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# Growth Forecasts, Belief Manipulation and Capital Markets\*

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## Abstract

We analyze how a benevolent, privately-informed government agency would optimally release information about the economy's growth rate when the agents hold heterogeneous beliefs. We model two types of agents: "trusting" and "distrustful." The former has a prior that is identical to that of the government agency, whereas the latter has a prior that differs from that of the government agency. We identify both "revealing" and "nonrevealing" equilibria and demonstrate that the "nonrevealing" equilibria can dominate the "revealing" equilibria in terms of *ex-post* social welfare.

*Keywords:* Social welfare, information, forecasting,  
asset pricing, heterogeneous beliefs

*JEL codes:* D83, G11, G12

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# 1 Introduction

The credibility of governments, when dealing with financial markets, is at the very heart of most economic policies. The perhaps most striking example was in 2008, when the US government attempted to launch its bailout policy towards major industries on the brink of bankruptcy. Most opponents argued that this bailout policy came down to passing the bucket to taxpayers, whereas the government tried to convince economic agents that the capital used in those operations was mere loans to be repaid when times get better. Needless to say, the main task of the government at the time it tried to pass the bill was to convince markets and congressmen that better times will very soon be ahead, so that the opportunity cost of the bailout would be acceptable.

Another striking example comes from the build-up to the European debt crisis, where a number of EU member states allegedly bypassed the Maastricht Treaty, which was supposed to put a limit on deficits and debt levels, by hiding these deficits and debts through the use of elaborate currency and credit derivatives structures. Ex ante, this behavior must have been expected to make a huge difference: they were prepared not only to take the indirect cost of a potentially damaged reputation, but also a considerable pecuniary cost, as is apparent from the massive fees that the prominent US investment banks who helped design these structures received in return for their services (Simkovic, 2009, 2011).

These examples extend to many other economic situations, such as inflation target and unemployment rates announcements for instance, where the government or the lo-

cal central bank may seek to induce optimistic beliefs. The point may not be to trigger speculative bubbles, but rather to prevent investors from precautionary measures leading them to withdraw capital from those markets. This way, enough liquidity remains available in financial markets and worse scenarios, such as systemic risk of widespread banking failures (as happened in 2009), are prevented. From the point of view of social welfare, is it not better that investors remain unaware of, e.g., upcoming recessions? Is it even socially desirable to publicly announce such forecasts?

One can easily anticipate that actual forecasts from official bodies with ties to the government would be overly optimistic, particularly right before elections. Indeed, there is ample empirical evidence confirming this assertion. Jonung and Larch (2006) find a significant upward bias in government forecasts of both real and potential GDP growth in Germany, Italy and France. Similarly, Ashiya (2007) finds a significant upward bias in the official government forecasts of real GDP growth in Japan. Boylan (2008) demonstrates that forecasts of state revenues in the US tend to be overly optimistic right before and right after gubernatorial elections, and he argues that this is due to a political incentive to satisfy a balanced budget requirement and avoid raising taxes. More specifically, he finds that, in gubernatorial election years, budget officials overstate the growth in state and US personal income.

Given this background, we analyze from a theoretical viewpoint how a privately-informed, benevolent government agency would choose to release information regarding future growth rates and its effects on asset prices in an exchange-only Lucas (1978) economy. For simplicity, we consider what is effectively a one-period model, but our results

readily extend to more general settings (e.g., repeated interaction under IID growth rates and “stationary” updating rules). The future growth rate may either be *high* or *low*. The government agency chooses between forwarding a private signal, which partially reveals the true future growth rate and sending an opposite signal. In doing so, it seeks to maximize a standard measure of *ex-post* social welfare.

We model two types of agents, which we call *trusting* and *distrustful* agents, respectively. The trusting agents have a prior that is identical to that of the government agency, whereas the distrustful agents have a prior that differs from that of the government agency. In this way, we intend to account for the circumstance that real-world agents often hold different *a priori* beliefs regarding the probabilities of events when these cannot be estimated through repeated experiments. Exactly how these *a priori* beliefs are determined is outside the scope of the paper; however, one can think of a setting where the government agency’s prior is common knowledge and the trusting agents align their prior with the government agency because they trust the government’s assessment and think that the agency’s prior is close to the true probability, whereas the distrustful agents think that the government’s probability assessments are arbitrary to a large extent, or, alternatively, they trust some other entity’s assessment. In the paper, all agents are rational Bayesian updaters and the heterogeneity in beliefs stems from differences in priors, but the model readily extends to other, behavioral assumptions on the agents’ updating process: similar equilibria would emerge in a setting where, e.g., a group of agents follows an irrational updating rule whereas another group of agents consists of rational Bayesian updaters.

In the case of homogeneous agents (i.e., 100% trusting or 100% distrustful agents), the

government agency is unable to affect ex-post social welfare through its signaling, because the prices adjust so that all agents hold on to their endowments. However, when both types of agents are present, the government agency may in some cases be able to affect the distribution of resources across the two future states (high and low growth) through its signaling, and thus be able to affect ex-post social welfare. Depending on parameter values, different types of equilibria can occur.

We identify two kinds of "revealing" equilibria: one in which the government agency always forwards its private signal and one in which it makes an announcement that is the opposite of its private signal. We also identify two kinds of "nonrevealing" equilibria: one in which the agents believe that there is an equal chance that the government agency forwards its private signal and that it makes an announcement which is the opposite of its private signal, and one in which the agents believe that the government agency always announces a high growth, regardless of its private signal. In the former, the agents pay no attention to the agency's public signal and thus, any public signal will meet its objective. In the latter, the agents stick to their priors along the equilibrium path, but still pay attention to the public signal, due to possible deviations from the equilibrium path.

We find that the "revealing" equilibria do not always result in a higher ex-post social welfare than the "nonrevealing" equilibria. In fact, the opposite can be true. This seems counter-intuitive at first sight, but it is in line with the central result in Crawford and Sobel (1982), as explained below.

We study the effects of the distrustful agents on the trusting agents' optimal strategies and asset prices. We find that, with respect to stock holdings, the mass of the distrustful

agents can affect both the sign (i.e., whether the trusting agents go long or short in the stock) and the magnitude, whereas with respect to bond holdings, the mass of the distrustful agents will only affect magnitudes. We also find that if the distrustful agents have a more pessimistic (optimistic) prior regarding dividend growth than the trusting agents, then the larger the mass of the distrustful agents, the higher (lower) the bond price, or, equivalently, the lower (higher) the interest rate. This is because if the distrustful agents have a more pessimistic (optimistic) prior regarding dividend growth than the trusting agents, then the bond appears more (less) attractive to the distrustful agents.

The paper is related to the literature on the social value of public information. Drèze (1960) and Hirshleifer (1971) identify the possibility that information may have a negative social value, which became known as the “Hirshleifer effect.” Since then, several authors have investigated the robustness of this result (e.g., Marshall, 1974; Ng, 1975; Green, 1981; Hakansson, Kunkel, and Ohlson, 1982; Schlee, 2001; Campbell, 2004). In the context of a “beauty contest” à la Keynes (1936), Morris and Shin (2002) show that greater provision of public information may not improve welfare. This result has gained attention in the media (Economist, 2004) and spurred academic debate (Svensson, 2006; Morris, Shin, and Tong, 2006). We contribute to this literature mainly by showing how the presence of heterogeneity among the agents can induce both revealing and nonrevealing equilibria, depending on the relative mass of the agents.

Another branch of the literature to which we contribute is that of cheap-talk equilibria (Crawford and Sobel, 1982; Farrell and Gibbons, 1989; Stein, 1989; Matthews, Okono-Fujiwara, and Postlewaite, 1991; Admati and Pfleiderer, 2004; Kawamura, 2011, among

others). In their seminal article, Crawford and Sobel (1982) explicitly model the choice to manipulate and allow it to be endogenously optimally chosen. They consider a better-informed sender who sends a signal to a receiver who then takes an action that affects the welfare of both. A central result in Crawford and Sobel (1982) is that “the more similar agents’ preferences, the more informative the equilibrium signal” (Crawford and Sobel, 1982, p. 1432). Our result that the “nonrevealing” equilibria can dominate the “revealing” ones in terms of social welfare is in line with this finding: our numerical results suggest that the mass of distrustful agents or the difference in priors needs to be sufficiently large in order for the “nonrevealing” equilibria to dominate and, further, we notice that, as the mass of the distrustful agents or the difference in priors becomes larger, the more different is the government agency’s objective from that of the average agent.

The paper is organized as follows. In Section 2, we introduce our model, and in Section 3, we present our theoretical results. Finally, the fourth section concludes the paper. All the proofs are in the Appendix.

## 2 Model

We consider what is effectively a one-period exchange-only Lucas (1978) economy but with three dates  $t = 1, 2, 3$ , as explained below. In this economy, there is a continuum of atomless agents with a total mass of one, who are *price takers*, and a government agency. There is one consumption good and two assets: a risk-free asset yielding one unit of consumption good at the last date  $t = 3$ , and a risky asset (stock) whose return at  $t = 3$



depends on one of the two realizations of nature (high  $h$ , or low  $l$ ) as follows. The risky asset returns a known dividend,  $D_2$ , at date  $t = 2$  and a second dividend,  $\tilde{D}_3 = D_2g$ , at the final date,  $t = 3$ , where, for simplicity, the growth rate  $g$  is a binary random variable, described below. Without loss of generality, we assume that the ownership of the risky asset is uniformly distributed across the agents; this ownership is their sole endowment.

According to the government agency's prior beliefs, the growth rate is *high* ( $g = g_h$ ) with probability  $p_h$  and *low* ( $g = g_l$ ) with probability  $1 - p_h$ . That is,

$$g = \begin{cases} g_h & \text{with probability } p_h \\ g_l & \text{with probability } 1 - p_h, \end{cases} \quad (1)$$

where  $p_h \in (0, 1)$  and  $0 < g_l < g_h$ .

