

Örsal, Deniz Dilan Karaman; Droge, Bernd

Working Paper

On the existence of the moments of the asymptotic trace statistic

SFB 649 Discussion Paper, No. 2009,012

Provided in Cooperation with:

Collaborative Research Center 649: Economic Risk, Humboldt University Berlin

Suggested Citation: Örsal, Deniz Dilan Karaman; Droge, Bernd (2009) : On the existence of the moments of the asymptotic trace statistic, SFB 649 Discussion Paper, No. 2009,012, Humboldt University of Berlin, Collaborative Research Center 649 - Economic Risk, Berlin

This Version is available at:

<https://hdl.handle.net/10419/25328>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

SFB 649 Discussion Paper 2009-012

On the Existence of the Moments of the Asymptotic Trace Statistic

Deniz Dilan Karaman Örsal*
Bernd Droge*



*Humboldt-Universität zu Berlin, Germany

This research was supported by the Deutsche Forschungsgemeinschaft through the SFB 649 "Economic Risk".

<http://sfb649.wiwi.hu-berlin.de>
ISSN 1860-5664

SFB 649, Humboldt-Universität zu Berlin
Spandauer Straße 1, D-10178 Berlin



SFB 649 ECONOMIC RISK BERLIN

On the Existence of the Moments of the Asymptotic Trace Statistic *

Deniz Dilan Karaman Örsal¹ and Bernd Droge²

Humboldt-Universität zu Berlin

18th February 2009

Abstract

In this note we establish the existence of the first two moments of the asymptotic trace statistic, which appears as weak limit of the likelihood ratio statistic for testing the cointegration rank in a vector autoregressive model and whose moments may be used to develop panel cointegration tests. Moreover, we justify the common practice to approximate these moments by simulating a certain statistic, which converges weakly to the asymptotic trace statistic. To accomplish this we show that the moments of the mentioned statistic converge to those of the asymptotic trace statistic as the time dimension tends to infinity.

Keywords: Cointegration, Trace statistic, Asymptotic moments, Uniform integrability.

JEL classification: C32, C33, C12

1 Motivation and Framework

Cointegration tests play an important role in the empirical analysis of long-run relationships among integrated variables, but they often suffer from low power properties due to the small time span of the available time series. The performance of the tests could be improved by enlarging the data basis, e.g. by considering additional cross-sectional units (individuals) with similar data. Therefore the cointegration methodology has been extended to the panel data framework. Similar to the case of testing for unit roots, panel cointegration tests may be based on standardizing the average of individual cointegration test statistics. By some central limit theorem, standard normal quantiles may then serve as critical values. However, the justification of such a procedure requires the existence of the first two moments of some distribution. For example, Larsson et al. (2001) used the likelihood framework to present a test for the cointegrating rank in heterogeneous panels. Their test, which they refer to as standardized LR-bar test, is based on the likelihood ratio (LR) test statistic developed by Johansen (1995) for vector autoregressive (VAR) models. Under the null hypothesis and as the time dimension approaches infinity, the LR statistic converges weakly to the asymptotic trace statistic, whose moments are thus used for standardizing the average of the individual LR test statistics.

*This research was supported by the Deutsche Forschungsgemeinschaft through the SFB 649 “Economic Risk”.

¹Institute for Statistics and Econometrics, School of Business and Economics, Humboldt-Universität zu Berlin, Spandauer Str. 1, 10099 Berlin, Germany, E-mail: karamand@staff.hu-berlin.de

²Institute for Statistics and Econometrics and CASE - Center for Applied Statistics and Economics, School of Business and Economics, Humboldt-Universität zu Berlin, Spandauer Str. 1, 10099 Berlin, Germany, E-mail: droge@wiwi.hu-berlin.de

The existence of the first two moments of the asymptotic trace statistic is claimed in Larsson et al. (2001), but their proof is incorrect as explained in Section 3. Therefore we provide a corrected version of the proof. Moreover, the asymptotic moments are usually approximated by simulating a certain statistic which converges weakly to the asymptotic trace statistic. To justify this approach we show that the first two moments of the mentioned statistic converge to those of the asymptotic trace statistic.

