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## Sensitivity to Calibrated Parameters

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# Sensitivity to Calibrated Parameters

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**cemmap** working paper  
CWP16/20

# Sensitivity to Calibrated Parameters\*

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## Abstract

Across many fields in economics, a common approach to estimation of economic models is to calibrate a sub-set of model parameters and keep them fixed when estimating the remaining parameters. Calibrated parameters likely affect conclusions based on the model but estimation time often makes a systematic investigation of the sensitivity to calibrated parameters infeasible. I propose a simple and computationally low-cost measure of the sensitivity of parameters and other objects of interest to the calibrated parameters. In the main empirical application, I revisit the analysis of life-cycle savings motives in [Gourinchas and Parker \(2002\)](#) and show that some estimates are sensitive to calibrations.

(JEL: C10, C52, C60)

**Keywords:** Sensitivity, Transparency, Structural Estimation, Calibration, Savings Motives.

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# 1 Introduction

Estimated dynamic economic models are now widely used across all fields of economics. The estimation of structural models is, however, notoriously time consuming and the estimation time increases drastically with the number of estimated parameters. A common approach to alleviate this computational burden is to calibrate a sub-set of the model parameters and keep them fixed while estimating the remaining parameters of interest.<sup>1</sup> The calibrated parameter values are often based on external sources such as previously published parameter estimates and will generally influence conclusions drawn from the estimated model.<sup>2</sup> Unfortunately, a systematic investigation of the sensitivity to calibrated parameters is often infeasible. If sensitivity is investigated, the current practice is to report results from a few re-estimated versions of the model. This approach is, however, generally very time consuming and the number of alternative calibrations thus typically low, ultimately reducing research transparency. In this paper, I propose a complementary approach that can greatly improve transparency of structural research.

I propose a low-cost measure of the sensitivity of any quantity of interest to the calibrated parameters. The sensitivity measure can often be calculated with little additional programming and without significant computational cost since it avoids re-estimation of the model parameters. The measure has a straightforward interpretation as the effect from a marginal change in the calibrated parameters and can e.g. be used to construct elasticities. Like most existing types of sensitivity and robustness analyses, the proposed measure is thus local. I find the encouraging result, however, that the measure provides a quite good approximation to even larger changes in the calibrated parameters in my main empirical application.

The sensitivity measure is based on the General Method of Moments (GMM) estimation framework, which includes most commonly used estimators. I rely on standard asymptotic theory and utilize that this type of estimator has an asymptotic linear representation. Importantly, I impose no restrictions on the calibrated parameters and they can e.g. be on the boundary of the parameter space. In the

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<sup>1</sup> See e.g. [Gourinchas and Parker \(2002\)](#); [Scholz, Seshadri and Khitrakun \(2006\)](#); [Cagetti and De Nardi \(2006\)](#); [De Nardi, French and Jones \(2010\)](#); [French and Jones \(2011\)](#); [Blundell, Dias, Meghir and Shaw \(2016\)](#); [Berger and Vavra \(2015\)](#); [Chiappori, Dias and Meghir \(forthcoming\)](#); [Huo and Ríos-Rull \(forthcoming\)](#) for a small sample of studies.

<sup>2</sup> See e.g. [Gourinchas and Parker \(2002\)](#); [Scholz, Seshadri and Khitrakun \(2006\)](#); [Cagetti and De Nardi \(2006\)](#); [De Nardi, French and Jones \(2010\)](#); [French and Jones \(2011\)](#); [Blundell, Dias, Meghir and Shaw \(2016\)](#); [Berger and Vavra \(2015\)](#); [Chiappori, Dias and Meghir \(forthcoming\)](#); [Huo and Ríos-Rull \(forthcoming\)](#) for a small sample of studies.

typical case in which calibrated parameters are assumed fixed at the true population values or are consistent estimators of the population values, the sensitivity measure is particularly simple to calculate.

The measure can be used to reduce the sensitivity to calibrations. The GMM framework weights different moments through a weighting matrix chosen by the researcher and the sensitivity measure thus depends on this weighting matrix. I discuss how to implement an “optimal” weighting matrix that e.g. minimizes parameter sensitivity to calibrations. Using the “optimal” weighting matrix will thus reduce sensitivity to calibrated parameters by potentially trading off efficiency.

The sensitivity of estimated parameters or other quantities of interest can be calculated straightforwardly. The objects of primary interest are often some functions of the parameters rather than the parameter estimates themselves. Such objects of interest could for example be outcomes from counter factual policy reforms, optimal policy design or welfare measures. The proposed measure can quantify the sensitivity of such model-based results to calibrations. In previous research – if the sensitivity of such results are investigated – the most common approach is to re-calculate the objects of interest from a change in the calibrated parameters while the estimated parameters remain fixed at their originally estimated values. This current approach completely ignores the effect on the estimated parameters and can produce misleading results. The sensitivity measure I propose takes the effect on the estimated parameters into account when approximating the effect from changing calibrations, without costly re-estimations of the model.

Generalizations of the estimated model can be investigated without estimating the richer and more complex model. A concrete application of the sensitivity measure is to investigate how sensitive results are to a particular generalization of the estimated model. For example, imagine that  $\sigma \geq 0$  measures the degree of heterogeneity in some model parameter of interest. Imagine having estimated a restricted version of the model *without* unobserved heterogeneity, i.e. with  $\sigma = 0$  fixed. The sensitivity measure can be used to assess the sensitivity of results to increasing  $\sigma$  *without* having to re-estimate the more computationally time demanding model with unobserved heterogeneity. The sensitivity measure thus increases transparency from the point of the reader but can also guide the research process itself.

The sensitivity measure can greatly improve transparency of work in many fields of economic research. I illustrate the usefulness of the approach through an application to importance of different savings motives over the life cycle, as studied in the seminal work by [Gourinchas and Parker \(2002\)](#). I show how the estimation results are especially sensitive to the calibrated value of the risk-free interest rate, a param-

eter not considered in their original robustness analysis. I compare the proposed sensitivity measures to a brute force re-estimation and find very encouraging results in the sense that the elasticities are close to the “true” percentage changes from changes in the calibrated parameters. This is also true for even larger changes in the risk-free interest rate although the sensitivity measure is based on a local linear approximation. Interestingly, I find that their main result that buffer-stock saving is the dominating savings motive early in working life while retirement saving or life cycle motives are more important later in the working life is insensitive to the calibrated parameters. A result that is also confirmed by a brute-force approach.

Finally, I also apply the sensitivity measure to recent research on home-ownership and the option value of migration in [Oswald \(2019\)](#). I find that the willingness to pay for the insurance value of migration against *regional* shocks are sensitive to calibrated parameters related to *idiosyncratic* uncertainty. This example shows the value of the sensitivity measure because the richness of the dynamic model prohibits many re-estimations. The sensitivity measure thus increases the transparency of such research significantly.

## 1.1 Existing Literature and Roadmap

The literature on sensitivity in economics is growing. Especially the sensitivity of estimators to the included moments in GMM-type estimators has received recent attention, see e.g. [Kitamura, Otsu and Evdokimov \(2013\)](#) and [Andrews, Gentzkow and Shapiro \(2017, 2018\)](#). To my knowledge, however, the only paper investigating the sensitivity of structural estimators to calibrated parameters is the recent paper by [Iskrev \(2019\)](#). That paper focuses on Bayesian approaches to estimation of macroeconomic models. I propose a measure based on a more commonly used type of estimator, derived from the minimization of a quadratic criterion, in a frequentist framework. Importantly, the approach in [Iskrev \(2019\)](#) requires identification of all (both calibrated and estimated) model parameters simultaneously and the availability of the covariance between the two sets of parameters. This e.g. precludes the use of external sources for calibration, which is a very common approach.<sup>3</sup> Another related study is by [Chernozhukov, Escanciano, Ichimura, Newey and Robins \(2018\)](#) who provide a setup for construction of *locally robust* moments that are *orthogonal* to the calibrated parameters. Their approach can completely eliminate sensitivity to calibrated parameters but relies on the availability of estimation information re-

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<sup>3</sup> A worst-case upper bound on the co-variance structure could be estimated following the approach suggested in [Cocci and Plagborg-Møller \(2019\)](#), however.

garding the calibrated parameters. Again, this would preclude calibration based on external sources.

Other recent related research includes [Bonhomme and Weidner \(2018\)](#); [Christensen and Connault \(2019\)](#); [Honoré, Jørgensen and de Paula \(2019\)](#); [Armstrong and Kolesár \(2019\)](#) and [Harenberg, Marelli, Sudret and Winschel \(2019\)](#). [Bonhomme and Weidner \(2018\)](#) study the sensitivity to model miss-specification and provide estimators to minimize the effect on quantities of interest. [Christensen and Connault \(2019\)](#) study counterfactual sensitivity to assumptions about unobserved heterogeneity. [Honoré, Jørgensen and de Paula \(2019\)](#) propose measures related to the effect on inference from e.g. changing the weighting put on moments in estimation. [Armstrong and Kolesár \(2019\)](#) study the sensitivity to moments included in estimation and proposes optimal weights that can reduce the sensitivity to included moments. While all these measures are local, [Harenberg, Marelli, Sudret and Winschel \(2019\)](#) suggest constructing a global approximation of the object of interest to reduce the computational time required for *global* sensitivity analysis. This approach has great potential for use in the context of structural estimation of dynamic economic models but still requires a significant number of re-estimations of the model. The local measure, I propose, could be used in junction with global methods as a low-cost approach to guide the placement of evaluation nodes in the approach proposed in [Harenberg, Marelli, Sudret and Winschel \(2019\)](#) to further reduce the time required to implement such an approach.

In engineering and operations research, sensitivity and uncertainty quantification of model outputs to model inputs have received substantial attention. Some measures in this literature (see e.g. [Borgonovo and Apostolakis, 2001](#)) bears resemblance to the one, I propose, but in completely different contexts. There is also a growing focus in this literature on global measures of sensitivity, see e.g. [Borgonovo and Plischke \(2016\)](#) for a recent review.