At date  $t = 1$ , the government agency has private information regarding the realization of the random variable  $g$ . This private information takes the form of a partially revealing signal,  $s_R$ . The government agency's private signal returns the correct growth rate with probability  $\xi \in (\frac{1}{2}, 1)$  and an incorrect growth rate with probability  $1 - \xi$ . We say that the signal  $s_R$  partially reveals the growth rate because the distribution of  $s_R$  conditional on the growth rate *depends* on the growth rate. Formally, we have<sup>1</sup>

$$s_R = \begin{cases} g & \text{with probability } \xi \\ \neg g & \text{with probability } 1 - \xi, \end{cases} \quad (2)$$

where  $\xi \in (\frac{1}{2}, 1)$ .

A group of agents with positive mass has a prior regarding the future growth rate that is identical to the government agency's prior. We call this group *trusting* agents.

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<sup>1</sup>Here, " $\neg g$ " means "not  $g$ ." That is, if  $g = g_h$ , then  $\neg g = g_l$  and if  $g = g_l$ , then  $\neg g = g_h$ .

In addition, there is a group of agents of mass  $v \in (0, 1)$ , whose prior beliefs deviate from the government agency's prior: Their initial probability of a high growth rate is  $p_h^d$ , where  $p_h^d \in (0, 1)$  and  $p_h^d \neq p_h$ . We call this group *distrustful* agents. We let  $\widehat{Pr}(\cdot)$  denote the trusting agents' probability operator, and we let  $\widehat{Pr}^d(\cdot)$  denote the distrustful agents' probability operator.

At  $t = 1$ , after receiving its private signal, the government agency decides on the probability by which it forwards its private signal  $s_R$ . That is, it sends a public signal  $s$ , where  $s = g_h$  or  $s = g_l$ , so that  $s = s_R$  or  $s = \neg s_R$ .

The government agency maximizes *ex-post* social welfare, as described below, by choosing the probability of forwarding its private signal to the agents. At date  $t = 1$ , these agents assign conditional probabilities  $\widehat{\theta}_h$  and  $\widehat{\theta}_l$  to the possibility that the government forwards its private signal, conditional on it being  $s_R = g_h$  and  $s_R = g_l$ , respectively.

At  $t = 2$ , the agents receive the government agency's previously chosen signal and they update their beliefs about  $g$  as a function of the received signal, as well as the probability that the government agency forwards its private signal, in a Bayesian manner. According to their newly formed beliefs, at date  $t = 2$  they allocate their current wealth between current consumption and the two assets yielding consumption at  $t = 3$ .

Every agent is a standard von Neumann–Morgenstern expected-utility maximizer, as of  $t = 2$ , with an intertemporal discount factor  $\beta \in (0, 1)$ ; that is, agents seek to maximize

$$u(C_2^i) + \beta E^{\widehat{P}^i} \left[ u \left( \widetilde{C}_3^i \right) \mid s \right] \quad i \in [0, 1], \quad (3)$$

where  $C_2^i$  denotes agent  $i$ 's initial consumption,  $\widetilde{C}_3^i$  denotes his final state-contingent

consumption, and  $\widehat{P}^i$  is his probability measure. The elementary utility function  $u$  is concave, strictly increasing, and twice-continuously differentiable. In addition, we assume that  $u'(c) \rightarrow +\infty$  as  $c \rightarrow 0$ . The agents' budget constraints are standard.

The objective of the government agency is to maximize social welfare from  $t = 1$  by choosing the appropriate signal to send while taking as given the individual demand functions and prices for which markets clear. In particular, the strategy for the agency is to choose a vector of probabilities,  $\theta = (\theta_h, \theta_l)$ , of forwarding its private signal, where  $\theta_h$  is the probability of forwarding the private signal, conditional on it being  $s_R = g_h$ , and, correspondingly,  $\theta_l$  is the probability of forwarding the private signal, conditional on it being  $s_R = g_l$ . Thus, the agency seeks to maximize the expressions

$$\mathcal{W}_j = E^P \left[ \int_{[0,1]} \left( u(C_2^i) + \beta u(\widetilde{C}_3^i) \right) di \mid s_R = g_j \right], \quad j = h, l \quad (4)$$

over  $\theta_h$  and  $\theta_l$  (where  $0 \leq \theta_h \leq 1$  and  $0 \leq \theta_l \leq 1$ ). Here,  $P$  denotes the agency's probability measure (under which its strategy  $\theta$  is known), and  $C_2^i$  and  $\widetilde{C}_3^i$  are the equilibrium consumption levels such that markets clear.

The structure of the signals,  $g_h$ ,  $g_l$ ,  $\xi$ , the agents' preferences and beliefs and the government agency's objective function are common knowledge.

Formally, we consider the following equilibrium concept.<sup>2</sup>

**Definition 1.** *An equilibrium for the economy described above is a set of consumption and investment decisions  $\{C_2^{i*}, \phi^{i*}\}_{i \in [0,1]}$ , a government agency strategy  $\theta^* = (\theta_h^*, \theta_1^*)$ , a set of prices  $(S, B)$  and a system of beliefs  $\widehat{Pr}(g = g_h | s)$  and  $\widehat{Pr}^d(g = g_h | s)$  such that*

*i)  $\{C_2^{i*}, \phi^{i*}\}_{i \in [0,1]}$  solve the agents' consumption and investment problems given  $\widehat{Pr}(g = g_h | s)$ ,  $\widehat{Pr}^d(g = g_h | s)$ ,  $S$  and  $B$ ;*

*ii)  $\theta^*$  solves the government agency's problem given  $\widehat{Pr}(g = g_h | s)$  and  $\widehat{Pr}^d(g = g_h | s)$ ;*

*iii) Markets clear,  $\int_0^1 C_2^{i*} di = D_2$  and  $\int_0^1 \phi^{i*} di = 1$ ; and*

*iv)  $\widehat{Pr}(g = g_h | s)$  and  $\widehat{Pr}^d(g = g_h | s)$  are computed according to Bayes' rule whenever possible. If  $\widehat{\theta}_h = 1$  and  $\widehat{\theta}_1 = 0$ , we let  $\widehat{Pr}(g = g_h | s = g_1) = \frac{p_h(1-\xi)}{p_h(1-\xi) + (1-p_h)\xi}$  and  $\widehat{Pr}^d(g = g_h | s = g_1) = \frac{p_h^d(1-\xi)}{p_h^d(1-\xi) + (1-p_h^d)\xi}$ . If  $\widehat{\theta}_h = 0$  and  $\widehat{\theta}_1 = 1$ , we let  $\widehat{Pr}(g = g_h | s = g_h) = \frac{p_h\xi}{p_h\xi + (1-p_h)(1-\xi)}$  and  $\widehat{Pr}^d(g = g_h | s = g_h) = \frac{p_h^d\xi}{p_h^d\xi + (1-p_h^d)(1-\xi)}$ .*

*v) Beliefs are consistent with the government agency's set of optimal policies, i.e.  $\widehat{\theta} \in \Theta^*$ , where  $\Theta^* = \{\theta^* | \theta^* \text{ solves the government agency's problem given } \widehat{Pr}(g = g_h | s) \text{ and } \widehat{Pr}^d(g = g_h | s)\}$*

Notice that in iv), we also specify posterior beliefs "off the equilibrium path," when Bayes' rule cannot be used. In the case when  $\widehat{\theta}_h = 1$  and  $\widehat{\theta}_1 = 0$ , observing the public signal  $s = g_1$  is a zero-probability event. After observing  $s = g_1$ , the agents must realize

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<sup>2</sup>Note that  $\theta^*$  (the vector of probabilities by which the government agency forwards its private signal) determines its equilibrium signal.

that something must be wrong about their beliefs: it cannot be that both  $\hat{\theta}_h = 1$  and  $\hat{\theta}_l = 0$ . In this case, we imagine that the agents still believe that, if the government agency receives  $s_R = g_h$ , it will forward this private signal with probability one; however, they revise their belief regarding the event that the government agency would forward  $s_R = g_l$ , i.e., they assign a non-zero probability to it. Similarly, in the case when  $\hat{\theta}_h = 0$  and  $\hat{\theta}_l = 1$ , observing  $s = g_h$  is a zero-probability event. In analogy to the previous case, we imagine that, upon observing  $s = g_h$  the agents still think that if the government agency receives  $s_R = g_l$ , it forwards this signal with probability one; however, they revise their belief regarding the event that the government agency would forward  $s_R = g_h$ . These types of revisions result in the conditional probabilities that we state in item iv) of the above definition (see the next section for derivations).

### 3 Results

In this section, we present our results. First, we consider the effect of heterogeneity on investment decisions and asset prices. Thereafter, we consider the effect on the government agency's decision, showing that the "nonrevealing" equilibria that we identify can in fact dominate the "revealing" equilibria from the point of view of social welfare.

Since  $g_h \neq g_l$ , markets are complete, and we can solve for the equilibrium using two Arrow–Debreu (AD) securities: The first AD security delivers one unit of consumption if the growth rate turns out to be high ( $g = g_h$ ) and zero units otherwise while the second AD security delivers one unit of consumption if the growth rate turns out to be low

( $g = g_1$ ) and zero units otherwise. The prices of these AD securities generically depend on the government agency's public signal. That is, the government agency can affect prices through its signaling. We denote the price of the first AD security by  $q_{hj}$  and the price of the second one by  $q_{lj}$ , where the index  $j$  indicates the realization of the signal, where  $j = h$  in case of a high-growth signal ( $s = g_h$ ) and  $j = l$  in case of a low-growth signal ( $s = g_l$ ).

