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Article

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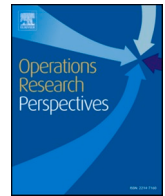
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Mixed Network DEA with Shared Resources: A Case of Measuring Performance for Banking Industry

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ABSTRACT

As a bank manager, not only the overall performance of the bank is considered, but also the performances of all divisions needed to be evaluated. Though there have been many works investigating the performance of banking by Data Envelopment Analysis (DEA) since the 1990s, the internal structure and operational processes of a banking system are never such simple as represented in existing DEA models. We propose a new DEA modelling technique to solve a mixed network structure, which consists of serial and parallel processes with shared input resources. Relational decomposition and additive aggregation network models are developed to measure the overall efficiency as well as the efficiencies of all components within the model. The empirical results obtained for the Taiwan banking industry acknowledges the discriminating power of our structure models compared to the ones proposed in existing literature. We also find evidence that traditional banking (lending) has been gradually outperformed by low-risk businesses (investment or non-interest-based services). Some managerial implications are then discussed to improve the system's efficiency.

1. Introduction

The liberalization and internationalization of financial market have led banks worldwide to diversify their financial services to generate additional sources of profit [1]. Stated that well-diversified banks will enjoy more benefits and competitive advantage in the banking market. Subsequently, not only the overall performance of the bank is of concern, but also the performances of all branches/processes need to be evaluated.

Many research works have investigated the performance of banking through using Data Envelopment Analysis (DEA) since the 1990s [2–5]. In order to illustrate the banking's performance, existing studies focus on two "bird's eye" views, being, "black-box" and "network structure". The former inherits the traditional DEA models, such as CCR [6] or BCC [7], and therefore, neglects the internal processes of banking performance [8–10]. Such models only provide information on the overall performance of a bank without taking into account the impact of intermediate products. On the other hand, the latter attempts to open the

"black box" by typically splitting it into two stages [2]. Constructed a two-stage network DEA in which the first stage profitability, determines the performance of the second stage, marketability. Based on the theories of multiple shared inputs [11], further developed a shared resources model to stress that inputs in certain network structures cannot be separated into independent components in the first stage and the second stage [12]. Reviewed a series of network models including parallel and serial processes and suggested approaches to solve them [13]. Utilized a two-stage process in which deposits are treated as the "bridge" between initial resources (labor and capital) and final outputs (desire and undesired outputs), to evaluate banks' network revenue performance [5]. Presented banking business as a serial three-stage process in which there exists an internal resource imbalance (IRI) between processes.

Though these researches contribute to the literature on the measurement of banking's performance, they appear to oversimplify the real world situation. Not only do such existing simplified models often generate insufficient information but also engender misleading analysis

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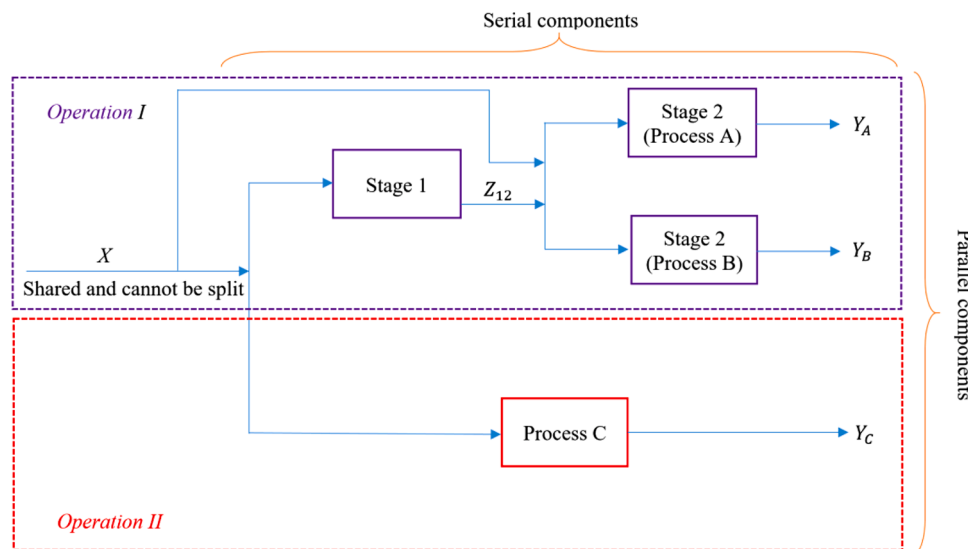


Figure 1. General mixed serial and parallel network structure

results for the bank’s managers. Therefore, they are limited to theoretical applications only. With the liberalization and internationalization of financial market, modern banking systems have become more diversified and complex than what were described before. For instance, on the one hand, it operates through a serial process with shared resources and intermediate products, while, on the other hand, there also exist some parallel processes or operations within the system. The importance of such models to the realistic operation of banks has not been well studied in previous investigations. Due to the shortage of literature on this aspect, we propose a mixed network structure for banking’s production, which consists of both series processes and parallel processes/operations with input resources shared by all components.

Figure 1 illustrates a mixed serial and parallel network structure with shared resources. Specifically, Operation I is a serial process consisting of two stages. In stage 1, a portion of resource (X) is used to obtain intermediate outputs (Z_{12}), which then along with another part of initial inputs (X), become the materials for stage 2. Stage 2 contains two parallel sub-processes (processes A and B), in which final outputs, namely, Y_A and Y_B , are produced, respectively. Operation II is an independent process (process C) that transfers multiple shared inputs (X) directly into final products Y_C .

Note that the literature of network DEA experienced some papers deriving solutions for either serial or parallel-processes systems. For instant, [14] and [15] both employ a serial stages network model with specific and shared resources to evaluate the efficiency of high-tech industries in China while a parallel structure of three processes, utilizing shared inputs is adopted in [16]. However, in the real situation, the relationships between stages/processes and resources are unfortunately not such simple [17]. Based on the operation of multimode bus transit, proposed a mixed structure wherein the two independent activities (parallel structure) jointly contribute to generate intermediate products for the sequential activity (serial structure).

It is important to note that such mixed structure networks are typical cases because they are flexible and mainly depends on the manner of the system’s operation. Therefore, we inherit the spirit of [17] to propound a compliance mixed network structure with modern banking’s activities. To the extent of our knowledge, there is no existing research ever tried the evaluate the performance of banking industry by utilizing such framework. Furthermore, differ from ([17]), in order to obtain the system efficiency and each component efficiency, two new network DEA techniques are proposed in this paper. Following the technique of [18, 19], a relational model is proposed to solve this mixed network structure

and it is shown that the system efficiency can be decomposed into divisional efficiencies. We also develop an additive aggregation model by which system efficiency and all branch efficiencies are also derived. Because this additive model cannot be converted into a linear program, we improve the parametric linear method of [20] to solve this model. The most advantage of proposed models is that they are not only helpful in evaluating every stages’ efficiency but also informative in disclosing how important is each stage contributes to the system’s score.

The commercial banking system in Taiwan is a chartered and regulated system very similar to that of western financial institutions. After the privatization and liberalization of financial market in 1989, the degree of Taiwan banking sector’s competition has gradually increased, which resulted in a low level of profits and high level of non-performing loans for banks ([21]). In 2002, Taiwanese authorities launched the first major financial reform in order to improve the performance of banking industry¹. Since then, the efficiency and productivity of this sector have become a matter of concern [22,23]. Inheriting the spirit of these studies, we employ the new proposed model to better illustrate the realistic operation of Taiwan banks and measure their performance. As of July 2017, Taiwan has 39 domestic commercial banks of which 16 are a part of financial holding companies (FHC).

The main purpose of our paper is to figure out the solution for the above mixed parallel-serial network with shared resources mentioned above and experiment its discriminating power with the sample of Taiwan banks. Equivalently, such questions will be explored in our work: (1) Is the explanatory power of our new models better than other traditional DEA models? (2) Is it possible to identify branches and sub-branches efficiencies in banks to derive effective policy implications for different divisions? (3) Can we distinguish new trends in bank business to guide new policy and strategy?

The remainder of the paper presents the following. Section 2 proposes a real-based mixed network structure in banks that consists of series processes and parallel processes with shared resources. A relational decomposition network DEA model, as well as an additive aggregation network DEA model, are developed to evaluate the performance of this banking system. Section 3 provides the empirical

¹ Compared to the revenue and profit from the traditional service businesses of loans and mortgages, the revenue and profit from banks’ consumer finance and wealth management departments have been growing much faster (Cook, Hababou and Tuenter, 2000).

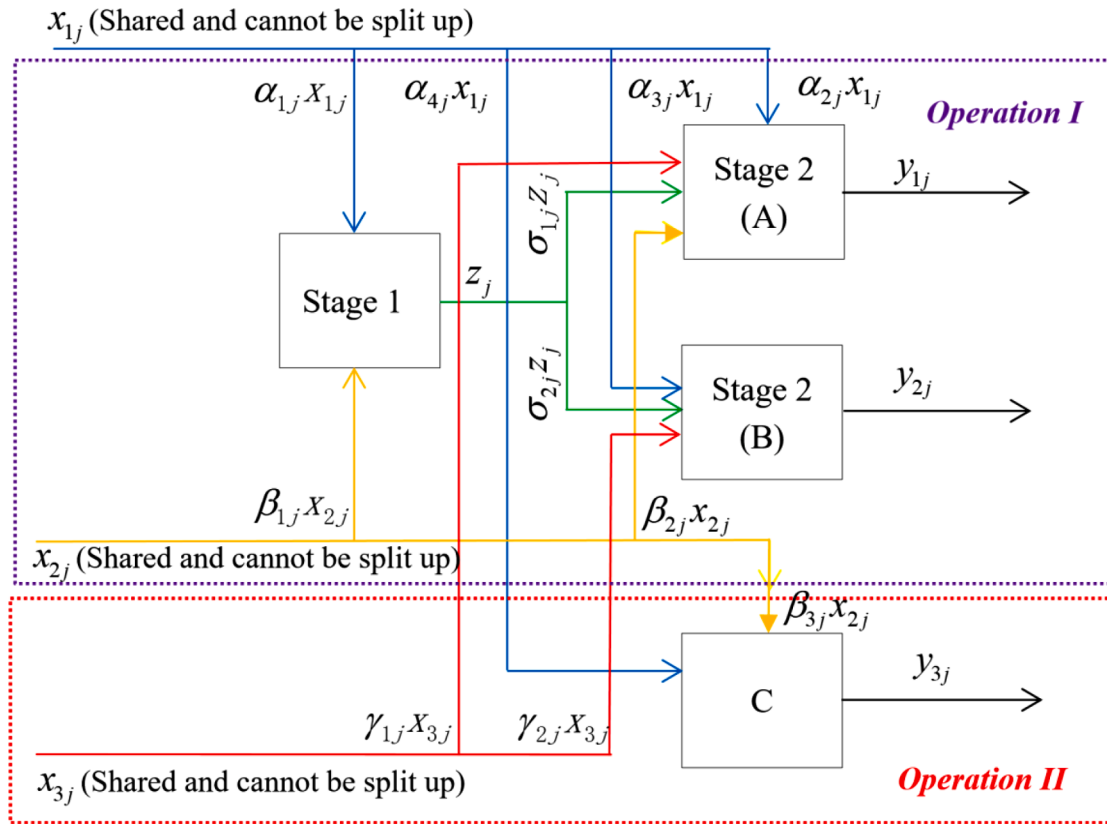


Figure 2. A symbolized mixed network structure with shared resources for banks

results of 26 banks in Taiwan associated with some managerial implications. Final section is the conclusion.

