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A pricing scheme for a human embryo storage service as a source for a secondary market



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ABSTRACT

A large reservoir of frozen human embryos is a negative outcome of their cryostorage for *in vitro* fertilization treatments. The existence of unused stored embryos, referred to herein as “remaining embryos”, places the service provider (usually a hospital) in a fragile situation. This is especially the case when the issue of discarding unused embryos arises or when unexpected failures associated with preserving the inventory occur. Moreover, the absence of intrinsic incentives for couples to donate embryos to the secondary market, as well as the increased operational costs for maintaining a viable storage service over time, further challenge this unique service. The present paper develops an optimization model that determines a charging scheme for the couples using the storage service and a payment scheme for couples who agree to provide their remaining embryos as a source for the secondary market. Couples who do not agree to receive payment for their remaining embryos are charged for discarding them (referred to herein as the “discarding payment”). While this model includes rewards and motivates couples to provide embryos for the secondary market, it also reduces the remaining inventory, thereby responding to the two key problems of embryo storage. The analytically developed model is used to derive optimal solution algorithms. A numerical example and a sensitivity analysis of the key parameters are presented.

1. Introduction

1.1. IVF process and human embryo storage

IVF (*in vitro* fertilization) is a treatment for infertility involving several phases, which enables external fertilization of an egg. Since the 1980s, cryostorage of human embryos has been a common procedure in IVF treatments [24]. The role played by this procedure is that excess embryos produced within a given treatment cycle can be transferred to a woman's uterus within a future cycle. Since the embryos are stored in deep freeze, the present paper categorizes them together with the cold storage literature. Cold storage is part of the cold chain [21, 33], which includes products such as food and blood. Storage technology for these products differs from that for embryos. The shelf life of blood and food is generally limited to days or weeks, whereas embryos have practically no expiration date and can be stored for many years (see, for example, [29]). Another distinction between embryos and other cold-chain products is the potential for unlimited storage capacity. Embryos are stored in liquid nitrogen tanks using test tubes. Each tank can store a large number of embryos owned by many different patients. Sufficient details are stamped on each tube in order to uniquely identify to whom the

embryos originally belonged.

1.2. Motivation

The entire process of IVF and human embryo storage is carried out by a clinic or hospital, which is the storage service provider. Currently, operational costs as well as ethical and legal issues mean that hospitals have no incentive to maintain an embryo storage service [5, 8]. Moreover, the donation of excess embryos to the secondary market is primarily voluntary and very infrequent [15, 23, 28] due to regulatory obstacles, ethical difficulties, and low willingness to donate. Couples who require embryo donation face a lack of available inventory [13]. This creates incentives for them to seek embryo donations outside their country of residence, potentially engaging in risky, expensive or illegal purchases to achieve pregnancy [13]. McMahon et al. [23] examine the concerns of couples with regard to donating their surplus embryos for medical research. Provoost et al. [28] examine how a couple's perception of their surplus embryos affects their decision as to whether to discard them or donate them to other couples.

Storage of frozen embryos as the byproduct of IVF treatments raises several economic, ethical and legal issues [16]. The growing number of

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embryos remaining in cryostorage is a distressing matter. It is estimated that there are more than 400,000 stored frozen embryos in the U.S.A [25]. Yet discarding excess inventory in an embryo bank may conflict with regulatory requirements and result in legal action against the provider [12].

In summary, the storage providers face increasing holding and operating costs while the embryo storage service is provided either for free or for a very limited payment. When unexpected technical failures of the storage facility occur, the providers are exposed to lawsuits and monetary penalties.

Thus, service providers do not currently have any economic incentive to maintain an ongoing service. The question of what should be done with the embryos after the original owners (the couples) have no further anticipated need to utilize them is barely addressed in the literature.

1.3. Literature survey

Since specific literature with respect to embryo storage services is scarce, the present paper mainly draws upon the literature regarding other types of service that share similar characteristics.

1.3.1. Limiting the storage period

A common practice in addressing some of the issues referred to above is to limit the length of time for which couples may store their embryos. According to Burton and Sanders [4], in Victoria, Australia, an embryo may only remain in storage for up to 5 years, although there are some provisions for an extension. In South Australia and the Northern Territory, embryo storage is limited to up to 10 years with no extensions. In 2001, Switzerland revised a law that prohibited the creation of embryos that will not be used for an immediate attempt at pregnancy. The revised law required all existing surplus of stored embryos to be discarded by the end of 2003. Later on, the law adopted a new deadline, the end of 2005, and allowed for creating a maximum of three embryos in each IVF cycle. Although *a priori* limiting of the storage period may be helpful, legal and ethical issues can still arise if some couples refuse to discard their embryos. In addition, the limited amount of donations and the lack of economic incentive remain.

1.3.2. Revenue management for similar storage services

During the last decade, IVF pricing policies have become an important focus of attention for researchers. With respect to the marketing aspect of IVF promotion, offering a money-back guarantee has become an increasingly common practice. Another pricing policy is discussed by Connolly et al. [8]. They consider the impact of introducing patient co-payments for IVF treatment in Germany on patient demand. Cooke [9] discusses the costs of IVF treatments in the context of public health provision by governments. Our work is highly relevant to the IVF pricing discussion.

This paper addresses the aforementioned problems associated with embryo storage by suggesting an operational-research and revenue-management approach, which, to the best of our knowledge, has not yet been undertaken in the literature. Pricing schemes are a common tool for revenue management [2, 22], as they impact consumer purchasing decisions. In special cases, pricing schemes may assist in controlling congestion or in shifting the purchasing incentive from one product to another. In the context of the present problem, increased prices could diminish the willingness of couples to keep unused embryos in storage.

The most closely related work in the existing literature consists of models for data storage services (e.g., [30, 34]), in which the goal is to devise a pricing model that maximizes profits. Other related studies include models that propose a dynamic pricing scheme to maximize the profit of service providers of wireless data, internet access, and communications services (e.g., [1, 20, 26]). Lee [17] considers an ex-post payment scheme where the service provider charges a price that is proportional to the length of time a customer spends in the system.

Although they differ, some similarities are also found in models of parking-lot services [14, 32] in which the goal is to maximize the parking operator's profits or minimize operational costs.

1.4. The current paper

The present paper follows the work mentioned above in the sense that it focuses on the objective of profit maximization. We suggest a revenue management approach to the surplus embryo market. The objective is to maximize the profits of the storage service provider, which is the hospital, over a finite period of time, T . The couples who request the embryo storage service enter into a contract with the hospital in which they are allowed to store one or more embryos for a defined period of time. The suggested model includes several components designed to address the difficulties presented above:

- (a) Allowing a secondary market for embryos: Consumers in the primary market may sell their remaining embryos to the service provider at a predetermined price and the service provider then sells some or all of these embryos to couples in the secondary market. The regulator may consider subsidizing the payment made by the secondary market.
- (b) Creating a pricing incentive for the service provider (i.e., hospital) by implementing charging schemes for both markets.
- (c) Creating a pricing incentive for the primary market (i.e., couples requesting a storage service for their embryos) through a payment scheme offered by the service provider in exchange for surplus embryos.
- (d) Decreasing the amount of remaining embryos: The present model incorporates a discarding payment for couples who do not utilize all of their stored embryos within the contract period and who refuse to sell their remaining embryos to the service provider at a predetermined price.

In summary, the present paper develops and analyzes a pricing optimization model that motivates couples in the primary market to elect for their remaining embryos to be a source of frozen human embryos for the secondary market. The hospital's additional purpose is to minimize the amount of frozen embryos remaining in storage.

The remainder of this paper is organized as follows. Section 2 presents the general model and its assumptions. Section 3 analyzes the model for both the special case where couples from the primary market must discard all remaining unused embryos, and for the general case that allows a secondary market. Section 4 presents illustrative numerical examples, a sensitivity analysis of the key parameters, and a comparison between the proposed model and two other modes of providing the embryo cryostorage service. Section 5 summarizes and concludes the paper.

