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America

Maria Jose Rodriguez
Banco de México

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Investment for the Demographic Window in Latin America*

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Abstract: This paper studies the behavior of investment during demographic transitions. In particular, I focus on the period where the working age to population ratio reaches its maximum, namely the demographic window. I document that in Europe, Asia, and Oceania investment rates are higher 15 years before and during the window than in other periods, whereas in Latin America they are lower. To understand the relation between investment and a demographic window, I build an overlapping generations model with demographic change and variable degree of financial openness. Within this framework, I conduct several exercises and counterfactuals involving potential drivers of the investment behavior. I find that the demographic behavior in conjunction with the region-specific financial openness can explain the main pattern of investment for the demographic window in Latin America vis-a-vis Europe and Asia.

Keywords: Investment, Demographic Window, Latin America

JEL Classification: F21, E22, J10, O54, F41

Resumen: Este documento estudia el comportamiento de la inversión durante transiciones demográficas. En particular, se enfoca en el periodo en el cual la razón de población en edad de trabajar a población total alcanza su máximo, es decir la ventana demográfica. Se documenta que en Europa, Asia y Oceanía las tasas de inversión son mayores 15 años antes y durante la ventana demográfica que en otros periodos, mientras que en América Latina son menores. Para entender la relación entre inversión y ventana demográfica, se propone un modelo de generaciones traslapadas con cambio demográfico y grado de apertura financiera variable. Con este modelo se realizan varios ejercicios y contrafactuales que involucran potenciales determinantes del comportamiento de la inversión. Se encuentra que el comportamiento demográfico en conjunto con la apertura financiera específica de la región puede explicar el principal patrón de inversión para la ventana demográfica en América Latina vis-a-vis Europa y Asia.

Palabras Clave: Inversión, Ventana Demográfica, América Latina

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1 Introduction

Regions of the world undergo a common demographic transition that occurs in three stages: first the child-dependent cohort is large, then the working-age cohort is at its maximum size, and finally, an aging-dependent population increases. A *demographic window* is defined by the United Nations (UN) as a period when “the fraction of the population under age 15 falls below 30% and the fraction of the population 65 years and older is still below 15%”.¹ Thus, during a demographic window there is a temporary increase in the working age population, as a percent of the total population, moving to its peak earning years; while the dependent population is still low.

For policy-makers, like the multilateral organizations, this window is seen as a of opportunity because of the *demographic dividend*: the potential increase in resources per capita that might result from having a smaller dependent population and a larger working age population. During a demographic window, dependency ratios fall to their minimum level,² freeing up resources for alternative uses other than consumption. In this sense the demographic window is seen as a temporary moment where dependency is the lowest and there are many workers to invest in. After this window, the needs of the elderly will increase and there will no longer be a large labor force.

The aforementioned argument of a decrease in dependency and an increase in working age population has been used to emphasize the importance of investment in preparation and during the demographic window. Case studies of East Asian countries, which had their demographic windows from the 1960s to the 1990s, have documented positive demographic effects on investment rates and GDP per capita growth.³ Motivated by these findings, multilateral organizations like the United Nations,⁴ the World Bank (WB),⁵ and the International

¹In the *World population to 2300* report (United-Nations, 2004).

²For example, in Latin America, during the middle of the demographic window, the total dependency ratio (the number of old and young dependents per worker) is projected to be at its minimum. See figure 9 .

³For example, Mason (2001) reports positive contributions of investment induced by demographics to GDP per capita growth rates in Japan, Taiwan, South Korea, and Thailand for the period of 1965 to 1990.

⁴*State of World Population Report* (United-Nations, 2014).*Remarks of Chuma (2015)*, International Labor Organization (ILO), in the framework of the *2030 Agenda for sustainable development* of the United Nations.

⁵*Realizing the Demographic Dividend: Challenges and Opportunities*, Ministry of Finance and Development 2011.

Monetary Fund (IMF)⁶ have issued as a general policy recommendation to increase investment in capital, both human and physical, before and during a demographic window to take full advantage of the demographic dividend.

Despite the fact that stages of a demographic transition are generally common across countries, their speed and timing are not. For instance, Europe and some Asian countries have already gone through their demographic windows and are now aging, while, in contrast, Latin America is just entering its window and Africa has yet to do so. In this paper I focus on Latin America to analyze a relevant current economic phenomenon: the demographic change this region is undergoing and its relationship with the observed patterns of investment rates.

I analyze the macroeconomic impact of a country's population structure passing through its demographic window, and I focus on the behavior of investment leading up to and during the demographic window in Latin America compared to other regions of the world. I document that in Europe, Asia, and Oceania investment rates are higher for the 15 years before and during the demographic window than in other periods. However, this is not the case for Latin America, where investment rates decrease during this period.

I build an overlapping generations (henceforth OLG) model with variable degree of financial openness (on international capital flows) and demographic change that can rationalize the documented pattern of investment. The main result is that in financially open economies, investment rates decrease in preparation for and during the demographic window, while in closed economies they increase. The main mechanism behind this result is that in financially open economies, investment decreases because of the decrease in the growth rate of the labor force that comes with a demographic window. On the other hand, in closed economies⁷ the decrease in child dependency that triggers the window generates an increase in savings rates and therefore in investment rates.⁸

⁶*What is the demographic dividend?* (Lee and Mason, 2006).

⁷Mason (1981) analyzed the effect of the demographic transition in a cross-section of countries for the 1960s reporting that Japan and Korea (referring to these as with rapid economic growth) experienced high aggregate savings rate as consequence of declining population growth. Fry and Mason (1982) documented that demographic change had significant effects of savings in seven Asian countries –Korea, Singapore, Taiwan, Malaysia, Philippines, India, and Burma– where lower dependency ratios increases aggregates savings rate.

⁸Savings rates increase because of the decrease in child dependency, the fact that the bulk of the population

In terms of the observed patterns, the fact that European and Asian countries are less open during their windows can help to explain why they have higher investment rates before and during those. Similarly, greater financial openness during the window in Latin America can explain why investment rates are lower during this period. While it might seem surprising that Europe and Asia are considered as relatively closed financially and Latin America as relatively open, the data support this claim.⁹ In addition, it is important to keep in mind that I am comparing Europe and East Asia in the 1970s to Latin America in the present days.¹⁰

The contribution of this paper is to document the relationship between investment and the demographic window for different regions of the world and to try to explain the differences behind this relationship, with a special focus on Latin America. Additionally, this paper addresses and provides empirical context to a concern of both governments of Latin American countries and international organizations: lack of investment for the demographic window.

The model proposed is consistent with documented facts for both Latin America and Europe, Asia, and Oceania, and it cannot rationalize the general recommendation to increase investment for Latin America. Per the model, in an open economy the privately optimal response to a demographic window is to decrease investment rates since the economy is slowing down.

As the the asynchronicity of demographic transitions has grown larger across countries and regions the literature on the demographic windows has become richer, new country/region case studies have been developed and added to the literature's early case studies of East Asian and European countries. The latest region to undergo a demographic transition has been Latin America and there are only few studies about it, most of which are descriptive of the demographic phenomenon and only briefly discuss the effects of such transitions on empirical estimations of percapita GDP growth rates. Some of this studies are de Carvalho (1997), Bloom et al. (2001), Van Der Ven and Smits (2011), Saad (2011), Guzmán et al. (2006). To

moves towards middle age savers and that labor income profile is increasing hence so are savings for retirement.

⁹The average Chinn-Ito index of financial openness is higher in the demographic windows of the countries of Latin America (0.58), than the average index in the windows of countries of Europe, Asia, and Oceania (0.49).

¹⁰The 1970s is the average of the European and East-Asian windows. In my sample these windows happened from the 1960s to the early 1990s and in the data some European windows date back to the early 1900s.

my knowledge, there are no systematic empirical estimations of differences in the relationship between investment and the current demographic transition of Latin America and the equivalent of Europe, Asia, and Oceania. This paper tries to contribute to the literature documenting the aforementioned fact and providing a theoretical framework for analyzing it.

The rest of the paper is organized as follows. Section 2 describes common trends that are observed before and during a demographic window. Section 3 documents the empirical findings of investment rates for the demographic window. Section 4 describes the model and the equilibrium. Section 5 reports and discusses the results of the model. Section 6 concludes.

2 The Demographic Window

In this section I describe the worldwide long run demographic transition, I define the concept of a demographic window, and describe the common demographic trends that are observed in both the demographic transition and the demographic window.

2.1 The Demographic Transition

Countries undergo a common long run demographic transition that occurs in three stages: first the child-dependent cohort is large, then the working-age cohort is at its largest and finally an aging-dependent population increases.

The first stage, characterized by a large child cohort, is usually reached during a country's development phase when it is able to meet basic needs in terms of food and health, mortality rates drop,¹¹ and population growth accelerates. In terms of demographic indicators, during this stage the population pyramid is *expansive*: triangular and horizontally expanding at its base. The growth rate of newborns is high,¹² the child to population ratio is increasing, and the working age to total population ratio is decreasing.

¹¹For example, according to Lee (2003) the first stages of mortality reduction are caused by a decrease in diseases and better nutrition. Also, according to the United-Nations (2014) "mortality rates among children fall because of interventions like safe water and sanitation."

¹²The annual growth rate of the child population (population aged 0-14) is high and not drastically decreasing.

In general, what triggers the movement from the first to the second stage is a reduction in fertility rates. The growth rate of newborns starts a sustained, sometimes sharp, decreasing trend and population growth slows down. As the growth of the youngest cohort decreases and the last and large child cohorts from stage one scale up the population pyramid, the working age population becomes the widest group. The pyramid is growing towards its center.¹³ The child to population ratio is decreasing and the working age to total population ratio is increasing— a reversal of the previous trend. It is important to mention that during this stage, and from there on, even though the working age to total population ratio might be increasing, the growth rate of the working age population is not. This growth rate is decreasing since each year there are fewer children who will be part of the future working age population.

Finally, the aging process begins, and countries transition from the second to the third stage. During this transition, the large working age population cohort of the second stage joins the elderly population. In the data there is an increasing elderly population to total population ratio, and the working age to total population ratio shifts from increasing to decreasing. As countries enter the third stage, the increasing elderly population makes it to the top of the demographic pyramid as a result of increases in life expectancy. The pyramid of this stage is thus widening at the top.¹⁴

2.2 The Demographic Window

The aforementioned transition is part of what the concept of the demographic window captures. Officially, a *demographic window* (DW) is defined by the United Nations as a period when “the fraction of the population under age 15 falls below 30% and the fraction of the population 65 years and older is still below 15%”. In terms of the stages described above we can think of a demographic window as a phenomenon that occurs as countries transition into the second stage and before they reach the third one— a phenomenon where the child cohort

¹³Here there are two options: the population pyramid will be *stationary* as the growth rate of the cohort of newborns relative to the previous period approaches zero, or it will be *contracting*— shrinking at the base—if the growth rate of the cohort of newborns relative to the previous period becomes negative. In Mexico and Latin America, for instance, the latter is the case, and in the United States the former applies.

¹⁴Japan is an example of a country that has this type of population pyramid.

is shrinking, the working age population cohort is expanding, and the elderly population has not increased substantially yet.

It is true that other phenomena could generate temporary demographic windows, like the immigration of young people and baby booms.^{15,16} However, in this paper I focus on demographic windows that are part of this long run worldwide demographic transition because they are more commonly occurring around and have more permanent consequences. These windows are triggered by a sustained reduction in the number of newborns and characterized by a temporary increase in the working age to total population ratio, which reaches its maximum during the window. I use these two features to generate and characterize a demographic window in the model of this paper.

Finally, it is important to mention that while the stages of a demographic transition are generally common across countries, their speed and timing are not. For instance, Europe and some Asian countries have already gone through their windows and are now aging, while Latin America (LatAm) is just entering its window and Africa will do so in the future (see figure 12). In this paper I focus on LatAm, the region currently undergoing this process. Figure 1 shows the demographic indicators and trends described above for this region.

In the next section, I document the different relationship between the demographic window and investment in LatAm and the rest of the world.¹⁷ In the model I use Mexico as a benchmark case for LatAm since their demographic transitions are very similar (see figure 10) and because of data availability to perform growth accounting exercises required for the model, also I analyze the case of the demographic transition of Brazil.¹⁸

¹⁵These phenomena can also contribute to an increase in the duration of a demographic window, like in the United States (U.S). The demographic transition of the U.S. has been characterized by long stages because of migration that keeps the country mostly comprised of working age population.

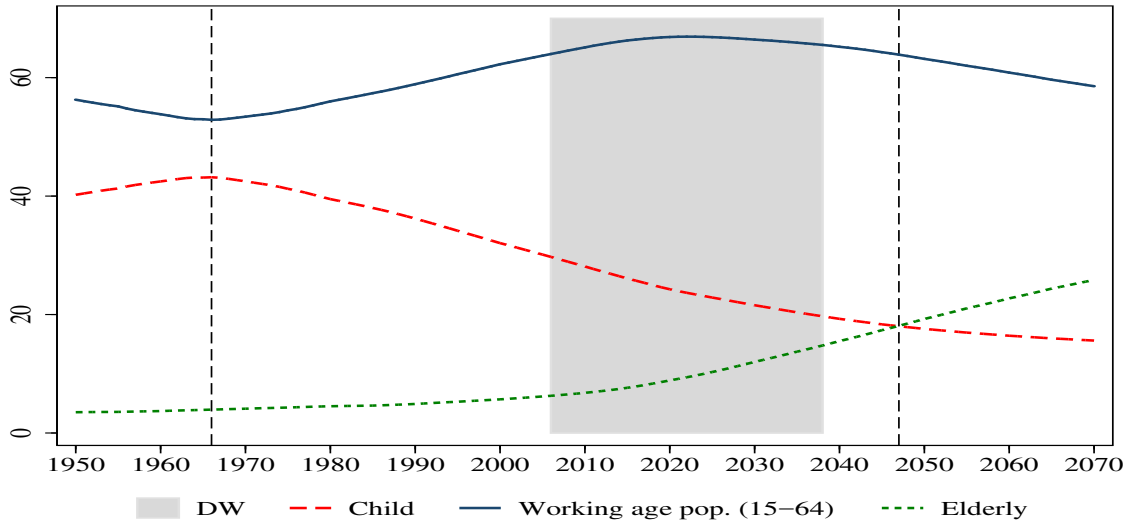
¹⁶And extreme cases, like an always-steady population, the sudden death of all the elderly or all the children, could also generate or extend demographic windows. This kind of cases are however rarely seen.

¹⁷In the data I do not find conclusive significant patterns for the current account. In this regard, there are other factors that have effects on the current account behavior besides demographics, among which shocks on terms to trade, commodity prices and its production, shocks to exchange rates have been analyzed in the literature.

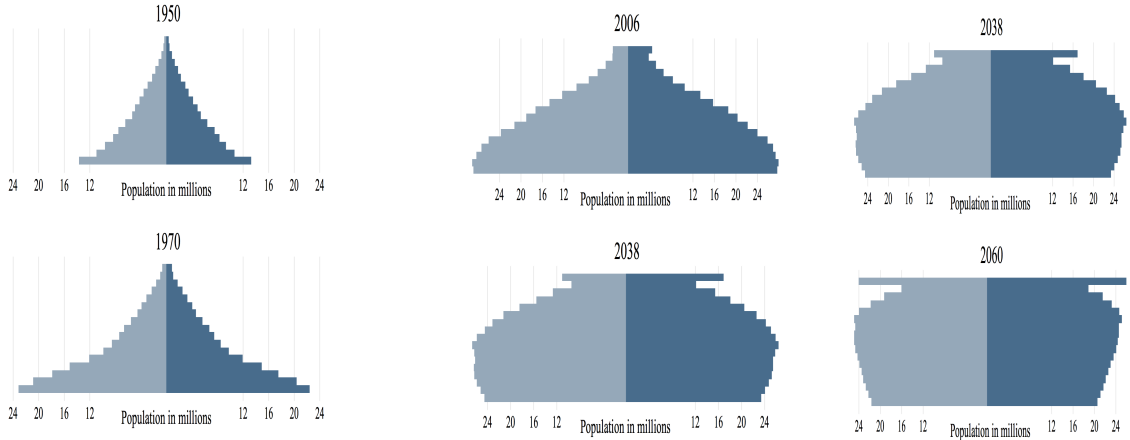
¹⁸These exercises are for Mexico and Brazil, the biggest countries of Latin America in terms of population.

Figure 1: Demographic transition in Latin America

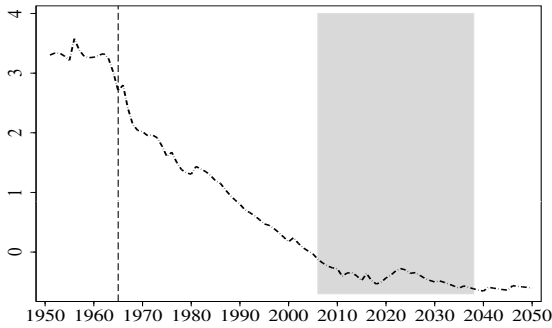
(a) Population by age group (%)



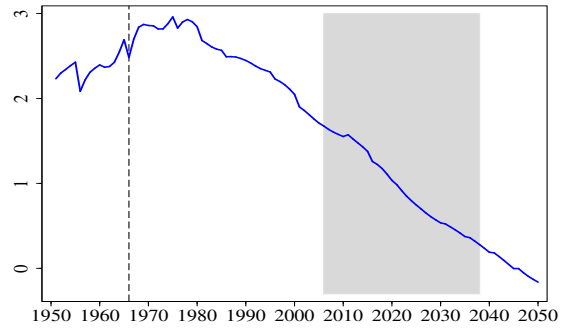
(b) Population pyramids



(c) Annual growth rate of pop. aged 0-14 (%)



(d) Annual growth rate of working age pop. (%)



Notes: The gray areas indicate the years of the demographic transition. The dotted vertical lines in panel (a) indicate the approximate years where stages 1 and 2 end respectively. Panels (a) and (b) are aligned horizontally to show the change in population structure in terms of both age groups and population pyramids. Source: 2012 World Population Projects Database, UN.

3 Data and Empirical Motivation

In this section, I first document that there are different patterns of investment for the demographic window across regions. Specifically, in Latin America investment rates before and during their demographic windows are lower than outside of these periods, whereas elsewhere the opposite is true. I then provide suggestive evidence of the relationship between financial openness and investment for the demographic window and its importance for the different patterns across regions.

3.1 Data Description

I use six databases for the empirical analysis: World Development Indicators of WB (WDI), International Financial Statistics of IMF (IFS), the 2012 World Population Projects Database of United Nations (WPP), Penn World Tables (7.1, 8.0, and 8.1), the Chinn and Ito (2008) index, the Total Economy Database (2016), and the LABORSTA database (ILO). The core variables in the analysis are gross capital formation, gross domestic product, number of people by age, labor force, and financial openness. Due to availability of the variables of interest, for the main sample I kept 51 countries with demographic transitions in Europe, Asia, Oceania, and Latin America, and the years 1960–2013.¹⁹

3.2 Investment and the Demographic Window

The main empirical result is that in Europe, Asia, and Oceania, *investment for the demographic window* (i.e., investment rates observed during the window and in the 15 years before it) is higher than investment (rates) outside of this period, while in Latin America, the opposite is observed. To derive this result, I grouped countries in two regions: (1) Europe, Asia, and Oceania; and (2) Latin America.^{20,21} As a first approach to the question: *are investment rates for the demographic window in Latin America different from other regions?*, I

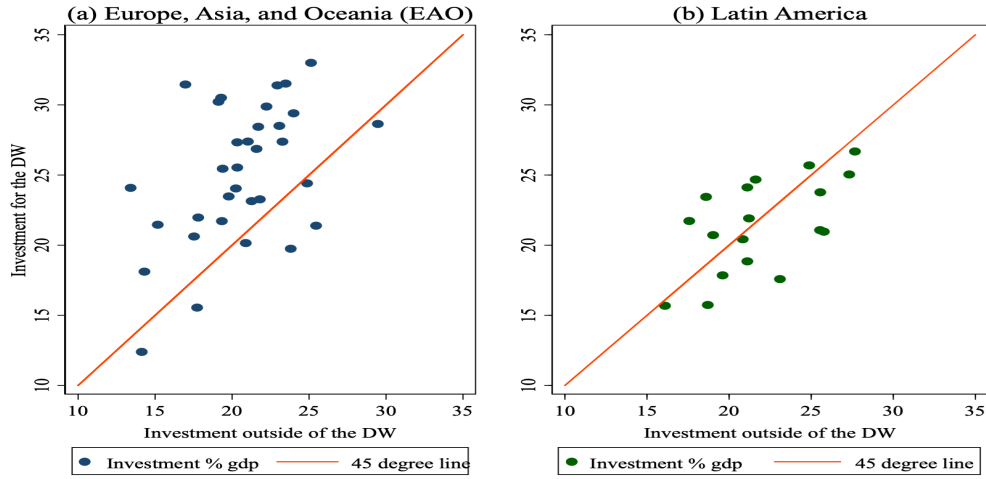
¹⁹Countries of Africa and those with economic systems designated as transitional by IMF and WB are not considered in the main sample.

²⁰Latin America is “Latin America and the Caribbean” using the UN geographic sub-regional classification.

²¹For a further description of the grouping of these two regions see section A.1 in the appendix.

compared average investment rates for the demographic window against average investment rates outside of this period.²² Figure 2 provides a visual illustration. The conclusion from this initial analysis is that investment rates for the demographic window in Europe, Asia, and Oceania are on average and in general higher than investment rates outside of this period. For Latin America, the pattern is different.

Figure 2: Investment rates for the demographic window (DW)



The figure plots average across years of investment rates: in the DW and the 15 years before it on the horizontal axis and that outside of this period on the vertical axis. Each dot is a country. Source: WDI and WPP.²³

Then, to formally obtain my main finding I regressed investment rates on dummies for region and demographic window, controlling for country and year fixed effects:

$$Inv_{c,t} = \beta_0 + \beta_1 dw_{c,t} + \beta_2 dw_{c,t} \cdot LA_c + f_c + f_t + \epsilon_{c,t}, \quad (1)$$

where c denotes the country and t the year. $dw_{c,t} = 1$ if the country is in its demographic window or in the 15 years before it and 0 otherwise, and $LA_c = 1$ if the country is in Latin America. f_c and f_t denote country and time fixed effects.²⁴ The results are that in Europe, Asia, and Oceania, investment rates for the demographic window are associated with

²²Average is across years. For this analysis *investment rate* is the ratio of gross capital formation to GDP.

²³Related figure 11 in the appendix .

²⁴For the econometric analysis, investment rates are defined as the trend component of gross capital formation (GCF) over gross domestic product (GDP). The ratio of GCF to GDP was hp-filtered and the cyclic component removed. The idea behind this procedure is to consider long term variations of investment.

an increase of 4.16 percentage points. In contrast, in Latin America, investment rates for the demographic window are associated with a decrease of 0.76 percentage points.

Table 1:

Regression Estimates			Results		
$\beta_1 (dw)$	4.16***	(0.24)	value	4.16	P-val null.
$\beta_2 (dw \times LA)$	-4.92***	(0.42)	$\beta_1 > 0$	0.00	
Observations	2445		$\beta_1 + \beta_2 < 0$	-0.76	0.01

Robust standard errors in parentheses, *** $p < .01$.
Source: Author's estimations, details described above.

The estimates and hypothesis testing in table 1 illustrate these findings. The coefficient estimate of the variable dw_{ct} is positive and significant, indicating that the period for the demographic window is associated with an increase of investment rates in Europe, Asia, and Oceania. This hypothesis can not be rejected, as the P-value of the null hypothesis is zero. For Latin America, the effect of being in the period for the demographic window is given the coefficient of dw_{ct} plus its interaction with the region $dw_{ct} \times LA_c$. The sum of these coefficients is negative, indicating that for Latin America the period for the demographic window is associated with a decrease in investment rates; and this hypothesis can not be rejected at the 1% level (P-value of 0.006).

These results are robust and they hold under different specifications. In particular, they hold when using different definitions of investment for the demographic window, and when dropping postwar years to consider possible effects of World War II (see tables 4 and 8 for these and other specifications, including different set of controls and samples).