2. Modelling Network DEA

After conducting targeted surveys with selected banking experts we construct a symbolized mixed network structure shown in Figure 2. All experts surveyed were senior executives at the management level in Taiwan banking industry. They were chosen to answer questions in banking operation decomposition. All of the questions are about processes and resources of banking and only have yes/no answers. Finally, we consult these experts about our framework to get advice. In conjunction with the financial intermediate approach², [24] we choose labor, capital, and funds as inputs and loans, investments, and non-interest income as outputs, respectively. Deposits are employed as the intermediate product.

We assume that there is a set of n DMUs (decision-making units), and that for each DMU_j ($j=1,2,\dots, n$) the system consists of two parallel operations. Operation I is a series process consisting of two stages. In stage 1 (fund-rising stage), multiple shared inputs x_{1j} (labor) and x_{2j} (capital) are used to produce output z_j (deposits), which is an intermediate product that is also an input in stage 2. Stage 2 (fund-spreading stage) has two parallel processes A (lending) and B (investing), in which the former generates final output y_{1j} by using shared inputs x_{1j} , x_{2j} , and x_{3j} (funds) along with the intermediate measure z_j , while the latter results in the final output y_{2j} by utilizing shared inputs x_{1j} and x_{3j} and intermediate product z_j . Operation II has only process C (non-interest based services), the shared inputs x_{1j} and x_{2j} are utilized to directly

produce the final output y_{3j} .

Since inputs x_{1j} , x_{2j} , and x_{3j} are shared by both operations, input x_{1j} is divided into four portions: $\alpha_{1j}x_{1j}$, $\alpha_{2j}x_{1j}$, $\alpha_{3j}x_{1j}$, and $\alpha_{4j}x_{1j}$ ($0 \leq \alpha_{1j}, \alpha_{2j}, \alpha_{3j}, \alpha_{4j} \leq 1$ and $\alpha_{1j} + \alpha_{2j} + \alpha_{3j} + \alpha_{4j} = 1$), corresponding to the portions used by stage 1, processes A, B, and C, respectively. The shared input x_{2j} is divided into three portions: $\beta_{1j}x_{2j}$, $\beta_{2j}x_{2j}$, and $\beta_{3j}x_{2j}$ ($0 \leq \beta_{1j}, \beta_{2j}, \beta_{3j} \leq 1$, and $\beta_{1j} + \beta_{2j} + \beta_{3j} = 1$), corresponding to the portions used by stage 1, process A, and process C, respectively. The shared input x_{3j} is divided into $\gamma_{1j}x_{3j}$ and $\gamma_{2j}x_{3j}$ ($0 \leq \gamma_{1j}, \gamma_{2j} \leq 1$ and $\gamma_{1j} + \gamma_{2j} = 1$), which are used respectively by processes A and B. Intermediate measure z_j is also shared by the two processes A and B and is divided into $\sigma_{1j}z_j$ and $\sigma_{2j}z_j$ ($0 \leq \sigma_{1j}, \sigma_{2j} \leq 1$ and $\sigma_{1j} + \sigma_{2j} = 1$). As in Chen et al. (2010), we assume that $\alpha_{1j}, \alpha_{2j}, \alpha_{3j}$ and α_{4j} have upper and lower bounds as $L_{ij}^\alpha \leq \alpha_{ij} \leq U_{ij}^\alpha$ ($i=1, 2, 3, 4$). Similarly, β_{1j}, β_{2j} and β_{3j} have upper and lower bounds as $L_{ij}^\beta \leq \beta_{ij} \leq U_{ij}^\beta$ ($i=1, 2, 3$); γ_{1j} and γ_{2j} have upper and lower bounds as $L_{ij}^\gamma \leq \gamma_{ij} \leq U_{ij}^\gamma$ ($i=1, 2$); and σ_{1j} and σ_{2j} have upper and lower bounds as $L_{ij}^\sigma \leq \sigma_{ij} \leq U_{ij}^\sigma$ ($i=1, 2$).

If we consider operation I (stage 1), processes A and B, and operation II (process C) as isolated “black-boxes”, based on the standard input-oriented CCR model, the multiplier form of their efficiencies for a specific unit, namely DMU_o , are:

$$\begin{aligned}
 e_o^1 &= \max \left(\frac{\eta^1 z_o}{\alpha_{1o} v_1^1 x_{1o} + \beta_{1o} v_2^1 x_{2o}} \right) \\
 s.t. \quad & \eta^1 z_j \leq \alpha_{1j} v_1^1 x_{1j} + \beta_{1j} v_2^1 x_{2j} \forall j \\
 & L_{1j}^\alpha \leq \alpha_{1j} \leq U_{1j}^\alpha \forall j \\
 & L_{1j}^\beta \leq \beta_{1j} \leq U_{1j}^\beta \forall j \\
 & v_1^1, v_2^1, \eta^1 \geq 0
 \end{aligned} \tag{1}$$

² There are several ways to model bank behavior and to set the input and output variables. The production approach, financial intermediate approach, and value added approach are the most common ways [3], Paradi et al., 2011, and Avkrian, 2015).

$$e_o^A = \max\left(\frac{u_1 y_{1o}}{\alpha_{2o} v_1^H x_{1o} + \beta_{2o} v_2^H x_{2o} + \gamma_{1o} v_3^I x_{3o} + \sigma_{1o} \eta^H z_o}\right)$$

s.t. $u_1 y_{1j} \leq \alpha_{2j} v_1^H x_{1j} + \beta_{2j} v_2^H x_{2j} + \gamma_{1j} v_3^I x_{3j} + \sigma_{1j} \eta^H z_j \forall j$

$$L_{2j}^\alpha \leq \alpha_{2j} \leq U_{2j}^\alpha \forall j$$

$$L_{2j}^\beta \leq \beta_{2j} \leq U_{2j}^\beta \forall j$$

$$L_{1j}^\gamma \leq \gamma_{1j} \leq U_{1j}^\gamma \forall j$$

$$L_{1j}^\sigma \leq \sigma_{1j} \leq U_{1j}^\sigma \forall j$$

$$v_1^H, v_2^H, v_3^I, \eta^H, u_1 \geq 0$$
(2)

$$e_o^B = \max\left(\frac{u_2 y_{2o}}{\alpha_{3o} v_1^H x_{1o} + \gamma_{2o} v_3^H x_{3o} + \sigma_{2o} \eta^H z_o}\right)$$

s.t. $u_2 y_{2j} \leq \alpha_{3j} v_1^H x_{1j} + \gamma_{2j} v_3^H x_{3j} + \sigma_{2j} \eta^H z_j \forall j$

$$L_{3j}^\alpha \leq \alpha_{3j} \leq U_{3j}^\alpha \forall j$$

$$L_{2j}^\gamma \leq \gamma_{2j} \leq U_{2j}^\gamma \forall j$$

$$L_{2j}^\sigma \leq \sigma_{2j} \leq U_{2j}^\sigma \forall j$$

$$v_1^H, v_3^H, \eta^H, u_2 \geq 0$$
(3)

$$e_o^C = \max\left(\frac{u_3 y_{3o}}{\alpha_{4o} v_1^V x_{1o} + \beta_{3o} v_2^H x_{2o}}\right)$$

s.t. $u_3 y_{3j} \leq \alpha_{4j} v_1^V x_{1j} + \beta_{3j} v_2^H x_{2j} \forall j$

$$L_{4j}^\alpha \leq \alpha_{4j} \leq U_{4j}^\alpha \forall j$$

$$L_{3j}^\beta \leq \beta_{3j} \leq U_{3j}^\beta \forall j$$

$$v_1^V, v_2^H, u_3 \geq 0$$
(4)

where η^a (a = I to II), v_1^b (b = I to IV), v_2^c (c = I to III), v_3^d (d = I to II), u_e (e = 1 to 3) are the weights for intermediate measures (z_j), shared inputs, (x_{1j}, x_{2j}, x_{3j}), and outputs (y_{1j}, y_{2j}, y_{3j}), respectively.

2.1. Relational decomposition network DEA

Several researchers (e.g., [11,25]) have concluded that separately optimizing models (1), (2), (3) and (4) results in a misleading conclusion of the intermediate measure z_j due to the conflict in its role (z_j is treated as input and output). The centralized perspective, therefore, attempts to determine a set of optimal weights on the intermediate measures that maximize the aggregate or global efficiency score.

Similar to the assumptions of [26], as well as the centralized model in [27], we assume that the weights for the intermediate measures (z_j) are equal for the two stages ($\eta^I = \eta^{II} = \eta$). We also assume that the weights for all possible divisions of shared inputs (x_{1j}, x_{2j}, x_{3j}) are the same - that is, $v_1^I = v_1^{II} = v_1^{III} = v_1^V = v_1$, $v_2^I = v_2^{II} = v_2^{III} = v_2$, and $v_3^I = v_3^{II} = v_3$ because these are the same types of inputs. The idea of solving the whole system efficiency and divisional efficiency can be following by [12].

In detail, we start from the overall efficiency for stage 2 that can be defined as a weighted average of the efficiencies of the two processes A and B: $e_o^2 = w^A e_o^A + w^B e_o^B$ where w^A and w^B are user-specified weights such that $w^A + w^B = 1$. Since these weights are utilized to reflect the relative importance of the performances of processes A and B in the second stage (operation I), one rational selection of each weight is the fraction of resources devoted to each process to total resources utilized for stage 2.

