

Caporale, Guglielmo Maria; Gil-Alana, Luis A.

**Working Paper**

## A multivariate long-memory model with structural breaks

CESifo Working Paper, No. 1950

**Provided in Cooperation with:**

Ifo Institute – Leibniz Institute for Economic Research at the University of Munich

*Suggested Citation:* Caporale, Guglielmo Maria; Gil-Alana, Luis A. (2007) : A multivariate long-memory model with structural breaks, CESifo Working Paper, No. 1950, Center for Economic Studies and ifo Institute (CESifo), Munich

This Version is available at:

<https://hdl.handle.net/10419/25995>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

# A MULTIVARIATE LONG-MEMORY MODEL WITH STRUCTURAL BREAKS

GUGLIELMO MARIA CAPORALE  
LUIS A. GIL-ALANA

CESIFO WORKING PAPER NO. 1950  
CATEGORY 10: EMPIRICAL AND THEORETICAL METHODS  
MARCH 2007

*An electronic version of the paper may be downloaded*

- *from the SSRN website:* [www.SSRN.com](http://www.SSRN.com)
- *from the RePEc website:* [www.RePEc.org](http://www.RePEc.org)
- *from the CESifo website:* [www.CESifo-group.de](http://www.CESifo-group.de)

# A MULTIVARIATE LONG-MEMORY MODEL WITH STRUCTURAL BREAKS

## Abstract

This paper introduces a multivariate long-memory model with structural breaks. In the proposed framework, time series exhibit possibly fractional orders of integration which are allowed to be different in each subsample. The break date is endogenously determined using a procedure which minimises the residual sum of squares (RSS). Monte Carlo experiments show that this method for detecting breaks performs well in large samples. As an illustration, we estimate a trivariate VAR including prices, employment and GDP in both the US and Mexico. For the subsample preceding the break our findings are similar to those of earlier studies based on a standard VAR approach in both countries, in the sense that the variables exhibit integer degrees of integration. On the contrary, the series are found to be fractionally integrated after the break, with the fractional differencing parameters being higher than 1 in the case of Mexico.

JEL Code: C22.

Keywords: multivariate models, fractional integration, structural breaks.

*Guglielmo Maria Caporale*  
*Brunel University*  
*Uxbridge, Middlesex UB8 3PH*  
*United Kingdom*  
*Guglielmo-Maria.Caporale@brunel.ac.uk*

*Luis A. Gil-Alana*  
*University of Navarra*  
*Faculty of Economics*  
*31080 Pamplona*  
*Spain*  
*alana@unav.es*

February 2007

The second-named author gratefully acknowledges financial support from the Ministerio de Ciencia y Tecnología (SEJ2005-07657, Spain).

## 1. Introduction

Mandelbrot (1969) and Mandelbrot and van Ness (1968) found that, although many time series exhibit a persistent trend-cyclical behaviour over a certain time span, when the same data are examined for a longer period, the persistent behaviour tends to disappear. This phenomenon was first noticed in a hydrological context, and called the Hurst effect, in honour of the hydrologist Hurst, (Hurst, 1951, 1957), who, when studying the records on the level of the river Nile, noticed this pattern in its behaviour. In particular, he observed that the autocorrelations took far longer to decay to zero than the exponential rate associated with other classic models such as Auto-Regressive Moving Average (ARMA) ones. This type of process is characterised by long memory, with significant time dependence even between distant observations.

Two alternative definitions of long memory can be provided. Given a discrete covariance stationary time series,  $\{x_t, t = 0, \pm 1, \dots\}$ , with autocorrelation function  $E[(x_t - Ex_t)(x_{t-j} - Ex_{t-j})] = \gamma_j$ , according to McLeod and Hipel (1978) the process exhibits long memory if:

$$\lim_{T \rightarrow \infty} \sum_{j=-T}^T |\gamma_j|$$

is infinite. The second definition is based on the frequency domain. Assuming that  $\{x_t\}$  has a continuous spectral distribution, so that it has a spectral density function  $f(\lambda)$ , one can say that  $\{x_t\}$  displays the property of long memory if  $f(\lambda)$  has a pole at some frequency  $\lambda_0$  in the interval  $[0, \pi)$ . A model with the above features is the Fractional Gaussian Noise one, analysed in Mandelbrot and van Ness (1968). Another simple model, very popular among time series analysts, is the fractionally integrated or I(d) one, where the number of differences required to obtain I(0) stationarity is not necessarily an integer, but might be instead any real value. Accordingly, one can define  $\{x_t\}$  as an I(d) process if:

$$(1 - L)^d x_t = u_t, \quad t = 1, 2, \dots, \quad (1)$$

where  $L$  is the lag-operator (i.e.,  $Lx_t = x_{t-1}$ );  $d$  can be any real value, and  $u_t$  is  $I(0)$ , defined as a covariance-stationary process with spectral density function that is positive and finite at any frequency. Note that the fractional differencing polynomial in (1) can be expressed in terms of its Binomial expansion, such that, for all real  $d$ ,

$$(1 - L)^d = \sum_{j=0}^{\infty} \psi_j L^j = \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j L^j = 1 - dL + \frac{d(d-1)}{2} L^2 - \dots,$$

and therefore

$$(1 - L)^d x_t = x_t - dx_{t-1} + \frac{d(d-1)}{2} x_{t-2} - \dots,$$

implying that the higher the parameter  $d$  is, the higher the level of association will be between the observations. These processes were introduced by Granger (1980, 1981), Granger and Joyeux (1980) and Hosking (1981), and they were theoretically justified in terms of aggregation of autoregressive (AR) processes with random heterogeneous coefficients by Robinson (1978) and Granger (1980). Parke (1999) justifies fractional integration with an error duration model, while Diebold and Inoue (2001) relates fractional integration with regime-switching models (see Baillie, 1996, for an extensive review of  $I(d)$  processes).<sup>1</sup>

Most of the literature on the estimation and testing of  $I(d)$  models focuses on the univariate case. Parametric methods can be found in Fox and Taqqu (1986), Dahlhaus (1989), Sowell (1992b), Robinson (1994), etc. Semiparametric approaches are followed by Geweke and Porter-Hudak (1983), Robinson (1995), Velasco (1999), Shimotsu and Phillips (2005), etc. By contrast, the literature on multivariate models is rather limited:

---

<sup>1</sup> Empirical applications of fractional integration to macro series can be found in Diebold and Rudebusch (1989), Sowell (1992a) and Gil-Alana and Robinson (1997) among others.

Gil-Alana (2003a,b) extends the univariate tests of Robinson (1994) in the frequency domain, while Nielsen (2005) proposes similar tests in the time domain.

In this paper we add to the multivariate literature by proposing a multivariate procedure for estimating the fractional differencing parameters in the presence of structural breaks. For simplicity we focus on the case of a single break, though the analysis can be easily extended to multiple breaks. More specifically, our contribution is twofold. Firstly, we consider a more general (fractional) VAR model, which includes as a special case the standard VAR specification when the orders of integration are equal to 0 or 1. Secondly, we allow for endogenously determined breaks. The layout of the paper is as follows. Section 2 describes the statistical model and outlines the parameter estimation procedure. Section 3 contains the Monte Carlo simulation results. An empirical application is carried out in Section 4, while Section 5 concludes the paper.

