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Foreign Shocks as Granular Fluctuations

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November 2020



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Abstract

This paper uses a data set covering the universe of French firm-level sales, imports, and exports over the period 1993-2007 and a quantitative multi-country model to study the international transmission of business cycle shocks at both the micro and the macro levels. The largest firms are both important enough to generate aggregate fluctuations (Gabaix 2011), and most likely to be internationally connected. This implies that foreign shocks are transmitted to the domestic economy primarily through the largest firms. We first document a novel stylized fact: larger French firms are significantly more sensitive to foreign GDP growth. We then implement a quantitative framework calibrated to the full extent of observed heterogeneity in firm size, exporting, and importing. We simulate the propagation of foreign shocks to the French economy and report one micro and one macro finding. At the micro level, heterogeneity across firms predominates: 40-85 percent of the impact of foreign fluctuations on French GDP is accounted for by the “foreign granular residual”—the term capturing the fact that larger firms are more affected by the foreign shocks. At the macro level, firm heterogeneity dampens the impact of foreign shocks, with the GDP responses 10-20 percent larger in a representative firm model compared to the baseline model.

Key words: granularity, shock transmission, aggregate fluctuations, input linkages, international trade

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To view the authors’ disclosure statements, visit https://www.newyorkfed.org/research/staff_reports/sr947.html.

1 Introduction

This paper studies the international transmission of business cycle shocks at the firm and the aggregate levels. After decades of globalization, the structure of production is increasingly international, with supply chains overlapping with country borders. An important feature of this internationalization of production is that the largest firms are responsible for the bulk of international trade linkages in a typical economy (e.g., [Freund and Pierola, 2015](#)). As a result, while only a minority of firms have direct trade linkages with foreign countries, those firms tend to account for a large share of aggregate economic activity ([di Giovanni et al., 2017, 2018](#)).

We quantify the consequences of this phenomenon for international shock transmission. Our point of departure is that even purely aggregate foreign shocks affect firms differentially depending on the extent and nature of their international linkages. In that sense, an aggregate shock to a country’s trading partners manifests itself as a set of heterogeneous shocks to individual firms. Our analysis combines a dataset covering the universe of French firm sales and country-specific imports and exports over the period 1993-2007 and a quantitative multi-country multi-sector model with heterogeneous firms. We present one micro finding and one macro finding.

The micro result is that foreign shocks are predominantly granular fluctuations. To make this statement precise, let $d \ln Y^F$ be the log change in France’s GDP following a foreign shock. By definition, $d \ln Y^F$ is a weighted average of individual firms’ value added changes. As in [Gabaix \(2011\)](#) and [Gabaix and Koijen \(2019\)](#), the GDP change can be decomposed into the simple average value added change across all firms in France \mathcal{E}^F , and the *foreign granular residual* Γ^F :

$$d \ln Y^F = \mathcal{E}^F + \Gamma^F. \tag{1}$$

The superscript F on all three objects calls attention to the fact that these are all changes in response to a foreign rather than a domestic shock. The foreign granular residual Γ^F captures the deviation of larger firms’ value added changes from the unweighted average value added change. It is proportional to the cross-sectional covariance between firm size and firm response to the shock. Because the foreign shocks affect predominantly the largest firms in France, they lead to granular fluctuations. We subject our model French economy to foreign shocks, and find that Γ^F accounts for 40 – 85% of the resulting fluctuations in French GDP, depending on the shock.¹

¹Various meanings have been attached to the word “granular” in the literature. To be precise, what we mean by “granular” in this paper is that the term Γ^F in (1) is quantitatively important. More broadly, we use this adjective to capture the notion that foreign shocks produce domestic aggregate fluctuations driven disproportionately by larger firms. It has been understood since [Gabaix \(2011\)](#) that the granular residual can in principle arise from idiosyncratic shocks to large firms, or from a differential response of larger firms to common shocks. While [Gabaix \(2011\)](#) explores the former, this paper emphasizes the latter. A distinct question is whether the observed firm size distribution comes from fat-tailed underlying distributions (what one might term “heterogeneity”), or from idiosyncratic draws that deviate from those underlying distributions (the meaning that [Gaubert and Itskhoki \(2020\)](#) attach to the word “granular”). As will become clear, an advantage of our approach of using actual firms in the quantification is that we never need to take a stand on which of these forces leads to the observed firm data. Our results are not sensitive to the relative importance of “heterogeneity” vs. “granularity” (in this narrower sense) in the data.

The macro result is that the observed heterogeneity across firms dampens the impact of foreign shocks. Following the same foreign shock, the GDP change in an economy with identical amounts of trade and output, but no within-sector firm heterogeneity is 10 – 20% *larger* than the GDP change in the baseline economy. Thus, the micro structure affects aggregate outcomes.

We begin by documenting a novel stylized fact: larger French firms are significantly more sensitive to foreign GDP growth. This empirical regularity is *prima facie* evidence that larger firms are more susceptible to foreign fluctuations, suggesting that the foreign granular residual Γ^F could be quantitatively important.² We also document that in our data (i) there is a great deal of heterogeneity in both import and export participation among French firms; and (ii) larger firms are systematically more likely to trade internationally, consistent with a large body of previous literature.

The econometric estimates do not lend themselves well to aggregation or to performing counterfactuals, as they yield the relative impact of foreign GDP growth across firms, but not the overall impact. That is, the regression evidence relates the variation in sensitivity to foreign GDP to firm size, but does not pin down either the level of individual firm-level value added changes, nor the terms in (1). Thus, we employ a quantitative framework to simulate the effects of foreign shocks on the French economy.

The model is calibrated to the observed firm-level information for France, and to the sector-level information for France’s trading partners from the World Input-Output Database (WIOD). A distinctive feature of our framework is that it is implemented directly on firm-level data. In other words, objects inside the model are actual firms in France. This means we capture the full extent of the joint heterogeneity across French firms in size and international linkages, without relying on common shortcuts like integrating over assumed parametric underlying productivity distributions. Importantly, our model is solved in general equilibrium with discrete firms, implying that shocks experienced by individual firms can move equilibrium objects such as wages, prices, and GDP. Thus, it is the appropriate environment to quantify the impact of the micro heterogeneity on aggregate outcomes.

The transmission mechanisms in the model are standard. Following a positive foreign productivity shock, firms importing foreign inputs experience a fall in the prices of those inputs, and thus expand production. Changes in foreign demand (which could be due to a foreign productivity shock or a foreign demand shock) affect the firms’ export sales. External shocks are transmitted

²This result is reduced-form evidence of the relationship between firm size and sensitivity to foreign shocks. It is not driven by differences in overall procyclicality, as larger firms are not differentially more sensitive to the domestic GDP growth. In our quantitative model, the sensitivity to foreign shocks arises from import and export links. Our previous work looks directly at the link between trade at the firm level and comovement with foreign countries, providing micro evidence for transmission of shocks through trade linkages. Di Giovanni et al. (2014) shows that firms exporting to foreign countries are subject to demand shocks from those countries. Di Giovanni et al. (2018) documents that firms importing from, and exporting to, a foreign country are more correlated with GDP growth in that country.

inside the French economy via domestic input-output linkages and general equilibrium effects on the domestic goods and factor prices. Thus, even purely domestic firms in France are in principle affected by foreign shocks.

We quantify the decomposition (1) in two ways. First, we subject our world economy to hypothetical foreign shocks: a 10% productivity shock to all the countries other than France, and a 10% foreign demand shock for French goods. Following these shocks, the foreign granular residual is responsible for 40 – 85% of the total GDP change, depending on the shock. Second, we simulate the response of the economy to actual foreign productivity shocks, sourced from the Penn World Table. Foreign TFP shocks can account for about one tenth of the actual GDP fluctuations in France. More importantly for us, the standard deviation of the foreign granular residual is 90% of the standard deviation of overall fluctuations in French GDP generated by foreign shocks. All in all, both of our quantitative exercises show that foreign shocks manifest themselves as largely granular fluctuations.

To establish the macro result, we compare the change in GDP following a foreign shock to the change in GDP in a counterfactual model that suppresses all within-sector heterogeneity in both importing and exporting. We refer to this alternative as the homogeneous firm model. It is common in international macro and trade, and can be implemented with only sector-level data such as the WIOD. The homogeneous firm model produces GDP changes that are 10 – 20% larger than the baseline. Surprisingly, the granularity of the economy dampens the GDP responses to foreign shocks.

We build intuition for this finding via a combination of theoretical and numerical results. The baseline model differs from the homogeneous firm model in two respects: (i) heterogeneous firm sales, and (ii) heterogeneous production functions across firms within a sector, reflected in firm-specific imported intermediate input shares. We investigate the consequences of these two sources of heterogeneity in turn. First, we prove analytically that if production functions are identical among firms within a sector, the real GDP change due to a foreign shock is invariant to the distribution of market shares across firms. This theoretical result provides a sharp characterization of the source of the dampening effect: a necessary condition for dampening is heterogeneity in the use of foreign inputs.

We next provide a heuristic illustration for how this dimension of heterogeneity generates dampening. Raising a firm’s imported input share lowers its impact on domestic GDP. This is because mechanically, a higher imported input share means lower demand for domestic value added by the firm. At the same time, raising a firm’s imported input share increases its exposure to foreign shocks. Thus, relative to a representative firm world, introducing heterogeneity in imported input shares leads to a negative covariance in the cross section of firms between impact on domestic GDP and exposure to foreign shocks. This negative covariance is the source of the dampening effect of

production function heterogeneity. Because this dampening effect of firm heterogeneity is to our knowledge new in the literature, we illustrate it using a simple 2-firm model as well as a variation of the full-fledged quantitative model.

We conclude that heterogeneity across firms in the responsiveness to foreign shocks is pervasive at the micro level, and relevant for macro adjustment. Reallocation of market shares towards firms more exposed to imported inputs following a positive foreign shock dampens the aggregate response of the economy.

Related literature. The paper draws from and contributes to the active literature on the micro origins of aggregate fluctuations. [Carvalho \(2010\)](#) and [Acemoglu et al. \(2012\)](#) modernized the research program on shock propagation through the input networks that dates back to [Long and Plosser \(1983\)](#). A number of papers enriched the theory and quantification of the sectoral input network models (see, among others, [Foerster et al., 2011](#); [Acemoglu et al., 2016](#); [Atalay, 2017](#); [Grassi, 2017](#); [Baqae, 2018](#); [Baqae and Farhi, 2019a,b](#); [Foerster et al., 2019](#); [Bigio and La'O, 2020](#)). At the same time, the seminal contribution of [Gabaix \(2011\)](#) drew attention to the role of large firms in the macroeconomy, which has been further quantified and formalized by [di Giovanni et al. \(2014\)](#), [Carvalho and Grassi \(2019\)](#), and [Gaubert and Itskhoki \(2020\)](#) among others. [Atkeson and Burstein \(2008\)](#), [Eaton et al. \(2012\)](#), and [Burstein et al. \(2020\)](#) explore the consequences of discreteness in environments with variable markups. The research agendas on input networks and firm granularity are merging, with the latest modeling and measurement exercises capturing network interactions at the firm level (e.g., [Barrot and Sauvagnat, 2016](#); [Huneus, 2018](#); [Lim, 2018](#); [Taschereau-Dumouchel, 2019](#); [Carvalho et al., 2020](#); [Dhyne et al., 2020](#)).

We apply the insights and tools from this literature to the international transmission of shocks. [Hummels et al. \(2001\)](#), [Yi \(2003\)](#), and [Johnson and Noguera \(2012, 2017\)](#) document the importance of international input trade, while [Burstein et al. \(2008\)](#), [Bems et al. \(2010\)](#), [Johnson \(2014\)](#), [Eaton et al. \(2016b\)](#), and [Eaton et al. \(2016a\)](#), among others, model and quantify international shock transmission through input trade. [Baqae and Farhi \(2019c\)](#) and [Huo et al. \(2020\)](#) develop theoretical and quantitative treatments of the international input network model. The international business cycle literature has by and large not used firm-level data in empirical and quantitative assessments of shock transmission.³ The few recent exceptions include [di Giovanni and Levchenko \(2012\)](#), [Kleinert et al. \(2015\)](#), [Cravino and Levchenko \(2017\)](#), [Blaum et al. \(2018\)](#), [di Giovanni et al. \(2018\)](#), [Blaum \(2019\)](#), and [Boehm et al. \(2019\)](#). Our paper combines empirics, quantification,

³[Ghironi and Melitz \(2005\)](#) and [Alessandria and Choi \(2007\)](#) provide quantitative assessments of the transmission of aggregate shocks using international real business cycle models with heterogeneous firms. In these papers, firm heterogeneity is handled by tracking the moments of the firm size distribution, whereas in our work each actual firm is an object in the model. These papers explore the role of the extensive margin whereas we focus on the intensive margin in the context of heterogeneous export and import participation. The intensive margin is quantitatively more important for aggregate fluctuations and cross-border business cycle comovement in environments with fat-tailed firm-size distributions, as is the case in the data ([di Giovanni et al., 2014, 2018](#)).

and analytical results to highlight the role of different types of heterogeneity at the micro and the macro levels.

2 The Foreign Granular Residual

To set the stage for the empirical and quantitative exercises that follow, we set up a simple accounting framework that introduces the concept of the foreign granular residual and illustrates the consequences of heterogeneity for the aggregates. Let Y_m denote real GDP in country m , and let $Y_{f,m}$ denote the real value added of firm f . GDP is just the sum of firm-level value added:

$$Y_m = \sum_f Y_{f,m}. \quad (2)$$

We are interested in understanding the change in GDP following some foreign shock. Denote by $d \ln Y_m^F$ the log change in m 's GDP following that foreign shock, and by $\omega_{f,m,-1} \equiv \frac{Y_{f,m,-1}}{Y_{m,-1}}$ the pre-shock share of firm f 's value added in total GDP. The aggregate GDP change is the weighted sum of firm-level log changes $d \ln Y_{f,m}^F$:

$$d \ln Y_m^F = \sum_f \omega_{f,m,-1} d \ln Y_{f,m}^F.$$

The GDP change can then be written as:

$$d \ln Y_m^F = \mathcal{E}^F + \Gamma^F,$$

where the superscript F on all the values highlights the fact that all of these are changes following a foreign shock. The component $\mathcal{E}^F \equiv \frac{1}{N} \sum_f d \ln Y_{f,m}^F$ is the unweighted average value added change across all N firms in the economy. The foreign granular residual Γ^F is the size-weighted firm deviation from the unweighted average, as in [Gabaix \(2011\)](#) and [Gabaix and Koijen \(2019\)](#):

$$\Gamma^F \equiv \sum_f \omega_{f,m,-1} \left(d \ln Y_{f,m}^F - \frac{1}{N} \sum_f d \ln Y_{f,m}^F \right). \quad (3)$$

To build intuition on the meaning of the granular residual, note that with some manipulation it can be rewritten as a covariance between firm size and the firm value added change:

$$\Gamma^F = \text{Cov} \left(\frac{\omega_{f,m,-1}}{\bar{\omega}}, d \ln Y_{f,m}^F \right), \quad (4)$$

where $\bar{\omega} \equiv \frac{1}{N} \sum_f \omega_{f,m,-1} = \frac{1}{N}$. Writing Γ^F this way helps illustrate the role of granularity in international shock transmission. Since the largest firms are more likely to be internationally connected, we would expect them to have a larger increase in value added following a positive foreign shock, and thus the covariance in (4) is positive.

From here, we proceed as follows. After introducing the dataset, [Section 3](#) provides reduced-form regression evidence that the covariance [\(4\)](#) is positive, by estimating the differential sensitivity of larger firms to foreign shocks. [Section 4](#) then sets up a multi-country general equilibrium model of trade that captures this reduced-form pattern through differences across firms in foreign market participation. [Section 5](#) quantifies the size of the foreign granular residual following foreign shocks, and presents the main macro dampening result. [Section 6](#) concludes.

3 Data and Basic Facts

We combine administrative data on the universe of French firms’ value added, imports, and exports with standard multi-country sector-level databases of production and trade. The use of micro data for one country allows us to capture the heterogeneous exposure of individual firms to foreign shocks. While such heterogeneity obviously exists in all countries, firm-level information at this level of detail and coverage is not available for multiple countries at once. As a consequence, we will study the impact of firm heterogeneity using the French firm-level data, suppressing heterogeneity within sectors in the rest of the country sample.