To be more specific, we consider, as Larsson et al. (2001), a sample of N cross-sections (individuals) observed over T time periods and suppose that for each individual i ($i = 1, \dots, N$) the K -dimensional time series y_{it} is generated by the following heterogeneous VAR(p_i) model:

$$y_{it} = \sum_{j=1}^{p_i} A_{ij} y_{i,t-j} + e_{it}, \quad i = 1, \dots, N; \quad t = 1, \dots, T, \quad (1)$$

where the initial values $y_{i,-p_i+1}, \dots, y_{i0}$ are fixed, A_{ij} are $(K \times K)$ coefficient matrices and the errors e_{it} are stochastically independent across i and t with $e_{it} \sim N_K(0, \Omega_i)$ for some nonsingular covariance matrices Ω_i . The components of the process y_{it} are assumed to be integrated at most of order one and cointegrated with cointegrating rank r_i with $0 \leq r_i \leq K$. The error correction representation of model (1) is

$$\Delta y_{it} = \Pi_i y_{i,t-1} + \sum_{j=1}^{p_i-1} \Gamma_{ij} \Delta y_{i,t-j} + e_{it}, \quad i = 1, \dots, N; \quad t = 1, \dots, T,$$

where the $(K \times K)$ parameter matrices $\Gamma_{ij} = -(A_{i,j+1} + \dots + A_{i,p_i})$ describe the short-run dynamics, and the $(K \times K)$ matrix $\Pi_i = -(I_K - A_{i1} - \dots - A_{i,p_i})$ can be written as $\Pi_i = \alpha_i \beta_i'$ with $(K \times r_i)$ matrices α_i and β_i of full column rank.

Interest is in testing whether in all of the N cross-sections there are at most r cointegrating relations among the K variables. Thus, the null hypothesis

$$H_0(r) : \text{rank}(\Pi_i) = r_i \leq r, \quad \text{for all } i = 1, \dots, N,$$

is tested against the alternative

$$H_1 : \text{rank}(\Pi_i) = K, \quad \text{for all } i = 1, \dots, N.$$

According to Johansen (1988), the cointegrating rank of the process may be determined by a sequential procedure. First, $H_0(0)$ is tested, and if this null hypothesis is rejected then $H_0(1)$ is tested. The procedure continues until the null hypothesis is not rejected or $H_0(K-1)$ is rejected.

The standardized LR-bar statistic for the panel cointegrating rank test is defined by

$$\Upsilon_{\overline{\text{LR}}}(r) = \frac{\sqrt{N} \left[\frac{1}{N} \sum_{i=1}^N \left(-T \sum_{j=r+1}^K \ln(1 - \hat{\lambda}_{ij}) \right) - \mathbb{E}(Z_d) \right]}{\sqrt{\text{Var}(Z_d)}},$$

where $\hat{\lambda}_{ij}$ is the j th largest eigenvalue to a suitable eigenvalue problem for the i th cross-section defined in Johansen (1995). Moreover, $\mathbb{E}(Z_d)$ and $\text{Var}(Z_d)$ denote the mean and the variance, respectively, of the asymptotic trace statistic

$$Z_d = \text{tr} \left[\left(\int_0^1 W(s) dW(s)' \right)' \left(\int_0^1 W(s) W(s)' ds \right)^{-1} \int_0^1 W(s) dW(s)' \right], \quad (2)$$

where $W(s)$ is a d -dimensional standard Brownian motion with $d = K - r$. Note that (2) is the limiting null distribution of the trace statistic (LR statistic) for a given individual i , i.e. of $-T \sum_{j=r+1}^K \ln(1 - \hat{\lambda}_{ij})$; see, e.g., Johansen (1995).

Under the null hypothesis and assuming suitable conditions, Larsson et al. (2001) applied a central limit theorem to establish the asymptotic normality of their standardized LR-bar statistic, so that standard normal quantiles may serve as critical values for the test. Moreover, they approximated the first two moments of the asymptotic trace statistic Z_d for different values d by simulation as sample moments of

$$Z_{T,d} = \text{tr} \left[\frac{1}{T} \sum_{t=1}^T \varepsilon_t X'_{t-1} \left(\frac{1}{T^2} \sum_{t=1}^T X_{t-1} X'_{t-1} \right)^{-1} \frac{1}{T} \sum_{t=1}^T X_{t-1} \varepsilon'_t \right], \quad (3)$$

where $\varepsilon_t \sim N_d(0, I_d)$ i.i.d. and $X_t = \sum_{i=1}^t \varepsilon_i$ for $t = 1, \dots, T$. This is motivated by the weak convergence of $Z_{T,d}$ to Z_d as $T \rightarrow \infty$. Consequently, the proposed procedure relies crucially on the fact that the first two moments of the asymptotic trace statistic exist and may be obtained as limits of the corresponding moments of $Z_{T,d}$.

