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Revisiting the public debt stability condition: Rethinking the Domar condition

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**REVISITING THE PUBLIC DEBT
STABILITY CONDITION: RETHINKING
THE DOMAR CONDITION**

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Abstract

This paper revisits the fiscal sustainability condition proposed by Domar (1944). The Domar condition is derived from the government budget constraint and thus focuses on the supply side of the government bond. By considering the demand for the government bond, this paper finds that public debt sustainability depends on interest rate sensitivity to changes in government bond supply and demand. Data shows that the prediction of our model on public debt sustainability is consistent with the cases of Greece and Japan.

Keywords: Domar condition, fiscal sustainability, demand for government bonds

JEL Classification: E12, E62, H63

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1. INTRODUCTION

Fiscal sustainability has been a central consideration in fiscal policy analysis. The recent Coronavirus pandemic pushed many countries to provide emergency government funds to those who had been affected by the shutdowns of businesses. Huge spending means huge increases in government debt, which will increase the danger of debt explosion.

To examine whether public debt is sustainable or not, the literature often uses the Domar condition. The Domar condition determines fiscal sustainability by comparing the interest rate of government bonds and the growth rate of the economy. Recently, Krugman (2020) used the Domar condition and claimed that a fiscal explosion can be avoided since the United States (US) has a very low interest rate, which is lower than growth rate of the economy.

This paper revisits the Domar condition by considering the demand for government bonds. As the Domar condition is obtained from the government budget constraint, it focuses only on the supply of government bonds and does not take into account the demand for government bonds. Since the United States uses the key international currency and the demand for government bonds comes from everywhere in the world, it may be justifiable to use the Domar condition to examine the fiscal sustainability in the US. However, many other countries do not issue US-denominated government bonds and local currency bonds have limited demand. Thus, it may not be appropriate to use the Domar condition in those countries.

This paper shows that the stability condition for government budgets will be different from the Domar condition once the demand for government bonds is taken into account. We find that public debt stability depends on interest rate sensitivity to changes in the government bond supply and demand. If an increase in the interest rate in order to supply one more unit of the government bond is higher than that of demand, the interest burden of bond supply becomes larger than the demand side. Thus, the government has to pay a higher interest rate, which leads to an explosion of budget deficits. In contrast, if an increase in the interest rate on the demand side is higher than that of supply side, buyers would pay more to purchase one unit of the government bond than suppliers, and the interest burden for the government will decline. Thus, the stability of the budget will be achieved.

We also examine whether the prediction of our model is empirically plausible by looking at the cases of Greece and Japan. We choose these two economies because they both have high debt-to-GDP ratios but differ in their fiscal sustainability. We find that the prediction of our fiscal sustainability condition is consistent with the fact that while Greece went bankrupt, Japan's fiscal stability is still sustainable, although Japan's debt-to-GDP ratio is higher than that of Greece. This result is explained by the difference between the two countries that lies on the demand side of their government bonds.

The remainder of the paper is organized as follows. Section 2 presents the Domar condition. In Section 3, we describe our model and derive the revised Domar condition by explicitly considering the demand for government bonds. Section 4 presents some empirical analyses. Section 5 concludes.

2. THE DOMAR CONDITION

The Domar condition is often used to judge whether the budget deficit is sustainable. The Domar condition is obtained from the government budget constraint:

$$G_t + r_t^B B_{t-1} = DB_t + T_t,$$

where G_t is government spending, B_t is the stock of public debt, T_t is total tax revenues, and r_t is the interest rate for public debt. By dividing (1) by GDP Y_t , we can obtain

$$b_t - b_{t-1} = g_t - t_t + \frac{r_t - \eta_t}{1 + \eta_t} b_{t-1},$$

where $b_t \equiv B_t/Y_t$, $g_t \equiv G_t/Y_t$, $t_t \equiv T_t/Y_t$, and $\eta_t \equiv \Delta Y_t/Y_t$.

When the primary balance is in equilibrium, if the interest rate exceeds growth ($r_t > \eta_t$), the budget deficit will increase. In contrast, if the interest rate is lower than the growth rate ($r_t < \eta_t$), the budget deficit will decrease—in a stable manner. However, as the government budget constraint focuses only on the supply of government bonds, the Domar condition does not consider the demand for government bonds.

3. THE MODEL

3.1 Demand for Government Bonds

In order to consider the demand for government bonds, we consider the portfolio choice of investors (households).¹ We assume that the utility function of investors depends on the rate of return r_t and risk σ_t :

$$U(r_t, \sigma_t) = r_t - \beta \sigma_t^2,$$

where β is the weight for the risk. A larger β implies that investors give more weight to the risk.