Hence, $w^A = \frac{\alpha_{2o} v_1 x_{1o} + \beta_{2o} v_2 x_{2o} + \gamma_{1o} v_3 x_{3o} + \sigma_{1o} \eta z_o}{(\alpha_{2o} + \alpha_{3o}) v_1 x_{1o} + \beta_{2o} v_2 x_{2o} + \gamma_{2o} v_3 x_{3o} + \sigma_{2o} \eta z_o}$ and $w^B = \frac{\alpha_{3o} v_1 x_{1o} + \gamma_{2o} v_3 x_{3o} + \sigma_{2o} \eta z_o}{(\alpha_{2o} + \alpha_{3o}) v_1 x_{1o} + \beta_{2o} v_2 x_{2o} + \gamma_{2o} v_3 x_{3o} + \sigma_{2o} \eta z_o}$, where the numerators indicate the “size” of or amount of resources consumed by processes A and B, respectively, while denominators reflects the total size of stage 2. The network model

for stage 2 in operation I can be measured as:

$$e_o^2 = \frac{u_1 y_{1o} + u_2 y_{2o}}{(\alpha_{2o} + \alpha_{3o}) v_1 x_{1o} + \beta_{2o} v_2 x_{2o} + \gamma_{2o} v_3 x_{3o} + \sigma_{2o} \eta z_o}$$
(5)

In a similar manner, we define the weights w^1 for stage 1 and w^2 for stage 2 in the series process (operation I) as: $w^1 = \frac{\alpha_{1o} v_1 x_{1o} + \beta_{1o} v_2 x_{2o}}{(\alpha_{1o} + \alpha_{2o} + \alpha_{3o}) v_1 x_{1o} + (\beta_{1o} + \beta_{2o}) v_2 x_{2o} + \gamma_{3o} v_3 x_{3o} + \eta z_o}$, and $w^2 = \frac{(\alpha_{2o} + \alpha_{3o}) v_1 x_{1o} + \beta_{2o} v_2 x_{2o} + \gamma_{2o} v_3 x_{3o} + \sigma_{2o} \eta z_o}{(\alpha_{1o} + \alpha_{2o} + \alpha_{3o}) v_1 x_{1o} + (\beta_{1o} + \beta_{2o}) v_2 x_{2o} + \gamma_{3o} v_3 x_{3o} + \eta z_o}$. Here, the relation between series (operation I) efficiency and divisions’ efficiencies can be expressed as $1 - e_o^{1,2} = w^1(1 - e_o^1) + w^2(1 - e_o^2)$. Thus, we propose model (6) to measure the efficiency of the series process (operation I).

$$e_o^{1,2} = \frac{u_1 y_{1o} + u_2 y_{2o}}{(\alpha_{1o} + \alpha_{2o} + \alpha_{3o}) v_1 x_{1o} + (\beta_{1o} + \beta_{2o}) v_2 x_{2o} + \gamma_{3o} v_3 x_{3o} + \eta z_o}$$
(6)

Since our structure contains of two parallel operations, we finally define the weights for each operation, that are $w^{1,2} = \frac{(\alpha_{1o} + \alpha_{2o} + \alpha_{3o}) v_1 x_{1o} + (\beta_{1o} + \beta_{2o}) v_2 x_{2o} + \gamma_{3o} v_3 x_{3o}}{v_1 x_{1o} + v_2 x_{2o} + v_3 x_{3o}}$ for operation I, and $w^C = \frac{\alpha_{4o} v_1 x_{1o} + \beta_{3o} v_2 x_{2o}}{v_1 x_{1o} + v_2 x_{2o} + v_3 x_{3o}}$ for operation II (process C).

We now obtain the relationship between system efficiency and the two operations’ efficiencies as: $e_o = w^{1,2} e_o^{1,2} + w^C e_o^C$. Therefore, the network model for this parallel system is proposed as:

$$e_o = \max\left(\frac{u_1 y_{1o} + u_2 y_{2o} + u_3 y_{3o}}{v_1 x_{1o} + v_2 x_{2o} + v_3 x_{3o}}\right)$$

s.t. $\eta z_j \leq \alpha_{1j} v_1 x_{1j} + \beta_{1j} v_2 x_{2j} \forall j$

$$u_1 y_{1j} \leq \alpha_{2j} v_1 x_{1j} + \beta_{2j} v_2 x_{2j} + \gamma_{1j} v_3 x_{3j} + \sigma_{1j} \eta z_j \forall j$$

$$u_2 y_{2j} \leq \alpha_{3j} v_1 x_{1j} + \gamma_{2j} v_3 x_{3j} + \sigma_{2j} \eta z_j \forall j$$

$$u_3 y_{3j} \leq \alpha_{4j} v_1 x_{1j} + \beta_{3j} v_2 x_{2j} \forall j$$

$$\alpha_{1j} + \alpha_{2j} + \alpha_{3j} + \alpha_{4j} = 1 \forall j$$

$$\beta_{1j} + \beta_{2j} + \beta_{3j} = 1 \forall j$$

$$\gamma_{1j} + \gamma_{2j} = 1 \forall j$$

$$\sigma_{1j} + \sigma_{2j} = 1 \forall j$$

$$L_{ij}^\alpha \leq \alpha_{ij} \leq U_{ij}^\alpha i = 1, 2, 3, 4 \forall j$$

$$L_{ij}^\beta \leq \beta_{ij} \leq U_{ij}^\beta i = 1, 2, 3 \forall j$$

$$L_{ij}^\gamma \leq \gamma_{ij} \leq U_{ij}^\gamma i = 1, 2 \forall j$$

$$L_{ij}^\sigma \leq \sigma_{ij} \leq U_{ij}^\sigma i = 1, 2 \forall j$$

$$v_1, v_2, v_3, \eta, u_1, u_2 \geq 0$$
(7)

We next adopt the Charnes-Cooper transformation [28] to convert the fractional model (7) to model (8):

$$e_o = \max(\mu_1 y_{1o} + \mu_2 y_{2o} + \mu_3 y_{3o})$$

s.t. $\omega_1 x_{1o} + \omega_2 x_{2o} + \omega_3 x_{3o} = 1$

$$\pi z_j - \alpha_{1j} \omega_1 x_{1j} - \beta_{1j} \omega_2 x_{2j} \leq 0 \forall j$$

$$\mu_1 y_{1j} - \alpha_{2j} \omega_1 x_{1j} - \beta_{2j} \omega_2 x_{2j} - \gamma_{1j} \omega_3 x_{3j} - \sigma_{1j} \pi z_j \leq 0 \forall j$$

$$\mu_2 y_{2j} - \alpha_{3j} \omega_1 x_{1j} - \gamma_{2j} \omega_3 x_{3j} - \sigma_{2j} \pi z_j \leq 0 \forall j$$

$$\mu_3 y_{3j} - \alpha_{4j} \omega_1 x_{1j} - \beta_{3j} \omega_2 x_{2j} \leq 0 \forall j$$

$$\alpha_{1j} + \alpha_{2j} + \alpha_{3j} + \alpha_{4j} = 1 \forall j$$

$$\beta_{1j} + \beta_{2j} + \beta_{3j} = 1 \forall j$$

$$\gamma_{1j} + \gamma_{2j} = 1 \forall j$$

$$\sigma_{1j} + \sigma_{2j} = 1 \forall j$$

$$L_{ij}^\alpha \leq \alpha_{ij} \leq U_{ij}^\alpha i = 1, 2, 3, 4 \forall j$$

$$L_{ij}^\beta \leq \beta_{ij} \leq U_{ij}^\beta i = 1, 2, 3 \forall j$$

$$L_{ij}^\gamma \leq \gamma_{ij} \leq U_{ij}^\gamma i = 1, 2 \forall j$$

$$L_{ij}^\sigma \leq \sigma_{ij} \leq U_{ij}^\sigma i = 1, 2 \forall j$$

$$\omega_1, \omega_2, \omega_3, \pi, \mu_1, \mu_2, \mu_3 \geq 0$$
(8)

By letting $\alpha'_{ij} = \alpha_{ij}\omega_1 (i = 1, 2, 3, 4)$, $\beta'_{ij} = \beta_{ij}\omega_2 (i = 1, 2, 3)$, $\gamma'_{ij} = \gamma_{ij}\omega_3 (i = 1, 2)$, and $\sigma'_{ij} = \sigma_{ij}\pi (i = 1, 2)$, model (8) can be linearized as:

$$\begin{aligned}
 e_o &= \max(\mu_1 y_{1o} + \mu_2 y_{2o} + \mu_3 y_{3o}) \\
 \text{s.t. } &\omega_1 x_{1o} + \omega_2 x_{2o} + \omega_3 x_{3o} = 1 \\
 &\pi z_j - \alpha'_{1j} x_{1j} - \beta'_{1j} x_{2j} \leq 0 \forall j \\
 &\mu_1 y_{1j} - \alpha'_{2j} x_{1j} - \beta'_{2j} x_{2j} - \gamma'_{1j} x_{3j} - \sigma'_{1j} z_j \leq 0 \forall j \\
 &\mu_2 y_{2j} - \alpha'_{3j} x_{1j} - \gamma'_{2j} x_{3j} - \sigma'_{2j} z_j \leq 0 \forall j \\
 &\mu_3 y_{3j} - \alpha'_{4j} x_{1j} - \beta'_{3j} x_{2j} \leq 0 \forall j \\
 &\alpha'_{1j} + \alpha'_{2j} + \alpha'_{3j} + \alpha'_{4j} = \omega_1 \forall j \\
 &\beta'_{1j} + \beta'_{2j} + \beta'_{3j} = \omega_2 \forall j \\
 &\gamma'_{1j} + \gamma'_{2j} = \omega_3 \forall j \\
 &\sigma'_{1j} + \sigma'_{2j} = \pi \forall j \\
 &L_{ij}^\alpha \omega_1 \leq \alpha'_{ij} \leq U_{ij}^\alpha \omega_1 \quad i = 1, 2, 3, 4 \forall j \\
 &L_{ij}^\beta \omega_2 \leq \beta'_{ij} \leq U_{ij}^\beta \omega_2 \quad i = 1, 2, 3 \forall j \\
 &L_{ij}^\gamma \omega_3 \leq \gamma'_{ij} \leq U_{ij}^\gamma \omega_3 \quad i = 1, 2 \forall j \\
 &L_{ij}^\sigma \pi \leq \sigma'_{ij} \leq U_{ij}^\sigma \pi \quad i = 1, 2 \forall j \\
 &\omega_1, \omega_2, \omega_3, \pi, \mu_1, \mu_2, \mu_3 \geq 0
 \end{aligned} \tag{9}$$