3.3 Empirical Relationship Between Investment for the Demographic Window and Openness

In this section I document the empirical relationship between investment for the demographic window and the degree of financial openness measured by the Chinn-Ito index of capital openness.²⁵ To do so, I consider the following specification²⁶ including as explana-

²⁵The main reasons for using the Chinn and Ito (2008) index is that it is one of the few that covers many countries and many years. There is another index of capital openness by Quinn (1997) which also includes many countries but it is only available until 2004. Other indices with a wide country coverage are more restrictive in terms of the years, for instance Fernandez et al. (2015) covers 100 countries and the period of 1995-2013.

²⁶See table 10 in appendix for other specifications.

tory variables of investment the Chinn-Ito index and its interaction with the window:

$$Inv_{c,t} = \beta_0 + \beta_1 dw_{c,t} + \beta_2 dw_{c,t} kaopen_{c,t} + \beta_3 kaopen_{c,t} \cdot dw_{c,t} + f_c + f_t + \epsilon_{c,t}, \quad (2)$$

where $kaopen$ is the normalized Chinn-Ito index of capital account openness. The normalization assigns a value of 1 to the highest degree of financial openness and 0 to the lowest; the lesser the restrictions on cross-border financial transactions, the higher the index.

Table 2: Demographic window and openness

β_1 (dw)	3.70***	(0.379)
β_2 (Chinn-Ito index)	1.45***	(0.487)
β_3 (Chinn-Ito index \times DW)	-2.21***	(0.564)
Observations	2064	

Robust standard errors in parentheses, *** $p < .01$.

Source: Author's estimations, details described above.

The effect of financial openness on investment for the demographic window is given by β_3 , negative and significant (the null hypothesis of $\beta_3 < 0$ has a p-value of 0), i.e. investment for the demographic window and financial openness display a negative significant relationship.

Regarding the level of financial openness, in Latin America the sample average openness using the normalized Chinn-Ito index in the 15 years before and during the window is 0.58, whereas outside of that period the level is 0.34, while in Europe Asia and Oceania corresponding figures are 0.49 and 0.71. The fact that countries in Latin America are more open before and during their windows than Europe, Asia, and Oceania could account for differences in investment behavior across regions. It should be mentioned however that the empirical analysis performed in this paper abstracts for other factors behind secular behaviors and other country-specific alternative hypotheses not explored insofar.

In this section I have documented that investment rates in Latin America display a different pattern in the presence of demographic changes, and that there is a negative relationship between investment for the demographic window and the degree of financial openness of the economies, when openness is measured by the Chinn-Ito index. I find this evidence suggestive of the importance of openness in both (1) the relationship between the demographic window and investment rates, and (2) the potential of openness in accounting for the documented

differing behavior of investment for the demographic window across regions. Motivated by this evidence, in the next section I develop an OLG model with demographic change and variable degree of financial openness.

4 The Model

In order to study the relationship between investment and the demographic window, I build an overlapping generations model with demographic change and a variable degree of financial openness. In this section, I first present the general model. Then, to provide intuition, I discuss the mechanisms of the two extreme cases: the fully-open and the fully-closed economies.

4.1 The Environment

An economy consists of households, firms, banks, and a government. In every period, four generations of households coexist. Generation t has N_t identical households who live for four periods and make economic choices during three of them. They feed their children, consume, save, and supply labor (inelastically). When the economy is financially open, they save in domestic capital and in foreign bonds; and when it is fully closed, all savings are directed to domestic capital.²⁷ Banks take the savings, transform them into capital, and rent that capital to firms that produce goods using capital and labor services. Banks are also in charge of international savings or borrowing, and for that they pay taxes. The degree of financial openness of an economy is captured by the level of these taxes. There are two taxes, one on interest payments for foreign debt (τ_1) and another on total returns for foreign assets holdings (τ_2).²⁸ These taxes are paid at home, and they are collected by the government, and redistributed to the workers in a lump sum fashion. The economy is small relative to the rest of the world in the sense that it does not influence the price of foreign bonds.

The evolution of the population is determined by a law of motion for newborns. There is

²⁷Both instruments, domestic capital and foreign bonds, have the same net return in equilibrium.

²⁸I allow for taxes on capital inflows (τ_1) and capital outflows (τ_2) to be different since a country might have a different degree of financial openness for capital inflows and outflows.

no early death; however, in the numerical exercises, growth rates of newborns are adjusted to growth rates of working age population and thus include mortality between these age groups.

Demographic Structure

In each period there are four generations alive: children, young workers, middle-aged workers, and retirees. Because of no early death, the size of each generations (N) is indexed by date of birth subscript.²⁹

At any given period t , total population is:

$$Pop_t = \underbrace{N_t}_{\text{children}} + \underbrace{N_{t-1}}_{\text{Young workers}} + \underbrace{N_{t-2}}_{\text{Middle-aged workers}} + \underbrace{N_{t-3}}_{\text{retireees}}.$$

The evolution of newborns is given by the law of motion:

$$N_t = (1 + g_{nt}) N_{t-1}.$$

The ratio of working age to total population, or WAP ratio is:

$$WAP_t : \frac{N_{t-1} + N_{t+2}}{Pop_t} = \left[\frac{(1 + g_{nt})(1 + g_{nt-1})}{2 + g_{nt-1}} + 1 + \frac{1}{(1 + g_{nt-2})(2 + g_{nt-1})} \right]^{-1},$$

where N_t is the number of newborns in t , and g_{nt} is the growth rate of N_t .

Remark 4.1. *A decrease in the growth rate of newborns leads to a temporary increase in the WAP ratio.*

This is an important remark, since it allows me to generate a demographic window (characterized by a temporary increase in the WAP ratio) by imposing a decreasing growth rate on newborns, as observed in the data.

The growth rate of the working age population is:

$$g_{WAPt} = \frac{(1 + g_{nt-2})(2 + g_{nt-1})}{(2 + g_{nt-2})} - 1.$$

²⁹The number of households of generation t alive in period s N_s^t is equal to the number of households of generation t born in $t \equiv N_t$. Note that choice variables are indexed by time subscript and generation superscript.

Remark 4.2. *If the growth rate of newborns is decreasing, then the growth rate of the working age population is also decreasing.*

In the model, in equilibrium, the working age population will be equal to the labor force.

Therefore a demographic window, generated by a decrease in the growth rate of newborns, will generate a decrease in the growth rate of labor force in the next period.³⁰

Regarding the choice of the number of generations: One must consider at least three generations (one being children) in the model to have decreasing growth rates of newborns before and during the demographic window and increasing WAP ratios. Furthermore, one must have at least four generations to have WAP ratios higher than 50% during the demographic window; this being the case, four generations provides a better demographic fit.³¹

Households

Households live for four periods and make economic choices during three of them. Because they care about their children, they feed their offspring. In this way, children consume what their parents give them. The discounted utility of a representative agent of generation $t-1$ is:

$$U^{t-1} = u(c_t^{t-1}) + \gamma \phi_{t-1} u(c_t^t) + \beta u(c_{t+1}^{t-1}) + \beta^2 u(c_{t+2}^{t-1}).$$

There is no disutility of working, but there is exogenous retirement in the latest period of life. The one-period utility over consumption is of the constant relative risk aversion (CRRA) form, i.e., $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$.³² Altruism towards children is captured by the parameter γ , which is the weight of each child's utility in their parents' utility.³³ In the benchmark model, γ is set to 0.5 using the "OECD equivalence scale."³⁴ ϕ_{t-1} is the number of children per worker of generation $t-1$, and since households are representative, $\phi_{t-1} = \frac{N_t}{N_{t-1}} = (1 + g_{nt})$.³⁵

³⁰See section A.3.5. In steady state both growth rates are equal and in the transition the decrease in the growth rates of newborns will generate a decrease in the growth rate of future labor force.

³¹Given that the model excludes early death, adjusting the growth rate of newborns to the growth rate of age cohorts gives a better demographic fit with four generations than with three.

³²The assumption of no disutility of working is sensible since in the model it implies that the growth rate of the labor force is equal to the growth rate of the working age population. This has as empirical counterpart that both these rates are similar in levels and trends. See section A.3.5.

³³Parents care equally for each of their children.

³⁴For an explanation of this choice see section A.3.1 and for a sensitivity analysis on γ , see section A.3.2.

³⁵See section A.3.1 for a proof of this result and for a more detailed expression of these preferences.

A representative agent of generation $t-1$ chooses (once she becomes active) in t : her labor supply, her consumption during working years and retirement, her savings, and consumption by her children (c_t^t). I.e. in t , generation $t-1$ chooses³⁶ $\{\ell_t^{t-1}, \ell_{t+1}^{t-1}, c_t^{t-1}, c_{t+1}^{t-1}, c_{t+2}^{t-1}, a_{t+1}^{t-1}, a_{t+2}^{t-1}, c_t^t\}$ to:

$$\begin{aligned}
\max \quad & u(c_t^{t-1}) + \gamma \phi_{t-1} u(c_t^t) + \beta u(c_{t+1}^{t-1}) + \beta^2 u(c_{t+2}^{t-1}) \\
s.t. \quad & c_t^{t-1} + \phi_{t-1} c_t^t + a_{t+1}^{t-1} \leq w_t \ell_t^{t-1} + TR_t^{t-1} \\
& c_{t+1}^{t-1} + a_{t+2}^{t-1} \leq w_{t+1} \ell_{t+1}^{t-1} + (1 + r_{t+1}^b) a_{t+1}^{t-1} + TR_{t+1}^{t-1} \\
& c_{t+2}^{t-1} \leq (1 + r_{t+2}^b) a_{t+2}^{t-1} \\
& \ell_t^{t-1} \leq 1, \quad \ell_{t+1}^{t-1} \leq 1,
\end{aligned}$$

where TR_t^{t-1} and TR_{t+1}^{t-1} are transfers from the government in t and $t+1$ respectively.

Banks

A representative bank transforms domestic and international savings into capital, rents this capital to the firms, buys international bonds, and it is competitive. This bank can finance itself from national and international markets, paying taxes domestically on external interest payments or earnings. I assume foreign investors have access to a similar bank in their home countries and thus international returns on assets and capital are equal ($r_{t+1}^{b*} = r_{t+1}^{k*} - \delta \equiv r_t^w$) and exogenous to this economy. Because of the tax structure, I distinguish between external borrowing D_{t+1} and lending B_{t+1} . There are two cases to consider. If the economy is an:

- (i) External borrower ($D_{t+1} > 0$): The bank will issue national and international debt promising to repay the principal and interests in the next period. It will then transform this debt into capital K_{t+1} and rent this capital to the firm in $t+1$. For each unit of capital, the bank will earn a revenue of r_{t+1}^k and a liquidation value of $(1-\delta)$. Thus raising $(1+r_{t+1}^k - \delta)$ and repaying $(1+r_{t+1}^b)$ at home, and $(1+r_{t+1}^w)$ abroad. In addition it pays a tax of $\tau_1 r_{t+1}^w$ at home. In this way the total cost of external financing is $r_{t+1}^w (1 + \tau_1)$.

³⁶Choices are indexed by time subscript and date of birth superscript.

- (ii) External lender ($B_{t+1} > 0$): The bank will issue national debt, collecting savings (a_{t+1}^{FII}) from domestic households. It will then allocate some of this savings to domestic capital K_{t+1} and some to foreign bonds B_{t+1} , raising $(1+r_{t+1}^k - \delta)$ and $(1+r_{t+1}^w)$ respectively and paying a tax of $\tau_2 r_{t+1}^w$. Thus the net external return on international bonds is $r_{t+1}^w (1 - \tau_2)$.

The bank chooses national K_{t+1}^{FII} , a_{t+1}^{FII} and international assets D_{t+1} , B_{t+1} to solve:

$$\begin{aligned} \max \quad & (1+r_{t+1}^k - \delta) K_{t+1}^{FII} + (1+r_{t+1}^w (1-\tau_2)) B_{t+1} - (1+r_{t+1}^b) a_{t+1}^{FII} - (1+r_{t+1}^w (1+\tau_1)) D_{t+1} \\ \text{s.t.} \quad & K_{t+1}^{FII} \leq a_{t+1}^{FII} - B_{t+1} + D_{t+1} \\ & D_{t+1} > 0 \text{ and } B_{t+1} = 0 \quad \text{or} \quad B_{t+1} > 0 \text{ and } D_{t+1} = 0. \end{aligned}$$

Firms

A representative firm is competitive and produces goods using capital and labor with a constant returns to scale technology with labor augmenting technological change.³⁷ It rents capital and labor services from the bank and the households respectively to solve:

$$\begin{aligned} \min_{\{K_t, L_t\}} \quad & w_t L_t + r_t^k K_t \\ \text{s.t.} \quad & Y_t \leq K_t^\alpha (A_t L_t)^{1-\alpha}. \end{aligned}$$

The Government

Taxes are paid domestically and the government collects its revenues and redistributes them back to the each worker in a lump sum fashion. Every period the government budget balances:

$$D_t r_t^w \tau_1 + B_t r_t^w \tau_2 = N_{t-1} TR_t^{t-1} + N_{t-2} TR_t^{t-2}, \quad (3)$$

where TR_t^{t-1} and TR_t^{t-2} are transfers in t to each young and middle-aged worker respectively.

³⁷Technology evolves according to $A_t = (1 + g_{at}) A_{t-1}$, where g_{at} is the growth rate of A_t .

Definition 4.3. Taxed Economy Equilibrium: Given sequences of productivity A_t , the interest rate of the rest of the world r_t^w , population dynamics N_t, ϕ_t , taxes τ_1, τ_2 , initial domestic bonds a_1^0, a_1^{-1} , and initial capital stock K_1^0 , a competitive equilibrium for this economy is: (i) consumption plans, savings (asset holdings), and labor supply choices from households of generation $t-1$ $\{c_t^{t-1}, c_t^t, c_{t+1}^{t-1}, c_{t+2}^{t-1}\}_{t=1}^\infty, \{a_{t+1}^{t-1}, a_{t+2}^{t-1}\}_{t=1}^\infty, \{\ell_t^{t-1}, \ell_{t+1}^{t-1}\}_{t=1}^\infty$; (ii) production plans of the firm $\{L_t^{FI}, K_t^{FI}\}_{t=1}^\infty$; (iii) capital allocations, national bonds, purchases and sales of international bonds of the bank $\{K_{t+1}^{FII}, a_{t+1}^{FII}, D_{t+1}, B_{t+1}\}_{t=0}^\infty$; (iv) transfers to young and middle-aged workers $\{TR_t^{t-1}, TR_t^{t-2}\}_{t=1}^\infty$; and (v) prices $\{w_t, r_t^b, r_t^k\}_{t=1}^\infty$, such that:

1. Given prices and transfers, choices of generation $t-1$ $\{c_t^{t-1}, c_t^t, c_{t+1}^{t-1}, c_{t+2}^{t-1}, a_{t+1}^{t-1}, a_{t+2}^{t-1}, \ell_t^{t-1}, \ell_{t+1}^{t-1}\}$ solve the household problem.
2. Given prices, banks' choices $\{K_{t+1}^{FII}, a_{t+1}^{FII}, D_{t+1}, B_{t+1}\}$ solve the bank's problem.
3. Given prices, firms' choices $\{K_t^{FI}, L_t^{FI}\}$ solve the goods producing firm problem.
4. Given prices, the government chooses transfers $\{TR_t^{t-1}, TR_t^{t-2}\}_{t=1}^\infty$ to satisfy its budget constraint.
5. The domestic bonds market clears:

$$\sum_{i=1}^{N_{j-1}} a_{t+1}^{j-1} + \sum_{i=1}^{N_{j-2}} a_{t+1}^{j-2} = a_{t+1}^{FII}.$$

6. The capital and labor inputs markets clear:

$$\begin{aligned} K_t^{FI} &= K_t^{FII} \\ L_t^{FI} &= N_{t-1} \cdot \ell_t^{t-1} + N_{t-2} \cdot \ell_t^{t-2}. \end{aligned}$$

7. The goods market clears:

$$N_{t-1}c_t^{t-1} + N_t c_t^t + N_{t-2}c_t^{t-2} + N_{t-3}c_t^{t-3} + K_{t+1} - (1-\delta)K_t + B_{t+1} - (1+r^w)B_t - D_{t+1} + (1+r^w)D_t = Y_t$$

8. The assets market clears:

$$\sum_{i=1}^{N_{j-1}} a_{t+1}^{j-1} + \sum_{i=1}^{N_{j-2}} a_{t+1}^{j-2} = K_{t+1} + B_{t+1} - D_{t+1}.$$

Characterization of the Equilibrium

The solution to the household problem can be summarized by her assets holdings:

$$a_{t+1}^{t-1} = \frac{\left[\beta^{\frac{1}{\sigma}} (1+r_{t+1})^{\frac{1}{\sigma}-1} + \beta^{\frac{2}{\sigma}} [(1+r_{t+1}^b)(1+r_{t+2}^b)]^{\frac{1}{\sigma}-1} \right] (w_t \ell_t^{t-1} + T R_t^{t-1}) - (1 + \gamma^{\frac{1}{\sigma}} \phi_t) \frac{w_{t+1} \ell_{t+1}^{t-1} + T R_{t+1}^{t-1}}{1+r_{t+1}^b}}{1 + \gamma^{\frac{1}{\sigma}} \phi_t + \beta^{\frac{1}{\sigma}} (1+r_{t+1}^b)^{\frac{1}{\sigma}-1} + \beta^{\frac{2}{\sigma}} [(1+r_{t+1}^b)(1+r_{t+2}^b)]^{\frac{1}{\sigma}-1}} \quad (4)$$

$$a_{t+2}^{t-1} = \frac{\beta^{\frac{2}{\sigma}} [(1+r_{t+1}^b)(1+r_{t+2}^b)]^{\frac{1}{\sigma}-1} [(1+r_{t+1}^b)(w_t \ell_t^{t-1} + T R_t^{t-1}) + w_{t+1} \ell_{t+1}^{t-1} + T R_{t+1}^{t-1}]}{1 + \gamma^{\frac{1}{\sigma}} \phi_t + \beta^{\frac{1}{\sigma}} (1+r_{t+1}^b)^{\frac{1}{\sigma}-1} + \beta^{\frac{2}{\sigma}} [(1+r_{t+1}^b)(1+r_{t+2}^b)]^{\frac{1}{\sigma}-1}}, \quad (5)$$

her optimal labor supply choice $\ell_t^{t-1} = 1$ (since there is no disutility of working), and her consumption demand functions.

Capital and labor demands from the goods-producing firm are given by:

$$(1 - \alpha) A_t k_t^\alpha = w_t \quad (6)$$

$$\alpha k_t^{\alpha-1} = r_t^k, \quad (7)$$

where $k_t \equiv \frac{K_t}{A_t L_t}$ is defined as capital per effective units of labor (p.e.u.l henceforth).

From the bank's allocations, we have the no-arbitrage conditions of domestic and international bonds, where net returns on domestic capital are equalized to after-tax returns abroad:

$$(1 + r_{t+1}^k - \delta) = (1 + r_{t+1}^b) \quad (8)$$

$$(1 + r_{t+1}^k - \delta) = [1 + r_{t+1}^w (1 + \tau_1)] \quad \text{if } D_{t+1} > 0 \quad (9)$$

$$(1 + r_{t+1}^k - \delta) = [1 + r_{t+1}^w (1 - \tau_2)] \quad \text{if } B_{t+1} > 0. \quad (10)$$

Conditions that determine whether an economy will be borrowing, lending, or in autarky are:

$$D_{t+1} > 0 \text{ if } F'_1(k_{t+1}^{aut}, 1) - \delta > r_{t+1}^w (1 + \tau_1) \quad \text{and} \quad K_{t+1} = a_{t+1}^{FII} + D_{t+1} \quad (11)$$

$$B_{t+1} > 0 \text{ if } F'_1(k_{t+1}^{aut}, 1) - \delta > r_{t+1}^w (1 - \tau_2) \quad \text{and} \quad K_{t+1} + B_{t+1} = a_{t+1}^{FII} \quad (12)$$

$$\text{Autarky if } (1 - \tau_2) r_{t+1}^w \leq F'_1(k_{t+1}^{aut}, 1) - \delta \leq r_{t+1}^w (1 + \tau_1) \text{ or } K_{t+1} < a_{t+1}^{FII} + D_{t+1} - B_{t+1} \quad (13)$$

where k_{t+1}^{aut} is the capital (p.e.u.l) of being in financial autarky in $t+1$, given the history of capital and transfers.³⁸

Let $s_t^{t-1} \equiv \frac{a_{t+1}^{t-1}}{A_t}$ and $s_{t+1}^{t-1} \equiv \frac{a_{t+2}^{t-1}}{A_{t+1}}$ be savings per effective units. Using the assets market clearing condition and the optimal savings and prices, the amount of borrowing/lending (p.e.u.l) is:

$$\frac{1}{(1+g_{at+1})(1+g_{Lt+1})} \left[\frac{1+g_{nt-1}}{2+g_{nt-1}} s_t^{t-1} + \frac{1}{2+g_{nt-1}} s_t^{t-2} \right] - k_{t+1} = \begin{cases} -d_{t+1} & \text{if borrower} \\ b_{t+1} & \text{if lender} \\ 0 & \text{if autarky.} \end{cases} \quad (14)$$

Capital per effective unit of labor is pinned down by the after-tax interest rate on international assets if the economy is open, and by a law of motion derived from optimal savings, prices, and equation (14) if it is closed:

$$k_t = \left(\frac{r_t^w(1+\tau_1)+\delta}{\alpha} \right)^{\frac{1}{\alpha-1}} \quad \text{if } d_{t+1} > 0 \quad (15)$$

$$k_t = \left(\frac{r_t^w(1-\tau_2)+\delta}{\alpha} \right)^{\frac{1}{\alpha-1}} \quad \text{if } b_{t+1} > 0 \quad (16)$$

$$k_t = k_{t+1}^{aut} (\{k_t\}_t, \{tr_t^j(d_t, b_t, \tau)\}_t) \quad \text{if in autarky,} \quad (17)$$

where the terms in brackets are included to emphasize the dependence of the history of capital and transfers on savings. tr_t^j are transfers per effective worker, which are the tax revenues divided equally between young and middle-aged workers ($\theta_t^y = \theta_t^m = 0.5$):

$$tr_t^{t-1} = \theta_t^y \frac{d_t L_t r_t^w \tau_1}{N_{t-1}} \quad tr_t^{t-2} = \theta_t^m \frac{d_t L_t r_t^w \tau_1}{N_{t-2}} \quad \text{if } d_t > 0 \quad (18)$$

$$tr_t^{t-1} = \theta_t^y \frac{b_t L_t r_t^w \tau_2}{N_{t-1}} \quad tr_t^{t-2} = \theta_t^m \frac{b_t L_t r_t^w \tau_2}{N_{t-2}} \quad \text{if } b_t > 0. \quad (19)$$

This completes the characterization of the equilibrium.³⁹ I now turn to an important remark regarding the determination of openness of the economies.

Remark 4.4. *From conditions (11), (12), and (13), given the demographics of an economy,*

³⁸See equation 20 in the next section.

³⁹See section A.3.7 for a description of the algorithm of the computation of the equilibrium.

there are thresholds on the taxes τ_1 and τ_2 such that for a tax level higher than the threshold, the economy will be closed and capital (p.e.u.l) will be determined endogenously. For lower tax levels, capital (p.e.u.l) will be given by the after-tax return on international assets.

This is important because for the demographic window exercises, the driver behind the different results in the fully-open and the fully-closed economies is also whether capital (p.e.u.l) is determined endogenously or exogenously. Therefore, it suffices to discuss the extreme cases which will extend to the various degrees of openness, depending on which side of the threshold the taxes of the economy fall.

4.2 Fully-Closed and Fully-Open Economies

Since the mechanisms in the model with variable degree of openness are those of the extreme cases,⁴⁰ in this section I lay down the main equations of the fully-closed and fully-open economies to provide intuition. In the next section I fully explore and discuss the effect of the demographic window in these cases.

In the fully-closed economy, capital is equal to the savings of young and middle-aged workers, and both are determined simultaneously. In contrast, in the fully-open economy, capital (p.e.u.l) is determined by the interest rate of the rest of the world, independently of savings, and foreign assets are the difference between national savings and this capital.