The remainder of the paper is organized as follows. In the following section, I specify the estimation framework and define the sensitivity measure. In Section 3, I discuss how the proposed measure relates to some recent sensitivity measures. In Section 4, I apply the measure to an empirical analysis of the relative importance of alternative savings motives over the life cycle. In section 5, I apply the approach to a rich model of home-ownership and migration before concluding in Section 6. Python code generating all results in this paper is available from the author's web-page.

## 2 Framework and Sensitivity

I focus on situations in which interest lies in estimating a  $K \times 1$  vector of parameters,  $\theta$ , given some  $L \times 1$  vector of calibrated parameters,  $\hat{\gamma}$ . Interest may then be in using these estimates to subsequently analyze different model outcomes and predictions. While the proposed sensitivity measure has general applicability, I will focus attention on estimation of dynamic economic models because such models typically are time-consuming to estimate.

I assume that the estimation approach employed is of the form

$$\hat{\theta} = \arg \min_{\theta \in \Theta} g_n(\theta | \hat{\gamma})' W_n g_n(\theta | \hat{\gamma})$$

where  $g_n(\theta | \hat{\gamma}) = \frac{1}{n} \sum_{i=1}^n f(\theta | \hat{\gamma}, \mathbf{w}_i)$  is some  $J \times 1$  vector valued function of the parameters and data,  $\mathbf{w}_i$  for  $i = 1, \dots, n$ , specified by the researcher.  $W_n$  is a  $J \times J$  positive semi-definite weighting matrix. When estimating dynamic economic models, evaluating  $g_n(\theta | \hat{\gamma})$  typically involves solving some model numerically. I assume that the objective function satisfies standard regularity conditions and abstract from any numerical approximation error associated with solving the model.<sup>4</sup> In particular, I assume that there exists unique population parameters  $\theta_0$  and  $\gamma_0$  such that  $g(\theta_0 | \gamma_0) \equiv \mathbb{E}[f(\theta_0 | \gamma_0, \mathbf{w}_i)] = 0$ .

This framework covers a wide range of frequently used approaches such as minimum distance, general method of moments (GMM), and maximum likelihood estimation (MLE). Simulated versions of such estimators, such as simulated minimum distance (SMD) and Indirect Inference ([Smith, 1993](#); [Gouriéroux, Monfort and Renault, 1993](#)) are also included in this class of estimators. The latter two approaches are often employed when estimating dynamic economic models.

I am interested in constructing a measure of how sensitive results derived from such an estimator are to the calibrated parameters. I will be more precise below but quantities of interest could be the estimator itself or some other function of both sets of parameters. By sensitivity I refer to questions of the form: How much does the quantity of interest change from a marginal change in the calibrated parameters?

Importantly, I do not study the effect of the uncertainty regarding the calibrated parameters on the inference on  $\hat{\theta}$ . This has been studied before in e.g. [Newey and McFadden \(1994\)](#). While such inference corrections are related, they generally require more structure on the calibration of  $\hat{\gamma}$ . In particular, I imagine situations

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<sup>4</sup> see e.g. [Newey and McFadden \(1994\)](#).

in which  $\hat{\gamma}$  could be estimated from different data-sources, by other researchers, or published in other papers. In other words, the estimation function and uncertainty related to  $\hat{\gamma}$  is likely not available to the researcher.

## 2.1 Current Practice to Sensitivity

In existing research the sensitivity to calibrated parameters is often not investigated, albeit the likely importance for subsequent results derived from the estimated model. If sensitivity to calibration is investigated the current practice is to report  $M$  estimation results,  $\{\tilde{\theta}_m\}_1^M$ , from  $M$  alternative calibrations,  $\{\tilde{\gamma}_m\}_1^M$ , in a robustness exercise where

$$\tilde{\theta}_m = \arg \min_{\theta \in \Theta} g_n(\theta | \tilde{\gamma}_m)' W_n g_n(\theta | \tilde{\gamma}_m).$$

While this approach has the potential to investigate alternative relevant calibrations, it is somewhat arbitrary and to a great extend rely on the researcher's priors on what might be important or interesting to investigate. If the reader has a different prior than the researcher on what the relevant alternatives to include in  $\tilde{\gamma}$  are, the reader cannot infer the implications of her own prior from the reported robustness results. I view this as a lack of transparency.

Importantly, the current practice involves re-estimating the model  $M$  times, which is potentially computationally time consuming if a dynamic economic model is solved every time the objective function is evaluated. This, in turn, often leads to a low value of  $M$ , reducing the transparency drastically. The severity of this limitation increases in the complexity of the underlying model and the dimensions of  $\theta$  and  $\gamma$ .<sup>5</sup> Paradoxically, the need for transparency is also increasing in the complexity and/or the number of parameters. This motivates the low-cost measure I propose below.

An example of the current practice is also found in the main application below in which I revisit the seminal work of [Gourinchas and Parker \(2002\)](#). Among other things, they investigate the sensitivity of their estimation results to changing the income shock variances and find that results are quite insensitive to these parameters. I confirm that finding using my proposed sensitivity measure. Interestingly, however, the calibrated parameter that I find to be most important (the risk-free interest rate) is not considered in their robustness exercise.

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<sup>5</sup> One approach could be to re-estimate the model for a changed value of each calibrated parameter at a time while keeping the remaining fixed. This would lead to  $L = \dim \gamma$  re-estimations of the model. Since  $L$  is typically large, this simple strategy quickly becomes prohibitively time consuming.

## 2.2 Sensitivity Measure

I propose a systematic sensitivity measure that can be calculated without significant additional computational cost. The interpretation of the measure is an approximation of the change in  $\theta$  from a marginal change in the calibrated parameters. The main sensitivity measure is local but I also discuss an alternative approach that can in principle be used to emulate the current practice discussed above without the computational burden of re-estimating the model for various  $\hat{\gamma}$ .

The sensitivity measure is motivated by a standard representation used in asymptotic derivations. In particular, I use that estimators within the current framework has an asymptotic linear form ([Newey and McFadden, 1994](#)), such that the estimator can be represented as

$$\hat{\theta}(\hat{\gamma}) = \theta_0 + \Lambda_n(\hat{\gamma})g_n(\theta_0|\hat{\gamma}) + o_p(n^{-\frac{1}{2}}) \quad (1)$$

where  $\Lambda_n(\hat{\gamma}) = -(G'_n W_n G_n)^{-1} G'_n W_n$  and  $G_n = \frac{\partial g_n(\theta|\hat{\gamma})}{\partial \theta'} \Big|_{\theta=\theta_0}$  is the  $J \times K$  Jacobian w.r.t. the estimated parameters. Although suppressed in the notation here,  $G_n$  depends on the calibrated parameters,  $\hat{\gamma}$ . Under fairly standard regularity conditions, similar in spirit to those employed in e.g. [Newey and McFadden \(1994\)](#) and [Andrews, Gentzkow and Shapiro \(2017\)](#), the estimator converges in probability to

$$\theta(\gamma) = \theta_0 + \Lambda(\gamma)g(\theta_0|\gamma) \quad (2)$$

where  $g(\theta_0|\gamma) \equiv \mathbb{E}[f(\theta_0|\gamma, \mathbf{w}_i)]$ ,  $\Lambda(\gamma) = -(G'WG)^{-1}G'W$  with  $G = \mathbb{E}\left[\frac{\partial f(\theta|\gamma, \mathbf{w}_i)}{\partial \theta'}\Big|_{\theta=\theta_0}\right]$  and  $\gamma \equiv \text{plim}_{n \rightarrow \infty} \hat{\gamma}$  is the probability limit of the calibrated parameters. This setup e.g. allows for cases in which  $\gamma \neq \gamma_0$  and thus  $g(\theta_0|\gamma) \neq 0$ .  $\hat{\theta}$  is a consistent estimator if  $g(\theta_0|\gamma) = 0$ , which is the case if the calibrated parameters are consistent estimators of the population values,  $\text{plim}_{n \rightarrow \infty} \hat{\gamma} = \gamma_0$ , or simply assumed fixed at their population values,  $\hat{\gamma} = \gamma_0$ .

Motivated by this formulation, I propose the sensitivity measure  $\frac{\partial \theta}{\partial \gamma} = (\frac{\partial \theta}{\partial \gamma_{(1)}}, \dots, \frac{\partial \theta}{\partial \gamma_{(L)}})$ . This is a  $K \times L$  Jacobian matrix with the derivative of  $\theta(\gamma)$  with respect to the  $l$ th element in  $\gamma$  being a  $K \times 1$  vector

$$\frac{\partial \theta}{\partial \gamma_{(l)}} = \frac{\partial \Lambda(\gamma)}{\partial \gamma_{(l)}} g(\theta_0|\gamma) + \Lambda(\gamma) \frac{\partial g(\theta_0|\gamma)}{\partial \gamma_{(l)}} \quad (3)$$

with

$$\begin{aligned}\frac{\partial \Lambda(\gamma)}{\partial \gamma_{(l)}} &= -(G'WG)^{-1}[\nabla_l'WG + G'W\nabla_l](G'WG)^{-1}G'W + (G'WG)^{-1}\nabla_l'W \\ &= -\Lambda\nabla_l\Lambda + (G'WG)^{-1}\nabla_l'W(I_{J \times J} + G\Lambda)\end{aligned}$$

being a  $K \times J$  matrix where  $\nabla_l \equiv \frac{\partial G}{\partial \gamma_{(l)}} = \mathbb{E} \left[ \frac{\partial^2 f(\theta|\gamma, \mathbf{w}_i)}{\partial \gamma_{(l)} \partial \theta'} \Big|_{\theta=\theta_0} \right]$  is a  $J \times K$  Jacobian.