2.2. Additive aggregation network DEA

Based on [29], we develop an additive network model to measure the efficiency of the network structure shown in Figure 2. Different to the relational model, we define the overall efficiency of this system as a weighted average of the serial and parallel component efficiencies: $e_o = \lambda^{1,2} [\lambda^1 e_o^1 + \lambda^2 (\lambda^A e_o^A + \lambda^B e_o^B)] + \lambda^C e_o^C$, where $\lambda^A, \lambda^B, \lambda^1, \lambda^2, \lambda^{1,2}, \lambda^C$ are constant weights selected by the user, which can be regarded as preferences over the processes, that $\lambda^A + \lambda^B = 1; \lambda^1 + \lambda^2 = 1; \lambda^{1,2} + \lambda^C = 1$. Thus, the additive network model is developed as:

$$\begin{aligned}
 e_o &= \max \left\{ \lambda^C \frac{u_3 y_{3o}}{\alpha_{4o} v_1 x_{1o} + \beta_{3o} v_2 x_{2o}} + \lambda^{1,2} \left[\lambda^1 \frac{\eta z_o}{\alpha_{1o} v_1 x_{1o} + \beta_{1o} v_2 x_{2o}} \right. \right. \\
 &\quad \left. \left. + \lambda^2 \left(\lambda^A \frac{u_1 y_{1o}}{\alpha_{2o} v_1 x_{1o} + \beta_{2o} v_2 x_{2o} + \gamma_{1o} v_3 x_{3o} + \sigma_{1o} \eta z_o} \right. \right. \right. \\
 &\quad \left. \left. \left. + \lambda^B \frac{u_2 y_{2o}}{\alpha_{3o} v_1 x_{1o} + \gamma_{2o} v_3 x_{3o} + \sigma_{2o} \eta z_o} \right) \right] \right\} \\
 \text{s.t. } &\eta z_j \leq \alpha_{1j} v_1 x_{1j} + \beta_{1j} v_2 x_{2j} \forall j \\
 &u_1 y_{1j} \leq \alpha_{2j} v_1 x_{1j} + \beta_{2j} v_2 x_{2j} + \gamma_{1j} v_3 x_{3j} + \sigma_{1j} \eta z_j \forall j \\
 &u_2 y_{2j} \leq \alpha_{3j} v_1 x_{1j} + \gamma_{2j} v_3 x_{3j} + \sigma_{2j} \eta z_j \forall j \\
 &u_3 y_{3j} \leq \alpha_{4j} v_1 x_{1j} + \beta_{3j} v_2 x_{2j} \forall j \\
 &\alpha_{1j} + \alpha_{2j} + \alpha_{3j} + \alpha_{4j} = 1 \forall j \\
 &\beta_{1j} + \beta_{2j} + \beta_{3j} = 1 \forall j \\
 &\gamma_{1j} + \gamma_{2j} = 1 \forall j \\
 &\sigma_{1j} + \sigma_{2j} = 1 \forall j \\
 &L_{ij}^\alpha \leq \alpha_{ij} \leq U_{ij}^\alpha \quad i = 1, 2, 3, 4 \forall j \\
 &L_{ij}^\beta \leq \beta_{ij} \leq U_{ij}^\beta \quad i = 1, 2, 3 \forall j \\
 &L_{ij}^\gamma \leq \gamma_{ij} \leq U_{ij}^\gamma \quad i = 1, 2 \forall j \\
 &L_{ij}^\sigma \leq \sigma_{ij} \leq U_{ij}^\sigma \quad i = 1, 2 \forall j \\
 &\lambda^A + \lambda^B = 1 \\
 &\lambda^1 + \lambda^2 = 1 \\
 &\lambda^{1,2} + \lambda^C = 1 \\
 &v_1, v_2, v_3, \eta, u_1, u_2 \geq 0
 \end{aligned} \tag{10}$$

To solve this type of additive models, both [30] and [20] respectively proposed two heuristic methods to obtain approximate optimal solutions. In this study we extend the parametric linear method of [20] to a mixed additive model (10).

First, let denote $\alpha'_{ij} = \alpha_{ij} v_1 (i = 1, 2, 3, 4)$, $\beta'_{ij} = \beta_{ij} v_2 (i = 1, 2, 3)$, $\gamma'_{ij} = \gamma_{ij} v_3 (i = 1, 2)$, $\sigma'_{ij} = \sigma_{ij} \eta (i = 1, 2)$ and consider $\theta_1 = \frac{\eta z_o}{\alpha'_{1o} x_{1o} + \beta'_{1o} x_{2o}}$, $\theta_2 = \frac{u_1 y_{1o}}{\alpha'_{2o} x_{1o} + \beta'_{2o} x_{2o} + \gamma'_{1o} x_{3o} + \sigma'_{1o} z_o}$, $\theta_3 = \frac{u_2 y_{2o}}{\alpha'_{3o} x_{1o} + \gamma'_{2o} x_{3o} + \sigma'_{2o} z_o}$. Then, by employing the Charnes-Cooper transformation, we obtain the following parametric linear model regarding $\theta_i (i = 1, 2, 3)$.

$$\begin{aligned}
 e_o &= \max \{ \lambda^C (u_3 y_{3o}) + \lambda^{1,2} [\lambda^1 \theta_1 + \lambda^2 (\lambda^A \theta_2 + \lambda^B \theta_3)] \} \\
 \text{s.t. } &\alpha'_{4o} x_{1o} + \beta'_{3o} x_{2o} = 1 \\
 &\eta z_o - \theta_1 (\alpha'_{1o} x_{1o} + \beta'_{1o} x_{2o}) = 0 \\
 &u_1 y_{1o} - \theta_2 (\alpha'_{2o} x_{1o} + \beta'_{2o} x_{2o} + \gamma'_{1o} x_{3o} + \sigma'_{1o} z_o) = 0 \\
 &u_2 y_{2o} - \theta_3 (\alpha'_{3o} x_{1o} + \gamma'_{2o} x_{3o} + \sigma'_{2o} z_o) = 0 \\
 &\eta z_j - \alpha'_{1j} x_{1j} - \beta'_{1j} x_{2j} \leq 0 \forall j \\
 &u_1 y_{1j} - \alpha'_{2j} x_{1j} - \beta'_{2j} x_{2j} - \gamma'_{1j} x_{3j} - \sigma'_{1j} z_j \leq 0 \forall j \\
 &u_2 y_{2j} - \alpha'_{3j} x_{1j} - \gamma'_{2j} x_{3j} - \sigma'_{2j} z_j \leq 0 \forall j \\
 &u_3 y_{3j} - \alpha'_{4j} x_{1j} - \beta'_{3j} x_{2j} \leq 0 \forall j \\
 &\alpha'_{1j} + \alpha'_{2j} + \alpha'_{3j} + \alpha'_{4j} = v_1 \forall j \\
 &\beta'_{1j} + \beta'_{2j} + \beta'_{3j} = v_2 \forall j \\
 &\gamma'_{1j} + \gamma'_{2j} = v_3 \forall j \\
 &\sigma'_{1j} + \sigma'_{2j} = \eta \forall j \\
 &L_{ij}^\alpha v_1 \leq \alpha'_{ij} \leq U_{ij}^\alpha v_1 \quad i = 1, 2, 3, 4 \forall j \\
 &L_{ij}^\beta v_2 \leq \beta'_{ij} \leq U_{ij}^\beta v_2 \quad i = 1, 2, 3 \forall j \\
 &L_{ij}^\gamma v_3 \leq \gamma'_{ij} \leq U_{ij}^\gamma v_3 \quad i = 1, 2 \forall j \\
 &L_{ij}^\sigma \eta \leq \sigma'_{ij} \leq U_{ij}^\sigma \eta \quad i = 1, 2 \forall j \\
 &\lambda^A + \lambda^B = 1 \\
 &\lambda^1 + \lambda^2 = 1 \\
 &\lambda^{1,2} + \lambda^C = 1 \\
 &0 < \theta_i \leq 1 \quad i = 1, 2, 3 \\
 &v_1, v_2, v_3, \eta, u_1, u_2, u_3 \geq 0
 \end{aligned} \tag{11}$$

[20] provided the upper and lower bounds of their searching parameter by using leader-follower models ([27]). However, this technique for the basic two-stage model cannot be used on a mixed multistage model (11), since the shared resources related to each stage may lead to an unidentifiable relationship between leader and follower. Fortunately, we know that the stage efficiencies of the network model (13) do not exceed those of their corresponding black box - that is, $\theta_1 \leq e_o^{1*}$, $\theta_2 \leq e_o^{A*}$, and $\theta_3 \leq e_o^{B*}$, where e_o^{1*} , e_o^{A*} , and e_o^{B*} are the optimal efficiency values of models (1.1-1.3), respectively. The parameters must satisfy these relationships: $\theta_1 \in (0, e_o^{1*}]$, $\theta_2 \in (0, e_o^{A*}]$, and $\theta_3 \in (0, e_o^{B*}]$.

Based on professional judgement by experts, who are senior executives at the management level, we set the weights $\lambda^{1,2} = 0.8, \lambda^C = 0.2$, and $\lambda^1 = \lambda^2 = \lambda^A = \lambda^B = 0.5$ (that is, process A, process B, stage 1, and stage 2 have the same weights in the objective function). In the calculations, we set parameters $\theta_i, i = 1, 2, 3$ in model (13) as $\theta_1 = e_o^{1*} - 0.001 * t_1$, $\theta_2 = e_o^{A*} - 0.001 * t_2$, and $\theta_3 = e_o^{B*} - 0.001 * t_3$, respectively. The number of iterations is $t_1 * t_2 * t_3$. When all the three parameters $\theta_i, i = 1, 2, 3$ approach the lower bounds of 0, the algorithm ends. We treat the maximal value of the $t_1 * t_2 * t_3$ calculations as the global optimal solution for model (11).