3.1 Firm-Level Variables

We make use of an administrative dataset that contains balance sheet information collected from individual firms’ tax forms, and includes sales, value added, total exports, the cost structure, as well as its sector of activity for the universe of French firms over 1993-2007.⁴ This source is complemented with customs data on bilateral export and import flows at the firm level. The resulting dataset is described in greater detail in [di Giovanni et al. \(2018\)](#). [Table A1](#) reports the distribution of firms across sectors in 2005. Interestingly, the largest sector in terms of its contribution to aggregate value added is the one providing “Business Activities” to the rest of the economy. This underscores how important input-output relationships are to the functioning of modern economies. More generally, non-traded sectors are a large share of the French economy, accounting for more than 80% of firms and 72% of the value added in our sample. The comparison of these two numbers indicates that non-traded sector firms tend to be relatively small. There are some exceptions, however. For instance, firms in the “Post and Telecommunications” or the “Air Transport” sectors are relatively large.

When describing the variables in this section, we anticipate the notation used in the quantitative framework ([Section 4](#)) throughout. Following [di Giovanni et al. \(2014\)](#), we harmonize customs and tax form data to obtain sales by destination market ($X_{f,mn,j}$ for $m = France$). The tax files contain information on total sales and total exports, which we use to allocate total sales by the

⁴We work with data for this period because after 2011 import data at firm-product level for France are substantially left-censored. Our sample ends in 2007 to sidestep the 2008 trade collapse as well.

firm to the domestic or all foreign markets. We then use customs data to apportion total exports to specific destination markets. We perform a similar exercise for firm inputs. The tax data contain information on total input purchases. We combine it with customs data on the value of imports by origin country and type of product to build values for source- and sector-specific input expenditures. The customs data do not include trade in services. As a consequence, we have no choice but to treat all services as non-tradables and adjust the calibration accordingly. [Appendix A](#) and [di Giovanni et al. \(2014\)](#) provide further detail on apportioning sectors into tradables and non-tradables, and the construction of firm-level trade and factor shares.

3.2 Aggregate and Sectoral Variables

The main source of data at the country-sector level is the World Input Output Database (WIOD) ([Timmer et al., 2015](#)). This dataset combines national input-output tables and data on bilateral trade flows to build the matrix of all intra- and international flows of goods and services between sectors and final consumers. We use the 2013 release of the dataset which covers 40 countries plus a rest of the world aggregate and 35 sectors classified according to the ISIC Revision 3 nomenclature. These data are available over 1995 to 2011 and the benchmark year for the calibration of the quantitative model is 2005.

The WIOD dataset is used to recover: i) final consumption spending ($P_n C_n$); ii) the value of bilateral sales by sector ($X_{mn,j}$); and iii) the sectoral production function parameters, which are used whenever more disaggregated data are not available. We use these data to measure the share of labor in country n sector j 's total costs ($\pi_{n,j}^l$) as well as the components of the input-output matrix, as measured by the share of inputs sourced from country m sector j by firms operating in country n sector i ($\pi_{mn,ji}^M$). The IO coefficients are readily available from the WIOD. Labor shares are measured by the ratio of value added over output, to be consistent with the interpretation of L as “equipped labor.”

The French administrative data and the WIOD data must be made consistent with each other, as the final dataset must feature firm-level trade flows that aggregate up to the sector-level bilateral trade flows reported in WIOD. In addition, shares of value added in total output implied by the French data must match those implied by WIOD for France. [Appendix A](#) describes in detail the harmonization procedure.

3.3 Basic Facts

Fact 1: Larger firms are more sensitive to foreign GDP growth. We establish this stylized fact by means of the following heuristic regression:

$$d \ln Y_{f,m,j,t} = \beta_0 d \ln Y_{W,t} + \beta_1 \ln Y_{f,m,j,t-1} \times d \ln Y_{W,t} + \beta_2 \ln Y_{f,m,j,t-1} + \delta_{j,t} + \epsilon_{f,t}, \quad (5)$$

where $d \ln Y_{f,m,j,t}$ is the log change in firm value added, $\ln Y_{f,m,j,t-1}$ is its initial log level, $d \ln Y_{W,t}$ is the GDP growth in the world outside of France, and $\delta_{j,t}$ is the sector-time effect. The coefficient of interest β_1 captures whether firms of different sizes have differential elasticity of value added growth with respect to foreign GDP.

Table 1 reports the results. The first column presents estimates of (5) without any fixed effects.⁵ Column 2 adds year effects, which implies that we can no longer estimate the main effect of foreign GDP growth. Columns 3-4 include interacted sector-year effects, implying that the coefficient of interest is estimated from the variation across firms within a sector along the size dimension. The coefficient of interest is strongly positive and significant: larger firms are more sensitive to foreign growth. The point estimate falls when sector-year effects are added, but remains significant at 1%. It is sizeable in magnitude, implying that a doubling of firm size increases the elasticity of firm growth to world GDP growth by about 0.08.

Next, we check whether larger firms are more sensitive to the foreign business cycle, or simply more procyclical. Column 4 adds an interaction between firm size and French GDP growth. It is clear that larger firms are more sensitive to foreign growth specifically: the interaction term of firm size with respect to the domestic GDP growth is a precisely estimated zero. The elasticity with respect to foreign growth is the same whether or not we control for the domestic growth interaction term.

Fact 2: Larger firms are more likely to both export and import. **Figure 1(a)** plots the cumulative distribution function of firm-level share of exports in total sales. Similarly, **Figure 1(b)** plots the distribution of the intensity of imported input use, summarized by the share of foreign inputs in firms' total input expenditure ($\sum_{n \neq m} \sum_{i \in T} \pi_{f,mn,ij}^M$). In both plots, the solid (red) line depicts the unweighted distribution and the (blue) circles the distribution weighted by the firms' share in overall value added.

We stress two features of these figures, both of which are known in the trade literature and are confirmed in our data. First, there is a great deal of heterogeneity across firms in both export intensity and imported input use. Overall, 58% of the firms producing tradable goods do not export in our data. Among the firms that do export, many have sales that are strongly biased towards the domestic market. Still, about 6% of firms have export/total sales shares above 50%, and are thus quite exposed to foreign demand shocks. Similarly, more than 85% of firms source the entirety of their inputs locally, thus isolating themselves from (direct) foreign input price shocks. At the other end of the spectrum, about 2% of firms source more than 40% of their inputs from abroad.

⁵The main effect of foreign GDP growth is negative. However, the main effect coefficient must be interpreted jointly with the size interaction. Combining the main effect with the size interaction, the impact of foreign growth on firm value added turns positive above $\ln Y_{f,m,j,t}$ of 7, corresponding to annual value added of about 1mln euros (the value added variable is in thousands). Note that this main effect coefficient should be interpreted with caution, as this specification does not include any fixed effects and thus omitted factors could be affecting the estimates.

Table 1. Sensitivity to Foreign GDP Growth by Firm Size

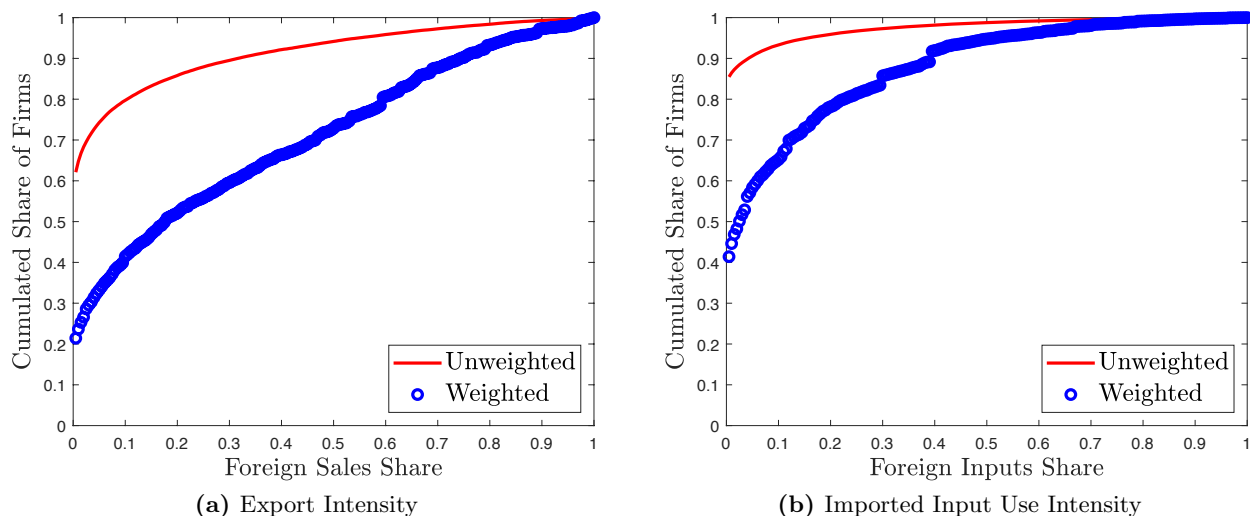
	(1)	(2)	(3)	(4)	(5)	(6)
					Model	
Dep. Var.: $d \ln Y_{f,m,j,t}$					World	World
					Prod.	Prod.
					Shock	Shock
	Data					
$\ln Y_{f,m,j,t-1} \times d \ln Y_{W,t}$	0.139 ^a (0.020)	0.160 ^a (0.020)	0.078 ^a (0.021)	0.077 ^a (0.022)	0.020 ^a (0.0001)	0.333 ^a (0.001)
$\ln Y_{f,m,j,t-1}$	-0.015 ^a (0.001)	-0.015 ^a (0.001)	-0.015 ^a (0.001)	-0.015 ^a (0.001)		
$d \ln Y_{W,t}$	-0.962 ^a (0.121)					
$\ln Y_{f,m,j,t-1} \times d \ln Y_{FRA,t}$				0.003 (0.017)		
Observations	1,345,729	1,345,729	1,345,729	1,345,729	385,926	385,926
# years	11	11	11	11	1	1
# firms	122,339	122,339	122,339	122,339	385,926	385,926
Adjusted R^2	0.004	0.012	0.019	0.019	0.444	0.432
Fixed Effects	–	Year	Sector \times Year	Sector \times Year	Sector	Sector

Notes: This table reports the estimates of [Equation \(5\)](#). Standard errors clustered at the firm level in parentheses with ^a, ^b and ^c denoting coefficients significantly different from zero at the 1, 5 and 10% levels, respectively. $d \ln Y_{FRA,t}$ denotes French GDP growth.

Second, participation in foreign markets is heavily tilted towards larger firms. This is illustrated in [Figure 1](#) by the comparison between the weighted and unweighted distributions. In both cases, the cdfs of the weighted distributions are substantially below the unweighted ones, meaning that on average larger firms have higher export and import intensities. For instance, the 6% of firms with more than 50% of their turnover abroad represent as much as 30% of the overall value added in tradable sectors. On the import side, the 15% of firms that source some inputs from abroad account for nearly 60% of aggregate value added, and firms sourcing more than 40% of their inputs abroad account for 10% of aggregate value added. In unreported results, we checked that the heterogeneity is not driven by cross-sector differences in overall exposure. While non-traded good sectors tend to be relatively less dependent on foreign inputs, most of the heterogeneity is actually driven by the within-sector variation.

Mechanisms and existing evidence. The patterns illustrated in Facts 1 and 2 have a natural connection: the import and export linkages to foreign countries make the larger firms more responsive to foreign shocks. The quantitative framework in the following section models these linkages formally and simulates the economy’s response to foreign shocks in an environment with

Figure 1. Distributions of Export and Imported Input Use Intensities Across French Firms



Notes: The left panel plots the cumulative distribution of firms according to their degree of exporting intensity, defined by the share of their sales going to foreign markets. The right panel plots the cumulative distribution of firms according to their share of inputs coming from other countries. The solid (red) lines correspond to the unweighted distributions and the (blue) circles to the weighted distributions, where firms' weights are defined according to their share in aggregate value added. The left panel is restricted to firms in traded good sectors. Source: French customs and balance sheet data for 2005.

firms heterogeneous in both size and trade participation.

It is a natural question why Fact 1 documents the reduced-form relationship with respect to size, rather than the impact of exporting and importing on comovement with foreign GDP at the firm level. In this paper we focus on the differential sensitivity with respect to firm size for two reasons. First, the foreign granular residual is the covariance between firm size and firm-level responses to foreign shocks. Thus, our Fact 1 regressions is the most direct way to get at the object encapsulated by (4).

Second, there is already a large body of evidence, from our work and others', that demonstrates convincingly using micro data that participation in international trade makes units (firms or sectors) more correlated with foreign GDP. To give a partial review of this body of work, [di Giovanni and Levchenko \(2010\)](#) shows that international trade synchronizes sectoral output across countries if those sectors use each other as intermediate inputs. [Di Giovanni et al. \(2014\)](#) shows that firms exporting to foreign countries are subject to demand shocks from those countries. [Di Giovanni et al. \(2018\)](#) provides econometric evidence that firms importing from, and exporting to, a foreign country are more correlated with GDP growth in that country. The latter two papers use the same French micro data as in this paper. [Boehm et al. \(2019\)](#) demonstrates that US firms that imported inputs subject to an exogenous shock (the 2011 Tohoku earthquake) contracted their

output dramatically. Thus, to avoid redundancy, we do not revisit these types of exercises in this paper.

4 Quantitative Framework

This section builds a heterogeneous-firm, multi-country, multi-sector model of trade. Crucially, we allow for heterogeneity in both input linkages and destination-specific sales at the firm level. The model features endogenous factor supply so that we can analyze how domestic and foreign shocks are transmitted to aggregate fluctuations.

4.1 Setup

The world is comprised of \mathcal{M} countries and \mathcal{J} sectors. Countries are denoted by m , n , and k , sectors by i and j , and firms by f and g . The notation follows the convention that the first subscript always denotes exporting (source) country, and the second subscript the importing (destination) country.

Households. There are \bar{L}_n households in country n . Each one consumes goods and supplies labor. Preferences over consumption and leisure are GHH (Greenwood et al., 1988):

$$U(c_n, l_n) = \nu \left(c_n - \frac{\psi_0}{\psi} l_n^{\bar{\psi}} \right),$$

where c_n is per-capita consumption, l_n the per-capita labor supply, and the function ν is increasing and concave. Note that the l_n should be thought of as “equipped labor” (Alvarez and Lucas, 2007), and thus captures the supply of all the primary factors.

The final consumption aggregate is Cobb-Douglas in the j sectors, with expenditure shares $\vartheta_{n,j}$:

$$c_n = \prod_j c_{n,j}^{\vartheta_{n,j}},$$

where $c_{n,j}$ is the per capita final consumption of sector j . Therefore, the ideal consumption price index is:

$$P_n = \prod_j \left(\frac{P_{n,j}}{\vartheta_{n,j}} \right)^{\vartheta_{n,j}}, \quad (6)$$

where $P_{n,j}$ is the price index of sector j goods in country n . Straightforward steps lead to the following labor supply:

$$L_n = \left(\frac{1}{\psi_0} \frac{w_n}{P_n} \right)^{\frac{1}{\psi-1}} \bar{L}_n,$$

where w_n is the price of equipped labor in country n .

Denote by $C_n \equiv c_n \bar{L}_n$ the aggregate final consumption in country n , and let $C_{n,j} \equiv c_{n,j} \bar{L}_n$ be the aggregate final consumption of sector j . Countries m sell (export) to country n . Origin-specific

output is apportioned to consumption and intermediate input usage. Let each sector's consumption be aggregated from origin-specific components:

$$C_{n,j} = \left[\sum_m \mu_{mn,j}^{\frac{1}{\sigma}} C_{mn,j}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}},$$

where $C_{mn,j}$ is final consumption of imports from country m in sector j , country n . Then the price index for consumption in sector j , country n is:

$$P_{n,j} = \left[\sum_m \mu_{mn,j} P_{mn,j}^{1-\sigma} \right]^{\frac{1}{1-\sigma}},$$

where $P_{mn,j}$ is the price index for exports from m to n in sector j , defined below. Final demand for goods from m is:

$$P_{mn,j} C_{mn,j} = \frac{\mu_{mn,j} P_{mn,j}^{1-\sigma}}{P_{n,j}^{1-\sigma}} P_{n,j} C_{n,j} = \frac{\mu_{mn,j} P_{mn,j}^{1-\sigma}}{P_{n,j}^{1-\sigma}} \vartheta_{n,j} P_n C_n.$$

Denote by Π_n the aggregate profits of firms owned by households in n , and by D_n any trade imbalance. Then the final expenditure in n on goods coming from country m sector j is:

$$P_{mn,j} C_{mn,j} = \frac{\mu_{mn,j} P_{mn,j}^{1-\sigma_j}}{P_{n,j}^{1-\sigma_j}} \vartheta_{n,j} \left[w_n \left(\frac{1}{\psi_0} \frac{w_n}{P_n} \right)^{\frac{1}{\psi-1}} \bar{L}_n + \Pi_n + D_n \right].$$

Note that we use the French customs data for imports at the firm level, and thus every import transaction is associated with a French firm (which may be a wholesaler or a retailer). Thus, French final consumers are never observed to import final consumption goods directly, and as a result French final consumption is composed only of domestically-supplied final goods.⁶ For all the other countries, we do not have firm-level data on imports, but instead have final consumption data by source country from WIOD. Thus, we assume that foreign consumers import final goods directly.