2 Results

On account of the weak convergence of $Z_{T,d}$ to the asymptotic trace statistic Z_d , the first two moments of Z_d exist if the sequence $\{Z_{T,d}^2\}$ is uniformly integrable. A sufficient condition for this is established in Lemma 2, which states that the fourth moments of $Z_{T,d}$ are uniformly bounded in T . We start with showing that all moments of $Z_{T,d}$ exist. To ensure that the inverted matrix appearing in (3) is nonsingular with probability one, we assume $T > d$.

Lemma 1. *Assume that $T > d$. Then all moments of $Z_{T,d}$ defined by (3) exist.*

Proof. As Larsson et al. (2001), we introduce the $(T \times d)$ matrices $\varepsilon = (\varepsilon_1, \varepsilon_2, \dots, \varepsilon_T)'$ and $X = (X_1, X_2, \dots, X_T)'$ as well as the $(T \times T)$ matrices

$$A = \begin{pmatrix} 1 & 0 & \cdots & \cdots & 0 \\ 1 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 1 & \cdots & \cdots & \cdots & 1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 0 & \cdots & \cdots & \cdots & 0 \\ 1 & 0 & \cdots & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 & 0 \end{pmatrix}.$$

Then, $X = A\varepsilon$ and the $(d \times d)$ matrices appearing in (3) can be rewritten as

$$A_T := \frac{1}{T^2} \sum_{t=1}^T X_{t-1} X'_{t-1} = \frac{1}{T^2} \varepsilon' A' B' B A \varepsilon, \quad B_T := \frac{1}{T} \sum_{t=1}^T X_{t-1} \varepsilon'_t = \frac{1}{T} \varepsilon' A' B' \varepsilon. \quad (4)$$

Defining $D = BA$ and $Y = D\varepsilon$, we obtain therefore

$$Z_{T,d} = \text{tr}(B'_T A_T^{-1} B_T) = \text{tr}[\varepsilon' D \varepsilon (\varepsilon' D' D \varepsilon)^{-1} \varepsilon' D' \varepsilon] \quad (5)$$

$$= \text{tr}(\varepsilon' P_Y \varepsilon) \leq \text{tr}(\varepsilon' \varepsilon), \quad (6)$$

where $P_Y = Y(Y'Y)^{-1}Y'$ denotes the projection matrix onto the column space of Y . The assumption $\varepsilon_t \sim N_d(0, I_d)$ i.i.d. now implies $\text{tr}(\varepsilon' \varepsilon) = \sum_{t=1}^T \varepsilon'_t \varepsilon_t \sim \chi_{Td}^2$, which completes the proof, since all moments of a χ^2 -distributed random variable exist. ■

Note that inequality (6) cannot be used to bound the moments of $Z_{T,d}$ uniformly in T , because the moments of a χ^2 -distributed random variable depend on the degrees of freedom.

Lemma 2. *Let $Z_{T,d}$ be defined as in (3). Then there exist some constants a and b such that, for all $T > d$,*

$$(i) \quad \mathbb{E} \left(Z_{T,d}^2 \right) < a,$$

$$(ii) \quad \mathbb{E} \left(Z_{T,d}^4 \right) < b.$$

Proof. Using an inequality of Coope (1994), we get, on account of (5),

$$Z_{T,d} = \text{tr}(A_T^{-1} B_T B_T') \leq \text{tr}(A_T^{-1}) \text{tr}(B_T B_T'), \quad (7)$$

since A_T^{-1} and $B_T B_T'$ are symmetric and nonnegative definite matrices of the same order.