2. Model formulation and assumptions

2.1. System description

Consider a primary market where each couple signs a contract with a hospital for a certain finite time, T . The hospital provides a storage service for one or more embryos in its facility and the couple accordingly agrees to pay for the service. Specifically, for each embryo, the couple pays a one-time lump sum plus an additional annual fee that increases (or decreases) over time to reflect changing insurance expenses. For example, the annual fee may increase due to a greater risk of a storage failure over time. The contract period represents the maximum duration for which the couples in the primary market may use their stored embryos for themselves. Once the initial contract has expired, a second phase begins that requires a decision to be made by each couple. The decision concerns whether to discard the remaining embryos for an additional payment or to sell them at a predetermined

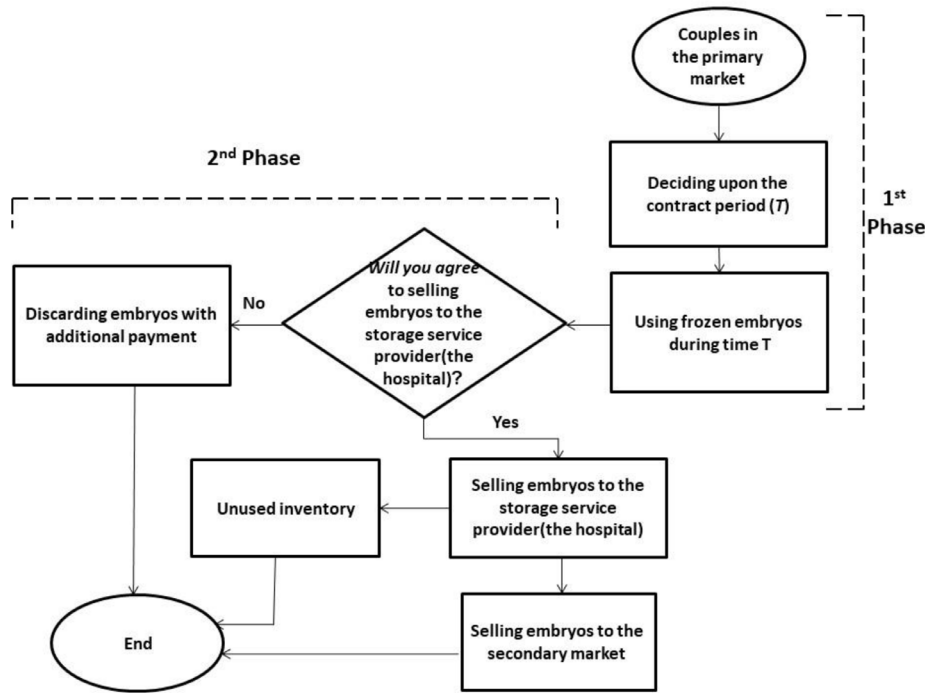


Fig. 1. The phases of the storage service.

price to the storage service provider, with the final intention that the embryos be a source for the secondary market. Thus, at time T , the only options are either to sell the surplus embryos to the hospital or to discard them. The goal of the suggested model is to maximize the profits of the storage service provider (the hospital) while reducing the number of remaining unused embryos. This goal is achieved firstly by giving the service provider economic incentives for providing the service. In addition, economic incentives are provided to the couples in the primary market in order to create a supply for the secondary market and thus decrease the remaining inventory. Fig. 1, below, presents the two stages of the entire decision process.

The notations of the model are provided below.

Indices:

t - The time elapsed from the beginning of the contact period

Parameters:

- T - The time period in the contract signed by the couple and the hospital
- θ_c - The estimated proportion of embryos used by the couple within the first phase (i.e., in period T)
- θ_d - The estimated proportion of embryos discarded by the end of period T
- θ_b - The estimated proportion of embryos needed to supply the secondary market at the end of period T
- c_d - The discarding payment per embryo
- p_b - The price at which couples can sell each surplus embryo to the provider.

- The annual increase in the holding cost over time
- The holding cost for each embryo for a unit of time (at $t = 0$)

Decision variables:

- K - The fixed and one-time payment fee for the entire period, T , for each embryo that is stored
- p_0 - The initial annual fee that is paid for storing each embryo
- α - The change over time in the annual fee for each embryo

p_s - The selling price for each embryo when offering it to the secondary market.

Variables:

- $p(t)$ - The storage price per unit (embryo) for a unit of time at time t
- $h(t)$ - The holding cost per unit (embryo) for a unit of time at time t
- $d(K, p_0, \alpha)$ - The annual demand for the storage service over the contract period
- $d_s(p_s)$ - The demand for the remaining embryos in the secondary market at the end of the planning horizon

Thus, the pricing policy is specified by the four decision variables defined above. The model is simplified by assuming that:

- A1. The contract time period, T , is fixed and is identical for all consumers.
- A2. After the contract period ends, the couples do not enter another period of storage service.
- A3. The demand function in the first phase is linear and decreasing with respect to each decision variable [18, 35].
- A4. The ratios θ_c and θ_d are not affected by the pricing policy.
- A5. The only options available to the couples in the primary market at time T are discarding or selling their surplus embryos.

2.2. Problem formulation

The storage price per unit at time t for a unit of time (i.e., the service price) is given by

$$p(t) = p_0 + \alpha t \tag{1}$$

where is the rate at which the annual fee increases (or decreases) over time. The holding cost per unit (embryo) for a unit of time at time t is modeled by

$$h(t) = h_0 + \delta \cdot t \tag{2}$$

where represents the holding cost for each embryo for a unit of time (at $t = 0$), and is a constant coefficient representing the annual increase in

the holding cost over time. This factor takes into consideration the increasing insurance costs over time due to increasing risk of failure. It is assumed that the annual demand for the storage service over the entire planning horizon (the contract period) is given by

$$d(K, p_0, \alpha) = n_0 - \beta_1 K - \beta_2 p_0 - \beta_3 \alpha, \tag{3}$$

where

- n_0 - Represents the maximal annual quantity (demand) for the storage service when all kinds of charges are eliminated
- β_1 - The one-time lump sum payment factor ($\beta_1 > 0$)
- β_2 - The annual fee factor ($\beta_2 > 0$)
- β_3 - The time coefficient factor ($\beta_3 > 0$).

It is assumed that these last three factors are positive, i.e., the demand decreases with each of the three decision variables. Note that in the case where the model allows a secondary market, $\theta_d < 1 - \theta_c$ and $\theta_c + \theta_d + \theta_b = 1$. Otherwise, $\theta_b = 0$ and $\theta_c + \theta_d = 1$.

Time-dependent demand functions that represent the short life-cycle pattern along all phases of a product's life-cycle, and that are linear in price, are assumed by Gan et al. [11]. In order to simplify the model, we assume that the ratios θ_b and θ_d are both determined by the service provider. Practically, their values can be estimated with the aid of historical data about the maximal secondary market size and the proportion of consumers who rejected any other use of the remaining embryos. The current model opens up the possibility of supplying an existing need from the secondary market. Although this route has always been theoretically possible, it is rarely available to couples in real life [19] due to the fact that it currently relies on donations. In contrast, in the present model, just before the storage contract ends, couples can either sell each surplus embryo to the provider at a price or discard each of the surplus embryos with an additional payment of c_d . The subscript "b" in p_b denotes the "buying" price. The disadvantage of the general model in comparison with the special case where $\theta_b = 0$ (i.e., the model is "discarding only") is that unused embryos that are neither sold to the secondary market nor discarded might remain in storage. To enable the possibility of supplying the secondary market, the service provider must determine the selling price, p_s , of each embryo when offering it to this market. The subscript "s" in p_s denotes the "selling" price. Similar to the vast literature on revenue management, we assume that the selling price in the secondary market is one of the key factors influencing the demand. The demand for the remaining embryos in the secondary market at the end of the planning horizon, $d_s(p_s)$, is assumed to be a known function of the form:

$$d_s(p_s) = \beta_5 e^{-\beta_6 p_s} \tag{4}$$

where

- β_5 - The quantity demanded by the secondary market in the case where the embryos are donated by the hospital (i.e., the price is) ($\beta_5 > 0$)
- β_6 - The sensitivity to the price, p_s ($\beta_6 > 0$).

A negative exponentially-decaying demand function [6, 7] is assumed in order to express a strong sensitivity to the price. Increasing the price increases the direct revenue for each embryo sold. However, at the same time, it decreases the demand. Therefore, the model should find the best price that maximizes profits.