The sensitivity measure can be calculated using eq. (3) above in principle without placing restrictions on the value of  $\hat{\gamma}$ . Since the asymptotic linear representation is an approximation around the true parameter,  $\theta_0$ , the approximation is most accurate close to  $\theta_0$ . The approximation might thus be less accurate for values of  $\hat{\gamma}$  far from  $\gamma_0$ . Under the frequently employed assumption that  $\gamma = \gamma_0$ , such that  $g(\theta_0|\gamma) = g(\theta_0|\gamma_0) = 0$ , the measure simplifies considerably.<sup>6</sup>

**Definition 1** (Sensitivity of estimated parameters). Under the assumption that  $g(\theta_0|\gamma) = 0$ , such that  $\hat{\theta}$  is a consistent estimator of  $\theta_0$ , the sensitivity of the estimated parameters to the calibrated parameters is the  $K \times L$  matrix

$$S = \Lambda D \tag{4}$$

where  $D = \mathbb{E} \left[ \frac{\partial f(\theta_0|\gamma, \mathbf{w}_i)}{\partial \gamma'} \right]$  is a  $J \times L$  Jacobian w.r.t. the calibrated parameters.

The sensitivity measure can be estimated at low cost by plugging in estimates of  $\hat{\theta}$  and  $\hat{\gamma}$  as

$$\hat{S}_n = \hat{\Lambda}_n \hat{D}_n. \tag{5}$$

where  $\hat{\Lambda}_n = -(\hat{G}_n'W_n\hat{G}_n)^{-1}\hat{G}_n'W_n$  with  $\hat{G}_n = \frac{\partial g_n(\theta|\hat{\gamma})}{\partial \theta'} \Big|_{\theta=\hat{\theta}}$  and  $\hat{D}_n = \frac{\partial g_n(\hat{\theta}|\hat{\gamma})}{\partial \hat{\gamma}'}$ . Importantly, all elements of  $\hat{\Lambda}_n$  are already constructed when calculating the asymptotic covariance matrix of  $\hat{\theta}$  and only  $\hat{D}_n$  needs to be calculated.<sup>7</sup> For example, if forward finite differences are used to construct  $\hat{D}_n$  numerically, calculating  $\hat{S}_n$  only requires  $L = \dim(\gamma)$  additional evaluations of the objective function. A brute force alternative approach to calculating  $\frac{\partial \theta}{\partial \gamma'}$  could be to re-estimate  $\theta$  for a small increase in each element in  $\gamma$  and calculate the change in the estimated  $\theta$ . This approach, however,

<sup>6</sup> The measure  $S$  in equation (2) is similar to that derived in Newey and McFadden (1994) but with the weight  $W$  included here. Newey and McFadden (1994) suggests this measure to determine if the asymptotic variance of a two-step estimator should be corrected for the uncertainty associated with the first-step estimator.

<sup>7</sup> In fact, if asymptotic standard errors are corrected for the two-step estimation approach, as in Gourinchas and Parker (2002), all elements of the sensitivity measure is already calculated.

requires  $L$  *re-estimations* of the model. The brute force approach is thus generally much more time consuming. In the main application below, I compare the proposed sensitivity measure to such a brute-force approach and find only minor differences.

The sensitivity measure has a straightforward interpretation and the elasticity of the  $k$ th estimated parameter to the  $l$ th calibrated parameter can be calculated as

$$\mathcal{E}_{(k,l)} = S_{(k,l)} \gamma_{(l)} / \theta_{(k)} \quad (6)$$

assuming that  $\gamma_{(l)}, \theta_{(k)} \neq 0$ .

**Example** (Linear Regression). Consider a simple linear regression model with two mean-zero explanatory variables,  $X_1$  and  $X_2$ , and measurement error,  $\varepsilon$ ,

$$Y_i = \beta_1 X_{1,i} + \beta_2 X_{2,i} + \varepsilon_i$$

where  $\mathbb{E}[\varepsilon|X_1, X_2] = 0$  is the identifying assumption. Imagine fixing the second parameter to  $\beta_2$  and only estimating  $\beta_1$ ,

$$\hat{\beta}_1 = \arg \min_{\beta_1} g_n(\beta_1|\beta_2)^2$$

with a single moment in  $g_n(\beta_1|\beta_2) = \frac{1}{n} \sum_{i=1}^n (Y_i - \beta_1 X_{1,i} - \beta_2 X_{2,i}) X_{1,i}$  and  $W = 1$ . This estimator can be found in closed form as

$$\hat{\beta}_1 = \frac{\sum_{i=1}^n X_{1,i} (Y_i - \beta_2 X_{2,i})}{\sum_{i=1}^n X_{1,i}^2}. \quad (7)$$

In this setting  $G_n = -\sum_{i=1}^n X_{1,i}^2$  and  $D_n = -\sum_{i=1}^n X_{1,i} X_{2,i}$  and the sensitivity measure is

$$S_n = -\frac{\sum_{i=1}^n X_{1,i} X_{2,i}}{\sum_{i=1}^n X_{1,i}^2}$$

which converges in probability to  $-\mathbb{E}[X_1 X_2] \cdot \mathbb{E}[X_2^2]^{-1}$ . This is the negated regression coefficient in a regression of  $X_2$  on  $X_1$  with the sample covariance between  $X_1$  and  $X_2$  in the nominator. We see the intuitive result that if they are positively (negatively) correlated, increasing  $\beta_2$  would lead to a reduced (increased)  $\hat{\beta}_1$ . We also see that if they are uncorrelated, the estimator of  $\beta_1$  is completely insensitive to  $\beta_2$ .

Because the linear asymptotic representation is exact in this example,  $S_n = \frac{\partial \hat{\beta}_1}{\partial \beta_2}$ . In general, however, such a direct derivative cannot be calculated in closed form and I thus propose to use the approximation instead.

## 2.3 Sensitivity of other Quantities of Interest

The primary interest is often not the parameter estimates themselves but rather some statistics based on the model. Concretely, denote the  $F \times 1$  vector of quantities of interest as  $h(\theta, \gamma) = \frac{1}{n} \sum_{i=1}^n h_i(\theta, \gamma | \mathbf{w}_i)$ . These statistics potentially involve simulations from variations of the estimated model. Examples include counter factual policy simulations and welfare measures. In turn, the reader might be mostly interested in the sensitivity of  $h(\bullet)$  to the calibrated parameters.

The proposed measure can easily be used to construct a sensitivity measure of such other quantities of interest, as shown in the definition below.

**Definition 2** (Sensitivity of quantities of interest). The sensitivity of a  $F \times 1$  vector of statistics,  $h(\theta, \gamma)$ , to the calibrated parameters is given by the  $F \times L$  matrix

$$H = A + BS \quad (8)$$

where  $A = \mathbb{E} \left[ \frac{\partial h_i(\theta_0, \gamma | \mathbf{w}_i)}{\partial \gamma'} \right]$  is a  $F \times L$  Jacobian matrix of  $h(\bullet)$  w.r.t.  $\gamma$  and  $B = \mathbb{E} \left[ \frac{\partial h_i(\theta, \gamma | \mathbf{w}_i)}{\partial \theta'} \Big|_{\theta=\theta_0} \right]$  is a  $F \times J$  Jacobian matrix of  $h(\bullet)$  w.r.t.  $\theta$ .

This generalizes the proposed measure above: If the object of interest is the estimated parameters,  $h(\theta, \gamma) = \theta$ , we have that  $F = K$ ,  $A = 0_{K \times 1}$  and  $B = 1_{K \times 1}$  such that  $H = S$ . Like above, the sensitivity measure can be estimated by plugging in the estimated values.

The derivatives  $A$  and  $B$  are often relatively fast to calculate numerically if not known in closed form. For illustrative purposes, assume that forward finite differences are used to construct all numerical derivatives. A naïve approach would require  $L$  evaluations of the objective function to calculate  $S$  and  $L + K$  evaluations of  $h(\bullet)$  to calculate  $A$  and  $B$ . If a dynamic economic model is solved once per evaluation of  $g(\bullet)$  and  $h(\bullet)$ , the model is solved  $2L + K$  additional times to calculate the sensitivity measure of any quantity of interest. If, however, the model solutions are stored, the additional number of model solutions to calculate is only  $L$ .

An alternative approach to calculating  $H$  could be to calculate the numerical derivative of  $h(\bullet)$  by brute force through re-estimating the model for small changes in  $\gamma$ . This would, however, require  $L$  *re-estimations* of the model. If estimating the model requires  $I(K)$  iterations per re-estimation, the sensitivity measure is  $I(K)$  times faster than the brute force approach. In many empirical applications,  $I(K)$  is likely to be relatively large even if good starting values are used making such an approach infeasible. Reassuringly, I find only minor differences between the

suggested sensitivity measure and this brute force approach in the main application below – where it is computationally feasible to compare the two approaches.

## 2.4 Extensions

**Sensitivity to Arbitrary Changes in  $\gamma$ .** The sensitivity measure above can be viewed as a linear approximation of the true effect of changing the calibrated parameters *one at a time*. The sensitivity to an arbitrary change in  $\gamma$ , say  $\Delta\gamma = \tilde{\gamma} - \hat{\gamma}$  could be constructed as a sum of the individual derivatives (see e.g. [Borgonovo and Apostolakis, 2001](#)). Denote the  $K \times 1$  vector of sensitivity measures of  $\theta$  to the  $j$ th element in  $\gamma$  as  $S_{(j)}$ , the sensitivity of  $\theta$  to a change  $\Delta\gamma$  could then be calculated as

$$S_{\Delta\gamma} = \sum_{j=1}^J S_{(j)} \Delta\gamma_{(j)}.$$

Although this measure is based on a local asymptotic approximation, it performs very well in the main empirical application below.

An alternative approach to investigating sensitivity to arbitrary changes in  $\gamma$ , could be to directly use the asymptotic linear representation to get

$$\begin{aligned} S_{\Delta\gamma} &= \hat{\theta}(\tilde{\gamma}) - \hat{\theta}(\hat{\gamma}) \\ &= \Lambda_n(\tilde{\gamma}) g_n(\hat{\theta}(\hat{\gamma}) | \tilde{\gamma}) - \Lambda_n(\hat{\gamma}) g_n(\hat{\theta}(\hat{\gamma}) | \hat{\gamma}) \end{aligned} \quad (9)$$

which only requires re-calculating  $g_n(\hat{\theta}(\hat{\gamma}) | \tilde{\gamma})$  and  $\Lambda_n(\tilde{\gamma})$  at the new set of calibrated parameters. The latter entails calculating the gradient of the objective function with respect to the  $K$  parameters in  $\theta$  for each alternative  $\tilde{\gamma}$ . While this is still an approximation, and potentially computationally more demanding than the measure in eq. (4), it allows for arbitrary changes in  $\gamma$  simultaneously without costly re-estimations of the model.<sup>8</sup> In turn, this is a low-cost approximation of the current practice outlined above. The same idea can be used to calculate the sensitivity of other quantities of interest,  $H_{\Delta\gamma} = h(\hat{\theta}(\tilde{\gamma}), \tilde{\gamma}) - h(\hat{\theta}, \hat{\gamma})$ .