Investors could purchase government bonds and invest. The rate of return on investment is denoted by r_t^I . Then, the total rate of return is

$$r_t = \alpha r_t^B + (1 - \alpha) r_t^I, \quad (2)$$

where α is the ratio of allocation between the government bond and investment.

The aggregate risk is

$$\sigma_t^2 = \alpha^2 (\sigma_t^B)^2 + (1 - \alpha)^2 (\sigma_t^I)^2 + 2\alpha(1 - \alpha) \sigma_t^{BI}.$$

where σ^B is risk associated with the government bond and σ^I is risk for investment, and $\sigma^{BI} = \sigma^B \sigma^I$.

¹ The problem of investors' portfolio choices is similar to that of Yoshino, Taghizadeh-Hesary, and Nakahigashi (2019).

Investors choose the amount to allocate between government bonds and investment to maximize their utility. Thus, the problem of investors is

$$\max_a U(r_t, s_t),$$

subject to (2) and (3). Then, the optimal allocation between government bonds and investment is

$$a^* = \frac{\frac{1}{b}(r_t^B - r_t^I) + (s_t^I)^2 - 2s_t^{BI}}{2(s_t^B)^2 + 2(s_t^I)^2 - 4s_t^{BI}}.$$

This implies that the demand for government bonds depends on the difference between the interest rate of government bonds and the rate of return on investment ($r_t^B - r_t^I$) as well as the risk associated with government bonds and investment. Thus, we have the following demand function for government bonds:

$$DB_t^d = b_0 + b_1(s_t^B, s_t^I)(r_t^B + r_t^I). \quad (4)$$

Similarly, the investment function is obtained by

$$I_t^d = i_0 + i_1(s_t^B, s_t^I)(r_t^B - r_t^I).$$

3.2 Government Bond Market

The supply of government bonds is described by the government budget constraint:

$$G_t + r_t^B B_{t-1} = DB_t^s + T_t + \Delta M_t, \quad (5)$$

where ΔM_t is the supply of money that is printed when government bonds are purchased from the market in an open market operation.

From equations (4) and (5), we have the equilibrium interest rate of government bonds:

$$r_t^{B*} = \frac{G_t - T_t - \Delta M_t - b_0 + b_1(s_t^B, s_t^I)r_t^I}{b_1 - B_{t-1}}. \quad (6)$$

3.3 Equilibrium GDP

The disposable income Y_t^D is income Y plus the interest from the government bond minus tax payments. As the disposable income is divided into consumption C_t and savings S_t , we have

$$Y_t^D = Y_t + r_t^B B_{t-1} - T_t = C_t + S_t. \quad (7)$$

As we discussed above, households save through purchasing government bonds and investment. Thus, we have

$$S_t = DB_t^d + I_t^d. \quad (8)$$

We assume that consumption depends on disposable income, the interest rate of government bonds, the rate of return on investment, and the risk associated with government bonds and investment.

$$C_t = c_0 + c_1 Y_t^D + c_2 (s_t^B, s_t^I)(r_t^B - r_t^I).$$

From equations (7) and (8), we have

$$Y_t = T_t - r_t^B B_{t-1} + C_t + D B_t^d + I_t^d.$$

As $Y_t = C_t + I_t + G_t$, with the above equation, we have

$$Y_t = C_t + I_t + G_t = T_t - r_t^B B_{t-1} + C_t + D B_t^d + I_t^d.$$

With the equilibrium interest rate r_t^{B*} , the equilibrium income is

$$Y_t^* = \frac{1}{1 - c_1} \{ c_0 + c_1 [-T_t + r_t^B B_{t-1}] + c_2 (s_t^B, s_t^I)(r_t^B - r_t^I) + i_0 + i_1 (s_t^B, s_t^I)(r_t^B - r_t^I) + G_t \}.$$

Note that instead of using the LM curve, this model uses the equilibrium condition of the bond market, which is often called BB curve. Thus, our framework is the IS-BB model.²

3.4 Revised Domar Condition

By substituting (6) into equation (5), we have

$$D B_t = G_t + r_t^{B*} B_{t-1} - \bar{T}_t - D M_t. \tag{9}$$

This yields

$$\frac{D B_t}{B_{t-1}} = \frac{r_t^{B*}}{B_{t-1}} B_{t-1} + r_t^{B*}, \tag{10}$$

where

$$\frac{r_t^{B*}}{B_{t-1}} = \frac{G_t - \bar{T}_t - D M_t - b_0 + b_1 (s_t^B, s_t^I) r_t^I}{[b_1 (s_t^B, s_t^I) - B_{t-1}]^2} = \frac{r_t^{B*}}{b_1 (s_t^B, s_t^I) - B_{t-1}}.$$

Then, (10) can be rewritten as

$$\frac{D B_t}{B_{t-1}} = \frac{1}{b_1 (s_t^B, s_t^I) - B_{t-1}} r_t^{B*}. \tag{11}$$

² Stiglitz and Weiss. (1981) use a similar framework to analyze credit rationing.