The profit per contract period, $\Pi = TR - TC$, includes two components: the total revenue, TR , and the total costs, TC . The objective function of the hospital is based on the profit per unit time over the contract period, $\pi = \frac{\Pi}{T}$, which is represented by $\pi(K, p_0, \alpha)$. $K + \int_0^T (p_0 + \alpha t) dt$ represents the revenue for the entire period due to charging fees, and $\int_0^T (h_0 + \delta \cdot t) dt$ represents the holding cost for the

entire period. There are two additional components in the objective function. The first component is cost per unit time due to acquiring surplus embryos from couples who decide to sell them to the service provider by the end of period T . Some or all of these embryos are later sold to the secondary market. Multiplying the demand for the entire period, $d(K, p_0, \alpha)$, by the entire expression within the inner parentheses in Eq. (5) results in the net profit over the entire period from the primary market. This net profit includes the stage of both revenues due to discarding and costs due to buying the remaining embryos from the couples. The second component which is associated with the secondary market is the additional revenue per unit time $\frac{1}{T} p_s d_s(p_s)$ from selling the embryos to consumers in the secondary market at the end of period T . The problem formulation is:

$$\begin{aligned} \text{Max}_{K, p_0, \alpha, p_s} \pi &= \frac{1}{T} \left[d(K, p_0, \alpha) \left(\theta_d c_d T - \theta_b p_b T + K + \int_0^T ((p_0 + \alpha t) - (h_0 + \delta \cdot t)) dt \right) + p_s d_s(p_s) \right] \\ \text{s. t} \\ (5.1) \quad n_0 - \beta_1 K - \beta_2 p_0 - \beta_3 \alpha &\geq 0 \\ (5.2) \quad d_s(p_s) &\leq \theta_b T (n_0 - \beta_1 K - \beta_2 p_0 - \beta_3 \alpha) \\ (5.3) \quad p_s &\geq 0 \end{aligned} \tag{5}$$

Constraint (5.1) ensures a non-negative demand rate for any chosen pricing scheme. Constraint (5.2) prevents the size of the secondary market from exceeding the remaining available inventory that is intended for sale to the secondary market. Constraint (5.3) ensures that the selling price, p_s , does not take on a negative value.

Note that the optimization model proposed in this study is deterministic. We justify this choice in a similar way to Bitran and Caldentey [3], who argue as follows: "First of all, deterministic models are easy to analyze, and they provide a good approximation for the more realistic yet complicated stochastic models... The second reason is that deterministic models are commonly used in practice." Furthermore, as noted by Sattenspiel [31], the implementation of simulations that are based on deterministic models, through adjustment of the input variables and other parameters, can provide important insights into the quantitative and qualitative features of the phenomenon under investigation.

3. Mathematical analysis of the model

3.1. The discarding model

The analysis begins by considering a special case. At the end of period T , the hospital charges all couples a fee for discarding each unused embryo, c_d . Thus, the model assumes that $\theta_b = 0$ and assumption A5 implies that the only option available to couples in the primary market is to discard. In this scenario, neither donations nor sales to the secondary market are admissible. According to assumption A5, $\theta_c + \theta_d = 1$.

Thus, the problem formulation in this case is:

$$\begin{aligned} \text{Max}_{K, p_0, \alpha} \pi &= \frac{1}{T} \left[d(K, p_0, \alpha) \left(\theta_d c_d T + K + \int_0^T ((p_0 + \alpha t) - (h_0 + \delta \cdot t)) dt \right) \right] \\ \text{s. t} \\ (6.1) \quad n_0 - \beta_1 K - \beta_2 p_0 - \beta_3 \alpha &\geq 0 \end{aligned} \tag{6}$$

The advantage of this policy over the general model (in which sales to the secondary market are allowed) is that it ensures that there are no embryos remaining at the end of the storage service horizon. In this subsection, we analyze this specific problem and provide an optimal solution. The objective is explicitly represented by:

$$\begin{aligned} \pi(K, p_0, \alpha) &= \frac{1}{T} \left[(n_0 - \beta_1 K - \beta_2 p_0 - \beta_3 \alpha) \left(K + p_0 T - h_0 T + \frac{1}{2}(\alpha - \delta)T^2 + \theta_d c_d T \right) \right]. \end{aligned} \tag{7}$$

To simplify the analysis, we first address a *reduced* version of problem (6) that does not consider the non-negativity of the demand (i.e., constraint (6.1) is omitted). The other alternative implies an *active* constraint, that is, $d(K, p_0, \alpha) = 0$, and accordingly, $\pi(K, p_0, \alpha) = 0$. Appendix A details the solution method, the proof of which is provided in Appendix B.

We summarize below the *solution algorithm* for solving the original problem (6):

- Step 0** (Initialization). Given θ_c , set the coefficients $n_0, \beta_1, \beta_2, \beta_3, h_0, \delta, \theta_d = 1 - \theta_c$.
- Step 1** Obtain the solution for the reduced problem (6), $(K^{red}, p_0^{red}, \alpha^{red})$ by (A.2), where we use superscript *red* to reflect the solution for the reduced problem (6).
- Step 2** If condition (A.3) does not hold, then $(K^{red}, p_0^{red}, \alpha^{red})$ is not feasible. Go to Step 4.
- Step 3** The optimal solution is $\pi^* = \pi(K^{red}, p_0^{red}, \alpha^{red})$ where π is computed in (6). Stop.
- Step 4** Obtain an active solution $(K^{act}, p_0^{act}, \alpha^{act})$ by finding a root of the constraint (6.1) as an equality, $n_0 - \beta_1 K - \beta_2 p_0 - \beta_3 \alpha = 0$. We use superscript *act* to reflect the active solution for problem (6). For example, $(K^{act}, p_0^{act}, \alpha^{act}) = (\frac{n_0}{\beta_1}, 0, 0)$.
- Step 5** The optimal solution is $\pi^* = \pi(K^{act}, p_0^{act}, \alpha^{act})$ where π is computed in (6).
- Step 6** End

We summarize this case in the following theorem, which is implemented using the above algorithm:

Theorem 1. Denote conditions (B.1)–(B.3) in Appendix B as the technical conditions. Denote also the solution (A.2) by $(K^{red}, p_0^{red}, \alpha^{red})$ and denote by $(K^{act}, p_0^{act}, \alpha^{act})$ any solution that makes constraint (6.1) active, which would imply that it is a root of the equation $n_0 - \beta_1 K - \beta_2 p_0 - \beta_3 \alpha = 0$. If $(K^{red}, p_0^{red}, \alpha^{red})$ is valid under condition (A.3), and if $|A^{-1}|$ exists,

- (a) then solution $(K^{red}, p_0^{red}, \alpha^{red})$ is an optimal solution (not necessarily global) of problem (6); otherwise $(K^{act}, p_0^{act}, \alpha^{act})$ is an optimal solution (not necessarily global) of problem (6).
- (b) and if technical conditions (B.1)–(B.3) hold, then solution $(K^{red}, p_0^{red}, \alpha^{red})$ is a global maximum solution of problem (6); otherwise $(K^{act}, p_0^{act}, \alpha^{act})$ is a global maximum solution of problem (6).

Proof. The proof is introduced in Appendix B.

The advantage of this model is that all remaining embryos are discarded. The disadvantages of this model are that couples do not have the privilege of refusing to discard their remaining embryos and that there is no secondary market. An alternative possibility would be to offer couples in the primary market two options: either to discard their remaining embryos or sell them to the provider for utilization by the secondary market. This model is discussed in the following section.

3.2. The extended model: allowing a secondary market

Let us define a Lagrange multiplier, λ_2 , for constraint (5.2), which is the FOC

$$\lambda_2 [d_s(p_s) - \theta_b T(n_0 - \beta_1 K - \beta_2 p_0 - \beta_3 \alpha)] = 0 \tag{8}$$

We first analyze a reduced version of problem (5) that only considers the second the second constraint.