**Sensitivity to Generalizations.** The sensitivity measure can be used to investigate general versions of the estimated model. The key insight is again to use the asymptotic linear form of the estimator for  $\theta$  to avoid estimating of the general model. In turn, the general model can be significantly more time consuming to

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<sup>8</sup> The accuracy of this approach can be improved by including higher order terms in the asymptotic approximation of  $\hat{\theta}$ .

solve and thus estimate.

To formalize this idea, partition  $\gamma$  into  $\gamma = (\gamma_1, \gamma_2)$  and denote the general model as  $\mathcal{M}(\theta, \gamma_1, \gamma_2)$ . Nested in the general model is the estimated model with  $\gamma_2 = 0$  and associated moment function  $g(\theta | \mathcal{M}(\theta, \gamma_1, 0))$ , with a slight abuse of notation. The sensitivity to a general version of the model is thus given by eq. (5) with  $D = \frac{\partial g(\theta | \mathcal{M}(\theta, \gamma_1, \gamma_2))}{\partial \gamma_2} \Big|_{\gamma_2=0}$ . Alternatively, the approach above in eq. (9) could be employed. The general model must be used when constructing the sensitivity measure but this does not involve re-estimation. This is thus a low cost approach to investigating the sensitivity to general versions of the model.

A particular example of interest could be individual or group-level heterogeneity. Imagine e.g. that  $\sigma \geq 0$  measures the degree of heterogeneity in some model parameter of interest. Imagine having estimated a restricted version of the model *without* unobserved heterogeneity, i.e. with  $\sigma = 0$  fixed. We could evaluate the sensitivity to  $\sigma$  as done above by simply using the model with heterogeneity when calculating the derivative  $D$ . This approximation can shed light on the importance of allowing for heterogeneity in the model without having to actually estimate the enriched model. As such, this can also provide a valuable guidance tool in the research process, providing researchers with information on which model extensions could be expected to generate large effects on the results and thus warrant further exploration.

**Choice of Weighting Matrix.** The degree of sensitivity depends on the weighting matrix,  $W$ , used in estimation. In fact, the sensitivity measure can be used to guide the choice of weighting matrix to be optimal in the sense that it minimizes the sensitivity of the estimator to calibrated parameters. Since the sensitivity measure can have both positive and negative entries, one object to minimize could be the quadratic  $SS' = \Lambda D D' \Lambda'$  or the trace thereof.

The optimal weighting matrix that minimizes  $SS'$  is  $W_{opt} = (DD')^{-1}$ , provided the inverse exists. To see this, recall that the weighting matrix that minimizes the asymptotic variance,  $\Lambda V \Lambda'$  where  $V = \text{Var}(f(\theta_0 | \gamma_0, \mathbf{w}_i))$ , is  $V^{-1}$  (Hansen, 1982). Swapping  $DD'$  for  $V$  gives the optimal weight matrix in terms of minimizing sensitivity. This reduced sensitivity would come at the cost of reduced efficiency. An estimator of  $W_{opt}$  could be based on an initial guess of  $\theta$  at which  $D$  is estimated. Alternatively, an iterative procedure can be implemented.

Other quantities than  $SS'$  could be minimized but would in general not have closed form solutions. Note, however, that given some value of  $\theta$ , the objective function does not need to be re-evaluated while searching for the optimal weighting matrix. One alternative, similar in spirit to that proposed by [Armstrong and](#)

Kolesár (2019) to minimize the sensitivity to moment miss-specification, could be to minimize the asymptotic variance of  $\hat{\theta}$ , under some (worst-case) bound on the sensitivity.

### 3 Relation to Some Existing Sensitivity Measures

I here discuss some of the existing measures closest related to what I propose. The sensitivity measure proposed by Andrews, Gentzkow and Shapiro (2017) is related to the current measure. In particular, they propose to report  $\Lambda$  as a local measure of the sensitivity of  $\theta$  to the included estimation moments in  $g_n(\bullet)$ . They do not consider the topic of the current paper and thus do not discuss sensitivity to calibrated parameters. The measure that I propose addresses this by weighting  $\Lambda$  by the effect of the calibrated parameters on each included moment through  $D$  in equation (4).

Another important contribution to the improvement of transparency is the recent work by Iskrev (2019). One of the sensitivity measures proposed in that study also measures how the estimated parameters are influenced by calibrated parameters. However, Iskrev (2019) focuses on Bayesian approaches and uses that the posterior distribution of  $\theta$  and  $\gamma$  is asymptotically jointly Normal to construct a local measure of sensitivity. Denote  $\Sigma_\theta$  and  $\Sigma_\gamma$  as the covariance matrices in the marginal asymptotic Normal distributions of  $\theta$  and  $\gamma$ , respectively, and  $\Sigma_{\theta,\gamma}$  as the covariance matrix between the two sets of parameters. From the asymptotic approximate Normal distribution, we have that the conditional mean vector of the estimated parameters, given the calibrated parameters, is

$$\mathbb{E}[\hat{\theta}|\hat{\gamma}] \stackrel{a}{=} \theta_0 + \Sigma_{\theta,\gamma}\Sigma_\gamma^{-1}(\hat{\gamma} - \gamma_0)$$

and Iskrev (2019) proposes the sensitivity measure

$$\Sigma_{\theta,\gamma}\Sigma_\gamma^{-1}. \quad (10)$$

A drawback of this measure is, however, that it requires the calculation of the covariance matrix between  $\theta$  and  $\gamma$ ,  $\Sigma_{\theta,\gamma}$ . This covariance is, unfortunately, often not readily available in many empirical applications considered in the current study. If e.g. multiple data-sources or externally calibrated parameters are included in  $\gamma$ , calculating the covariance between  $\gamma$  and  $\theta$  is not straight forward – if not practically impossible.<sup>9</sup> Besides the examples given in Footnote ??, an example of such

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<sup>9</sup> One strategy to uncovering *upper bounds* on the measure in (10) could be to use the worst-

a situation is [Gourinchas and Parker \(2002\)](#). In that study, the Panel Study of Income Dynamics (PSID) is used to calibrate the exogenous income process in a first step and then subsequently estimate preference parameters using the Consumer Expenditure Survey (CEX), given the income process parameters. When the authors subsequently calculate standard errors of the estimated preference parameters, the authors assume that  $\gamma$  and  $\theta$  are uncorrelated, implying that  $\Sigma_{\theta,\gamma} = 0$ .

The measure in equation (10) is furthermore derived under the assumption that both sets of parameters are identified simultaneously, while mine is not. This is also evident from the linear regression example above where  $\beta_1$  and  $\beta_2$  cannot be identified simultaneously from the one moment condition used. I view this as a strength of my approach because one motivation for using an externally calibrated  $\gamma$  could be due to the unavailability of data that could identify  $\gamma$ .

There is also a literature focusing on *global* sensitivity measures of quantities of interest to model inputs. One approach could be to simulate  $\gamma$ 's from some assumed distribution and investigate the resulting distribution of  $\hat{\theta}$  from re-estimation of the model for each value of the drawn  $\gamma$ s.<sup>10</sup> These methods would often require re-estimation of the second-step parameters relatively many times making such approaches computationally prohibitively expensive to apply to rich dynamic economic models, as I focus on here. The approach outlined in (9) could, however, be utilized to reduce this computational complexity.

Recently, [Harenberg, Marelli, Sudret and Winschel \(2019\)](#) have proposed a *polynomial chaos expansion* to alleviate the computational burden associated with global sensitivity (or uncertainty quantification) approaches. However, building on series expansions, that approach also requires the re-estimation of the dynamic economic model at  $M$  evaluation nodes to construct a global approximation of  $\hat{\theta}$ . If the dimension of the parameter space is large and/or the model complex, this can be quite computationally time demanding if a reasonable approximation is desired. Combining the local low-cost measure, that I propose, with the approach proposed in [Harenberg, Marelli, Sudret and Winschel \(2019\)](#) could potentially reduce the computational time required to perform global sensitivity analysis significantly: The local measure can guide researchers in which parameters are likely to require more evaluation nodes for a given degree of approximation accuracy.

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case upper bound on the co-variance structure following the approach suggested in [Cocci and Plagborg-Møller \(2019\)](#).

<sup>10</sup>See, e.g. [Borgonovo and Plischke \(2016\)](#) for a recent literature review.

## 4 Application: Life-Cycle Savings Motives

In a seminal paper, [Gourinchas and Parker \(2002\)](#) estimate a dynamic structural model of life cycle consumption and saving using data for the US. They use the estimated model to study the importance of life cycle (retirement) and buffer (risk) related motives for saving over the life cycle. Here, I illustrate the usefulness of the sensitivity measure through my implementation of that analysis.