Fully-Closed Economy

When the economy is fully closed, the total capital stock is equal to the supply of domestic savings of the workers. The law of motion of capital per effective units of labor reads:

$$(1 + g_{at+1})(1 + g_{Lt+1})k_{t+1} = \underbrace{\frac{N_{t-1}}{L_t}}_{\% \text{ of young savers}} s_t^{t-1} + \underbrace{\frac{N_{t-2}}{L_t}}_{\% \text{ of middle-age savers}} s_t^{t-2}, \quad (20)$$

⁴⁰See remark 4.4, figure 4, and numerical exercise in Section 5.1.

where s_t^{t-1} and s_{t+1}^{t-1} are functions of capital (p.e.u.l), demographic variables and parameters. The equilibrium can be summarized by a sequence of capital per effective units of labor.⁴¹ The rest of the equilibrium allocations are expressed in terms of this variable.

In the closed economy, the demographic window will operate via savings and the drivers of the savings will be the drivers of investment for the demographic window.

Fully-Open Economy

For the extreme case of a fully-open economy, assume there are no taxes on capital flows (i.e., $\tau_1=\tau_2=0$). The no-arbitrage conditions for international assets are reduced to:

$$(1 + r_{t+1}^k - \delta) = (1 + r_{t+1}^w). \quad (21)$$

Since the interest rate of the rest of the world is given, the rental price of domestic capital is also determined by the rest of the world. Therefore, the firm demands capital up to the point where its net marginal product⁴² equalizes the return abroad and the expression of capital (p.e.u.l) is:

$$k_t = \left(\frac{r_t^w + \delta}{\alpha} \right)^{\frac{1}{\alpha-1}}. \quad (22)$$

Capital per effective units of labor is pinned down by the return abroad, savings are determined independently of capital (p.e.u.l), and foreign assets holdings are the difference between effective savings and domestic capital (p.e.u.l):⁴³

$$(1 + g_{at+1}) (1 + g_{Lt+1}) [k_{t+1}(r_{t+1}^w) + b_{t+1} - d_{t+1}] = \frac{N_{t-1}}{L_t} s_t^{t-1}(r^w) + \frac{N_{t-2}}{L_t} s_t^{t-2}(r^w). \quad (23)$$

The main drivers of the results of a demographic window will be the facts that savings are independent of capital p.e.u.l and that in open economies investment and savings need not to be equal.

⁴¹The sequence can be reduced to a third-order difference equation for a numerical solution, see section A.3.7.

⁴²Net of depreciation.

⁴³Expression 23 (and expression 20) is derived from the assets market clearing condition.

5 The Demographic Window: Main Results of the Models

Recall that in the data there is a decreasing child to population ratio both before and during the demographic window, and that during the window there is a temporary increase in the working age to total population ratio.⁴⁴ In the model, I generate a demographic window by imposing an decrease in the growth rate of newborns, which generates a temporary increase of the WAP ratio, and of course a decreasing (and low) children to population ratio.⁴⁵

This paper studies the behavior of investment leading up to and during the demographic window. I do so through the lens of the models by analyzing the response of the investment rate to this decrease in the growth rate of newborns. The exercise consists of calculating the transition between two steady states and analyzing the change in investment rates before and during the demographic window, which is part of this transition. The first steady state has a high newborns' growth rate and the second one, a low growth rate. The shock consists of imposing a decreasing growth rate of newborns g_{nt} through which the large cohort of children becomes part of the working age population, generating a demographic window. As the population stabilizes, the economy moves out of the demographic window and into the second steady state. The change in the growth rate of newborns is perfectly foreseen by everyone alive during the transition.⁴⁶

In this section I explore the effects of different variables on investment for the demographic window. I begin by analyzing openness, then I analyze changes in the return on foreign assets and in productivity, and the effects of capital depletion after World War II. The conclusion is that openness is the determining factor behind the documented differences in investment behavior. Finally, I compare the results of the benchmark model to investment rates observed in Brazil.

⁴⁴These is true in the data in general, and in particular for Latin America.

⁴⁵Recall that in this paper I focus on the demographic windows that are part of the long-run demographic transition and are more frequently observed. These windows are triggered by a reduction in fertility rates and characterized by a temporary increase in the WAP ratio, which reaches its maximum level during the window. There are other shocks besides a decreasing growth rate for newborns that could generate a demographic window in the framework of this model, however I do not study those cases since they are not the focus of this paper.

⁴⁶The foreseeable nature of the shock is sensible assumption given the focus of this paper is on the demographic windows that are part of long run worldwide demographic transition.

5.1 Investment and openness

The main result of the model is that in open economies, investment rates decrease before and during the demographic window, whereas in closed economies they temporarily increase. The investment rates, as a function of capital (p.e.u.l), can be expressed as:

$$ir_t = (1 + g_{Lt+1})(1 + g_{at+1}) \frac{k_{t+1}}{k_t^\alpha} - (1 - \delta)k_t^{1-\alpha}, \quad (24)$$

In the open economy, capital (p.e.u.l) is determined by the exogenous return on foreign assets, hence investment rates move together with the growth rate of the effective labor force. The growth rate of the labor force is in turn decreasing during the demographic transition since in each period fewer children will join the labor force (remark 4.1.).⁴⁷ Even considering improvements in labor productivity, the growth rate of the effective labor force does not increase since, in the data, demographic transitions are stronger than technological changes.⁴⁸

In the closed economy, investment is determined by national savings, and the rates of both are equal. The first term in the above investment equation is the savings rate of workers, and the second is the dissavings rate:⁴⁹

$$ir_t = \underbrace{\frac{N_{t-1}}{L_t}}_{\% \text{ young savers}} sr_t^{t-1} + \underbrace{\frac{N_{t-2}}{L_t}}_{\% \text{ middle savers}} sr_t^{t-2} - \underbrace{(1 - \delta)k_t^{1-\alpha}}_{\text{dissavings rate}}. \quad (25)$$

Workers' savings rates⁵⁰ increase for three reasons: there are fewer mouths to be fed; the bulk of the labor force moves towards middle-aged workers, who are the ones saving the most;⁵¹ and labor earnings increase because of price effects.⁵² Dissavings rate increases more slowly, and therefore the overall investment rate increases. This last result follows from capital (p.e.u.l) being increasing, as it is the only vehicle of savings.⁵³ As the economy moves to the long run, savings stabilize and investment rate decreases to its long-run value. Overall,

⁴⁷For a more detailed discussion on this see section A.3.5.

⁴⁸See section 5.2 for specific exercises that allow changes in productivity and a detailed discussion on this.

⁴⁹The second term is the dissavings rate since in OLG each period agents sell their undepreciated capital.

⁵⁰For the complete expressions of sr_t^{t-1} and sr_t^{t-2} , take equations 28 and 29 of A.3.6 and divide by k_t^α .

⁵¹See figure 17 in appendix A.3.6.

⁵²Price effects do not seem to be the determining factor behind the behavior of savings, as discussed below.

⁵³For a visual illustration see figure 18 in appendix A.3.6.

there is a temporary increase in investment rates before and during the demographic window.

The cause for the aforementioned result in investment, per the model, is that in small open economies there is a complete separation between savings and investment decisions. In a demographic window, the former are modified by changes in child dependency and in savers' cohort sizes and the latter by the growth rate of effective labor; they move in opposite directions.⁵⁴ In closed economies, however, savings and investment move in the same direction, so it is possible to have increasing investment rates.

Figure 3 shows the benchmark numerical exercise for Mexico. The first steady state is characterized by the average growth rate of newborns from 1950–1970, and the second by the projection for 2050. I plot this growth rate in panel (a).⁵⁵ Panel (b) shows the change in the WAP ratio that this generates. Notice that the ratio peaks in the window. Panels (c) and (d) show investment and savings rates of the closed and the open (frictionless) economies.

The evolution of savings rates in the closed and the open economies is similar—they increase before and during the demographic window and decrease as the economy converges to the long run— this suggests that price changes are not the dominant effects. In the open economy, savings of young workers increase since they bring less money from the future for their children's consumption (decrease of dissavings) and as a result of more available income for themselves. Savings of middle-aged workers also increase, because of the latter.

In the closed economy total worker's savings also increase. However, young workers' savings decrease the period before and during the demographic window, while middle-age savings increase. Savings of young workers, increase in the first period of the transition because of the reduction in child dependency.⁵⁶ After that, once price effects kick in, savings of the young decrease through the rest of the transition. Borrowing increases since the labor income profile is increasing through the transition, and the cost of financing decreases, making it cheaper to bring the increased future labor income to the present and smooth consumption. However, the magnitude of this change is small and the young savers cohort shrinks at the

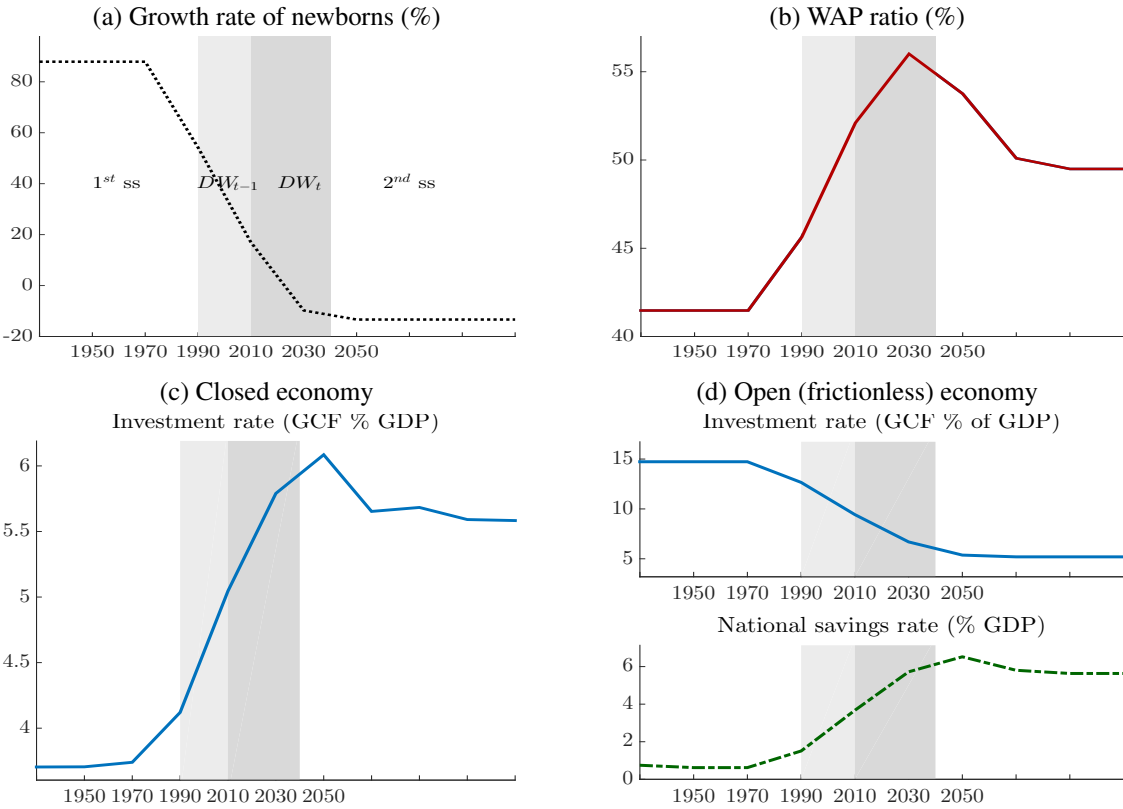
⁵⁴Child dependency is measured by children per worker in this model.

⁵⁵20-year cumulative growth rate: average for 1950-1970, observed for 1990 and 2010, projection for 2050.

⁵⁶The first period of the demographic transition is 1970-1990 and the period before the window is 1990-2010.

expense of the middle-aged workers' cohort, which makes their savings the determinant ones. Savings of middle-aged workers increase through the entire transition. In the first period, this occurs because the decrease in child dependency increases the available income. Further on, this higher available income compounds with higher labor earnings to generate an increase in savings. Even though returns for savings decrease, savings increase since that is the only way to consume during retirement.

Figure 3: Benchmark exercise, Mexico



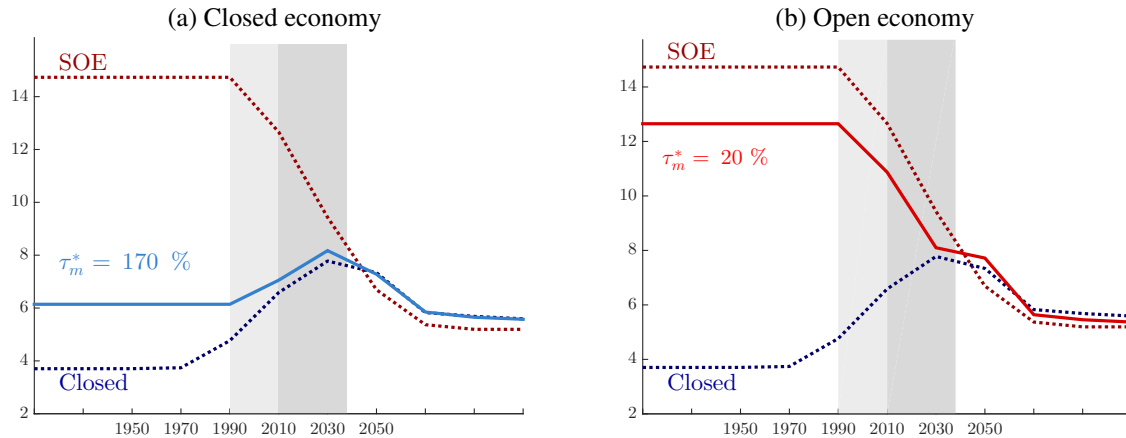
Notes: This figure plots the Mexican demographic transition (i.e., the reduction in the growth rates of newborns in panel) (a) and the demographic window that this generates in panel (b). The dark gray area indicates the years of the demographic window and the light gray area, the 15 years before the window. In panels (c) and (d) I plot investment and savings rates (equal to each other in the closed economy case c) throughout this transition.

Recall that the results of the extreme cases extend to the model with variable degrees of openness. Figure 4 illustrates this point, with the previous exercise for Mexico using the model with variable degree of openness for high and low taxes.⁵⁷ When taxes are high,

⁵⁷Figure 19 in appendix A.3.6 plots the capital (p.e.u.l) of this exercise.

investment for the demographic window increases as in the fully-closed economy, and when taxes are low it decreases as in the fully-open economy.⁵⁸

Figure 4: **Benchmark exercise for Mexico: varying degree of openness**



Notes: This figure plots the Mexican demographic transition in a framework of variable openness. The dark gray bar indicates the years of the demographic window and the light gray bar, the 15 years before it. In panels (a) and (b) I plot the investment rate throughout this transition in the taxed-closed and the taxed-open economies. The solid line is the investment rate for the model with taxes and dotted lines investment rates in the frictionless model. In the closed economy $\tau_1 = 20\%$ and $\tau_2 = 5\%$ and in the open economy $\tau_1 = 170\%$, and $\tau_2 = 7\%$.

In these numerical exercises, the growth rate of technology and the rate of return on the rest of the world are set to the average between 1950 and 2014. Both variables were calculated through growth accounting exercises on Mexico and on the United States (U.S.)— which represents the rest of the world (this country is not part of those undergoing a demographic transition in the time studied).⁵⁹ In the next section I relax these assumptions.

5.2 Robustness to alternative confounding factors

In this section, I allow for changes in productivity, international returns, and negative shocks on the capital stock. I explore whether these variables can generate, on their own, different responses of investment for the demographic window and whether they can change the main result of openness.

⁵⁸For more details of the open taxed case see the open economy section in appendix A.3.7.

⁵⁹I compute the growth rate of total factor productivity (TFP) of Mexico using the model specification and retrieving TFP as a residual. The rate of return of the rest of the world is the marginal productivity of capital (p.e.u.l) net of depreciation of the US. In the growth accounting exercises, a la Kehoe, I use data on investment, consumption of fixed capital, GDP, and WAP, with a depreciation of 5% and a capital share of income of 0.3.

Return on international assets

I first relax the assumption of a constant rate of return on international assets and examine the effects that the observed decrease in this rate of return could have on investment for the demographic window. The exercise consists on introducing the observed decreasing path of the rate of return of the rest of the world, measured as the net marginal productivity of capital of the U.S., and computing the equilibrium in the open (frictionless) economy through the Mexican demographic transition.⁶⁰ Figure 5 shows the results of this exercise and compares it to the benchmark.⁶¹

The decrease in the return of international assets generates an increase in domestic capital (p.e.u.l) to lower its marginal productivity and equalizes the return of domestic and international assets (see panels (a) and (b) of figure 5). Investment rates increase in the first period of the demographic transition because of the increasing capital (p.e.u.l). However, once the effects of the demographic window kick in, investment rates decrease. In particular, during the period before and in the first period of the demographic window, investment rates go down since the decrease in future labor force associated with the demographic window is large enough to offset the increase in domestic capital (p.e.u.l) generated by the changes in the interest rate.⁶² The end result, as depicted in panel (d) of figure 5, is a decrease of investment for the demographic window.

To summarize, the observed decrease in international returns, absent of the demographic window, would generate an increase of investment rates. Since it makes it relatively less worthwhile to invest abroad, households shift part of their international assets to domestic capital which increases investment rates. However, once the demographic window effect is taken into account, namely the large decrease in growth rate of effective labor force, the aforementioned result is more than offset decreasing investment rates. The conclusion is that even

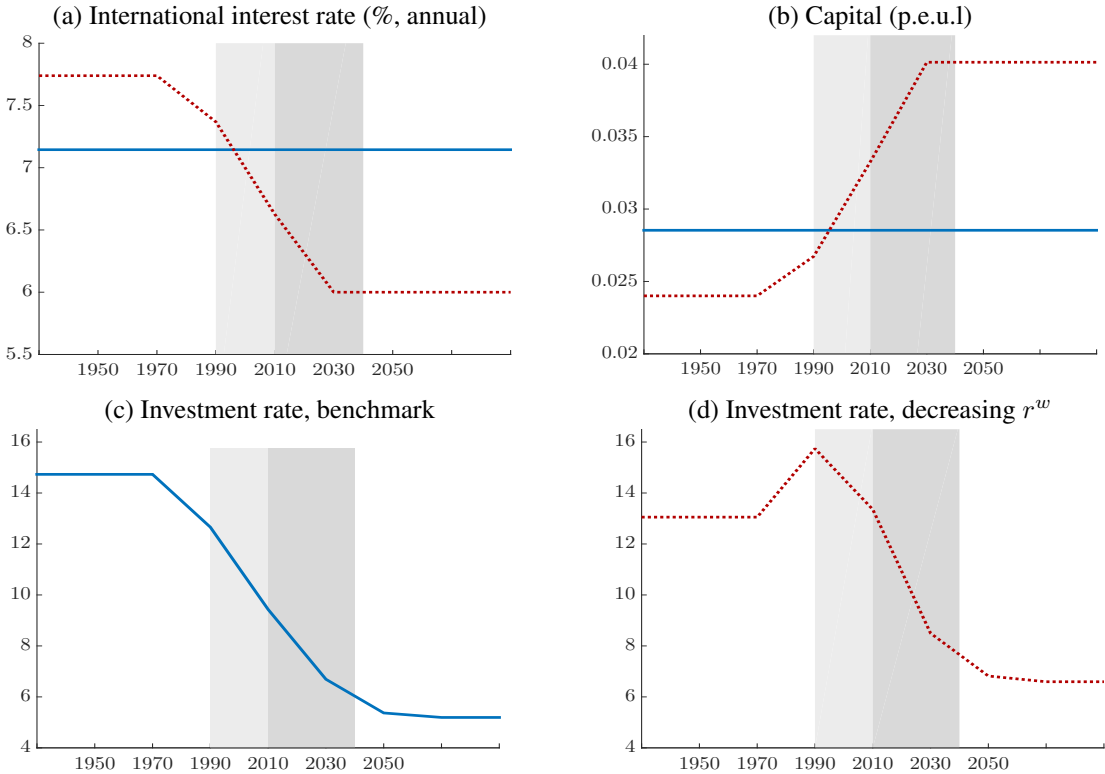
⁶⁰The annualized international rates of return are 7.7% (1950-1970), 7.4% (1970-1990), 6.6% (1990-2010), and 6% (2030 on). I chose 6% for 2030 on to abstract from the drop in rates of the Great Recession (the value of 2007 was 6.14% and afterwards rates dropped to 5.5%). A second criterium is to use the average of the most recent decade, 2004–2014, which is also around 6% (5.8%).

⁶¹To see the results on savings rate, refer to figure 20 in appendix A.3.6.

⁶²In these two periods, the decreases in the future growth rate of the labor force are the biggest, since they are the decreases in current labor force that happen during the demographic window itself.

considering the decrease in returns on international assets, investment for the demographic window decreases in the open economy.

Figure 5: **Decreasing international rate of return**



Notes: This figure shows capital (p.e.u.l) and investment rates (GCF % GDP) through the Mexican demographic transition under two scenarios. The first scenario, shown by the dashed lines, is characterized by the observed decreasing international return until 2010 with a projection of 6% for 2030 on. The second scenario, shown by the solid lines, is the benchmark exercise with an annual constant return of 7.15% (the average of 1950-2014).

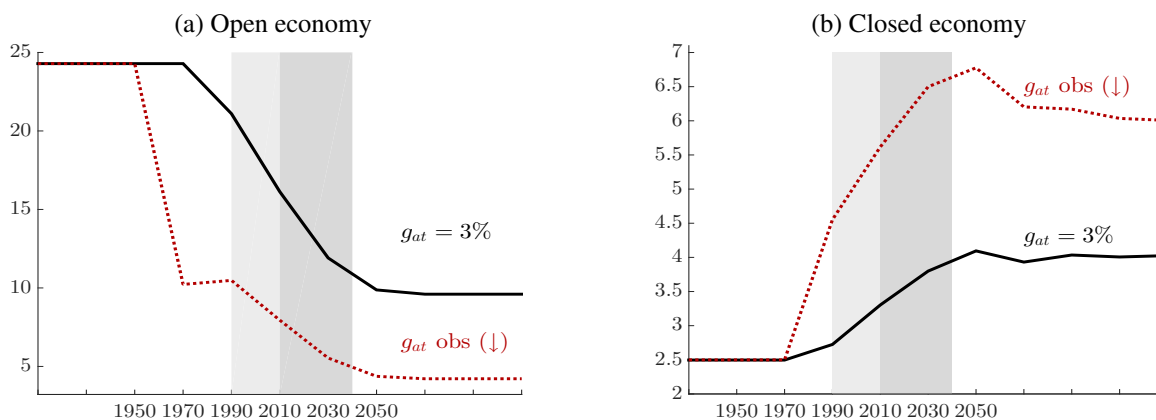
Productivity Growth

Recall that in open economies investment rates grow with effective labor force. In principle, increases in the growth rate of productivity could increase investment rates. However, once we consider demographic changes, investment for the DW does not increase with technological progress since in order to offset demographic effects, changes in the growth rate of productivity need to be implausibly high.⁶³ To illustrate this point, in figure 6(a) I compare

⁶³The growth rate of working age population decreases more than the increase in productivity during this period. In Latin America annual growth rate of the working age population decreases from 3% at the beginning of the demographic transition to 0% towards the end of the window. See figure 21(a) in appendix A.3.6.

a scenario with high technological growth (the average growth of Mexico between 1950 and 1970), with the observed decreasing path.⁶⁴ Note that in both scenarios investment rate is decreasing in the transition regardless of these big technological changes.⁶⁵

Figure 6: Investment rates and technology growth



Notes: This figure shows investment rates (GCF % GDP) through the Mexican demographic transition under two scenarios. Shown by the solid line: a “high technological growth rate,” where $g_{at} = 3\%$ annually (e.g. the observed average for the first steady state 1950-1970). Shown by the dashed line: the “observed technological growth” until 2010 (see footnote 62) with a projection of 0% for 2030 on.

In particular, consider the case of most Western European countries and Japan.⁶⁶ These countries experienced high growth from the postwar era until the mid 1970s, followed by a slowdown.⁶⁷ In all the countries in my sample that belong to this group, this period of high productivity, falls during their demographic windows.⁶⁸ One could therefore ask if we should think of these countries as open economies where investment for the DW increased because of high TFP, but, as explained above, through the lens of the model we cannot.

In the same spirit, it is not possible to undo the result of the closed economy with productivity changes. In closed economies, changes in the growth rate of TFP modify the level

⁶⁴Annualized TFP growth rates are 2.84%(1950-1970), -0.82%(1970-1990), -0.11%(1990-2010), 0%(2030 on).