(a) In this case, the reduced problem can be decomposed into two simpler independent maximization sub-problems, i.e.,

$$Max_{\pi} \pi(K, p_0, \alpha, p_s) = Max_{\pi_1} \pi_1(K, p_0, \alpha) + Max_{\pi_2} \pi_2(p_s), \tag{9}$$

where the two optimization problems are:

$$\begin{aligned} &Max_{K, p_0, \alpha} \pi_1 \\ &= \frac{1}{T} \left[d(K, p_0, \alpha) \left(\theta_d c_d T - \theta_b p_b T + K + \int_0^T ((p_0 + \alpha t) - (h_0 + \delta \cdot t)) dt \right) \right] \\ &s. t \\ &(10.1) n_0 - \beta_1 K - \beta_2 p_0 - \beta_3 \alpha \geq 0 \end{aligned} \tag{10}$$

and

$$\begin{aligned} &Max_{p_s} \pi_2 = \frac{1}{T} [p_s d_s(p_s)] \\ &s. t \\ &(11.1) p_s \geq 0 \end{aligned} \tag{11}$$

The optimal solution to problem (10) is presented in (A.2) and is considered in the case where it is feasible under (A.3), with the only

modification being the replacement of vector $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ with $\begin{bmatrix} b'_1 \\ b'_2 \\ b'_3 \end{bmatrix}$, where

$$\begin{bmatrix} b'_1 \\ b'_2 \\ b'_3 \end{bmatrix} = \begin{bmatrix} \beta_3 \left((\theta_d c_d - \theta_b p_b) T - h_0 T - \frac{1}{2} \delta \cdot T^2 \right) - \frac{1}{2} T^2 n_0 \\ \beta_1 \left((\theta_d c_d - \theta_b p_b) T - h_0 T - \frac{1}{2} \delta \cdot T^2 \right) - n_0 \\ \beta_2 \left((\theta_d c_d - \theta_b p_b) T - h_0 T - \frac{1}{2} \delta \cdot T^2 \right) - n_0 T \end{bmatrix}. \tag{12}$$

After substituting demand function (4) into the objective of sub-problem (11), one obtains

$$\begin{aligned} &Max_{p_s} \pi = \frac{\beta_5}{T} (p_s e^{-\beta_6 p_s}) \\ &s. t \\ &(13.1) p_s \geq 0 \end{aligned} \tag{13}$$

The FOC for the objective of (13), i.e., $\partial \pi / \partial p_s = 0$, results in

$$p_s = \frac{1}{\beta_6} \tag{14}$$

Lemma 1. $p_s = \frac{1}{\beta_6}$ is the global unique maximum of sub-problem (11).

Proof. The proof is introduced in Appendix C.

(b) Following (8), we can calculate from

$$\begin{aligned} \beta_5 e^{-\beta_6 p_s} &= \theta_b T(n_0 - \beta_1 K - \beta_2 p_0 - \beta_3 \alpha) \\ p_s &= \frac{-1}{\beta_6} \ln \left(\frac{\theta_b T(n_0 - \beta_1 K - \beta_2 p_0 - \beta_3 \alpha)}{\beta_5} \right) \end{aligned} \tag{15}$$

In order that solution (15) is feasible under $p_s \geq 0$, the following condition is required:

$$\theta_b T(n_0 - \beta_1 K - \beta_2 p_0 - \beta_3 \alpha) \leq \beta_5 \tag{16}$$

By substitution of (15), problem (5) reduces to

Table 1
List of parameters.

Parameters	n_0	β_1	β_2	β_3	β_5	β_6	h_0	δ	θ_d	θ_b	p_b	c_d	T
Values (discarding model)	20,000	1	5	20	N/A	N/A	1000	500	0.4	0	N/A	1000	5
Values (secondary market model)	20,000	1	5	20	5000	0.0001	1000	500	0.2	0.2	1000	1000	5

$$Max_{K, p_0, \alpha} \pi = \frac{1}{T} \left[d(K, p_0, \alpha) \left(\theta_d c_d T - \theta_b p_b T + K + \int_0^T ((p_0 + \alpha t) - (h_0 + \delta \cdot t)) dt \right) - \frac{\theta_b T (n_0 - \beta_1 K - \beta_2 p_0 - \beta_3 \alpha)}{\beta_6} \ln \left(\frac{\theta_b T (n_0 - \beta_1 K - \beta_2 p_0 - \beta_3 \alpha)}{\beta_5} \right) \right]$$

s. t

(17.1) $n_0 - \beta_1 K - \beta_2 p_0 - \beta_3 \alpha \geq 0$

(17.2) $\theta_b T (n_0 - \beta_1 K - \beta_2 p_0 - \beta_3 \alpha) \leq \beta_5$ (17)

Constraints (17.1) and (17.2) can shortly be re-written as

$$0 \leq n_0 - \beta_1 K - \beta_2 p_0 - \beta_3 \alpha \leq \frac{\beta_5}{\theta_b T}$$
 (18)

Violation of (18) means that too many embryos are available in the secondary market, and as a result, there will be embryos remaining that are neither purchased nor discarded. The optimal solution of problem (17) is obtained numerically. After solving this problem, decision variables p_0 , α and K are substituted into (15) to obtain.

We summarize below the *solution algorithm* for solving the original problem (5):

Step 0 (Initialization). Given θ_c , set the coefficients. $n_0, \beta_1, \beta_2, \beta_3, \beta_5, \beta_6, h_0, \delta, p_b, \theta_b < 1 - \theta_c, \theta_d = 1 - \theta_c - \theta_b$.

Step 1 Assign $\begin{bmatrix} b_1' \\ b_2' \\ b_3' \end{bmatrix}$ according to (12). Obtain $(K^{red}, p_0^{red}, \alpha^{red})$ by

(A.2). Set $p_s^{red} = \frac{1}{\beta_6}$.

Step 2 If condition (A.3) does not hold, then $(K^{red}, p_0^{red}, \alpha^{red}, p_s^{red})$ is not feasible. Go to Step 4.

Step 3 If (5.2) holds, then the optimal solution is $\pi^* = \pi(K^{red}, p_0^{red}, \alpha^{red}, p_s^{red})$ where π is computed in (5). Stop.

Step 4 Obtain $(K^{act}, p_0^{act}, \alpha^{act})$ by numerically solving problem (17).

$$Set p_s^{act} = \frac{-1}{\beta_6} \ln \left(\frac{\theta_b (n_0 - \beta_1 K^{act} - \beta_2 p_0^{act} - \beta_3 \alpha^{act}) T}{\beta_5} \right).$$

Step 5 The optimal solution is $\pi^* = \pi(K^{act}, p_0^{act}, \alpha^{act}, p_s^{act})$ where π is computed in (5).

Step 6 End

Note that the algorithm presented in Section 3.1 to solve problem (6) is not a special case of the algorithm presented above for solving problem (5).

Theorem 2. Denote conditions (B.1)–(B.3) in Appendix B as the technical conditions. Denote also by $(K^{red}, p_0^{red}, \alpha^{red}, p_s^{red})$ the solution (A.2) where

vector $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ is replaced with $\begin{bmatrix} b_1' \\ b_2' \\ b_3' \end{bmatrix}$ and where $p_s^{red} = \frac{1}{\beta_6}$. If $(K^{red}, p_0^{red}, \alpha^{red}, p_s^{red})$ is feasible,

(a) then solution $(K^{red}, p_0^{red}, \alpha^{red}, p_s^{red})$ is an optimal solution (not necessarily global) of problem (5). Otherwise, denote by $(K^{act}, p_0^{act}, \alpha^{act})$ any solution of (17) and let $p_s^{act} = \frac{-1}{\beta_6} \ln \left(\frac{\theta_b (n_0 - \beta_1 K^{act} - \beta_2 p_0^{act} - \beta_3 \alpha^{act}) T}{\beta_5} \right)$; then solution $(K^{act}, p_0^{act}, \alpha^{act}, p_s^{act})$ is an optimal solution (not necessarily global) of problem (5).

(b) If technical conditions (B.1)–(B.3) hold, then solution $(K^{red}, p_0^{red}, \alpha^{red}, p_s^{red})$ is a global maximum solution of problem (6); otherwise $(K^{act}, p_0^{act}, \alpha^{act}, p_s^{act})$ is a global maximum solution of problem (6).

Proof. Since the subtraction of the constant $\theta_b p_b T$ within the inner parentheses of (10) does not alter the Hessian matrix, the proof directly follows from the optimal search algorithm above and is similar to the proof of Theorem 1. \square

4. Numerical example and sensitivity analysis

In this section we demonstrate the significance of the models and the applicability of the method suggested for solving them. We present the optimal solution for a given set of parameters, as well as a sensitivity analysis of the optimal solution when changing key parameters, to show the robustness of the results. Table 1 presents the data used for the example.