The recursive form of the model is

$$\begin{aligned}
 V_t(M_t, P_t) &= \max_{C_t \in (0, M_t]} v_t \frac{C_t^{1-\rho}}{1-\rho} + \beta \mathbb{E}_t[V_{t+1}(M_{t+1}, P_{t+1})] \\
 &\text{s.t.} \\
 M_{t+1} &= (1+r)(M_t - C_t) + Y_{t+1} \\
 Y_{t+1} &= P_{t+1} U_{t+1} \\
 P_{t+1} &= G_{t+1} P_t N_{t+1} \\
 \log N_{t+1} &\sim \mathcal{N}(0, \sigma_n^2) \\
 U_{t+1} &= \begin{cases} \tilde{U}_{t+1} & \text{with probability } 1-p \\ 0 & \text{with probability } p \end{cases} \\
 \log \tilde{U}_{t+1} &\sim \mathcal{N}(0, \sigma_u^2)
 \end{aligned}$$

for  $t \leq T$  where  $\beta$  is the discount factor and  $\rho$  is the coefficient of constant relative risk aversion (CRRA). Initial wealth,  $W_{26} = (1+r)(M_{25} - C_{25})$ , is drawn from a log-normal distribution such that  $\log W_{26} \sim \mathcal{N}(\omega_{26}, \sigma_{\omega_{26}}^2)$  and initial permanent income is  $P_{26}$  for all households. Consumers face log-normal permanent and transitory income shocks, denoted  $N_t$  and  $U_t$ , respectively. Furthermore, consumers experience a transitory zero-income shock with probability  $p$ . The income growth factor,  $G_t$ , and taste shifter associated with family composition,  $v_t$ , evolves deterministically and are perfectly foreseeable by consumers. At retirement, a simple linear consumption function is assumed to apply,  $c_{T+1} = \gamma_0 + \gamma_1 m_{T+1}$  where  $c_{T+1} = C_{T+1}/P_{T+1}$  and  $m_{T+1} = M_{T+1}/P_{T+1}$  are normalized consumption and marked resources, respectively. Further details of the economic model as well as the numerical solution approach is given in the Supplemental Material and in the original paper.

The authors estimate  $\theta = (\beta, \rho, \gamma_0, \gamma_1)$  and keep all other parameters (which I denote  $\gamma$ ) fixed at calibrated values using simulated minimum distance,

$$\hat{\theta} = \arg \min_{\theta} g(\theta|\gamma)' W g(\theta|\gamma)$$

where

$$g(\theta|\gamma) = \begin{pmatrix} \log \bar{C}_{26}^{data} - \log \bar{C}_{26}^{sim}(\theta|\gamma) \\ \vdots \\ \log \bar{C}_{65}^{data} - \log \bar{C}_{65}^{sim}(\theta|\gamma) \end{pmatrix}^{'}$$

with  $\log \bar{C}_a^{sim}(\theta|\gamma)$  being the log average simulated consumption of age-group  $a$  for a given  $\theta$  and  $\log \bar{C}_a^{data}$  is the log average consumption of age group  $a$  in the Consumer Expenditure Survey, net of family and cohort variation. The preferred weight is diagonal with the inverse of the variance of the empirical moments on the diagonal.<sup>11</sup>

The calibrated parameters are reported in Table 1 and Figure A1 in the Supplemental Material where  $\tilde{\omega}_{26} = \exp(\omega_{26})$ . I use values identical to [Gourinchas and Parker \(2002\)](#). I re-estimate  $\beta$  and  $\rho$  using my implementation while fixing  $\gamma_0 = 0.0015$  and  $\gamma_1 = 0.071$  to the estimated values in [Gourinchas and Parker \(2002\)](#) and use the re-estimated model throughout. The estimates are reported in the bottom of the left panel of Figure A2 in the Supplemental Material and are slightly different compared to those in [Gourinchas and Parker \(2002\)](#). The model fit is also reported in the Supplemental Material.

Table 1: Calibrated Parameters.

$\sigma_n$	$\sigma_u$	$p$	$\tilde{\omega}_{26}$	$\sigma_{\omega_{26}}$	$r$
0.0212	0.044	0.00302	0.061	1.784	0.0344

## 4.1 Sensitivity of Parameter Estimates

Table 2 reports the sensitivity measure (in elasticities) in columns 2–3 together with brute-force elasticities based on re-estimation of the model in columns 4–5. The brute force calculated effect on the  $k$ th element of  $\theta$  of a  $\epsilon$  percent increase in the  $l$ th element in  $\gamma$  is calculated as  $(\tilde{\theta}_{(k)}^l - \hat{\theta}_{(k)})/\hat{\theta}_{(k)} \cdot 100$  where  $\tilde{\theta}_{(k)}^l$  is the  $k$ th element in  $\tilde{\theta}^l = \arg \min_{\theta \in \Theta} g_n(\theta|\tilde{\gamma}^l)'W_n g_n(\theta|\tilde{\gamma}^l)$  with  $\tilde{\gamma}^l = \hat{\gamma}(1_L + \iota_l \cdot \epsilon/100)$  being the original calibration with an increase of  $\epsilon$  percent in the  $l$ th calibrated parameter. All sensitivity measures are very close to the brute force calculations without being

<sup>11</sup>For each value of  $\theta$ , I solve the model using the endogeneous grid method (EGM), proposed by [\(Carroll, 2006\)](#), rather than time iteration used in [Gourinchas and Parker \(2002\)](#). The EGM is faster and more accurate than time iteration (see e.g. [Jørgensen, 2013](#)). The Supplemental Material contains a detailed description of the implementation. All Python code and data used herein are available online. I am grateful to Pierre-Olivier Gourinchas, Jonathan Parker and Isaiah Andrews for supplying some of their original code and data.

based on re-estimations of the model. This suggests that the *finite sample* behavior of the sensitivity measure is quite good for this application.

I find that the discount rate is relatively insensitive to the fixed parameters while the CRRA coefficient,  $\rho$ , is sensitive to all calibrated parameters. As also found in [Gourinchas and Parker \(2002\)](#), I find that lowering the transitory and permanent income shock variances have minuscule effects on the value of the estimated discount factor and only slightly affect the estimated risk aversion coefficient.

Table 2: Sensitivity of Parameters.  
Elasticities.

	Sensitivity measure		Re-estimation (brute force)	
	$\hat{\beta}$	$\hat{\rho}$	$\hat{\beta}$	$\hat{\rho}$
$\sigma_n$	-0.001	-0.023	-0.003	0.055
$\sigma_u$	0.001	-0.069	0.001	-0.063
$p$	0.008	-0.361	0.009	-0.408
$r$	-0.001	-1.365	-0.010	-0.945
$\tilde{\omega}_{26}$	-0.010	0.435	-0.010	0.413
$\sigma_{\omega_{26}}$	-0.016	0.670	-0.013	0.599

*Notes:* The table reports the sensitivity of the estimated parameters in  $\theta$  to the calibrated parameters in  $\gamma$ . The left panel reports the proposed sensitivity measure as elasticities. The right panel shows the same statistics calculated “brute force” as the percentage change relative to the baseline with re-estimated  $\theta$  parameters.

The CRRA coefficient is particularly sensitive to the probability of a zero-income shock,  $p$ , the initial wealth distribution,  $\tilde{\omega}_{26} = \exp(\omega_{26})$  and  $\sigma_{\omega_{26}}$ , and the risk-free interest rate,  $r$ . Like the CRRA coefficient, the zero-income shock probability,  $p$ , affects the curvature of the consumption function for lower levels of resources (see, e.g. the discussion in [Carroll, 1992, 1997](#)). Increasing either  $\rho$  or  $p$  would tend to lower consumption for low levels of resources. In turn, if  $p$  is increased,  $\rho$  would have to decrease to match the observed consumption profile. On the other hand, if the mean initial level of wealth is increased either through an increase in  $\tilde{\omega}_{26}$  or  $\sigma_{\omega_{26}}$ , the CRRA coefficient,  $\hat{\rho}$ , would increase to maintain the fit of the observed consumption profile. Such a positive relationship is also found in [Gourinchas and Parker \(2002\)](#).

The parameter to which the estimates are most sensitive (in percentage terms)

is the risk-free interest rate,  $r$ , with an elasticity of around  $-1.4$ . This parameter is not varied in the original study. The sign is negative because increasing the risk free interest rate increases the value of holding wealth through a dominating substitution effect, decreasing consumption.<sup>12</sup> The same is true with the CRRA coefficient. In turn, increasing the interest rate will lead to a reduction in  $\hat{\rho}$  in order to match the consumption age profile in the data.

Is the sensitivity measure also a good approximation to larger changes in the calibrated parameters? Table 3 investigates this question by increasing the risk-free interest rate from one to five percent. The table shows the sensitivity measure elasticities from (6) in the top panel through linear extrapolation together with the actual (brute-force) percentage change in the estimated parameter values in the bottom panel. The latter brute-force approach requires re-estimation of the model for each new value of  $r$  but measures the “true” finite sample effects of the interest changes considered. Since the sensitivity measure is a derivative of an asymptotic approximation calculated at the baseline  $r$ , one would expect it to approximate the percentage change best for small changes. This is confirmed by a very good approximation to a one-percent increase in  $r$  and a slight reduction in the quality of the approximation to a five-percent increase in  $r$ . Overall, however, the approximation performs very well.

## 4.2 Sensitivity of Savings Motives.

A key result in [Gourinches and Parker \(2002\)](#) is the decomposition of the saving motives over the life cycle. In particular, the estimated model suggests that before age 40, households save predominantly due to buffer against income shocks, referred to as buffer savings. In the remaining working life until age 65, the primary savings motive is to sustain a desired consumption level in retirement, referred to as life-cycle savings. To investigate the sensitivity of this result, I construct a measure of the difference in the two savings-motives below, following the approach in [Gourinches and Parker \(2002\)](#).

Denote savings as the change in end-of-period wealth  $s_{j,t} = A_{j,t} - A_{j,t-1}$ . Solving and simulating an alternative model without income uncertainty and a modified retirement consumption rule, the life-cycle saving is defined as

$$s_{j,t}^{LC} = A_{j,t}^{LC} - A_{j,t-1}^{LC}$$

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<sup>12</sup>See e.g. the discussion in [Carroll, Slacalek and Sommer \(2019\)](#).

Table 3: Sensitivity of Estimates to Large Changes in  $r$ . Elasticities.

