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Article

GARCH Generated Volatility Indices of Bitcoin and CRIX

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Abstract: In this paper, the pricing performance of the generalised autoregressive conditional heteroskedasticity (GARCH) option pricing model is tested when applied to Bitcoin (BTCUSD). In addition, implied volatility indices (30, 60- and 90-days) of BTCUSD and the Cryptocurrency Index (CRIx) are generated by making use of the symmetric GARCH option pricing model. The results indicate that the GARCH option pricing model produces accurate European option prices when compared to market prices and that the BTCUSD and CRIx implied volatility indices are similar when compared, this is consistent with expectations because BTCUSD is highly weighted when calculating the CRIx. Furthermore, the term structure of volatility indices indicate that short-term volatility (30 days) is generally lower when compared to longer maturities. Furthermore, short-term volatility tends to increase to higher levels when compared to 60 and 90 day volatility when large jumps occur in the underlying asset.

Keywords: cryptocurrency index; Bitcoin; GARCH; volatility index

1. Introduction

The use of volatility indices (often referred to as fear indices) as a measure of market sentiment has become popular in recent years. According to [Fernandes et al. \(2014\)](#), the Chicago Board Options Exchange (CBOE) has published the Volatility Index (VIX) since 1993. The VIX is calculated using near term (30 calendar days) volatility implied by options on the S&P 500 index. Furthermore, the VIX is based on a model-free estimator of implied volatility and therefore does not rely on a particular option pricing framework.

Cryptocurrencies have recently gained a lot of attention from finance researchers and practitioners. Currently, there is not a cryptocurrency volatility index. Furthermore, cryptocurrencies do not have a well-established derivatives market. In a recent paper, [Alexander and Imeraj \(2019\)](#) addressed this problem by comparing two methods to construct a Bitcoin volatility index. The first is based on the standard geometric formula for the sum of squared log price increments; this is consistent with the CBOE VIX methodology. The second (arithmetic) approach represents a fair value for the average sum of squared log price increments.

According to [Bouri et al. \(2017\)](#), short horizon investment in Bitcoin can serve as a hedge against global equity market uncertainty (form of electronic gold)—hence the need for a Bitcoin volatility index with different time horizons. In this paper, by making use of the generalised autoregressive conditional heteroskedasticity (GARCH) option pricing model, the Bitcoin and Cryptocurrency Index (CRIx) volatility indices are estimated. The CRIx implied volatility index will give a more holistic

view of cryptocurrency volatility (30, 60, and 90-day). The estimation of a volatility index for CRIX in the absence of a derivatives market was considered by [Kolesnikova \(2018\)](#), this was based on an exponentially weighted moving average approach. The rest of this paper is structured as follows: Section 2 reviews the recent and relevant literature. Section 3 focuses on the theoretical framework; this section is divided into two parts. The first focuses on the GARCH option pricing framework and the second elaborates on the GARCH volatility index. Thereafter, the statistical properties of Bitcoin and CRIX are illustrated. This is followed by the empirical results. Finally, concluding remarks are considered.

2. Literature Review

This section focuses on recent and relevant literature, and is divided into three subsections. The first subsection focuses on cryptocurrency indices, and the second reviews relevant literature based on GARCH models applied to cryptocurrencies. Finally, studies based on cryptocurrency volatility indices are considered.

2.1. Cryptocurrency Indices

According to [Chu et al. \(2017\)](#), with the exception of Bitcoin, there is not much literature focused on the application of GARCH models to cryptocurrencies. Therefore, the CRIX is also considered in this paper. According to [Abboud \(2017\)](#), there have been several attempts to construct a cryptocurrency index. Most cryptocurrency index attempts make use of empirical models from traditional financial markets with arbitrary parameters fitted to cryptocurrencies. The indices include capitalisation weighted indices like CRIX, Bletchley, TaiFu30, Crypto30, LBI, and Smith + Crown SCI. Furthermore, capped capitalisation indices include: CRYPTO20, CCX30, and BIT20. Finally, the smoothed capitalisation weighted index, such as the CCI30.

According to [Kim et al. \(2019\)](#), the CRIX is comparable to the S&P 500 index (reflection of the current state of the US market) because it gives an indication of the current state of the cryptocurrency market. Furthermore, [Kim et al. \(2019\)](#) explain that the CRIX provides a statistically backed (the number of constituents is determined by the explanatory power of each cryptocurrency has over market movements, this is based on the Akaike information criterion) market measure, which distinguishes it from other cryptocurrency indices. Therefore, the CRIX is used in this study to give an indication of the volatility of the cryptocurrency market as a whole. The CRIX was also used as a proxy for the cryptocurrency market by [Elendner et al. \(2018\)](#); [Hafner \(2020\)](#); [Klein et al. \(2018\)](#).