Consider couples in the primary market who sign a five-year contract. When it expires, they must (according to the discarding model) pay 1000 NIS to discard each remaining embryo. However, according to the extended model, they must decide whether to pay 1000 NIS for discarding each remaining embryo or alternatively, to sell each embryo to the service provider for 1000 NIS. Such a sale also enables the provision of a source of embryos for the secondary market. For the purposes of this example, it is estimated that 60% of the entire demand consists of embryos used by couples in the primary market within the first phase of period T (i.e., $\theta_c = 0.6$). Further, as shown by the values in Table 1, the service provider estimates that 50% of the remaining embryos (i.e., 20% of the entire demand) should be allocated to supply the secondary market at the end of period T .

4.1. Optimal pricing policy

We utilized the optimal searching algorithms presented above to obtain the solutions. Table 2 presents the optimal policy and the

Table 2
Optimal solutions.

	K^*	p_0^*	α^*	p_s^*	π^*	Available reservoir for the secondary market M^S	Remaining embryos I_T
Optimal solution for the discarding model	0	2960	0	N/A	$5.772 \cdot 10^6$	0	0
Optimal solution for the secondary-market model	0	3120	0	10,000	$7.507 \cdot 10^6$	4400	2560

associated optimal profit for the above two models. Given the parameters presented in Table 1 for the discarding model, the optimal pricing policy for the service provider that maximizes its profits includes a fixed annual payment for storing each embryo for the entire period of 2960 NIS and annual profits of $5.772 \cdot 10^6$ NIS. Allowing the possibility of a secondary market (i.e., $\theta_b > 0$, the second set of values in Table 1) alters the optimal pricing policy. In this case, the service provider charges a fixed annual payment for storing each embryo for the entire period of 3120 NIS and sells each embryo to couples who belong to the secondary market for 10,000 NIS, where the annual profits are $7.507 \cdot 10^6$ NIS. The results indicate that in the case where there is the potential for a secondary market, the service provider to some extent exploits its power in determining the prices by increasing the service price for couples in the primary market.

On the other hand, in the secondary-market model, the service provider is left with unsold embryos. Specifically, for the given parameters, out of the 4400 embryos available for the secondary market, M^S , only 1840 are eventually sold, thus leaving 2560 remaining embryos. It is interesting to note that for these two examples, although the pricing model allows three degrees of freedom in setting the optimal price, only the fixed annual payment p_0^* is positive.

4.2. Sensitivity analysis of the parameters

Table 3 presents the effect of the parameters' values given in Table 2 on several key performance measures, where these changes are expressed in percentage terms. The performance measures in question are the optimal profit π^* , the available reservoir for the secondary market

M^S , and the remaining embryos I_T . In the case where $n_0 = 15,000$ (i.e., 25% smaller than the initial value), the optimal policy changes into a two-part tariff (for the primary market) with $K^* = 10, 100, p_0^* = 620$ (and $\alpha^* = 0, p_s^* = 10, 216$). In this case, the demand in the secondary market exceeds the available reservoir, and thus the optimal solution does not leave any unused embryos (i.e., $I_T = 0$ and the percentage change in I_T is -100).

The results presented in Table 3 indicate that the optimal profit per unit time π^* increases with the parameters β_5, c_d and θ_d , while it decreases with the following parameters: β_1 (until a specified threshold), β_2 (until a specified threshold), β_3 (until a specified threshold), β_6, δ, T, p_b and h_0 . Interestingly, the optimal profit per unit time π^* also shows a general increasing trend with parameter n_0 , although this curve includes a single local minimum. The results further indicate that the available reservoir for the secondary market M^S increases with parameter n_0 and decreases with parameters β_1 (until a specified threshold), β_2 (until a specified threshold), β_3 (until a specified threshold), δ, θ_d and h_0 . Finally, the results indicate that the number of remaining embryos, I_T , increases with parameter n_0 and decreases with the parameters β_1 (until a specified threshold), β_2 (until a specified threshold), β_3 (until a specified threshold), $\beta_5, \delta, \theta_d$ and h_0 .

Each of the three performance measures is insensitive to changes in the parameters β_1, β_2 and β_3 , as long as these changes are positive. Interestingly, M^S and I_T show an unusual trend with the contract period T ; namely, there is more than a single local maximum. The service provider's profit is strongly affected by the price sensitivity parameters in both the primary and secondary markets. The results shown in Table 3 strongly motivate the service provider to offer shorter contract

Table 3
The effect of the deviation from the original parameter value (in%) on three optimal variables, expressed in terms of the percentage change.

Change in parameter value (%) Parameter	-50	-25	-10	+10	+25	+50	Optimal variable
n_0	-15.52	7.39	-20.69	25.95	74.88	183.02	π^*
	-70.45	-59.09	-26.36	20.0	59.09	118.18	M^S
	-100.0	-100.0	-45.30	34.36	101.53	203.07	I_T
β_1	224.25	67.61	21.11	0	0	0	π^*
	63.63	27.27	9.09	0	0	0	M^S
	109.35	46.86	15.62	0	0	0	I_T
β_2	224.25	67.61	21.11	0	0	0	π^*
	63.63	27.27	9.09	0	0	0	M^S
	109.35	46.86	15.62	0	0	0	I_T
β_3	49.71	0	0	0	0	0	π^*
	27.27	0	0	0	0	0	M^S
	46.86	0	0	0	0	0	I_T
β_5	-24.50	-12.25	-4.90	4.90	12.25	24.50	π^*
	0	0	0	0	0	0	M^S
	35.92	17.95	7.18	-7.18	-17.95	-35.92	I_T
β_6	49.00	16.33	5.44	-4.45	-9.80	-16.34	π^*
	0	0	0	0	0	0	M^S
	0	0	0	0	0	0	I_T
δ	42.91	19.83	7.51	-7.36	-16.58	-29.92	π^*
	36.36	18.18	9.09	-9.09	-18.18	-36.36	M^S
	62.48	31.24	15.62	-15.62	-31.24	-62.48	I_T
T	336.36	109.34	35.07	-11.49	-26.39	-46.25	π^*
	-9.09	9.09	6.36	0	2.27	-4.54	M^S
	-15.62	15.62	10.93	0	3.90	-7.81	I_T
p_b	5.91	2.93	1.17	-1.17	-2.93	-5.70	π^*
	9.09	0	0	0	0	-9.09	M^S
	15.62	0	0	0	0	-15.62	I_T
θ_d	-11.03	-5.70	-2.34	2.34	5.91	12.30	π^*
	36.36	13.63	10.0	-10.0	-18.18	-45.45	M^S
	62.48	23.43	17.18	-17.18	-31.24	-78.10	I_T
c_d	-5.70	-2.93	-1.17	1.16	2.93	5.91	π^*
	-9.09	0	0	0	0	9.09	M^S
	-15.62	0	0	0	0	15.62	I_T
h_0	33.30	15.50	5.91	-5.70	-13.59	-24.99	π^*
	27.27	18.18	9.09	-9.09	-18.18	-27.27	M^S
	46.86	31.24	15.62	-15.62	-31.24	-46.86	I_T

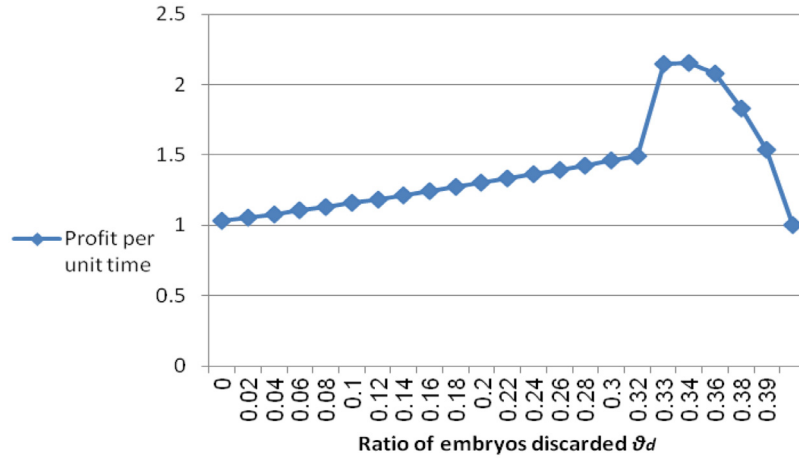


Fig. 2. Profit per unit time (relative to the profit in the case of no secondary market) as a function of the estimated proportion of embryos discarded, θ_d .

periods, as this would have a significant positive effect on profit.