⁶⁵In the open economy, savings rates increase like in the closed economy, see figure 22 in appendix A.3.6.

⁶⁶I group Japan with Europe since the demographic window of Japan happened around the same time as those of other European countries (1960 to 1996).

⁶⁷High growth in terms of GDP per capita and GDP per working age population, which in a balanced growth path is equivalent to productivity growth. For some countries this high growth lasted until 1973 and for others until 1978, the years of the oil shocks.

⁶⁸This group being Western Europe and Japan. See figure 23 in appendix A.3.6 for the evolution of GDP per capita, because of data availability, which is similar to the evolution of GDP per working age population.

of investment rates, but these changes do not modify the response of investment rates to demographic changes. We can see this in figure 6(b). Again, we might ask if we should think of Latin America as a closed economy where decreasing TFP explains decreasing investment rates. The answer is no, since not only is the behavior of investment for the demographic window unchanged, but decreasing growth rates of TFP increase investment rates, the opposite of what I document for this region.

The fact that a decrease in the growth rate of TFP increases investment rates is a standard result in closed models with technological and population growth. In a simple representative agent model of this type, a permanent decrease in the growth rate of either variable increases the steady-state level of capital (p.e.u.l).⁶⁹ At the beginning of the transition, accumulation is fast, overcompensating the decrease in the growth rate of TFP (see equation 24), and therefore increasing investment rates. As the economy converges to the long run, capital accumulation slows down. The same logic extends to the case of a decreasing sequence of growth rates of TFP, which implies convergence towards increasing capital (p.e.u.l) steady states and therefore increasing investment rates.⁷⁰

In the model of this paper, there are similar mechanisms in place regarding steady state levels. A decreasing TFP growth rate increases capital (p.e.u.l) of the steady state.⁷¹ About convergence, there is an increase in savings and therefore in investment rates.⁷² For young workers, since future labor income grows more slowly, less is brought to the present, decreasing dissavings. For middle-aged workers, since the growth of labor income was higher when they were young, they increase savings for retirement to smooth consumption. The dissavings rate increases more slowly since the economy is converging to a higher steady state. In short, in the closed economy, the response to a decreasing productivity growth is to increase savings and investment rates to smooth consumption since labor income growth slows down.

⁶⁹In a balanced growth path model with CRRA utility over consumption and Cobb-Douglas labor augmenting technology, the steady state capital (p.e.u.l) is $k_{ss} = \frac{\alpha}{\frac{\sigma}{\beta} - (1-\delta)} \frac{1}{1-\alpha}$, where G is the growth rate of technology and labor. Note that a decrease in G will increase k_{ss} for $\sigma \geq 0$.

⁷⁰This result does not depend on the parameters of the model; in particular it holds for $\sigma \geq 0$.

⁷¹For standard parameters, i.e., $\forall \sigma > 0, \forall \delta, \forall \gamma \geq 0, \forall \beta > 0$.

⁷²See savings equations 28 and 29 in appendix A.3.6.

Finally, I analyze the possible effects of the change in the long-run growth rate of Western Europe and some Asian countries. In the 1800s, when the United Kingdom was the industrial leader, these countries generally grew at an approximate annual rate of 1%. Later, in the 1900s, when the U.S. took over as the lead country, they grew at an approximate rate of 2%.⁷³ For the countries of my sample, the change in growth trend happened between 1880 and 1920.⁷⁴

In the model, the aforementioned change in long-run growth is a one-time increase in the growth rate of technology that can be incorporated as a change to a higher balanced growth path.⁷⁵ In the open economy framework, when the change in growth rate happens close to or during the demographic window, this change is offset by an even larger demographic change (see equation 24). To give a sense of demographic changes in this region, for instance, the annual growth rate of the working age population decreased in Japan from 2% before its demographic window to around 0% towards its end.⁷⁶ This is a bigger change than the movement from a 2% BGP to a 1% one.

In the closed economy framework, a one-time increase in the TFP growth rate generates on its own a decrease in the investment rate. Therefore this change in long run growth cannot help to explain the documented increase of investment for the demographic window in these region. Once we add the demographic window on the top of the technological change, the increasing pattern of investment emerges, as depicted in figure 6(b), where the only difference is that the level of investment is lower than what it would be without technological changes.⁷⁷

Capital Depletion and Subsequent Accumulation (WWII)

In this section I address the question of whether the increase in investment for the demographic window in Europe was a result of postwar capital destruction and subsequent recon-

⁷³2% is the average growth rate of GDP per working age population of the US over the 20th century.

⁷⁴See figure 24 in appendix A.3.6.

⁷⁵Note that this is different from the discussion of the behavior of the growth rate of technology during the period of 1950s to 1970s. Here, the focus is on a one-time change in long-run growth, as opposed to an increasing or decreasing path. This is could be of interest in the closed economy. See figure 25 in appendix.

⁷⁶I use the case of Japan since its demographic window started in 1960 and finished in 1996, and thus there is full data availability. See fig 21(b) in appendix.

⁷⁷For an exercise that replicates the exact steps followed in this explanation see figure 25 in appendix.

struction. Many European countries were in their demographic windows around the World War II (WWII). In the empirical analysis, the main findings documented in this paper (i.e., the different behaviors of investment for the demographic window across regions) are robust to possible effects of the war on investment rates. As mentioned in the empirical section, the results hold even in a subsample where I dropped post-war years, e.g, the 1960s and the 1970s, for Europe, Asia, and Oceania.

To analyze possible war effects on investment for the demographic window in the model, I consider the destruction and subsequent accumulation of capital. To do so, I assume that in the first period, the economy is below its balanced growth path. I perform this exercise in the open economy framework, in the same spirit as the productivity exercises, since the closed economy already generates increasing investment rates.

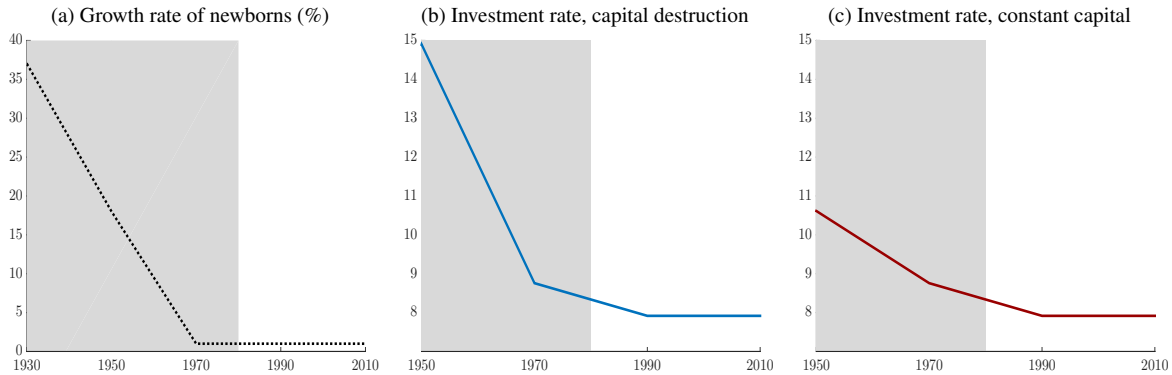
As an example of a European demographic transition I consider the demographic window of Great Britain (1920–1980).⁷⁸ This transition is introduced as a decreasing growth rate of newborns starting one period before 1950, since Great Britain was already in its window by then.⁷⁹ This growth rate stabilizes by 1990, which I take as the second steady state. I assume that in 1950 capital (p.e.u.l) was 50% below the steady state-level and that it recovered by 1970, after twenty years (one period in this model) and thus I do not include adjustment costs.

The result, shown in figure 7, is that the initial investment rate is higher (compared to a scenario without capital destruction) in the first period, and afterwards it decreases towards the long-run equilibrium. Thus, even considering capital destruction and reconstruction, in an open economy, investment rates decrease in the demographic window. According to the model, the increase in investment for the demographic window in Europe was not a result of postwar capital destruction and its subsequent reconstruction in an open economy framework.

⁷⁸I exclude Scotland and use census data of England and Wales because of data availability and because their transition exhibits the general characteristics discussed in section 2, see figure 26 in appendix A.3.6. According to McDaniel and Zimmer (2013) its transition “was triggered by a rapid drop in fertility in the early 1900s”.

⁷⁹I considered the period of 1930-1990 and used observed values of 1931, 1971, and 1991, under the same methodology of the Mexican exercises. For 1951 I used the average of 1931 and 1971 because of missing data.

Figure 7: Investment rates, WWII exercise



Notes: In panel (a), I plot the demographic transition of Great Britain as introduced by reduction in the growth rate of newborns. In panels (b) and (c), I plot the investment rate through this transition in a frictionless open economy under two scenarios. The first one is the WWII case, where capital (p.e.u.l) in 1950 is 50% below the steady-state level. The second one is a *constant capital scenario* without capital destruction.

Mechanically, in the first period capital is below the steady state and it increases back to the open economy level in one period. This is the reason why the initial investment rate is high.⁸⁰ From the second period onwards, capital has recovered and it is given by the after-tax foreign interest rate, independently of demographic changes, so investment rates decrease with the effective labor force. The assumption behind this result is that capital (p.e.u.l) recovers to prewar levels by 1970.

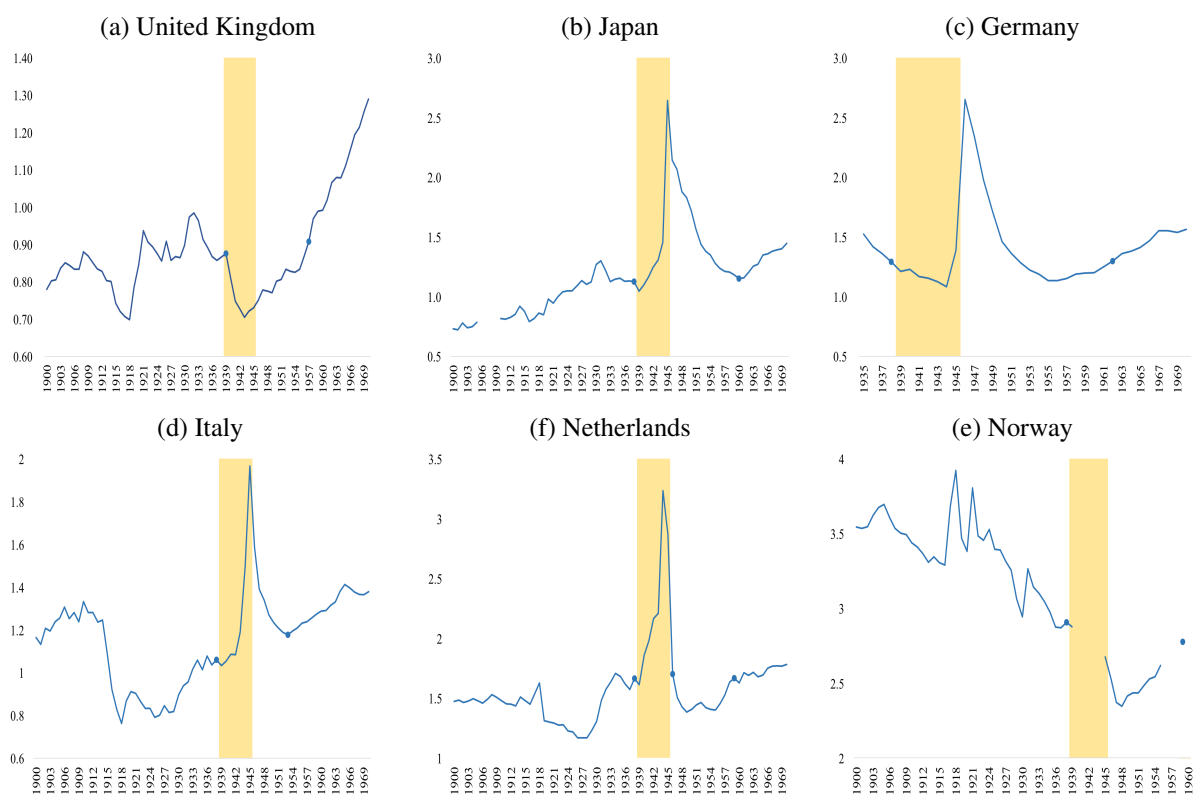
In figure 8, I plot estimates of the non-residential capital stock to output ratio of some European countries and Japan for the years before, during, and after World War II.⁸¹ Notice that this ratio returns to pre-war (1938) levels by the late 1950s and early 1960s.⁸² For example, in Germany, one of the most affected countries, the ratio recovers by 1962. This supports the assumption that capital (p.e.u.l) recovers at latest by 1970.

⁸⁰We can see this directly from the investment equation (24) where $k_t = 0.5k_{t+1}$, $k_{t+1}(r^w, \tau)$, $k_t < k_{t+1}$.

⁸¹For a longer time series and a comparison with the United States, see figure 27 in appendix A.3.6.

⁸²Total non-residential capital stock recovers even faster (see figure 28 in appendix A.3.6).

Figure 8: Capital stock to output ratio and WWII



Notes: This figure plots the capital to output ratio of some European countries and Japan. The shaded area indicates the years of WWII. This ratio is the estimated non residential capital stock divided by GDP. The estimate for the Netherlands is the gross stock of structures, machinery, and equipment from Groote et al. (1996); for the UK, Japan, Germany, it is from Maddison (1994). For Italy the estimate is the net stock of machinery and equipment, construction, and means of transport from Baffigi (2011). Level series are in 1990 international dollars, except for Italy's, which is in 2011 international dollars because of data availability. GDP in 1990 and in 2011 international dollars comes from the Maddison and World Economics Global GDP databases, respectively.

5.3 The Case of Brazil

In this section I analyze the case of Brazil, one of the biggest countries in Latin America. I compare the results of the benchmark model to the investment rates⁸³ observed in Brazil, in the 15 years before and during its demographic window.

⁸³Regarding savings behavior, the model predicts an increase of savings rates of throughout the demographic transition that compares with data of 20.85, 20.98 and 18.44 in the respective periods with only an increase in the transition from the first steady state to the second one. This observation is related to the fact that no significant consistent pattern was found for the current account to GDP ratio. Regarding more general data patterns on savings, the model has strong predictions of savings rates in demographic transitions, despite evidence being mixed in this prediction, as mentioned in the introduction East Asian case studies have documented such; in the model of this paper predictions on savings for both the closed and the open economy are the same.

In Brazil, investment rates decreased from an average of 22% in the 1970s and early 1980s, to an average of 20% 15 years before their window, and an average of 18% up to this point in their window (table 3). This pattern of decreasing investment rates for the demographic window is consistent with the open economy result. The fact that the model implies a greater decline in investment rates than the data is suggestive of its potential in explaining the documented decrease of investment for the demographic window in Latin America. Table 3 shows the results of the open frictionless benchmark model and a comparison with the data.

Table 3: **Investment rates (%): the model and the data**

(a) Brazil: model		(c) Brazil: data	
period	open	period	inv. rate
1 st ss	16.96	1970–1984	21.95
DW-15	13.58	1985–1999	19.86
<i>DW</i>	9.18	2000–2013*	17.89
2 nd ss	6.5		
Decrease of inv. for DW_1	10.42		4.06

This table shows the investment rates generated in the open frictionless economy model (benchmark exercise for Brazil) in panel (a), and the observed investment rates of Brazil in panel (b). The first row is the period before the demographic transition: in the model it is the first steady state, for Brazil it is the period of 1970 to 1984.⁸⁴ The second row (DW-15) reports the investment rate of the 15 years before the demographic window: in the model it is the average of the years of 2000 and 2040 first and the last year of that period, and in the data is the average of the period indicated in the first column. The third row (*DW*) reports the investment rate of the demographic window up to 2013* for consistency with the empirical section (see note below): in the model it is the average of the years of 2000 and 2040, and in the data⁸⁵ the average of the period indicated in the first column. The last row “Decrease of investment for the *DW*” reports the decrease of investment rates from the period before the demographic transition up the first years of a demographic window.

In the benchmark of Brazil I consider 1960 as the first steady state and 2040 as the second one. I introduce the demographic transition as the decreasing growth rate of newborns (adjusted for mortality). Analogous to the benchmark exercise for Mexico, I use the average growth rate of TFP of 1968-2011. (see figure 29 for an illustration of the result).

⁸⁴The choice of the start year is to be as consistent as possible with other periods in the table, i.e. years before the period “DW-15”. Note that for Brazil there is no liberalization process in the 70s, emphasizing the need for incorporating a time varying degree of financial openness in the model. I elaborate on this in the conclusions.

⁸⁵Note: investment rate is GCF as percentage of GDP build using data from WDI (2014 wave for consistency with the empirical section, for Brazil because of data availability, the years of 2014 to 2015 can be computed using the growth rate of GCF of the following WD indicators wave yielding 17.64.

6 Conclusions

In this paper, I have documented that different regions display different patterns of investment before and during a demographic window. In particular, LatAm displays a different behavior in that investment rates are lower for the 15 years before and during their demographic window than outside of this period, whereas in Europe, Asia, and Oceania these rates are higher. Additionally, I documented a negative relationship between investment for the demographic window and the degree of financial openness of the economies, when financial openness is measured by the Chinn-Ito index. This fact indicates that financial openness is an important consideration for the relationship between investment and the demographic window.

Motivated by this finding I developed an OLG model with a variable degree of financial openness and demographic change that can rationalize the documented pattern of investment. The main result of the model is that in open economies investment rates decrease in preparation for and during the demographic window, while in closed economies they increase.

The demographic windows studied in this paper, which are part of the long-run worldwide demographic change, are triggered by a reduction in the number of newborns and characterized by a temporary increase in the working age to total population ratio. There are three important effects associated with these windows: a reduction in child dependency, an increase in the population share of middle-aged savers, and a decreasing growth rate of the labor force.

These effects generate opposite changes in investment rates depending on the level of financial openness of each economy. In open economies, since capital (p.e.u.l) is pinned down by the net return on international assets, and this is a growth path model, investment rates grow with the growth rate of the effective labor force, which is decreasing in a demographic window. In closed economies, the reduction in child dependency and the increase in the share of middle-aged savers increases savings rates and therefore investment rates. The reduction in child dependency increases available income for self-spending and decreases dissavings of young workers associated with the consumption of their children.

This emphasizes the importance of the fact that before and during a demographic window, the growth rate of the working age population is not increasing, even though the fraction and

the total working age population might be so. The growth rate of the working age population is decreasing since each year there are fewer children that will be part of the future working age population, and this translates into a decreasing growth rate of the labor force. Therefore even though dependency is lower during a demographic window and there are many workers to invest in, the investment to GDP ratio does not necessarily need to increase. What matters for investment rates in open economies is the growth rate of labor force, not the level, and this rate is decreasing.

In the model, I also explored alternative variables and hypotheses that could generate different responses of investment for the demographic window. I considered the decrease in the rate of return of international assets, both short- and long-run changes in productivity growth, and the effects of capital depletion and subsequent accumulation after WWII. The conclusion from these exercises is that these factors cannot undo the main result of openness nor can they generate results in line with the documented patterns in investment on their own.

The model proposed in this paper is consistent with the documented facts for LatAm, Europe, Asia and Oceania. In particular, the fact that Europe and some Asian countries were less financially open during their windows can help to explain why they have higher investment rates in those period. Similarly, being more financially open during the ongoing demographic window in LatAm can explain why investment rates are decreasing. This framework is a first step in understanding such phenomenon, I acknowledge however that it has limitations; while it explains investment patterns it cannot account for the behavior of the current account.⁸⁶

The framework developed in this paper can be extended along a number of dimensions. In terms of future work, the first item in my research agenda is to perform a full calibration in order to fully assess the model's ability to account for changes in investment rates associated with a demographic window.⁸⁷ In the preliminary exercise I conducted to compare the performance of the model to the data, I find that the observed decrease of investment before

⁸⁶No patterns were found at conclusive significant levels for the current account. In that sense it is not possible to rule out alternative hypotheses.

⁸⁷Depreciation rate is the parameter calibrated to target data moments, taxes were chosen to simulate open and closed economies and to make the point that results are similar to those of fully-open and -closed economies.

and during the demographic window of Brazil is 4.1 percentage points and 10.4 percentage points in the model of the fully open benchmark economy. The observed pattern of decreasing investment rates for the demographic window is consistent with the open economy result, and the exercise is suggestive of the potential of the model in explaining the documented decrease of investment for the demographic window in Latin America.

Related to this, an exercise where the degree of openness changes over time is also necessary to compare the predictions of the model with the data, specially for countries with demographic windows starting in the early 2000s, and in order to achieve a more realistic approach regarding the classification of an economy as open or closed. Finally, welfare calculations to make policy recommendations are left as a future task.

In terms of the assumptions of the model, in this paper I take the reduction in fertility rates as exogenous, from the data, to generate a demographic window. An alternative to this approach is to model the developing process, where countries undergo through the demographic change as income increases and endogenize fertility choices. Endogenizing such choices would not change the analysis of the model as that would be the first step to model the shock that makes the economy transition from the first to the second steady state. Additionally, I assume that there is no early death and adjust the growth rate of newborns to the growth rate of working-age groups. This assumption can be relaxed by including a survival probability, and it would allow for the incorporation of the decrease in mortality of the demographic transition which is a determinant of the length of a demographic window.

Another assumption is that young and middle-aged workers are equally productive. The demographic framework of the model —having two different cohorts of workers— allows for the incorporation of income profiles. For countries with hump-shaped income profiles, this can be modeled as a higher efficiency of the middle-aged workers to young workers. Since in a demographic window the bulk of the labor force moves to the middle-aged workers, this will be an additional source of increase in savings. This extension could be an important determinant of patterns of investment across regions since, as some authors like Lagakos et al. have documented, experience-wage profiles are steeper in more developed countries

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A Appendix

A.1 Data Analysis

Sample selection

The main sample has 51 countries with demographic transitions and includes data from the years of 1960 to 2013. I classify a country as having a *demographic transition* if it has data on investment and GDP during both (1) its demographic window (DW), and (2) the 15 years preceding the DW. Countries with less than 3 observations in either of these two periods were dropped from the sample, as well as countries with no data on financial openness during both the two aforementioned periods and potential outliers.⁸⁸ Additionally, countries with transitional economic systems according to IMF and WB classifications were excluded. African countries were also excluded since the demographic window of the continent is projected to begin in 2059. For Sections 2 and 5 I used observed data and projections of working age population and dependency for the years of 1950-2050, and observed data and projections of labor force for the years of 1950-2020.

Investment rates

For the econometric analysis, investment rates are the trend component of the ratio of GCF to GDP. This ratio was HP-filtered using a smooth parameter of 6.25 corresponding to yearly frequency, and the cycle component removed to consider long term variations of investment.⁸⁹

⁸⁸Criteria for dropping potential outliers is: i) removing countries where the increase of investment (the difference between average investment rates for the demographic window (DW) and average investment rates outside of this time period depicted in figure 2) is two or more standard deviations away from the average of each region (Europe, Asia, and Latin America, independently); and ii) excluding countries where the level of average investment rates in the period for the DW and outside of it is two or more standard deviations away from the average of each region. The first criterion corresponds to the baseline econometric specification and the second one to the econometric specification where country fixed effects are pooled into region fixed effects, columns one and two of tables 4 and 7, main results hold even if we consider those in the analysis.

⁸⁹Using the non HP-filtered database, the results of the baseline econometric specification (equation 1): coefficients and robust standard errors for dw and $dw \times LA$ are 4.14 (0.28)*** and -4.80 (0.51)*** respectively, the sum of these coefficients is -0.66 and the hypothesis of it being negative cannot be rejected at 5% level.

Different specifications of the model

Table 4 reports the coefficient estimates of regressing investment rates on region and demographic window dummies for the baseline econometric specification of equation 1 under different sets of controls, samples, and definitions of investment for the demographic window. Column 1 contains the estimates for the main sample and the baseline model. In column 2, I report the estimates of pooling the country-fixed effects by region on the dummy LA. Columns 3 and 4 contain the results of dividing the sample by region and performing a regression only on the variable DW. In columns 5 and 6, I drop the 1960s and the 1970s for Europe, Asia, and Oceania to consider possible effects of WWII on investment. In column 7, I include transitional economic systems. In column 8, I use an alternative definition of the demographic window by defining DW investment as that which occurs during the window and 10 years before it; in column 9, I do the same but for the DW and the 20 years before it. In column 10, I control for growth rates of constant GDP per capita.