The agents' posterior beliefs are of crucial importance. Item iv) in the definition of the equilibrium (Definition 1) states that these should be computed according to Bayes' rule whenever possible and specifies beliefs in cases when it is not possible to use Bayes' rule. The following proposition determines posterior beliefs in cases when it is possible to apply Bayes' rule.

**Proposition 1.** *According to Bayes' rule, the agents' posterior beliefs regarding the growth rate are given by*

$$\hat{\mu}_{hh}^d \equiv \widehat{Pr}^d(g = g_h | s = g_h) = \frac{p_h^d \xi \hat{\theta}_h + p_h^d (1 - \xi)(1 - \hat{\theta}_1)}{(p_h^d \xi + (1 - p_h^d)(1 - \xi))\hat{\theta}_h + ((1 - p_h^d)\xi + p_h^d(1 - \xi))(1 - \hat{\theta}_1)}; \quad (5)$$

$$\hat{\mu}_{hl}^d \equiv \widehat{Pr}^d(g = g_h | s = g_l) = \frac{p_h^d \xi (1 - \hat{\theta}_h) + p_h^d (1 - \xi)\hat{\theta}_1}{(p_h^d \xi + (1 - p_h^d)(1 - \xi))(1 - \hat{\theta}_h) + ((1 - p_h^d)\xi + p_h^d(1 - \xi))\hat{\theta}_1}; \quad (6)$$

$$\hat{\mu}_{hh} \equiv \widehat{Pr}(g = g_h | s = g_h) = \frac{p_h \xi \hat{\theta}_h + p_h (1 - \xi)(1 - \hat{\theta}_1)}{(p_h \xi + (1 - p_h)(1 - \xi))\hat{\theta}_h + ((1 - p_h)\xi + p_h(1 - \xi))(1 - \hat{\theta}_1)}; \quad (7)$$

$$\hat{\mu}_{hl} \equiv \widehat{Pr}(g = g_h | s = g_l) = \frac{p_h \xi (1 - \hat{\theta}_h) + p_h (1 - \xi)\hat{\theta}_1}{(p_h \xi + (1 - p_h)(1 - \xi))(1 - \hat{\theta}_h) + ((1 - p_h)\xi + p_h(1 - \xi))\hat{\theta}_1}. \quad (8)$$

If  $\hat{\theta}_h = 1$  and  $\hat{\theta}_l = 0$ , then observing  $s = g_l$  is a zero-probability event for both types of agents and, similarly, if  $\hat{\theta}_h = 0$  and  $\hat{\theta}_l = 1$ , then observing  $s = g_h$  is a zero-probability event for both types of agents. In these cases, Bayes' rule cannot be used, but we still

want to specify updated probabilities "off-the-equilibrium-path" where the agents would observe  $s = g_l$  and  $s = g_h$ , respectively. This is done in item iv) of the definition of the equilibrium (Definition 1). The discussion below Definition 1 provides the intuition behind the conditional probabilities that are specified in item iv) of the definition.

In order to achieve tractability, we solely focus on logarithmic utility functions. From the calculations in the Appendix, we find that, in equilibrium, the ratios of state-contingent consumption equal the ratios of beliefs,

$$C_{3hj}/C_{3hj}^d = \hat{\mu}_{hj}/\hat{\mu}_{hj}^d \quad (9)$$

$$C_{3lj}/C_{3lj}^d = (1 - \hat{\mu}_{hj})/(1 - \hat{\mu}_{hj}^d), \quad (10)$$

meaning that, if the trusting agents believe that a state is more likely than the distrustful agents do, they will also allocate more resources to that state than the distrustful agents.<sup>3</sup> Since the trusting and the distrustful agents only differ in terms of their priors, one group will always be more optimistic (namely, the one with the highest prior probability of a high growth rate), so if, e.g., the trusting agents have a higher prior probability of a high growth rate, then the members of this group will always allocate more resources to the high-growth state than the distrustful agents, regardless of the government agency's public signal. However, the magnitude of the difference is generically affected by the government agency's public signal.

The state-contingent consumptions of trusting and distrustful agents can be reached by trading the stock and the bond. Focusing on the trusting agents, we find that their

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<sup>3</sup>Note that they start out with identical endowments.

optimal number of stocks is given by<sup>4</sup>

$$\phi_j = \frac{\widehat{\mu}_{hj}(1 - \widehat{\mu}_{hj})(g_h - g_l) + v(\widehat{\mu}_{hj} - \widehat{\mu}_{hj}^d)((1 - \widehat{\mu}_{hj})g_l + \widehat{\mu}_{hj}g_h)}{(g_h - g_l)(v\widehat{\mu}_{hj}^d + (1 - v)\widehat{\mu}_{hj})(v(1 - \widehat{\mu}_{hj}^d) + (1 - v)(1 - \widehat{\mu}_{hj}))} \quad (11)$$

and their optimal number of bonds is given by

$$\eta_j = \frac{D_2 g_h g_l v (\widehat{\mu}_{hj}^d - \widehat{\mu}_{hj})}{(g_h - g_l)(v\widehat{\mu}_{hj}^d + (1 - v)\widehat{\mu}_{hj})(v(1 - \widehat{\mu}_{hj}^d) + (1 - v)(1 - \widehat{\mu}_{hj}))}. \quad (12)$$

Here, we can see that provided that the trusting agents have a higher prior probability of a high growth rate than the distrustful agents, they will short sell the bond in order to invest more in the stock; conversely, if they have a lower prior probability, they will have a long position in the bond. The government agency's public signal will then generically affect magnitudes.

### 3.1 Optimal asset holdings and distrustful agents

We now analyze how the mass of distrustful agents impact the overall distribution of bond and stock holdings.

Taking the derivative of the trusting agents' optimal bond holding with respect to  $v$ , we have that

$$\frac{\partial \eta_j^u}{\partial v} = \frac{D_2 g_h g_l (\widehat{\mu}_{hj}^d - \widehat{\mu}_{hj}) ((1 - \widehat{\mu}_{hj}) \widehat{\mu}_{hj} + (\widehat{\mu}_{hj} - \widehat{\mu}_{hj}^d)^2 v^2)}{(g_h - g_l)(v\widehat{\mu}_{hj}^d + (1 - v)\widehat{\mu}_{hj})^2 (v(1 - \widehat{\mu}_{hj}^d) + (1 - v)(1 - \widehat{\mu}_{hj}))^2}. \quad (13)$$

Hence, the sign of the derivative depends on the sign of  $\widehat{\mu}_{hj}^d - \widehat{\mu}_{hj}$ , meaning that, if the distrustful agents have a more pessimistic (optimistic) prior, then an increase in the

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<sup>4</sup>We can solve for their optimal asset holdings from  $\phi_j D_2 g_h + \eta_j \cdot 1 = C_{3hj}$  and  $\phi_j D_2 g_l + \eta_j \cdot 1 = C_{3lj}$ , where  $\phi_j$  is the number of stocks and  $\eta_j$  is the number of bonds held by a trusting agent.



mass of distrustful agents will lead the trusting agents to decrease (increase) their bond holdings. From (12), we know that in the case when the distrustful agents have a more pessimistic prior, the bond holdings of the trusting agents are negative, whereas in the case when the distrustful agents have a more optimistic prior, the bond holdings of the trusting agents are positive. Thus, with respect to bond holdings, the mass of distrustful agents will only affect magnitudes, not signs.

We can also study how the mass of the distrustful agents affects the trusting agents' optimal stock holding:

$$\frac{\partial \phi_j^u}{\partial v} = \frac{\widehat{\mu}_{hj} - \widehat{\mu}_{hj}^d}{g_h - g_l} \left( \frac{\widehat{\mu}_{hj} g_h}{(v \widehat{\mu}_{hj}^d + (1-v) \widehat{\mu}_{hj})^2} + \frac{(1 - \widehat{\mu}_{hj}) g_l}{(v(1 - \widehat{\mu}_{hj}^d) + (1-v)(1 - \widehat{\mu}_{hj}))^2} \right). \quad (14)$$

That is, the sign of this derivative depends on the sign of  $\widehat{\mu}_{hj} - \widehat{\mu}_{hj}^d$ . This is opposite the result we obtained regarding the trusting agents' bond holdings. Thus, if the distrustful agents have a more pessimistic (optimistic) prior, then the effect of an increase in the mass of the distrustful agents will increase (decrease) the trusting agents' stock holdings. Here, the mass of the distrustful agents can affect signs, as can be seen in (11).

We next notice that the government's signal does not affect stock prices, while it directly affects holdings.

**Lemma 1.** *In equilibrium, the stock price is independent of individual beliefs.*

This result is specific for the log utility case. For risk aversions different from one, agents' beliefs and thus the government agency's signal will generically affect the stock price. However, the bond price will generically depend on beliefs also under log utility.

We can consider the effect of a larger mass of distrustful agents on the bond price:

$$\frac{\partial B_j}{\partial v} = \beta \left( \frac{1}{g_l} - \frac{1}{g_h} \right) (\widehat{\mu}_{hj} - \widehat{\mu}_{hj}^d). \quad (15)$$

If the distrustful agents have a more pessimistic (optimistic) prior regarding dividend growth than the trusting agents, then the bond is more (less) attractive to the distrustful agents, and thus, the larger the mass of distrustful agents, the higher (lower) the bond price, or, equivalently, the lower (higher) the interest rate.