To deal with A_T , let $\lambda_1 \geq \dots \geq \lambda_{T-1} \geq \lambda_T \geq 0$ and v_1, \dots, v_T be the eigenvalues and the associated orthonormal eigenvectors, respectively, of the symmetric and nonnegative definite $(T \times T)$ matrix $F = D'D$. Then, for any $m \in \{1, \dots, T-1\}$,

$$F = \sum_{t=1}^T \lambda_t v_t v_t' \succeq \lambda_m \sum_{t=1}^m v_t v_t' =: F_m, \quad (8)$$

where \succeq denotes the Löwner partial ordering for symmetric matrices. Because of the orthonormality of the matrix $V = (v_1, \dots, v_T)$, $V'\varepsilon$ has the same distribution as ε , that is, with the notation of Muirhead (1982), $V'\varepsilon \sim N(0, I_T \otimes I_d)$. This implies

$$\varepsilon' F_m \varepsilon = \lambda_m U, \quad \text{with } U := \sum_{t=1}^m \varepsilon' v_t v_t' \varepsilon \sim W_d(m, I_d),$$

and thus, in view of (4) and (8),

$$A_T = \frac{1}{T^2} \varepsilon' F \varepsilon \succeq \frac{1}{T^2} \varepsilon' F_m \varepsilon = \frac{\lambda_m}{T^2} U =: A_{T,m}. \quad (9)$$

Clearly, $A_{T,m}$ is almost surely positive definite if $m \geq d$. Then (9) leads to $A_{T,m}^{-1} \succeq A_T^{-1}$, so that we arrive at

$$\text{tr}(A_T^{-1}) \leq \text{tr}(A_{T,m}^{-1}) = \frac{T^2}{\lambda_m} \text{tr}(U^{-1}). \quad (10)$$

Observing

$$D = BA = \begin{pmatrix} 0 & 0 & \dots & \dots & 0 \\ 1 & 0 & \dots & \dots & 0 \\ 1 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 1 & 1 & \dots & 1 & 0 \end{pmatrix}, \quad F = D'D = \begin{pmatrix} T-1 & T-2 & T-3 & \dots & 1 & 0 \\ T-2 & T-2 & T-3 & \dots & 1 & 0 \\ T-3 & T-3 & T-3 & \dots & 1 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 1 & 1 & \dots & 1 & 0 \\ 0 & 0 & 0 & \dots & 1 & 0 \end{pmatrix},$$

it follows $\lambda_T = 0$, and $\lambda_1, \dots, \lambda_{T-1}$ are the eigenvalues of the positive definite matrix \tilde{F} obtained from F by deleting the last column and the last row. This matrix can be represented

as the inverse of a tridiagonal Minkowski matrix (see Neumann, 2000, and Yueh, 2006), i.e.

$$\tilde{F} = \begin{pmatrix} T-1 & T-2 & T-3 & \dots & 2 & 1 \\ T-2 & T-2 & T-3 & \dots & 2 & 1 \\ T-3 & T-3 & T-3 & \dots & 2 & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 2 & 2 & 2 & \dots & 2 & 1 \\ 1 & 1 & 1 & \dots & 1 & 1 \end{pmatrix} = (-1) \begin{pmatrix} -1 & 1 & 0 & \dots & 0 & 0 \\ 1 & -2 & 1 & \dots & 0 & 0 \\ 0 & 1 & -2 & \ddots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \ddots & -2 & 1 \\ 0 & 0 & 0 & \dots & 1 & -2 \end{pmatrix}^{-1}.$$

Using Theorem 2 of Yueh (2005), the positive (ordered) eigenvalues of F can be represented as

$$\lambda_t = \frac{1}{2 \left[1 - \cos \left(\frac{(2t-1)\pi}{2T-1} \right) \right]} \quad \text{for } t = 1, \dots, T-1.$$

The series expansion of the cosine function provides, for a fixed $m \in \{1, \dots, T-1\}$ and as $T \rightarrow \infty$,

$$1 - \cos \left(\frac{(2m-1)\pi}{2T-1} \right) = \frac{(2m-1)^2 \pi^2}{2(2T-1)^2} + o(T^{-3})$$

and therefore

$$\frac{T^2}{\lambda_m} \xrightarrow{T \rightarrow \infty} \frac{(2m-1)^2 \pi^2}{4} =: c_1 < \infty. \quad (11)$$

Note that, for fixed m and $T \rightarrow \infty$, λ_m is of the same order T^2 as the sum of all eigenvalues of F , since $\sum_{t=1}^T \lambda_t = \text{tr}(F) = T(T-1)/2$.