Sectors. Sectors are populated by heterogeneous, monopolistically-competitive firms. Not all firms sell to all destinations. Denote by $\Omega_{mn,j}$ the set of firms from country m , sector j that sell to country n . The CES aggregate of output in sector j of firms from m selling in country n is:

$$Q_{mn,j} = \left[\sum_{f \in \Omega_{mn,j}} \xi_{f,mn,j}^{\frac{1}{\rho}} Q_{f,mn,j}^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}, \quad (7)$$

⁶Formally, when $n = \text{France}$, $\mu_{mn,j} = 0 \forall m \neq n$, $P_{n,j} = P_{nn,j}$, and $P_{nn,j} C_{nn,j} = P_{n,j} C_{n,j} = \vartheta_{n,j} \left[w_n \left(\frac{1}{\psi_0} \frac{w_n}{P_n} \right)^{\frac{1}{\psi-1}} \bar{L}_n + \Pi_n + D_n \right]$, where $P_{nn,j}$ is the ideal price index of output produced by French firms in France.

where $Q_{f,mn,j}$ is the quantity of firm f 's good from country m and sector j selling to country n .⁷ The taste shock to a firm's destination-specific sales $\xi_{f,mn,j}$ is at this point left unrestricted. It could be allowed to have a firm-specific global component, and/or a source-destination-sector common component across firms. The latter would be isomorphic to $\mu_{mn,j}$ in the cross section. The price level of the aggregate of sector j sellers from m in destination n is:

$$P_{mn,j} = \left[\sum_{f \in \Omega_{mn,j}} \xi_{f,mn,j} p_{f,mn,j}^{1-\rho} \right]^{\frac{1}{1-\rho}},$$

where $p_{f,mn,j}$ is the price charged by firm f in country n .

Let X denote expenditure (at each level of aggregation). Then demand faced by firm f in country n is:

$$X_{f,mn,j} = \xi_{f,mn,j} \frac{p_{f,mn,j}^{1-\rho}}{P_{mn,j}^{1-\rho}} X_{mn,j}.$$

Thus, $X_{mn,j}$ is the total value of exports from m to n in sector j , and $X_{f,mn,j}$ is the value of exports by firm f .

Firms. Firms face downward-sloping demand and set price equal to a constant markup $\frac{\rho}{\rho-1}$ over the marginal cost. (Below we show that the results are robust to allowing variable markups following [Atkeson and Burstein \(2008\)](#), such that larger firms have both higher and more flexible markups.) Firms located in m face an iceberg cost of $\tau_{mn,j}$ to export to n . They have a total factor productivity a_f , and the cost of the input bundle

$$b_{f,m,j} = \left[\alpha_{f,m,j} w_m^{1-\phi} + (1 - \alpha_{f,m,j}) (P_{f,m,j}^M)^{1-\phi} \right]^{\frac{1}{1-\phi}},$$

where $\alpha_{f,m,j}$ is a firm-specific parameter governing the firm's labor share. The cost of intermediate inputs $P_{f,m,j}^M$ is firm-specific, and given by:

$$P_{f,m,j}^M = \left[\sum_i \sum_k \gamma_{f,km,ij} P_{km,i}^{1-\eta} \right]^{\frac{1}{1-\eta}},$$

where $\gamma_{f,km,ij}$ is the parameter governing the use of inputs sourced from country k sector i by firm f operating in country m , sector j . That is, firms in m use inputs from potentially all countries k in each sector i , with firm-specific taste shifters $\gamma_{f,km,ij}$. Some of these will be zero, i.e. the firm does not use inputs from a particular sector and country. Sales by firm f from country m in destination n are then

$$X_{f,mn,j} = \xi_{f,mn,j} \frac{\left(\frac{\rho}{\rho-1} \frac{\tau_{mn,j} b_{f,m,j}}{a_f} \right)^{1-\rho}}{P_{mn,j}^{1-\rho}} X_{mn,j}.$$

⁷In the counterfactual experiments below, we assume that following a foreign shock, the sets of firms serving each market $\Omega_{mn,j}$ are unchanged. See [di Giovanni et al. \(2014, 2018\)](#) for evidence that the extensive margin adjustments are not quantitatively important at the business cycle frequency.

Equilibrium. Market clearing for exports from m to n in sector j is:

$$X_{mn,j} = \frac{\mu_{mn,j} P_{mn,j}^{1-\sigma}}{P_{n,j}^{1-\sigma}} \vartheta_{n,j} \left[w_n \left(\frac{1}{\psi_0} \frac{w_n}{P_n} \right)^{\frac{1}{\psi-1}} \bar{L}_n + \Pi_n + D_n \right] \quad (8)$$

$$+ \sum_i \sum_{f \in i} \frac{\rho-1}{\rho} (1 - \pi_{f,n,i}^l) \pi_{f,mn,ji}^M \sum_k \frac{\xi_{f,nk,i} \left(\frac{\rho}{\rho-1} \frac{\tau_{nk,i} b_{f,n,i}}{a_f} \right)^{1-\rho}}{P_{nk,i}^{1-\rho}} X_{nk,i},$$

where $\pi_{f,m,j}^l$ and $\pi_{f,km,ij}^M$ are firm f 's expenditure shares on labor and input from sector i , country k , respectively:

$$\pi_{f,m,j}^l = \frac{\alpha_{f,m,j} w_m^{1-\phi}}{\alpha_{f,m,j} w_m^{1-\phi} + (1 - \alpha_{f,m,j}) \left(P_{f,m,j}^M \right)^{1-\phi}}$$

$$\pi_{f,km,ij}^M = \frac{\gamma_{f,km,ij} P_{km,i}^{1-\eta}}{\sum_i \sum_n \gamma_{f,nm,ij} P_{nm,i}^{1-\eta}}.$$

In Equation (8), the first line is the final demand, and the second is the intermediate demand. Note that the intermediate demand is a summation of firm-level intermediate demands, and thus captures the notion that not all firms, even within the same sector, will import inputs from a particular foreign sector-country with the same intensity. The price indices are:

$$P_{mn,j} = \left[\sum_{f \in \Omega_{mn,j}} \xi_{f,mn,j} \left(\frac{\rho}{\rho-1} \frac{\tau_{mn,j} b_{f,m,j}}{a_f} \right)^{1-\rho} \right]^{\frac{1}{1-\rho}}. \quad (9)$$

Total labor compensation in the sector is the sum of firm-level expenditures on labor:

$$w_n L_{n,j} = \frac{\rho-1}{\rho} \sum_{f \in j} \pi_{f,n,j}^l \sum_k X_{f,nk,j}$$

$$= \frac{\rho-1}{\rho} \sum_{f \in j} \pi_{f,n,j}^l \sum_k \frac{\xi_{f,nk,j} \left(\frac{\rho}{\rho-1} \frac{\tau_{nk,j} b_{f,n,j}}{a_f} \right)^{1-\rho}}{P_{nk,j}^{1-\rho}} X_{nk,j}.$$

Labor market clearing ensures that real wages adjust to equate the aggregate labor demand (right-hand side) with labor supply:

$$\left(\frac{1}{\psi_0} \frac{w_n}{P_n} \right)^{\frac{1}{\psi-1}} \bar{L}_n = \sum_j L_{n,j} \quad (10)$$

$$= \frac{\rho-1}{\rho} \frac{1}{w_n} \sum_j \sum_{f \in j} \pi_{f,n,i}^l \sum_k \frac{\xi_{f,nk,j} \left(\frac{\rho}{\rho-1} \frac{\tau_{nk,j} b_{f,n,j}}{a_f} \right)^{1-\rho}}{P_{nk,j}^{1-\rho}} X_{nk,j}.$$

Equations (8), (9), and (10) are a system of equations that define equilibrium wages, prices, and expenditures.

Heterogeneity. In the cross-section, heterogeneity in firm size is thus driven by productivity, taste/quality, and differences in input sourcing across firms. To illustrate, the share of firm f 's sales in total sales by domestic firms to the home market in sector j is:

$$\pi_{f,mm,j} = \frac{\xi_{f,mm,j} a_f^{\rho-1} \left[\alpha_{f,m,j} w_m^{1-\phi} + (1 - \alpha_{f,m,j}) \left(P_{f,m,j}^M \right)^{1-\phi} \right]^{\frac{1-\rho}{1-\phi}}}{\sum_{g \in \Omega_{mm,j}} \xi_{g,mm,j} a_g^{\rho-1} \left[\alpha_{g,m,j} w_m^{1-\phi} + (1 - \alpha_{g,m,j}) \left(P_{g,m,j}^M \right)^{1-\phi} \right]^{\frac{1-\rho}{1-\phi}}}.$$

Sales dispersion across firms in the same market is generated by differences in productivity a_f , the taste shifter $\xi_{f,mm,j}$, and the fact that input sourcing shifters $\gamma_{f,km,ij}$ differ across firms (even though we assume that all firms face the same input prices $P_{km,i}$). As will become clear below, we will not need to take a stand on the levels of a_f , $\xi_{f,mm,j}$, and $\gamma_{f,km,ij}$. Instead the counterfactual exercises will use the observed shares such as $\pi_{f,mm,j}$ directly to calibrate the model at the baseline period and then use the equilibrium conditions to compute the changes in those $\pi_{f,mm,j}$'s between the baseline and the counterfactual equilibrium.

Following a shock, what are the reasons that firms will differ in their value added growth rates $d \ln Y_{f,m}^F$? With some manipulation, we can write the approximate log change in value added of firm f as:

$$\begin{aligned} d \ln Y_{f,m}^F \approx & (1 - \rho) \left[\pi_{f,m,j,-1}^l d \ln w_m + \sum_i \sum_k (1 - \pi_{f,m,j,-1}^l) \pi_{f,km,ij,-1}^M d \ln P_{km,i} \right] \\ & + \sum_n s_{f,mn,j,-1} d \ln \left[\xi_{f,mn,j} \left(\frac{\tau_{mn,j}}{P_{mn,j}} \right)^{1-\rho} X_{mn,j} \right], \end{aligned} \quad (11)$$

where $s_{f,mn,j,-1}$ is the pre-shock share of market n in the total gross sales of firm f . Thus, a firm that only serves the domestic market has $s_{f,mm,j,-1} = 1$ and $s_{f,mn,j,-1} = 0 \forall n \neq m$.

The first term in (11) captures the change in the firm's costs, and the second term the change in the firm's demand following any external shock. Equation (11) highlights the sources of differential responses to shocks. On the cost side, following a shock in country k , only firms that import from k – $\pi_{f,km,ij}^M \neq 0$ – directly experience a change in input costs. At the same time, the change in foreign demand – be it from the price-adjusted foreign expenditure $X_{mn,j}/P_{mn,j}^{1-\rho}$, or from a taste ($\xi_{f,mn,j}$) or trade cost shock – will to first order affect only firms that export to country n , and even among those firms will vary with the sales share to that market.

At the same time, this expression underscores the general-equilibrium channels that will operate and thus should be accounted for. To the extent that the foreign shock changes domestic wages ($d \ln w_m$), all firms in m will be affected. Also, all firms sell domestically. Thus, if the foreign shock affects domestic demand $d \ln \left(X_{mm,j}/P_{mm,j}^{1-\rho} \right)$, it will reach all firms in m . Finally, even the non-importing firms' input prices $d \ln P_{mm,i}$ change through second-order input linkages and general equilibrium effects.

It is ultimately an empirical and quantitative question how much $d \ln Y_{f,m}^F$ varies across firms, and how it covaries with firm size. [Section 3.2](#) provides econometric evidence that $d \ln Y_{f,m}^F$ is indeed heterogeneous in its comovement with foreign GDP. The reduced-form results are however silent on the relative importance of the direct effects on the connected firms and the general equilibrium effects on all firms in the economy. The quantitative analysis addresses this question.

GDP accounting in the model. GDP is real value added. Following the national accounting conventions, in the main text we report the results for real GDP obtained using the double-deflation procedure.⁸ This definition of real GDP corresponds to the notion of the change in the physical final output produced by the economy. The procedure for computing real GDP implicitly defines the GDP deflator, which we take to be the measure of the aggregate price level change. The GDP deflator is required to compute real value added changes for individual firms following a shock. Thus, in implementing the decomposition [\(1\)](#), we deflate each firm’s nominal value added growth with the GDP deflator. This procedure ensures that aggregate real GDP is the sum of all firms’ real value added. [Appendix B.1](#) presents the complete set of definitions and formulas underlying the construction of the real GDP and the GDP deflator, which mimic national accounts procedures.

As an alternative, we can deflate nominal GDP change by the CPI (P_m in [Equation \(6\)](#)). The CPI-deflated GDP incorporates changes in prices of imported goods following a foreign shock. This notion of real GDP corresponds to the change in the real purchasing power of a country’s final output from the perspective of the consumer. Thus, this concept of real GDP will increase following a reduction in the prices of imports, even if the physical quantities of every good produced by the economy were unchanged. [Table A2](#) reports the main results for CPI-deflated real GDP.

4.2 Calibration

To perform counterfactuals that simulate the impact of foreign shocks on domestic firms and the aggregate economy, we follow the approach of [Dekle et al. \(2008\)](#) and express the equilibrium conditions in terms of gross changes in endogenous variables, to be solved for as a function of shocks expressed in gross changes, and initial (pre-shock) observables. [Appendix B.2](#) describes the procedure in detail.

Importantly, each actual firm in France is an object inside the model, and the solution is implemented directly on the observed firm-level data for France. Doing so requires data on firm-destination-specific sales shares $\pi_{f,nk,j}$, firm-source-specific sectoral input expenditure shares $\pi_{f,mn,ji}^M$, as well a firm-specific primary factor shares $\pi_{f,n,i}^l$. We only have this information for France, and thus for the other countries the model collapses to the standard international trade model with sector-level input-output linkages (see, e.g. the Handbook chapter by [Costinot and](#)

⁸See also [Kehoe and Ruhl \(2008\)](#), [Burstein and Cravino \(2015\)](#), and [Baqae and Farhi \(2019a,b,c\)](#).

Rodríguez-Clare, 2014). For the other countries we use WIOD to obtain sector-level counterparts of these shares. For French firms, $\pi_{f,mn,ji}^M$'s are available for imported inputs, but not domestic ones. The domestic input-output linkages are inferred using firm-level data on overall input usage and sector-level information on domestic IO linkages. See Section 3 and Appendix A for details on the construction of all firm- and sector-specific shares.

In addition to initial-period values taken from the data, solving the model requires a small number of structural parameters. Table 2 summarizes the calibration. We set the elasticity of substitution between firms in the same sector selling to the same destination to $\rho = 3$, a common value according to standard methodologies (see e.g. Broda and Weinstein, 2006). We set the Armington elasticity of substitution between goods coming from different source countries to $\sigma = 1.5$. This is the value favored by the international business cycle literature following Backus et al. (1995), and is supported by the recent estimates by Feenstra et al. (2018). We set the labor supply parameter to $\bar{\psi} = 3$, implying a Frisch labor supply elasticity of 0.5, as advocated by Chetty et al. (2013). In the baseline, we set the production function elasticities $\eta = \phi = 1$ (Cobb-Douglas), as is standard in the literature. In the robustness analysis we implement both higher and lower values of each of these parameters.

Our model does not feature endogenous deficits. In all our experiments, we thus assume that the change in deficits is zero: $\hat{D}_n = 0$. We adopt a similar approach to profits: $\hat{\Pi}_n = 0$. In the absence of a model of multinational production and ownership of firms, in an open economy like France changes in profits are not pinned down in our framework. This is because the aggregate profits in Equation (8) refer to those used by French residents for domestic consumption spending. These are not the same as profits of firms operating in France, both because French residents own French multinationals operating abroad and thus have claims on those foreign-generated profits, and because not all firms operating in France are domestically-owned, and the profits of foreign multinational affiliates operating in France are not available to French residents for consumption spending. Since the profit share of GDP is under 10%, and for our counterfactuals what matters is not the level of profit share but the change, as an approximation we abstract from the impact of changes in profits on final consumption in our counterfactuals. Section 5.2 checks robustness to an alternative specification of the profit change.