2.2. GARCH Models Applied to Cryptocurrencies

When it comes to the topic of time-varying volatility, most financial modelling researchers and practitioners will agree that the GARCH model is the most popular. GARCH models applied to cryptocurrencies have gained a lot of attention in recent years (as mentioned previously, most of this work has been based on Bitcoin). In an attempt to forecast Bitcoin risk, [Agyarko et al. \(2019\)](#) made use of univariate symmetric and asymmetric GARCH models. Their empirical results indicate that the symmetric GARCH(1,1) model provides the best fit. This is also consistent with the argument by [Hansen and Lunde \(2005\)](#), that it is difficult to find a model that consistently outperforms the GARCH(1,1) model because it is highly robust and parsimonious. With regard to forecasting risk, [Agyarko et al. \(2019\)](#) explain that no model clearly emerged as superior. Therefore, the study indicates that it is reliable to use the best fitted model when forecasting volatility (symmetric GARCH, in the case of Bitcoin).

[Chen et al. \(2018\)](#) performed an econometric analysis of the CRIX for portfolio investment. The empirical analysis included the application of autoregressive integrated moving average (ARIMA), univariate GARCH, and multivariate GARCH models. Their empirical results illustrate that the GARCH(1,1) model is sufficient to explain the heteroskedasticity of the CRIX. [Chen et al. \(2018\)](#) also consider alternate GARCH specifications. To capture the leverage effect (negative relationship between

return shocks and subsequent shocks to volatility), the exponential GARCH (EGARCH) model was estimated. However, McAleer and Hafner (2014) show that leverage is not possible for the EGARCH model. Chen et al. (2018) conclude that the symmetric GARCH(1,1) model with a Student- t error distribution is the best performing univariate model when applied to the CRIX.

In order to determine the effect weather has on the cryptocurrency market, Kathiravan et al. (2019) made use of a GARCH(1,1) model, Johansen cointegration, and a Granger causality test. The Coinbase index was used as a proxy for the cryptocurrency market in this study. The GARCH analysis showed that temperature is the only weather factor that is statistically significant when modelling cryptocurrency volatility.

To give an indication of the best performing volatility model when applied to the cryptocurrencies market (not focused on Bitcoin only), Chu et al. (2017) applied twelve GARCH models (eight different error distributions) to the seven most popular cryptocurrencies. The models were compared based on the goodness of fit, forecasting performance, and acceptability of value-at-risk estimates. Their empirical results indicate that the normal distribution provides the best fitting GARCH model in most cases. Furthermore, the symmetric integrated GARCH(1,1) (IGARCH(1,1)) model with normal innovation was the best fitting model for most cryptocurrencies.

In a recent study, Hafner (2020) made use of GARCH models to test for the existence of speculative bubbles in the cryptocurrency market. The empirical analysis made use of eleven of the largest cryptocurrencies and the CRIX. The estimated parameters of the GARCH models indicate that volatility clustering is important and significant when modelling cryptocurrency volatility and, unlike equities, cryptocurrencies do not have asymmetric news impact curves. More specifically, the asymmetry terms of asymmetric GARCH models are generally statistically insignificant when applied to cryptocurrencies; this is consistent with other findings in the literature (see, e.g., Baur and Dimpfl 2018). According to Hafner (2020), there is general evidence that speculative bubbles exist in cryptocurrency markets.

Gyamerah (2019) made use of the symmetric GARCH(1,1), threshold-GARCH(1,1) (TGARCH(1,1)), and IGARCH(1,1) models to model the volatility of Bitcoin returns. With regard to the error distribution, Gyamerah (2019) considered the Student- t , generalised error, and normal inverse Gaussian distributions. The different models were compared based on the Akaike and Bayesian information criteria. Their empirical results indicate that the asymmetric TGARCH(1,1) model with a normal inverse Gaussian error distribution is the best fitting model when modelling volatility of Bitcoin returns. This implies that incorporating asymmetry in the GARCH model specification, and skewness and kurtosis in the error distribution, can improve the fit of a GARCH model when applied to Bitcoin.

In order to determine the best performing model when forecasting exchange rate and cryptocurrency (Bitcoin, Ethereum, and Dash) volatility, Peng et al. (2018) made use of the following univariate GARCH models: GARCH(1,1), EGARCH(1,1) and the Glosten, Jagannathan and Runkle GARCH(1,1) (GJR-GARCH(1,1)) model. Three different error distributions were considered: normal, Student- t , and skewed Student- t distributions. In addition, a support vector regression (SVR) GARCH(1,1) model was also estimated. Their empirical results show that the SVR-GARCH(1,1) model is superior when compared to the other models considered. Furthermore, when the traditional GARCH models are compared, the GJR-GARCH(1,1) performed slightly better when compared to the symmetric GARCH(1,1) and EGARCH(1,1) models. The different error distributions yielded similar results. This illustrates that different GARCH specifications can offer better results when applied to exchange rate and cryptocurrency volatility.

2.3. Cryptocurrency Volatility Indices

Studies based on cryptocurrency volatility indices are limited; this is because there is not a well established cryptocurrency derivatives market. Volatility indices are used based on implied volatility obtained from the option market (e.g., the CBOE VIX). Alexander and Imeraj (2019) constructed a Bitcoin volatility index by making use of the VIX methodology (geometric variance swap), Bitcoin

option data were obtained from the Deribit exchange. In addition, [Alexander and Imeraj \(2019\)](#) note that Bitcoin prices tend to jump, therefore the fair value of geometric variance swaps are underestimated using this method. Hence, the method based on arithmetic variance swaps was also employed. [Alexander and Imeraj \(2019\)](#) recommend the use of the arithmetic index for horizons of one month or more. However, the volatility index based on arithmetic or geometric (VIX methodology) variance swaps is dependent on an established derivatives market, this is not the case for all cryptocurrencies and therefore a different approach is required.