## 2. The statistical model

We start by analysing the case of a single break. Specifically, we consider a model of the form:

$$D^a y_t = u_t, \quad t = 1, 2, \dots, T_b, \quad (2)$$

$$D^b y_t = u_t, \quad t = T_b + 1, \dots, T, \quad (3)$$

where  $y_t$  is the  $(n \times 1)$  vector of observed data;  $u_t$  is a  $(n \times 1)$  vector of  $I(0)$  processes<sup>2</sup>;  $D^a$  and  $D^b$  are  $(n \times n)$  diagonal matrices with  $i^{\text{th}}$  elements  $(1-L)^{d_i^a}$  and  $(1-L)^{d_i^b}$  respectively;  $T_b$  is the break date. The procedure minimises the residual sum of squares for a grid of values of the fractional differencing parameters  $d_i^a$  and  $d_j^b$ ,  $i, j = 1, \dots, n$ . Thus, for a given partition,  $T_b$ , and given initial values of the fractional

---

<sup>2</sup> An  $I(0)$  vector process is defined as a covariance stationary process with spectral density matrix that is finite and positive definite.

differencing parameters  $d_o^a = (d_{10}^a, d_{20}^a, \dots, d_{no}^a)^T$ ;  $d_o^b = (d_{10}^b, d_{20}^b, \dots, d_{no}^b)^T$ , the objective function is  $RSS(T_b; d_o^a, d_o^b)$ , and minimising this expression for all values of  $d_o^a$  and  $d_o^b$ , one obtains:

$$RSS(T_b) = \arg \min_{\{k\}} RSS(T_b; d_{ik}^a, d_{jk}^b), \quad i, j = 1, 2, \dots, n.$$

Then, the estimated break date,  $\hat{T}_k$ , is such that

$$\hat{T}_k = \arg \min_{i=1, \dots, m} RSS(T_i),$$

where the minimisation is done over all partitions  $T_1, T_2, \dots, T_m$ , such that  $T_i - T_{i-1} \geq |\varepsilon T|$ . Hence, the estimated fractionally differencing parameters associated to the estimated k-partition are:

$$\hat{d}_i^a = \hat{d}_i^a(\{\hat{T}_k\}) \quad \text{and} \quad \hat{d}_j^b = \hat{d}_j^b(\{\hat{T}_k\}),$$

for  $i, j = 1, 2, \dots, n$ .

Clearly, the model can be extended to the case of multiple breaks by considering the following specification:

$$D^j y_t = u_t, \quad t = T_{j-1} + 1, \dots, T_j,$$

for  $j = 1, \dots, m+1$ ,  $T_0 = 0$  and  $T_{m+1} = T$ , with the parameter  $m$  representing the number of breaks. The break dates ( $T_1, \dots, T_m$ ) are treated as unknown and for  $i = 1, \dots, m$ , we have  $\lambda_i = T_i/T$ , with  $\lambda_1 < \dots < \lambda_m < 1$ . Following the same approach as in the previous case, for each j-partition,  $\{T_1, \dots, T_j\}$ , denoted  $\{T_j\}$ , the estimates of the  $d$ 's are obtained by minimising the sum of squared residuals in the  $d$ -differenced models, i.e.,

$$\sum_{j=1}^{m+1} \sum_{t=T_{j-1}+1}^{T_j} (D^j y_t)^T (D^j y_t),$$

where  $\hat{d}(T_j)$  denotes the resulting estimates. Substituting into the new objective function and denoting the sum of squared residuals as  $RSS_T(T_1, \dots, T_m)$ , the estimated break dates  $(\hat{T}_1, \hat{T}_2, \dots, \hat{T}_m)$  are obtained as follows

$$\min_{(T_1, T_2, \dots, T_m)} RSS_T(T_1, \dots, T_m)$$

where the minimisation is again obtained over all partitions  $(T_1, \dots, T_m)$ .

### 3. A Monte Carlo simulation study

The data generating process is a trivariate model given by:

$$\begin{pmatrix} (1-L)^{d_1^a} & 0 & 0 \\ 0 & (1-L)^{d_2^a} & 0 \\ 0 & 0 & (1-L)^{d_3^a} \end{pmatrix} \begin{pmatrix} y_{1,t} \\ y_{2,t} \\ y_{3,t} \end{pmatrix} = \begin{pmatrix} u_{1,t} \\ u_{2,t} \\ u_{3,t} \end{pmatrix}, \quad t = 1, 2, \dots, T_b, \quad (4)$$

$$\begin{pmatrix} (1-L)^{d_1^b} & 0 & 0 \\ 0 & (1-L)^{d_2^b} & 0 \\ 0 & 0 & (1-L)^{d_3^b} \end{pmatrix} \begin{pmatrix} y_{1,t} \\ y_{2,t} \\ y_{3,t} \end{pmatrix} = \begin{pmatrix} u_{1,t} \\ u_{2,t} \\ u_{3,t} \end{pmatrix}, \quad t = T_b + 1, \dots, T, \quad (5)$$

with white noise  $u_t$ , and initially (in Table 1) we assume that  $d_1^a = 0.2$ ;  $d_2^a = 0.4$ ;  $d_3^a = 0.3$ , while  $d_1^b = 0.6$ ;  $d_2^b = 0.9$  and  $d_3^b = 1.4$ . Thus, the orders of integration are higher in the second subsample for the three series, which are stationary before the break, occurring at  $T_b = T/2$ , and nonstationary afterwards. To conduct this experiment, we generate Gaussian series using the routines GASDEV and RAN3 of Press, Flannery, Teukolsky and Wetterling (1986), with sample sizes  $T = 100, 200, 300, 500$  and  $1000$ . The number of replications is 1000 in each case.

We followed the procedure described in Section 2 for a grid of values of the  $d$ 's from 0 to 2 with 0.2 increments and estimated break dates  $T^* = T/10, T/10 + 1, \dots$ ,



$9T/10 - 1$  and  $9T/10$ . Table 1 reports the percentage of cases corresponding to the minimum RSS. In order to save space we only display cases with a probability higher than 0.01.

It can be seen that all such probabilities correspond to the values  $T^* = T/2 - 2$ ,  $T/2 - 1$ ,  $T/2$ ,  $T/2 + 1$  and  $T/2 + 2$ , the highest probability mass being concentrated around the true break date  $T/2$ . If  $T = 100$ , the probability of correctly determining the break date along with the six fractional differencing parameters is very small (0.031), though it is the highest of all cases. This indicates that, for this small size, the probabilities are spread widely across all cases. When increasing the sample size, the probabilities corresponding to the true cases substantially increase, being equal to 14.6% with  $T = 200$ ; 33.8% with  $T = 300$ ; 77.6% if  $T = 500$  and higher than 90% with  $T = 1000$ .

**INSERT TABLES 1 – 3 ABOUT HERE**

Table 2 displays similar results for a model with  $d^a = [0.8, 1.2, 0.6]^T$  and  $d^b = [0.2, 0.4, 0.2]^T$  and the break occurring at  $T/4$ . The probabilities of correctly determining the break date along with the fractional differencing parameters are slightly smaller than before, though again increasing with the sample size. However, the probability mass around the true break is now higher: even for a sample size of  $T = 100$  the percentage around  $T/4$  is 0.750.

In Table 3 we combine stationary and nonstationary processes for each subsample,  $d^a = [0.6, 0.3, 0.5]^T$ ,  $d^b = [0.2, 0.9, 0.5]^T$  and  $T^b = 3T/4$ . The results are similar to those presented in Table 2, with the highest probabilities corresponding to the true model in all cases, and increasing with  $T$ .