5 Quantitative Results

5.1 Micro: the Granular Origins of International Shock Transmission

We start by simulating the impact on the French economy of a 10% productivity improvement in every foreign country in the sample. The left panel of Table 3 presents the results of the decomposition (1). As discussed above, we report real GDP changes deflated by the GDP deflator.

Table 2. Parameter Values

Param.	Value	Source	Related to
ρ	3	Broda and Weinstein (2006)	subst. elasticity btw. firms
σ	1.5	Feenstra et al. (2018)	Armington elasticity
η	1	standard	subst. elasticity btw. inputs
ϕ	1	standard	subst. elasticity btw. inputs and labor
$\bar{\psi}$	3	Chetty et al. (2013)	Frisch elasticity
$\pi_{f,n,i}^l, \pi_{f,mn,ji}^M$	}	Our calculations based on French data and WIOD	labor and intermediate shares
$\vartheta_{n,j}$			final consumption shares
$\pi_{mn,j}^c$			final trade shares
$\pi_{f,nk,j}$			intermediate use trade shares

Notes: This table summarizes the parameter values used in the calibration.

French GDP increases by 2.7% following a 10% world productivity shock. This is a sizeable GDP change considering that France itself does not experience the productivity shock, and thus the entire effect is due to it being transmitted to France via goods trade linkages.

Our central micro result concerns not so much the overall magnitude, but the role of heterogeneity. Decomposing the aggregate elasticity into the unweighted mean and the granular residual, we find that the latter is positive as expected and quite large. The Γ^F term is responsible for 85% of the overall effect of a world shock. Thus, our results reveal a quantitatively large role of the heterogeneity in firm-level international linkages in business cycle transmission across countries. [Table A2](#) presents the results when deflating by CPI. The change in GDP is larger at 6.3% following the world shock. It is not surprising that deflating by the CPI produces a larger real GDP change, as the CPI includes reductions in the prices of imported goods. Since the movement in the aggregate price level is larger for the CPI than the GDP deflator, and enters entirely in \mathcal{E}^F , the \mathcal{E}^F term is also larger. Nonetheless, the granular residual is still responsible for 43% of the total GDP change for the world shock.

Next, we evaluate whether in the baseline model, the heterogeneity that drives the high covariance term is within or across sectors. To that end, we take the results from the baseline model, and instead of writing the decomposition (1) at the firm level, write it at the sector level:

$$d \ln Y^F = \mathcal{E}_{\mathcal{J}}^F + \Gamma_{\mathcal{J}}^F. \quad (12)$$

where $\Gamma_{\mathcal{J}}^F \equiv \sum_j \omega_{j,m,-1} d \ln Y_{j,m}^F - \frac{1}{\mathcal{J}} \sum_f d \ln Y_{j,m}^F$ is the granular residual defined based on sectoral value added growth rates $d \ln Y_{j,m}^F$ and shares $\omega_{j,m,-1}$, and $\mathcal{E}_{\mathcal{J}}^F$ is the unweighted average sectoral growth rate. Importantly, we implement this decomposition on the baseline model featuring the full heterogeneity across firms, but compute the sector-level shares and value added changes. The

Table 3. Responses of French Real GDP to 10% Foreign Productivity and Demand Shocks

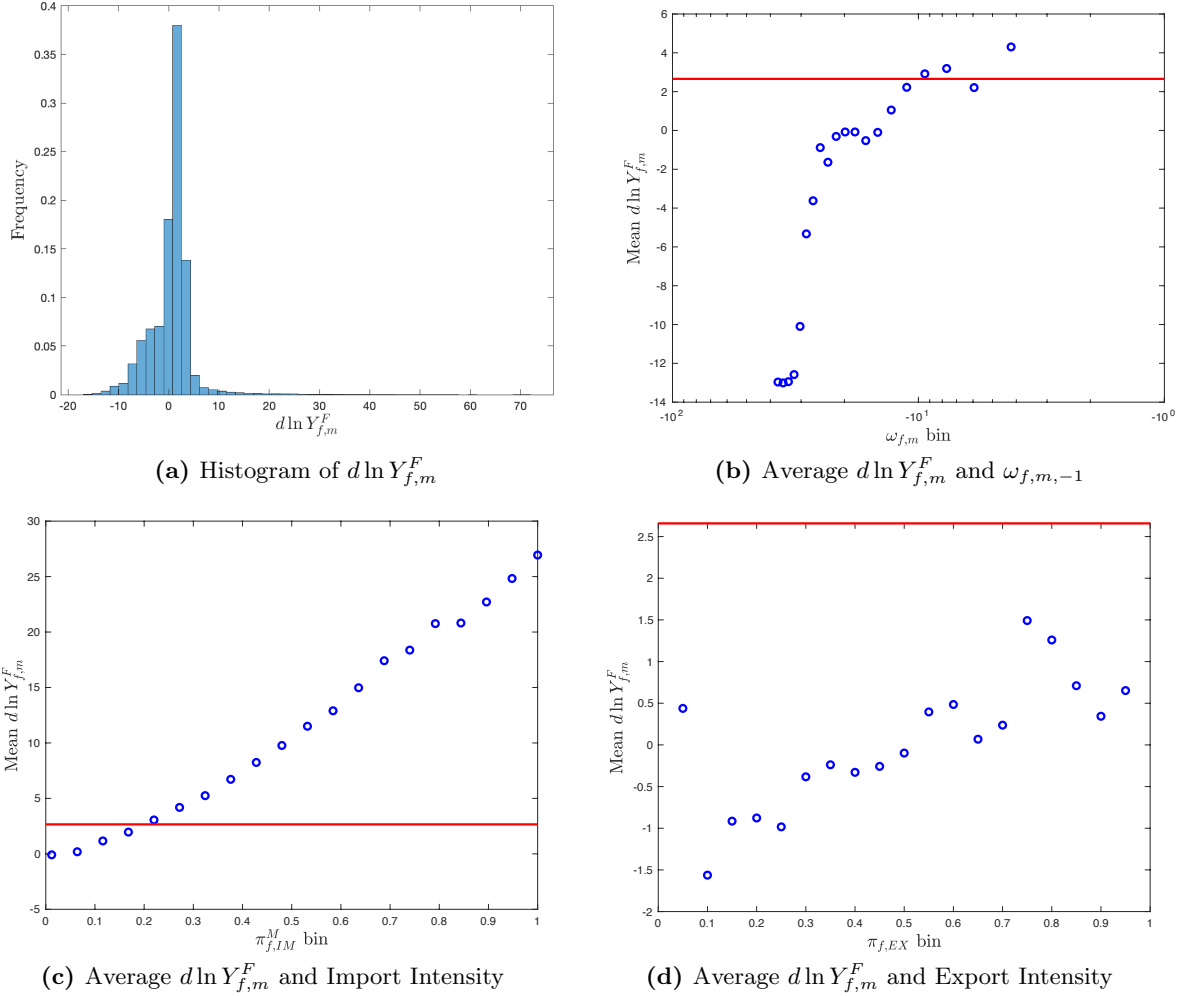
	$d \ln Y^F$	\mathcal{E}^F	Γ^F	$d \ln Y^F$	\mathcal{E}^F	Γ^F
Shock:	Productivity			Demand		
Baseline	2.66	0.39	2.27	0.35	0.20	0.15
<i>Share:</i>		<i>0.148</i>	<i>0.852</i>		<i>0.572</i>	<i>0.428</i>
Homogeneous firms	3.13	3.07	0.06	0.37	0.38	-0.01
<i>Share:</i>		<i>0.982</i>	<i>0.018</i>		<i>1.025</i>	<i>-0.025</i>
Sector-Level Decomposition						
	$d \ln Y^F$	$\mathcal{E}_{\mathcal{J}}^F$	$\Gamma_{\mathcal{J}}^F$	$d \ln Y^F$	$\mathcal{E}_{\mathcal{J}}^F$	$\Gamma_{\mathcal{J}}^F$
Baseline	2.66	2.05	0.60	0.35	0.60	-0.25
<i>Share:</i>		<i>0.773</i>	<i>0.227</i>		<i>1.699</i>	<i>-0.699</i>

Notes: This table reports the change in French GDP, in percentage points, following a 10% productivity shock (left panel) or a 10% foreign demand shock for French goods (right panel) in every other country in the world, in both the baseline model and the alternative model that suppresses firm heterogeneity. The table reports the decomposition of the the GDP change into the unweighted average and granular residual terms as in (1).

results are presented in the panel labeled “Sector-Level Decomposition” of Table 3. By construction, the overall GDP change $d \ln Y^F$ is exactly the same as in the top panel of the table. The sector-level granular residual term is 23% of the total, much smaller than the firm-level granular residual, suggesting that the impact of heterogeneity is to a large extent not captured by the sectoral dimension. Note that standard multi-sector models of international shock transmission would capture the sectoral granular residual. Thus, the sectoral granular residual is a natural benchmark for our firm-level results.

To illustrate the main results, Figure 2(a) plots the histogram of firm-level value added changes in the baseline model for the world shock. The dispersion in firm-level growth rates is evident. While most firm value added changes are positive, there is substantial density below zero as well – some firms shrink in response to a positive shock in the rest of the world. At the same time, there is an upper tail as well, as the density of $d \ln Y_{f,m}^F$ above 10 percentage points change is visible. Next, Figure 2(b) presents the average $d \ln Y_{f,m}^F$ for firms of different sizes $\omega_{f,m}$. We break firm shares

Figure 2. Micro Responses to a 10% World Productivity Shock



Notes: This figure displays the firm responses to a 10% world productivity shock in the baseline model. Panel (a) is the histogram of $d \ln Y_{f,m}^F$. Panels (b), (c), and (d) plot the mean $d \ln Y_{f,m}^F$, in percentage points, over firm size bins, total imported intermediate input intensity, and overall export intensity, respectively.

in aggregate value added into size bins, and plot the mean $d \ln Y_{f,m}^F$ in each size bin. This figure is a graphical illustration of the positive granular residual term. As highlighted in Equation (4), the granular residual is a covariance between the firm-level value added growth and firm size. The horizontal line plots the aggregate GDP change $d \ln Y_m^F$. It is notable that it is towards the top of the plot, coinciding with the $d \ln Y_{f,m}^F$ of the largest firms.

The variation in firm-specific elasticities with respect to foreign shocks has the expected relationship to the intensity of intermediate input purchases from abroad. Figure 2(c) plots the average $d \ln Y_{f,m}^F$ for each value of total imported input share, $\pi_{f,IM}^M \equiv \sum_{n \neq m} \sum_i \pi_{f,mn,ji}^M$. There is a pro-

nounced positive relationship. **Figure 2(d)** plots the average $d \ln Y_{f,m}^F$ against the total export intensity of each firm, defined as the ratio of total firm exports to total firm sales, $\pi_{f,EX} \equiv \sum_{n \neq m} s_{f,mn,j}$. Over a range of export intensity values, there is an evident positive relationship, with more export-oriented firms having higher elasticities to the foreign shock. On the other hand, unlike the relationship with $\pi_{f,IM}^M$, the unweighted mean $d \ln Y_{f,m}^F$ in each export intensity bin is actually below the aggregate country-level $d \ln Y_m^F$, and the most extreme export intensity values do not fit the upward-sloping pattern. This is an illustration of the granularity in the data, and the mechanisms behind the variation in $d \ln Y_{f,m}^F$. Exporting in and of itself does not necessarily lead the firms to expand due to a foreign productivity improvement. Following a foreign productivity shock, there are two reasons why firms can shrink: (i) non-importing firms lose domestic market share to the importing firms, who enjoy the direct benefit of lower imported input prices; and (ii) firms lose market share to foreign firms in both home and foreign markets when substitution effects due to lower competitors' prices dominate positive income effects of higher productivity. Firms at the bottom of the distribution of export intensity mostly suffer from (i) but firms that export a lot are more exposed to (ii). The only firms that really benefit from higher foreign productivity are the input importers that enjoy a large enough direct cost reduction. These happen to be large exporters on average.

Next, we evaluate the propagation of a foreign demand shock to France. To that end, we simulate an increase in the taste shock $\xi_{f,mn,j}$ to all firms in $m = \text{France}$ in all foreign markets $n \neq m$. Examining **Equation (7)**, it is clear that an increase in the taste for all French firms abroad amounts to a $\widehat{\xi_{mn,j}^{\frac{1}{\rho-1}}}$ productivity increase for French exports abroad, and thus an increase in demand for French goods by foreign firms and consumers. (We assume that this is a purely external shock, such that the French domestic demand shifter $\xi_{f,mm,j}$ is unchanged.) We thus simulate a 10% shift in demand for French goods, namely $d \ln \widehat{\xi_{mn,j}^{\frac{1}{\rho-1}}} = 0.1$.

The right panel of **Table 3** reports the results. In the baseline, a 10% demand shock for French goods abroad raises French real GDP by 0.35%. This is a smaller GDP change than following a foreign productivity shock, but that is because the overall shock is much smaller, as it affects only the French tradable sector. The granular residual accounts for 43% of the overall impact for the foreign demand shock. When deflating by CPI, the foreign demand shock raises French GDP by 0.49%, with the contribution of the foreign granular residual of 31% (**Table A2**).

The bottom panel reports the average-granular residual decomposition at the sector level for the foreign demand shock. Not only is the covariance term not positive, it is actually strongly negative, accounting for -70% of the overall effect for the world demand shock. Evidently, sectors with the highest positive elasticities with respect to foreign demand shocks tend to actually be relatively smaller in size. This is sensible, as some of the largest sectors in our data are non-tradable, and thus by construction not experiencing the positive foreign demand shock.

Finally, we run the heuristic regression (5) inside the model. The results are reported in Table 1, columns 5 (for the world productivity shock) and 6 (world demand shock). Since the model simulation is of a single year’s growth rate, there are fewer firms in this regression, and sector-time fixed effects become sector fixed effects. The model reproduces the pattern in the data qualitatively. Larger firms are more sensitive to both the world productivity and world demand shocks. Interestingly, the coefficient of interest is much smaller than in the data in the productivity shock simulation, but much larger than in the data in the demand shock simulation. Given that actual world GDP is a mix of productivity and demand shocks, we should not expect a single shock inside the model to replicate the data coefficient. The fact that the data coefficient is between those for productivity and demand shocks is perhaps telling that foreign shocks experienced by France are a mixture of the two.

5.1.1 Responses to Country-Specific Shocks

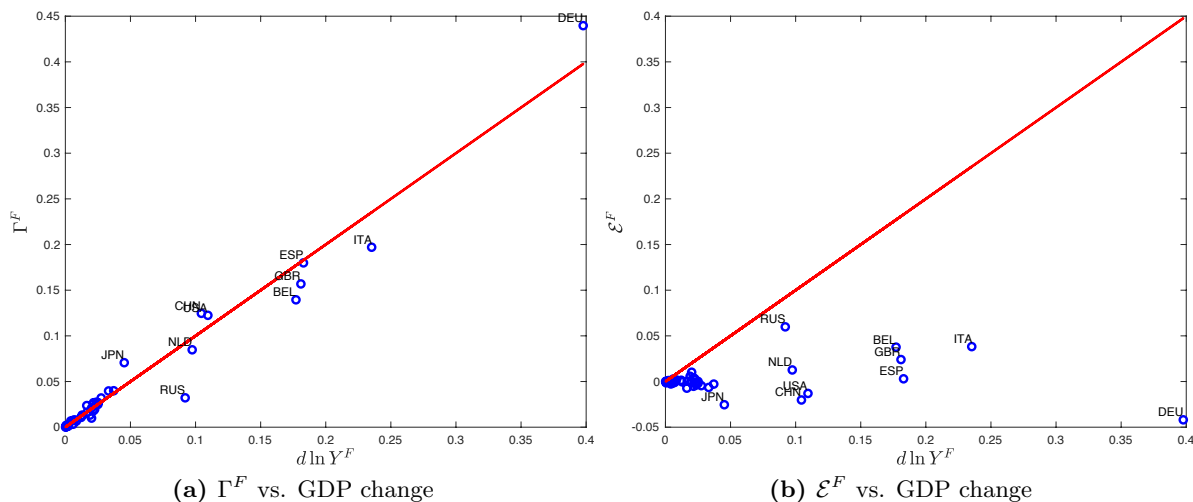
We can also subject our model to shocks in each foreign country separately, and perform the decomposition (1) of the French GDP change in response to country-specific shocks. Figure 3 displays the results for 10% productivity shocks. On the x-axis of both panels is the change in GDP. Not surprisingly, French GDP responds by different magnitudes to shocks in different countries, with the size of the response conditioned by country size and level of trade integration with France. The largest by a wide margin is the GDP response to a shock in Germany (DEU), which produces a 0.4% change in French GDP. Smaller and more distant countries produce negligible GDP changes.