It is interesting to consider the effect of changes in θ_d , the estimated proportion of embryos discarded by the end of period T . The profits presented in Tables 2 and 3 show, at first sight, some contradictory results. In Table 2, opening up the possibility of a secondary market (second set of values) increases annual profits by approximately 30% in comparison to those obtained in the model where the only option is to discard unused embryos (first set of values). In Table 3, however, it can be seen that further decreasing θ_d increases the available reservoir for the secondary market and decreases annual profits. Since this parameter is controlled by the service provider, it is worth examining its influence in greater detail. Fig. 2 shows the change in the optimal profit per unit time as a function of θ_d , where the baseline profit (i.e., value of 1) is the result obtained in Table 2 for the model in which there is no secondary market (where $\theta_d = 0.4$).

Fig. 2 implies a quasi-concave profit function with the proportion of embryos allocated by the service provider to supply the secondary market, θ_b (since $\theta_b = 1 - \theta_c - \theta_d$). The service provider obtains the highest profits when $\theta_d = 0.34$ (i.e., when $\theta_b = 0.06$), meaning that only 6% of the total market demand is allocated to the secondary market. In mathematical terms, the "jump" seen near $\theta_d = 0.33$ is associated with the fact that constraint (5.2) becomes active. Since the regulator might disagree with an allocation for the secondary market that is as low as $\theta_b = 0.06$, in practice, regulations that restrict the minimal θ_b or that increase monetary incentives for the service provider and for the couples in the primary market to sell their surplus embryos could be introduced.

4.3. Comparing strategies

This sub-section presents a comparison between the common modes of providing embryo cryostorage services (for free or with an annual fee for storing each embryo) with the suggestion in the present paper (according to the extended model) of allowing a secondary market. For the comparison, we utilize the second set of values presented in Table 1. The following expressions define several performance measures for evaluating the pricing strategies:

(a) Profit of the service provider, π

$$\pi = \frac{1}{T} \left[\begin{aligned} & d(K, p_0, \alpha) \left(\theta_d c_d T - \theta_b p_b T + K + \int_0^T ((p_0 + \alpha t) - (h_0 + \delta \cdot t)) \right. \\ & \left. dt \right) \\ & + p_s d_s(p_s) \end{aligned} \right]$$

(b) Available reservoir for the secondary market M^S

$$M^S = \theta_b(n_0 - \beta_1 K - \beta_2 p_0 - \beta_3 \alpha) T$$

(c) Number of remaining embryos I_T

$$I_T = \max\{0, \theta_b(n_0 - \beta_1 K - \beta_2 p_0 - \beta_3 \alpha) T - \beta_5 e^{-\beta_6 p_s}\}$$

for the suggested strategy.

$$I_T = \theta_c(n_0 - \beta_1 K - \beta_2 p_0 - \beta_3 \alpha) T + \max\{0, \theta_b(n_0 - \beta_1 K - \beta_2 p_0 - \beta_3 \alpha) T - \beta_5 e^{-\beta_6 p_s}\}$$

for all other strategies.

(d) Primary market size

$$I^P = (1 - \theta_d - \theta_b)(n_0 - \beta_1 K - \beta_2 p_0 - \beta_3 \alpha) T$$

(e) Secondary market size I^S

$$I^S = \min\{\theta_b(n_0 - \beta_1 K - \beta_2 p_0 - \beta_3 \alpha) T, \beta_5 e^{-\beta_6 p_s}\}$$

(f) Cost that the primary market pays C^P

$$C^P = (n_0 - \beta_1 K - \beta_2 p_0 - \beta_3 \alpha) \left(K + \int_0^T (p_0 + \alpha t) dt + \theta_d c_d T - \theta_b p_b T \right)$$

(g) Cost that the secondary market pays C^S

$$C^S = \min\{p_s \theta_b(n_0 - \beta_1 K - \beta_2 p_0 - \beta_3 \alpha) T, p_s \beta_5 e^{-\beta_6 p_s}\}$$

In computing the performance measures for the different strategies (see Table 4), it is assumed that in all strategies other than the suggested model, the service provider does not charge the couples for discarding unused embryos and does not offer them any payment for purchasing their remaining embryos after utilizing the service. It is further assumed that in all strategies other than the suggested model, the remaining embryos are not discarded and become a source for the secondary market.

All comparative strategies result in significant losses to the service provider, and thus highlight the lack of intrinsic incentives it has to offer the storage service at all or to supply the secondary market. The

Table 4
Performance measures of several charging strategies.

Mode	Neither market is charged	The primary market is charged and donates its surplus $\alpha = K = 0$ embryos* $p_0 = 1460, p_s = 0$	Only the primary market exists $\theta_b = 0,$ $\theta_d = 0.4$ $\alpha = K = 0$ $p_0 = 1460, p_s = 0$	Both markets are charged $\alpha = K = 0$ $p_0 = 1460$ $p_s = 5000$	Proposed policy
π	$-45 \cdot 10^6$	$-10.03 \cdot 10^6$	$-10.03 \cdot 10^6$	$-7 \cdot 10^6$	$7.507 \cdot 10^6$
M^S	20,000	12,700	0	12,700	4400
I_T	35,000	20,400	25,400	22,367	2560
I^P	60,000	38,100	38,100	38,100	13,200
I^S	5000	5000	0	3033	1840
C^P	0	$92.71 \cdot 10^6$	$92.71 \cdot 10^6$	$92.71 \cdot 10^6$	$66.44 \cdot 10^6$
C^S	0	0	0	$15.16 \cdot 10^6$	$18.4 \cdot 10^6$

* as published by The Concept Fertility Clinic in London, UK.

suggested model achieves the lowest level of remaining embryos in inventory (i.e., 2560), far less than the competing strategies. This result illustrates one of the key motivations for developing the model, as this relatively low inventory level is likely to reduce the likelihood of conflicts between couples, the service provider, and the regulator. According to the first strategy, which is commonly implemented in current practice, couples are not charged at all either in the primary or the secondary market. This strategy results in the highest remaining inventory as well as a significant loss for the service provider. The charging scheme in the suggested model also reduces the size of the primary market. This is important since the primary market is the source of remaining embryos and the source of the losses suffered by most of the service providers. The fifth line in Table 4 shows that the suggested pricing scheme results in a moderate secondary market of 1840 embryos. However, while the other three strategies result in a larger secondary market, they are not profitable. Thus, such strategies may lead the hospital to withdraw from offering the service at all. Moreover, the first two strategies, which achieve the largest secondary market of 5000 embryos, assume that the primary market does not receive any payment for its donations (i.e., $C^S = 0$). This assumption does not reflect real-life scenarios which are commonly associated with low willingness to donate to the secondary market.

5. Discussion and conclusions

This study analyzes a unique problem of the storage of frozen embryos following IVF treatments – a problem that has economic, ethical and social aspects. The present model contributes to the existing literature by developing and solving an optimization model. The model combines the interests of (a) couples in the primary market who utilize the storage service; (b) couples who, for many reasons, wish to pursue embryo donation through the secondary market; (c) the service provider, which would like to maximize its profit; and (d) the social planner, which would like to decrease the quantity of unused embryos while maintaining the option of embryo donation either for the secondary market or for research purposes. Through a numerical example and sensitivity analysis, the present paper examines the applicability of the suggested model and the significance of each of the influencing parameters on the optimal solution and other performance measures.

An optimization model is developed for maximizing the profit of the storage service provider. In the extended version of this model, couples in the primary market have the options of either discarding surplus inventory at the end of a particular time period or selling it to the provider, thus opening up a secondary market. Optimal solutions with accordingly optimal searching algorithms are analytically obtained for the extended model. It is shown that utilizing the suggested pricing policy may lead to benefits not only for the service provider, in terms of maximizing its profit, but also for the other players. In particular, the model significantly reduces the amount of remaining inventory relative to the alternative schemes. This is because the model encourages the

service provider, through appropriate selection of parameters p_b and θ_b , to increase the supply of embryos for the secondary market from embryos remaining in cryostorage, while decreasing the overall unused inventory. The results show that the service provider is strongly motivated to offer shorter contract periods, as this has a significant, positive effect on its profit. Reducing the contract period decreases the risk of preserving inventory for long durations and encourages couples to lower their primary demand. Accordingly, it also lowers the unused inventory.

