illustrated in panel a of Figure 5) is revealed once and for all in period $t = 1$.
- **Initial state:** $\{B_0, K_0, \mathcal{M}_0, \mu_0(z)\}$ is given.
- **Outer loop:** Guess C_1 . Bisection update using transversality condition.
- **Inner loop:** Guess $\{w_t, p_t^X, p_t^N, K_{t+1}\}_{t=1}^T$.
 - **Households.**
 - * Get $\{p_t^T\}_{t=1}^T$ using (C.10), $\{p_t\}_{t=1}^T$ using (C.9), $\{L_t\}_{t=1}^T$ using (C.1), $\{C_{t+1}\}_{t=1}^T$ using (C.2), $\{p_t^I\}_{t=1}^T$ using (C.32), $\{r_{t+1}^k\}_{t=1}^T$ using (C.3), $\{I_t\}_{t=1}^T$ using (C.4), $\{C_t^N, C_t^T, C_t^X, C_t^M\}_{t=1}^T$ using (C.5)-(C.8), $\{ID_t^N, ID_t^M\}_{t=1}^T$ using (C.30)-(C.31), $\{\phi_{jt}\}_{t=1}^T, j = l, h$ using (C.11).
 - Set period $t = T$ (final steady state) value function vector $V_T(z)$.
 - **Iterate backwards.** For $t = T - 1 : -1 : 1$
 - * Compute value functions and cutoffs via (C.12)-(C.16).
 - * Use (C.17) to get the mass of entrants \mathcal{M}_{et} .
 - **Iterate forward.** For $t = 1 : T$
 - * Get mass \mathcal{M}_t and distribution $\mu_t(z)$ using (C.18)-(C.19).
 - **Aggregation.**
 - * Get $\{L_t^X, K_t^X, pX_t^X, Y_t^X\}_{t=1}^T$ using (C.22), (C.23), (C.21) and (C.33).
 - * Get $\{Y_t^N, L_t^N, K_t^N\}_{t=1}^T$ using (C.34), (C.28) and (C.27).
 - * Get $\{Y_t^C, L_t^{Co}, K_t^{Co}\}_{t=1}^T$ using (C.26), (C.24) and (C.25).
 - **Updating: model-implied sequences.**
 - * $\{p_t^N\}_{t=1}^T$ using (C.29).
 - * $\{p_t^X\}_{t=1}^T$ using (C.20).
 - * $\{w_t\}_{t=1}^T$ using (C.35).
 - * $\{K_{t+1}\}_{t=1}^T$ using (C.36).
 - **Iterate** over $\{w_t, p_t^X, p_t^N, K_{t+1}\}_{t=1}^T$ until convergence. Exit inner loop.
- **Definitions:** Get $\{\Phi_t, \mathcal{F}_t, Y_t\}_{t=1}^T$ using (C.42), (C.43), (C.44).
- **Trade balance:** Get $\{X_t, M_t, TB_t, REN_t\}_{t=1}^T$ using (C.40), (C.41), (C.38), (C.39).
- **NFA:** Get $\{B_{t+1}\}_{t=1}^T$ from (C.37).
- **Iterate** over C_1 until $\{B_{t+1}\}_{t=1}^T$ is stable in the long run. Exit outer loop.

D Steady State System

The Euler equation for bond holdings collapses to $\beta = \frac{1}{1+r^*}$. I solve for an initial steady state given a net foreign asset position level B from the data.

$$\begin{aligned}
\text{Endogenous (45)} &= \{C^N, C^T, C^X, C^M, Y^X, Y^N, Y^{Co}, pX^X\} = 8 \\
&= \{L^N, L^X, L^{Co}, K^N, K^X, K^{Co}, ID^N, ID^M\} = 8 \\
&= \{C, I, K, L, Y, X, M, TB, REN, r^k, w, p, p^N, p^T, p^X, p^I\} = 16 \\
&= \{V(z), V_{sj}(z), \mu(z), \bar{z}_d, \bar{z}_h, \bar{z}_x, \mathcal{M}, \mathcal{M}_e, \phi_j, \mathcal{F}\} = 10 \\
&= \{YCY, TBY, RENY\} = 3
\end{aligned}$$