The second notable feature of the figure is that virtually all the variation in the overall GDP response is accounted for by the variation in the foreign granular residual (left panel). The observations are near the 45-degree line. In a few instances, including Germany, the foreign granular residual is actually slightly more than 100% of the total GDP response, implying that the unweighted average change across firms is negative. The right panel is the scatterplot of $d \ln Y^F$ against the unweighted change \mathcal{E}^F . This term is on average close to zero, and does not correlate well with the overall GDP change. Thus, differences in GDP responses to shocks in different foreign countries are accounted for by the granular residual rather than the unweighted average change.

5.1.2 Simulating Actual Foreign GDP Growth

The above results explore the propagation into France of hypothetical shocks. To provide a closer comparison to actual GDP data, in this section we subject the French economy to actual foreign GDP growth rates. Because France trades with many partner countries, to compute the French economy’s responses to worldwide economic conditions requires simulating shocks to multiple countries at once. We do this in two ways. First, we feed the TFP shocks to foreign countries from the Penn World Tables into the model. In the second approach, we obtain actual GDP growth for all

Figure 3. GDP Changes in Response to 10% Country-Specific Productivity Shocks



Notes: This figure plots the real GDP change in France on the y-axis following a country-specific shock against the Γ^F (left panel) and \mathcal{E}^F (right panel). A 45-degree line is added to both plots. All units are in percentage points.

the countries in our sample from the World Development Indicators. To compute the propagation of foreign GDP growth rates into France, we re-express the model directly in terms of elasticities of French firms to foreign GDP. The advantage of the latter approach is that it in principle accounts for all of GDP movements abroad, not just the movements in measured TFP. The disadvantage is that it implicitly attributes all of the foreign GDP changes to TFP, which may not be accurate. [Appendix B.3](#) details the two procedures.

[Table 4](#) reports the results for two time periods: 1975–2014, and 1991–2007. There are two reasons to focus on the shorter time period. The first is that for this time period we can report the standard deviation of the overall French granular residual (Γ), sourced from our earlier work ([di Giovanni et al., 2014](#)).⁹ Second, our model is implemented on the trade and production data from this period, and it is not clear that the cross-border trade linkages we assume are realistic prior to the 1990s. The first two columns report the standard deviations of actual French GDP growth and the granular residual. The middle panel reports the standard deviations $d \ln Y_m^F$, \mathcal{E}^F , and Γ^F generated purely by foreign TFP shocks. Foreign shocks by themselves can generate about 10% of the observed GDP fluctuations of France. More importantly for us, the standard deviation of the foreign granular residual Γ^F is 91 to 94% of the overall standard deviation of the foreign shock-induced GDP fluctuations. By contrast, the standard deviation of the unweighted average

⁹The overall granular residual is the contribution of all firm-level idiosyncratic shocks, including domestic ones, to aggregate fluctuations. Hence, it is more volatile than Γ^F , which is generated solely from propagation of aggregate foreign shocks.

Table 4. Standard Deviations of Actual and Foreign-Induced GDP Growth and Its Components, Percentage Points

Period	Data		Foreign TFP			Foreign GDP		
	$d \ln Y_m$	Γ	$d \ln Y_m^F$	\mathcal{E}^F	Γ^F	$d \ln Y_m^F$	\mathcal{E}^F	Γ^F
1975–2014	1.54		0.14	0.02	0.13	0.11	0.01	0.10
1991–2007	1.11	0.96	0.14	0.03	0.12	0.07	0.02	0.06

Notes: The panel labeled “Data” reports the standard deviations of actual French GDP growth ($d \ln Y_m$), and the actual French granular residual (Γ). The panels labeled “Foreign TFP” and “Foreign GDP” report the standard deviations of French GDP generated purely from observed foreign TFP and GDP, respectively, and standard deviations of each component of (1). Foreign TFP growth rates are taken from the Penn World Tables, the French and foreign GDP growth from the World Development Indicators, and Γ from di Giovanni et al. (2014).

component \mathcal{E}^F is 16 to 21% of the total standard deviation. Thus, foreign shocks are indeed predominantly granular fluctuations. The right-most panel reports the results of feeding in GDP growth. The relative contribution of the foreign granular residual to the overall foreign impact is similarly close to 90%.

Using different approaches, Gabaix (2011), di Giovanni et al. (2014), and Carvalho and Grassi (2019) document that a significant fraction of GDP fluctuations is driven by idiosyncratic shocks to individual firms. The contribution of firm idiosyncratic shocks to aggregate fluctuations is captured by the granular residual. Beyond accounting for aggregate fluctuations, the granular residual is an object of interest in other contexts; see for instance its use as an instrument (Gabaix and Koijen, 2019). Because of the systematically heterogeneous cross-border linkages across firms, foreign shocks are a quantitatively important contributor to the granular residual, and are thus one of the sources of granular fluctuations.

5.2 Macro: the Dampening Effect of Firm Heterogeneity

We compare the baseline model to an alternative implementation that suppresses all within-sector firm heterogeneity: domestic and foreign sales shares (the $\pi_{f,nk,j}$'s) and intermediate import usage ($\pi_{f,mn,ji}^M$) are made identical across firms in each sector. To preserve comparability to the baseline, this model still has firms, that are homogeneous in their importing and exporting intensities. The $\pi_{f,mn,ji}^M$'s are set to match the sector-level imported input coefficients, and the export shares $\pi_{f,nk,j}$ are set to match aggregate export shares in each sector. Importantly, this exercise preserves the overall levels of imports and exports, by sector, so this alternative model features the exact same level of trade openness as the baseline. This implies that the imported input coefficients in this

implementation are lowered for the firms that in the data actually import inputs, but raised for firms that in the data do not. Similarly, firms that in the data export nothing in this scenario export to all countries. This model can be implemented using only the WIOD sectoral production and trade data, and does not require any firm-level information.

Table 3 reports the results in the row labeled “Homogeneous firms.” The main macro finding is that the aggregate GDP change following the world productivity shock is about 20% larger in the homogeneous firm model than in the baseline. The dampening effect also appears for the foreign demand shock, though here the disparity is smaller at 6%. In all cases, the average-granular decomposition shows that the entirety of the GDP change is now accounted for by the unweighted average value added change \mathcal{E}^F , with zero contribution of the granular residual. Not surprisingly, the representative firm model is very different at the micro level.

This dampening effect is not unique to our preferred calibration. **Table A3** presents the comparison of GDP changes in the baseline and homogeneous models following the world productivity and demand shocks, while raising and lowering each key elasticity in the model. The finding that GDP changes are larger in the homogeneous model obtains for every alternative parameter value considered in the table. The proportional differences in GDP changes between the homogeneous and baseline models are also similar to the main calibration, which does not stand out in terms of the relative magnitude of the dampening effect.

Next, **Table A3** reports the results under flexible markups as in [Atkeson and Burstein \(2008\)](#). In this environment, firms take into account the impact of their own pricing decisions on the sectoral price index, and thus markups are heterogeneous across firms, with larger firms having higher markups. Flexible markups attenuate somewhat the difference between the homogeneous and heterogeneous models, as expected, but substantial dampening effect still remains. Finally, we examine what happens when changes in aggregate profits Π_n contribute to final demand. As we do not have firm ground to stand on when evaluating aggregate profit changes, in the baseline we assume that aggregate profits do not change following the foreign shock. In this robustness check, we instead assume that the total profits of firms operating in France equal to the variable profits, which in turn are a constant fraction of aggregate sales. Note that this approach gives profits the highest chance to make a difference, by assuming that variable profits are total profits, that is, there are no fixed costs. The last line of **Table A3** reports the results, and shows that the dampening effect of heterogeneity persists.

5.2.1 Understanding the Mechanisms

The baseline model differs from the homogeneous firm model in two respects: (i) heterogeneous sales across firms by destination, and (ii) heterogeneous production functions across firms within a sector, reflected in firm-specific labor and input shares. We investigate the consequences of these

two sources of heterogeneity in turn. First, we prove analytically that if production functions are identical across firms within a sector, the real GDP change due to a foreign shock is invariant to the distribution of market shares across firms. This theoretical result provides a sharp characterization of where the dampening effect comes from: a necessary condition for dampening is heterogeneity in the production functions. Though we do not have an analytical result on how production function heterogeneity affects the size of the GDP response to foreign shocks, we next provide a heuristic illustration for how this dimension of heterogeneity generates dampening.

Exporting/sales heterogeneity.

Proposition 1 *If $\gamma_{f,mn,ij} = \gamma_{mn,ij}$ and $\alpha_{f,n,j} = \alpha_{n,j} \forall f$, the real GDP change $d \ln Y_m^F$ following a foreign productivity shock or a non-firm-specific foreign demand shock is invariant to the distribution of firm-level destination-specific sales shares $\pi_{f,nk,j}$.*

Proof: See [Appendix B.4](#). ■

The proof proceeds to show that as long as within-sector production functions are identical across firms, the sector-destination-level equations that must be satisfied in equilibrium do not have sales shares $\pi_{f,nk,j}$ in them, and therefore the macro aggregates are independent of either initial or post-shock $\pi_{f,nk,j}$'s. The proof covers all distributions of $\pi_{f,nk,j}$'s, including zero market shares. This implies that any extensive margin differences across model implementations, whereby firms do or do not serve all or some markets have no effect on GDP changes due to foreign shocks if these firms have the same production functions.

The proposition applies in our quantitative framework, which is general in some respects – such as unrestricted distributions of $\pi_{f,nk,j}$ and foreign input usage by source country and sector – but relies on some key assumptions, notably constant markups. If larger firms had systematically different markups, as in [Atkeson and Burstein \(2008\)](#) for instance, then the GDP change would not be invariant to the size distribution within a sector even if all firms had identical production functions. Nonetheless, the constant markup case is an important benchmark, and [Proposition 1](#) clarifies the conditions under which different types of firm heterogeneity matter. In robustness exercises above we showed that variable markups a la [Atkeson and Burstein \(2008\)](#) do not overturn the macro dampening result.

Importing/production function heterogeneity. Having established that sales heterogeneity will not deliver different GDP responses to foreign shocks absent production function heterogeneity, we now investigate how production function heterogeneity can lead to dampening.

The intuition is as follows. Raising a firm's imported input share lowers its impact on domestic GDP. This is because mechanically, a higher imported input share means lower demand for domestic

value added by the firm. At the same time, raising a firm's imported input share increases its exposure to foreign shocks. Thus, relative to a representative firm world, introducing heterogeneity in imported input shares leads to a negative covariance in the cross section of firms between impact on domestic GDP and exposure to foreign shocks. This negative covariance is the source of the dampening effect of production function heterogeneity.

To make this more precise, we introduce the following notation. Define firm f 's *influence* as the elasticity of GDP with respect to a productivity shock in that firm: $\lambda_f \equiv \frac{d \ln Y_m}{d \ln a_f}$ (Acemoglu et al., 2012).¹⁰ Recall that $\pi_{f,IM}^M \equiv \sum_{n \neq m} \sum_i \pi_{f,mn,ji}^M$ denotes the combined imported input share of firm f . Below, we will show by means of numerical illustrations that holding firm size fixed, influence decreases in $\pi_{f,IM}^M$: $\partial \lambda_f / \partial \pi_{f,IM}^M < 0$. That is, all else equal a firm that has a higher import share has a lower influence on domestic GDP. This is intuitive since a higher import share means a lower share of domestic value added in production.

At the same time, a firm with a higher import share also experiences a *de facto* bigger shock following a foreign productivity improvement. Heuristically, differences across firms in value added growth following a foreign shock come from differential reductions in input prices:

$$d \ln Y_{f,m,j} \propto (1 - \rho) \sum_i \sum_k (1 - \pi_{f,m,j,-1}^l) \pi_{f,km,ij,-1}^M d \ln P_{km,i}, \quad (13)$$

since, modulo differences in labor shares, the other terms that enter value added growth – such as market-specific demand changes – are common to all firms. Firms with larger import shares $(1 - \pi_{f,m,j,-1}^l) \pi_{f,km,ij,-1}^M$, $k \neq m$ benefit more from foreign cost reductions. Denote by \tilde{a}_f a hypothetical productivity shock to firm f that would produce a marginal cost reduction equal to the fall in input prices:

$$d \ln \tilde{a}_f^{-1} \equiv \sum_i \sum_k (1 - \pi_{f,m,j,-1}^l) \pi_{f,km,ij,-1}^M d \ln P_{km,i}. \quad (14)$$

The hypothetical productivity shock is increasing in the firm's import share: $\partial d \ln \tilde{a}_f / \partial \pi_{f,IM}^M > 0$.

Now compare a homogeneous to a heterogeneous firm model. In the homogeneous firm model, there is no variation across firms (within a sector) in either λ_f or $d \ln \tilde{a}_f$. By contrast, in the heterogeneous firm model, there is a negative relationship across firms between $d \ln \tilde{a}_f$ and λ_f conditional on size: firms importing a lot of inputs have a larger marginal cost change following a foreign shock, but a lower domestic influence. The change in GDP due to a vector of hypothetical productivity shocks $d \ln \tilde{a}_f$ is $d \ln Y_m = \sum_f \lambda_f d \ln \tilde{a}_f$. We can write it as the sum of averages and a covariance: $d \ln Y_m = \lambda \frac{1}{N} \sum_f d \ln \tilde{a}_f + N \text{Cov}(\lambda_f, d \ln \tilde{a}_f)$, where λ is the elasticity of GDP with respect to an aggregate domestic productivity shock. While in the homogeneous firm model the

¹⁰By definition, following a set of firm specific productivity shocks a_f , the total change in GDP is to first order given by

$$d \ln Y_m = \sum_f \lambda_f d \ln a_f.$$

covariance term is zero, in the heterogeneous model $\text{Cov}(\lambda_f, d \ln \tilde{a}_f) < 0$, conditional on size. Thus, the negative relationship between influence and exposure to the shock drives down the response of GDP to foreign shocks in the heterogeneous model.

Since this mechanism has not to our knowledge been previously pointed out, we start by illustrating it via the simplest possible example: a model with 2 countries (France and the Rest of the World), 2 sectors (Tradables and Non-Tradables), and 2 firms in each sector. To isolate the impact of heterogeneity in imported input intensity, we assume that within each sector these firms have the same sales to all markets, and are thus the same size. We start with the homogeneous firm model, in which both firms in each sector have the exact same imported input coefficients. These input coefficients are reported in the top panel of [Table 5](#). In the homogeneous firm model, 24% of a Tradable sector firm’s total costs (intermediates plus primary factors) are spent on foreign inputs, with the remaining 76% on domestic intermediates and labor. In the Non-Tradable sector, 8% of total costs go to pay for foreign inputs. These values correspond to the WIOD data when collapsed to 2 sectors and 2 countries, France and ROW.

We then progressively reassign foreign inputs to Firm 1 in each sector, so that in the final simulation, in the Tradable sector 47% of Firm 1’s costs are spent on foreign inputs (bottom of [Table 5](#)). As we do this, we keep the sector-level share of spending on imported inputs constant in the Tradable sector at 24%. Thus, Firm 2’s share of imported inputs is now 1% (recall that these firms have the same sales). The same reassignment of import shares occurs in the Non-Tradable sector. While we kept this economy’s overall trade openness constant, we made import participation heterogeneous.

Even in this simple example, we obtain the same result as in the full quantitative model that the GDP change is larger in the homogeneous case than in the heterogeneous one. [Figure 4](#) plots Tradable sector Firm 1’s $d \ln \tilde{a}_f$ and λ_f as a function of its imported input intensity on the x-axis. As argued above, increasing a firm’s input intensity lowers its domestic influence (solid line), while at the same time increasing the size of the shock that it experiences (dashed line).

While in the $2 \times 2 \times 2$ example we could keep the size of all firms the same, in the quantitative model firms also differ dramatically in size. That creates an extra effect: making foreign input shares identical across firms raises the influence λ_f of the larger firms, and lowers it for smaller firms. This is because in the data larger firms import relatively more, and homogenizing production functions means reducing their foreign input shares. This in turn raises their λ_f . The opposite occurs with smaller firms: making foreign input shares identical tends to raise their foreign input shares, and hence lower their λ_f .