With the notation $\varepsilon_t = (\varepsilon_{t1}, \dots, \varepsilon_{td})'$, the last term in inequality (7) may be written as

$$\begin{aligned} \text{tr}(B_T B_T') &= \frac{1}{T^2} \text{tr}(\varepsilon' D' \varepsilon \varepsilon' D \varepsilon) = \sum_{i=1}^d \sum_{j=1}^d \alpha_{ij}^2, \quad \text{where} \\ \alpha_{ij} &= \frac{1}{T} \sum_{s=1}^T \sum_{t=s+1}^T \varepsilon_{sj} \varepsilon_{ti}. \end{aligned} \quad (12)$$

To prove (i), we first take the second power in (7) and apply the Cauchy-Schwarz inequality, which gives

$$\mathbb{E}(Z_{T,d}^2) \leq \mathbb{E} \left[\text{tr}(A_T^{-1}) \text{tr}(B_T B_T') \right]^2 \leq \left\{ \mathbb{E}[\text{tr}(A_T^{-1})]^4 \mathbb{E}[\text{tr}(B_T B_T')]^4 \right\}^{1/2}. \quad (13)$$

Consequently, it suffices to verify that both expectations on the right-hand side of inequality (13) are uniformly bounded in $T > d$.

In view of (12), $\mathbb{E}[\text{tr}(B_T B_T')]^4$ is uniformly bounded in T if, for $i, j \in \{1, \dots, d\}$, $\sup_T \mathbb{E}(\alpha_{ij}^8) < \infty$. But this follows from $\varepsilon_t \sim N(0, I_d)$ i.i.d. and

$$\mathbb{E}(\alpha_{ij}^8) = \frac{1}{T^8} \left[\sum_{s_1=1}^T \dots \sum_{s_8=1}^T \sum_{t_1=s_1+1}^T \dots \sum_{t_8=s_8+1}^T \mathbb{E}(\varepsilon_{s_1 j} \dots \varepsilon_{s_8 j} \varepsilon_{t_1 i} \dots \varepsilon_{t_8 i}) \right],$$

because $\mathbb{E}(\varepsilon_{s_1 j} \dots \varepsilon_{s_8 j} \varepsilon_{t_1 i} \dots \varepsilon_{t_8 i}) = 0$ if more than eight of the subscripts $s_1, \dots, s_8, t_1, \dots, t_8$ differ.

Finally, to bound $\mathbb{E}[\text{tr}(A_T^{-1})]^4$ uniformly, we recall $U \sim W_d(m, I_d)$ and use results of von Rosen (1988, 1997) on moments for the inverted Wishart distribution. In particular it is known that the q th moments of U^{-1} exist if $m - d - 2q + 1 > 0$. Consequently,

$$\mathbb{E}[\text{tr}(U^{-1})]^4 \leq c_2 < \infty \quad \text{for } m \geq d + 8, \quad (14)$$

so that an application of inequality (10) for $m = d + 8$ (assuming $T > m$) together with (11), (14) and Lemma 1 yields the desired result.

The proof of (ii) is analogous to that of (i) and thus only sketched. First, the Cauchy-Schwarz inequality provides, using (7),

$$\mathbb{E}(Z_{T,d}^4) \leq \mathbb{E}[\text{tr}(A_T^{-1})\text{tr}(B_T B_T')]^4 \leq \{\mathbb{E}[\text{tr}(A_T^{-1})]^8 \mathbb{E}[\text{tr}(B_T B_T')]^8\}^{1/2}.$$

It is easy to see that $\mathbb{E}[\text{tr}(B_T B_T')]^8$ is uniformly bounded in T , since $\sup_T \mathbb{E}(\alpha_{ij}^{16}) < \infty$. Finally, $\mathbb{E}[\text{tr}(A_T^{-1})]^8$ is uniformly bounded by choosing $m = d + 16$ and applying (10) together with (11), because then $\mathbb{E}[\text{tr}(U^{-1})]^8 \leq c_3 < \infty$. This completes the proof. ■

Theorem. *It holds that $\mathbb{E}(Z_d^2) < \infty$ and $\lim_{T \rightarrow \infty} \mathbb{E}(Z_{T,d}^q) = \mathbb{E}(Z_d^q)$ for $q = 1, 2$.*