In Table 4 we extend the model to allow for weak parametric autocorrelation, and consider the model in (4) and (5) with  $d^a = [0.2, 0.4, 0.3]^T$ , and  $d^b = [0.6, 0.9, 1.4]^T$ ,  $T^b = T/2$ , and  $u_t$  following a VAR(1) process of the form:

$$\begin{pmatrix} u_{1,t} \\ u_{2,t} \\ u_{3,t} \end{pmatrix} = \begin{pmatrix} 0.2 & 0.1 & 0.6 \\ 0.2 & 0.3 & 0.1 \\ 0.1 & 0.2 & 0.2 \end{pmatrix} \begin{pmatrix} u_{1,t-1} \\ u_{2,t-1} \\ u_{3,t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{pmatrix}, \quad t = 1, 2, \dots, T_b$$

$$\begin{pmatrix} u_{1,t} \\ u_{2,t} \\ u_{3,t} \end{pmatrix} = \begin{pmatrix} 0.6 & 0.1 & 0.3 \\ 0.4 & 0.5 & 0.5 \\ 0.1 & 0.1 & 0.5 \end{pmatrix} \begin{pmatrix} u_{1,t-1} \\ u_{2,t-1} \\ u_{3,t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{pmatrix}, \quad t = T_b + 1, \dots, T,$$

with white noise  $\varepsilon_t$ .

#### **INSERT TABLE 4 ABOUT HERE**

It is apparent that in this case, if the sample size is small ( $T = 100$ ), the probability of correctly determining the break date and the fractional differencing parameters is negligible (smaller than 0.01), though the probability mass around the true break date is 44.8%. If  $T = 200$  the probability around the true values is still small (5.2%), though it is the highest of all. As in the previous cases, when increasing  $T$ , the probabilities around the true values also increase, and, if  $T = 1000$ , the true probability is around 75.7%, while the probabilities for the remaining cases never exceed 5%.

#### **4. An empirical application**

In this section we apply the procedure outlined above to analyse the linkages between prices (inflation), employment and GDP in the US and Mexico in a fractional multivariate model allowing for a single, endogenously determined break. We choose

these two countries as representative of the G-7 and emerging economies respectively, and as they most likely exhibit breaks at different points in time.

Two types of models are estimated in the existing literature. The first is the standard VAR, namely,

$$D(L)y_t^* = u_t, \quad t = 1, 2, \dots, \quad (6)$$

$$y_t^* = \begin{pmatrix} y_{1,t}^* \\ y_{2,t}^* \\ y_{3,t}^* \end{pmatrix} = \begin{pmatrix} (1-L)^{d_1} y_{1,t} \\ (1-L)^{d_2} y_{2,t} \\ (1-L)^{d_3} y_{3,t} \end{pmatrix}, \quad t = 1, 2, \dots,$$

where integer degrees of differentiation  $d_1$ ,  $d_2$  and  $d_3$  are implicitly assumed. In other words, the series are individually pre-tested using classical methods (Dickey and Fuller, 1979; Phillips and Perron, 1988; etc.) to determine if they should be included in (6) in levels ( $d = 0$ ) or in first differences ( $d = 1$ ) (Sims, 1980). Examples of this type of analysis are the papers of Debenedictis (1997), Aucremanne and Wouters (1999), Claus (2000), Canova (2002), etc.

The second approach is based on cointegration, assuming that there are some common trends implying  $I(0)$  equilibrium relationships between the variables (Johansen, 1988, 1991). Here the implicit assumption is again that at least two of the series are nonstationary  $I(1)$ . Papers along these lines include those of Cameron, Hum and Simpson (1996), Pétursson and Slok (2001) and Reade (2005). The present study falls within the first category, since it is a generalisation of the VAR model to the fractional VAR case including a structural break as well.

#### **4.1 The US case**

The series analysed in this subsection are the Consumer Price Index (for all urban consumers, all items), and civilian employment (sixteen years and over), both obtained

from the U.S. Department of Labor (Bureau of Labour Statistics), and GDP, provided by the U.S. Department of Commerce (Bureau of Economic Analysis), all quarterly, seasonally adjusted, for the time period 1948q1 2006q2. All series in the estimated model are in logs.

### **INSERT TABLE 5 ABOUT HERE**

We perform the procedure described in Section 2 for values of  $(d^a, d^b)$  in (4) and (5) in the interval -2 to 2, with 0.01 increments, and estimated break date  $T^* = T/10, T/10 + 1, \dots, 9T/10 - 1, 9T/10$ .<sup>3</sup> Table 5 reports the results based on white noise disturbances. It can be seen that the break takes place at 1974Q4 (1975Q1), which corresponds to the first OPEC oil price shock. In the first subsample the order of integration of the three series is 1 or close to 1 – more precisely, the order of integration of CPI and GDP ( $d_2$  and  $d_3$  respectively) is exactly equal to 1, whilst it is 1.01 for employment ( $d_1$ ). However, after the break, the three orders of integration are much higher than 1, ranging from 1.45 (CPI) to 1.52 (GDP). Therefore, there is a substantial increase in the degree of persistence of the three series after the break. Confidence intervals for the estimated fractional differencing parameters can be obtained using bootstrapping methods, although they are highly computationally expensive. Instead, we have computed Gil-Alana's (2003a,b) multivariate version of the univariate tests of Robinson (1994) for each subsample, which enables us to obtain confidence intervals for the fractional differencing parameters. The 95% confidence intervals are also displayed in Table 5. As expected, the unit root null hypothesis cannot be rejected for

---

<sup>3</sup> Following standard practice, we have not considered 10% of the observations at both ends of the sample.

any of the three series in the first subsample, though this hypothesis is decisively rejected after the break.

### **INSERT TABLE 6 ABOUT HERE**

Table 6 shows the results with VAR (1) disturbances. The break again occurs at 1974Q4 (1975Q1), but now in the first subsample the three orders of integration are 0 or close to 0 and the null hypothesis of  $I(0)$  cannot be rejected for any series. This is clearly because the time dependence is now captured by the VAR coefficients, which are close to 1 in all cases. After the break, the values of the  $d$  parameters range between 0 and 1, and the two  $I(0)$  and  $I(1)$  hypotheses are rejected in the cases of employment and GDP. Specifically, the values are 0.66 for employment; 0.25 for CPI and 0.43 for GDP. Therefore, in both cases (whether with white noise or autocorrelated disturbances), we observe an increase in the degree of persistence after the break in 1975.

#### **4.2 The Mexican case**

We analyse the same variables also for Mexico, more specifically consumer price index (1970 = 100), economically active population (EAP, in thousand values), and GDP (million 1970 PPP\$), annually, from 1900 to 2000. The data are taken from the Oxford Latin American Economic History Database, Latin American Centre at Oxford University (<http://oxlad.qeh.ox.ac.uk>). Similarly to the US case, the series are log-transformed.

Tables 7 and 8 are similar to Tables 5 and 6 above but refer to the Mexican case, that is, they correspond to the estimated multivariate fractional models with a single

break for the two cases of white noise and VAR(1) disturbances respectively. It can be seen that, if  $u_t$  is specified as a white noise, the break occurs in 1959, whilst the break date is 1962 with autocorrelated disturbances.