Table 4: **Investment rates for the demographic window by region**

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
DW	4.16*** (0.23)	4.68*** (0.27)	-1.88** (0.50)	4.00*** (0.24)	2.38*** (0.29)	2.40*** (0.29)	3.26*** (0.26)	3.77*** (0.24)	3.67*** (0.23)	4.11*** (0.24)
DW × LA	-4.93*** (0.42)	-5.63*** (0.47)			-3.00*** (0.51)		-3.86** (0.46)	-4.31*** (0.46)	-5.02*** (0.40)	-5.28*** (0.43)
LA		2.04*** (0.29)								
gr. GDP										0.17*** (0.03)
Fixed effects										
Year	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Country	Y	N	Y	Y	Y	Y	Y	Y	Y	Y
Sampling										
Lat Am	N	N	Y	N	N	N	N	N	N	N
EAO	N	N	N	Y	N	Y	N	N	N	N
– 60-70s	N	N	N	N	Y	Y	N	N	N	N
+ Trans.	N	N	N	N	N	N	Y	N	N	N
For DW										
-15 years	Y	Y	Y	Y	Y	Y	Y	N	N	Y
-10 years	N	N	N	N	N	N	N	Y	N	N
-20 years	N	N	N	N	N	N	N	N	Y	N
Controls										
GDP pc	N	N	N	N	N	N	N	N	N	Y
N	2445	2445	861	1584	1649	788	2855	2426	2717	2279

Robust standard errors in parentheses. * $p < .10$, ** $p < .05$, *** $p < .01$. EAO stands for Europe, Asia, and Oceania
Source: Author's estimations, details described above.

Two regions: Latin America, and Europe Asia and Oceania

I analyzed each region independently, using the specification reported in column 2 of table 4. Results are reported in table 5 below, showing that in Europe and Asia, alone, investment rates for the DW are associated with an increase whereas in LatAm the opposite is true (in LatAm: P-value of null hypothesis is 0.008 and P-value of the dw coefficient below is 0.018).⁹⁰

⁹⁰Samples of Europe and Oceania have less variations, related to the first one the specification reported in column (1) of Table 4 is not significant when considering only Europe, and for the second one the region is

Table 5:

	Europe	Asia	Latin America
$\beta_1 (dw)$	1.43*** (0.33)	5.12*** (0.52)	1.17** (0.49)
Observations	731	804	861
P-val of null to $\beta_1 > 0$	0.0000	0.0000	.
P-val of null to $\beta_1 < 0$.	.	0.0088

Robust standard errors in parentheses, *** $p < .01$, ** $p < .05$.

Source: Author's estimations, details described above.

Changes before and after the demographic window

I also analyzed changes in investment for the demographic window while distinguishing between the periods before and after the window. Regarding changes in investment before the “lagged demographic window” (the window and the 15 years before it), in Asia (excluding Japan) and Oceania, investment rates increased in preparation (15 years before) and during the window as compared to previous years.⁹¹ In LatAm the opposite is true. This result is supported by dividing the sample by countries with data before and after the window and for LatAm, Asia, and Oceania estimating a regression of investment rates on a dummy for the period before the lagged demographic window $bdw_{c,t}$ and a dummy for region:

$$Inv_{c,t} = \beta_0 + \beta_1 bdw_{c,t} + \beta_2 bdw_{c,t} \cdot LA_c + f_c + f_t + \epsilon_{c,t}. \quad (26)$$

Table 6:

Regression Estimates:			Results:		
$\beta_1 (bdw)$	-2.71***	(0.48)	value	P-val null	
$\beta_2 (bdw \times LA)$	5.07***	(0.47)	$\beta_1 < 0$	-2.71	0.00
Observations	1671		$\beta_1 + \beta_2 > 0$	2.36	0.00

Robust standard errors in parentheses *** $p < .01$

The fact that the coefficient estimate of β_1 is negative while the sum of $\beta_1 + \beta_2$ is positive indicates that in Europe and Asia investment rates increases before and during the demographic window, while in LatAm they decrease. Neither of these hypotheses can be rejected at the 1% level. Table 7 reports the estimates of this specification (equation 26) using different controls, samples, and definitions for the demographic window; it is analogous to table 4.

grouped with Europe and Asia, see table 7.

⁹¹I exclude Japan from this analysis since its demographic window started in 1960 and finished in 1996.

Table 7: Investment rates and the period before the lagged demographic window

	(1)	(2)	(3)	(4)	(5)	(6)	(7) [†]	(8)	(9)	(10)
b_dw	-2.71*** (0.48)	-4.74*** (0.49)	1.88** (0.50)	-1.76*** (0.57)	-1.81*** (0.58)	3.26*** (0.74)	-4.90*** (0.57)	-2.03*** (0.51)	-1.20*** (0.421)	-2.63*** (0.52)
b_dw × LA	5.07*** (0.47)	6.03*** (0.53)			3.26*** (0.58)		6.46*** (0.61)	4.28*** (0.48)	4.55*** (0.45)	5.40*** (0.50)
LA		-3.23*** (0.41)					-5.11 (0.45)			
gr. GDP										0.17*** (0.03)
Fixed effects										
Year	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Country	Y	N	Y	Y	Y	Y	Y	Y	Y	Y
Sampling										
Lat Am	N	N	Y	N	N	N	N	N	N	N
AO	N	N	N	Y	N	Y	N	N	N	N
- 60-70s	N	N	N	N	Y	Y	N	N	N	N
+ Trans.	N	N	N	N	N	N	Y	N	N	N
For DW										
-15 years	Y	Y	Y	Y	Y	Y	Y	N	N	Y
-10 years	N	N	N	N	N	N	N	Y	N	N
-20 years	N	N	N	N	N	N	N	N	Y	N
Controls										
GDP pc	N	N	N	N	N	N	N	N	N	Y
N	1671	1671	861	810	1244	383	1839	1652	1943	1512

Robust standard errors in parentheses. * $p < .10$, ** $p < .05$, *** $p < .01$. AO stands for Asia and Oceania

† Note: for the sample that consider transitional countries the model specification of equation (1) is not significant, the table reports that analogous to column 2 of table 6.

Source: Author's estimations, details described above.

Hypothesis Testing

The main empirical result is that in Europe, Asia, and Oceania, investment rates for the demographic window are associated with an increase, whereas in LatAm they are associated with a decrease. Table 8 reports the P-values of testing the hypotheses behind these results

for different specifications of the model.⁹² The specification of column (2) additionally indicates that investment rates for the demographic window are higher in Europe, Asia, and Oceania, as compared to those in LatAm, in the period for the demographic window. The results of columns (3) and (4) indicate that even taking the smallest sample, the effect of the demographic window is positive in Europe, Asia, and Oceania and negative in LatAm. Column (6) indicates that the results are robust even in the smallest sample and controlling for possible effects of the war.

Table 9 reports the hypothesis testing that support the result about changes in investment before the lagged demographic window: in Latin America investment rates decrease before and during its demographic window but increase elsewhere and it is analogous to table 8.

Table 8: Hypothesis testing of investment for the DW

	(1)	(2)	(3)	(4)	(5)	(6)
β_1	4.16	4.68	-1.88	4.00	2.38	2.41
P-val of null to $\beta_1 > 0$	0.00	0.00	.	0.00	0.00	0.00
P-val of null to $\beta_1 < 0$.	.	0.00	.		
$\beta_1 + \beta_2$	-0.76	-0.95	.	.	-0.64	.
P-val of null to $(\beta_1 + \beta_2) < 0$	0.01	0.00	.	.	0.05	.
$\beta_2 + \beta_3$.	-3.59	.	.		
P-val of null to $(\beta_2 + \beta_3) < 0$.	0.00	.	.		

Source: Author's estimations, details described above.

Table 9: Hypothesis testing of investment before the lagged DW

	(1)	(2)	(3)	(4)	(5)	(6)
β_1	-2.71	-4.74	1.81	-2.85	-1.81	-3.26
P-val of null to $\beta_1 < 0$	0.00	0.00	.	0.00	0.00	0.00
P-val of null to $\beta_1 > 0$.	.	0.00	.		
$\beta_1 + \beta_2$	2.36	1.29	.	.	1.45	
P-val of null to $(\beta_1 + \beta_2) > 0$	0.00	0.01	.	.	0.00	
$\beta_2 + \beta_3$.	2.80	.	.		
P-val of null to $(\beta_2 + \beta_3) < 0$.	0.00	.	.		

Source: Author's estimations, details described above.

⁹²The number at the top of each column corresponds to the model specification of the columns of table 4.

Openness

Table 10 reports the coefficient estimates of regression (2), excluding Europe from the sample.

Table 10: **Demographic window and openness**

β_1 (<i>dw</i>)	1.44***	(0.487)
β_2 (Chinn-Ito index)	2.37***	(0.577)
β_3 (Chinn-Ito index \times <i>DW</i>)	-2.85***	(0.697)
Observations	1377	

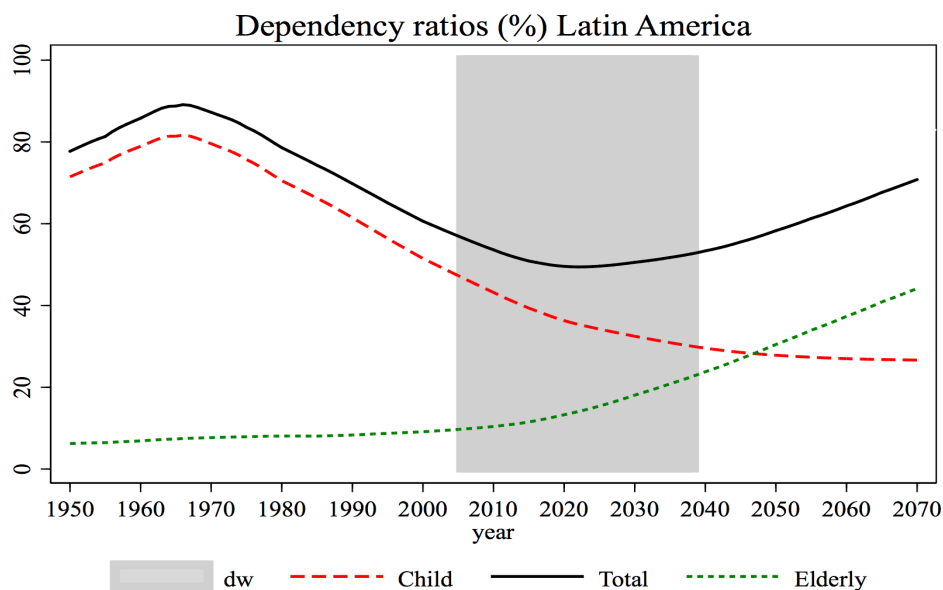
Robust standard errors in parentheses, *** $p < .01$.

Source: Author's estimations, details described above.

A.2 Demographics

An indicator that is sometimes used to describe the demographic window and to motivate its importance is the total dependency ratio. This ratio is defined as the child plus elderly population divided by the working age population. Dependency ratios usually reach their minimum during a demographic window, since the working age population is increasing, the child population is decreasing, and the elderly population is low.

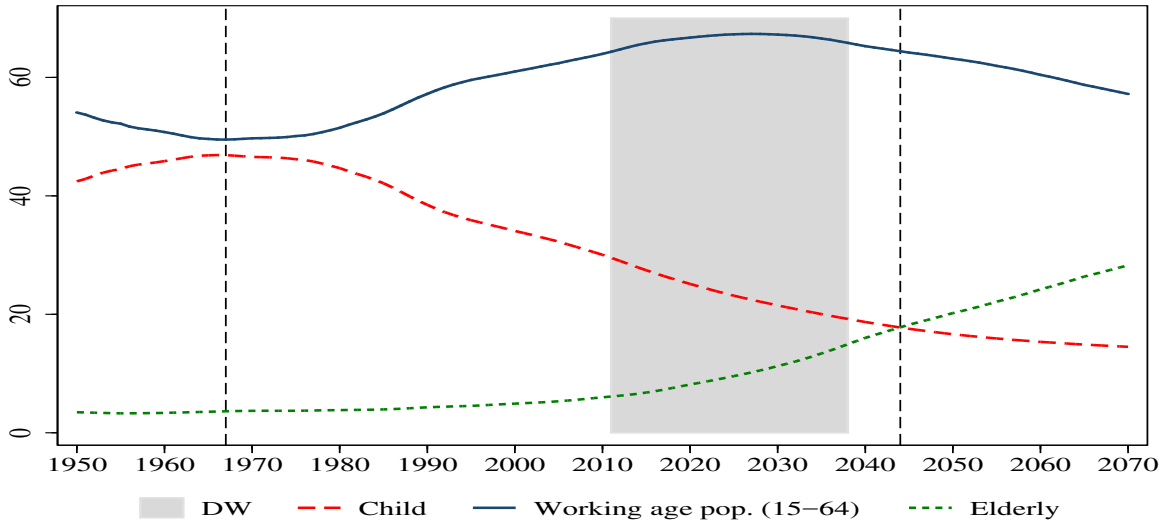
Figure 9: **Demographic window and dependency rates**



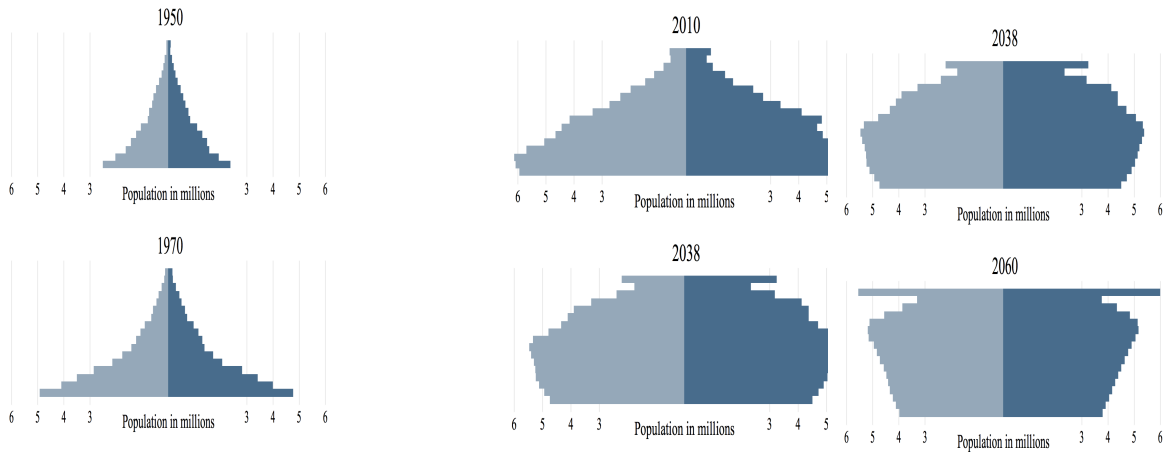
Notes: The figure plots the dependency ratios. The child dependency ratio is the ratio of child population over working age population, and the elderly ratio is the ratio of elderly population over working age population. Source: 2012 World Population Projects Database, UN.

Figure 10: Demographic transition in Mexico

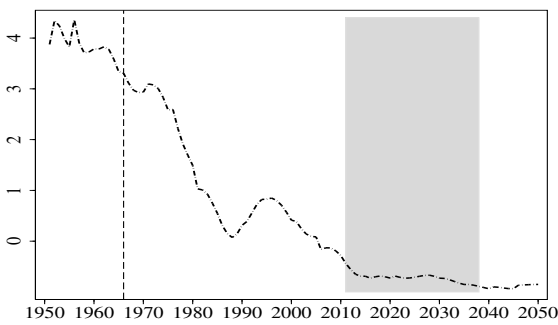
(a) Population by age group (%)



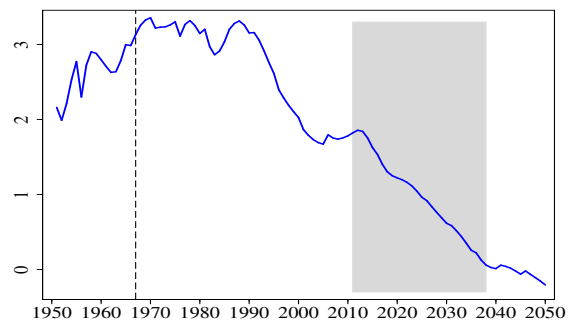
(b) Population pyramids



(c) Annual growth rate of pop. aged 0-14 (%)

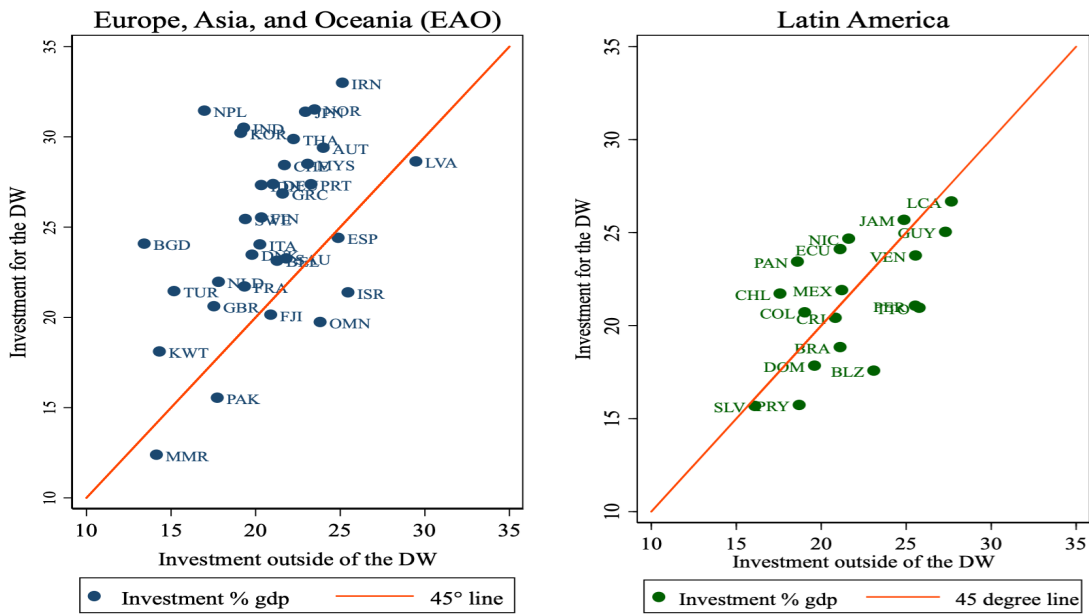


(d) Annual growth rate of working age pop. (%)



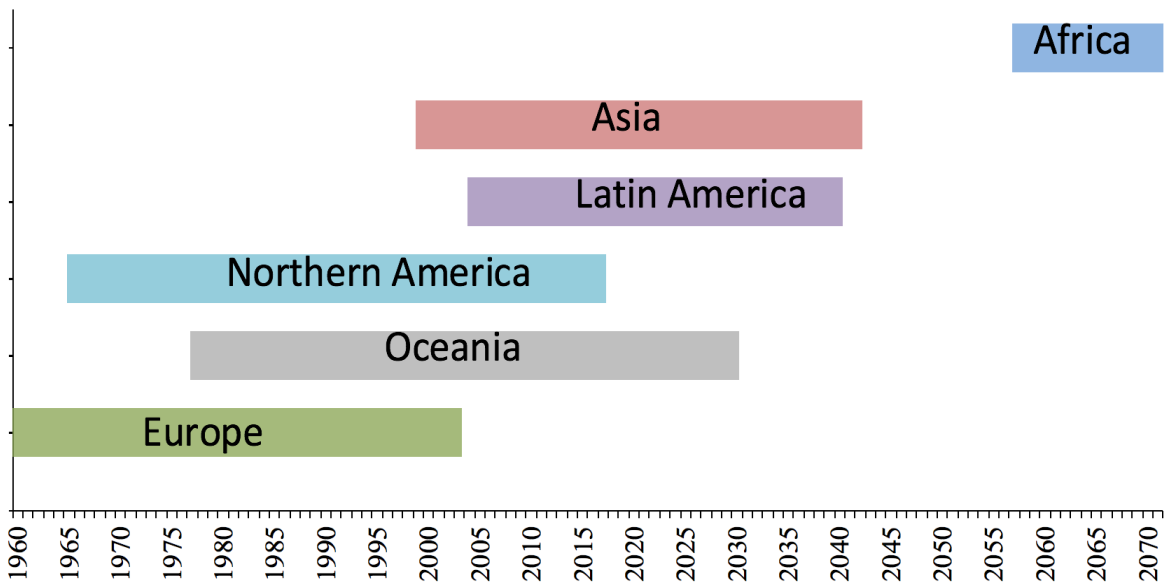
Notes: The gray areas indicate the years of the demographic transition. The dotted vertical lines in panel (a) indicate the approximate years where stages 1 and 2 end respectively. Panels (a) and (b) are aligned horizontally to show the change in population structure in terms of both age groups and population pyramids. Source: 2012 World Population Projects Database, UN.

Figure 11: Investment rates for the demographic window



Notes: The figure plots the average across years of investment rates during the demographic window and the 15 years before it on the horizontal axis, and the average across years outside of this period on the vertical axis.
 Source: WDI and WPP.

Figure 12: Timing of the Demographic Window



Notes: The figure plots the years of the demographic windows of each continent starting in 1960 (the first year in my database). Note that the demographic window of Europe begins before the timeline shown in this graph: for example, as discussed in 5.2, the window for England and Wales started in 1920. Source: demographic indicators from 2012 World Population Projects Database, UN and regional classifications from WB.

A.3 Model Appendix

A.3.1 Household Preferences over Children

The utility representation of households of generation $t - 1$,

$$U^{t-1}(c_t^{t-1}, c_t^t, c_{t+1}^{t-1}, c_{t+2}^{t-1}) = u(c_t^{t-1}) + \gamma\phi_{t-1}u(c_t^t) + \beta u(c_{t+1}^{t-1}) + \beta^2 u(c_{t+2}^{t-1}),$$

is a simplification of the following specification:

$$U_k^{t-1}(c_t^{t-1}, c_t^t, c_{t+1}^{t-1}, c_{t+2}^{t-1}) = u(c_t^{t-1}) + \sum_{j=1}^{N_k} \gamma_j u(c_t^{k,j}) + \beta u(c_{t+1}^{t-1}) + \beta^2 u(c_{t+2}^{t-1}),$$

where U_k^{t-1} denotes the discounted utility of agent k of generation $t - 1$, and: $c_t^{t,j}$ is the consumption of the j -th children, γ_j is the valuation of the j -th child's utility in his parents utility, N_k is the number of children of this household.

Since parents care equally about each of their children, $\gamma_i = \gamma_j \equiv \gamma \forall i, j \Rightarrow c_t^{t,j} = c_t^{t,i} \equiv c_t^t \forall i, j$. Hence: $U_k^{t-1}(c_t^{t-1}, c_t^t, c_{t+1}^{t-1}, c_{t+2}^{t-1}) = u(c_t^{t-1}) + \gamma \cdot N_k \cdot u(c_t^t) + \beta u(c_{t+1}^{t-1}) + \beta^2 u(c_{t+2}^{t-1})$. And since households are representative, the number of children in each household is the same ($N_{k_1} = N_{k_2} \equiv N_k \quad \forall k_1, k_2$) and is equal to the total child population over the population of generation $t-1$: $N_k = \frac{N_t}{N_{t-1}} = \phi_{t-1}$.

Choice of the value of altruism towards children γ

In the benchmark model, γ is set to 0.5 using the *OECD equivalence scales* that assign a value of 0.5 to each child in a one-household member. Since, to my knowledge, there are no official estimations of equivalence scales for Mexico, and no consistent estimations across countries, specially for Latin America, I chose a scale from an international organization. According to the OECD (1982), their scale could be used in "countries which have not established their own scales." Their scale was used in the 1980s and the early 1990s, and since my data covers 1960 to 2013, I found it suitable for reflecting an intermediate year in this time frame.