Change in interest rate, $r$					
	1 pct.	2 pct.	3 pct.	4 pct.	5 pct.
<i>Sensitivity measure</i>					
$\hat{\beta}$	-0.001	-0.002	-0.003	-0.004	-0.005
$\hat{\rho}$	-1.365	-2.731	-4.096	-5.462	-6.827
<i>Re-estimated <math>\theta</math> (brute force)</i>					
$\hat{\beta}$	-0.010	-0.023	-0.033	-0.051	-0.063
$\hat{\rho}$	-0.945	-1.774	-2.696	-3.338	-4.225

*Notes:* The table reports the sensitivity of  $\theta$  to the risk-free interest rate,  $r$ . In the top panel is the proposed sensitivity measure reported as elasticities in percent. In the bottom panel, the brute-force percentage increase relative to the baseline  $r$  in estimated parameters are reported. All other parameters are fixed at their calibrated/estimated values.

where the parameters of this model is  $\sigma_n = \sigma_u = p = 0$  and  $\gamma_1 = 0.0615$ .<sup>13</sup> I also allow for borrowing in this version of the model up to 5 times the level of permanent income. The buffer saving is then given as  $s_{j,t}^B = s_{j,t} - s_{j,t}^{LC}$ .

Figure 1 shows the average age profiles of these measures in the left panel and the average age profile of wealth split by life cycle and buffer wealth in the right panel. Let  $s_{30}^{LC}$  denote the saving due to life-cycle motives at age 30 and let  $s_{30}^B$  be the savings due to buffer motives at age 30. I then calculate the difference  $h_{30} = s_{30}^B - s_{30}^{LC}$  and similarly at age 60,  $h_{60}$ . Table 4 shows the elasticities of these statistics with respect to the calibrated parameters.

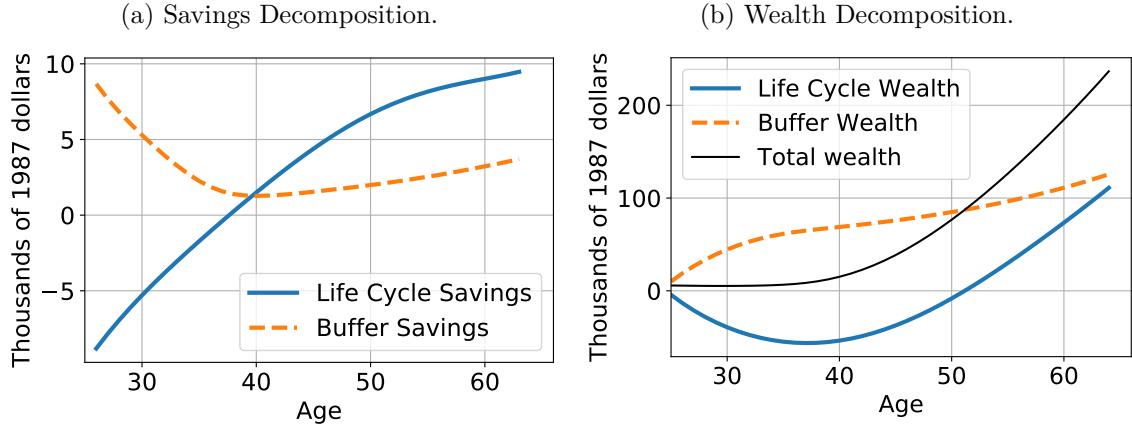
The savings motives decomposition is rather insensitive to the calibrated parameters. While the estimated parameters are sensitive to e.g. the interest rate, the effect of the changed interest rate on the savings motives are counter-balanced by the adjustment in  $\theta$  from the change in  $\gamma$ . In the current application, this happens

<sup>13</sup>The retirement consumption function is modified such that there is full certainty after retirement,

$$\gamma_1 = \frac{1 - \hat{\beta}^{\frac{1}{\rho}} (1 + r)^{\frac{1}{\rho} - 1}}{1 - \left( \hat{\beta}^{\frac{1}{\rho}} (1 + r)^{\frac{1}{\rho} - 1} \right)^{D-T}}$$

where  $D = 88$  and  $T = 65$ .

Figure 1: Savings Motives Decomposition.



*Notes:* The left panel illustrates the saving decomposition into life cycle savings and buffer savings, comparable to the top panel of figure 7 in [Gourinches and Parker \(2002\)](#). The right panel illustrates the wealth decomposition into life cycle wealth and buffer wealth, comparable to the bottom panel of figure 7 in [Gourinches and Parker \(2002\)](#).

to such a degree that the savings motive decomposition is hardly affected by the calibrated parameters. The reason for this is likely that the age profile of consumption (mirror of savings) is included in the estimation moments. In turn, roughly speaking, the estimator basically adjusts the estimated parameters to changes in  $\gamma$  as to leave the savings profile unaffected.<sup>14</sup>

The low-cost sensitivity measure is almost identical to the brute-force elasticities, calculated from re-estimating the model. This is very encouraging because it suggests that, at least in the current application, the proposed sensitivity measure has the potential to capture the complex effects on  $h(\bullet)$  from changing  $\gamma$  through the direct effect ( $A$ ) and the indirect effect ( $B \cdot S$ ) without having to re-estimate the model.

Table 5 shows the sensitivity measure of the savings-motives from larger changes in the risk-free interest rate,  $r$ . I include the brute-force re-estimation results in the middle panel and a third measure in the bottom panel based on the change in the savings motives from changing  $r$  while keeping  $\theta$  fixed at their baseline estimated values. This latter statistic is sometimes reported as a low-cost analysis of the

<sup>14</sup>This result is similar in spirit to that found in [Druedahl and Jørgensen \(2017\)](#). Simulation results in that paper based on the buffer-stock model suggests that a slight miss-calibration of the persistence parameter in the income process leads to significant bias in the subsequent parameter estimates but does not seem to significantly alter the marginal propensity to consume from the estimated model.

Table 4: Sensitivity of Saving Motives.  
Elasticities.

	Sensitivity measure		Re-estimation (brute force)	
	$H_{30}^e$	$H_{60}^e$	$H_{30}^e$	$H_{60}^e$
$\sigma_n$	0.046	-0.008	0.046	-0.008
$\sigma_u$	0.011	0.000	0.012	0.001
$p$	0.009	0.013	0.013	0.012
$r$	0.011	-0.120	-0.012	-0.127
$\tilde{\omega}_{26}$	0.006	-0.011	0.003	-0.012
$\sigma_{\omega_{26}}$	-0.006	-0.046	-0.039	-0.061

*Notes:* The table reports the sensitivity of the difference between the level of buffer and life-cycle savings at age 30 and 60. The left panel reports the proposed sensitivity measure as elasticities. The right panel shows the same statistics calculated as the percentage change relative to the baseline with re-estimated  $\theta$  parameters.

sensitivity to calibrated parameters. I denote this measure as  $B^e$  since it is closely related to  $B$  in (8).

Again, the results are extremely encouraging. The sensitivity measure is very close to the “true” brute-force percentage changes, even for larger interest rate increases. On the other hand, the bottom panel shows that results from changing  $r$  while keeping  $\theta$  fixed leads to significant *overestimation* of the effect of a change in the interest rate. This is because this latter measure does not take into account that the estimated  $\theta$  will adjust to such a change in the calibration and thus also affect the calculated statistics.

Table 5: Sensitivity of Saving Motives to Large Changes in  $r$ . Elasticities.

		Change in interest rate, $r$				
		1 pct.	2 pct.	3 pct.	4 pct.	5 pct.
<i>Sensitivity measure</i>						
$H_{30}^e$	0.011	0.022	0.034	0.045	0.056	
$H_{60}^e$	-0.120	-0.239	-0.359	-0.479	-0.599	
<i>Re-estimated <math>\theta</math> (brute force)</i>						
$H_{30}^e$	-0.012	-0.020	-0.029	-0.016	-0.025	
$H_{60}^e$	-0.127	-0.254	-0.384	-0.505	-0.639	
<i>Fixed <math>\theta</math></i>						
$B_{30}^e$	-1.005	-2.004	-2.998	-3.987	-4.971	
$B_{60}^e$	-0.523	-1.037	-1.542	-2.040	-2.528	

*Notes:* The table reports the sensitivity of the difference between the level of buffer and life-cycle savings at age 30 and 60. The top panel reports the proposed sensitivity measure as elasticities. The middle panel shows the same statistics calculated as the percentage change relative to the baseline with re-estimated  $\theta$  parameters for the various values of  $r$ . The bottom panel illustrates the percentage change in the statistics from the change in  $r$  while keeping  $\theta$  fixed at their baseline estimated values.

## 5 Application: Home-ownership and the Option Value of Regional Migration

The previous application facilitated a direct comparison of the sensitivity measure with a more brute force approach. In this second application, I illustrate the usefulness of the measure by applying it to a rich model that requires significant computational time to solve and estimate.<sup>15</sup> In turn, the brute-force approach is for all practical purposes infeasible.

Motivated partly by the empirical fact that homeowners migrate less than renters, [Oswald \(2019\)](#) estimates a rich dynamic programming model of home-ownership and migration. He then uses the estimated model to show that although the frequency of migration among homeowners is relatively low, they still value the migration option

<sup>15</sup>I am grateful to Florian Oswald for supplying  $A_n$ ,  $B_n$ ,  $D_n$ ,  $G_n$ , and  $W_n$  in his application, making my analysis possible.

because this option acts as an insurance against adverse regional shocks.

In the model, individuals choose in which region to live,  $d_j \in D$ , where  $j$  denotes age. Simultaneously, they choose whether to own a house,  $h_j \in \{0, 1\}$  and how much to consume,  $c_j$ , and thus how much wealth to carry over to the following period,  $a_{j+1}$ . I refer the reader to the original paper for a detailed description of the model and give only a brief outline of the model here. Individuals make their optimal choices taking into account 10 state variables in  $x_j = (a_j, z_j, s_j, \mathbf{F}_j, h_{j-1}, d_{j-1}, \tau, j)$  denoting, respectively, assets, an individual income shock, household size, an aggregate 2-dimensional price vector, housing status coming into the current period, current region index, time-invariant moving cost type and age.<sup>16</sup> I denote the transition density as  $x_{j+1} \sim \Gamma(x_j, d_j, h_j, c_j)$ .