In a recent study, [Kim et al. \(2019\)](#) construct a cryptocurrency volatility index based on the CRIX. The purpose of the index is to offer a forecast for the mean annualised volatility of the next month. Due to the shortcomings of the cryptocurrency derivatives market, [Kim et al. \(2019\)](#) make use of a proxy for implied volatility, therefore rolling volatility is used; this is based on historical volatility of the underlying. To get forward looking estimates (for the next 30 days) of rolling volatility, [Kim et al. \(2019\)](#) made use of GARCH family models, the Heterogeneous Auto-Regressive (HAR) model, and a neural network-based Long short-term memory cell; the performance of the different models was compared based on the mean squared error and the mean absolute error. Their empirical results show that the HAR model is the best performing model when forecasting rolling volatility of the CRIX. However, rolling volatility is based on historical volatility and not risk-neutral volatility. Therefore, the GARCH option pricing model is used in this study in order to estimate the implied volatility index (risk-neutral) in the absence of a well established derivatives market.

3. Theoretical Framework

In this section, the theoretical framework applied in this paper is discussed. This section is divided into two parts; the first deals with the GARCH option pricing framework, while the second focuses on the GARCH volatility index.

Option Pricing Model

[Duan \(1995\)](#) explains that, when using a GARCH process to model the log returns of an asset, the following is assumed,

$$\ln \left(\frac{S_t}{S_{t-1}} \right) = r + \lambda \sigma_t - \frac{1}{2} \sigma_t^2 + \epsilon_t, \tag{1}$$

and

$$\epsilon_t = \sigma_t \eta_t, \tag{2}$$

where r is the unique risk-free rate (continuously compounded), and λ is the constant unit risk premium. Furthermore, η_t is assumed to be identically and independently distributed with mean zero and variance equal to one under the real world measure \mathbb{P} .

[Wilmott \(2007\)](#) explains that the value of an option can be shown to be the expectation of the discounted future payoff under the risk-neutral measure ($\tilde{\mathbb{P}}$). Consider the following definition from [Duan \(1995\)](#):

Definition 1. *A pricing measure $\tilde{\mathbb{P}}$ satisfies the locally risk-neutral valuation relationship (LRNVR) if the measure $\tilde{\mathbb{P}}$ is absolutely continuous with respect to measure \mathbb{P} , S_t/S_{t-1} is lognormally distributed, with conditional expectation and variance under the risk-neutral measure*

$$\tilde{\mathbb{E}} \left[\frac{S_t}{S_{t-1}} \mid \Omega_{t-1} \right] = \exp \{r\}, \tag{3}$$

and

$$\text{Var}^{\tilde{\mathbb{P}}} \left[\ln \left(\frac{S_t}{S_{t-1}} \right) \mid \Omega_{t-1} \right] = \text{Var}^{\mathbb{P}} \left[\ln \left(\frac{S_t}{S_{t-1}} \right) \mid \Omega_{t-1} \right], \tag{4}$$

almost surely with respect to measure \mathbb{P} , where Ω_t is the information set available at time t .

The above definition allows us to derive the following theorem:

Theorem 1. Under pricing measure $\tilde{\mathbb{P}}$, the LRNVR implies

$$\ln \left(\frac{S_t}{S_{t-1}} \right) = r - \frac{1}{2} \sigma_t^2 + \zeta_t, \tag{5}$$

where

$$\zeta_t = \sigma_t \tilde{\eta}_t,$$

and $\tilde{\eta}_t$ is a standard normal random variable under the risk-neutral measure. This implies that

$$\zeta_t \mid \Omega_{t-1} \sim \mathcal{N}(0, \sigma_t^2).$$

Proof. Given that S_t/S_{t-1} is log-normal under measure $\tilde{\mathbb{P}}$, it can be written as

$$\ln \left(\frac{S_t}{S_{t-1}} \right) = \nu_t + \zeta_t, \tag{6}$$

where ν_t is the conditional mean, and ζ_t is a $\tilde{\mathbb{P}}$ -normal random variable, with conditional mean zero and variance σ_t^2 . This follows that

$$\begin{aligned} \tilde{\mathbb{E}} \left[\frac{S_t}{S_{t-1}} \mid \Omega_{t-1} \right] &= \tilde{\mathbb{E}} [\exp \{ \nu_t + \zeta_t \} \mid \Omega_{t-1}] \\ &= \exp \left\{ \nu_t + \frac{1}{2} \sigma_t^2 \right\} \end{aligned}$$