In the late 1990s, the EUROSTAT⁹³ adopted the use of the *OECD-modified scales* (which were proposed by Hagenars et al. (1994)) which assign a value of 0.3 to each child. I opted to use the original scales not only because they better suited the timeframe of my data, but

⁹³OECD (2009) provides a description of the evolution across time of the more commonly used scales.

also since the value given to a child is higher and adapts better to the number of years–20– that a generation is part of the child population. In the model children are dependent on their parents until they are 19, which makes their consumption higher than what it would be if they were children until age 14. Finally since the focus of this paper is LatAm, mostly comprised by developing countries, an additional reason for using a higher scale is that in these countries the amount of income spent in a children as a fraction of their parents’ income is around 40 to 50%⁹⁴(Deaton, 1997).⁹⁵ Nonetheless, in the next subsection, I perform a sensitivity analysis on this parameter and show that the main results are not affected by its exact choice.

A.3.2 Sensitivity Analysis

In this section, I discuss the sensitivity of the results of the model to the parametrization used in the numerical exercises. Table 11 shows the parameter values chosen, and the source or target for these. For this analysis I focus on two parameters from the households’ preferences, since the rest of the parameters are standard. I first analyze the benevolence of parents towards their children and then the intertemporal elasticity of substitution (EIS).

Table 11: **Baseline parameter values**

Parameter	Value	Source/Target
β	0.96 [†] *	Annual return on risk-free bonds
σ	2	Reciprocal of IES: Conesa and Garriga (2008)
γ	0.5	OECD equivalence scale
α	0.3	Bergoeing et al. (2002)
δ	0.05 [†] *	Consumption of Fixed Capital/GDP
τ_1 / τ_2	20% / 5%	Open economy
τ_1 / τ_2	170% / 7%	Closed economy**

[†] Values of β and δ are 0.44 and 0.64 respectively (20 year equivalents of the annual values of 0.96 and 0.05).

* The value of δ of Mexico (0.0458) was calibrated to match the average GFK/GDP ratio of 1970-2013.

** The minimum tax τ_1 to close the economy is 160%.

A.3.3 Benevolence Towards Children

A decrease in the valuation of children in their parents’ utility increases savings rates, which in the closed economy increases investment rates and in the open one leaves them unchanged.

⁹⁴Per additional children and to maintain constant adult welfare.

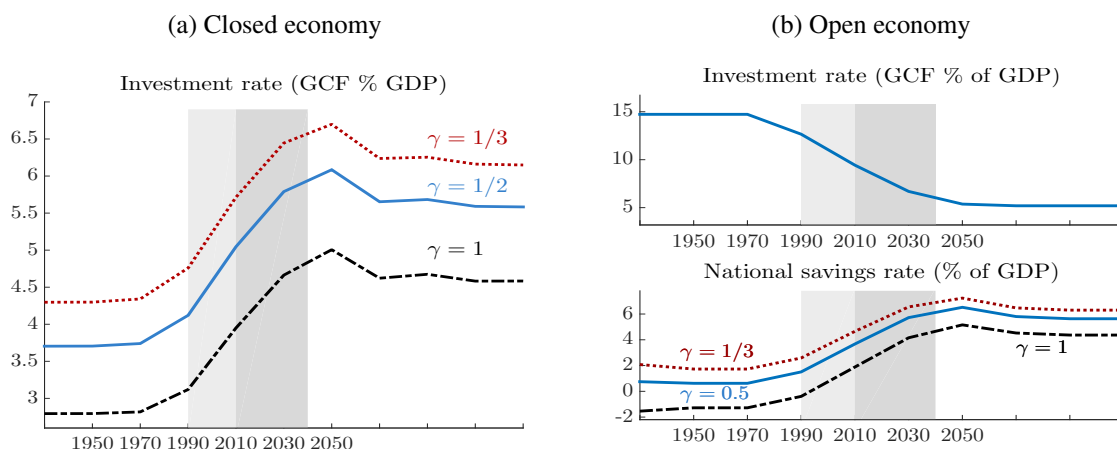
⁹⁵Olken (2005) finds the value for developing countries even higher than the originally proposed by Deaton.

Recall that households care about the utility of their children and because of that, during the first period of their working life they feed their offspring. Given the utility specification,⁹⁶ households equalize the marginal utility of consumption of the first period of their working life to the marginal utility of each of their children weighted by the parameter γ .

This implies that as young workers they dissave for their own consumption, during that period, and for the consumption of their children. For each unit of consumption in the first period of their working life, these young workers set aside $\gamma^{\frac{1}{\sigma}} \phi_{t-1}$ units for their children.⁹⁷ Therefore, a decrease in the valuation of children increases savings rates for two reasons. First, it decreases dissavings of young workers. Second, it increases savings rates in general by freeing up resources for self-spending which are distributed across periods, via savings.

In the closed economy the increase in savings rates generates an increase in investment rates, in the open economy it does not and investment rates remain unchanged.⁹⁸

Figure 13: Sensitivity to the benevolence towards children γ



Notes: This figure plots investment and savings rates through the Mexican demographic transition for different values of γ . The results of the closed economy are depicted in panel (a) and the results of the open economy in panel (b). The solid line shows the benchmark exercise of figure 3, while the dotted line shows a lower value and it illustrates the explanation provided above. The dashed line shows a higher (γ) value.

⁹⁶ $U^{t-1} = u(c_t^{t-1}) + \gamma \phi_{t-1} u(c_t^t) + \beta u(c_{t+1}^{t-1}) + \beta^2 u(c_{t+2}^{t-1})$

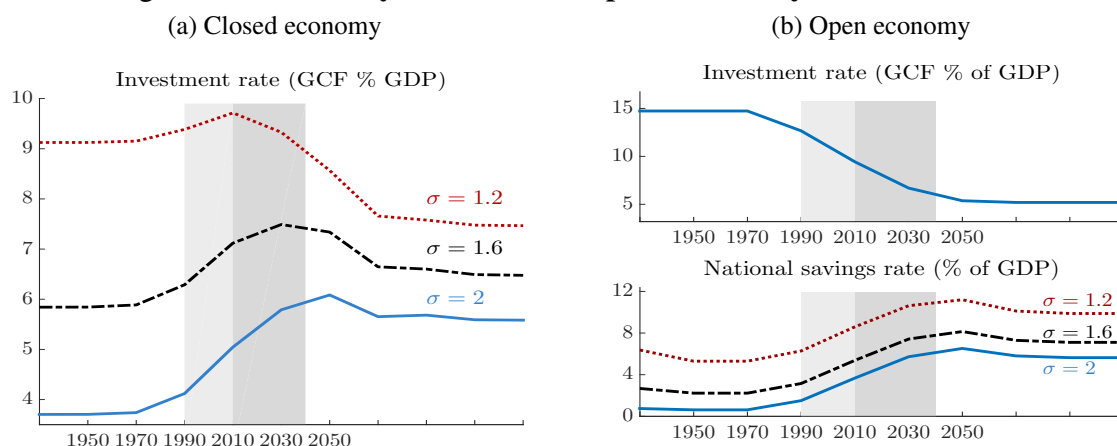
⁹⁷See consumption equation 27 in appendix A.3.6.

⁹⁸In the closed economy, effective savings increase, hence future capital (p.e.u.l) (see equation 20) increases. The rate of capital accumulation (percentage increase in capital (p.e.u.l)) before and during the window seems unchanged, thus the investment rate shifts up without affecting its dynamics before and during the window.

A.3.4 Elasticity of Intertemporal Substitution

In this section I discuss the effects of increasing the EIS. There are two effects: First, a higher EIS increases savings rate levels. Second, if there are general equilibrium price effects, a higher EIS reduces the effect of the demographic window: it reduces the increase in investment rates that happens before and during the window. In the open economy only the first effect is observed in savings rates, while in the closed economy both effects impact savings rates and therefore investment rates.

Figure 14: Sensitivity to the intertemporal elasticity of substitution σ



Notes: This figure plots investment and savings rates through the Mexican demographic transition for different σ values. Panels (a) and (b) show the results of the closed and open economies. The solid line shows the benchmark exercise of figure 3, and the dotted and dashed lines show lower values.

The specification of the utility function (CRRA) implies that an increase in the EIS is equivalent to a decrease in σ . In the open economy, a lower σ shifts up the savings rate, as it increases the effective discount factor $(\beta^{\frac{t}{\sigma}}(1+r)^{\frac{t}{\sigma}-1})$.⁹⁹ Note that since there are no general equilibrium price effects in the open economy, savings rates' dynamics remain unaffected throughout the demographic transition. These changes do not affect investment rates.

In the closed economy, in addition to the increase in levels, a higher σ lowers the effect of the demographic window. The increase in savings rates that happens before and during the window is bigger the lower the σ , because during the transition capital (p.e.u.l) and therefore

⁹⁹We can see this in equations 28 and 29 or in the numerical exercises of figure 14(b) of appendix A.3.6.

labor income increases. The bigger the σ , the more important the response of savings to income effects as opposed to substitution effects. Thus lower values of σ decrease the response of savings to the increasing labor income profile generated by the demographic window. Investment rates decrease and the effect of the demographic window decreases as well.¹⁰⁰

A.3.5 Labor Force

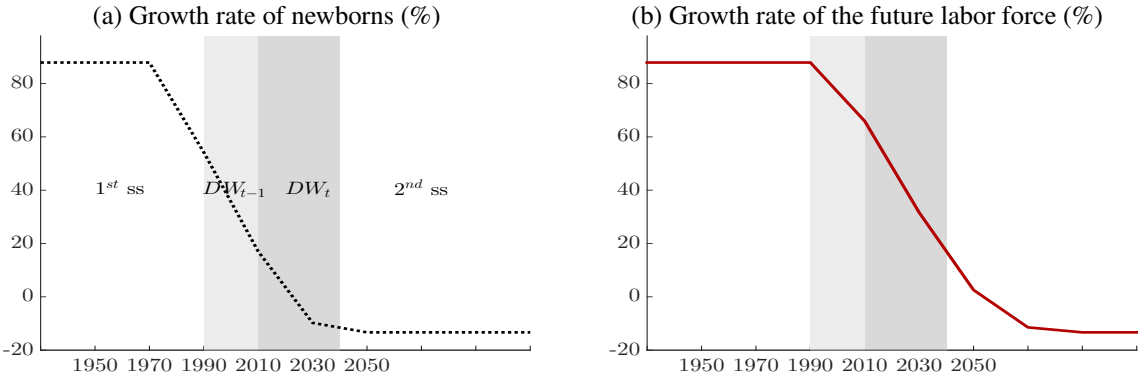
The demographic transition under consideration in this paper, i.e., one triggered by a decrease in the growth rate of newborns, generates a decrease in the growth rate of labor in since each period there are fewer children that will join the future labor force.

Using the demographic specification the model, the growth rate of labor force is:

$$g_{Lt} = \frac{L_t}{L_{t-1}} - 1 = \frac{(1+g_{nt-2})(2+g_{nt-1})}{(2+g_{nt-2})} - 1,$$

which is decreasing if the growth rate of the labor force is decreasing (remark 4.1). Figure 15 gives a graphical illustration of this for the exercise of the Mexican demographic change:

Figure 15: Effect of the demographic window on the growth rate of labor



Notes: This figure plots the 20-year growth rate of newborns used in the benchmark exercise for Mexico in panel (a). In panel (b), I plot the 20-year growth rate of future the labor force, g_{Lt+1} .

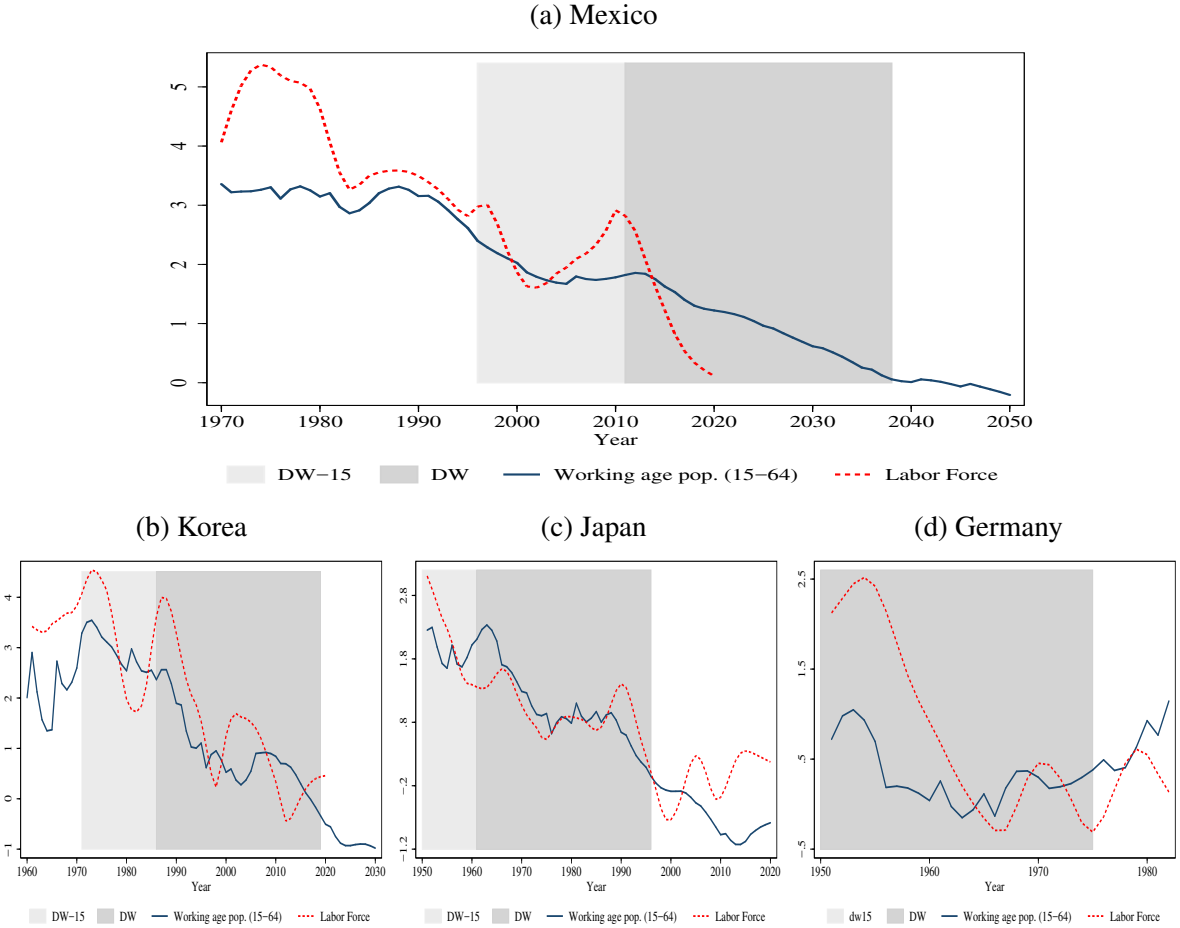
Labor force and working age population

This decrease in the growth rate of labor force is the main driver of the result in the open economy, i.e., a decrease in investment rates for the demographic window. It is important to

¹⁰⁰See figure 14(a) of appendix A.3.6.

mention that in the model, since there is no disutility of working, the labor force is equal to the working age population. However, this is a sensible assumption since in the data the growth rates of the working age population and of the labor force behave similarly: decreasing before and during the demographic window. Figure 16 shows the evolution of these variables for the case of Mexico, as well as for the EAO region.

Figure 16: Annual growth rates of the working age population and labor force



Notes: for each country, this figure plots the annual growth rate of the working age population in the solid line and the annual growth rate of the labor force in dashed line. The dark gray area indicates the years of the demographic window and the light gray area, the 15 years preceding it. The labor force was HP-filtered and the cyclical component removed to reflect long term trends. Labor force data was taken from the Total Economy Database for 1950–2015 and from the projections of the LABORSTA database for 2016–2020.

A.3.6 Additional Equations and Figures

The consumption demand of a young worker from generation t-1 in the closed economy is:

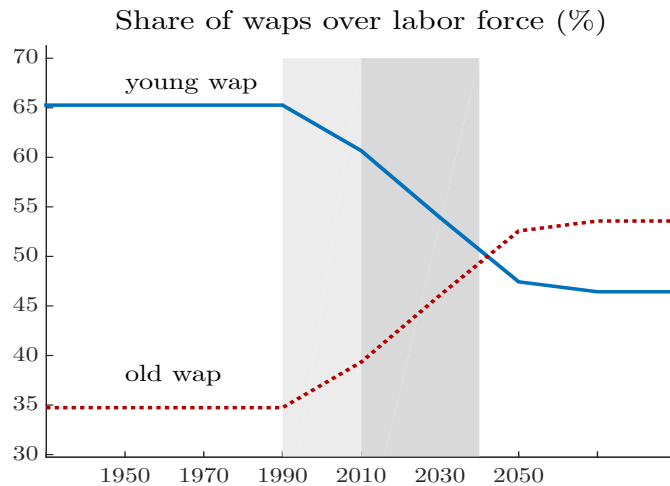
$$c_t^{t-1} = \frac{w_t + \frac{w_{t+1}}{(1+r_{t+1}^b)}}{1 + \gamma^{\frac{1}{\sigma}} \phi_{t-1} + \beta^{\frac{1}{\sigma}} (1 + r_{t+1}^b)^{\frac{1}{\sigma}-1} + \beta^{\frac{2}{\sigma}} [(1 + r_{t+1}^b) (1 + r_{t+2}^b)]^{\frac{1}{\sigma}-1}}. \quad (27)$$

The expression for savings (of households of generation t-1) per effective unit of the closed economy is given as

$$s_t^{t-1} = \frac{\beta^{\frac{1}{\sigma}} (1 + \alpha k_{t+1}^{\alpha-1} - \delta)^{\frac{1}{\sigma}-1} \left[1 + \beta^{\frac{1}{\sigma}} (1 + \alpha k_{t+2}^{\alpha-1} - \delta)^{\frac{1}{\sigma}-1} \right] (1 - \alpha) k_t^\alpha - \left(1 + \gamma^{\frac{1}{\sigma}} \phi_t \right) \frac{1 + g_{at+1}}{1 + \alpha k_{t+1}^{\alpha-1} - \delta} (1 - \alpha) k_{t+1}^\alpha}{1 + \gamma^{\frac{1}{\sigma}} \phi_t + \beta^{\frac{1}{\sigma}} (1 + \alpha k_{t+1}^{\alpha-1} - \delta)^{\frac{1}{\sigma}-1} + \beta^{\frac{2}{\sigma}} [(1 + \alpha k_{t+1}^{\alpha-1} - \delta) (1 + \alpha k_{t+2}^{\alpha-1} - \delta)]^{\frac{1}{\sigma}-1}} \quad (28)$$

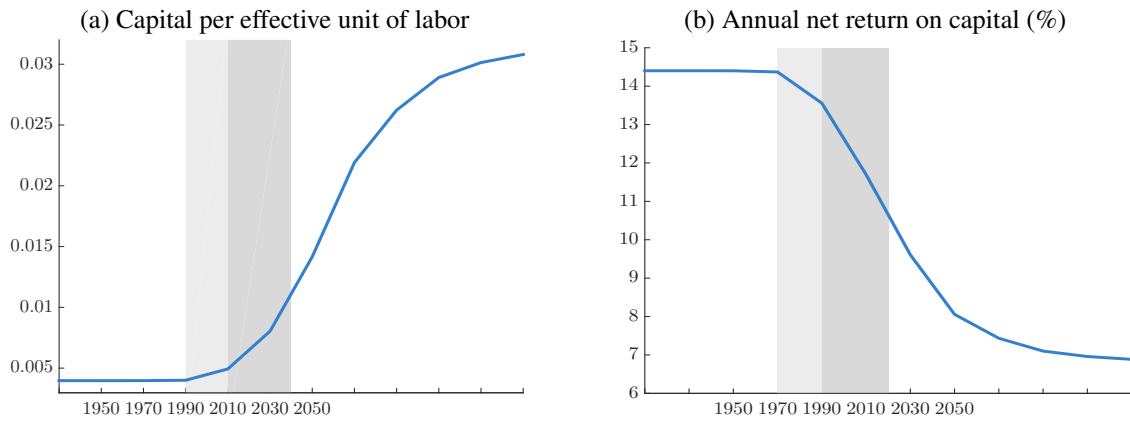
$$s_t^{t-2} = \frac{\beta^{\frac{2}{\sigma}} [(1 + \alpha k_t^{\alpha-1} - \delta) (1 + \alpha k_{t+1}^{\alpha-1} - \delta)]^{\frac{1}{\sigma}-1} \left[\frac{1 + \alpha k_t^{\alpha-1} - \delta}{1 + g_{at}} (1 - \alpha) k_{t-1}^\alpha + (1 - \alpha) k_t^\alpha \right]}{1 + \gamma^{\frac{1}{\sigma}} \phi_{t-1} + \beta^{\frac{1}{\sigma}} (1 + \alpha k_t^{\alpha-1} - \delta)^{\frac{1}{\sigma}-1} + \beta^{\frac{2}{\sigma}} [(1 + \alpha k_t^{\alpha-1} - \delta) (1 + \alpha k_{t+1}^{\alpha-1} - \delta)]^{\frac{1}{\sigma}-1}}. \quad (29)$$

Figure 17: Share of savers in the economy



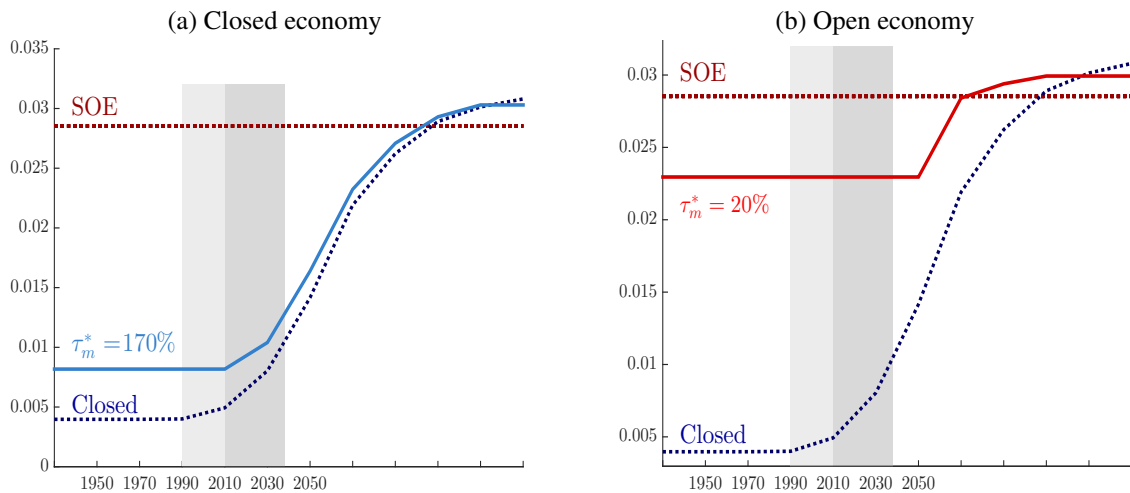
Notes: this figure plots the shares of young and middle age workers as a fraction of the total population. The dark gray area indicates the years of the demographic window and the light gray area, the 15 years before it. The solid line is the share of young workers and the dotted line the share of middle-age workers.

Figure 18: **Capital and interest rate in the closed economy (benchmark exercise)**



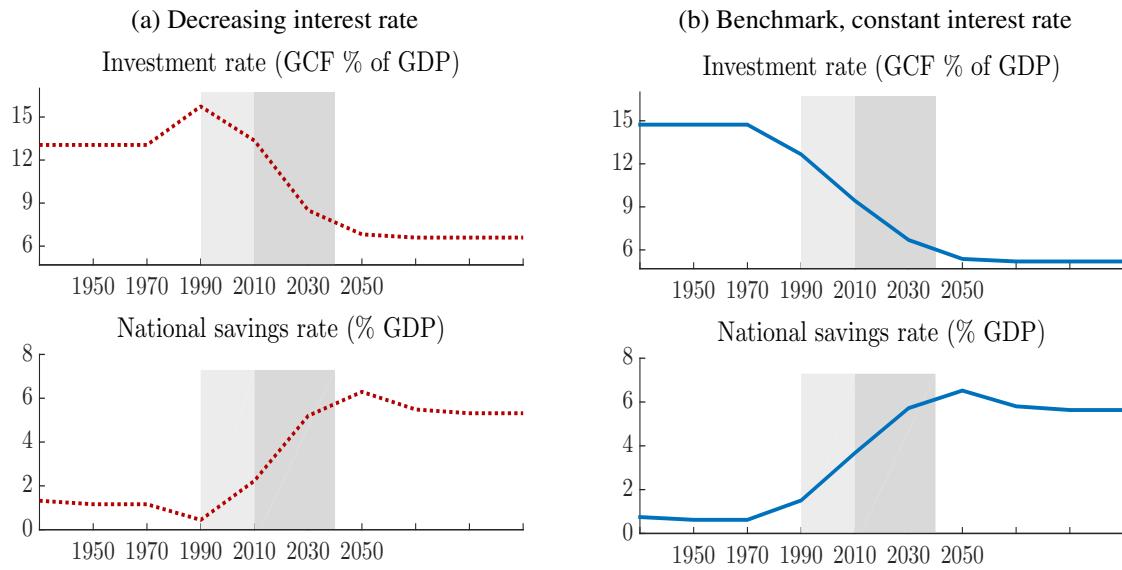
Notes: This figure plots capital (p.e.u.l) and annualized return on capital net of depreciation of the closed economy, throughout the Mexican demographic transition of the benchmark numerical exercise (depicted in figure 3). Note that net return of capital is simply marginal productivity of the capital (p.e.u.l) net of depreciation.