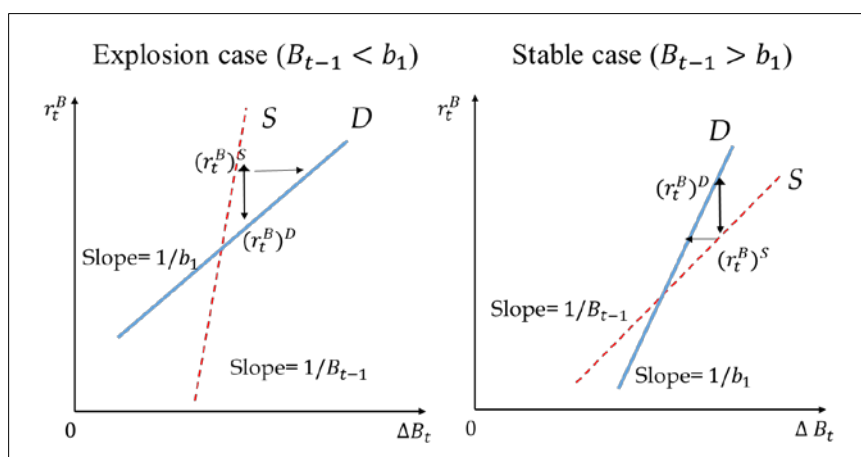
This implies

$$\frac{\frac{\partial \Delta B_t}{\partial r_t^B} > 0}{\frac{\partial B_{t-1}}{\partial r_t^B} < 0} \hat{U} > 1 \frac{\partial B_{t-1}}{\partial r_t^B} < \frac{\partial \Delta B_t}{\partial r_t^B} < \frac{1}{b_1}$$

This result is interpreted as follows. In order to increase the bond supply by one unit, the rate of the government bond has to increase by $1/B_{t-1}$, which is derived from Equation (9). On the other hand, in order to increase government bond demand by one unit, the interest rate of the government bond has to rise by $1/b_1$. If the increase in the interest rate from the supply side is higher than that of demand, namely $1/B_{t-1} > 1/b_1$, the interest burden of the bond supply becomes larger than the demand side. The government that supplies the bond has to pay a higher interest rate. It will lead to an explosion of budget deficits. When the increase in the interest rate to buy one more unit of the government bond is higher than that of supply, namely $1/b_1 > 1/B_{t-1}$, the interest burden for the supplier (government) will become smaller. Since buyers would pay more to purchase one unit of the government bond than the supplier, the interest burden for the government will decline. The stability of the budget will be achieved.

This can be easily understood by looking at Figure 1. In figure 1, dashed and solid lines show the supply curve of the government bond and the demand curve of the government bond, respectively. From equations (4) and (5), the demand and supply for government bonds increase when the interest rate rises. Thus, both demand and supply curves are upward-sloping. Note that the slope of the demand curve is $1/b_1$ and that of the supply curve is $1/B_{t-1}$. When the supply curve is steeper than the demand curve, the change in the interest rate of government bonds in order to change government bond supply by one unit is larger than the change in the interest rate from the demand side. Thus, this leads to an explosion of budget deficits. When the demand curve is steeper than the supply curve, the opposite takes place.

Figure 1: Graphical Explanation of Stability Condition



Source: Authors.

4. EMPIRICAL ANALYSIS

This section empirically examines the validity of our fiscal sustainability condition by looking at the cases of Greece and Japan. We choose these two economies because they both have high debt-to-GDP ratios but differ in their fiscal sustainability. While

Greece went bankrupt, Japan’s fiscal stability is still sustainable although Japan’s debt-to-GDP ratio is higher than Greece. We compare the values of b_1 and B_{t-1} for these two economies.

For simplicity, in order to obtain the value of b_1 , we estimate the following simple linear regression model:

$$\Delta B_{t-1} = b_0 + b_1 r_t^B + e_t,$$

where e_t is the error term.