### **INSERT TABLES 7 AND 8 ABOUT HERE**

The results based on white noise disturbances are very similar to the US ones. Before the break, the orders of integration are close to 1 for the three series: 1.06 for prices, 0.97 for employment, and 0.99 for GDP and the unit root cannot be rejected for any of the series. However, after the break, they are significantly above 1, ranging from 1.37 (GDP) to 1.77 (employment), and the unit root null hypothesis is rejected for the three series. If a VAR(1) structure is incorporated into the model, they are around 0 before the break, once more reflecting the competition with the VAR parameters in capturing nonstationarity, while, after the break, the corresponding values are strictly above 1 for all three series considered.<sup>4</sup>

Overall, in the subsample before the break, the results for the three series in the two countries are consistent with the standard approach employed in the literature, that is, the series are found to be either I(1) (if no autocorrelation is allowed for) or I(0) (when modelling the disturbances as VAR(1) processes). However, after the breaks, the series are clearly fractionally integrated: for the US, the values range between 1 and 2 without autocorrelation in the disturbances, and lie between 0 and 1 with autocorrelated  $u_t$ . In Mexico, though, regardless of whether or not autocorrelation is allowed for, the series are in all cases I(d) with d lying between 1 and 2.

---

<sup>4</sup> Note, however, that the null hypothesis of a unit root cannot be rejected for any of the series in the second subsample.

## 5. Conclusions

This paper introduces a multivariate long-memory model with structural breaks. In the proposed framework, time series exhibit possibly fractional orders of integration which are allowed to be different in each subsample. The break point is endogenously determined using a procedure which minimises the residual sum of squares (RSS). Monte Carlo experiments show that this method for detecting the break performs well in large samples. Both the multivariate fractional nature of the suggested model, and the endogenous determination of breaks represent innovative features of the present study compared to earlier ones, which are based on standard VAR models with or without structural breaks (e.g., McConnell and Perez-Quiros, 2000; Chauvet and Potter, 2001; Kim, Nelson and Piger, 2001, etc.)

As an illustration, we apply our procedure to estimate a trivariate VAR including prices, employment and GDP both in the US and Mexico. For the subsample preceding the break our findings are similar to those of earlier studies based on a standard VAR approach in both countries, in the sense that the variables exhibit integer degrees of integration. On the contrary, the series are found to be fractionally integrated after the break, with the fractional differencing parameters being higher than 1 for the three series in the case of Mexico.

The present study can be extended in several ways. First, deterministic components like intercepts, linear trends or even non-linear structures (Caporale and Gil-Alana, 2007) can be included in the regression models, therefore allowing for fractionally integrated regression errors. Moreover, allowance can be made for multiple breaks, though, given the long memory characteristic of the processes analysed here, a long span of data would be required. Finally, the fractional polynomials used in the model specification adopted in this paper imply the existence of a pole or singularity in

the spectral density function at the zero frequency - this can clearly be generalised to the case of a pole at a frequency other than zero for some or all series considered. This could also lead to the analysis of seasonal multivariate fractional models, which to date have not been extensively investigated.



## References

- Aucremanne, L. and R. Wouters, 1999, A structural VAR approach to core inflation and its relevance for monetary policy, in BIS (eds.), Measures of Underlying Inflation and their Role in the Conduct of Monetary Policy, Proceedings of the Workshop of Central Bank Model Builders, Amsterdam.
- Baillie, R.T., 1996, Long memory processes and fractional integration in econometrics. *Journal of Econometrics* 73, 5-59.
- Cameron, N., D. Hum and W. Simpson, 1996, Stylized facts and stylized illusions: inflation and productivity revisited, *Canadian Journal of Economics* 29, 152-162.
- Canova, F., 2002, G7 Inflation forecasts, ECB Working Paper No. 151.
- Chauvet, M. and S. Potter, 2001, Recent changes in the US business cycle, Federal Reserve Bank of New York, Staff Reports, n. 126.
- Caporale, G.M. and L.A. Gil-Alana, 2007, Non-linearities and fractional integration in the US unemployment rate, *Oxford Bulletin of Economics and Statistics*, forthcoming.
- Claus, I., 2000, Estimating potential output for New Zealand. A structural VAR approach, Reserve Bank of New Zealand, Discussion Paper 2000/03.
- Dahlhaus, R., 1989, Efficient parameter estimation for self similar processes, *Annals of Statistics* 17, 1749-1766.
- Debenedictis, L.F., 1997, A vector autoregressive model of the British Columbia regional economy, *Applied Economics* 29, 877-888.
- Dickey, D. and W. Fuller, 1979, Distribution of the estimators for autoregressive time series with a unit root, *Journal of the American Statistical Association*, 74, 427-431.
- Diebold, F.X. and A. Inoue, 2001, Long memory and regime switching, *Journal of Econometrics* 105, 131-159.

Diebold, F.X. and G.D. Rudebusch, 1989, Long memory and persistence in aggregate output, *Journal of Monetary Economics* 24, 189-209.

Fox, R. and M.S. Taqqu, 1986, Large sample properties of parameter estimates for strongly dependent stationary Gaussian time series, *Annals of Statistics* 14, 517-532.

Geweke, J. and S. Porter-Hudak, 1983, The estimation and application of long memory time series models, *Journal of Time Series Analysis* 4, 221-238.

Gil-Alana, L.A., 2003a, Multivariate tests of nonstationary hypotheses, *South African Statistical Journal* 37, 1-28.

Gil-Alana, L.A., 2003b, A fractional multivariate long memory model for the US and the Canadian real output, *Economics Letters* 81, 355-359.

Gil-Alana, L.A. and P.M. Robinson, 1997, Testing of unit roots and other nonstationary hypotheses in macroeconomic time series, *Journal of Econometrics* 80, 241-268.

Granger, C.W.J., 1980, Long memory relationships and the aggregation of dynamic models, *Journal of Econometrics* 14, 227-238.

Granger, C.W.J., 1981, Some properties of time series data and their use in econometric model specification, *Journal of Econometrics* 16, 121-130.

Granger, C.W.J. and R. Joyeux, 1980, An introduction to long memory time series and fractional differencing, *Journal of Time Series Analysis* 1, 15-29.

Hosking, J.R.M., 1981, Fractional differencing, *Biometrika* 68, 165-176.

Hurst, H.E., 1951, Long term storage capacity of reservoirs, *Transactions of the American Society Civil Engineering* 116, 770-779.

Hurst, H.E., 1957, A suggested statistical model of some time series which occur in nature, *Nature* 180, 494.

Johansen, S., 1988, Statistical analysis of cointegration vectors, *Journal of Economic Dynamics and Control*, 12, 231-254

Johansen, S., 1991, Estimation and hypothesis testing of cointegration vectors in Gaussian vector autoregressive models, *Econometrica*, 59, 1551-1580.

Kim, C., C. Nelson and J. Piger, 2001, The less volatile US economy. A Bayesian investigation of breadth and potential explanations, Unpublished paper, Board of Governors of the Federal Reserve System.

Mandelbrot, B., 1969, Long run linearity, locally Gaussian processes, H-spectra and infinite variances, *International Economic Review* 75, 828-835.

Mandelbrot, B. and J.W. van Ness, 1968, Fractional Brownian motions, fractional noises and applications, *SIAM Review* 10, 422-437.

McConnel, M.M. and G. Perez-Quiros, 2000, Output fluctuations in the United States. What has changed since the early 1980s, *American Economic Review* 90, 1464-1476.

McLeod, A.I. and K.W. Hipel, 1978, Preservation of the rescaled adjusted range. A reassessment of the Hurst phenomenon, *Water Resources Research* 14, 491-507.

Nielsen, M.O., 2005, Efficient inference in multivariate fractionally integrated time series models, *Econometric Journal*, Royal Economic Society 7, 63-97.

Parke, W.R., 1999, What is fractional integration?, *Review of Economics and Statistics* 81, 632-638.

Pétursson, T.G. and T. Slok, 2001, Wage formation and employment in a cointegrated VAR model, *The Econometrics Journal* 4, 191-209.

Phillips, P.C.B. and P. Perron, 1988, Testing for a unit root in a time series regression, *Biometrika*, 75, 335-346.