by the LRNVR, $\sigma_t^2 = \text{Var}^{\tilde{\mathbb{P}}} \left[\ln \left(\frac{S_t}{S_{t-1}} \right) \mid \Omega_{t-1} \right] = \text{Var}^{\mathbb{P}} \left[\ln \left(\frac{S_t}{S_{t-1}} \right) \mid \Omega_{t-1} \right]$. Furthermore, because Equation (3) holds, it follows that

$$\nu_t = r - \frac{1}{2} \sigma_t^2.$$

This completes the proof. \square

The assumption regarding the specification of the conditional variance is the symmetric GARCH(1,1) process of [Bollerslev \(1986\)](#). According to [Hansen and Lunde \(2005\)](#), it is difficult to find a volatility model that consistently outperforms the GARCH(1,1) due to its numerical stability and parsimony. The assumption of a GARCH(1,1) model is also appropriate based on previous findings in the literature (see, e.g., [Agyarko et al. 2019](#)). Furthermore, the asymmetry terms of asymmetric GARCH models are usually statistically insignificant when applied to cryptocurrencies (see, e.g., [Baur and Dimpfl 2018](#); [Hafner 2020](#)); this will lead to an inefficient option pricing model. According to [Brooks \(2014\)](#), the GARCH(1,1) model is specified as follows:

$$\sigma_t^2 = \omega + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2, \tag{7}$$

with coefficient constraints, $\omega \geq 0$, $0 \leq \alpha, \beta < 1$, and $\alpha + \beta \leq 1$. The above equation specifies the dynamics of the volatility under the real world measure \mathbb{P} . As mentioned previously, the value of an option is the discounted expected payoff under the risk-neutral measure. The risk-neutral dynamics of volatility is given by the following theorem:

Theorem 2. *If σ_t^2 takes on a GARCH(1,1) specification, the LRNVR implies*

$$\sigma_t^2 = \omega + \alpha (\zeta_{t-1} - \lambda\sigma_{t-1})^2 + \beta\sigma_{t-1}^2. \tag{8}$$

Proof. By making use of Equations (3) and (6), it is clear that

$$r + \lambda\sigma_t - \frac{1}{2}\sigma_t^2 + \epsilon_t = r - \frac{1}{2}\sigma_t^2 + \zeta_t,$$

which suggests that

$$\epsilon_t = \zeta_t - \lambda\sigma_t.$$

if the above is substituted into Equation (7) and (8) is obtained. This completes the proof. \square

It is clear from the above that, irrespective of how the conditional variance (σ_t^2) is specified, the variable ϵ_t is always replaced by $\zeta_t - \lambda\sigma_t$. According to Asteriou and Hall (2015), the GARCH(1,1) model usually leads to a good fit and estimation is fairly easy. Furthermore, according to Dyhrberg (2016), the volatility of Bitcoin reacts similarly to positive and negative shocks. Therefore, the use of the symmetric GARCH(1,1) model is appropriate in this case. In addition, the assumption of a normal distribution is appropriate based on the findings by Chu et al. (2017).

4. GARCH Volatility Index

To be consistent with the CBOE VIX, the BTCUSD and CRIX volatility indices are based on implied volatility in the following 30 calendar days (22 trading days). Hao and Zhang (2013) assume that the volatility index is calculated as the expected arithmetic average of the variance in the n subperiods of the following 30, 60, or 90 calendar days:

$$\left(\frac{\vartheta_t}{100}\right)^2 = \frac{252}{n} \sum_{k=1}^n E_t^{\mathbb{P}}[\sigma_{t+k}^2].$$

In this study, the dynamics of Bitcoin and CRIX are assumed to be consistent with a square-root stochastic autoregressive volatility (SR-SARV) model, which is defined below (Meddahi and Renault 2004):

Definition 2. *A stationary, square integrable process $\{\epsilon_t, t \in \mathbb{Z}\}$ is called a SR-SARV(p) (of order p) process with respect to filtration \mathcal{F}_t , if:*

1. ϵ_t is a martingale difference sequence with respect to \mathcal{F}_t ($\mathbb{E}[\epsilon_t | \mathcal{F}_t] = 0$).
2. The conditional variance process f_t of ϵ_{t+1} given \mathcal{F}_t is a marginalisation of a stationary \mathcal{F}_t -adapted vector autoregressive process of dimension p :

$$f_t = \text{Var}[\epsilon_{t+1} | \mathcal{F}_t] = c'F_t$$

$$F_t = \delta + \Gamma F_{t-1} + V_t,$$

where $\mathbb{E}[V_t | \mathcal{F}_t] = 0$, $c \in \mathbb{R}^c$, $\delta \in \mathbb{R}^c$ and the eigenvalues of Γ have a modulus smaller than one.

Hao and Zhang (2013) show that, if the underlying asset (Bitcoin or CRIX) follows a SR-SARV(p) process under the risk neutral measure \mathbb{P} , the implied volatility index has the analytical formula:

$$\vartheta_t = \zeta + \psi \sigma_t^2,$$

where

$$\zeta = \frac{\Omega}{1 - \Gamma}(1 - \psi),$$

$$\psi = \frac{1 - \Gamma^n}{n(1 - \Gamma)}.$$

The implied volatility index of the GARCH(1,1) option pricing model is easily obtained using the general form above. The important studies required for the theory applied in this paper are summarised in Table 1 below:

Table 1. Summary of important studies.