### 3.2 Government's reaction

We now consider the problem of the government agency in this economy. The agency seeks to maximize *ex-post* social welfare by choosing the probability by which it forwards its revealing private signal. The agency's objective function is given in (4). With two types of log-utility agents (trusting and distrustful) of mass  $v$  and  $(1 - v)$ , respectively, we can write it as

$$\begin{aligned} \mathcal{W}_j = & \theta_j (v(\ln C_2^d + \beta[\widehat{\mu}_{hj}^G \ln C_{3hj}^d + (1 - \widehat{\mu}_{hj}^G) \ln C_{3lj}^d]) \\ & + (1 - v)(\ln C_2 + \beta[\widehat{\mu}_{hj}^G \ln C_{3hj} + (1 - \widehat{\mu}_{hj}^G) \ln C_{3lj}])) \\ & (1 - \theta_j) (v(\ln C_2^d + \beta[\widehat{\mu}_{hj}^G \ln C_{3h\neg j}^d + (1 - \widehat{\mu}_{hj}^G) \ln C_{3l\neg j}^d]) \\ & + (1 - v)(\ln C_2 + \beta[\widehat{\mu}_{hj}^G \ln C_{3h\neg j} + (1 - \widehat{\mu}_{hj}^G) \ln C_{3l\neg j}]))), \quad (16) \end{aligned}$$

where the index  $j$  indicates whether the government agency has observed  $s_R = g_h$  ( $j = h$ ) or  $s_R = g_l$  ( $j = l$ ),  $\neg j$  stands for "not  $j$ ", so that  $\neg h = l$  and  $\neg l = h$ , and  $\widehat{\mu}_{hj}^G$  is the government agency's posterior probability, after having observed  $s_R = g_j$ . Using Bayes'

rule, we can calculate the agency's posteriors as

$$\widehat{\mu}_{hh}^G = \frac{p_h \xi}{p_h \xi + (1 - p_h)(1 - \xi)} \quad (17)$$

$$\widehat{\mu}_{hl}^G = \frac{p_h(1 - \xi)}{p_h(1 - \xi) + (1 - p_h)\xi}. \quad (18)$$

From (16), we see that, conditional on observing  $s_R = g_j$ , the government agency's optimal policy depends on the sign of

$$\begin{aligned} H_j &\equiv v \left( \widehat{\mu}_{hj}^G \ln \frac{C_{3hj}^d}{C_{3h-j}^d} + (1 - \widehat{\mu}_{hj}^G) \ln \frac{C_{3lj}^d}{C_{3l-j}^d} \right) \\ &\quad + (1 - v) \left( \widehat{\mu}_{hj}^G \ln \frac{C_{3hj}}{C_{3h-j}} + (1 - \widehat{\mu}_{hj}^G) \ln \frac{C_{3lj}}{C_{3l-j}} \right) \\ &= \widehat{\mu}_{hj}^G \left( \ln \frac{v \widehat{\mu}_{h-j}^d + (1 - v) \widehat{\mu}_{h-j}}{v \widehat{\mu}_{hj}^d + (1 - v) \widehat{\mu}_{hj}} + v \ln \frac{\widehat{\mu}_{hj}^d}{\widehat{\mu}_{h-j}^d} + (1 - v) \ln \frac{\widehat{\mu}_{hj}}{\widehat{\mu}_{h-j}} \right) \\ &\quad + (1 - \widehat{\mu}_{hj}^G) \left( \ln \frac{v(1 - \widehat{\mu}_{h-j}^d) + (1 - v)(1 - \widehat{\mu}_{h-j})}{v(1 - \widehat{\mu}_{hj}^d) + (1 - v)(1 - \widehat{\mu}_{hj})} + v \ln \frac{1 - \widehat{\mu}_{hj}^d}{1 - \widehat{\mu}_{h-j}^d} + (1 - v) \ln \frac{1 - \widehat{\mu}_{hj}}{1 - \widehat{\mu}_{h-j}} \right) \end{aligned} \quad (19)$$

If (19) is positive, then social welfare is increasing in the probability of a revealing signal, and the government agency's optimal policy is to forward its private signal with probability one ( $\theta_j^* = 1$ ). If the above expression is negative, the optimal policy is to send a signal that is the opposite of its private signal with probability one ( $\theta_j^* = 0$ ), and if the above expression is zero, then the optimal solution is  $\theta_j^* \in [0, 1]$ . Simple numerical tests show that the sets of parameters generating positive, negative and zero values on  $H_j$  are all nonempty. In particular, if agents believe that there is always a 50% probability that the government agency announces a high growth rate, regardless of its private signal, then their beliefs are not affected by the public signal. That is, in this case, the government agency is unable to affect social welfare through its public signal, and thus, there are infinitely many equilibria.

**Proposition 2.** *If agents hold beliefs  $\hat{\theta}_h = \hat{\theta}_l = \frac{1}{2}$ , then the government agency is unable to affect ex-post social welfare through its public signal. Thus, such beliefs support infinitely many equilibria in which  $\theta_j^* \in [0, 1]$  ( $j = h, l$ ).*

Depending on the parameter values, there is either a unique equilibrium or infinitely many equilibria. That is, in contrast to the equilibria we consider in the above proposition, there are other equilibria in which the government agency's optimal strategy is unique. Since the agents' beliefs need to be consistent with the set of optimal government agency strategies, and the government agency's optimal strategy is unique in a unique equilibrium, its optimal strategy must coincide with the agents' beliefs in such an equilibrium.

**Lemma 2.** *In a unique equilibrium,  $\theta^*$  coincides with  $\hat{\theta}$ .*

The result follows directly from item v) of the equilibrium definition (Definition 1).

Since the expression in (19) does not depend on the government agency's strategy, we can conclude that, in unique equilibria, the government agency's strategy must be a "bang-bang" solution, which leaves us with four possible combinations:  $(1, 1)$ ,  $(0, 0)$ ,  $(1, 0)$  and  $(0, 1)$ . However, as one might suspect, given that the agents believe that the agency lies and only says "low growth" regardless of its private signal, and their belief "off-the-equilibrium path" that a high-growth public signal comes from the government agency's forwarding of  $s_R = g_h$  (as specified in the fourth item of the equilibrium definition), the government agency can increase ex-post social welfare by deviating and sending a high-growth public signal in the case that it receives a high-growth private signal. Thus, we can rule out that the government agency strategy  $(0, 1)$  is part of a unique equilibrium.

**Proposition 3.** *The government agency strategy  $\theta = (\theta_h, \theta_l) = (0, 1)$  cannot be part of a unique equilibrium.*

Out of the extreme agency strategies that we mention above,  $(1, 1)$  and  $(0, 0)$  are both revealing the government agency's private signal to the public (the latter by always sending a public signal that is the opposite of the private one). It therefore seems natural that if the agents' beliefs coincide with these agency strategies, the strategies will give rise to the same ex-post social welfare. As the following proposition shows, this is indeed the case.

**Proposition 4.** *Given that  $\hat{\theta} = (1, 1)$  and  $\hat{\theta} = (0, 0)$ , respectively, the strategies  $\theta = (1, 1)$  and  $\theta = (0, 0)$  result in the same ex-post social welfare.*

If the government agency plays  $(1, 0)$  and the agents' beliefs coincide with this strategy, i.e., if the government agency always says "high growth" regardless of its private signal, then, provided that the government agency sticks to this strategy, its public announcement is not valuable and the agents stick to their priors. This is not the same as saying that they do not pay attention to the government agency's announcement, because they would still update their beliefs if the government agency were to deviate from this strategy. However, in the case that we consider above, in which the agents believe that there is always a 50% probability of the government agency announcing a high growth rate, the agents do not pay attention to the government agency's public announcement. We show below that these two situations give rise to the same ex-post social welfare.

**Proposition 5.** *Given that  $\hat{\theta} = (1, 0)$  and  $\hat{\theta} = (\frac{1}{2}, \frac{1}{2})$ , respectively, the strategies  $\theta = (1, 0)$*

and  $\theta = (s, t) \in [0, 1] \times [0, 1]$  result in the same ex-post social welfare.

One would think that the "revealing" unique equilibria where  $\theta^* = (1, 1)$  and  $\theta^* = (0, 0)$  would dominate the unique equilibrium where the government agency always announces a high growth ( $\theta^* = (1, 0)$ ). However, as Figures 1–4 show, the unique equilibria where  $\theta^* = (1, 1)$  and  $\theta^* = (0, 0)$  do not always result in an ex-post social welfare that is higher than the unique equilibrium in which  $\theta^* = (1, 0)$ . In fact, the opposite can be true. The following proposition summarizes this finding.<sup>5</sup>

**Proposition 6.** *The unique equilibrium in which  $\theta^* = (1, 0)$  can result in a higher ex-post social welfare than the "revealing" unique equilibria in which  $\theta^* = (1, 1)$  and  $\theta^* = (0, 0)$ , respectively.*

As seen in Figure 1b, given that the agents believe that the government agency forwards its private signal with probability one, the strategy of forwarding the private signal in the case in which it says "low growth" breaks down, provided that the fraction of distrustful agents is sufficiently large. Moreover, from Figure 3b, we see that –given the assumed parameter values– the strategy of always announcing a high growth can only be an equilibrium if the fraction of distrustful agents is sufficiently large. Our interpretation is that if the mass of distrustful agents is sufficiently large, the government agency's objective function deviates sufficiently from the average agent in order for it not to forward

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<sup>5</sup>Of course, a necessary condition for  $\theta^* = (1, 0)$  to achieve higher ex-post social welfare than the "revealing" unique equilibria is that the mass of distrustful agents be positive ( $v > 0$ ). As mentioned earlier, if  $v = 0$ , then the government agency cannot affect ex-post social welfare.

valuable information.<sup>6</sup> Table 1 also suggests that, if the distrustful agents' prior differs "sufficiently much" from the trusting agents' prior, the only possible unique equilibrium is the nonrevealing one, in which the government agency's signal always says "high growth." A difference between the trusting and distrustful agents' priors will of course also induce a difference between the government agency's objective and that of the average agent. This is in line with Crawford and Sobel's (1982) result that "the more similar agents' preferences [sender and receiver], the more informative the equilibrium signal" (Crawford and Sobel, 1982, p. 1432).