To illustrate this, we consider an alternative homogeneous counterfactual model, in which production functions are identical across firms, but firm sizes (governed by $\pi_{f,mn,j}$ ’s) are still given by the data. By [Proposition 1](#), when production functions are identical across firms, the GDP change

Table 5. Input Coefficients and Domar Weights in the $2 \times 2 \times 2$ Model

	Tradable		Non-Tradable	
	Firm 1	Firm 2	Firm 1	Firm 2
	Homogeneous Input Shares			
Share of inputs from:				
France	0.76	0.76	0.92	0.92
ROW	0.24	0.24	0.08	0.08
Domar weight	0.21	0.21	0.52	0.52
	Heterogeneous Input Shares			
Share of inputs from:				
France	0.53	0.99	0.86	0.99
ROW	0.47	0.01	0.14	0.01
Domar weight	0.21	0.21	0.52	0.52

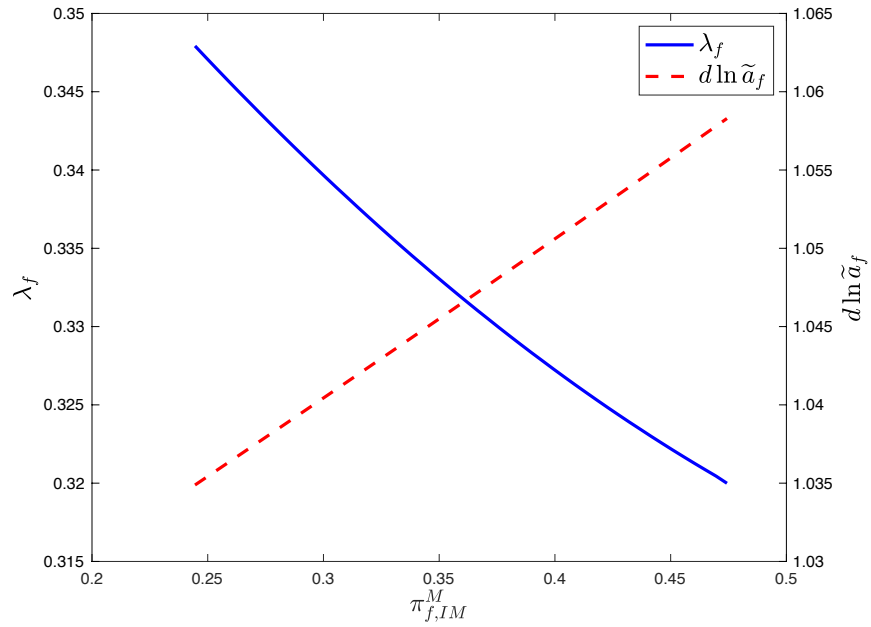
Notes: This table reports the firm-specific input coefficients and Domar weights in the simplified $2 \times 2 \times 2$ model.

following a foreign shock is invariant to the distribution of $\pi_{f,mn,j}$, and hence the distribution of firm size. Thus, the GDP change in this intermediate model is identical to the GDP change in the “Homogeneous firms” model reported in [Table 3](#). At the same time, because the firm sales distribution in this counterfactual model coincides with the fully heterogeneous firm baseline, each firm’s Domar weight is also exactly the same in this counterfactual and the baseline. Firm-specific production functions are the only difference between the two scenarios.

[Figure 5](#) plots the mean ratio of λ_f in the homogeneous relative to the heterogeneous model ($\lambda_f^{HOM}/\lambda_f^{HET}$) across firm size deciles. In the top size decile, this ratio is above 1: the domestic GDP influence of the largest firms is higher in the homogeneous production function model compared to the baseline. The relationship is monotonic across the size distribution, so that progressively smaller firms experience a greater reduction in their influence when production functions are made identical.

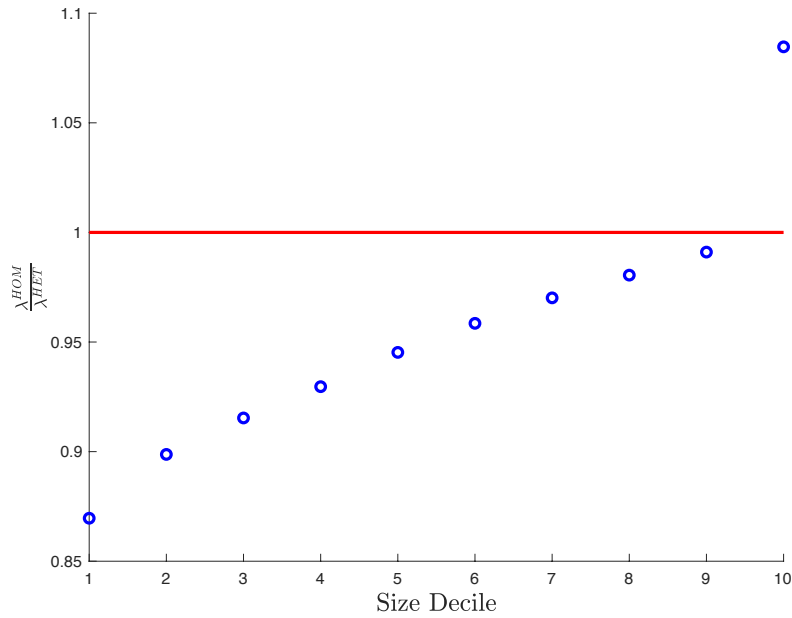
Relationship to [Hulten \(1978\)](#) and [Baqae and Farhi \(2019c\)](#). A classic result in macroeconomics provides an analytical solution to the influence vector λ_f in a closed economy with fixed factor supplies and perfect competition: λ_f is equal to the Domar weight, i.e. the ratio of the firm’s gross sales to aggregate value added ([Hulten, 1978](#); [Acemoglu et al., 2012](#); [Baqae and Farhi, 2019a](#)).

Figure 4. Influence and Shock Size as Import Intensity Changes



Notes: This figure displays $d \ln \tilde{a}_f$ and λ_f for Firm 1 in the tradable sector as a function of imported input intensity.

Figure 5. Ratio of Influences in the Homogeneous to the Baseline Model and Firm Size



Notes: This figure displays the mean of the ratio $\lambda_f^{HOM}/\lambda_f^{HET}$ for each size decile, where λ_f^{HOM} is firm f 's influence on GDP in the homogeneous production function case, and λ_f^{HET} is the same firm's influence on GDP in the baseline case.

Baqae and Farhi (2019c) extend this result to the open economy setting, and show that under the same assumptions – fixed factor supply and perfect competition – the result that λ_f equals the Domar weight continues to hold. This property is remarkable in that a sector/firm’s import and export intensity are irrelevant for its influence on domestic GDP. Shocks to two producers (firms or sectors) with identical total sales have identical GDP impact even if one uses mostly foreign inputs, while the other uses only domestic ones, for example.

This invariance result does not hold in our framework. In the illustrative $2 \times 2 \times 2$ model, we keep total firm sales unchanged as input coefficients vary. Thus the Domar weights, reported in Table 5, are constant for each firm by construction. However, Figure 4 shows that the influence λ_f changes with the firm’s import intensity. Figure 5 further underscores the departure of our model from the classic benchmark where λ_f equals the Domar weight. Because all the Domar weights are exactly the same in the two scenarios, the ratio of Domar weights is simply constant at 1 by construction, and depicted by the horizontal line in Figure 5. However, changing production functions affects the true influence of firms, systematically along the size distribution.

Conceptually, the two reasons that λ_f is not equal to the Domar weight are endogenous factor supply and profits. Huo et al. (2020) derive the influence vector in a multi-country input network model with variable factor supply. When factor supply is fixed, they recover the Baqae and Farhi (2019c) result that a sector’s influence is its Domar weight. However, under variable factor supply, a sector’s influence is no longer its Domar weight, but rather a function of the entire global input-output matrix. Thus, changes in that matrix – say, as we shuffle firms’ import intensities – will generically affect each firm’s influence.¹¹

The presence of profits implies that the real GDP change cannot be written as a sum of Domar-weighted changes in true TFP and primary factors. Instead, because profits are a fraction of gross sales, total intermediate input usage enters measured GDP. Thus, heterogeneity in foreign input usage will generally induce heterogeneity in a firm’s influence on GDP. A firm that imports foreign inputs demands less from other domestic firms, and thus a positive shock to it generates less profits for other firms in the home country. Hence, holding sales constant, increasing a firm’s input share lowers its influence. This effect bears some relation to Arkolakis and Ramanarayanan (2009) and Liao and Santacreu (2015), who show greater transmission of foreign shocks to domestic GDP in the presence of profits. We experimented with alternative values of parameters that govern the elasticity of labor supply and the size of profits in the economy. Qualitatively and quantitatively, either endogenous labor supply or profits alone are enough to generate the effect illustrated Figure 4 that λ_f falls in the imported input share.

¹¹We stress that this departure from Hulten (1978)-Baqae and Farhi (2019c) is not due to second-order terms. Endogenous factor supply generates first-order departures from Hulten (1978), as shown by Huo et al. (2020).

6 Conclusion

Large firms are more likely to import and export. A natural conjecture is that this greater participation in international markets also makes the large firms more sensitive to foreign shocks. In this paper, we explored both the micro and the macro implications of this joint heterogeneity in size and international linkages. We first provided firm-level econometric evidence that larger firms are indeed more correlated with foreign GDP growth. We then implemented a quantitative multi-country model in which French firms exhibit the observed joint distribution of size, importing, and exporting.

We reported one micro and one macro finding. The micro finding is that foreign shocks manifest themselves as largely granular fluctuations in France. Large firms are thus the key channel through which foreign shocks propagate to the domestic economy. The macro finding is that the heterogeneity in trade participation actually dampens the impact of a given foreign shock on French GDP. This is because heterogeneity in importing behavior induces a negative covariance between the size of the shock experienced by the firm and its contribution to domestic GDP, controlling for size.

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Appendix A Data

A.1 Harmonizing French Firm-Level Data with Global Sectoral Data

The firm’s sector in the French data is reported in the *Nomenclature d’Activités Françaises* classification, which we convert into the 35 sectors of the WIOD nomenclature. Note that the balance-sheet data do not cover Financial Activities and Private Households with Employed Persons (sectors J and P in WIOD), and thus those sectors are dropped from the analysis. We also dropped the “Public Administration” sector (sector L) which represents 23 firms and less than 0.1% of overall value added in our data.

Data on individual bilateral imports, together with information on each firm’s cost structure, are used to recover the firm-specific input shares. Firm-specific labor shares $\pi_{f,n,j}^l$ are defined as the ratio of value added over sales, both available in the balance-sheet data. In order to ensure comparability with the rest of the sample, in which labor shares are calibrated using WIOD for each country and sector, the distribution of firm-level labor shares is rescaled sector-by-sector in a way that preserves the heterogeneity but ensures that the weighted average across firms matches the corresponding information in the WIOD. Namely:

$$\pi_{f,n,j}^l = \tilde{\pi}_{f,n,j}^l \frac{\pi_{n,j}^l}{\tilde{\pi}_{n,j}^l}.$$

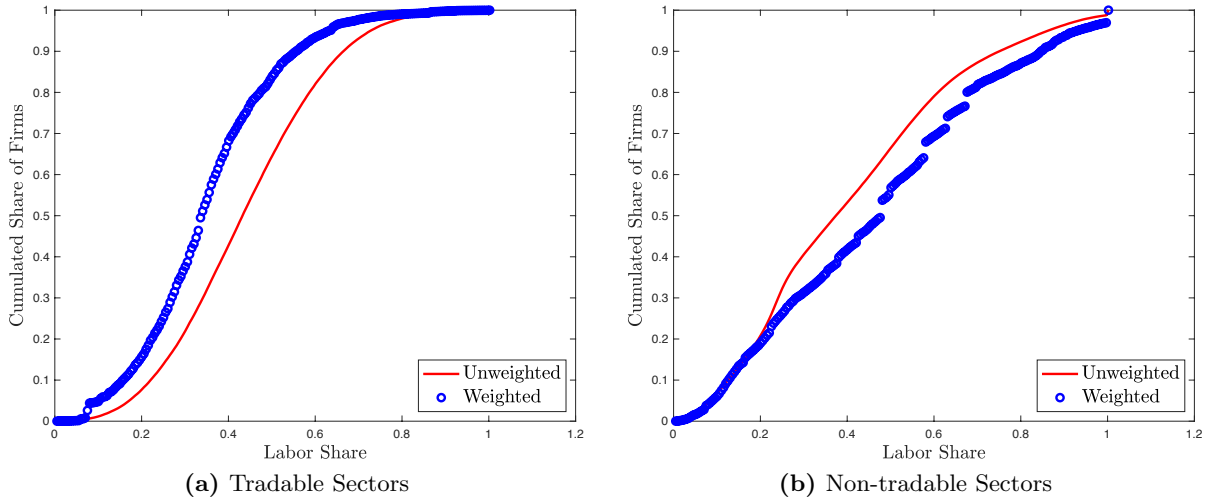
In this equation, $\pi_{f,n,j}^l$ and $\tilde{\pi}_{f,n,j}^l$ are the rescaled and original firm-level coefficients, respectively, and $\pi_{n,j}^l$ is the sectoral counterpart recovered from the WIOD data. Finally, $\tilde{\pi}_{n,j}^l$ is a weighted average of the original firm-level coefficients, where each firm is weighted according to its share $\omega_{f,n,j}^S$ in sectoral sales: $\tilde{\pi}_{n,j}^l = \sum_{f \in (n,j)} \omega_{f,n,j}^S \tilde{\pi}_{f,n,j}^l$.¹²

Figure A1 displays the cumulative distribution of labor shares, distinguishing between tradable and non-tradable sectors. The solid (red) line correspond to the unweighted distributions and the (blue) circles to the weighted ones. These distributions show a high degree of heterogeneity across firms, both within and across broad sectors. In traded good sectors, large firms tend to be less labor intensive, although the pattern is not systematic in all individual sectors and is not very strong. On the contrary, large firms in non-traded good sectors are often more labor-intensive than smaller ones.¹³

¹²The rescaling procedure implies that some rescaled firm-level coefficients end up lying outside of the range of possible values ($[0, 1]$). The corresponding coefficients are winsorized at the maximum and minimum values. This affects less than 0.02% of the firms in the total sample. The rescaling procedure is applied to all sectors but three, namely Wholesale; Retail, including Motor Vehicles; and Fuel. For these three sectors, the average labor share is low in the French data compared to the WIOD. This comes from the treatment of merchandise, which we categorize as intermediates while WIOD does not. Our approach is consistent with the model in the case of France, where we assume that consumers never interact directly with foreign firms. From that point of view, all merchandise imported from abroad is used as inputs by a French firm which ultimately sells to the final consumer. Because this is all the more important for retailing and wholesaling activities, we decided to keep the distribution of measured $\pi_{f,n,j}^l$ unchanged in these sectors.

¹³In tradable sectors, the correlation between the firm’s labor share and its size varies between 0 and -0.09 (Wood

Figure A1. Distribution of Labor Shares Across French Firms



Notes: This figure plots the cumulative distributions of firm-level labor shares ($\pi_{f,n,j}^l$), in tradable and in non-tradable sectors. The solid (red) lines correspond to the unweighted distribution and the (blue) circles to the weighted distribution, where firms’ weights are defined according to their share in aggregate value added. Calculated from French balance-sheet data together with the WIOD information on sectoral labor shares, for 2005.

Total input usage at the firm level equals one minus the labor share (in our setting “labor” stands for the composite of primary factors). We further disaggregate total input usage across sectors and source countries using the information on imports, by product. This allows us to recover the $\pi_{f,mn,ij}^M$ coefficients for $n = France$. While in principle straightforward, calibrating these parameters entails two key difficulties: i) it requires the use of two sources of firm-level data, which raises concerns regarding comparability; and ii) not all of these coefficients can be recovered from the firm-level data. In particular, we do not have detailed information on inputs purchased domestically and thus need to infer their sectoral breakdown using (more aggregated) information from WIOD. We proceed as follows.

For each sector i among the subset of tradable sectors and each source country $m \neq n$, we first compute the share of bilateral imports of goods produced by country m , sector i in the firm’s total input expenses.¹⁴ Since this ratio uses data collected from two databases, the overall import share obtained from the summation of these $\pi_{f,mn,ij}^M$ coefficients over all tradable sectors and foreign

products) and is often significant. In non-tradable sectors, it is positive and significant in 10 sectors out of 18 and is as high as 0.13 for Post and Telecommunication Services.

¹⁴This requires the conversion of product-level import data expressed in the highly disaggregated *Harmonized System* into broader sectoral categories. Since the customs data do not allow us to distinguish between the import of intermediates and merchandise (goods that are not further processed before being sold by the firm), we measure the firm’s input expenses accordingly as the sum of raw materials *and* merchandise purchases (taking into account changes in inventories). See [Blaum et al. \(2018\)](#) for a similar treatment of the data.

countries is larger than one in some cases (for less than 1% of firms). Whenever this happens, the import share is winsorized to one and the bilateral sectoral coefficients rescaled accordingly.