Press, W.H., B.P. Flannery, S.A. Teukolsky and W.T. Wetterling, 1986, *Numerical recipes: The Art of Scientific Computing*, Cambridge University Press, Cambridge.

Reade, J., 2005, A cointegrated VAR analysis of employment, Project for Econometrics Summer School in Copenhagen.

- Robinson, P.M., 1978, Statistical inference for a random coefficient autoregressive model, *Scandinavian Journal of Statistics* 5, 163-168.
- Robinson, P.M., 1994, Efficient tests of nonstationary hypotheses, *Journal of the American Statistical Association* 89, 1420-1437.
- Robinson, P.M., 1995, Gaussian semiparametric estimation of long range dependence, *Annals of Statistics* 23, 1630-1661.
- Shimotsu, K. and P.C.B. Phillips, 2005, Exact local Whittle estimation of fractional integration, *Annals of Statistics* 33, 1890-1933.
- Sims, C., 1980, Macroeconomics and reality, *Econometrica* 48, 1- 48.
- Sowell, F., 1992a, Modelling long run behaviour with the fractional ARIMA model, *Journal of Monetary Economics* 29, 277-302.
- Sowell, F., 1992b, Maximum likelihood estimation of stationary univariate fractionally integrated time series models, *Journal of Econometrics* 53, 165-188.
- Velasco, C., 1999, Gaussian semiparametric estimation of nonstationary time series, *Journal of Econometrics* 20, 87-127.

TABLE 1											
Probabilities of detecting the true break date in a model with $T_b = T/2$ and $d^a = [0.2, 0.4, 0.3]^T$ and $d^b = [0.6, 0.9, 1.4]^T$											
First subsample			Second subsample			Sample size					
T/2	d <sub>1</sub>	d <sub>2</sub>	d <sub>3</sub>	d <sub>1</sub>	d <sub>2</sub>	d <sub>3</sub>	100	200	300	500	1000
T/2-2	0.2	0.4	0.3	0.6	0.9	1.4	-----	0.014	0.026	0.018	0.003
T/2-1	0.2	0.4	0.1	0.6	0.9	1.4	-----	-----	0.010	-----	-----
	0.2	0.4	0.3	0.6	0.9	1.4	0.010	0.031	0.052	0.036	0.022
T/2	0.0	0.4	0.3	0.6	0.9	1.4	0.016	0.017	0.035	0.013	-----
	0.2	0.2	0.1	0.6	0.9	1.4	-----	0.010	-----	-----	-----
	0.2	0.2	0.3	0.6	0.9	1.4	0.014	0.029	0.021	0.016	-----
	0.2	0.4	0.1	0.6	0.9	1.4	0.014	0.017	0.020	0.007	-----
	0.2	0.4	0.3	0.4	0.9	1.4	0.011	0.024	0.017	0.006	-----
	0.2	0.4	0.3	0.6	0.7	1.4	0.016	0.020	0.016	0.011	-----
	0.2	0.4	0.3	0.6	0.9	1.2	0.011	0.019	0.030	0.011	-----
	<b>0.2</b>	<b>0.4</b>	<b>0.3</b>	<b>0.6</b>	<b>0.9</b>	<b>1.4</b>	<b>0.031</b>	<b>0.146</b>	<b>0.338</b>	<b>0.776</b>	<b>0.913</b>
	0.2	0.4	0.3	0.6	0.9	1.6	-----	0.025	0.020	-----	-----
	0.2	0.4	0.3	0.6	1.1	1.4	0.012	0.017	0.013	0.013	-----
	0.2	0.4	0.3	0.8	0.9	1.4	-----	0.014	0.019	0.013	-----
	0.2	0.4	0.5	0.6	0.9	1.4	-----	0.026	0.019	0.011	-----
	0.2	0.6	0.3	0.6	0.9	1.4	0.012	0.017	0.014	0.003	-----
0.4	0.4	0.3	0.6	0.9	1.4	-----	0.021	0.014	-----	-----	
T/2+1	0.2	0.4	0.3	0.6	0.9	1.4	-----	0.023	0.050	0.033	0.012
T/2+2	0.2	0.4	0.3	0.6	0.9	1.4	-----	0.015	0.040	0.033	0.009
Percentage of cases with the break at T/2 - 2							0.081	0.072	0.051	0.032	0.003
Percentage of cases with the break at T/2 - 1							0.140	0.113	0.102	0.056	0.025
Percentage of cases with the break at T/2							0.599	0.648	0.710	0.942	0.951
Percentage of cases with the break at T/2 + 1							0.102	0.092	0.082	0.045	0.012
Percentage of cases with the break at T/2 + 2							0.079	0.075	0.055	0.015	0.009

**TABLE 2**

**Probabilities of detecting the true break date in a model with  $T_b = T/4$  and  $d^a = [0.8, 1.2, 0.6]^T$  and  $d^b = [0.2, 0.4, 0.2]^T$**

First subsample			Second subsample			Sample size						
T/4-1	d <sub>1</sub>	d <sub>2</sub>	d <sub>3</sub>	d <sub>1</sub>	d <sub>2</sub>	d <sub>3</sub>	100	200	300	500	1000	
T/4	0.6	1.0	0.4	0.2	0.4	0.2	0.011	0.009	-----	-----	-----	
	0.6	1.0	0.6	0.2	0.4	0.0	0.010	-----	-----	-----	-----	
	0.6	1.0	0.6	0.2	0.4	0.2	0.018	0.016	0.016	-----	-----	
	0.6	1.0	0.8	0.2	0.4	0.2	0.015	-----	-----	-----	-----	
	0.6	1.2	0.4	0.2	0.4	0.2	0.015	0.015	0.011	-----	-----	
	0.8	1.2	0.8	0.2	0.4	0.2	0.020	0.056	0.073	0.053	0.024	
	0.6	1.2	0.8	0.2	0.4	0.2	0.010	0.010	0.017	-----	-----	
	0.6	1.4	0.4	0.2	0.4	0.2	0.012	-----	-----	-----	-----	
	0.6	1.4	0.6	0.2	0.4	0.2	-----	0.013	0.013	-----	-----	
	0.8	1.4	0.8	0.2	0.4	0.2	0.010	-----	-----	-----	-----	
	0.8	1.0	0.4	0.2	0.4	0.2	0.016	0.022	0.015	-----	-----	
	0.8	1.0	0.6	0.2	0.4	0.2	0.018	0.053	0.064	0.061	0.025	
	0.8	1.0	0.8	0.2	0.4	0.2	0.015	0.017	0.011	-----	-----	
	0.8	1.2	0.4	0.0	0.4	0.2	-----	0.010	-----	-----	-----	
	0.8	1.2	0.4	0.2	0.4	0.2	0.021	0.060	0.077	0.054	0.026	
	0.8	1.2	0.6	0.0	0.4	0.2	0.010	0.012	0.013	-----	-----	
	0.8	1.2	0.6	0.2	0.2	0.2	-----	0.011	-----	-----	-----	
	0.8	1.2	0.6	0.2	0.4	0.0	-----	0.012	0.015	-----	-----	
	0.8	<b>0.8</b>	<b>1.2</b>	<b>0.6</b>	<b>0.2</b>	<b>0.4</b>	<b>0.2</b>	<b>0.020</b>	<b>0.136</b>	<b>0.287</b>	<b>0.565</b>	<b>0.847</b>
	0.8	0.8	1.2	0.6	0.2	0.6	0.4	-----	0.014	-----	-----	-----
	0.8	0.8	1.2	0.6	0.2	0.6	0.2	-----	-----	0.013	0.011	-----
	0.8	0.8	1.2	0.6	0.4	0.4	0.2	-----	0.013	-----	-----	-----
	0.8	0.8	1.2	0.8	0.2	0.4	0.2	0.014	0.049	0.044	0.042	0.024
	0.8	0.8	1.4	0.4	0.2	0.4	0.2	0.012	0.013	0.010	-----	-----
0.8	0.8	1.2	0.8	0.2	0.4	0.2	0.014	0.029	0.040	0.048	0.019	
1.0	1.0	1.0	0.4	0.2	0.4	0.2	0.011	-----	-----	-----	-----	
1.0	1.0	1.0	0.6	0.2	0.4	0.2	-----	0.017	-----	-----	-----	
1.0	1.0	1.2	0.4	0.2	0.4	0.2	0.011	-----	0.013	-----	-----	
1.0	1.0	1.2	0.6	0.2	0.4	0.2	0.016	0.033	0.061	0.045	0.024	
1.0	1.0	1.2	0.8	0.2	0.4	0.2	0.014	0.016	0.013	-----	-----	
1.0	1.0	1.4	0.4	0.2	0.4	0.2	0.010	-----	-----	-----	-----	
Percentage of cases with the break at T/4 - 2							0.017	0.005	0.003	0.002	0.000	
Percentage of cases with the break at T/4 - 1							0.013	0.005	0.003	0.005	0.001	
Percentage of cases with the break at T/4							0.750	0.878	0.983	0.992	0.998	
Percentage of cases with the break at T/4 + 1							0.013	0.007	0.004	0.001	0.001	
Percentage of cases with the break at T/4 + 2							0.007	0.0067	0.004	0.000	0.000	