## 4 Conclusion

This paper analyzes how a benevolent, privately-informed government agency would optimally convey information regarding future growth rates and the effects of this information on agents' strategies and equilibrium asset prices in a Lucas (1978) exchange-only economy where the growth rate can be either *high* or *low*. The government agency chooses between forwarding its private signal, which partially reveals the true future growth rate, and sending an opposite signal. We model two types of agents: trusting and distrustful. The trusting agents hold the same prior beliefs as the government agency, whereas the distrustful agents differ in their prior beliefs. In our model, both types of agents are rational Bayesian updaters, but our results readily extend to settings where a group of

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<sup>6</sup>Recall that both the agency's and the trusting agents' prior probability of a high growth rate is  $p_h$ , while the distrustful agents' prior is  $p_h^d \neq p_h$ .

agents or all agents follow a behavioral updating rule. In deciding on what public signal to send, the government agency seeks to maximize a standard *ex-post* measure of social welfare.

When both types of agents are present, the government agency is in some cases able to affect the distribution of resources across the two future states (high and low growth). We identify both "revealing" and "nonrevealing" equilibria. However, the "revealing" equilibria do not always result in a higher ex-post social welfare than the "nonrevealing" equilibria. Some numerical examples suggest that the mass of distrustful agents or the difference in priors between the two types of agents needs to be sufficiently large in order for the "nonrevealing" equilibria to dominate. With a larger mass of distrustful agents or a larger difference in priors, the government agency's objective becomes more different from that of the average agent. Hence, this result is in line with the conclusion in Crawford and Sobel (1982) that "the more similar agents' preferences [sender and receiver], the more informative the equilibrium signal" (Crawford and Sobel, 1982, p. 1432).

We also analyze how the mass of the distrustful agents affects agents' investment decisions and equilibrium asset prices. One result is that, if the distrustful agents have a more pessimistic (optimistic) prior regarding dividend growth than the trusting agents, then the larger the mass of the distrustful agents, the higher (lower) the bond price, or, equivalently, the lower (higher) the interest rate. The reason is that if the distrustful agents have a more pessimistic (optimistic) prior regarding dividend growth than the trusting agents, then the bond appears more (less) attractive to the distrustful agents.



# Appendix

## Agents' equilibrium consumption

After having determined the agents' posterior beliefs, we can consider the agents' optimization problems. We can write the distrustful agents' problem as

$$\begin{aligned}
 & \max_{C_{2j}^d, C_{3hj}^d, C_{3lj}^d} u(C_{2j}^d) + \beta [\widehat{\mu}_{hj}^d u(C_{3hj}^d) + (1 - \widehat{\mu}_{hj}^d) u(C_{3lj}^d)] & (20) \\
 & \text{s.t. } C_{2j}^d + q_{hj} C_{3hj}^d + q_{lj} C_{3lj}^d = D_2 + q_{hj} D_2 g_h + q_{lj} D_2 g_l \\
 & & C_{2j}^d \geq 0 \\
 & & C_{3hj}^d \geq 0 \\
 & & C_{3lj}^d \geq 0
 \end{aligned}$$

Since  $u'(c) \rightarrow +\infty$  as  $c \rightarrow 0$ , the last three constraints are not binding. Hence, the corresponding Lagrangian is

$$\begin{aligned}
 \mathcal{L}_j^d &= u(C_{2j}^d) + \beta (\widehat{\mu}_{hj}^d u(C_{3hj}^d) + (1 - \widehat{\mu}_{hj}^d) u(C_{3lj}^d)) \\
 &+ \lambda_j (D_2 + q_{hj} D_2 g_h + q_{lj} D_2 g_l - C_{2j}^d - q_{hj} C_{3hj}^d - q_{lj} C_{3lj}^d), \quad (21)
 \end{aligned}$$

where  $\lambda_j$  is the Lagrange multiplier. The first-order conditions with respect to initial and state-contingent consumption are

$$C_{2j}^d : u'(C_{2j}^d) - \lambda_j = 0, \quad (22)$$

$$C_{3hj}^d : \beta \widehat{\mu}_{hj}^d u'(C_{3hj}^d) - \lambda_j q_{hj} = 0, \quad (23)$$

$$C_{3lj}^d : \beta (1 - \widehat{\mu}_{hj}^d) u'(C_{3lj}^d) - \lambda_j q_{lj} = 0. \quad (24)$$

Assuming constant relative risk aversion, so that  $u'(c) = c^{-\gamma}$ , we have that

$$C_{2j}^d = \lambda_j^{-\frac{1}{\gamma}}, \quad (25)$$

$$C_{3hj}^d = \left( \frac{\lambda_j q_{hj}}{\beta \widehat{\mu}_{hj}^d} \right)^{-\frac{1}{\gamma}}, \quad (26)$$

$$C_{3lj}^d = \left( \frac{\lambda_j q_{lj}}{\beta(1 - \widehat{\mu}_{hj}^d)} \right)^{-\frac{1}{\gamma}}. \quad (27)$$

From the budget constraint, it follows that

$$\lambda_j^{-\frac{1}{\gamma}} = \frac{D_2(1 + q_{hj}g_h + q_{lj}g_l)}{1 + \beta^{\frac{1}{\gamma}}((\widehat{\mu}_{hj}^d)^{\frac{1}{\gamma}} q_{hj}^{1-\frac{1}{\gamma}} + (1 - \widehat{\mu}_{hj}^d)^{\frac{1}{\gamma}} q_{lj}^{1-\frac{1}{\gamma}})}. \quad (28)$$

We can use similar elementary lines to determine the optimal initial and state-contingent consumption of the trusting agents. With those findings, we can now move to analyzing the government's optimal signal.

In order to achieve tractability, we solely focus on logarithmic utility functions ( $\gamma = 1$ ).

In this case, the initial and state-contingent consumption of the distrustful agents looks as follows.

$$C_{2j}^d = \frac{D_2(1 + q_{hj}g_h + q_{lj}g_l)}{1 + \beta}, \quad (29)$$

$$C_{3hj}^d = \frac{\beta \widehat{\mu}_{hj}^d D_2(1 + q_{hj}g_h + q_{lj}g_l)}{q_{hj}(1 + \beta)}, \quad (30)$$

$$C_{3lj}^d = \frac{\beta(1 - \widehat{\mu}_{hj}^d) D_2(1 + q_{hj}g_h + q_{lj}g_l)}{q_{lj}(1 + \beta)}. \quad (31)$$

Similarly, the consumption of the trusting agents is given by

$$C_{2j} = \frac{D_2(1 + q_{hj}g_h + q_{lj}g_l)}{1 + \beta}, \quad (32)$$

$$C_{3hj} = \frac{\beta \widehat{\mu}_{hj} D_2(1 + q_{hj}g_h + q_{lj}g_l)}{q_{hj}(1 + \beta)}, \quad (33)$$

$$C_{3lj} = \frac{\beta(1 - \widehat{\mu}_{hj}) D_2(1 + q_{hj}g_h + q_{lj}g_l)}{q_{lj}(1 + \beta)}. \quad (34)$$

The market-clearing conditions can be summarized as

$$vC_{2j}^d + (1 - v)C_{2j} = D_2, \quad (35)$$

$$vC_{3hj}^d + (1 - v)C_{3hj} = D_2g_h, \quad (36)$$

$$vC_{3lj}^d + (1 - v)C_{3lj} = D_2g_l, \quad (37)$$

where, by Walras' law, market clearing in any two of these markets implies market clearing also in the third.

The first market-clearing condition in (35) can be rewritten as  $q_{hj}g_h + q_{lj}g_l = \beta$ . Thus, if we combine this with the second market clearing condition in (36), we obtain the prices of the AD securities;

$$q_{hj} = \frac{\beta(v\widehat{\mu}_{hj}^d + (1 - v)\widehat{\mu}_{hj})}{g_h} \quad (38)$$

and

$$q_{lj} = \frac{\beta(v(1 - \widehat{\mu}_{hj}^d) + (1 - v)(1 - \widehat{\mu}_{hj}))}{g_l}. \quad (39)$$

Here, we note that a stronger belief in a high growth rate among both agents leads to a higher price of consumption in the high-growth state and a lower price of consumption in the low-growth state.

The equilibrium consumption of the distrustful agents is given by

$$C_{2j}^d = D_2, \quad (40)$$

$$C_{3hj}^d = \frac{\widehat{\mu}_{hj}^d}{v\widehat{\mu}_{hj}^d + (1 - v)\widehat{\mu}_{hj}} D_2g_h, \quad (41)$$

$$C_{3lj}^d = \frac{(1 - \widehat{\mu}_{hj}^d)}{v(1 - \widehat{\mu}_{hj}^d) + (1 - v)(1 - \widehat{\mu}_{hj})} D_2g_l, \quad (42)$$

and, similarly, the trusting agents' equilibrium consumption is

$$C_{2j} = D_2, \quad (43)$$

$$C_{3hj} = \frac{\widehat{\mu}_{hj}}{v\widehat{\mu}_{hj}^d + (1-v)\widehat{\mu}_{hj}} D_2 g_h, \quad (44)$$

$$C_{3lj} = \frac{(1 - \widehat{\mu}_{hj})}{v(1 - \widehat{\mu}_{hj}^d) + (1-v)(1 - \widehat{\mu}_{hj})} D_2 g_l. \quad (45)$$

## Proof of Lemma 1

By no-arbitrage, the prices of the stock<sup>7</sup> and the bond are given by

$$S_j = D_2(q_{hj}g_h + q_{lj}g_l) = \beta D_2 \quad (46)$$

$$B_j = q_{hj} + q_{lj} = \beta \left( \frac{\widehat{\mu}_{hj} - v(\widehat{\mu}_{hj} - \widehat{\mu}_{hj}^d)}{g_h} + \frac{1 - \widehat{\mu}_{hj} + v(\widehat{\mu}_{hj} - \widehat{\mu}_{hj}^d)}{g_l} \right). \quad (47)$$

Here, we see that the stock price does not depend on beliefs. Hence, the government agency's signal does not affect the price of the risky asset.