Beyond comparability issues between the two firm-level sources, the introduction of these firm-level import shares into the broader multi-country model also means we must ensure consistency with the sectoral coefficients in the global data. As we did with the labor shares, this implies rescaling the overall distribution of firm-level coefficients to the mean observed in the WIOD data:

$$\pi_{f,mn,ij}^M = \tilde{\pi}_{f,mn,ij}^M \frac{\pi_{mn,ij}^M}{\tilde{\pi}_{mn,ij}^M},$$

where $\pi_{f,mn,ij}^M$ and $\tilde{\pi}_{f,mn,ij}^M$ denote the rescaled and original firm-level coefficients, respectively, $\pi_{mn,ij}^M$ is the sectoral counterpart measured with the WIOD data, and $\tilde{\pi}_{mn,ij}^M$ is the weighted average of the firm-level original coefficients, where each firm is weighted according to its share $\omega_{f,n,j}^M$ in sectoral input purchases: $\tilde{\pi}_{mn,ij}^M = \sum_{f \in (n,j)} \omega_{f,n,j}^M \pi_{f,mn,ij}^M$. The normalization preserves as much heterogeneity across firms as possible, while avoiding overestimating the international transmission of shocks through foreign input purchases via an exaggeration of the degree to which French firms actually rely on foreign inputs. From that point of view, our calibration is conservative.

By definition, the remaining input purchases, those not sourced abroad, include tradable goods purchased in France and all expenses on non-tradable inputs. While we do not have any information on how these domestic expenses are spread across sectors, we can recover the firm-level share of individual input purchases as $\sum_i \pi_{f,nn,ij}^M = 1 - \sum_{m \neq n} \sum_{i \in T} \pi_{f,mn,ij}^M$. This domestic input share is then assigned to domestic input sectors using information in the WIOD:¹⁵

$$\pi_{f,nn,ij}^M = \frac{\pi_{nn,ij}^M}{\sum_i \pi_{nn,ij}^M} \times \sum_i \pi_{f,nn,ij}^M.$$

We have tested an alternative calibration strategy in which the input coefficients for non-traded sectors are all set exactly to their values in the WIOD. The remaining (homogeneous) share in input purchases is then spread across tradable sectors and countries using the bilateral import shares available at the firm level. The residual which corresponds to tradable inputs purchased domestically is spread across sectors using the WIOD coefficients. Note that this strategy tends to underestimate the share of tradable goods that are purchased domestically, i.e., it overestimates the participation of French firms to foreign input markets. For this reason, we have chosen to use the more conservative strategy described above as our benchmark.

¹⁵Our definition of non-tradable (NT) sectors is somewhat unconventional since we de facto exclude from the tradable sector all services that are potentially traded but that we do not observe in the customs data. As a consequence, some of our NT sectors might display strictly positive foreign input shares in WIOD, i.e. $\pi_{mn,ij}^M \neq 0$ for $j \in NT$. We adjust the WIOD data to make them consistent with our definition of non-tradable sectors by allocating all purchases from a NT sector to the same French sector, i.e.: $\pi_{nn,ij}^M = \sum_m \pi_{mn,ij}^M$ and $\pi_{mn,ij}^M = 0, \forall i \in NT$. We apply the same adjustment to the other countries in the sample, to ensure comparability.

Appendix B Theory and Quantification

B.1 The GDP Deflator Construction in the Model

This Appendix describes how we replicate the procedures used by the system of national accounts to compute changes in real GDP and the GDP deflator. The GDP deflator is an implicit deflator that is defined as the ratio of nominal and real GDP changes. In turn, the real GDP is computed using the “double deflation” method that records output net of inputs when both are evaluated at base prices. Specifically, define real GDP, evaluated at base prices (prices at -1) by:

$$Y_m = \sum_{j=1}^{\mathcal{J}} (P_{m,j,-1} Q_{m,j} - P_{m,j,-1}^M M_{m,j}),$$

where $Q_{m,j}$ is the gross physical output in sector j , $M_{m,j}$ is the physical use of inputs in the sector, $P_{m,j,-1}$ is the gross output base price, and $P_{m,j,-1}^M$ is the base price of inputs in that sector.

Denote by a “hat” a gross proportional change in a variable relative to its base value: $\hat{x} \equiv x/x_{-1}$. The gross change in real GDP is then:

$$\hat{Y}_m = \sum_{j=1}^{\mathcal{J}} \omega_{m,j,-1}^D \left(\hat{Q}_{m,j} - \pi_{m,j,-1}^M \hat{M}_{m,j} \right), \quad (\text{B.1})$$

where $\omega_{m,j,-1}^D \equiv \frac{P_{m,j,-1} Q_{m,j,-1}}{Y_{m,-1}}$ is the base period Domar weight of sector j , that is, the ratio of the sector’s gross sales to aggregate value added, and $\pi_{m,j,-1}^M$ is the base period sector-level share of input spending in gross output. Since $\omega_{m,j,-1}^D$ and $\pi_{m,j,-1}^M$ are both nominal beginning-of-period values, they are easily constructable from data.

To measure changes in physical quantities $\hat{Q}_{m,j}$ and $\hat{M}_{m,j}$, in practice national statistical agencies measure sectoral nominal gross sales and PPIs, and deflate the gross sales changes by PPI changes. That is, the pieces of data at the disposal of the statistical agencies are: nominal output in a sector, call it $P_{m,j} Q_{m,j}$, and a change in PPI, call it $\hat{P}_{m,j}$. Then:

$$\hat{Q}_{m,j} = \frac{1}{\hat{P}_{m,j}} \times \frac{P_{m,j} Q_{m,j}}{P_{m,j,-1} Q_{m,j,-1}}.$$

For inputs, the mechanics are the same, but we have to know the change in the input price deflator in every sector, call it $\hat{P}_{m,j}^M$. Then:

$$\hat{M}_{m,j} = \frac{1}{\hat{P}_{m,j}^M} \times \frac{P_{m,j}^M M_{m,j}}{P_{m,j,-1}^M M_{m,j,-1}}.$$

For the implementation inside our model, it is trivial to compute the sectoral nominal output and

input spending growth relative to pre-shock values:

$$\begin{aligned}\frac{P_{m,j}Q_{m,j}}{P_{m,j,-1}Q_{m,j,-1}} &= \frac{\sum_n \sum_{f \in \Omega_{mn,j}} X_{f,mn,j}}{\sum_n \sum_{f \in \Omega_{mn,j}} X_{f,mn,j,-1}} \\ \frac{P_{m,j}^M M_{m,j}}{P_{m,j,-1}^M M_{m,j,-1}} &= \frac{\sum_n \sum_{f \in \Omega_{mn,j}} \left(1 - \pi_{f,m,j}^l\right) X_{f,mn,j}}{\sum_n \sum_{f \in \Omega_{mn,j}} \left(1 - \pi_{f,m,j,-1}^l\right) X_{f,mn,j,-1}}.\end{aligned}$$

For price indices, in best practice of the statistical agencies, $\hat{P}_{m,j}$ is just the PPI change. There is some heterogeneity across countries in whether the PPI includes export prices or not. For us, PPI will include exports, and will be computed as

$$\hat{P}_{m,j} = \sum_n \sum_{f \in \Omega_{mn,j}} \omega_{f,mn,j,-1}^j \hat{P}_{f,mn,j}, \quad (\text{B.2})$$

where $\omega_{f,mn,j,-1}^j \equiv \frac{X_{f,mn,j,-1}}{\sum_n \sum_{f \in \Omega_{mn,j}} X_{f,mn,j,-1}}$ is the gross output weight of the firm's sales to n in sector j sales. Note that this is more comprehensive than what is actually done in practice, as the PPI is a survey that catches the minority of firms, and thus implementing (B.2) amounts to using more data than the statistical agencies do.

To construct the input price deflator $\hat{P}_{m,j}^M$, the statistical agencies use the PPI and the IO tables. We mimic this procedure by computing the input-share weighted change in input prices, where we use the PPI for the domestic inputs, and the foreign sectoral price changes for foreign inputs. The important thing is that we carry this out at the sector level, without using any firm-level information:

$$\hat{P}_{m,j}^M = \sum_i \sum_k \pi_{km,ij,-1}^M \hat{P}_{k,i}.$$

The $\pi_{km,ij,-1}^M$'s are the input shares coming from the WIOD. For the domestic components of the right-hand side of this expression, the $\hat{P}_{k,i}$ are just the PPI's we have in (B.2). For the foreign components, we assume that the foreign import prices are measured correctly, and use the import price indices from a particular country and sector, called $\hat{P}_{mn,j}$ in the main text.

Now we have all the ingredients to compute the real GDP change (B.1). Since the GDP deflator is defined implicitly as the ratio between the nominal and real GDP change, we need to compute the nominal GDP change. The nominal GDP change is a weighted sum of all firms' nominal value added changes. In particular, in our framework nominal value added associated with firm f 's sales to market n is a constant fraction of its sales there:

$$Y_{f,mn,j}^{NOM} = \frac{1 + \pi_{f,m,j}^l(\rho - 1)}{\rho} X_{f,mn,j},$$

and thus total firm value added is given by:

$$Y_{f,m,j}^{NOM} = \frac{1 + \pi_{f,m,j}^l(\rho - 1)}{\rho} \sum_n X_{f,mn,j}.$$

Nominal GDP is simply the sum over all firm-level value added, as in (2). The change in GDP is:

$$\widehat{Y}_m^{NOM} = \sum_f \sum_n \omega_{f,m,j,-1} s_{f,mn,j,-1} \widehat{X}_{f,mn,j}, \quad (\text{B.3})$$

where, as in Section 2, $\omega_{f,m,j,-1}$ is the pre-shock share of firm f 's value added in total GDP, and $s_{f,mn,j,-1}$ is the pre-shock share of sales to n in firm f 's total gross sales.

Finally, the GDP deflator is defined implicitly as the ratio of nominal and real GDP:

$$\widehat{P}_m^{GDP} = \frac{\widehat{Y}_m^{NOM}}{\widehat{Y}_m}.$$

B.2 A Shock Formulation of the Model

To perform counterfactuals that simulate the impact of foreign shocks on domestic firms and the aggregate economy, we follow the approach of Dekle et al. (2008) and express the equilibrium conditions in terms of gross changes $\widehat{x} = x/x_{-1}$ in endogenous variables, to be solved for as a function of shocks expressed in gross changes, and the pre-shock (“−1”) observables. Starting with (8), we write it as a function of observed expenditure shares:

$$\begin{aligned} X_{mn,j} = & \pi_{mn,j}^c \pi_{n,j}^c \left[w_n \left(\frac{1}{\psi_0} \frac{w_n}{P_n} \right)^{\frac{1}{\psi-1}} \bar{L}_n + \Pi_n + D_n \right] \\ & + \sum_i \frac{\rho-1}{\rho} \sum_{f \in i} (1 - \pi_{f,n,i}^l) \pi_{f,mn,ji}^M \sum_k \pi_{f,nk,i} X_{nk,i}, \end{aligned} \quad (\text{B.4})$$

where $\pi_{mn,j}^c$ is the share of final consumption spending on goods from m in the total consumption spending on goods in sector j , country n , $\pi_{n,j}^c = \vartheta_{n,j}$ is simply the share of sector j in total final consumption spending, and $\pi_{f,nk,i}$ is the share of firm f in the total exports from country n to country k in sector i . All of these π 's are observable when $n = \text{France}$. $\pi_{mn,j}^c$ and $\pi_{n,j}^c$ are observable in WIOD. $\pi_{f,nk,i}$ when neither n nor k are France is not observable, so would require an assumption on which firms use imported intermediates. Since we do not have firm-level information on other countries, we assume that in those countries there is a representative firm in each sector. Writing out the shares:

$$\begin{aligned} \pi_{n,j}^c &= \vartheta_{n,j}, \\ \pi_{mn,j}^c &= \frac{\mu_{mn,j} P_{mn,j}^{1-\sigma}}{\sum_k \mu_{kn,j} P_{kn,j}^{1-\sigma}}, \\ \pi_{f,nk,i} &= \frac{\xi_{f,nk,i} \left(\frac{\rho}{\rho-1} \frac{\tau_{nk,i} b_{f,n,i}}{a_f} \right)^{1-\rho}}{P_{nk,i}^{1-\rho}}. \end{aligned}$$

Then, in proportional changes relative to pre-shock values, (B.4) can be written as:

$$\begin{aligned} \widehat{X}_{mn,j} X_{mn,j,-1} &= \pi_{mn,j}^c \pi_{n,j}^c \left[\widehat{w}_n \left(\frac{\widehat{w}_n}{\widehat{P}_n} \right)^{\frac{1}{\psi-1}} s_{n,-1}^L + \widehat{\Pi}_n s_{n,-1}^\Pi + \widehat{D}_n s_{n,-1}^D \right] P_{n,-1} C_{n,-1} \\ &+ \sum_i \frac{\rho-1}{\rho} \sum_{f \in i} (1 - \pi_{f,n,i}^l) \pi_{f,mn,ji}^M \sum_k \pi_{f,nk,i} \widehat{X}_{nk,i} X_{nk,i,-1}, \end{aligned} \quad (\text{B.5})$$

where $s_{n,-1}^L$ is the pre-shock share of labor (more generally factor payments) in the total final consumption expenditure, and the same for $s_{n,-1}^\Pi$ and $s_{n,-1}^D$.

Equation (10) is expressed in changes as:

$$\sum_j \sum_{f \in j} \sum_k \frac{\rho-1}{\rho} \pi_{f,n,j,-1}^l \pi_{f,nk,j,-1} X_{nk,j,-1} \left[\widehat{\pi}_{f,n,j}^l \widehat{\pi}_{f,nk,j} \widehat{X}_{nk,j} - \widehat{w}_n^{\frac{\psi}{\psi-1}} \widehat{P}_n^{\frac{1}{1-\psi}} \right] = 0. \quad (\text{B.6})$$

The prices (9) are expressed in changes as:

$$\widehat{P}_{mn,j} = \left[\sum_{f \in \Omega_{mn,j}} \pi_{f,mn,j,-1} \widehat{\xi}_{f,mn,j} \left(\widehat{b}_{f,m,j} \widehat{a}_f^{-1} \right)^{1-\rho} \right]^{\frac{1}{1-\rho}}, \quad (\text{B.7})$$

$$\widehat{P}_{n,j} = \left[\sum_m \widehat{P}_{mn,j}^{1-\sigma} \pi_{mn,j,-1}^c \right]^{\frac{1}{1-\sigma}}, \quad (\text{B.8})$$

$$\widehat{P}_n = \prod_j \widehat{P}_{n,j}^\theta. \quad (\text{B.9})$$

Finally, the equations above require knowing post-shock π 's. These can be expressed as:

$$\pi_{mn,j}^c = \frac{\widehat{P}_{mn,j}^{1-\sigma} \pi_{mn,j,-1}^c}{\sum_k \widehat{P}_{kn,j}^{1-\sigma} \pi_{kn,j,-1}^c}, \quad (\text{B.10})$$

$$\pi_{f,nk,j} = \frac{\widehat{\xi}_{f,nk,j} \left(\widehat{b}_{f,n,j} \widehat{a}_f^{-1} \right)^{1-\rho} \pi_{f,nk,j,-1}}{\sum_{g \in \Omega_{nk,j}} \widehat{\xi}_{g,nk,j} \left(\widehat{b}_{g,n,j} \widehat{a}_g^{-1} \right)^{1-\rho} \pi_{g,nk,j,-1}}, \quad (\text{B.11})$$

$$\widehat{b}_{f,m,j} = \left[\pi_{f,m,j,-1}^l \widehat{w}_m^{1-\phi} + (1 - \pi_{f,m,j,-1}^l) \left(\widehat{P}_{f,m,j}^M \right)^{1-\phi} \right]^{\frac{1}{1-\phi}}, \quad (\text{B.12})$$

$$\widehat{P}_{f,m,j}^M = \left[\sum_i \sum_k \pi_{f,km,ij,-1}^M \widehat{P}_{km,i}^{1-\eta} \right]^{\frac{1}{1-\eta}}, \quad (\text{B.13})$$

$$\pi_{f,m,j}^l = \frac{\pi_{f,m,j,-1}^l \widehat{w}_m^{1-\phi}}{\pi_{f,m,j,-1}^l \widehat{w}_m^{1-\phi} + (1 - \pi_{f,m,j,-1}^l) \left(\widehat{P}_{f,m,j}^M \right)^{1-\phi}}, \quad (\text{B.14})$$

$$\pi_{f,km,ij}^M = \frac{\pi_{f,km,ij,-1}^M \widehat{P}_{km,i}^{1-\eta}}{\sum_i \sum_n \pi_{f,nm,ij,-1}^M \widehat{P}_{nm,i}^{1-\eta}}. \quad (\text{B.15})$$

B.2.1 Model Solution and Calibration

The model implementation involves solving equations (B.5)-(B.15). In particular, we solve for the following equilibrium variables:

1. Changes in trade values $\widehat{X}_{mn,j} \forall m, n, j$;
2. Changes in wages $\widehat{w}_n \forall n$;
3. Changes in the price indices $\widehat{P}_n \forall n, \widehat{P}_{n,j} \forall n, j, \widehat{P}_{mn,j} \forall m, n, j$;
4. Post-shock trade shares $\pi_{mn,j}^c \forall m, n, j, \pi_{f,nk,j} \forall k, n, j, f, \pi_{f,n,j}^l \forall n, j, f, \pi_{f,mn,ij}^M \forall n, m, i, j, f$.