The suggested model also has numerous managerial implications, including the following:

- (a) By utilizing the suggested model, the hospital responds to consumer demand with economic incentives. In particular, couples who own embryos that they no longer need act as suppliers of their remaining inventory, while couples in the secondary market who are waiting for embryo donations represent the demand side. Certainly, addressing this demand with minimal or no payment from that market requires an agreement between the service provider and the social planner.
- (b) The numerical example shows that the service provider has strong monetary incentives to shorten the contract period as well as to estimate a relatively small secondary market. The regulator should be aware of this and should establish restrictions with which the service provider must comply. For example, the regulator may determine a minimal payment, p_b , to the couple supplying the embryo or a maximal discarding proportion θ_d .
- (c) Numerical analysis shows very low sensitivity of the optimal solution to positive changes in the pricing factors $\beta_1, \beta_2,$ and β_3 . This observation is important since the demand function is only estimated. This implies that even if, in real-life applications, the demand function is inaccurately estimated, the optimal results remain similar.
- (d) The optimal pricing scheme in all of our experiments included no more than a two-part tariff for the primary market. This not only simplifies the pricing scheme, but it also increases the likelihood that the scheme will be accepted by the market.
- (e) Deterministic models, such as that proposed in this paper, cannot accurately capture all real-life scenarios. One of the key parameters in revenue management, the annual demand, is assumed to be known. Unlike other, much more widely researched problems in the revenue-management literature that address storage capacity (e.g., inventory and production planning problems or capacity resource planning problems), the storage capacity for embryos is ample. This means that even in a special case where the demand for the storage service is suddenly doubled, the hospital would not need additional "storage resources" to cope with such an unexpected scenario. This, of course, is not true for classical inventory and capacity problems; thus the use of the deterministic approach is better justified in the present case.

The novel strategy suggested in this model decreases the remaining reservoir of embryos both by discarding them and by supplying the secondary market. Yet, at the same time, the model assumes that the pricing policy of the hospital has no direct influence on the proportion of embryos discarded or the proportion sold at the end of the contract period, T . Thus, the social planner, who is assumed to be interested in social welfare and in avoiding ethical problems, may monitor both the intensity of the secondary market and the amount of unused embryos, and then may intervene as necessary. Specifically, the social planner can offer monetary assistance to the couples, and can provide subsidies or impose taxes and other restrictions on the service provider. Numerical evidence in support of the fact that a regulatory subsidy offered to a manufacturer can lead to higher social welfare is described in Qu et al. [27]. Such extensions to the model are suggested for further research.

We assume that the annual increase in the holding cost over time, is a constant coefficient. Yet this parameter, which is not easy to predict, may be subject to random variations in real-life scenarios, thus affecting the expected profit, the available reservoir for the secondary market, and the number of remaining embryos. Generalization of the proposed model to consider a random annual increase in the holding cost is

Appendix A. Solution method for the discarding model

The first-order condition (FOC) for local maximization is given by the solution of the following system of linear algebraic equations with three independent variables:

$$\begin{aligned} (-\beta_3 T^2)\alpha - \left(\beta_3 + \frac{1}{2}\beta_1 T^2\right)K - \left(\beta_3 T + \frac{1}{2}T^2\beta_2\right)p_0 &= \beta_3 \left(\theta_d c_d T - h_0 T - \frac{1}{2}\delta \cdot T^2\right) - \frac{1}{2}T^2 n_0 \\ - \left(\frac{1}{2}\beta_1 T^2 + \beta_3\right)\alpha - (2\beta_1)K - (\beta_1 T + \beta_2)p_0 &= \beta_1 \left(\theta_d c_d T - h_0 T - \frac{1}{2}\delta \cdot T^2\right) - n_0 \\ - \left(\frac{1}{2}\beta_2 T^2 + \beta_3 T\right)\alpha - (\beta_2 + \beta_1 T)K - (2\beta_2 T)p_0 &= \beta_2 \left(\theta_d c_d T - h_0 T - \frac{1}{2}\delta \cdot T^2\right) - n_0 T \end{aligned}$$

To simplify tracking, we replace the above system with the following matrix representation:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} \alpha \\ K \\ p_0 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}, \tag{A.1}$$

where

$$\begin{aligned} a_{11} &= -\beta_3 T^2, a_{12} = -\left(\beta_3 + \frac{1}{2}\beta_1 T^2\right), a_{13} = -\left(\beta_3 T + \frac{1}{2}T^2\beta_2\right), b_1 = \beta_3 \left(\theta_d c_d T - h_0 T - \frac{1}{2}\delta \cdot T^2\right) - \frac{1}{2}T^2 n_0 \\ a_{21} &= -\left(\beta_3 + \frac{1}{2}\beta_1 T^2\right), a_{22} = -(2\beta_1), a_{23} = -(\beta_1 T + \beta_2), b_2 = \beta_1 \left(\theta_d c_d T - h_0 T - \frac{1}{2}\delta \cdot T^2\right) - n_0 \\ a_{31} &= -\left(\beta_3 T + \frac{1}{2}\beta_2 T^2\right), a_{32} = -(\beta_1 T + \beta_2), a_{33} = -(2\beta_2 T), b_3 = \beta_2 \left(\theta_d c_d T - h_0 T - \frac{1}{2}\delta \cdot T^2\right) - n_0 T \end{aligned}$$

We denote the determinant of matrix A by $|A|$, and whenever $|A^{-1}|$ exists, according to Cramér's rule,

$$\alpha^{red} = \frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}, K^{red} = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}, p_0^{red} = \frac{\begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}. \tag{A.2}$$

In (A.2) we use the superscript *red* to reflect the solution for the reduced problem (6). We now address the feasibility of the solution under the constraint. Following (6.1) and (A.2), the condition of non-negativity of the demand is

$$\beta_1 \begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix} + \beta_2 \begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix} + \beta_3 \begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix} \leq n_0 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}. \tag{A.3}$$

suggested as a future research avenue. Other potential areas of future research could include: determining the optimal purchasing price that the service provider should offer couples in the primary market; extending the model to allow for negotiations concerning the contract period; and an empirical study to validate the model in a real-life situation.

CRedit authorship contribution statement

Avi Herbon: Methodology, Software, Writing - original draft. **Yael Lahav:** Writing - original draft, Software, Validation. **Uriel Spiegel:** Conceptualization, Investigation, Supervision, Writing - review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix B. Proof of Theorem 1

- (a) The optimality of problem (6) may result from two possible scenarios. The first, represented by (A.2), is associated with the case where constraint (A.3) is not violated under the solution of the reduced problem, $(K^{red}, p_0^{red}, \alpha^{red})$. The optimality and uniqueness of solution (A.2) under the existence of $|A^{-1}|$ is supported by Cramér's theorem [10]. In the case where constraint (A.3) is violated (the second scenario), the solution is represented by $(K^{act}, p_0^{act}, \alpha^{act})$, that is, a root of the equation $n_0 - \beta_1 K - \beta_2 p_0 - \beta_3 \alpha = 0$. We use superscript *act* to reflect the active solution for problem (6). In this scenario, constraint (6.1) is active and $\pi(K^{act}, p_0^{act}, \alpha^{act}) = 0$. Thus, to summarize, the feasibility under constraint (6.1) is ensured in both scenarios; however, $(K^{red}, p_0^{red}, \alpha^{red})$ additionally complies with (A.3) in order to be considered.
- (b) To ensure that an optimal solution is also the global maximum, a sufficient condition is that the Hessian matrix is negative definite and that the feasible domain defined by the constraints is a convex set. After taking the second order derivatives, the Hessian matrix is

$$\frac{1}{T} \begin{bmatrix} -2\beta_1 & -\beta_1 T - \beta_2 & -\frac{1}{2}\beta_1 T^2 - \beta_3 \\ -\beta_1 T - \beta_2 & -2\beta_2 T & -\frac{1}{2}\beta_2 T^2 - \beta_3 T \\ -\frac{1}{2}\beta_1 T^2 - \beta_3 & -\frac{1}{2}\beta_2 T^2 - \beta_3 T & -\beta_3 T^2 \end{bmatrix}$$