TABLE 3											
Probabilities of detecting the true break in a model with $T_b = 3T/4$ and $d^a = [0.6, 0.3, 0.5]^T$ and $d^b = [0.2, 0.9, 0.5]^T$											
First subsample			Second subsample			Sample size					
	$d_1$	$d_2$	$d_3$	$D_1$	$d_2$	$d_3$	100	200	300	500	1000
3T/4-2	0.6	0.3	0.5	0.2	0.9	0.5	-----	-----	0.016	0.030	0.031
3T/4-1	0.6	0.3	0.5	0.2	0.9	0.5	-----	-----	0.023	0.033	0.056
3T/4	0.4	0.1	0.4	0.2	0.9	0.5	-----	0.014	0.012	-----	-----
	0.6	0.3	0.5	0.0	0.7	0.3	0.010	-----	-----	-----	-----
	0.6	0.3	0.5	0.0	0.7	0.5	0.017	0.022	0.013	-----	-----
	0.6	0.3	0.5	0.0	0.9	0.3	0.015	0.050	0.041	0.050	0.019
	0.6	0.3	0.5	0.0	0.9	0.5	0.012	0.012	0.010	-----	-----
	0.6	0.3	0.5	0.0	1.1	0.3	-----	0.014	0.012	-----	-----
	0.6	0.3	0.5	0.0	1.1	0.7	-----	0.016	-----	-----	-----
	0.6	0.3	0.5	0.2	0.7	0.5	0.010	0.040	0.56	0.040	0.020
	0.6	0.3	0.5	0.2	0.7	0.7	-----	0.017	-----	-----	0.021
	0.6	0.3	0.5	0.2	0.9	0.3	0.011	0.036	0.058	0.046	-----
	<b>0.6</b>	<b>0.3</b>	<b>0.5</b>	<b>0.2</b>	<b>0.9</b>	<b>0.5</b>	<b>0.019</b>	<b>0.100</b>	<b>0.202</b>	<b>0.522</b>	<b>0.759</b>
	0.6	0.3	0.5	0.2	0.9	0.7	-----	0.026	0.036	0.050	0.012
	0.6	0.3	0.5	0.2	1.1	0.3	0.011	0.012	0.015	-----	-----
	0.6	0.3	0.5	0.2	1.1	0.5	0.010	0.031	0.037	0.045	0.013
	0.6	0.3	0.5	0.2	1.1	0.7	-----	-----	0.010	-----	-----
	0.6	0.3	0.5	0.4	0.9	0.3	0.010	-----	0.013	-----	-----
	0.6	0.3	0.5	0.4	0.9	0.5	-----	0.032	0.028	-----	0.023
	0.6	0.3	0.5	0.4	0.9	0.7	-----	0.011	-----	-----	-----
0.6	0.3	0.5	0.4	0.9	0.7	-----	-----	0.034	-----	-----	
0.6	0.3	0.5	0.4	1.1	0.3	-----	-----	0.013	-----	-----	
0.6	0.3	0.7	0.2	0.9	0.5	-----	0.013	-----	-----	-----	
0.8	0.3	0.5	0.2	0.9	0.5	-----	0.010	-----	-----	-----	
3T/4+1	0.6	0.3	0.5	0.2	0.7	0.5	-----	-----	0.011	-----	-----
	0.6	0.3	0.5	0.2	0.9	0.5	-----	0.017	0.022	0.037	0.059
3T/4+2	0.6	0.3	0.5	0.2	0.9	0.5	-----	-----	0.011	0.024	-----
Percentage of cases with the break at 3T/4 - 2							0.074	0.061	0.053	0.032	0.037
Percentage of cases with the break at 3T/4 - 1							0.108	0.084	0.084	0.044	0.071
Percentage of cases with the break at 3T/4							0.654	0.715	0.730	0.864	0.871
Percentage of cases with the break at 3T/4 + 1							0.093	0.085	0.076	0.034	0.065
Percentage of cases with the break at 3T/4 + 2							0.069	0.055	0.057	0.016	0.056

**TABLE 4**

**Probabilities of detecting the true break date in a model with  $T_b = T/2$  and  $d^a = [0.2, 0.4, 0.3]^T$  and  $d^b = [0.6, 0.9, 1.4]^T$  and a VAR(1) structure for the d-differenced process**