We further require several pre-shock data series, either at the firm or sector level. Specifically, we require information on:

1. Gross sales $X_{mn,j,-1} \forall m, n, j$;
2. Final consumption shares within a sector across sources $\pi_{mn,j,-1}^c \forall m, n, j$;
3. Firm-level within sector, within-destination trade shares $\pi_{f,nk,j,-1} \forall k, n, j, f$;
4. Final consumption spending $P_{n,-1}C_{n,-1}$;
5. Shares of labor (factor) income, pure profits, and deficits in final consumption spending $s_{n,-1}^L, s_{n,-1}^\Pi$ and $s_{n,-1}^D \forall n$;
6. Initial input shares $\pi_{f,n,j,-1}^l \forall n, j, f, \pi_{f,mn,ij,-1}^M \forall m, n, i, j, f$.

The construction of these variables and the relevant data sources are described in [Appendix A](#). The solution of the model further requires setting a small number of parameter values. These are summarized in [Table 2](#).

B.2.2 Satisfying Market Clearing

In order to proceed correctly with the hat algebra in each sector/country pair, in the pre-period the market clearing condition in levels must be satisfied:

$$X_{mn,j,-1} = \pi_{mn,j,-1}^c \pi_{n,j,-1}^c P_{n,-1} C_{n,-1} + \sum_i \frac{\rho-1}{\rho} \sum_{f \in i} (1 - \pi_{f,n,i,-1}^l) \pi_{f,mn,ji,-1}^M \sum_k \pi_{f,nk,i,-1} X_{nk,i,-1}. \quad (\text{B.16})$$

In the data, this is unlikely to be the case. We therefore adopt the following approach: in each mn, j , trivially we can find a wedge $\zeta_{mn,j,-1}$ such that conditional on all the other data, (B.16) does hold with equality:

$$X_{mn,j,-1} = \pi_{mn,j,-1}^c \pi_{n,j,-1}^c P_{n,-1} C_{n,-1} + \sum_i \frac{\rho-1}{\rho} \sum_{f \in i} (1 - \pi_{f,n,i,-1}^l) \pi_{f,mn,ji,-1}^M \sum_k \pi_{f,nk,i,-1} X_{nk,i,-1} + \zeta_{mn,j,-1}.$$

Then applying the hat algebra to this equation:

$$\begin{aligned} \widehat{X}_{mn,j} X_{mn,j,-1} &= \pi_{mn,j}^c \pi_{n,j}^c \left[\widehat{w}_n \left(\frac{\widehat{w}_n}{\widehat{P}_n} \right)^{\frac{1}{\psi-1}} s_{n,-1}^L + \widehat{\Pi}_n s_{n,-1}^\Pi + \widehat{D}_n s_{n,-1}^D \right] P_{n,-1} C_{n,-1} \\ &+ \sum_i \frac{\rho-1}{\rho} \sum_{f \in i} (1 - \pi_{f,n,i}^l) \pi_{f,mn,ji}^M \sum_k \pi_{f,nk,i} \widehat{X}_{nk,i} X_{nk,i,-1} \\ &+ \widehat{\zeta}_{mn,j} \zeta_{mn,j,-1}. \end{aligned} \quad (\text{B.17})$$

Next, we solve the entire model while feeding in a “shock” that eliminates this wedge, namely: $\widehat{\zeta}_{mn,j} = 0$. Finding the model solution will give the a set of $\widehat{X}_{mn,j}$ ’s that are required to arrive at a set of levels of $X_{mn,j}$ for which the market clearing condition is satisfied with equality for every mn, j . Then use these $X_{mn,j}$ as the starting (pre-shock) values for all the real counterfactuals we run. The antecedent of this approach is [Costinot and Rodríguez-Clare \(2014\)](#), who use a similar device to eliminate the trade deficits.

B.3 Simulating Actual Foreign GDP Growth

In any year in the data, there will be a vector of country-specific productivity shocks. Let $\epsilon_{f,n} \equiv d \ln Y_f^F / d \ln a_n$ denote the elasticity of value added of firm f to a productivity shock in country n (we drop the m subscript from $d \ln Y_f^F$, as it is understood that firm f is always in France). We obtain these elasticities for every firm in France and every partner country by simulating country-specific aggregate productivity shocks $d \ln a_n$ in the baseline model, and recording each firm’s responses to it. Firm f ’s real value added growth rate following a vector of foreign shocks is

$$d \ln Y_f^F = \sum_n \epsilon_{f,n} d \ln a_n. \quad (\text{B.18})$$

Then the change in French GDP due to a worldwide vector of foreign shocks is simply:

$$d \ln Y_m^F = \sum_f \omega_{f,m,-1} d \ln Y_f^F. \quad (\text{B.19})$$

We implement (B.18)-(B.19) in two ways. The first approach feeds the aggregate TFP shocks from the Penn World Tables directly into (B.18) to compute each firm’s response to those foreign TFP shocks. The second approach uses actual GDP growth rates. To compute the propagation of foreign GDP growth rates into France, we re-express (B.18) directly in terms of elasticities of French firms to foreign GDP. Specifically, instead of (B.18) we assume that firm growth rate following a country-specific shock is:

$$d \ln Y_f^F = \sum_n \tilde{\epsilon}_{f,n} d \ln Y_n, \quad (\text{B.20})$$

where $\tilde{\epsilon}_{f,n} \equiv d \ln Y_f^F / d \ln Y_n$ is the elasticity of firm f ’s value added growth to country n ’s GDP, rather than the TFP shock directly. The $\tilde{\epsilon}_{f,n}$ ’s can be computed by simulating a country-specific

shock and tracking the response of both firm f and the foreign country's GDP. Equation (B.20) is then combined with (B.19) to compute French GDP growth. Once we simulate the firm and aggregate growth rates due to actual changes in foreign TFP and GDP for a sample of years, we can compute the average-granular residual decomposition (1).

Note that implementing (B.18)-(B.19)-(B.20) amounts to the first-order approach, where firm and aggregate responses are linear functions of the vector of foreign shocks. Huo et al. (2020) analyze the properties of the linear solution in a similar environment, and show that the first-order solution is very close to the exact one.

B.4 Proof of Proposition 1

Since all firms have the same production function, their initial labor shares $\pi_{n,i,-1}^l$ and input shares $\pi_{mn,ji,-1}^M$ are identical within a sector. All firms face the same effective intermediate input price change: $\widehat{P}_{f,m,j}^M = \widehat{P}_{m,j}^M \forall f$ (see (B.13)). Then, it is immediate from (B.14) and (B.15) that the post-shock labor and input shares $\pi_{n,i}^l$ and $\pi_{mn,ji}^M$ are also identical within a sector. The market clearing condition (B.5) becomes:

$$\begin{aligned} \widehat{X}_{mn,j} X_{mn,j,-1} &= \pi_{mn,j}^c \pi_{n,j}^c \left[\widehat{w}_n \left(\frac{\widehat{w}_n}{\widehat{P}_n} \right)^{\frac{1}{\psi-1}} s_{n,-1}^L + \widehat{\Pi}_n s_{n,-1}^\Pi + \widehat{D}_n s_{n,-1}^D \right] P_{n,-1} C_{n,-1} \\ &\quad + \sum_i \frac{\rho-1}{\rho} (1 - \pi_{n,i}^l) \pi_{mn,ji}^M \sum_k \widehat{X}_{nk,i} X_{nk,i,-1}, \end{aligned}$$

and thus does not involve $\pi_{f,nk,j}$'s or $\pi_{f,nk,j,-1}$'s or any other firm-level objects.

Since all firms face the same input bundle cost change: $\widehat{b}_{f,m,j} = \widehat{b}_{m,j} \forall f$ (see (B.12)), the $\pi_{f,nk,j}$ updating Equation (B.11) becomes:

$$\begin{aligned} \pi_{f,nk,j} &= \frac{\widehat{\xi}_{f,nk,j} \left(\widehat{b}_{f,n,j} \widehat{a}_f^{-1} \right)^{1-\rho} \pi_{f,nk,j,-1}}{\sum_{g \in \Omega_{nk,j}} \widehat{\xi}_{g,nk,j} \left(\widehat{b}_{g,n,j} \widehat{a}_g^{-1} \right)^{1-\rho} \pi_{g,nk,j,-1}} \\ &= \frac{\pi_{f,nk,j,-1}}{\sum_{g \in \Omega_{nk,j}} \pi_{g,nk,j,-1}} \\ &= \pi_{f,nk,j,-1}, \end{aligned}$$

since taste and productivity shocks are not firm-specific, and the denominator sums to 1. Thus, sales shares are unchanged following a foreign shock: $\pi_{f,nk,j} = \pi_{f,nk,j,-1} \forall f, k$, or $\widehat{\pi}_{f,nk,j} = 1 \forall f, k$.

When labor shares $\pi_{n,i,-1}^l$ do not differ across firms, the labor market condition (B.6) also does not require firm-level shares, and simplifies to:

$$\sum_j \sum_k \frac{\rho-1}{\rho} \pi_{n,j,-1}^l X_{nk,j,-1} \left[\widehat{\pi}_{n,j}^l \widehat{X}_{nk,j} - \widehat{w}_n^{\frac{\bar{\psi}}{\psi-1}} \widehat{P}_n^{\frac{1}{1-\bar{\psi}}} \right] = 0,$$

which once again is independent of $\pi_{f,nk,j}$'s or $\pi_{f,nk,j,-1}$'s.

Finally, the price equation also has no $\pi_{f,nk,j}$ or $\pi_{f,nk,j,-1}$ terms if taste and productivity shocks are not firm-specific:

$$\begin{aligned}\widehat{P}_{mn,j} &= \left[\sum_{f \in \Omega_{mn,j}} \pi_{f,mn,j,-1} \widehat{b}_{m,j}^{1-\rho} \right]^{\frac{1}{1-\rho}} \\ &= \widehat{b}_{m,j}\end{aligned}$$

These equations define the equilibrium in changes, and thus $\widehat{X}_{mn,j}$'s and $\widehat{P}_{mn,j}$'s can be found without knowing the firm-level market shares $\pi_{f,nk,j}$'s or $\pi_{f,nk,j,-1}$'s.

Since markups are constant, all the firm-specific prices change by the same proportional amount: $\widehat{p}_{f,mn,j} = \widehat{p}_{mn,j} \forall f$. Because $\widehat{\pi}_{f,mn,j} = 1 \forall f, n$, all nominal sales changes are the same across firms within a sector: $\widehat{X}_{f,mn,j} = \widehat{X}_{mn,j}$. Therefore, none of the steps in constructing real GDP in [Appendix B.1](#) require firm-level sales shares. \square

Table A1. Summary Statistics of Firms, by Sector

WIOD sector	# firms	Share VA	Traded/ non-traded
Agriculture, Hunting, Forestry, Fishing	7,718	.0067	T
Mining, Quarrying	1,022	.0041	T
Food, Beverages, Tobacco	10,883	.0354	T
Textile Products	1,684	.0039	T
Leather, Footwear	2,501	.0058	T
Wood Products	3,045	.0044	T
Pulp, Paper, Publishing	7,721	.0202	T
Coke, Refined Petroleum, Nuclear Fuel	50	.0056	T
Chemical Products	2,051	.0358	T
Rubber and Plastics	2,992	.0155	T
Other Non-Metallic Minerals	2,607	.0127	T
Basic and Fabricated Metals	14,561	.0373	T
Machinery n.e.c.	6,442	.0243	T
Electrical, Optical Equipment	6,599	.0288	T
Transport Equipment	1,804	.0315	T
Manufacturing n.e.c.	4,946	.0086	T
Electricity, Gas, Water Supply	321	.0364	NT
Construction	54,428	.0664	NT
Wholesale and Retail Motor Vehicles and Fuel	25,975	.0218	NT
Wholesale Trade	49,166	.0867	NT
Retail Trade	76,069	.0739	NT
Hotels and restaurants	29,135	.0259	NT
Inland Transport	9,244	.0401	NT
Water Transport	171	.0017	NT
Air Transport	66	.0085	NT
Other Transport Activities	2,068	.0256	NT
Post and Telecommunications	276	.0488	NT
Real Estate	7,726	.0425	NT
Business Activities	31,605	.1849	NT
Education	1,569	.0037	NT
Health and Social Work	6,200	.0200	NT
Other Personal Services	15,283	.0324	NT
Total	385,928	1.000	

Notes: This table reports summary statistics on the numbers of firms and shares of aggregate value added by WIOD sector. The data are from INSEE-Ficus/Fare and correspond to year 2005.

Table A2. Responses of French Real GDP to 10% Foreign Productivity and Demand Shocks, CPI Deflation

	$d \ln Y^F$	\mathcal{E}^F	Γ^F	$d \ln Y^F$	\mathcal{E}^F	Γ^F
Shock:	Productivity			Demand		
Baseline	6.34	3.99	2.35	0.49	0.34	0.15
<i>Share:</i>		<i>0.630</i>	<i>0.370</i>		<i>0.691</i>	<i>0.309</i>
Homogeneous firms	7.08	7.02	0.06	0.52	0.53	-0.01
<i>Share:</i>		<i>0.992</i>	<i>0.008</i>		<i>1.018</i>	<i>-0.018</i>
Sector-Level Decomposition						
	$d \ln Y^F$	$\mathcal{E}_{\mathcal{J}}^F$	$\Gamma_{\mathcal{J}}^F$	$d \ln Y^F$	$\mathcal{E}_{\mathcal{J}}^F$	$\Gamma_{\mathcal{J}}^F$
Baseline	6.34	5.71	0.62	0.49	0.74	-0.25
<i>Share:</i>		<i>0.901</i>	<i>0.099</i>		<i>1.504</i>	<i>-0.504</i>

Notes: This table reports the change in French GDP, in percentage points, following a 10% productivity shock (left panel) or a 10% foreign demand shock for French goods (right panel) in every other country in the world, in both the baseline model and the alternative model that suppresses firm heterogeneity. The table reports the decomposition of the the GDP change into the unweighted average and granular residual terms as in (1). The real GDP is obtained by deflating by CPI.

Table A3. Robustness: GDP Responses in the Baseline vs. Homogeneous Models

Shock:	Productivity			Demand		
	Baseline	Homogeneous	Ratio H/B	Baseline	Homogeneous	Ratio H/B
Main calibration	2.66	3.13	1.18	0.35	0.37	1.06
ρ :						
high: 5	1.74	2.80	1.61	0.29	0.34	1.16
low: 1.5	3.53	3.68	1.04	0.44	0.45	1.02
Frisch:						
high: 2	14.22	15.57	1.09	0.58	0.62	1.06
low: 0.1	0.80	1.08	1.35	0.29	0.31	1.06
η :						
high: 1.5	2.18	2.48	1.14	0.51	0.53	1.04
low: 0.5	3.31	4.00	1.21	0.16	0.17	1.09
ϕ :						
high: 1.5	2.76	3.35	1.21	0.36	0.38	1.07
low: 0.5	2.51	2.85	1.14	0.35	0.37	1.06
σ :						
high: 3	0.40	0.63	1.60	0.97	1.00	1.04
low: 1.1	3.56	4.15	1.17	0.11	0.12	1.10
Flexible markups	2.77	3.13	1.13	0.36	0.37	1.03
Changing profits in final demand	3.14	3.50	1.11	0.37	0.39	1.07

Notes: This table reports the change in French GDP, in percentage points, following a 10% productivity shock (left panel) or a 10% foreign demand shock for French goods (right panel) in every other country in the world, both in the baseline model and the alternative model that suppresses firm heterogeneity, for alternative parameter values.