Letting  $\varepsilon_{j,d_j}$  denote a region-specific Extreme Value Type I taste shock, and stacking the  $|D|$  shocks in  $\varepsilon_j$ , the recursive form of the model is

$$\begin{aligned} V_j(x_j, \varepsilon_j) &= \max_{d_j \in D} \{v(x_j|d_j) + \varepsilon_{j,d_j}\} \\ v(x_j|d_j) &= \max_{c_j, h_j} u(c_j, h_j, d_j; x_j) + \beta \mathbb{E}_j[\bar{v}(x_{j+1})] \\ \bar{v}(x_{j+1}) &= \log \left( \sum_{k \in D} \exp(v(x_{j+1}|k)) \right) \\ x_{j+1} &\sim \Gamma(x_j, d_j, h_j, c_j). \end{aligned}$$

Oswald (2019) fixes  $L = 8$  calibrated parameters in  $\gamma = (\tilde{\gamma}, \beta, \rho, \sigma, \phi, \chi, r, r^m)$  and estimates  $K = 19$  parameters in  $\theta$  by SMD. Both sets of parameters are reproduced in Table 6. Adopting some of the notation from the original paper the estimator is

$$\hat{\theta} = \arg \min_{\theta} (m - \hat{m}(\theta|\hat{\gamma}))' W_n (m - \hat{m}(\theta|\hat{\gamma}))$$

where  $m$  is a set of  $J = 38$  moments calculated from the data and  $\hat{m}(\theta|\hat{\gamma})$  are similar moments calculated from simulated data from the model at  $\theta$ . The weighting matrix,  $W_n$ , is chosen to be diagonal.

The sensitivity measure elasticities of the estimated parameters are reported in Table A1 in the Supplemental Material. The estimated parameters are especially sensitive to the values of the first four parameters: The value of the risk aversion parameter,  $\tilde{\gamma}$ , the discount factor,  $\beta$ , the persistence of income shocks,  $\rho$ , and the standard error of idiosyncratic income shocks,  $\sigma$ . Interestingly, the parameter ad-

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<sup>16</sup>Oswald (2019) allows for cohort effects and indices in the original paper thus has a time-dimension denoted by  $t$ . For ease of exposition, I abstract from that here.

Table 6: Parameters in [Oswald \(2019\)](#).

Estimated parameters in $\theta$	value	Calibrated parameters in $\gamma$	value
<i>Utility function</i>			
Owner premium size 1	$\xi_1$	-0.009	CRRA coefficient
Owner premium size 2	$\xi_2$	0.003	Discount factor
Util. of cons. scale	$\eta$	0.217	AR(1) of pers. inc. shock
Continuation value	$\omega$	4.364	Std. of pers. inc. shock
<i>Moving costs</i>			
Constant	$\alpha_0$	3.165	Transaction cost
Age	$\alpha_1$	0.017	Down-payment proportion
Age <sup>2</sup>	$\alpha_2$	0.0013	Risk-free interest rate
Owner	$\alpha_3$	0.217	30-year mortgage rate
Household size	$\alpha_4$	0.147	$r^m$
Proportion of high type	$\pi_\tau$	0.697	0.055
<i>Amenities</i>			
New England	$A_{NwE}$	0.044	
Middle Atlantic	$A_{MdA}$	0.112	
Middle Atlantic	$A_{StA}$	0.168	
West North Central	$A_{WNC}$	0.090	
West South Central	$A_{WSC}$	0.122	
East North Central	$A_{ENC}$	0.137	
East South Central	$A_{ESC}$	0.063	
Pacific	$A_{pcf}$	0.198	
Mountain	$A_{Mnt}$	0.124	

justing the continuation value of a house at the terminal period,  $\omega$ , and the share of high-types,  $\pi_\tau$ , seem relatively insensitive to most calibrated parameters.

[Oswald \(2019\)](#) uses the estimated model to calculate the option value of migration. To do so, a welfare measure is simulated under the baseline economy as

$$\begin{aligned}
 V(\hat{\theta}, \hat{\gamma}) &= \frac{1}{JN} \sum_{i=1}^N \sum_{j=1}^J \max_{d_j \in D} \{v(x_{ij}, d_j; \hat{\theta}, \hat{\gamma}) + \varepsilon_{ij} d_j\} \\
 &= \frac{1}{JN} \sum_{i=1}^N \sum_{j=1}^J u(c_{ij}^*, h_{ij}^*, d_{it}^*; x_{ij}, \hat{\theta}, \hat{\gamma}) + \beta \mathbb{E}_j [\bar{v}(x_{i,j+1}; \hat{\theta}, \hat{\gamma})]
 \end{aligned}$$

where superscript  $*$  denotes optimal choices. Likewise, a similar welfare measure is simulated under an alternative economy in which the option to migrate is not available in the model but a consumption compensation of  $\delta$  is introduced:

$$\tilde{V}(\delta | \hat{\theta}, \hat{\gamma}) = \frac{1}{JN} \sum_{i=1}^N \sum_{j=1}^J u(\delta \cdot \tilde{c}_{ij}^*, \tilde{h}_{ij}^*, \tilde{d}_{ij}^*; \tilde{x}_{ij}, \hat{\theta}, \hat{\gamma}) + \beta \mathbb{E}_j [\bar{v}(\tilde{x}_{i,j+1}; \hat{\theta}, \hat{\gamma})]$$

where “ $\hat{\cdot}$ ” illustrates optimal behavior under this alternative regime. To calculate the consumption value of the option of migration  $\delta$  is chosen as

$$\hat{\delta}(\hat{\theta}, \hat{\gamma}) = \arg \min_{\delta} (V(\hat{\theta}, \hat{\gamma}) - \tilde{V}(\delta | \hat{\theta}, \hat{\gamma}))^2.$$

This exercise yields an estimated option value of migration of around  $\hat{\Delta} = (\hat{\delta} - 1) \cdot 100 = 19.2\%$  ([Oswald, 2019](#), Table 12). Table 7 illustrates the sensitivity of this measure to calibrated parameters. All numbers are elasticities. Interestingly, the option value of migration is clearly most sensitive to calibrated parameters related to risk: A one-percent increase in the risk aversion coefficient would increase the option value of migration with around one percent and a one-percent increase in the persistence of income shocks would decrease the option value with around half a percent.

Table 7: Sensitivity of the Option Value of Migration. Elasticities.

	$\tilde{\gamma}$	$\beta$	$\rho$	$\sigma$	$\phi$	$\chi$	$r$	$r^m$
$\hat{\Delta}^e$	1.349	-0.127	-0.524	-0.026	0.005	-0.053	-0.002	0.002

*Notes:* The table reports the sensitivity of the estimated option value of migration,  $\delta$ , in [Oswald \(2019\)](#). Elasticities are reported.

The sensitivity measure can increase transparency and improve our understanding of underlying mechanisms in complex models. The main component in the option value of migration is insurance against adverse *regional* shocks. If a consumer in the model is more risk averse, insurance against such risk is more valuable. On the other hand, if the consumer face greater *idiosyncratic* income risk due to more persistent income shocks (larger  $\rho$ ), such an insurance mechanism is relatively *less* valuable. The reason is that increased idiosyncratic income risk will lead to increased savings in order to buffer against this risk. With more buffer-stock savings, the consumer will also have more self-insurance against adverse *regional* shocks. In turn, the option value of migration would be lower.

## 6 Concluding Discussion

The complexity of estimated structural dynamic economic models has increased dramatically with the computational power of computers. To maintain credibility and transparency as the complexity of models increase, systematic low-cost measures of the sensitivity of results based on such estimated models should, in my opinion,

be developed and reported by researchers.

A standard approach to estimation of dynamic economic models is to calibrate a sub-set of the model parameters and keep them fixed while estimating the remaining parameters of interest. If the importance of such calibrations is investigated, it is now standard to re-estimate the model using a few permutations of the calibrated parameters. In this paper, I propose an alternative approach to this relative time-consuming approach that is applicable to most popular estimators. The sensitivity measure is simple and fast to implement, yet offers an easy interpretation of the sensitivity of any quantity of interest to the calibrated parameters. In turn, the proposed sensitivity measure can greatly improve transparency of structural research.

Applying the proposed measure to the seminal work by [Gourinchas and Parker \(2002\)](#) of savings motives over the life cycle, I illustrate the usefulness of the measure. The authors report re-estimated parameters varying a set of fixed parameters but do not consider e.g. the effect of the fixed risk-free interest rate. I find that especially the point estimate of the constant relative risk aversion is sensitive to several calibrated parameters – especially the risk-free interest rate. While the main sensitivity measure is a local asymptotic approximation, the main application shows very encouraging results in the sense that the low-cost sensitivity measure is very close to the “true” brute-force effects from re-estimating the model. This is true for even relative large changes in the interest rate.

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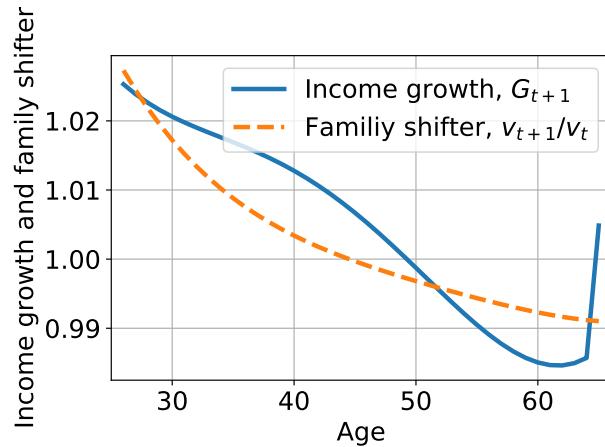
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## A Model Implementation: [Gourinchas and Parker \(2002\)](#)

### A.1 Additional figures

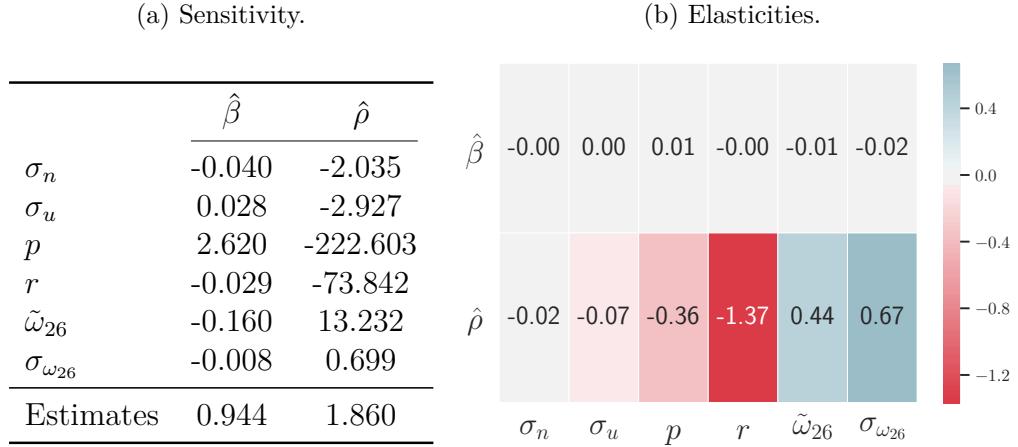
Figure A1: Income Growth and Family Shifter Calibration.