Proof. Recalling that $Z_{T,d}$ converges weakly to the asymptotic trace statistic Z_d (Johansen, 1995), the result follows if $\{Z_{T,d}^2\}$ is uniformly integrable (see Theorem A on p.14 in Serfling, 1980). A sufficient condition for the uniform integrability of $\{Z_{T,d}^2\}$ is that $\mathbb{E}|Z_{T,d}|^{2+\delta}$ is uniformly bounded for some $\delta > 0$, i.e. $\sup_T \mathbb{E}|Z_{T,d}|^{2+\delta} < \infty$. But this is an immediate consequence of Lemma 2 (ii), completing the proof. ■

3 Discussion

Several authors have used the first two moments of the asymptotic trace statistic to base panel cointegration tests on a standardized average of individual cointegration test statistics; see, for instance, Larsson et al. (2001), Groen & Kleibergen (2003) and Breitung (2005). Our Theorem provides a theoretical justification for such an approach. To the best of our knowledge, the only attempt to establish this result is due to Larsson et al. (2001). However, the proof of their Lemma 1, which coincides with our Lemma 2, is incorrect and has thus initiated this note. In what follows, we comment in more detail on the proof by Larsson et al. (2001).

In our notation, Larsson et al. (2001) assumed $\varepsilon_t \sim N_d(0, \Omega)$ i.i.d for defining $Z_{T,d}$ in (3). This seems to be unnecessary, but would not lead to complications in our proof. Moreover, they used the spectral decomposition of the (random) positive definite $(d \times d)$ matrix A_T (see (4)), i.e.

$$A_T = \frac{1}{T^2} \sum_{t=1}^T X_{t-1} X_{t-1}' = G' \Gamma G,$$

where G is an orthogonal $(d \times d)$ matrix and $\Gamma = \text{diag}(\gamma_1, \dots, \gamma_d)$, and defined $\tilde{\varepsilon}$ by $\varepsilon = \tilde{\varepsilon} G$. Then they rewrote (3) as

$$Z_{T,d} = \text{tr}(B_T' G' \Gamma^{-1} G B_T) = \sum_{i=1}^d H_{ii} \gamma_i^{-1},$$

(compare also (5)), where H_{ii} are the diagonal elements of $H = T^{-2}\tilde{\varepsilon}'D'\tilde{\varepsilon}\tilde{\varepsilon}'D\tilde{\varepsilon}$. Finally, they applied the triangle and Cauchy-Schwarz inequalities to get

$$\mathbb{E}(Z_{T,d}^2) \leq \sum_{i=1}^d \sum_{j=1}^d \left[\mathbb{E}(H_{ii}^4)\mathbb{E}(\gamma_i^{-4})\mathbb{E}(H_{jj}^4)\mathbb{E}(\gamma_j^{-4}) \right]^{\frac{1}{4}},$$

so that it remains to bound $\mathbb{E}(H_{ii}^4)$ and $\mathbb{E}(\gamma_j^{-4})$ (uniformly in T).

The major difficulty with the proof of Larsson et al. (2001) is that the authors seem to ignore the randomness of the matrix G . They argue, for example, that $\tilde{\varepsilon} = \varepsilon G'$ has the same distribution as ε since G is orthogonal; but G depends on ε (note that even for a deterministic G the assumption $\varepsilon \sim N(0, I_T \otimes \Omega)$ would generally imply a different distribution of $\tilde{\varepsilon}$: $\tilde{\varepsilon} \sim N(0, I_T \otimes G\Omega G')$). More importantly, to bound $\mathbb{E}(\gamma_j^{-4})$ they state that $\Gamma = GA_T G' = T^{-2}\tilde{\varepsilon}'D'D\tilde{\varepsilon}$ follows some d -variate Wishart distribution with $T - 1$ degrees of freedom. However, we do not see how the diagonal matrix Γ can be Wishart distributed. Probably, the authors believe that $A_T = T^{-2}\varepsilon'D'D\varepsilon$ is Wishart distributed and use the orthogonality of G . As before, complications arise from the randomness of G . Moreover, A_T would be Wishart distributed if, for instance, the rows of $D\varepsilon \sim N(0, DD' \otimes \Omega)$ are independent or $T^{-2}D'D$ is a projection matrix, but both statements do obviously not hold.