Table 1
Variable definitions and descriptive statistics

Variables	Definition (from TEJ)	Mean	Sta. error	Min	Max	
Inputs	Labor** (x_1)	Number of employees	4.724	2.943	0.873	12.848
	Capital* (x_2)	Total fixed assets	13.749	11.828	0.862	49.743
	Fund* (x_3)	Rest of borrowed funds excluding deposits	159.417	145.448	5.388	652.359
Intermediate	Deposit* (z)	Deposits from mutual loans accounts	985.918	729.112	120.881	2734.383
Outputs	Loan* (y_1)	Net loans, discounted and bills purchased	751.739	572.219	98.768	2011.474
	Investment* (y_2)	Investment	270.617	203.265	25.866	930.466
	Non-interest Income* (y_3)	Net commission income	5.459	5.775	0.458	34.694

Note: *: Measured by millions of New Taiwan dollars. **: Measured by thousands of people.

3. Empirical results of Taiwan Banks

There is some controversy in the literature when seeking the unique set of criteria for inputs and outputs of banking ([3]). It is common to select resources employed as inputs and some financial indices as outputs³. This paper follows the financial intermediate approach to choose inputs (labor, capital, and funds), outputs (loans, investments, and non-interest income) and intermediate product (deposits). We use data on 26 banks in Taiwan, obtained from Taiwan Economic Journal (TEJ) database from 2013 to 2015⁴, to show the characteristics of our two new models. The basic statistics of the key variables are shown in Table 1.

3.1. Results of mixed network framework

For the relational model (9), the empirical results are presented in Table 2 and Table 3. We find that not many DMUs can reach the overall efficient state unless all their components are efficient. Indeed, the numbers of efficient DMUs are only 1, 1, and 2 for year 2013, 2014 and 2015, respectively⁵.

The average performance of 26 banks from 2013 to 2015 are consistent with their performance in 2014, as shown in Table 3. For 2014 case, the only DMU to reach a unique overall efficiency score (e_0) is DMU 24, where all 3 sub-systems (e_0^1 , e_0^2 , and $e_0^{1,2}$) and all 3 sub-processes (e_0^A , e_0^B and e_0^C) are found to be efficient. The results fit completely with the argument of [19], who claimed that system efficiency can only be obtained if all decomposition efficiencies are also efficient.

For the 2nd highest efficient DMU (DMU 11), we find that the inefficiency of the B process in stage 2 (e_0^B) causes its stage-2 subsystem (e_0^2) to be inefficient, as a result its series process ($e_0^{1,2}$) is also inefficient. Therefore, the key factor for improving its system efficiency is its performance of the B process in stage 2 (e_0^B). Therefore, the advantage of the relational DEA model is its sharp discriminating power in identifying which branches are the critical factors to pull down overall efficiency.

For the additive aggregation model (11), the empirical results of 2014 and the average results of 2013 to 2015 are shown in Table 4 and Table 5. The numbers of efficient DMUs are 0, 1, and 2 for years 2013, 2014 and 2015, respectively.

The average performances of all banks from 2013 to 2015 are consistent with the performances in 2014, shown in Table 5. For 2014 case, the only DMU to reach an efficient system (e_0) is DMU 24 again with all its components being efficient. DMU 11 is again the 2nd highest

³ While labor and capital are the most common inputs variables, many studies also choose funds, non-interest expense, time, and/or number of transactions as additional inputs. Regarding outputs, it is common to set loans, investments, and non-interest income as key variables. However, interest income, checks, mutual funds, mortgages, share of stock, EPS, and so on are also selected as outputs (Seiford and Zhu, 1999).

⁴ Taiwan has 39 domestic commercial banks in which 16 are a part of financial holding companies (FHC) by 2017. Due to data constraints, we only collect the data of 26 banks from 2013 to 2015.

⁵ We only show the result of 2014 to save space. Empirical results of 2013 and 2015 are available upon request by email.

efficient DMU. The critical problem resulting in overall inefficiency is again the inefficiency of the B process in stage 2 (e_0^B), which leads to both e_0^2 and $e_0^{1,2}$ being inefficient.

Therefore, the additive model also has a sharp discriminating power in identifying which process and sub-branches diminish the overall efficiency.

The results found for all efficiency scores of the additive aggregation model are indeed very similar with those of the relational decomposition model. However, in most cases, we find that the scores generated from the relational model are comparatively smaller than those from the additive model.

3.2. Comparisons of 4 models

In order to emphasize the advantages and explanatory power of our two models, we compare the performance indices of the following four DEA models in 2014: (1) CCR model without considering intermediate processes; (2) a simple two-stage network DEA model (simplified as two-stage NDEA; [11]); (3) the relational decomposition network DEA model; and (4) the additive aggregation network DEA model.

Table 6 lists the results of banks' overall efficiency and main branches' efficiency scores in 2014. We find that the results of the first two models are similar, while the results of the other two models are very close. Statistically, the p-value of non-parametric paired Wilcoxon Signed Rank test of difference between relational model and additive model is 0.8119, which indicates regardless of model utilized to solve our proposed framework, bank's managers should have the same insight about the performance, and hence, relative ranking of their organization. Similarly, there is not enough evidence (only at most 10.24 percent) to declare that there is a significantly difference between "black-box" and "simple two-stage" models. The two-stage network framework theoretically has more logical meaning, though, when facing with real situation, it is not necessary to have better discriminable power (see Appendix Table 8).

Figure 3 illustrates the trends of system efficiency scores of the 4 models for all 26 DMUs. We find that the scores of our 2 models always lie below those of CCR or two-stage DEA models. Furthermore, the gap between the lines represented in our models and those of the two traditional models is apparent. Following the work of [31], we again conduct the Wilcoxon Signed Rank test and find that even at 0.1 percent of significant level, the efficiency scores derive from our reality framework are empirically lower than those of purely theoretical-based models. This finding is consistent with [11] and [25] who proved that when the internal process is neglected or oversimplified, optimistic information tends to be more frequently generated.

Our comparison can go even further using the descriptive statistics of the efficiency scores of the 4 models, as shown in Table 7. In terms of the number of efficient DMUs, the CCR model generates 6 and two-stages DEA produces 5, while both of our two new models only generate one each. In terms of average efficiency scores, CCR (0.920) and two-stage NDEA (0.925) are very close, likewise, the differences between our two new models are relatively small (0.818 and 0.808, respectively).

For the dispersion of the scores' distribution, firstly, the minimum

Table 2
Efficiency scores of the relational decomposition model – 2014

DMU	Name	e_0	e_0^1	w^1	e_0^A	w^A	e_0^B	w^B	e_0^2	w^2	$e_0^{1,2}$	$w^{1,2}$	e_0^C	w^C
1	Chang Hwa Bank	0.815	0.581	0.315	0.999	0.687	0.871	0.313	0.959	0.868	0.833	0.872	0.692	0.128
2	Standard Chartered	0.768	0.659	0.245	0.646	0.581	0.980	0.419	0.786	0.916	0.720	0.826	0.994	0.174
3	KTB	0.784	0.646	0.226	0.701	0.650	0.994	0.350	0.803	0.920	0.739	0.795	0.956	0.205
4	Taichung Bank	0.906	0.937	0.115	1.000	0.809	0.508	0.191	0.906	0.993	0.899	0.922	0.980	0.078
5	Taiwan Business Bank	0.844	0.591	0.335	0.998	0.752	0.966	0.248	0.990	0.863	0.854	0.899	0.757	0.101
6	Bank of Kaohsiung	0.952	0.905	0.115	0.994	0.721	0.923	0.279	0.974	0.989	0.963	0.951	0.727	0.049
7	KGI Bank	0.785	0.273	0.108	0.815	0.667	0.994	0.333	0.875	0.922	0.806	0.917	0.548	0.083
8	Union Bank of Taiwan	0.524	0.226	0.258	0.568	0.731	0.844	0.269	0.642	0.801	0.514	0.924	0.647	0.076
9	Far Eastern Bank	0.770	0.818	0.221	0.840	0.637	0.609	0.363	0.756	0.960	0.726	0.813	0.966	0.187
10	Ta Chong Bank	0.637	0.606	0.182	0.498	0.748	0.989	0.252	0.622	0.928	0.577	0.858	1.000	0.142
11	En Tie Bank	0.990	1.000	0.459	0.999	0.788	0.928	0.212	0.984	1.000	0.984	0.663	1.000	0.337
12	Shin Kong Bank	0.752	0.424	0.213	0.902	0.743	0.660	0.257	0.840	0.877	0.737	0.929	0.952	0.071
13	O-Bank	0.726	0.533	0.310	0.565	0.522	0.976	0.478	0.762	0.855	0.651	0.786	1.000	0.214
14	Cathay United Bank	0.797	0.715	0.231	0.695	0.616	1.000	0.384	0.812	0.934	0.759	0.832	0.986	0.168
15	Taipei Fubon Bank	0.869	0.810	0.221	0.823	0.693	0.999	0.307	0.877	0.958	0.840	0.820	1.000	0.180
16	Hua Nan Bank	0.905	0.882	0.244	0.970	0.693	1.000	0.307	0.979	0.971	0.951	0.837	0.669	0.163
17	China trust Bank	0.834	0.534	0.325	0.883	0.654	0.997	0.346	0.922	0.849	0.783	0.736	0.976	0.264
18	Mega Bank	0.919	0.994	0.480	0.961	0.773	0.667	0.227	0.894	0.997	0.891	0.744	1.000	0.256
19	First Bank	0.868	0.564	0.339	0.999	0.691	0.999	0.309	0.999	0.853	0.852	0.890	1.000	0.110
20	E. Sun Bank	0.869	0.628	0.292	0.900	0.624	0.999	0.376	0.938	0.892	0.836	0.799	1.000	0.201
21	Tai Hsin International Bank	0.693	0.384	0.331	0.707	0.701	0.999	0.299	0.794	0.796	0.632	0.835	1.000	0.165
22	Bank Sino Pac	0.859	0.722	0.281	0.924	0.630	0.940	0.370	0.930	0.922	0.857	0.819	0.868	0.181
23	Jih Sun Bank	0.738	0.172	0.217	0.894	0.731	0.983	0.269	0.918	0.821	0.753	0.941	0.490	0.059
24	Yuanta Bank	1.000	1.000	0.211	1.000	0.768	1.000	0.232	1.000	1.000	1.000	0.903	1.000	0.097
25	Taiwan Cooperative Bank	0.711	0.572	0.344	0.999	0.674	0.415	0.326	0.809	0.853	0.690	0.922	0.966	0.078
26	Shanghai Commercial and Savings Bank	0.941	0.925	0.250	0.920	0.646	1.000	0.354	0.948	0.981	0.931	0.859	1.000	0.141
Average		0.818	0.658	0.264	0.854	0.689	0.894	0.310	0.874	0.912	0.799	0.849	0.891	0.150

values of CCR (0.672) and the simple two-stage model (0.732) are higher than the ones of our two models (0.524 and 0.554, respectively). Secondly, the standard deviations of the efficiency scores from the first two models (0.086 and 0.076) are smaller than our two models (0.110 and 0.115).