Since constraint (6.1) defines a convex set (as also do (5.1)-(5.3)), it is sufficient to show that the Hessian is negative definite. The conditions are:

$$-2\beta_1 < 0 \tag{B.1}$$

$$4\beta_1\beta_2 T - (\beta_1 T + \beta_2)^2 > 0 \tag{B.2}$$

$$\begin{aligned} & -2\beta_1 \left[2\beta_2\beta_3 T^3 - \left(\frac{1}{2}\beta_2 T^2 + \beta_3 T \right)^2 \right] + \\ & (\beta_1 T + \beta_2) \left[\beta_3 T^2 (\beta_1 T + \beta_2) - \left(\frac{1}{2}\beta_2 T^2 + \beta_3 T \right) \left(\frac{1}{2}\beta_1 T^2 + \beta_3 \right) \right] - \\ & \left(\frac{1}{2}\beta_1 T^2 + \beta_3 \right) \left[(\beta_1 T + \beta_2) \left(\frac{1}{2}\beta_2 T^2 + \beta_3 T \right) - 2\beta_2 T \left(\frac{1}{2}\beta_1 T^2 + \beta_3 \right) \right] < 0 \end{aligned} \tag{B.3}$$

□

Appendix C. Proof of Lemma 1

The objective is $\pi(p_s) = \frac{\beta_5}{T} (p_s e^{-\beta_6 p_s})$. The first derivative with respect to the variable p_s is

$$\frac{\partial \pi(p_s)}{\partial p_s} = \frac{\beta_5}{T} [(e^{-\beta_6 p_s}) - p_s \beta_6 e^{-\beta_6 p_s}].$$

Equating the above expression to zero yields $p_s^* = \frac{1}{\beta_6}$. The second derivative is

$$\frac{\partial^2 \pi(p_s)}{\partial p_s^2} = \frac{\partial}{\partial p_s} \left(\frac{\beta_5}{T} (e^{-\beta_6 p_s}) - \frac{\beta_5}{T} p_s \beta_6 e^{-\beta_6 p_s} \right) = -\frac{\beta_5 \beta_6}{T} (e^{-\beta_6 p_s}) - \frac{\beta_5 \beta_6}{T} [(e^{-\beta_6 p_s}) - p_s \beta_6 e^{-\beta_6 p_s}]$$

Inserting $p_s = \frac{1}{\beta_6}$ into the above expression yields $\frac{\partial^2 \pi(p_s)}{\partial p_s^2} < 0$. We conclude that $p_s^* = \frac{1}{\beta_6}$ is maximum. □

References

[1] Acharya J, Yates RD. Resource and power costs in dynamic spectrum allocation. Proc. CISS 2008:938–43.

[2] Aziz HA, Saleh M, Rasmy MH, ElShishiny H. Dynamic room pricing model for hotel revenue management systems. Egyptian Inf. J. 2011;12(3):177–83.

[3] Bitran G, Caldentey R. An overview of pricing models for revenue management. Manufact. Serv. Operat. Manag. 2003;5(3):203–29.

[4] Burton PJ, Sanders K. Patient attitudes to donation of embryos for research in Western Australia. MJA 2004;180:559–61.

[5] Chambers GM, Adamson GD, Eijkemans MJ. Acceptable cost for the patient and society. Fertil. Steril. 2013;100(2):319–27.

[6] Chen JM, Chen LT. Periodic pricing and replenishment policy for continuously decaying inventory with multivariate demand. Appl. Math. Model. 2007;31(9):1819–28.

[7] Chou WS, Julian P. Inventory system of deteriorating items with exponential declining demand and partial backlogging. J. Interdiscipl. Math. 2015;18(1–2):89–96.

[8] Connolly MP, Griesinger G, Ledger W, Postma MJ. The impact of introducing patient co-payments in Germany on the use of IVF and ICSI: a price-elasticity of demand assessment. Human Reprod. 2009;24(11):2796–800.

[9] Cooke DL. Public and Low-Cost IVF. Organizat. Manag. IVF Units 2016:301–14. https://doi.org/10.1007/978-3-319-29373-8_16.

[10] Cramér H, Wold H. Some theorems on distribution functions. J Lond Mathemat Soc 1936;1(4):290–4.

[11] Gan SS, Pujawana IN, Widodob SB. Pricing decision model for new and re-manufactured short-life cycle products with time-dependent demand. Operat Res Perspect 2015;1:1–12.

[12] Gilson, H. (2012). Center for Reproductive Health (1986-1995). Embryo Project Encyclopedia (2008-09-30). ISSN: 1940-5030. <http://embryo.asu.edu/handle/10776/1946>.

[13] Golombok S, Wilkinson S, editors. Regulating reproductive donation. Cambridge University Press; 2016.

[14] Hodel TB, Cong S. Parking space optimization services, a uniformed web application architecture. Intelligent Transport Systems and Services. ITS World Congr Proc 2003:16–20.

[15] De Lacy S. Decisions for the fate of frozen embryos: fresh insights into patients' thinking and their rationales for donating or discarding embryos. Human Reprod 2007;22(6):1751–8.

[16] Koninckx PR, Schotsmans P. Frozen embryos: too cold to touch? Human Reprod 1996;11:1841–2.

[17] Lee DH. Note on the optimal pricing strategy in the discrete-time Geo/Geo/1 queuing system with sojourn time-dependent reward. Operat Res Perspect 2017;4:113–7.

[18] Liu X, Tang O, Huang P. Dynamic pricing and ordering decision for the perishable food of the supermarket using RFID technology. Asia Pacific J Market Logist 2008;20:7–22.

[19] Lyerly AD, Faden RR. Willingness to donate frozen embryos for stem cell research. Science 2007;317:46–7.

[20] Mackie-Mason JK, Varian HR. Pricing congestible network resources. IEEE J Selected Areas Commun 1995;13(7):1141–9.

[21] Coticchio G, Debrock S, Lundin K, Plancha CE, Prados F, Rienzi L, Verheyen G, Woodward B, Vermeulen N. Revised guidelines for good practice in IVF laboratories. Human Reproduct 2016;31(4):685–6.

[22] Manthari B, Nasser N, Hassanein H. Congestion pricing in wireless cellular networks. IEEE Commun Survey Tutor 2011;13(3):358–71.

[23] McMahon CA, Gibson FL, Leslie GI, Saunders DM, Porter KA, Tennant CC. Embryo donation for medical research: attitudes and concerns of potential donors. Human Reprod 2003;18(4):871–7.

[24] Michelmann HW, Nayudu P. Cryopreservation of human embryos. Cell Tissue Bank

- 2006;7:135–41.
- [25] Merrill JP. Embryos in limbo. *The New Atlantis* 2009;24:18–28.
- [26] Oren SS, Smith AS. Critical mass and tariff structure in electronic communications markets. *Bell J Econom* 1981;12(2):467–87.
- [27] Qu SJ, Zhou YY, Zhang YL, Wahab MIM, Zhang G, Ye YY. Optimal strategy for a green supply chain considering shipping policy and default risk. *Comput Ind Eng* 2019;131:172–86.
- [28] Provoost V, Pennings G, DeSutter P, Gerris J, Van der Velde A, DeLissnyder E, Dhont M. Infertility patients' beliefs about their embryos and their disposition preferences. *Human Reprod* 2009;24(4):896–905.
- [29] Provoost V, Pennings G, De Sutter P, Gerris J, Van de Velde A, Dhont M. Patients' conceptualization of cryopreserved embryos used in their fertility treatment. *Human Reprod* 2010;25(3):705–13.
- [30] Sambangi S. Cloud data storage services considering public audit for security. *Global J Comput Sci Technol* 2013;13(1):1–5. Online ISSN: 0975-4172 & Print ISSN: 0975-4350.
- [31] Sattenspiel L. *The geographic spread of infectious diseases: models and applications*. Princeton, NJ, USA: Princeton University Press; 2009. ISBN 978-0-691-12132-1.
- [32] Theodorovic D, Lucic P. Intelligent parking systems. *Eur J Oper Res* 2006;175:1666–81.
- [33] Thakker Y, Woods S. Storage of vaccines in the community: weak link in the cold chain. *Br Med J* 1992;304:756–8.
- [34] Tuviam J, Priyanka S. Security services of data storage in the cloud considering third party servers. *Int Res J Eng Technol* 2017;4(10):70–3.
- [35] You PS. Inventory policy for products with price and time-dependent demands. *J Operat Res Soc* 2005;56:870–3.