D.1 Household

$$p = \left[\chi_N (p^N)^{1-\epsilon} + (1 - \chi_N) (p^T)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} \quad (\text{D.1})$$

$$p^T = \left[\chi_X (p^X)^{1-\epsilon} + (1 - \chi_X) \right]^{\frac{1}{1-\epsilon}} \quad (\text{D.2})$$

$$p^I = \left[\chi_I (p^N)^{1-\xi} + (1 - \chi_I) \right]^{\frac{1}{1-\xi}} \quad (\text{D.3})$$

$$C^N = \chi_N \left(\frac{p}{p^N} \right)^\epsilon C \quad (\text{D.4})$$

$$C^T = (1 - \chi_N) \left(\frac{p}{p^T} \right)^\epsilon C \quad (\text{D.5})$$

$$C^X = \chi_X \left(\frac{p^T}{p^X} \right)^\epsilon C^T \quad (\text{D.6})$$

$$C^M = (1 - \chi_X) (p^T)^\epsilon C^T \quad (\text{D.7})$$

$$ID^N = \chi_I \left(\frac{p^I}{p^N} \right)^\xi I \quad (\text{D.8})$$

$$ID^M = (1 - \chi_I) (p^I)^\xi I \quad (\text{D.9})$$

$$I = \delta^k K \quad (\text{D.10})$$

$$\varphi L^{\zeta-1} = \frac{w}{p} \quad (\text{D.11})$$

$$r^k = \left(\frac{1}{\beta} - 1 + \delta^k \right) p^I \quad (\text{D.12})$$

D.2 Exportable Goods

$$\phi_j = \left(\frac{r^k}{\alpha_j}\right)^{\alpha_j} \left(\frac{w}{1-\alpha_j}\right)^{1-\alpha_j}, \quad j = l, h \quad (\text{D.13})$$

$$V(z) = \max\{V_{dl}(z), V_{dh}(z), V_{xl}(z), V_{xh}(z)\} \quad (\text{D.14})$$

$$V_{sj}(z) = \frac{\pi_{sj}(z)}{1 - (1-\delta)\beta}; \quad s = d, x; \quad j = l, h \quad (\text{D.15})$$

$$V_{dl}(\bar{z}_d) = 0 \quad (\text{D.16})$$

$$V_{dl}(\bar{z}_x) = V_{xl}(\bar{z}_x) \quad (\text{D.17})$$

$$V_{xl}(\bar{z}_h) = V_{xh}(\bar{z}_h) \quad (\text{D.18})$$

$$\bar{f}_e = \int_{\bar{z}_d}^{\infty} V(z)g(z)dz - \phi_e [\exp(\mathcal{M}_e - \bar{\mathcal{M}}_e) - 1] \quad (\text{D.19})$$

$$\mu(z) = \begin{cases} \frac{g(z)}{1-G(\bar{z}_d)}, & \text{if } z \geq \bar{z}_d \\ 0, & \text{otherwise} \end{cases} \quad (\text{D.20})$$

$$\delta\mathcal{M} = [1 - G(\bar{z}_d)] \mathcal{M}_e \quad (\text{D.21})$$

$$K^X = \mathcal{M} \left[\int_{\bar{z}_d}^{\bar{z}_x} k_{dl}^D(z) + \int_{\bar{z}_x}^{\bar{z}_h} (k_{xl}^D(z) + k_{xl}^X(z)) + \int_{\bar{z}_h}^{\infty} (k_{xh}^D(z) + k_{xh}^X(z)) \right] \mu(z) dz \quad (\text{D.22})$$

$$L^X = \mathcal{M} \left[\int_{\bar{z}_d}^{\bar{z}_x} l_{dl}^D(z) + \int_{\bar{z}_x}^{\bar{z}_h} (l_{xl}^D(z) + l_{xl}^X(z)) + \int_{\bar{z}_h}^{\infty} (l_{xh}^D(z) + l_{xh}^X(z)) \right] \mu(z) dz \quad (\text{D.23})$$

$$Y^X = \left[\mathcal{M} \left(\int_{\bar{z}_d}^{\bar{z}_x} (q_{dl}^D(z))^\rho + \int_{\bar{z}_x}^{\bar{z}_h} (q_{xl}^D(z))^\rho + \int_{\bar{z}_h}^{\infty} (q_{xh}^D(z))^\rho \right) \mu(z) dz \right]^{\frac{1}{\rho}} \quad (\text{D.24})$$

$$p^X = \frac{1}{\rho} \left[(\phi_l)^{1-\sigma} \tilde{z}_{dl} + \left(\frac{\phi_l}{\kappa_x}\right)^{1-\sigma} \tilde{z}_{xl} + \left(\frac{\phi_h}{\kappa_x \kappa_h}\right)^{1-\sigma} \tilde{z}_{xh} \right]^{\frac{1}{1-\sigma}} \iff$$

$$\tilde{z}_{dl} = \mathcal{M} \int_{\bar{z}_d}^{\bar{z}_x} z^{\sigma-1} \mu(z) dz$$

$$\tilde{z}_{xl} = \mathcal{M} \int_{\bar{z}_x}^{\bar{z}_h} z^{\sigma-1} \mu(z) dz$$

$$\tilde{z}_{xh} = \mathcal{M} \int_{\bar{z}_h}^{\infty} z^{\sigma-1} \mu(z) dz$$

$$p_x^X X^X = \mathcal{M} \left[\int_{\bar{z}_x}^{\bar{z}_h} p_{xl}(z) q_{xl}^X(z) \mu(z) dz + \int_{\bar{z}_h}^{\infty} p_{xh}(z) q_{xh}^X(z) \mu(z) dz \right] \quad (\text{D.25})$$