While the simple two-stages DEA model returns in no better explanatory power than the traditional black-box model, the two new proposed models provide superior results. Through constructing real and complex network structure of the production, the discriminating power of network DEA models can be significantly increased because the

simplified structures are more likely to overestimate the real performance, and result in a less reliable solution.

3.3. Management implication

For banking management, not only the overall performance but also division performances need to be evaluated in order to generate higher profits. However, it becomes more and more important in banking industries to identify division and sub-division efficiencies. The empirical findings of this paper help us distinguish the new trend in Taiwan

Table 3
Average efficiency scores of the relational decomposition model –2013-2015

DMU	Name	e_0	e_0^1	w^1	e_0^A	w^A	e_0^B	w^B	e_0^2	w^2	$e_0^{1,2}$	$w^{1,2}$	e_0^C	w^C
1	Chang Hwa Bank	0.824	0.635	0.292	0.946	0.701	0.911	0.299	0.934	0.892	0.833	0.881	0.778	0.119
2	Standard Chartered	0.721	0.506	0.296	0.645	0.589	0.993	0.411	0.788	0.850	0.670	0.840	0.992	0.160
3	KTB	0.767	0.589	0.252	0.716	0.645	0.958	0.355	0.800	0.895	0.718	0.806	0.984	0.194
4	Taichung Bank	0.939	0.963	0.088	1.000	0.789	0.681	0.211	0.938	0.997	0.935	0.941	0.993	0.059
5	Taiwan Business Bank	0.847	0.642	0.310	0.999	0.717	0.889	0.283	0.966	0.888	0.858	0.893	0.772	0.107
6	Bank of Kaohsiung	0.884	0.779	0.178	0.994	0.689	0.798	0.311	0.935	0.950	0.891	0.936	0.743	0.064
7	KGI Bank	0.814	0.539	0.080	0.754	0.626	0.996	0.374	0.836	0.972	0.812	0.941	0.824	0.059
8	Union Bank of Taiwan	0.542	0.274	0.206	0.568	0.732	0.848	0.268	0.643	0.847	0.543	0.917	0.598	0.083
9	Far Eastern Bank	0.767	0.700	0.274	0.838	0.682	0.754	0.318	0.800	0.908	0.720	0.809	0.966	0.191
10	Ta Chong Bank	0.632	0.523	0.220	0.650	0.606	0.706	0.394	0.640	0.890	0.568	0.853	1.000	0.147
11	En Tie Bank	0.995	1.000	0.463	1.000	0.773	0.965	0.227	0.992	1.000	0.992	0.676	1.000	0.324
12	Shin Kong Bank	0.841	0.669	0.138	0.947	0.744	0.699	0.256	0.878	0.945	0.831	0.938	0.960	0.062
13	O-Bank	0.796	0.563	0.263	0.726	0.408	0.958	0.592	0.853	0.884	0.755	0.818	0.968	0.182
14	Cathay United Bank	0.820	0.718	0.251	0.771	0.573	0.981	0.427	0.858	0.920	0.789	0.827	0.973	0.173
15	Taipei Fubon Bank	0.893	0.765	0.246	0.893	0.665	0.999	0.335	0.927	0.941	0.871	0.820	0.992	0.180
16	Hua Nan Bank	0.893	0.726	0.292	0.990	0.685	0.999	0.315	0.992	0.916	0.909	0.869	0.813	0.131
17	China trust Bank	0.849	0.599	0.308	0.890	0.655	0.924	0.345	0.901	0.876	0.790	0.705	0.992	0.295
18	Mega Bank	0.940	0.988	0.399	0.987	0.739	0.760	0.261	0.930	0.994	0.925	0.779	0.982	0.221
19	First Bank	0.872	0.624	0.326	0.999	0.676	1.000	0.324	1.000	0.869	0.869	0.878	0.935	0.122
20	E. Sun Bank	0.867	0.591	0.321	0.967	0.600	0.999	0.400	0.979	0.865	0.846	0.785	0.956	0.215
21	Tai Hsin International Bank	0.691	0.427	0.304	0.691	0.684	0.932	0.316	0.767	0.824	0.631	0.833	0.988	0.167
22	Bank Sino Pac	0.891	0.760	0.272	0.955	0.628	0.936	0.372	0.946	0.930	0.880	0.836	0.954	0.164
23	Jih Sun Bank	0.683	0.225	0.200	0.872	0.671	0.713	0.329	0.812	0.842	0.680	0.947	0.748	0.053
24	Yuanta Bank	0.963	0.949	0.188	1.000	0.705	0.913	0.295	0.977	0.990	0.967	0.899	0.943	0.101
25	Taiwan Cooperative Bank	0.711	0.552	0.356	0.999	0.697	0.424	0.303	0.823	0.840	0.692	0.920	0.929	0.080
26	Shanghai Commercial and Savings Bank	0.950	0.953	0.223	0.963	0.611	1.000	0.389	0.977	0.988	0.965	0.851	0.895	0.149
Average		0.823	0.664	0.259	0.875	0.665	0.874	0.335	0.880	0.912	0.805	0.854	0.911	0.146

Table 4
Efficiency scores of the additive aggregation model – 2014

DMU	Name	e_0	e_0^1	e_0^A	e_0^B	e_0^2	$e_0^{1,2}$	e_0^C
1	Chang Hwa Bank	0.745	0.581	0.999	0.871	0.935	0.758	0.692
2	Standard Chartered	0.790	0.659	0.657	0.980	0.819	0.739	0.994
3	KTB	0.793	0.646	0.720	0.994	0.857	0.752	0.956
4	Taichung Bank	0.880	0.937	1.000	0.546	0.773	0.855	0.980
5	Taiwan Business Bank	0.828	0.604	1.000	0.966	0.983	0.793	0.969
6	Bank of Kaohsiung	0.926	0.905	1.000	0.923	0.962	0.933	0.898
7	KGI Bank	0.586	0.273	0.843	0.994	0.918	0.596	0.548
8	Union Bank of Taiwan	0.554	0.280	0.718	0.844	0.781	0.531	0.647
9	Far Eastern Bank	0.816	0.818	0.906	0.575	0.741	0.779	0.966
10	Ta Chong Bank	0.750	0.606	0.550	0.989	0.769	0.688	1.000
11	En Tie Bank	0.986	1.000	1.000	0.928	0.964	0.982	1.000
12	Shin Kong Bank	0.713	0.524	0.902	0.660	0.781	0.653	0.952
13	O-Bank	0.764	0.533	0.778	0.976	0.877	0.705	1.000
14	Cathay United Bank	0.826	0.715	0.714	1.000	0.857	0.786	0.986
15	Taipei Fubon Bank	0.891	0.810	0.835	0.999	0.917	0.864	1.000
16	Hua Nan Bank	0.884	0.882	0.987	1.000	0.993	0.938	0.669
17	China Trust Bank	0.785	0.534	0.883	0.997	0.940	0.737	0.976
18	Mega Bank	0.931	0.994	1.000	0.667	0.833	0.914	1.000
19	First Bank	0.845	0.613	0.999	1.000	1.000	0.807	1.000
20	E. Sun Bank	0.831	0.628	0.900	1.000	0.950	0.789	1.000
21	Tai Hsin International Bank	0.714	0.361	0.849	0.999	0.924	0.642	1.000
22	Bank Sino Pac	0.835	0.722	0.924	0.940	0.932	0.827	0.868
23	Jih Sun Bank	0.603	0.309	0.926	0.983	0.954	0.632	0.490
24	Yuanta Bank	1.000	1.000	1.000	1.000	1.000	1.000	1.000
25	Taiwan Cooperative Bank	0.755	0.719	0.957	0.415	0.686	0.702	0.966
26	Shanghai Commercial and Savings Bank	0.966	0.925	0.979	1.000	0.990	0.957	1.000
Average		0.808	0.676	0.886	0.894	0.890	0.783	0.906

banking industry, and suggest that banking management may readjust their resource distribution and formulate new business strategies accordingly.

Fig. 4 and Fig. 5 illustrate the average divisional efficiency scores and their weights for 26 banks in Taiwan generated by relation model from 2013 to 2015. The results indicate some important implications for banking management.

First, the “fund-raising” process is not only less important but also worse performing than the “fund-spreading” process in a practical sense ($w^1 < w^2$ and $e^1 < e^2$). Furthermore, “fund-raising” is the least efficient

process among others in the banking’s operation (approximately 0.7). This suggests Taiwan banks can significantly enhance their performance by expanding the deposit mobilization activities through various channels by utilizing information and technology.