## Proof of Proposition 1

By Bayes' Theorem, we have that

$$\widehat{Pr}(g = g_h | s = g_h) = \frac{\widehat{Pr}(g = g_h \cap s = g_h)}{\widehat{Pr}(s = g_h)} = \frac{\widehat{Pr}(g = g_h) \widehat{Pr}(s = g_h | g = g_h)}{\widehat{Pr}(s = g_h)}. \quad (48)$$

Further, by the law of total probability,

$$\widehat{Pr}(s = g_h) = \widehat{Pr}(s_R = g_h) \widehat{Pr}(s = g_h | s_R = g_h) + \widehat{Pr}(s_R = g_l) \widehat{Pr}(s = g_h | s_R = g_l) \quad (49)$$

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<sup>7</sup>Note that the stock price is the price of a stock that has been stripped of its initial dividend.

and

$$\begin{aligned}\widehat{Pr}(s = g_h | g = g_h) &= \widehat{Pr}(s_R = g_h | g = g_h)\widehat{Pr}(s = g_h | g = g_h, s_R = g_h) \\ &\quad + \widehat{Pr}(s_R = g_l | g = g_h)\widehat{Pr}(s = g_h | g = g_h, s_R = g_l).\end{aligned}\quad (50)$$

The government agency does not know the true growth rate, it only has access to the partially revealing private signal  $s_R$ . Thus, conditioning  $s$  on  $s_R$  and  $g$  is equivalent to conditioning  $s$  on  $s_R$  only, and we have that

$$\begin{aligned}\widehat{Pr}(s = g_h | g = g_h) &= \widehat{Pr}(s_R = g_h | g = g_h)\widehat{Pr}(s = g_h | s_R = g_h) \\ &\quad + \widehat{Pr}(s_R = g_l | g = g_h)\widehat{Pr}(s = g_h | s_R = g_l).\end{aligned}\quad (51)$$

This implies that

$$\widehat{Pr}(s = g_h | g = g_h) = \xi\widehat{\theta}_h + (1 - \xi)(1 - \widehat{\theta}_l).\quad (52)$$

Again, by the law of total probability,

$$\widehat{Pr}(s_R = g_h) = \widehat{Pr}(g = g_h)\widehat{Pr}(s_R = g_h | g = g_h) + \widehat{Pr}(g = g_l)\widehat{Pr}(s_R = g_h | g = g_l).\quad (53)$$

and

$$\widehat{Pr}(s_R = g_l) = \widehat{Pr}(g = g_h)\widehat{Pr}(s_R = g_l | g = g_h) + \widehat{Pr}(g = g_l)\widehat{Pr}(s_R = g_l | g = g_l).\quad (54)$$

From (49), we have that

$$\begin{aligned}\widehat{Pr}(s = g_h) &= (p_h\xi + (1 - p_h)(1 - \xi))\widehat{\theta}_h + (p_h(1 - \xi) + (1 - p_h)\xi)(1 - \widehat{\theta}_l) \\ &= p_h(\xi\widehat{\theta}_h + (1 - \xi)(1 - \widehat{\theta}_l)) + (1 - p_h)((1 - \xi)\widehat{\theta}_h + \xi(1 - \widehat{\theta}_l)).\end{aligned}\quad (55)$$

Thus, one of the conditional probabilities that we are seeking is given by

$$\widehat{Pr}(g = g_h | s = g_h) = \frac{p_h(\xi\widehat{\theta}_h + (1 - \xi)(1 - \widehat{\theta}_1))}{p_h(\xi\widehat{\theta}_h + (1 - \xi)(1 - \widehat{\theta}_1)) + (1 - p_h)((1 - \xi)\widehat{\theta}_h + \xi(1 - \widehat{\theta}_1))}. \quad (56)$$

The conditional probability  $\widehat{Pr}(g = g_h | s = g_l)$  can be calculated using the same steps as outlined above. This results in

$$\widehat{Pr}(g = g_h | s = g_l) = \frac{p_h(\xi(1 - \widehat{\theta}_h) + (1 - \xi)\widehat{\theta}_1)}{p_h(\xi(1 - \widehat{\theta}_h) + (1 - \xi)\widehat{\theta}_1) + (1 - p_h)((1 - \xi)(1 - \widehat{\theta}_h) + \xi\widehat{\theta}_1)}. \quad (57)$$

In order to get to the distrustful agents' conditional probabilities, one just needs to take into account that they have a different prior,  $p_h^d$ .

## Proof of Proposition 2

If  $\widehat{\theta}_h = \widehat{\theta}_l = \frac{1}{2}$ , it follows from equations (5) through (8) that  $\widehat{\mu}_{hh}^d = \widehat{\mu}_{hl}^d = p_h^d$  and  $\widehat{\mu}_{hh} = \widehat{\mu}_{hl} = p_h$ . Thus, from (19), we see that  $H_j = 0$ , so that  $\theta_j^* \in [0, 1]$  ( $j = h, l$ ). Now, we can go through the equilibrium definition (Definition 1) and confirm that  $\theta_j^* \in [0, 1]$  ( $j = h, l$ ) holds in equilibrium.

## Proof of Proposition 3

We will prove this by contradiction. Suppose  $\theta = (\theta_h, \theta_l) = (0, 1)$  is part of a unique equilibrium. Then,  $\hat{\theta} = \theta^* = (0, 1)$  and the agents' posterior beliefs are given by

$$\hat{\mu}_{hl}^d = p_h^d \quad (58)$$

$$\hat{\mu}_{hh}^d = \frac{p_h^d \xi}{p_h^d \xi + (1 - p_h^d)(1 - \xi)} \quad (59)$$

$$\hat{\mu}_{hl} = p_h \quad (60)$$

$$\hat{\mu}_{hh} = \frac{p_h \xi}{p_h \xi + (1 - p_h)(1 - \xi)}. \quad (61)$$

Now, consider  $H_h$  in (19):

$$\begin{aligned} H_h &= \frac{1}{(1 - p_h)(1 - \xi) + p_h \xi} (p_h \xi (\ln(vp_h^d + (1 - v)p_h) - v \ln(p_h^d) - (1 - v) \ln(p_h)) \\ &\quad (1 - p_h)(1 - \xi) (\ln(v(1 - p_h^d) + (1 - v)(1 - p_h)) - v \ln(1 - p_h^d) - (1 - v) \ln(1 - p_h))) \\ &> 0 \quad (v \in (0, 1), p_h^d \in (0, 1), p_h \in (0, 1), \xi \in (0, 1)), \end{aligned} \quad (62)$$

where the inequality follows from the strict concavity of the logarithmic function. Thus,  $\theta_h^* = 1$ , which contradicts the supposition that  $\theta^* = (\theta_h^*, \theta_l^*) = (0, 1)$ .

## Proof of Proposition 4

By substituting  $\hat{\theta} = (1, 1)$  and  $\hat{\theta} = (0, 0)$ , respectively, into the posterior beliefs in (5) through (8), we see that

$$\mathcal{W}_j |_{\theta_j=1, \hat{\theta}=(1,1)} = \mathcal{W}_j |_{\theta_j=0, \hat{\theta}=(0,0)} \quad (j = h, l). \quad (63)$$

## Proof of Proposition 5

If  $\hat{\theta} = (1, 0)$ , the posterior beliefs conditional on observing a high growth public signal are  $\hat{\mu}_{hh} = p_h$  and  $\hat{\mu}_{hh}^d = p_h^d$ , respectively. Further, if  $\hat{\theta} = (\frac{1}{2}, \frac{1}{2})$ , then the posterior beliefs are  $\hat{\mu}_{hh} = \hat{\mu}_{hl} = p_h$  and  $\hat{\mu}_{hh}^d = \hat{\mu}_{hl}^d = p_h^d$ , respectively. Thus, we have that

$$\mathcal{W}_j \mid_{\theta=(1,0), \hat{\theta}=(1,0)} = \mathcal{W}_j \mid_{\theta=(s,t) \in [0,1] \times [0,1], \hat{\theta}=(\frac{1}{2}, \frac{1}{2})} \quad (j = h, l). \quad (64)$$

## Proof of Proposition 6

Consider the following parameter values:  $v = 0.5$ ,  $p_h^d = 0.60$ ,  $p_h = 0.80$ ,  $D_2 = 1$ ,  $g_h = 1.2$ ,  $g_l = 0.8$ ,  $\xi = 0.70$ ,  $\beta = 0.95$ . Under these parameter values, we have that  $H_h \mid_{\hat{\theta}=(1,1)} > 0$ ,  $H_h \mid_{\hat{\theta}=(0,0)} < 0$ ,  $H_h \mid_{\hat{\theta}=(1,0)} > 0$ ,  $H_l \mid_{\hat{\theta}=(1,1)} > 0$ ,  $H_l \mid_{\hat{\theta}=(0,0)} < 0$  and  $H_l \mid_{\hat{\theta}=(1,0)} < 0$ . Thus, the parameter values support all of the unique equilibria we mention in the proposition.