Study	Topic
Hansen and Lunde (2005)	GARCH(1,1) model
Duan (1995)	GARCH option pricing
Meddahi and Renault (2004)	SR-SARV processes
Hao and Zhang (2013)	GARCH implied volatility index
Trimborn and Härdle (2018)	The CRIX
Chu et al. (2017)	GARCH Modelling of cryptocurrencies
Hafner (2020)	GARCH Modelling of cryptocurrencies and CRIX

The preliminary data analysis is considered in the next section.

5. Data Analysis

In this section, the descriptive statistics of Bitcoin (BTCUSD) and CRIX are reported. The BTCUSD data were obtained from the Thomson Reuters Datastream databank. The CRIX historical data were obtained from thecrix.de. Daily data from 1 January 2016 to 3 January 2019 are used in this study. The line graphs of the BTCUSD and CRIX log-returns are plotted in Figure 1 below:

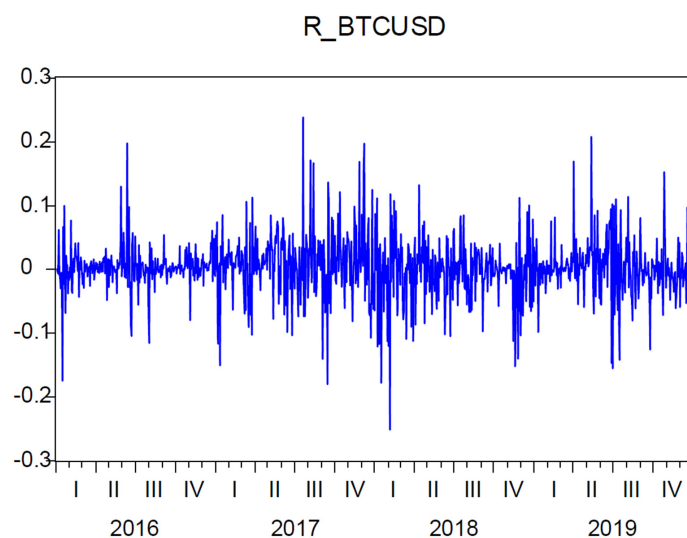


Figure 1. Cont.

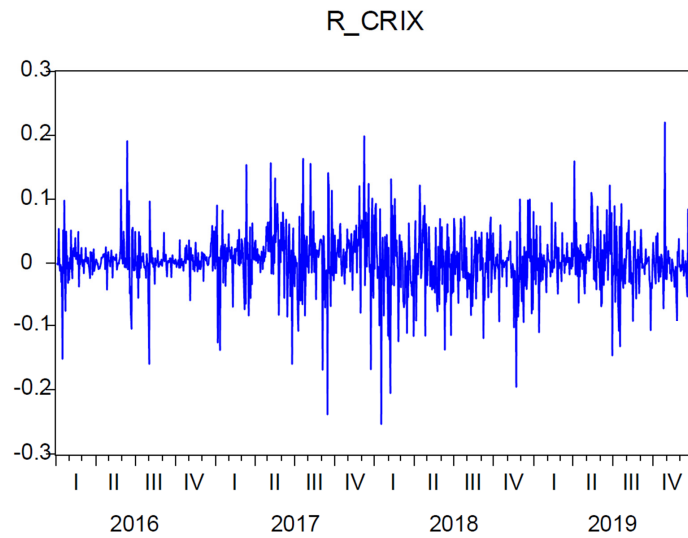


Figure 1. Line-graphs: BTCUSD and CRIX.

The log-returns of both BTCUSD and CRIX seem to show signs of volatility clustering. This is consistent with the stylised facts of financial time series (Cont 2001). The histograms of BTCUSD and CRIX are plotted in Figure 2 below:

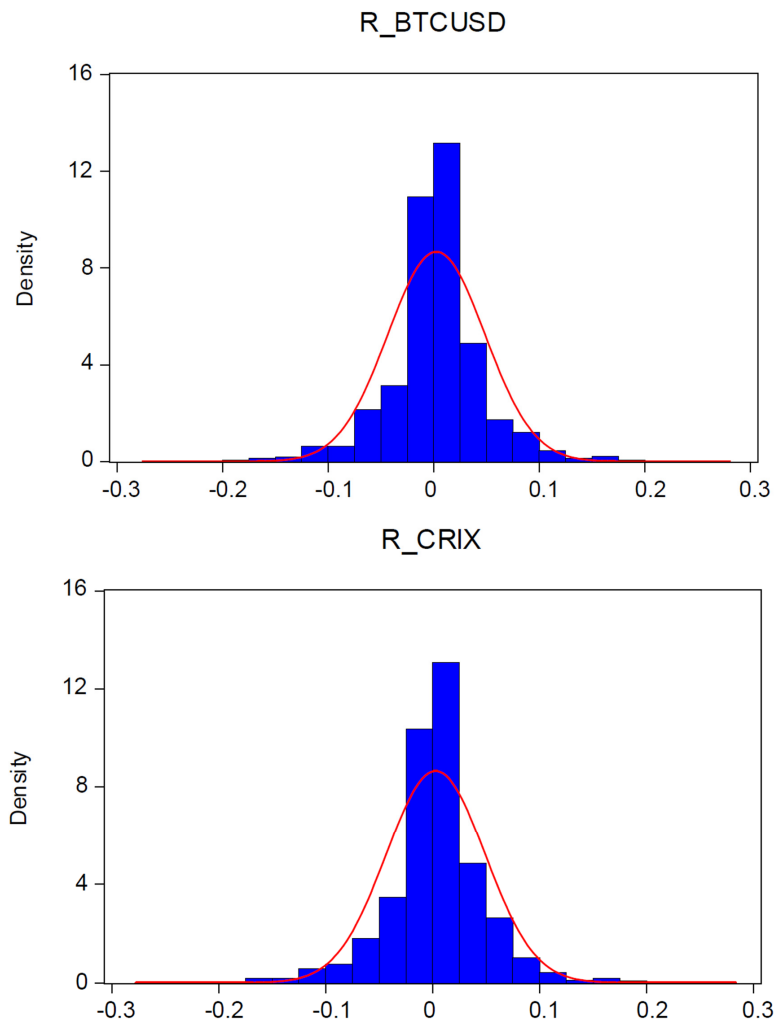


Figure 2. Histograms: BTCUSD and CRIX.

When compared to the normal distribution, both variables show signs of leptokurtosis (fat tails), which is also consistent with the stylised facts of financial time series. The descriptive statistics of BTCUSD and CRIX are reported in Table 2 below:

Table 2. Descriptive statistics (Log-returns).

	BTCUSD	CRIX
Mean	0.0027	0.0029
Median	0.0026	0.0032
Maximum	0.2384	0.2203
Minimum	−0.2514	−0.2533
Standard Deviation	0.046	0.0461
Skewness	0.0046	−0.3903
Kurtosis	7.1336	7.3798
Jarque–Bera	743.2889	860.9537
Observations	1044	1044

The descriptive statistics confirm expectations of leptokurtosis. Furthermore, the maximum, minimum, and standard deviation of the log-returns indicate that both return series are highly volatile. The GARCH volatility indices are considered in the next section.

6. Results

In this section, the pricing performance of the GARCH option pricing model when applied to BTCUSD is tested. In addition, the GARCH volatility indices for BTCUSD and CRIX are illustrated. The BTCUSD European option prices were obtained from Madan et al. (2019), the value date of the market prices is 29 June 2018. The GARCH(1,1) parameters when calibrated to BTCUSD log-returns (from 1 January 2016 to 28 June 2018) are reported in Table 3:

Table 3. GARCH(1,1) calibrated parameters.

	BTCUSD
ω	2.6×10^{-5}
α	0.1149
β	0.8837
λ	0.1116
AIC	−6.1810

The pricing performance of the GARCH option pricing model applied to BTCUSD is illustrated in Figure 3 and Table 4 below. The market price is the mid European put price calculated using the bid and ask prices.

The GARCH prices were obtained by simulating 50,000 realisations of Equation (5). As mentioned previously, the price of an option is the expectation of the discounted payoff under the risk-neutral measure. Small differences are obtained when the GARCH prices are compared to market European put option prices. Hence, the GARCH(1,1) model produces accurate BTCUSD option prices and is therefore appropriate to be used for the calculation of volatility indices.



Figure 3. BTCUSD European put option prices.

Table 4. BTCUSD European put option prices.

Market Price	GARCH Price	Difference	% Difference
583.665	589.96	6.295	1.0670%
853.5	850.65	-2.85	-0.3350%
1164.4	1163.1	-1.3	-0.1118%
1532.495	1519.59	-12.905	-0.8492%
1925.68	1911.84	-13.84	-0.7239%
2373	2332.64	-40.36	-1.7302%
2812.99	2773.88	-39.11	-1.4099%
3273.505	3229.46	-44.045	-1.3639%
4200.415	4170.21	-30.205	-0.7243%
5185.985	5134.92	-51.065	-0.9945%
6185.695	6112.5	-73.195	-1.1975%
7150.045	7096.55	-53.495	-0.7538%
8141.315	8084.57	-56.745	-0.7019%
9134.05	9074.67	-59.38	-0.6543%
14,115.325	14,038.73	-76.595	-0.5456%
19,106.87	19,009.72	-97.15	-0.5111%
24,098.41	23,981.97	-116.44	-0.4855%
29,089.95	28,954.7	-135.25	-0.4671%
34,082.955	33,927.67	-155.285	-0.4577%

For the calculation of the volatility indices, the parameters were calibrated to log returns from 1 January 2016 to 3 January 2019. The GARCH(1,1) calibrated parameters are reported in Table 5 below:

Table 5. GARCH(1,1) calibrated parameters.