Secondly, as for two traditional businesses of Taiwan banks, being credit and investment, the comparison of w^d and w^b illustrates that the “crediting” process contributes much more to efficiency than the “investing” process does. Regarding the efficiency of these processes, investing process slightly outperforms crediting process (0.89 vs 0.85). This encourages banks’ managers to restructure the resources in order to

Table 5
Average efficiency scores of the additive aggregation model – 2013-2015

DMU	Name	e_0	e_0^1	e_0^A	e_0^B	e_0^2	$e_0^{1,2}$	e_0^C
1	Chang Hwa Bank	0.785	0.635	0.968	0.911	0.940	0.787	0.778
2	Standard Chartered	0.749	0.554	0.653	0.993	0.823	0.689	0.992
3	KTB	0.772	0.589	0.739	0.958	0.849	0.719	0.984
4	Taichung Bank	0.925	0.963	1.000	0.705	0.853	0.908	0.993
5	Taiwan Business Bank	0.823	0.646	0.988	0.889	0.938	0.792	0.948
6	Bank of Kaohsiung	0.893	0.910	0.894	0.920	0.907	0.908	0.833
7	KGI Bank	0.699	0.452	0.844	0.922	0.883	0.667	0.824
8	Union Bank of Taiwan	0.555	0.324	0.709	0.848	0.778	0.551	0.572
9	Far Eastern Bank	0.814	0.754	0.852	0.742	0.797	0.775	0.966
10	Ta Chong Bank	0.706	0.539	0.659	0.792	0.725	0.632	1.000
11	En Tie Bank	0.993	1.000	1.000	0.965	0.983	0.991	1.000
12	Shin Kong Bank	0.836	0.773	0.947	0.704	0.825	0.799	0.983
13	O-Bank	0.840	0.700	0.838	0.992	0.915	0.808	0.968
14	Cathay United Bank	0.835	0.718	0.768	1.000	0.884	0.801	0.973
15	Taipei Fubon Bank	0.884	0.765	0.901	0.999	0.950	0.857	0.992
16	Hua Nan Bank	0.856	0.736	0.995	0.999	0.997	0.866	0.813
17	China Trust Bank	0.801	0.599	0.890	0.924	0.907	0.753	0.992
18	Mega Bank	0.944	0.988	1.000	0.760	0.880	0.934	0.982
19	First Bank	0.857	0.685	0.979	1.000	0.989	0.837	0.935
20	E. Sun Bank	0.876	0.768	0.880	0.999	0.940	0.854	0.964
21	Tai Hsin International Bank	0.707	0.419	0.777	0.932	0.855	0.637	0.988
22	Bank Sino Pac	0.873	0.760	0.955	0.936	0.945	0.853	0.954
23	Jih Sun Bank	0.615	0.291	0.873	0.875	0.874	0.582	0.748
24	Yuanta Bank	0.980	0.963	0.972	1.000	0.986	0.975	1.000
25	Taiwan Cooperative Bank	0.753	0.730	0.980	0.397	0.688	0.709	0.929
26	Shanghai Commercial and Savings Bank	0.957	0.953	0.983	1.000	0.992	0.972	0.895
Average		0.820	0.701	0.886	0.891	0.889	0.795	0.923

Table 6
System efficiency and ranking - Comparing 4 models - 2014

DMU	Name	CCR		Two-stage NDEA		Relational model		Additive model	
		Rank	e_0	Rank	e_0	Rank	e_0	Rank	e_0
1	Chang Hwa Bank	18	0.906	16	0.929	14	0.815	21	0.745
2	Standard Chartered	1	1.000	6	0.997	19	0.768	16	0.790
3	KTB	20	0.899	20	0.889	17	0.784	15	0.793
4	Taichung Bank	1	1.000	1	1.000	6	0.906	8	0.880
5	Taiwan Business Bank	8	0.986	10	0.970	12	0.844	12	0.828
6	Bank of Kaohsiung	1	1.000	1	1.000	3	0.952	5	0.926
7	KGI Bank	10	0.982	9	0.975	16	0.785	25	0.586
8	Union Bank of Taiwan	26	0.672	26	0.732	26	0.524	26	0.554
9	Far Eastern Bank	17	0.910	19	0.897	18	0.770	14	0.816
10	Ta Chong Bank	24	0.782	25	0.787	25	0.637	20	0.750
11	En Tie Bank	1	1.000	1	1.000	2	0.990	2	0.986
12	Shin Kong Bank	23	0.834	21	0.879	20	0.752	23	0.713
13	O-Bank	12	0.952	23	0.803	22	0.726	18	0.764
14	Cathay United Bank	21	0.872	18	0.901	15	0.797	13	0.826
15	Taipei Fubon Bank	16	0.913	14	0.933	8	0.869	6	0.891
16	Hua Nan Bank	9	0.986	8	0.988	7	0.905	7	0.884
17	China Trust Bank	13	0.932	15	0.930	13	0.834	17	0.785
18	Mega Bank	7	0.990	7	0.994	5	0.919	4	0.931
19	First Bank	14	0.927	13	0.937	10	0.868	9	0.845
20	E. Sun Bank	11	0.963	11	0.965	9	0.869	11	0.831
21	Tai Hsin International Bank	25	0.745	24	0.799	24	0.693	22	0.714
22	Bank Sino Pac	19	0.899	17	0.928	11	0.859	10	0.835
23	Jih Sun Bank	22	0.842	22	0.853	21	0.738	24	0.603
24	Yuanta Bank	1	1.000	1	1.000	1	1.000	1	1.000
25	Taiwan Cooperative Bank	15	0.925	12	0.953	23	0.711	19	0.755
26	Shanghai Commercial and Savings Bank	1	1.000	1	1.000	4	0.941	3	0.966

take advantage of investment business.

Third, our empirical result advocates the benefit of other fee-based services (e^c). With the growth in financial technology (Fintech) in Taiwan, numerous new financial products have been born (e.g., providing credit cards, cash cards, trusts, wealth management), and the quality of those products is continuously improving. This business enjoys the highest efficiency score when compared to other traditional businesses. As can be seen in Figure 4, the service activities of all banks in Taiwan always outperform other processes. Especially, in 2015, the efficiency score of this business of Taiwan banks rockets to a very high level of approximately 0.96. Unfortunately, in comparison with traditional banking services, the size of this non-interest earning activity is quite small (0.15 vs 0.85). In the context of intense competition in banking industry, it suggests that Taiwan banks should shift resources to strengthen the importance and contribution of this business.

Fourth, a close examination of “low-efficiency” banks (those whose scores are less than 0.7), suggests that while Banks 10 and 21 seem to streamline their non-interest services, those of Bank 8 are not only very

inefficient but also have poorly distributed resources (e^c and w^c are 0.65 and 0.076, respectively). Furthermore, it is apparent that the inefficiency of these banks is mostly due to “credit activities” (e^l s and e^a s of these banks are relatively low). Therefore, in order to catch up to the level of high performing banks, these banks should not only improve the quality of “collecting deposits” process, but also expand loan activities while focusing on controlling underperforming loans.

4. Conclusions and Future Research Directions

Internal operational processes in the banking system are complicated and have not been well studied in previous literature. This study proposes relational decomposition and additive aggregation network DEA models to measure the efficiency of a banking system, with a mixed network structure consisting of series processes and parallel processes with shared input resources. In the two approaches, the system efficiency as well as all component efficiencies can be reliably derived and obtained.

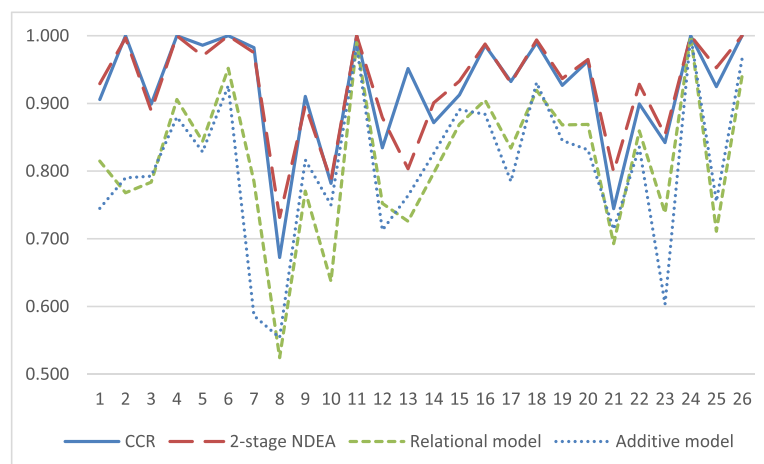


Fig. 3. System efficiency scores of the 4 models

Table 7
Descriptive statistic of efficiency score – Comparing 4 models - 2014

	CCR	Two-stage NDEA	Relational model	Additive model
Mean	0.920	0.925	0.818	0.808
Median	0.930	0.935	0.824	0.821
Sta. Error	0.086	0.076	0.110	0.115
Min	0.672	0.732	0.524	0.554
Max	1	1	1	1
# of efficient DMUs	6	5	1	1

By comparing the results of four types of models in the non-parametric Wilcoxon signed-ranks test, we find that: (1) the component efficiency scores of the relational decomposition model are closely consistent with those of the additive aggregation model; (2) the results of the conventional CCR model are consistent with the simple two-stage model; and (3) the results of the relational and additive models are different from those of CCR and two-stage models. Moreover, our empirical results of the two new models show that only one or two DMUs for each year can reach the overall efficient state, as all components must be efficient to achieve this. However, the CCR model and the two-stage DEA model produce greater numbers of efficient banks than our two new models. The results show the efficiency evaluations of the CCR models and the simple two-stage DEA models are overestimated due to the intermediate processes either being neglected or oversimplified.

Conversely, the discriminating power of our new network DEA models for mixed structure is significantly stronger.

Our research also has a significant contribution in terms of managerial implications for Taiwan banks. By evaluating the divisional efficiencies and their contribution to the overall efficiency, we propose that the quality of credit business of Taiwan banks can be improved, especially the “collecting deposit” process. Furthermore, bank’s managers should pay attention to re-allocating resources from traditional banking products to other fee-based services. As for performance evaluation experts, the similarity of our two models suggests that it is necessary to first build an appropriate assessment system based on real situation before extracting any technical measurement approaches.