Now, given these parameter values, consider ex-post social welfare in the case of a low-growth private signal:

$$\mathcal{W}_1 \mid_{\theta_1=1, \hat{\theta}=(1,1)} = \mathcal{W}_1 \mid_{\theta_1=0, \hat{\theta}=(0,0)} = 3.35 \cdot 10^{-3} < 4.49 \cdot 10^{-3} = \mathcal{W}_1 \mid_{\theta_1=0, \hat{\theta}=(1,0)}. \quad (65)$$

Thus, under the parameter values given in the proposition, ex-post social welfare conditional on a low-growth private signal is higher in the unique equilibrium in which  $\theta^* = (1, 0)$  than in the "revealing" unique equilibria in which  $\theta^* = (1, 1)$  and  $\theta^* = (0, 0)$ , respectively.



## References

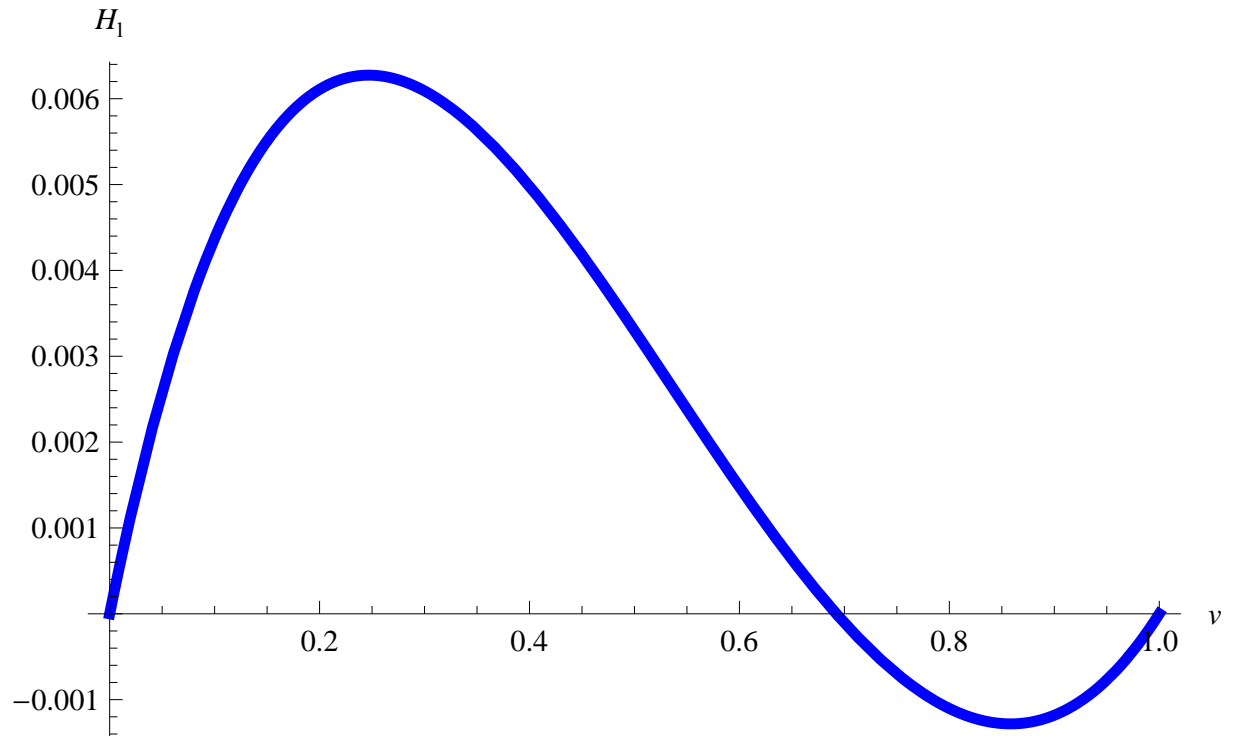
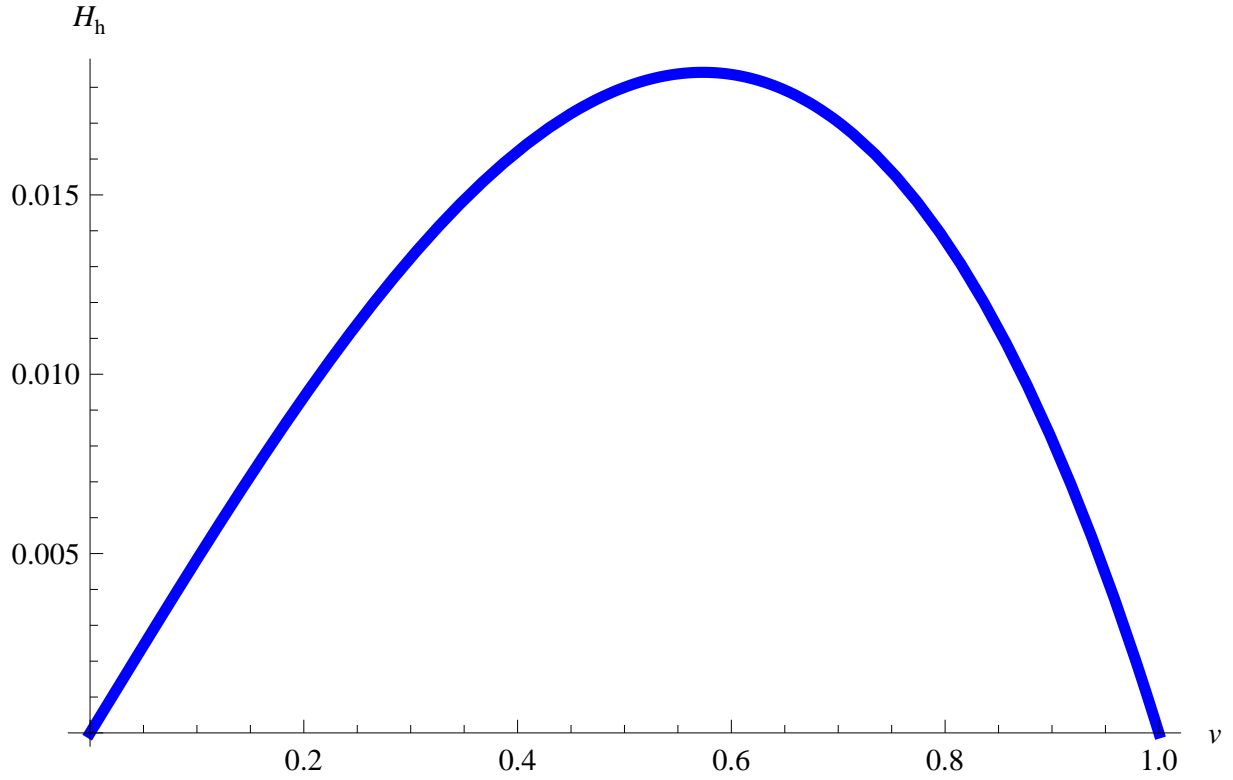
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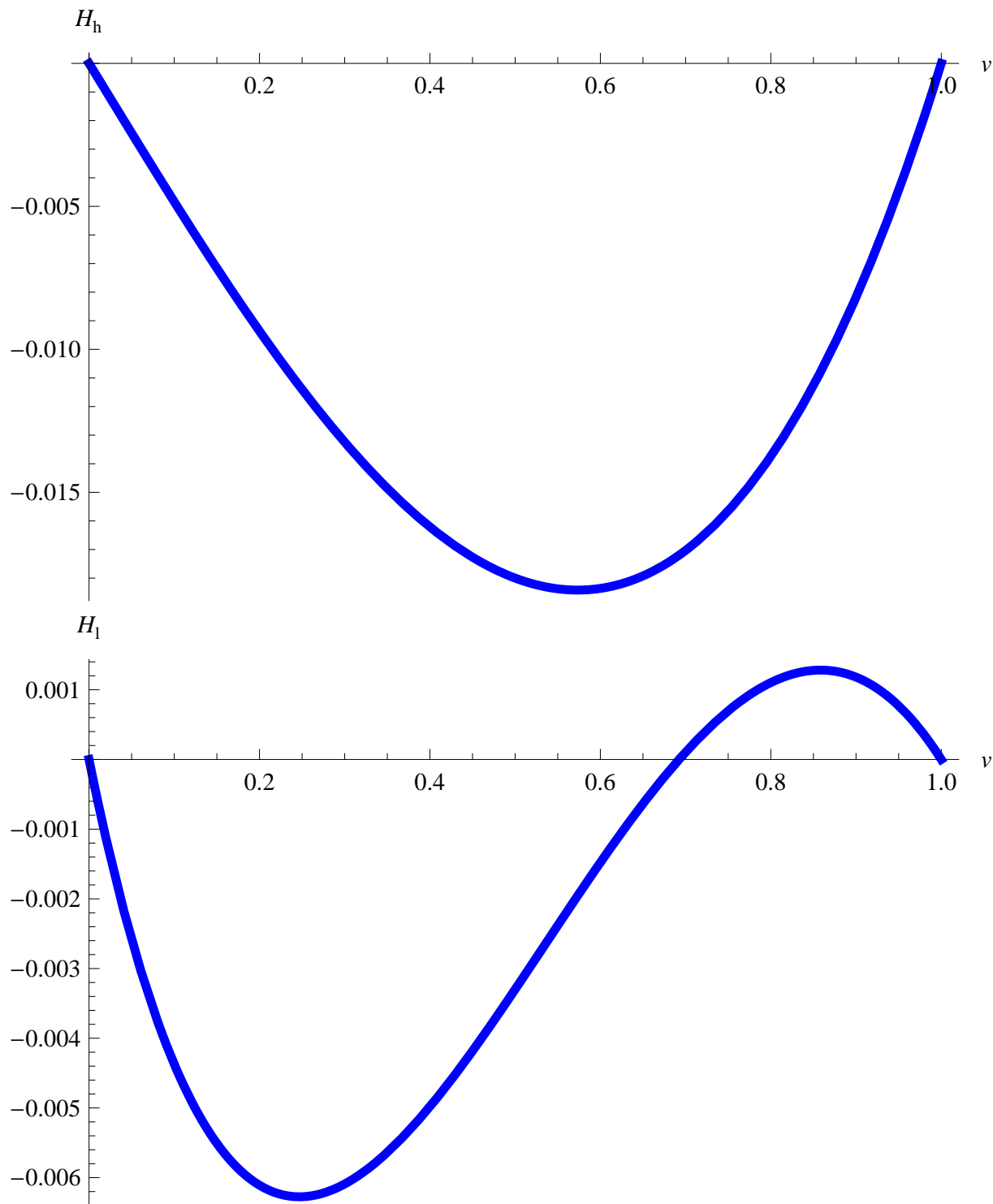
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Table 1: The table lists the the possible unique equilibrium strategies for the government agency ( $\theta^*$ ) for various values on the distrustful agents' prior ( $p_h^d$ ) when the fraction of distrustful agents ( $v$ ) is 30% and 60%, respectively. We have assumed that  $p_h = 0.80$  and  $\xi = 0.70$ . Note that the government agency's strategy depends on the agents' beliefs regarding the likelihood that the government agency forwards its private signal ( $\hat{\theta}$ ). In a unique equilibrium  $\theta^* = \hat{\theta}$ .