D.3 Commodity Good

$$Y^{Co} = \bar{R} \left[(K^{Co})^{\alpha^{Co}} (L^{Co})^{1-\alpha^{Co}} \right]^{\eta} \quad (D.26)$$

$$r^k = \eta(\alpha^{Co}) p^{Co} Y^{Co} / K^{Co} \quad (D.27)$$

$$w = \eta(1 - \alpha^{Co}) p^{Co} Y^{Co} / L^{Co} \quad (D.28)$$

D.4 Nontradable Good

$$p^N = \left(\frac{1}{A^N} \right) \left(\frac{r_t^k}{\alpha^N} \right)^{\alpha^N} \left(\frac{w_t}{1 - \alpha^N} \right)^{1-\alpha^N} \quad (D.29)$$

$$r^k = p^N \alpha^N Y^N / K^N \quad (D.30)$$

$$w = p^N (1 - \alpha^N) Y^N / L^N \quad (D.31)$$

D.5 Aggregation

$$Y^X = C^X \quad (D.32)$$

$$Y^N = C^N + ID^N \quad (D.33)$$

$$L = L^X + L^N + L^{Co} \quad (D.34)$$

$$K = K^X + K^N + K^{Co} \quad (D.35)$$

$$B = -\frac{(TB + REN)}{r^*} \quad (D.36)$$

$$TB \equiv X - M \quad (D.37)$$

$$X \equiv p^{Co} Y^{Co} + X^X \quad (D.38)$$

$$M \equiv C^M + ID^M + \mathcal{F} \quad (D.39)$$

$$REN = -(1 - \tau) p_t^{Co} Y_t^{Co} \quad (D.40)$$

$$\mathcal{F} = \mathcal{M}f_d + \mathcal{M}p_x f_x + \mathcal{M}p_h f_h + \mathcal{M}e f_e (\mathcal{M}_e) \quad (D.41)$$

$$Y = p^X Y^X + X^X + p^N Y^N + p^{Co} Y^{Co} - \mathcal{F} \quad (D.42)$$

$$YCY = \frac{p^{Co} Y^C}{Y} \quad (D.43)$$

$$TBY = \frac{TB}{Y} \quad (D.44)$$

$$RENY = \frac{REN}{Y} \quad (D.45)$$

D.6 Steady State Solution Algorithm

The details about the calibration strategy are in the main text (see Table 4). I provide here a pseudo-code to solve the nonlinear system.

$$\begin{aligned} \text{Precalibrated targets} &= \{TBY, RENY, YCY, Y\} \\ \text{Endogenized parameters} &= \{B, \tau, \bar{R}, \varphi\} \end{aligned}$$

Given the precalibrated targets and the exogenous commodity price p^{Co} (normalized to 1).

- Guess initial values for (p^N, p^X, K, C) .
 - From the Euler equation for bonds (not listed above) we have $\beta = 1/(1 + r^*)$.
 - Get (p, p^T, p^I) using (D.1), (D.2), (D.3).
 - Get (r^k, w) using (D.12) and (D.29).
 - Get (C^N, C^X, C^M, C^T) using (D.4), (D.5), (D.6) and (D.7)
 - Get I using (D.10) and (ID^N, ID^M) using (D.8)-(D.9).
 - Get (Y^{Co}, TB, REN) using (D.43), (D.44), (D.45).
 - Get ϕ_j using (D.13).
 - Get values and cutoffs using (D.14)-(D.18).
 - Get distribution $\mu(z)$ using (D.20). Get Y^X using (D.32)
 - Get $(\mathcal{M}, \mathcal{M}_e)$ using (D.24), (D.21). Get \mathcal{F} using (D.41).
 - Get (K^X, L^X, X^X) using (D.22), (D.23) and (D.25).
 - Get (K^{Co}, L^{Co}) using (D.27) and (D.28).
 - Get (Y^N, K^N, L^N) using (D.33), (D.30) and (D.31).
 - Get φ using (D.11). Get \bar{R} using (D.26). Get B using (D.36). Get τ using (D.40).
 - Get (L, K) using (D.34) and (D.35).
 - Get (X, M) using (D.38), (D.39).
 - Use a nonlinear solver to minimize the residuals given by the four remaining equilibrium equations:
 - * the X sector free entry condition (D.19),
 - * the capital market clearing condition (D.35),
 - * the GDP definition (D.42),
 - * the TB definition (D.37).
- Iterate over (p^N, p^X, K, C) until convergence.