As intended by Larsson et al. (2001), we establish the existence of the first two moments of the asymptotic trace statistic by showing that the sequence $\{Z_{T,d}^2\}$ is uniformly integrable. However, our corrected proof of their Lemma 1 uses basically inequality (8) and thus (9), where we have to choose a fixed value of m in an appropriate way. On the one hand the moments of the inverted Wishart variable U^{-1} (with m degrees of freedom) must exist, and on the other hand the eigenvalue λ_m must be of order T^2 , which requires a careful investigation of the eigenvalues of the matrix $F = D'D$.

References

- Breitung, J. (2005). A parametric approach to the estimation of cointegration vectors in panel data. *Econometric Reviews*, **24**, 1–20.
- Coope, I. D. (1994). On matrix trace inequalities and related topics for products of Hermitian matrices. *Journal of Mathematical Analysis and Applications*, **188**, 999–1001.
- Groen, J. J. J. & Kleibergen, F. (2003). Likelihood-based cointegration analysis in panels of vector error correction models. *Journal of Business and Economic Statistics*, **21**, 295–318.
- Johansen, S. (1988). Statistical analysis of cointegrating vectors. *Econometric Reviews*, **12**, 231–254.
- Johansen, S. (1995). *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models*. Oxford University Press: Oxford.
- Larsson, R., Lyhagen, J., & Löthgren, M. (2001). Likelihood-based cointegration tests in heterogeneous panels. *Econometrics Journal*, **4**, 109–142.
- Muirhead, R. J. (1982). *Aspects of Multivariate Statistical Theory*. New York: Wiley.
- Neumann, M. (2000). Inverses of Perron complements of inverse M-matrices. *Linear Algebra and its Applications*, **313**, 163–171.

- Serfling, R. J. (1980). *Approximation Theorems of Mathematical Statistics*. New York: Wiley.
- von Rosen, D. (1988). Moments for the inverted Wishart distribution. *Scandinavian Journal of Statistics*, **15**, 97–109.
- von Rosen, D. (1997). On moments of the inverted Wishart distribution. *Statistics*, **30**, 259–278.
- Yueh, W.-C. (2005). Eigenvalues of several tridiagonal matrices. *Applied Mathematics E-Notes*, **5**, 66–74.
- Yueh, W.-C. (2006). Explicit inverses of several tridiagonal matrices. *Applied Mathematics E-Notes*, **6**, 74–83.

SFB 649 Discussion Paper Series 2009

For a complete list of Discussion Papers published by the SFB 649, please visit <http://sfb649.wiwi.hu-berlin.de>.

- 001 "Implied Market Price of Weather Risk" by Wolfgang Härdle and Brenda López Cabrera, January 2009.
- 002 "On the Systemic Nature of Weather Risk" by Guenther Filler, Martin Odening, Ostap Okhrin and Wei Xu, January 2009.
- 003 "Localized Realized Volatility Modelling" by Ying Chen, Wolfgang Karl Härdle and Uta Pigorsch, January 2009.
- 004 "New recipes for estimating default intensities" by Alexander Baranovski, Carsten von Lieres and André Wilch, January 2009.
- 005 "Panel Cointegration Testing in the Presence of a Time Trend" by Bernd Droge and Deniz Dilan Karaman Örsal, January 2009.
- 006 "Regulatory Risk under Optimal Incentive Regulation" by Roland Strausz, January 2009.
- 007 "Combination of multivariate volatility forecasts" by Alessandra Amendola and Giuseppe Storti, January 2009.
- 008 "Mortality modeling: Lee-Carter and the macroeconomy" by Katja Hanewald, January 2009.
- 009 "Stochastic Population Forecast for Germany and its Consequence for the German Pension System" by Wolfgang Härdle and Alena Mysickova, February 2009.
- 010 "A Microeconomic Explanation of the EPK Paradox" by Wolfgang Härdle, Volker Krätschmer and Rouslan Moro, February 2009.
- 011 "Defending Against Speculative Attacks" by Tijmen Daniëls, Henk Jager and Franc Klaassen, February 2009.
- 012 "On the Existence of the Moments of the Asymptotic Trace Statistic" by Deniz Dilan Karaman Örsal and Bernd Droge, February 2009.

SFB 649, Spandauer Straße 1, D-10178 Berlin
<http://sfb649.wiwi.hu-berlin.de>

This research was supported by the Deutsche
Forschungsgemeinschaft through the SFB 649 "Economic Risk".