*Notes:* The figure shows the calibrated income growth,  $G_{t+1}$ , and the relative family shifter,  $v(Z_{t+1})/v(Z_t)$ .

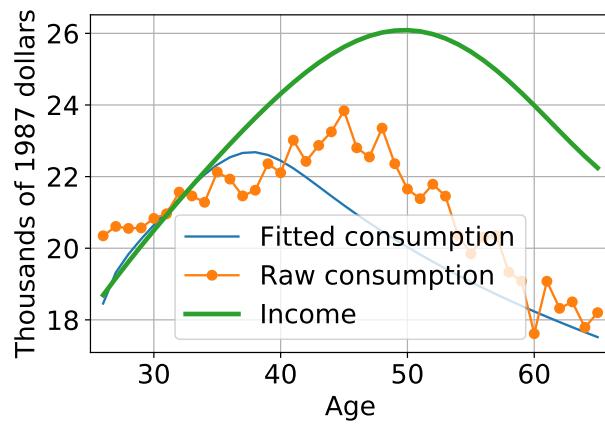
Figure A2 shows the proposed sensitivity measure. The left panel reports the raw sensitivity measure with respect to key calibrated parameters and the right panel shows the elasticities.

Figure A2: Sensitivity of Parameter Estimates.



*Notes:* The figure illustrates the sensitivity of  $\hat{\theta}$  with respect to the fixed parameters  $\gamma$ . The left panel shows the raw sensitivity measure,  $\hat{S}$ , together with my estimates of  $\beta$  and  $\rho$  from my implementation using the original data and weight matrix. The right panel shows elasticities,  $\hat{\mathcal{E}}$ , in a heatmap with estimated parameters on the y-axis and fixed parameters on the x-axis.

Figure A3: Model Fit.



*Notes:* The figure illustrates the observed average income and consumption age profiles together with simulated average consumption from the re-estimated model.

## A.2 Solution Approach

As [Gourinches and Parker \(2002\)](#), I use the Euler equation of a normalized model. From their Gauss code, and Appendix p.86 of the original paper, it seems that they assume that income is affected by the same taste shifter as consumption/utility. In the solution, I thus assume that  $Y_{t+1} = v_{t+1}^{1/\rho} P_{t+1} U_{t+1}$ . We can then normalize by  $P_t v_t^{1/\rho}$  to get the Euler equation in normalized terms as

$$c_t^{-\rho} = \max\{m_t^{-\rho}, \beta(1+r)\mathbb{E}[(G_{t+1}N_{t+1}f_{t+1})^{-\rho}c_{t+1}^{-\rho}]\}$$

where  $f_{t+1} = \left(\frac{v_{t+1}}{v_t}\right)^{\frac{1}{\rho}}$  adjusts for family composition. Normalized resources evolves according to

$$m_{t+1} = (1+r)a_t(G_{t+1}N_{t+1}f_{t+1})^{-1} + U_{t+1}$$

where  $a_t = m_t - c_t$  is end-of-period normalized wealth. Constructing a grid of end-of-period wealth as  $\vec{a}$  and approximating the expectation with two-dimensional Gauss-Hermite quadrature, optimal consumption can be found in closed form using the endogenous grid method (EGM) proposed by [Carroll \(2006\)](#) by inverting the Euler equation

$$c_t^* = \left[ (1+r)\beta \sum_{k=1}^Q \sum_{j=1}^Q [(G_{t+1}N^{(k)}f_{t+1})^{-\rho} (\check{c}_{t+1}^{(k,j)})^{-\rho}] \right]^{-\frac{1}{\rho}}$$

where  $\check{c}_{t+1}^{(k,j)} = \check{c}_{t+1}((1+r)(G_{t+1}N^{(k)}f_{t+1})^{-1}\vec{a} + U^{(j)})$  is the linearly interpolated next-period consumption for a given set of quadrature nodes  $(k, j)$ . The endogenous grid over resources is then  $\vec{m}_t = \vec{a} + c_t^*(\vec{m}_t)$ . The credit constraint can be handled by including a lower point of  $(m_{t+1}, c_{t+1}) = (0, 0)$  when interpolating the next-period solution. I use 300 points in the  $\vec{a}$  grid and  $Q = 10$  quadrature nodes in each dimension. The implied consumption function is illustrated in Figure 1 with the calibrated parameters given below. The retirement consumption function is given by  $c_{T+1}^*(m_{T+1}) = \gamma_0 + \gamma_1 m_{T+1}$ .

## A.3 Simulating Data

To simulate synthetic data (normalized by  $v_t^{1/\rho}$ ), I draw  $N_{sim} \times T$  standard normal shocks  $\{\tilde{n}_{j,t}, \tilde{u}_{j,t}\}_{1,1}^{N_{sim},T}$  together with uniform draws  $\{e_{j,t}\}_{1,1}^{N_{sim},T}$ . I can then construct

permanent and transitory income shocks, respectively, as

$$n_{j,t} = \exp(\sigma_n \tilde{n}_{j,t})$$

$$u_{j,t} = \exp(\sigma_u \tilde{u}_{j,t})(1 - p)^{-1} \mathbf{1}(e_{j,t} > p)$$

I also draw standard normal initial wealth  $\{\tilde{w}_{j,26}\}_1^{N_{sim}}$  and construct initial normalized resources as  $m_{j,26} = \exp(\omega_{26} + \sigma_{\omega_{26}} \tilde{w}_{j,26}) + u_{j,26}$ . Income is simulated as

$$P_{j,t} = \begin{cases} P_{26} & \text{if } t = 26 \\ G_t P_{j,t-1} n_{j,t} & \text{else} \end{cases}$$

$$Y_{j,t} = P_{j,t} u_{j,t}$$

and resources are

$$m_{j,t} = (1 + r)(m_{j,t-1} - c_{j,t-1})(G_t n_{j,t} f_t)^{-1} + u_{j,t}$$

where consumption is found as the linearly interpolated optimal consumption solved above,  $c_{j,t} = \check{c}_t(m_{j,t})$  and non-normalized consumption is then  $C_{j,t} = c_{j,t} \cdot P_{j,t}$ . All simulations are based on  $N_{sim} = 500,000$  simulated individuals.

## B Additional Figures and Tables [Oswald \(2019\)](#)

Table A1: Sensitivity of Parameters in [Oswald \(2019\)](#). Elasticities.

	$\tilde{\gamma}$	$\beta$	$\rho$	$\sigma$	$\phi$	$\chi$	$r$	$r^m$
$\hat{\xi}_1$	114.817	-86.300	-189.804	-3.443	1.378	-1.868	-0.179	-0.190
$\hat{\xi}_2$	-1050.357	209.728	2772.968	-2.366	-5.134	37.807	1.255	1.537
$\hat{\eta}$	-36.763	10.532	232.386	-2.831	-5.039	-1.397	-0.023	0.069
$\hat{\omega}$	0.165	0.162	-0.047	-0.003	-0.023	-0.006	-0.001	0.000
$\hat{\alpha}_0$	-1.174	-0.019	2.211	0.010	0.013	0.023	0.001	-0.000
$\hat{\alpha}_1$	-85.165	-32.862	547.621	-1.296	-7.673	0.002	-0.122	-0.028
$\hat{\alpha}_2$	1554.699	201.729	-3620.681	-21.701	18.066	-10.432	-0.093	-2.910
$\hat{\alpha}_3$	-0.936	3.430	20.203	0.129	0.015	0.016	0.008	0.005
$\hat{\alpha}_4$	37.091	-3.228	-17.754	-0.277	-0.088	-0.977	-0.058	0.033
$\hat{\pi}_\tau$	0.057	-0.003	-0.057	-0.000	-0.003	-0.002	-0.000	0.000
$\hat{A}_{NwE}$	62.353	-0.490	-75.947	0.373	2.561	-1.728	0.008	-0.134
$\hat{A}_{MdA}$	-127.703	11.875	437.474	-1.113	-5.718	2.360	0.088	0.019
$\hat{A}_{StA}$	-29.806	3.227	-9.579	0.042	-0.107	0.731	0.024	-0.032
$\hat{A}_{WNC}$	-237.423	6.282	-21.925	3.446	2.524	8.880	0.379	-2.884
$\hat{A}_{WSC}$	56.868	0.880	-54.668	0.416	0.827	-0.892	-0.024	0.050
$\hat{A}_{ENC}$	-4.929	0.411	12.863	-0.049	-0.035	0.074	-0.004	0.002
$\hat{A}_{ESC}$	214.474	18.102	-1198.287	-2.251	12.797	-4.375	0.642	0.463
$\hat{A}_{Pcf}$	-2.024	0.576	20.273	0.215	0.228	0.203	-0.008	0.019
$\hat{A}_{Mnt}$	281.108	131.541	-2120.151	37.155	-1.200	12.639	0.665	-0.504

*Notes:* The table reports the sensitivity of the estimated  $\theta$  parameters in [Oswald \(2019\)](#). Elasticities are reported.