	BTCUSD	CRIX
ω	0.0001	0.0001
α	0.1035	0.1504
β	0.8650	0.8203
λ	0.0744	0.0880
AIC	-6.9982	-6.9909

By making use of a similar approach to [Alexander and Imeraj \(2019\)](#), the 30-day, 60-day, and 90-day volatility indices are shown. The BTCUSD and CRIX GARCH volatility indices shown are in [Figures 4 and 5](#) below:

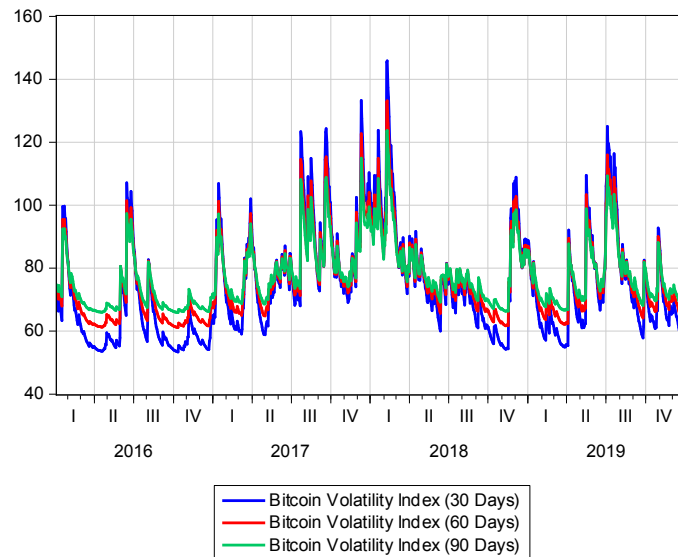


Figure 4. BTCUSD GARCH(1,1) Volatility indices.

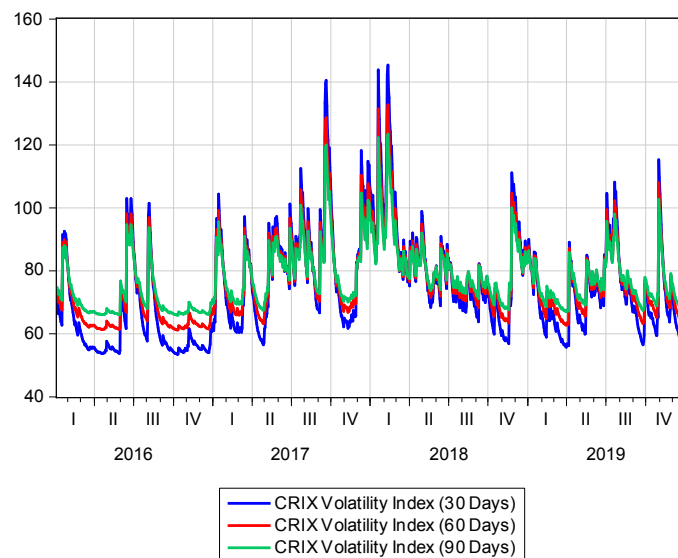


Figure 5. CRIX GARCH(1,1) Volatility indices.

It is evident from the above that GARCH volatility indices tend to increase after positive and negative shocks. This is consistent with findings by [Conrad et al. \(2018\)](#); [Dyhrberg \(2016\)](#). To illustrate how the term structure varies over time, the differences in volatility indices (left axis) and underlying assets (right axis) are plotted in [Figures 6 and 7](#) below:

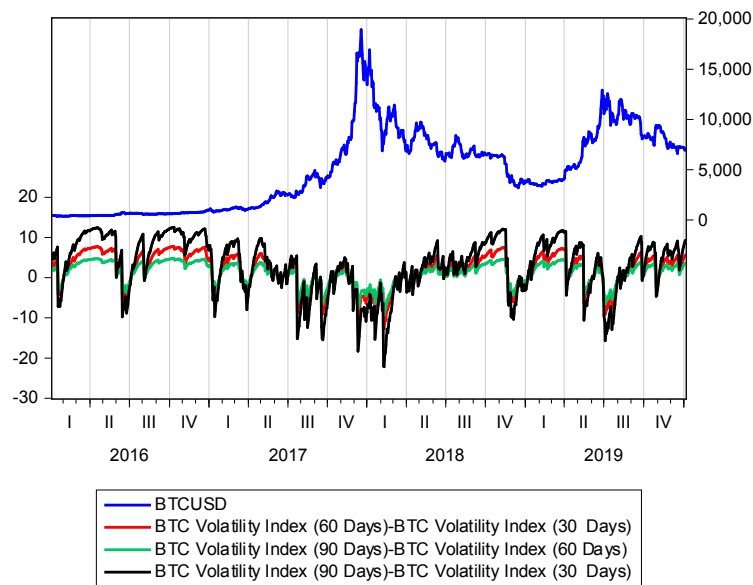


Figure 6. BTCUSD GARCH(1,1) Term structure.