First subsample			Second subsample			Sample size					
T/2	d <sub>1</sub>	d <sub>2</sub>	d <sub>3</sub>	d <sub>1</sub>	d <sub>2</sub>	d <sub>3</sub>	100	200	300	500	1000
T/2-1	0.2	0.4	0.3	0.6	0.9	1.4	-----	-----	0.010	0.023	0.052
T/2	0.0	0.2	0.1	0.4	0.7	1.2	0.024	0.012	-----	-----	-----
	0.0	0.2	0.1	0.4	0.7	1.4	0.015	-----	-----	-----	-----
	0.0	0.2	0.1	0.6	0.7	1.2	0.016	-----	-----	-----	-----
	0.0	0.2	0.1	0.6	0.9	1.4	0.011	0.035	0.026	0.020	-----
	0.0	0.2	0.3	0.4	0.7	1.2	0.012	0.012	0.010	-----	-----
	0.0	0.2	0.3	0.4	0.7	1.4	-----	0.011	-----	-----	-----
	0.0	0.2	0.3	0.6	0.9	1.4	-----	0.018	0.026	0.018	-----
	0.0	0.4	0.1	0.6	0.7	1.2	0.010	-----	-----	-----	-----
	0.0	0.4	0.1	0.6	0.9	1.4	-----	0.017	0.023	0.013	-----
	0.0	0.4	0.3	0.4	0.7	1.2	0.013	0.017	-----	-----	-----
	0.0	0.4	0.3	0.6	0.9	1.4	-----	0.029	0.032	0.047	-----
	0.2	0.2	0.1	0.6	0.9	1.4	-----	0.011	0.011	0.011	-----
	0.2	0.2	0.3	0.4	0.7	1.2	0.014	0.013	-----	-----	-----
	0.2	0.2	0.3	0.4	0.7	1.4	0.012	-----	-----	-----	-----
	0.2	0.2	0.3	0.6	0.9	1.4	0.010	0.038	0.057	0.089	0.044
	0.2	0.4	0.1	0.6	0.9	1.4	-----	0.014	-----	0.010	-----
	0.2	0.4	0.3	0.4	0.7	1.2	-----	0.014	0.032	0.027	-----
	0.2	0.4	0.3	0.4	0.7	1.4	-----	0.012	0.016	0.012	-----
	0.2	0.4	0.3	0.6	0.7	1.2	-----	-----	0.015	-----	-----
		<b>0.2</b>	<b>0.4</b>	<b>0.3</b>	<b>0.6</b>	<b>0.9</b>	<b>1.4</b>	-----	<b>0.052</b>	<b>0.161</b>	<b>0.422</b>
0.2	0.4	0.3	0.6	0.9	1.6	-----	-----	0.011	-----	-----	
0.2	0.4	0.3	0.8	1.1	1.6	-----	-----	-----	0.013	-----	
0.2	0.4	0.5	0.6	0.9	1.4	-----	-----	0.011	0.019	-----	
0.2	0.6	0.3	0.4	0.7	1.2	-----	0.011	-----	-----	-----	
0.2	0.6	0.3	0.6	0.9	1.4	-----	0.025	0.042	0.052	0.018	
0.4	0.4	0.3	0.6	0.9	1.4	-----	-----	0.021	0.015	-----	
0.4	0.4	0.5	0.6	0.9	1.4	-----	-----	0.015	-----	-----	
0.4	0.6	0.3	0.6	0.9	1.4	-----	-----	0.015	-----	-----	
T/2+1	0.2	0.4	0.3	0.6	0.9	1.4	-----	-----	0.010	0.017	0.031
T/2+2	0.2	0.4	0.3	0.6	0.9	1.4	-----	-----	-----	-----	0.025
Percentage of cases with the break at T/2 - 2							0.032	0.012	0.015	0.016	0.012
Percentage of cases with the break at T/2 - 1							0.068	0.046	0.061	0.046	0.060
Percentage of cases with the break at T/2							0.448	0.582	0.658	0.879	0.865
Percentage of cases with the break at T/2 + 1							0.037	0.042	0.043	0.042	0.038
Percentage of cases with the break at T/2 + 2							0.015	0.018	0.023	0.020	0.026



<b>TABLE 5</b>						
Estimated model using white noise disturbances. The US case						
First subsample			Break date	Second subsample		
d <sub>1</sub>	d <sub>2</sub>	d <sub>3</sub>		d <sub>1</sub>	d <sub>2</sub>	d <sub>3</sub>
1.01 (0.84, 1.32)	1.00 (0.81, 1.26)	1.00 (0.80, 1.30)	1974Q4	1.50 (1.21, 1.72)	1.45 (1.14, 1.65)	1.52 (1.19, 1.77)

d<sub>1</sub> is the order of integration of employment; d<sub>2</sub> is the order of integration of CPI, and d<sub>3</sub> is the order of integration of GDP.

<b>TABLE 6</b>						
Estimated model using VAR(1) disturbances. The US case						
First subsample			Break date	Second subsample		
d <sub>1</sub>	d <sub>2</sub>	d <sub>3</sub>		d <sub>1</sub>	d <sub>2</sub>	d <sub>3</sub>
0.02 (-0.31, 0.43)	0.00 (-0.37, 0.55)	0.00 (-0.22, 0.51)	1974Q4	0.66 (0.44, 0.98)	0.25 (-0.11, 0.43)	0.43 (0.11, 0.69)
VAR(1) coefficients				VAR(1) coefficients		
$\begin{pmatrix} 0.908 & -0.025 & 0.032 \\ 0.372 & 0.968 & -0.067 \\ 0.102 & -0.071 & 1.008 \end{pmatrix}$				$\begin{pmatrix} 0.524 & -0.063 & 0.125 \\ -0.224 & 0.806 & 0.288 \\ -0.019 & -0.043 & 1.065 \end{pmatrix}$		

<b>TABLE 7</b>						
Estimated model using white noise disturbances. The Mexican case						
First subsample			Break date	Second subsample		
d <sub>1</sub>	d <sub>2</sub>	d <sub>3</sub>		d <sub>1</sub>	d <sub>2</sub>	d <sub>3</sub>
1.06 (0.90, 1.21)	0.97 (0.88, 1.16)	0.99 (0.78, 1.23)	1959	1.52 (1.30, 1.81)	1.77 (1.37, 1.98)	1.37 (1.03, 1.54)

<b>TABLE 8</b>						
Estimated model using VAR(1) disturbances. The Mexican case						
First subsample			Break date	Second subsample		
d <sub>1</sub>	d <sub>2</sub>	d <sub>3</sub>		d <sub>1</sub>	d <sub>2</sub>	d <sub>3</sub>
0.13 (-0.32, 0.39)	0.03 (-0.31, 0.27)	0.00 (-0.41, 0.37)	1962	1.20 (0.98, 1.34)	1.15 (0.92, 1.44)	1.18 (0.99, 1.40)
VAR(1) coefficients				VAR(1) coefficients		
$\begin{pmatrix} 0.914 & -0.046 & 0.099 \\ -0.408 & 0.781 & 0.485 \\ 0.577 & -0.025 & 0.827 \end{pmatrix}$				$\begin{pmatrix} 0.477 & 0.024 & -0.101 \\ 6.176 & 0.620 & -1.876 \\ 0.452 & 0.025 & -0.080 \end{pmatrix}$		

# CESifo Working Paper Series

(for full list see [www.cesifo-group.de](http://www.cesifo-group.de))

---

- 1888 Timothy J. Goodspeed and Andrew Haughwout, On the Optimal Design of Disaster Insurance in a Federation, January 2007
- 1889 Wim Groot, Henriëtte Maassen van den Brink and Bernard van Praag, The Compensating Income Variation of Social Capital, January 2007
- 1890 Bas Jacobs, Ruud A. de Mooij and Kees Folmer, Analyzing a Flat Income Tax in the Netherlands, January 2007
- 1891 Hans Jarle Kind, Guttorm Schjelderup and Frank Stähler, Newspapers and Advertising: The Effects of Ad-Valorem Taxation under Duopoly, January 2007
- 1892 Erkki Koskela and Rune Stenbacka, Equilibrium Unemployment with Outsourcing under Labour Market Imperfections, January 2007
- 1893 Maarten Bosker, Steven Brakman, Harry Garretsen, Herman de Jong and Marc Schramm, The Development of Cities in Italy 1300 – 1861, January 2007
- 1894 Michel Beine, Oscar Bernal, Jean-Yves Gnabo and Christelle Lecourt, Intervention Policy of the BoJ: A Unified Approach, January 2007
- 1895 Robert S. Chirinko and Daniel J. Wilson, State Investment Tax Incentives: A Zero-Sum Game?, January 2007
- 1896 Theo S. Eicher and Oliver Roehn, Sources of the German Productivity Demise – Tracing the Effects of Industry-Level ICT Investment, January 2007
- 1897 Helge Berger, Volker Nitsch and Tonny Lybek, Central Bank Boards around the World: Why does Membership Size Differ?, January 2007
- 1898 Gabriel Felbermayr and Wilhelm Kohler, Does WTO Membership Make a Difference at the Extensive Margin of World Trade?, January 2007
- 1899 Benno Torgler and Friedrich Schneider, The Impact of Tax Morale and Institutional Quality on the Shadow Economy, January 2007
- 1900 Tomer Blumkin and Efraim Sadka, On the Desirability of Taxing Charitable Contributions, January 2007
- 1901 Frederick van der Ploeg and Reinhilde Veugelers, Higher Education Reform and the Renewed Lisbon Strategy: Role of Member States and the European Commission, January 2007
- 1902 John Lewis, Hitting and Hoping? Meeting the Exchange Rate and Inflation Criteria during a Period of Nominal Convergence, January 2007