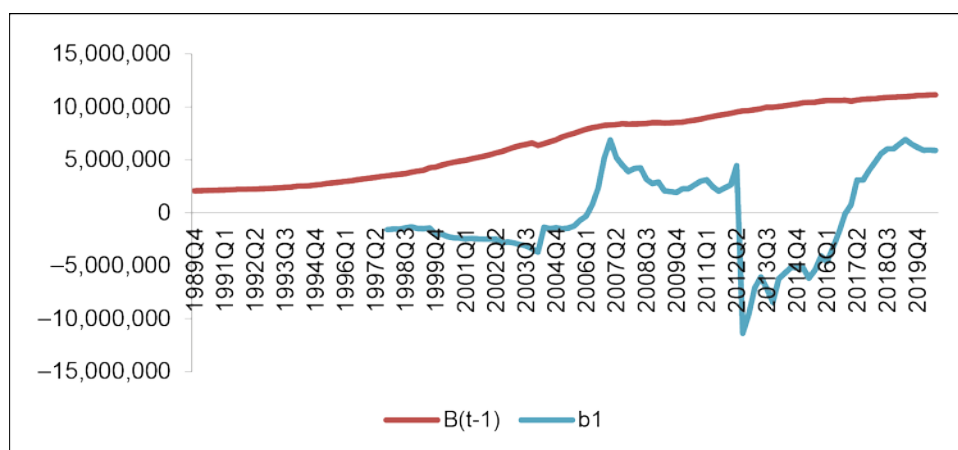
We use the quarterly data for the interest rate of the government bond and the outstanding government debt. The sample period spans the longest timeframe for which data is available. Data comes from the Bank of Japan, Japan’s Ministry of Finance, and Eurostat.

In order to assess the time variation in the relationship between r_t and ΔB_{t-1} , we consider a fixed rolling window estimation. In particular, we consider all possible 8 years subsamples starting at the beginning of the full sample and moving sequentially by one time period. The resulting time series are plotted in Figures 2 and 3 with the time series of B_{t-1} . The estimated parameters are not constant for both economies.

Figure 2 shows that over the sample period, the estimated b_1 is less than the lagged outstanding government debt in Japan. On the other hand, Figure 3 shows that the estimated b_1 was higher than the outstanding government debt in Greece in the period 2009–2010. This result is consistent with the fact that Japan’s fiscal condition is still sustained while Greece went bankrupt.

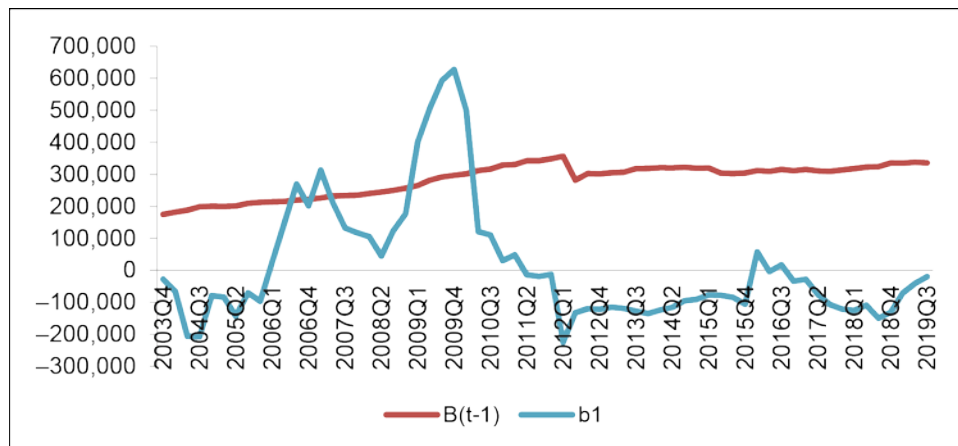
The key difference between Greece and Japan is the demand for the government bond. While more than 90% of Japan’s government bond is held by domestic investors, more than 70% of investors in Greece’s bond market were foreigners. This high ratio of overseas investors was the cause of the increase in the interest rate of government bonds in Greece, which led to the Greek government debt crisis.

Figure 2: Japan’s Case



Source: Authors’ calculation.

Figure 3: Greece's Case



Source: Authors' calculation.

5. CONCLUSION

This paper revisits the Domar fiscal sustainability condition by considering the demand for the government bond. Once the demand for the government bond is taken into account, the stability condition for the government's budget will be different from the Domar condition. Public debt sustainability depends on interest rate sensitivity to changes in the government bond supply and demand. Our empirical analysis finds that the prediction of our model is consistent with the cases of Greece and Japan on public debt sustainability.

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