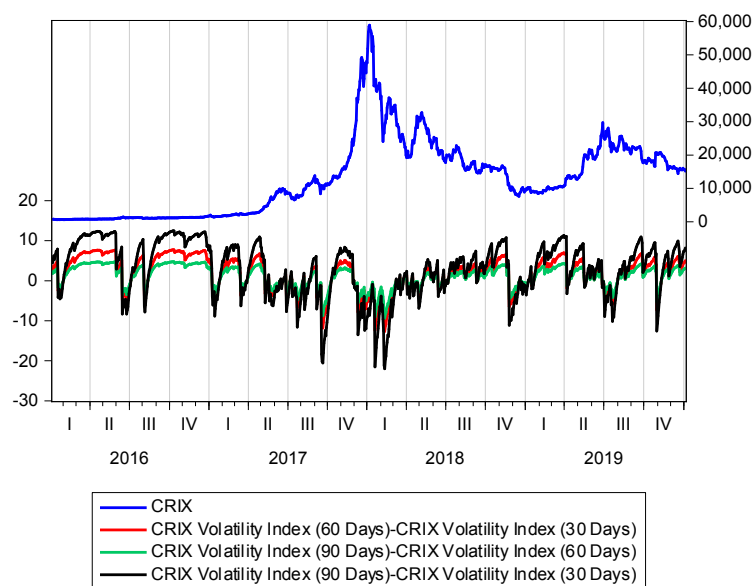


Figure 7. CRIX GARCH(1,1) Volatility indices.

When the term structure of volatility is considered, the 30-day volatility index for both BTCUSD and CRIX seem to be the lowest in most cases. This is consistent with expectations because there is more uncertainty over a longer period of time. However, when large jumps occur in the underlying asset, the short-term volatility index tends to increase to higher levels when compared to the 60-day and 90-day volatility indices (this is due to the fact that the volatility index is calculated as the expected arithmetic average of the variance in the n subperiods of the following 30, 60, or 90 calendar days). The 30-day GARCH volatility indices of BTCUSD and CRIX are compared in Figure 8 below:

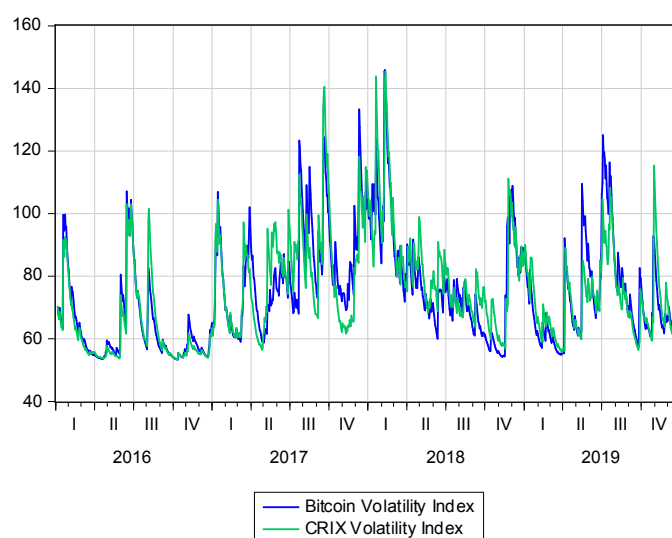


Figure 8. 30-Day GARCH(1,1) Volatility Indices.

Figure 8 indicates that the 30-day GARCH volatility index for BTCUSD is similar when compared to the 30-day CRIX GARCH volatility index.

7. Conclusions

In this paper, the pricing performance of the GARCH option pricing methodology is tested. In addition, the GARCH option pricing model is applied to BTCUSD and CRIX to estimate a GARCH volatility index. Volatility indices are usually estimated using a model-free approach. This approach has previously been applied to BTCUSD (Alexander and Imeraj 2019). In this paper, we rely on the symmetric GARCH volatility index. This is appropriate because previous findings indicate that BTCUSD volatility reacts similarly to positive and negative shocks.

The GARCH option pricing model produces accurate option prices when compared to market prices. As expected, the results indicate that the GARCH volatility indices also react similarly to positive and negative shocks. Furthermore, similar GARCH volatility indices are obtained when BTCUSD and CRIX are compared. This is consistent with expectations due to BTCUSD being highly weighted when calculating CRIX (Trimborn and Härdle 2018). The term structure of volatilities are consistent with expectations, with 30-day volatility being lower when compared to longer maturities. In addition, short-term volatility tends to increase to higher levels when compared to 60-day and 90-day volatility when large jumps occur in the underlying asset.

As per previous studies (Antonopoulos 2014; Böhme et al. 2015; Leong and Sung 2018; Leong et al. 2020), Bitcoin, as a digital currency, has huge potential in applications and advantages, such as lower fees, fraud protection, simpler international payments, etc. The findings of this paper, hopefully, would contribute to the development of future bitcoin research. Areas for future research include the use of different GARCH specifications (e.g., TGARCH(1,1) based on Gyamerah 2019; GJR-GARCH based on Peng et al. 2018) and error distributions (e.g., Student- t distribution based on the findings of Chen et al. 2018; normal inverse Gaussian based on Gyamerah 2019) applied to the modelling of cryptocurrency GARCH volatility indices. In addition, GARCH option pricing models with jump processes should also be considered because BTCUSD prices tend to jump excessively (Alexander and Imeraj 2019). Finally, the calibrated risk neutral GARCH processes can also be used for the pricing of derivatives written on cryptocurrency implied volatility indices.

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