This study discusses a mixed network structure in banking system composed of series processes and parallel processes, and all components of this system have shared resources. However, sometimes a DMU may have functions or special task forces in each component. In other words, real operation in banks may be more complex, not restricted as the mixed structure of serial and parallel processes. For example, general multistage serial and parallel processes, multistage processes with feedbacks studied in [32], and other general multistage processes ([33]). Generally, different components in banking system may have different operational processes in real-world situations. How to measure the efficiency of such banking systems and how to identify the efficiency relationships between the system and all components are challenges for

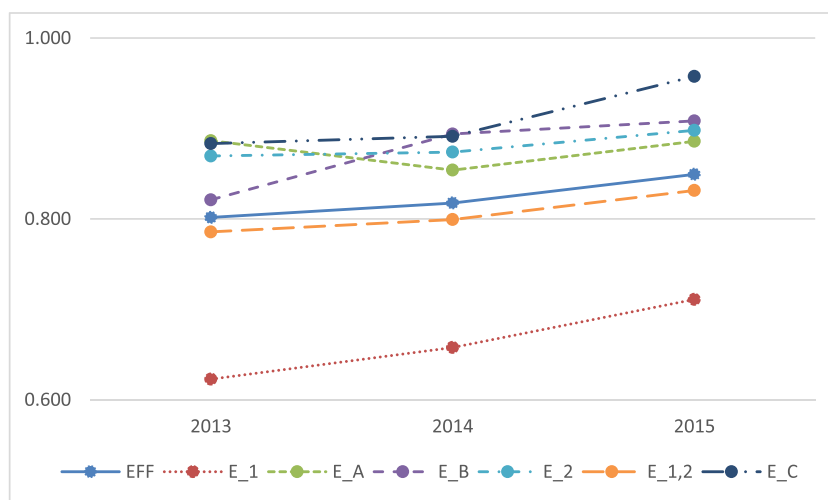


Fig. 4. Average system and divisional efficiency in relational model

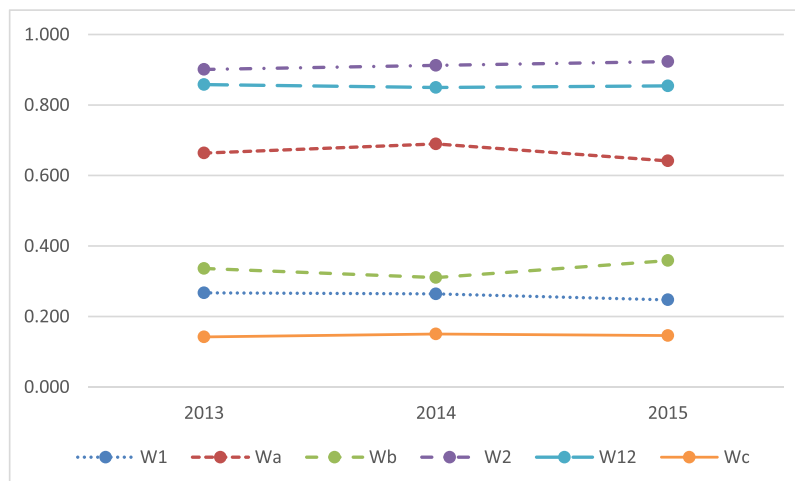


Fig. 5. Average division weights in relational model

Table 8
Wilcoxon Signed Rank Test

	CCR	Two-stage NDEA	Relational model	Additive model
CCR	-			
Two-stage NDEA	0.8976	-		
Relational model	0.0005***	0.0002***	-	
Additive model	0.0003***	0.0001***	0.8119	-

Note: (1) The numbers shown in the table are the *p*-value of the test; that is; $p > |z|$. (2) *** denotes that the null hypotheses are rejected at the 1% significance level.

our future research.

Declaration of interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A

Table 8.

References

[1] Nguyen TL. Diversification and Bank Efficiency in Six ASEAN Countries. *Global Finance Journal* 2018;37:57–78. <https://doi.org/10.1016/j.gfj.2018.04.004>.
 [2] Seiford LM, Zhu Joe. Profitability and marketability of the top 55 U.S. commercial banks. *Management Science* 1999;45(9):1270–88. <https://doi.org/10.1287/mnsc.45.9.1270>.
 [3] Holod D, Lewis HF. Resolving the deposit dilemma: A new DEA bank efficiency model. *Journal of Banking & Finance* 2011;35(11):2801–10. <https://doi.org/10.1016/j.jbankfin.2011.03.007>.
 [4] Avkiran NK. An illustration of dynamic network DEA in commercial banking including robustness tests. *Omega* 2015;55:141–50. <https://doi.org/10.1016/j.omega.2014.07.002>.
 [5] Zhou X, Xu Z, Chai J, Yao L, Wang S. Efficiency evaluation for banking systems under uncertainty: A multi-period three-stage DEA model. *Omega* 2018. <https://doi.org/10.1016/j.omega.2018.05.012>. Available online 30 May 2018.
 [6] Charnes AW, Cooper WW, Rhodes EL. Measuring the efficiency of decision making units. *European Journal of Operational Research* 1978;2(6):429–44. [https://doi.org/10.1016/0377-2217\(78\)90138-8](https://doi.org/10.1016/0377-2217(78)90138-8).
 [7] Banker R, Charnes A, Cooper W. Some models for estimating technical and scale inefficiencies in data envelopment analysis. *Management Science* 1984;30(9):1078–92. <https://doi.org/10.1287/mnsc.30.9.1078>.
 [8] Ariff M, Can L. Cost and Profit Efficiency of Chinese Banks: A Non-parametric Analysis. *China Economic Review* 2008;19:260–73. <https://doi.org/10.1016/j.chieco.2007.04.001>.
 [9] Lin, T., Lee, C., & Chiu, T. (2009). Application of DEA in Analyzing Bank’s Operating Performance. *Expert System with Applications*, 36(5), 8883–8891. doi: 10.1016/j.eswa.2008.11.018.
 [10] Kamarudin F, Sufian F, Nassir AM. Does Country Governance Foster Revenue Efficiency of Islamic and Conventional Banks in GCC Countries? *EuroMed Journal of Business* 2016;11(2):181–211. <https://doi.org/10.1108/EMJB-06-2015-0026>.
 [11] Chen Y, Du J, Sherman H, Zhu J. DEA model with shared resources and efficiency decomposition. *European Journal of Operational Research* 2010;207(1):339–49. <https://doi.org/10.1016/j.ejor.2010.03.031>.

[12] Kao C. Network data envelopment analysis: A review. *European Journal of Operational Research* 2014;239(1):1–16. <https://doi.org/10.1016/j.ejor.2014.02.039>.
 [13] Fukuyama H, Matousek R. Modelling Bank Performance: A Network DEA Approach. *European Journal of Operational Research* 2017;259:721–32. <https://doi.org/10.1016/j.ejor.2016.10.044>.
 [14] Wang Y, Pan J-f, Pei R-m, Yi B-W, Yang G-l. Assessing the Technological Innovation Efficiency of China’s High-tech Industries with a Two-stage. Network DEA Approach. *Socio-Economic Planning Sciences* 2020;71:100810. <https://doi.org/10.1016/j.seps.2020.100810>.
 [15] Zhang B, Luo Y, Chiu Y-H. Efficiency Evaluation of China’s High-tech Industry with a Multi-activity Network Data Envelopment Analysis Approach. *Socio-Economic Planning Sciences* 2019;66:2–9. <https://doi.org/10.1016/j.seps.2018.07.013>.
 [16] Stefaniec A, Hosseini K, Xie J, Li Y. Sustainability Assessment of Inland Transportation in China: A Triple Bottom Line-based Network DEA Approach. *Transportation Research Part D: Transport and Environment* 2020;80:102258. <https://doi.org/10.1016/j.trd.2020.102258>.
 [17] Yu Ming-Miin, Fan Chih-Ku. Measuring the performance of multimode bus transit: A mixed structure network DEA model. *Transportation Research Part E* 2009;45: 501–15. <https://doi.org/10.1016/j.tre.2008.10.001>.
 [18] Kao C. Efficiency decomposition in network data envelopment analysis: A relational model. *European Journal of Operational Research* 2009;192(3):949–62. <https://doi.org/10.1016/j.ejor.2007.10.008>.
 [19] Kao C. Efficiency decomposition for parallel production systems. *Journal of the Operational Research Society* 2012;63(1):64–71. <https://doi.org/10.1057/jors.2011.16>.
 [20] Ang S, Chen C-M. Pitfalls of decomposition weights in the additive multi-stage DEA model. *Omega* 2016;58:139–53. <https://doi.org/10.1016/j.omega.2015.05.008>.
 [21] Huang, T.-H., Hu, C.-N., & Chang, B.-G. (2018). Competition, efficiency, and innovation in Taiwan’s banking industry — An application of copula methods. *The Quarterly Review of Economics and Finance*, 67, 362-375. doi:10.1016/j.qref.2017.08.006.
 [22] Huang T-H, Wang M-H. Estimating Scale and Scope Economies with Fourier Flexible Functional Form – Evidence from Taiwan’s Banking Industry. *Australian Economic Papers* 2002;40(2):298–311. <https://doi.org/10.1111/1467-8454.00123>.
 [23] Peng J-L, Jeng V, Wang JL, Chen Y-C. The impact of bancassurance on efficiency and profitability of banks: Evidence from the banking industry in Taiwan. *Journal of Banking & Finance* 2017;80:1–13. <https://doi.org/10.1016/j.jbankfin.2017.03.013>.
 [24] Sealey CW, Lindley JT. Inputs, Outputs, and a Theory of Production and Cost at Depository Financial Institutions. *The Journal of Finance* 1977;32(4):1251–66. <https://doi.org/10.2307/2326527>.
 [25] Chen K, Zhu J. Second order cone programming approach to two-stage network data envelopment analysis. *European Journal of Operational Research* 2017;262(1):231–8. <https://doi.org/10.1016/j.ejor.2017.03.074>.
 [26] Kao C, Huang S-N. Efficiency decomposition in two-stage data envelopment analysis: An application to non-life insurance companies in Taiwan. *European Journal of Operational Research* 2008;185(1):418–29. <https://doi.org/10.1016/j.ejor.2006.11.041>.
 [27] Liang L, Cook WD, Zhu J. DEA models for two-stage processes: Game approach and efficiency decomposition. *Naval Research Logistics* 2008;229(1):643–53. <https://doi.org/10.1002/nav.20308>.
 [28] Charnes A, Cooper W. Programming with linear fractional functionals. *Naval Research Logistics Quarterly* 1962;9(3-4):181–6. [https://doi.org/10.1016/0304-4076\(90\)90048-X](https://doi.org/10.1016/0304-4076(90)90048-X).
 [29] Cook WD, Zhu J, Bi G, Yang F. Network DEA: Additive efficiency decomposition. *European Journal of Operational Research* 2010;207(2):1122–9. <https://doi.org/10.1016/j.ejor.2010.05.006>.
 [30] Lim S, Zhu J. Integrated data envelopment analysis: Global vs. local optimum. *European Journal of Operational Research* 2013;229(1):276–8. <https://doi.org/10.1016/j.ejor.2013.02.023>.
 [31] Chao, C.-M., Yu, M.-M., & Wu, H.-N. (2015). An Application of the Dynamic Network DEA Model: The Case of Banks in Taiwan. *Emerging Markets Finance and Trade*, 51(sup1), S133-S151. doi:10.1080/1540496X.2014.998887.
 [32] Liang L, Li Z-Q, Cook W, Zhu J. DEA Efficiency in Two-stage Networks with Feedback. *IIE Transactions* 2011;43:309–22. <https://doi.org/10.1080/0740817X.2010.509307>.
 [33] Kao C. Efficiency Decomposition and Aggregation in Network Data Envelopment Analysis. *European Journal of Operational Research* 2016;255(3):778–86. <https://doi.org/10.1016/j.ejor.2016.05.019>.