- 1903 Torben M. Andersen, The Scandinavian Model – Prospects and Challenges, January 2007
- 1904 Stephane Dees, Sean Holly, M. Hashem Pesaran and L. Vanessa Smith, Long Run Macroeconomic Relations in the Global Economy, January 2007
- 1905 Richard Jong-A-Pin and Jakob De Haan, Political Regime Change, Economic Reform and Growth Accelerations, January 2007
- 1906 Sascha O. Becker and Peter H. Egger, Endogenous Product versus Process Innovation and a Firm's Propensity to Export, February 2007
- 1907 Theo S. Eicher, Chris Papageorgiou and Oliver Roehn, Unraveling the Fortunes of the Fortunate: An Iterative Bayesian Model Averaging (IBMA) Approach, February 2007
- 1908 Liliana E. Pezzin, Robert A. Pollak and Barbara S. Schone, Efficiency in Family Bargaining: Living Arrangements and Caregiving Decisions of Adult Children and Disabled Elderly Parents, February 2007
- 1909 Christian Keuschnigg and Soren Bo Nielsen, Self-Selection and Advice in Venture Capital Finance, February 2007
- 1910 Rune Jansen Hagen and Gaute Torsvik, Irreversible Investments, Dynamic Inconsistency and Policy Convergence, February 2007
- 1911 Eric A. Hanushek and Ludger Woessmann, The Role of School Improvement in Economic Development, February 2007
- 1912 Bernard M. S. van Praag, Perspectives from the Happiness Literature and the Role of New Instruments for Policy Analysis, February 2007
- 1913 Volker Grossmann and Thomas M. Steger, Growth, Development, and Technological Change, February 2007
- 1914 Margarita Katsimi and Thomas Moutos, Human Capital and the Feldstein-Horioka Puzzle, February 2007
- 1915 Oliver Roehn, Theo S. Eicher and Thomas Strobel, The Ifo Industry Growth Accounting Database, February 2007
- 1916 Ian Babetskii, Aggregate Wage Flexibility in Selected New EU Member States, February 2007
- 1917 Burkhard Heer, Alfred Maussner and Paul D. McNelis, The Money-Age Distribution: Empirical Facts and Limited Monetary Models, February 2007
- 1918 Yin-Wong Cheung, Menzie D. Chinn and Eijii Fujii, The Overvaluation of Renminbi Undervaluation, February 2007

- 1919 Jim Malley, Apostolis Philippopoulos and Ulrich Woitek, To React or Not? Fiscal Policy, Volatility and Welfare in the EU-3, February 2007
- 1920 Mattias Polborn, Competing for Recognition through Public Good Provision, February 2007
- 1921 Lars P. Feld and Benno Torgler, Tax Morale after the Reunification of Germany: Results from a Quasi-Natural Experiment, February 2007
- 1922 Robert S. Chirinko and Huntley Schaller, Fundamentals, Misvaluation, and Investment: The Real Story, February 2007
- 1923 Benno Torgler and Friedrich Schneider, Shadow Economy, Tax Morale, Governance and Institutional Quality: A Panel Analysis, February 2007
- 1924 Adrian Pagan and M. Hashem Pesaran, On Econometric Analysis of Structural Systems with Permanent and Transitory Shocks and Exogenous Variables, February 2007
- 1925 Hans-Werner Sinn, The Welfare State and the Forces of Globalization, February 2007
- 1926 Michael Smart, Raising Taxes through Equalization, February 2007
- 1927 Øystein Foros, Kåre P. Hagen and Hans Jarle Kind, Price-Dependent Profit Sharing as an Escape from the Bertrand Paradox, February 2007
- 1928 Balázs Égert, Kirsten Lommatzsch and Amina Lahrèche-Révil, Real Exchange Rates in Small Open OECD and Transition Economies: Comparing Apples with Oranges?, February 2007
- 1929 Aleksander Berentsen and Cyril Monnet, Monetary Policy in a Channel System, February 2007
- 1930 Wolfgang Ochel, The Free Movement of Inactive Citizens in the EU – A Challenge for the European Welfare State?, February 2007
- 1931 James K. Hammitt and Nicolas Treich, Statistical vs. Identified Lives in Benefit-Cost Analysis, February 2007
- 1932 Wilhelm Kohler, The Bazaar Effect, Unbundling of Comparative Advantage, and Migration, February 2007
- 1933 Karsten Staehr, Fiscal Policies and Business Cycles in an Enlarged Euro Area, February 2007
- 1934 Michele Bernasconi and Paola Profeta, Redistribution or Education? The Political Economy of the Social Race, March 2007
- 1935 Axel Dreher, Martin Gassebner and Lars-H. R. Siemers, Does Terror Threaten Human Rights? Evidence from Panel Data, March 2007

- 1936 Naércio Aquino Menezes Filho and Marc-Andreas Muendler, Labor Reallocation in Response to Trade Reform, March 2007
- 1937 Gebhard Flaig and Timo Wollmershaeuser, Does the Euro-zone Diverge? A Stress Indicator for Analyzing Trends and Cycles in Real GDP and Inflation, March 2007
- 1938 Michael Funke and Michael Paetz, Environmental Policy Under Model Uncertainty: A Robust Optimal Control Approach, March 2007
- 1939 Byeongchan Seong, Sung K. Ahn and Peter A. Zadrozny, Cointegration Analysis with Mixed-Frequency Data, March 2007
- 1940 Monika Büttler and Michel André Maréchal, Framing Effects in Political Decision Making: Evidence from a Natural Voting Experiment, March 2007
- 1941 Giacomo Corneo and Olivier Jeanne, A Theory of Tolerance, March 2007
- 1942 Qing Hong and Michael Smart, In Praise of Tax Havens: International Tax Planning and Foreign Direct Investment, March 2007
- 1943 Yin-Wong Cheung, Dickson Tam and Matthew S. Yiu, Does the Chinese Interest Rate Follow the US Interest Rate?, March 2007
- 1944 Panu Poutvaara and Mikael Priks, Unemployment and Gang Crime: Could Prosperity Backfire?, March 2007
- 1945 Burkhard Heer, On the Modeling of the Income Distribution Business Cycle Dynamics, March 2007
- 1946 Christoph A. Schaltegger and Lars P. Feld, Are Fiscal Adjustments less Successful in Decentralized Governments?, March 2007
- 1947 Giovanni Facchini, Marcelo Olarreaga, Peri Silva and Gerald Willmann, Substitutability and Protectionism: Latin America's Trade Policy and Imports from China and India, March 2007
- 1948 C. Mirjam van Praag and Bernard M. S. van Praag, The Benefits of Being Economics Professor A (and not Z), March 2007
- 1949 Astrid Hopfensitz and Frans van Winden, Dynamic Choice, Independence and Emotions, March 2007
- 1950 Guglielmo Maria Caporale and Luis A. Gil-Alana, A Multivariate Long-Memory Model with Structural Breaks, March 2007