Figure 19: **Capital of the benchmark exercise with a varying degree of openness**



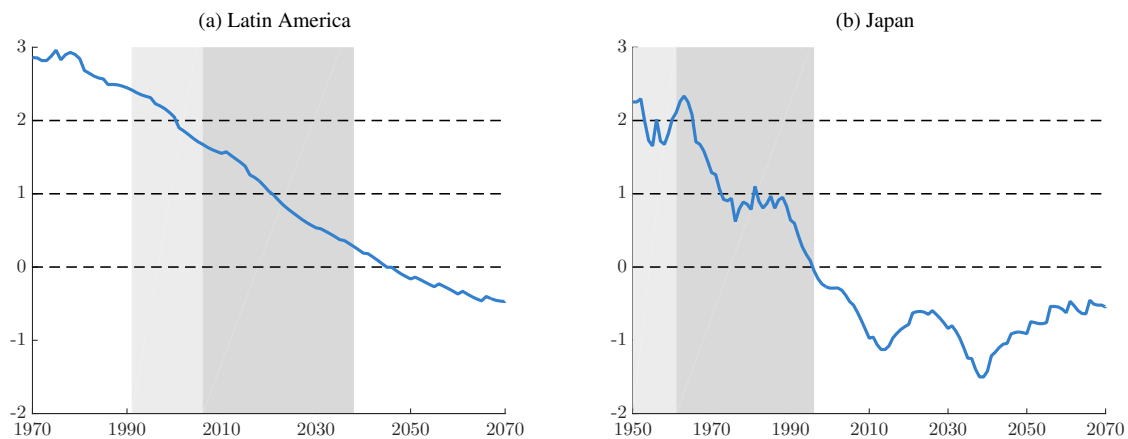
Notes: this figure plots the capital (p.e.u.l) of the benchmark exercise with a varying degree of openness (depicted in figure 4). The dotted line is the capital (p.e.u.l) in the frictionless model, the solid line in panel (a) is capital (p.e.u.l) in the taxed closed economy, and the solid line in panel (b) is capital (p.e.u.l) in the taxed open economy. In the closed economy $\tau_1 = 20\%$ and $\tau_2 = 5\%$, and in the open economy $\tau_1 = 170\%$ and $\tau_2 = 7\%$.

Figure 20: Investment and savings, decreasing international rate of return exercise



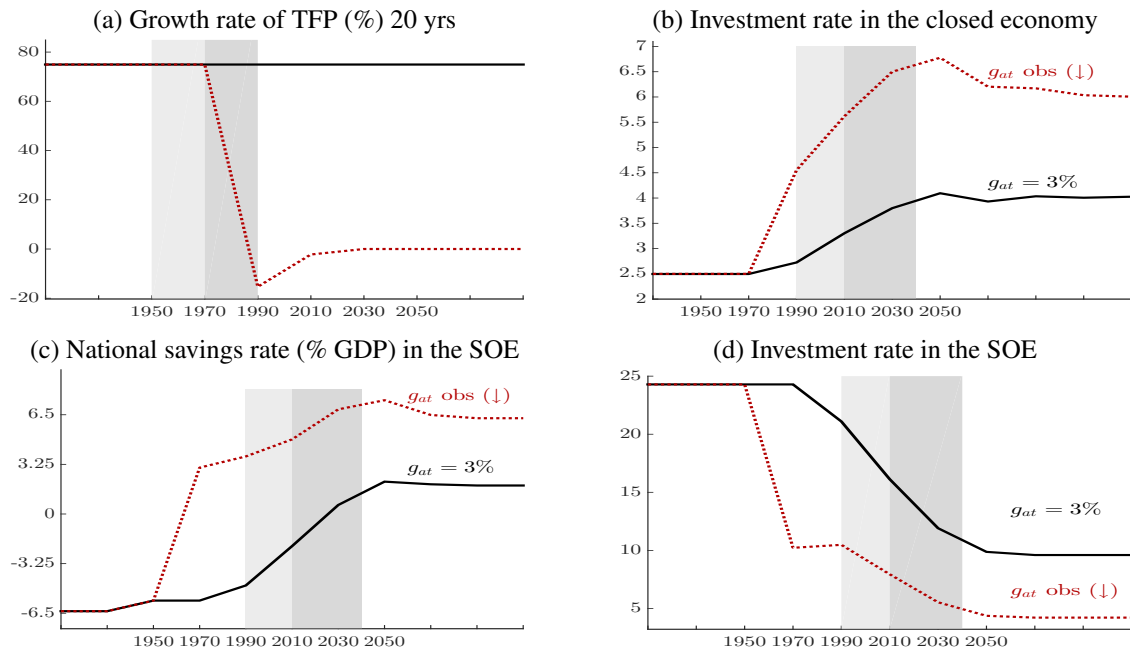
Notes: this figure plots investment and savings rates generated by the Mexican demographic transition under two scenarios. Panel (a) shows a scenario characterized by the observed decreasing international return until 2010, with a projection of 6% for 2030 onwards. Panel (b) shows the benchmark exercise with a constant annual return of 7.15% (average of 1950–2014).

Figure 21: Annual growth rate of the working age population (%)



Note: this figure plots the annual growth rate (in percentage points) of the working age population. The dark gray area indicates the years of the demographic window and the light gray area, the 15 years before it.

Figure 22: Investment and savings rate, technology growth



Notes: The panels in this figure show savings and investment rates generated by the Mexican demographic transition under two scenarios. The dashed line illustrates a scenario characterized by the observed decreasing growth rate of TFP until 2010, with a projection of 0% for 2030 onwards. The solid line depicts the benchmark scenario with a constant annual growth rate of 3% (average 1950–2014). This growth rate is shown in panel (a). In panels (b) and (d), I plot investment rates in the closed and open economies, respectively. In panel (c), I plot the savings rate in the open economy.

Figure 23: Growth in Western Europe and Japan (1950s–1970s)

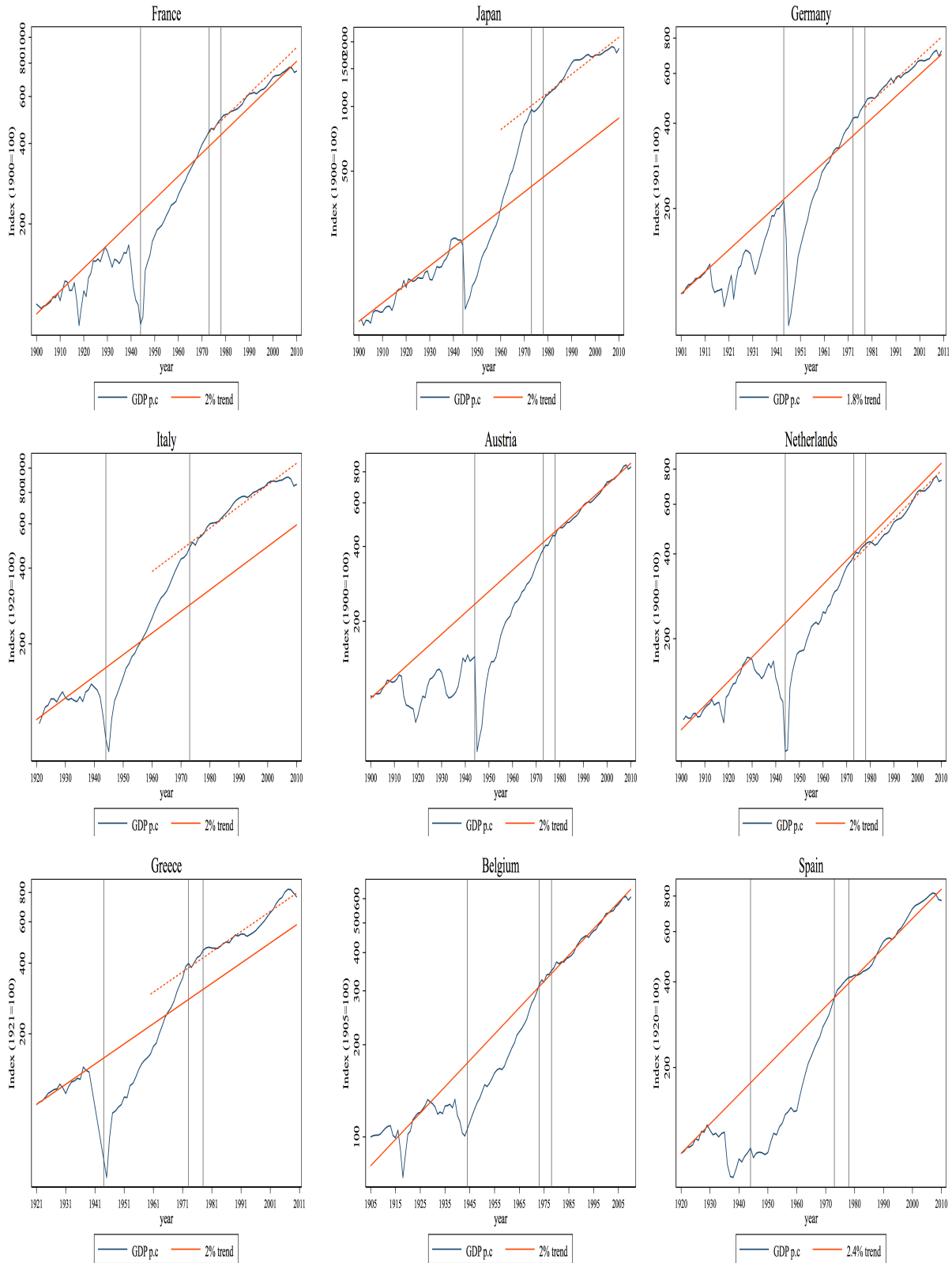
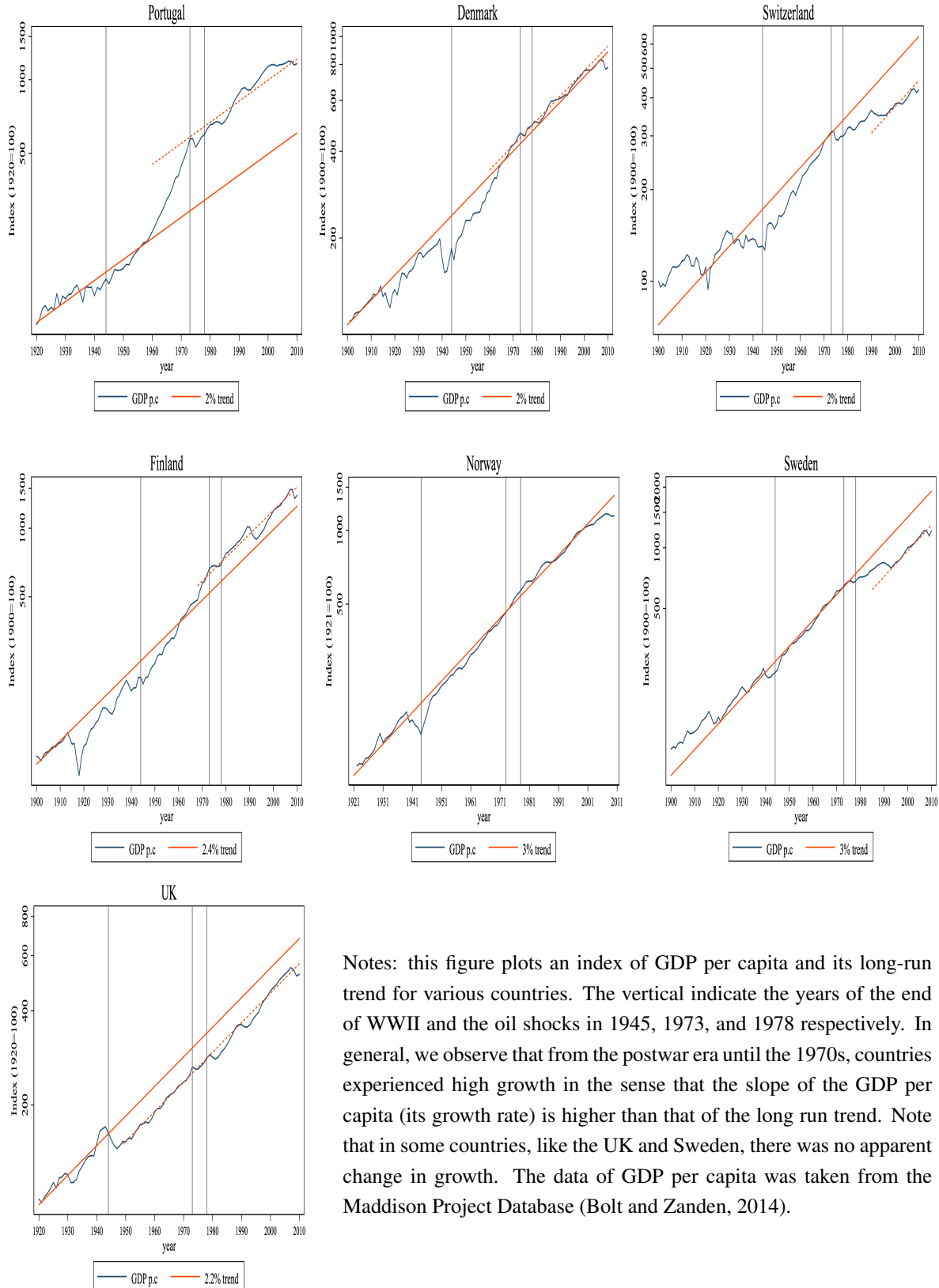


Figure 23 (Cont.): Growth in Western Europe and Japan (1950s–1970s)



Notes: this figure plots an index of GDP per capita and its long-run trend for various countries. The vertical indicate the years of the end of WWII and the oil shocks in 1945, 1973, and 1978 respectively. In general, we observe that from the postwar era until the 1970s, countries experienced high growth in the sense that the slope of the GDP per capita (its growth rate) is higher than that of the long run trend. Note that in some countries, like the UK and Sweden, there was no apparent change in growth. The data of GDP per capita was taken from the Maddison Project Database (Bolt and Zanden, 2014).

Figure 24: Growth in Western Europe and Japan (1800–1900s)

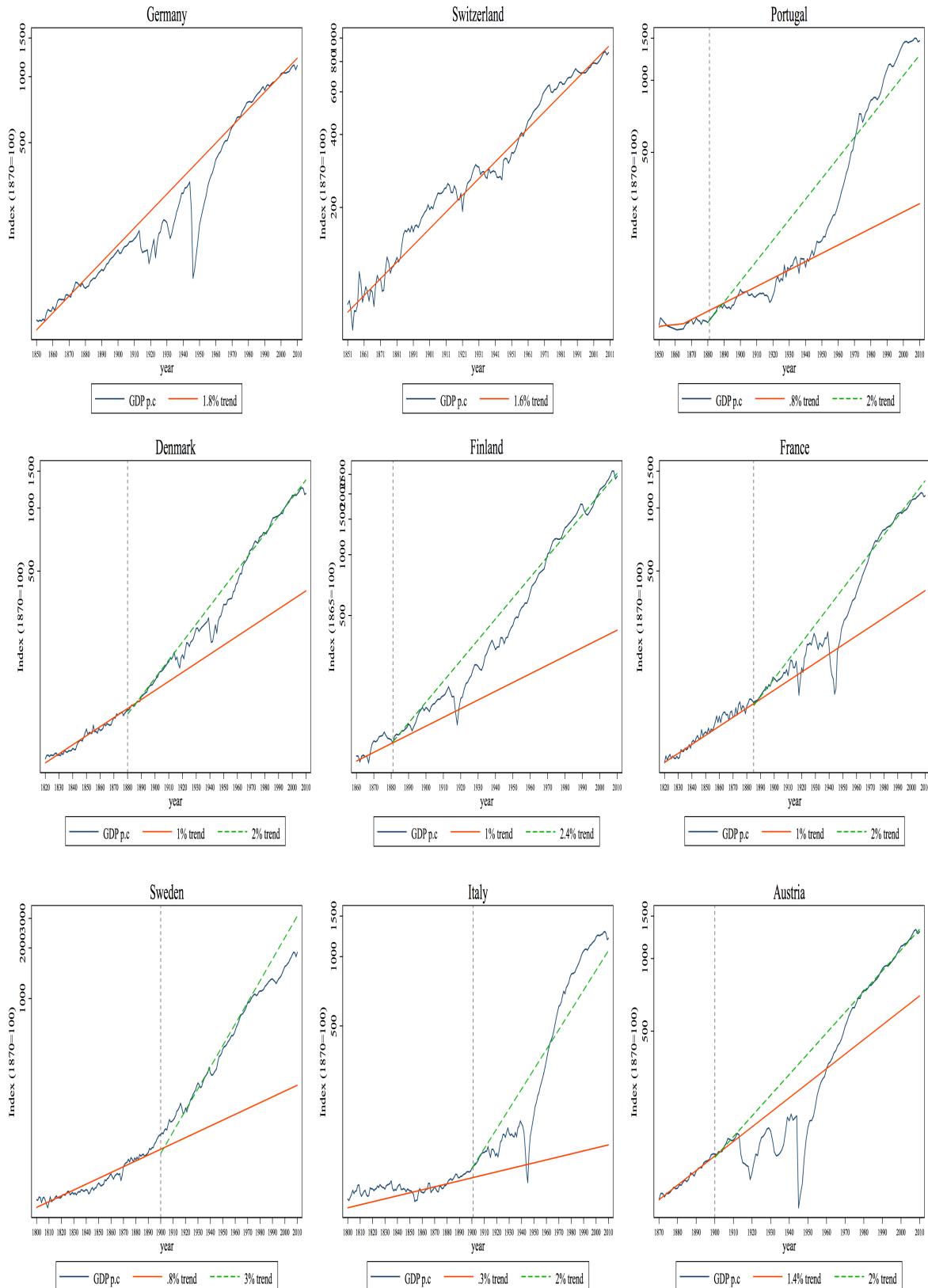
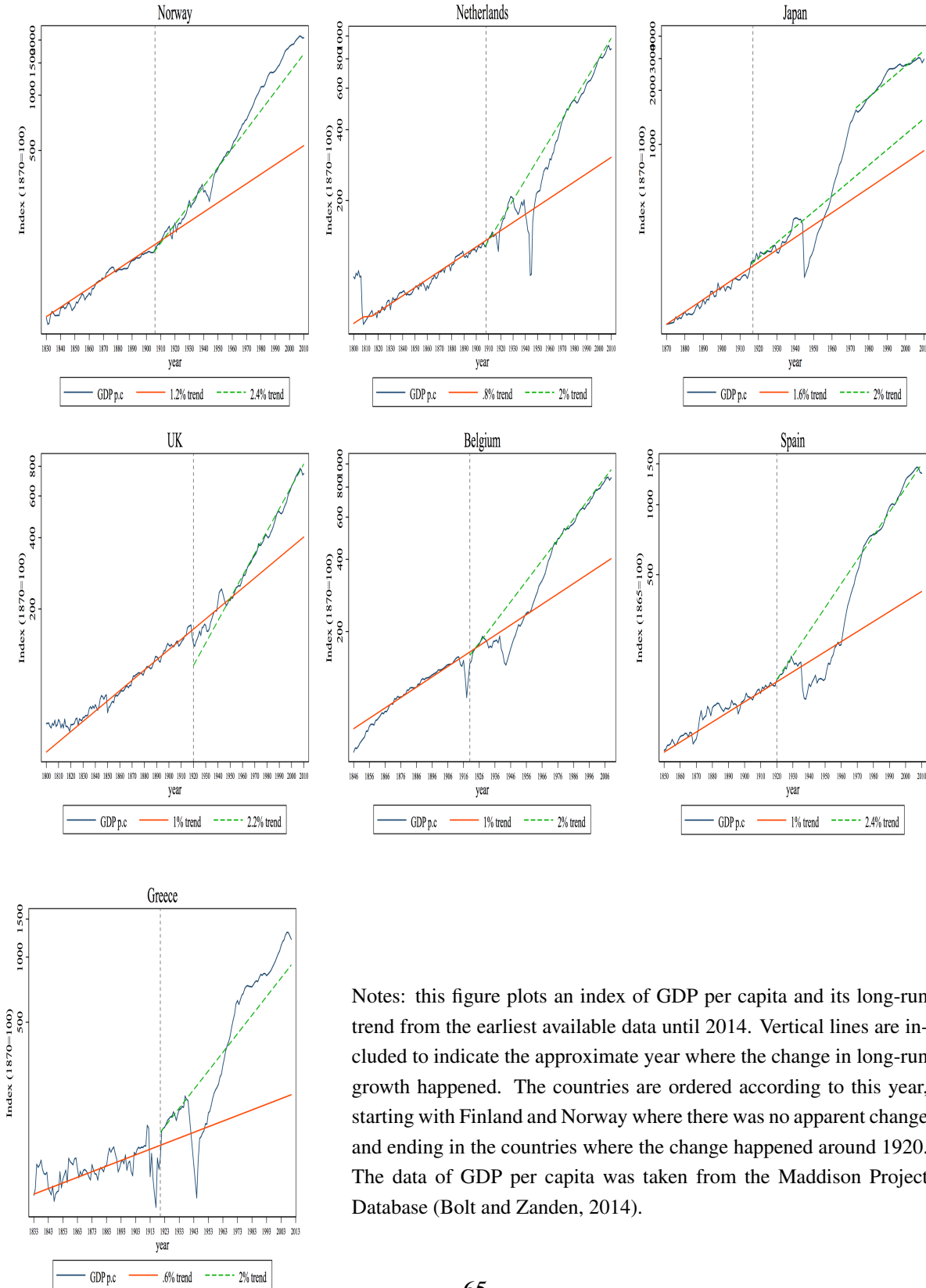


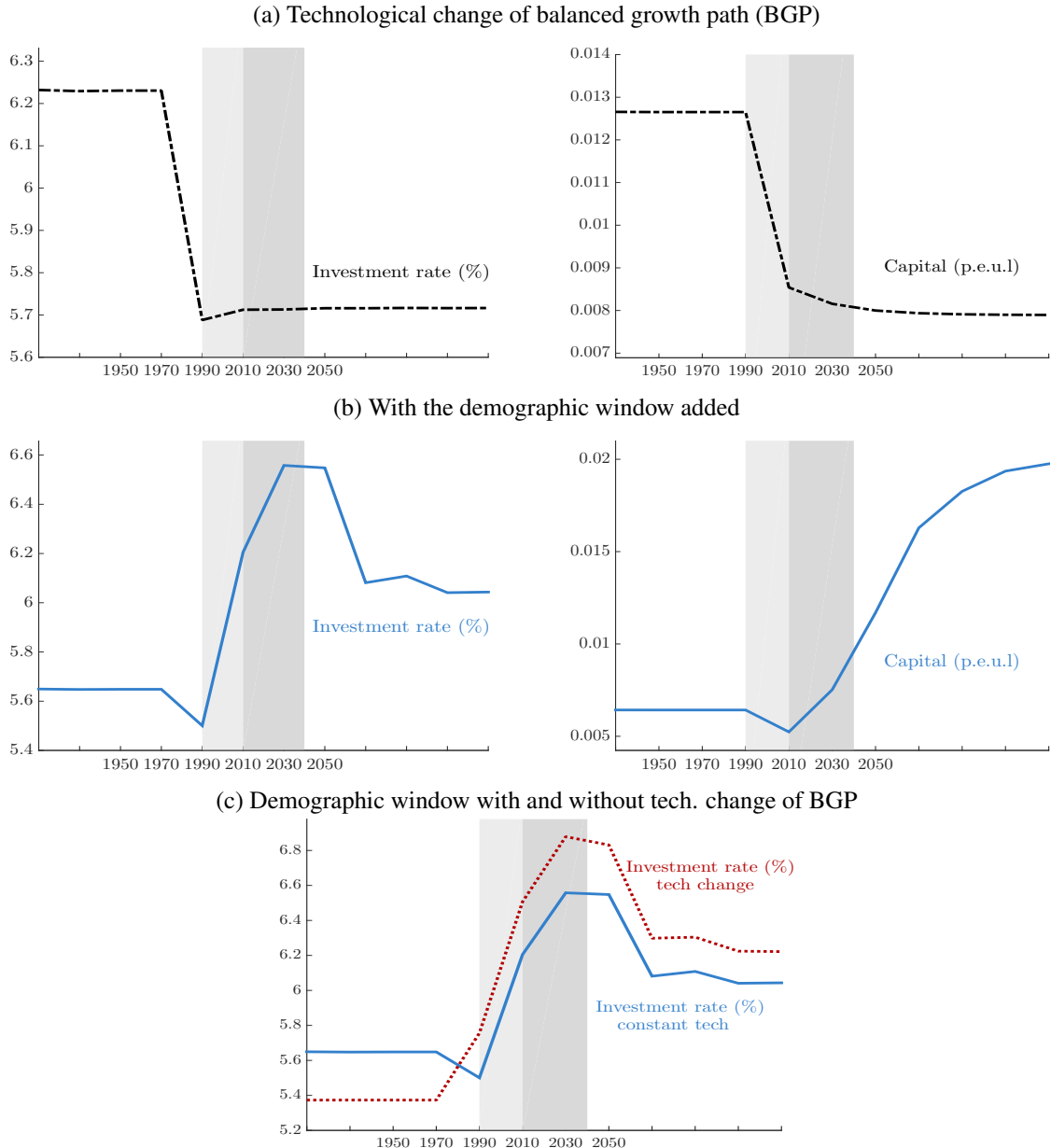
Figure 24 (Cont.): **Growth in Western Europe and Japan (1800–1900s)**



Notes: this figure plots an index of GDP per capita and its long-run trend from the earliest available data until 2014. Vertical lines are included to indicate the approximate year where the change in long-run growth happened. The countries are ordered according to this year, starting with Finland and Norway where there was no apparent change and ending in the countries where the change happened around 1920. The data of GDP per capita was taken from the Maddison Project Database (Bolt and Zanden, 2014).

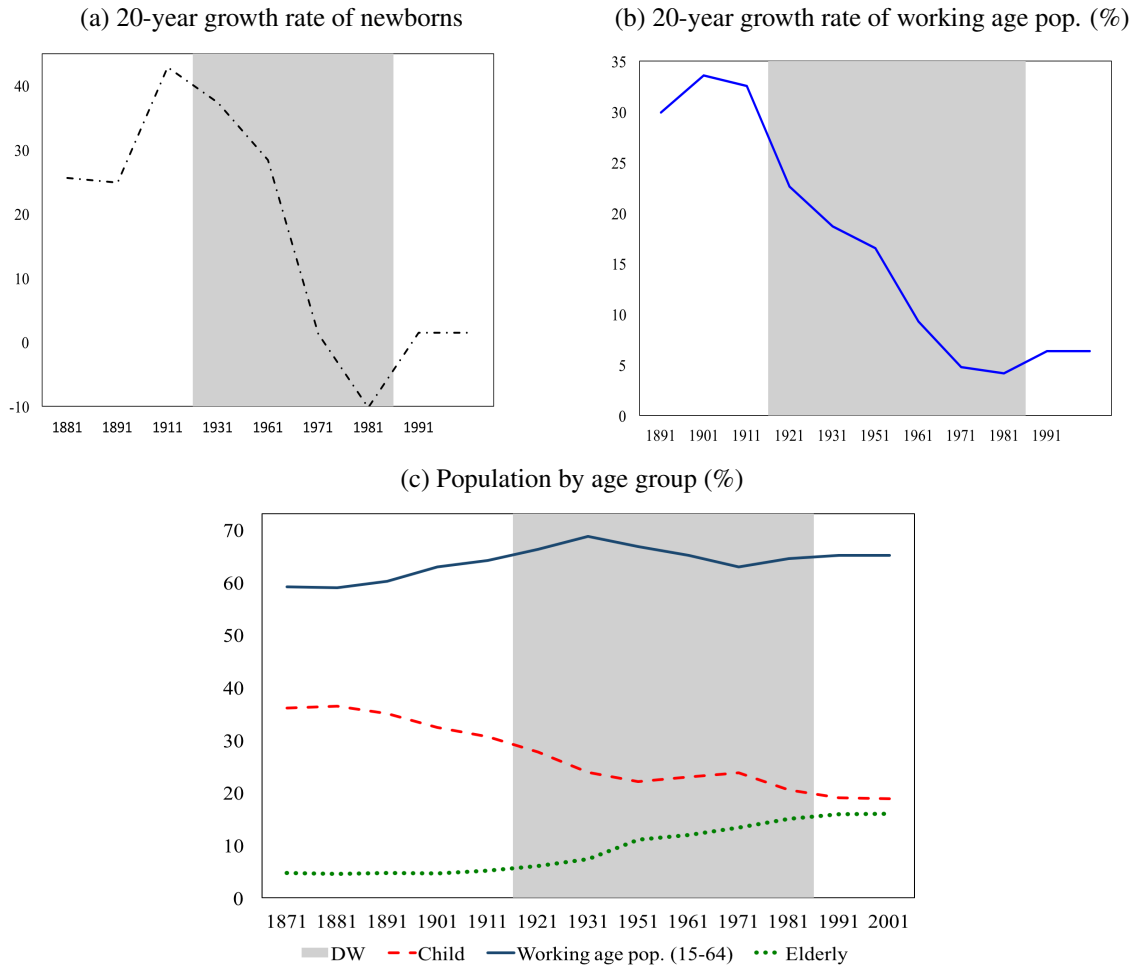
In this exercise, I first introduce a change in the growth rate of technology from 1% to 2%, keeping the population growth rate constant. Second I add, on the top of the technological exercise, the Mexican demographic change to generate a demographic window. Finally, I compare the results to an exercise where the growth rate of technology is fixed at 1.5% annually throughout the demographic transition.

Figure 25: Investment rates, long-run technological growth



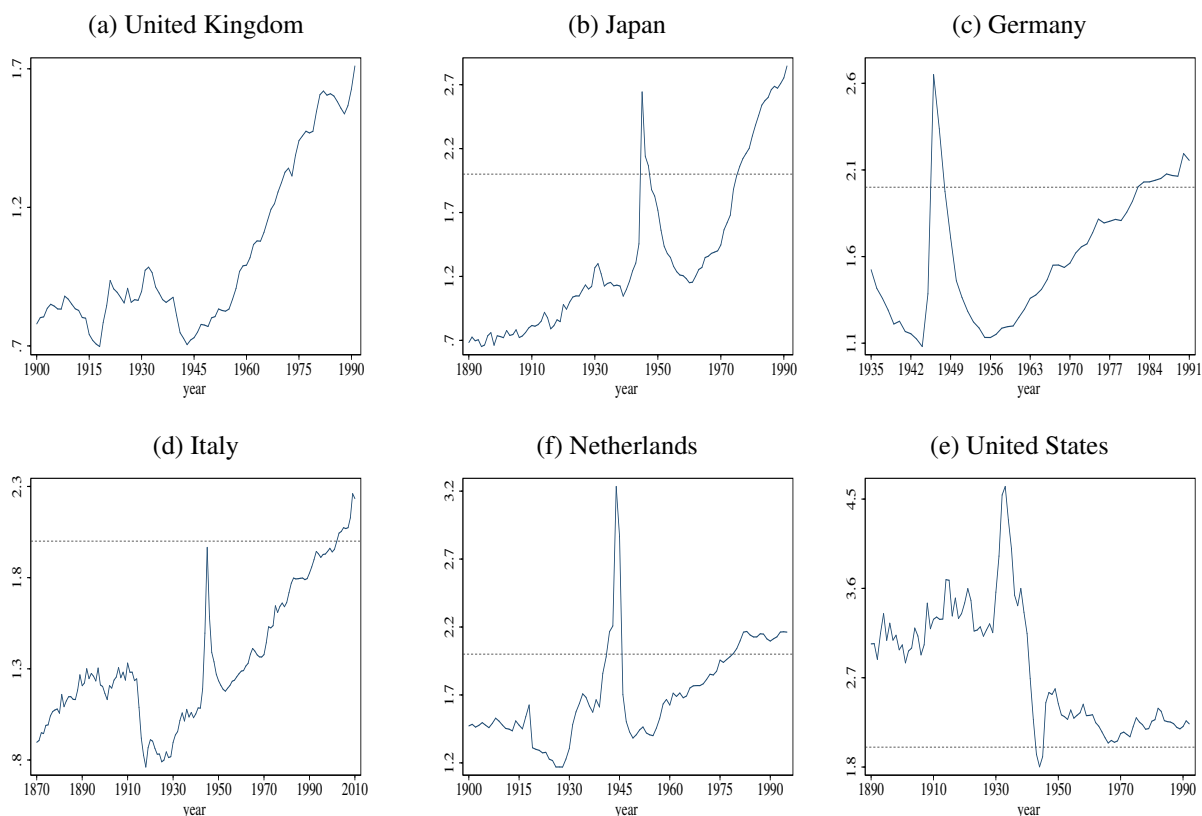
Note: in panel (a), I first introduce a change in the growth rate of technology from 1% to 2%, keeping the growth rate of population constant. Second, in panel (b) I add the Mexican demographic change. In panel (c) I compare the exercise of panel (b) to one with a constant 1.5% growth rate throughout the demographic transition.

Figure 26: Demographic transition in England and Wales



Notes: this figure plots the demographic transition of England and Wales. In panel (a), I plot the 20-year cumulative (%) growth rate of newborns used in the model. As explained in section 4.1, growth rates of newborns are adjusted to the growth rates of age cohorts to reflect mortality. In particular this is the forward (1 period ahead) growth rate of the cohort aged 40–59. In panel (b), I plot the 20-year cumulative (%) growth rate of the working age population (the cohort aged 15–64). In panel (c), I plot the child, working-age and elderly populations as percentage of the total population. The gray area indicates the years of the demographic window.

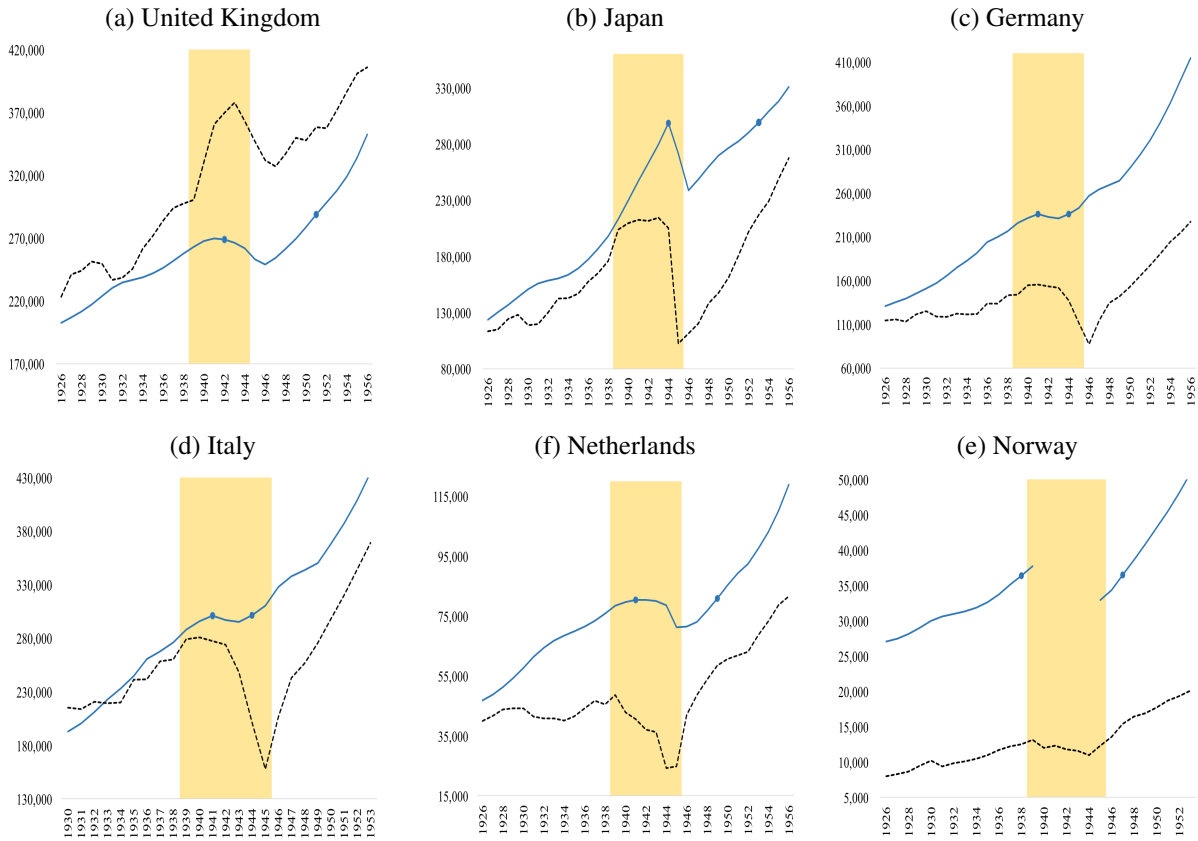
Figure 27: Non-residential capital to output ratio (selected countries)



Note: This figure shows the capital to output ratio of the United States and the countries of figure 8 that had available data from 1990 onwards (i.e., Norway is not included). The ratio is the estimated non residential capital stock divided by GDP. The estimate for the Netherlands is the gross stock of structures, machinery, and equipment from Groote et al. (1996); for the UK, Japan, Germany, and the United States¹⁰¹ it is from Maddison (1994). For Italy the estimate is the net stock of machinery and equipment, construction, and means of transport from Baffigi (2011). Level series are in 1990 international dollars, except for Italy's which is in international dollars of 2011 because of data availability. GDP in 1990 and in 2011 international dollars comes from the Maddison and World Economics Global GDP databases, respectively. The horizontal line is at 2.

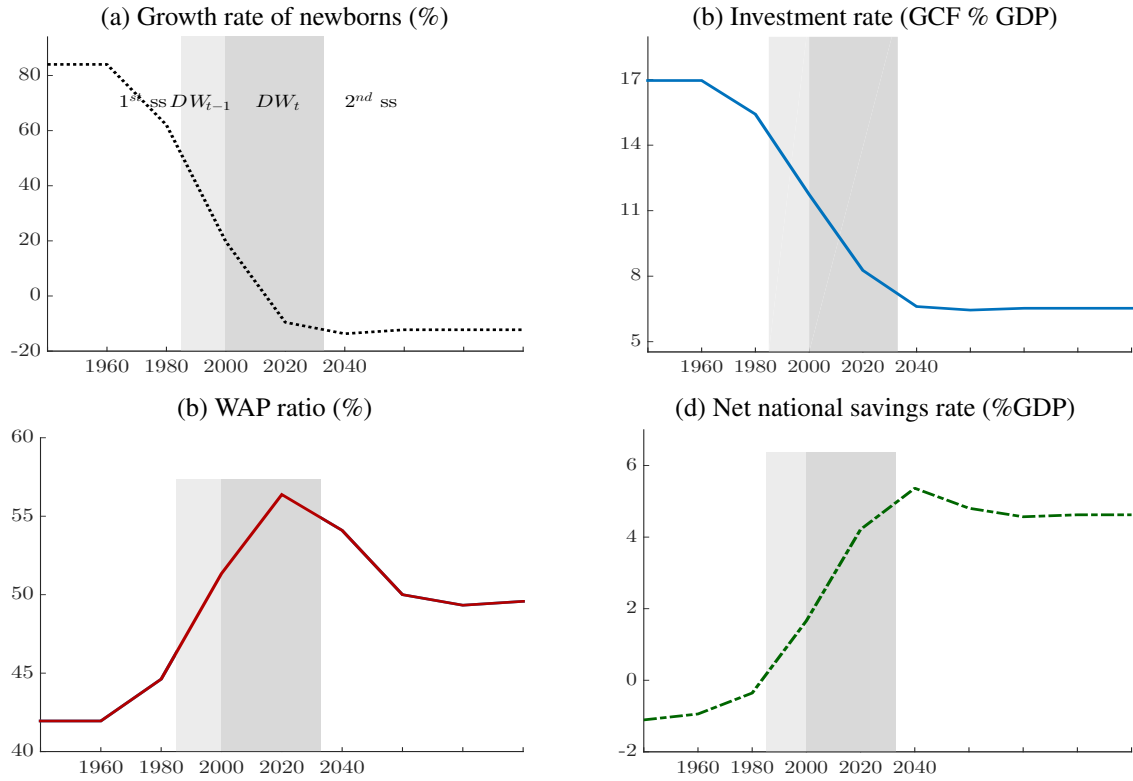
¹⁰¹From 1950 to 1990, the average ratio of capital stock estimated by Maddison for the United States is around 2.2. The most comparable official available series to this one is the net stock of fixed non-residential assets from the Bureau of Economic Analysis (BEA), which for the same time period is around 1.9 times GDP. Note that the latter does not include depreciation, which makes it a lower number. However, the numbers are close and I take this comparison as a check to reject the possibility of the estimates being far off the actual data. Also, notice that in later years in Japan and the rest of the European countries, the ratio of capital to GDP increases to levels that are close to the 2 of the United States. It is important to mention that the ratio of residential plus non-residential net capital stock to GDP of the BEA is around 3, a number the reader might be familiar with. However due to availability of data for prewar and interwar years, I present only non-residential capital stock.

Figure 28: Capital stock and GDP before and after WWII



Note: This figure shows estimates of non-residential capital and output of some European countries and Japan. The estimate for the Netherlands is the gross stock of structures, machinery, and equipment from Groote et al. (1996); for the UK, Japan, Germany, it is from Maddison (1994). For Italy the estimate is the net stock of machinery and equipment, construction, and means of transport from Baffigi (2011). Levels are in 1990 international dollars, except for Italy's which are in 2011 international dollars because of data availability. GDP in 1990 and in 2011 international dollars comes from the Maddison and World Economics Global GDP databases, respectively.

Figure 29: Benchmark exercise for Brazil in the open economy



Notes: This figure plots the Brazilian demographic transition (i.e., the reduction in the growth rates of newborns in panel) (a) and the demographic window that this generates in panel (c). The dark gray area indicates the years of the demographic window (2000-2033) and the light gray area, the 15 years before the window (1985-1999). In panels (b) and (d) I plot the investment and savings rates throughout this transition.

A.3.7 Computation of the Equilibrium

The closed economy

The equilibrium of the closed economy can be summarized by a sequence of capital per effective unit of labor (p.e.u.l) since the rest of the allocations can be written as a function of it, exogenous demographic variables, and the parameters of the economy. Using the law of motion of newborns and of capital (p.e.u.l) and the savings equations (28 and 29) of young and middle-aged workers, the equilibrium condition can be reduced to a third-order difference equation on $k_{t-1}, k_t, k_{t+1}, k_{t+2}$:

$$k_{t+1} = \frac{(1 + g_{nt-1}) s_t^{t-1} (k_t, k_{t+1}, k_{t+2}, g_{at+1}, g_{nt}) + s_t^{t-2} (k_{t-1}, k_t, k_{t+1}, g_{at}, g_{nt-1})}{(1 + g_{at+1}) (1 + g_{nt-1}) (2 + g_{nt})}.$$

To solve for steady-state growth paths, given productivity and demographic parameters of each state, I find the invariant capital (p.e.u.l) level that satisfies this nonlinear equation.¹⁰²

To compute the transition between steady states I use time path iteration. I first guess a transition path of capital (p.e.u.l) $\{k_t^g\}_{t=1}^N$ that goes from the first to the second steady state. Then, I compute the prices of this transition path and given these prices solve for the optimal savings of young and middle-aged workers (per effective unit) using their policy functions. Next, I aggregate these savings into capital $\{k_t^i\}_{t=1}^N$ using the demographic structure and the law of motion of capital. Finally, I compare the the guessed $\{k_t^g\}_{t=1}^N$ and the resulting capital paths $\{k_t^i\}_{t=1}^N$; if they do not coincide I update the guess, repeat, and iterate until convergence.

The open economy

In the open economy the variables that summarize the equilibrium are capital and foreign assets holdings (p.e.u.l). Capital is either pinned down by the after-tax interest rate of international assets or by the equilibrium equation of the closed economy given the history of transfers.¹⁰³ Foreign assets (p.e.u.l) f_t satisfy the assets market clearing condition:

$$k_{t+1} + f_{t+1} = \frac{s_t^{t-1}(\{k_{t+1}\}_{i=0}^2, g_{at+1}, \{g_{nt+i}\}_{i=-1}^1, f_t, f_{t+1})}{(1 + g_{at+1})(2 + g_{nt})} + \frac{s_t^{t-2}(\{k_{t+1}\}_{i=-1}^1, g_{at}, \{g_{nt+i}\}_{i=-2}^0, f_{t-1}, f_t)}{(1 + g_{at+1})(1 + g_{nt-1})(2 + g_{nt})},$$

where savings equations of young and middle-aged workers s_t^{t-1} and s_t^{t-2} depend on foreign assets and additional demographic variables (since transfers themselves depend on those). We thus have a third-order difference equation on $k_{t-1}, k_t, k_{t+1}, k_{t+2}$ and $f_{t-1}, f_t, f_{t+1}, f_{t+2}$.

To solve for the steady state, I first solve for capital (p.e.u.l), either equalizing the marginal productivity of capital to the after-tax international interest rate, or using the steady states of autarky. If the level of steady-state capital (p.e.u.l) of autarky is smaller (higher) than the level of capital (p.e.u.l) of the open economy and the net marginal productivity of autarkic capital is higher (smaller) than the after-tax foreign interest rate, the economy will be a borrower

¹⁰²Since, in this model, variables grow at the product of the growth rate of newborns and technology, we define detrended variables per effective unit of labor. In the steady states the detrended variables are constant and the aggregates grow at a constant rate of $(1 + g_n^{ss})(1 + g_a^{ss})$.

¹⁰³Equations 15, 16, and 17.

(lender).¹⁰⁴ Otherwise the capital of the steady states is the autarkic capital calculated above. Then, if the economy starts as being open, I solve for foreign assets by finding the invariant level f^* that satisfies this third-order difference equation.

To compute the transition in the open economy, for each period I use the conditions that determine if the economy will be borrowing, lending, or in autarky (eq. 11, 12, and 13). Then I solve for foreign assets holdings f_{t+1} using the above difference equation given f_{t-1} and f_t .

In particular in the numerical exercise for Mexico, for low levels of taxes, i.e. “the open taxed economy,” the economy begins as a borrower, then moves to autarky, and finally ends as a lender. In each period I compare the cost/return of being in autarky against the after-tax international interest rate and I check for feasibility of the assets market clearing condition. As the economy moves along the demographic transition, the autarkic return on capital increases to the point where the economy would rather be a lender. For this to happen, domestic savings need to increase beyond the capital of autarky, which takes some time. During this time the economy is in autarky until savings are high enough to become a borrower.

As soon as the autarkic return on capital rises beyond the after-tax international cost, the economy stops borrowing. Then, to determine if it will remain in international markets or move to autarky, I compute the savings of being a lender (given the history of transfers and capital) and check if savings are higher than the capital level of being a lender. If they are not, the economy cannot lend and it will move to autarky. Next, to determine how many periods it will remain in autarky, I guess the number of periods and compute savings given past and future transfers. This implies a path of capital, given by the law of motion of autarky along this transition and by the capital level of being a lender after the guessed period. Finally, I check if savings are higher than capital in the guessed period; if they are not, I update the guess and repeat.

¹⁰⁴If the supply of capital in autarky is smaller than the supply in open markets and the cost of financing capital inside of the economy is higher than the cost of borrowing, the economy will borrow. If instead the supply of capital in autarky is higher than in open markets and its return is higher in open markets, the economy will lend.