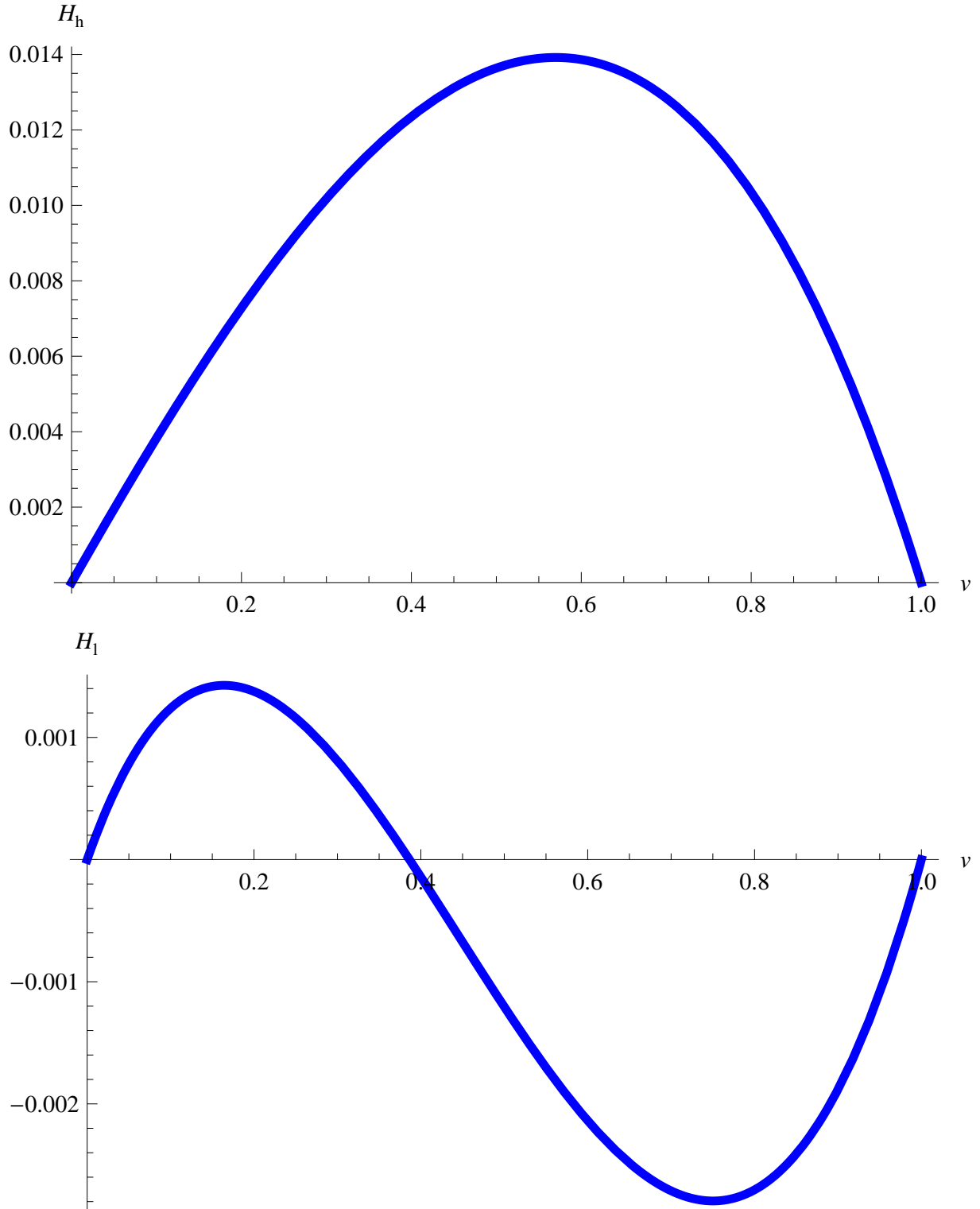
	<b>Distrustful agents' prior</b>	<b>Possible unique equilibria</b>
$v=0.30$	0.10	(1,0)
	0.30	(1,1),(0,0),(1,0)
	0.50	(1,1),(0,0),(1,0)
	0.70	(1,1),(0,0)
	0.90	(1,1),(0,0)
$v=0.60$	0.10	(1,0)
	0.30	(1,0)
	0.50	(1,0)
	0.70	(1,1),(0,0)
	0.90	(1,1),(0,0)



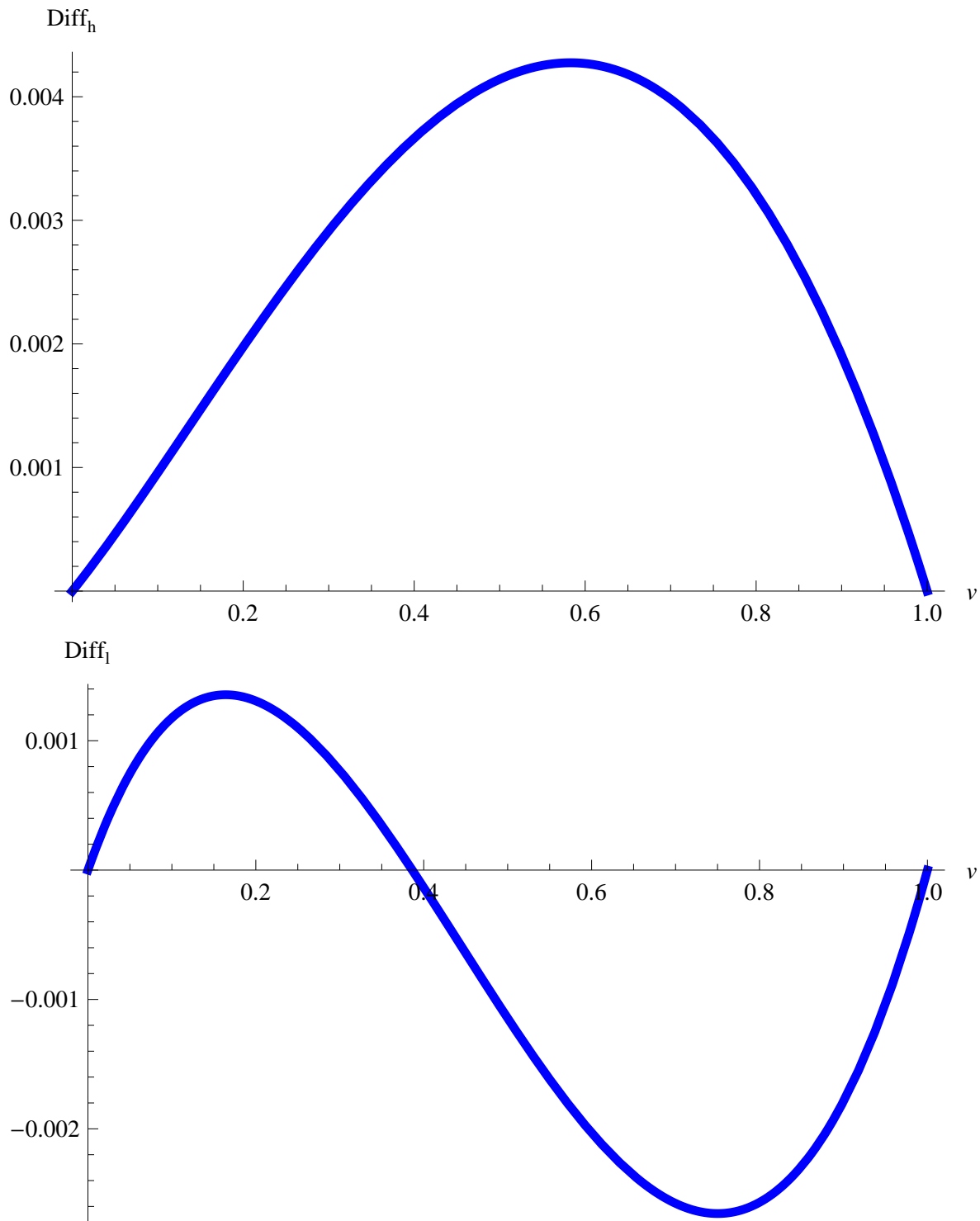
Figures 1a,b): The figures show  $H_h$  and  $H_1$  in (19) as a function of the mass of the distrustful agents ( $v$ ). A positive (negative) value means that social welfare is increasing (decreasing) in the probability that the government agency forwards its private signal. We have assumed the following parameter values:  $p_h^d = 0.60$ ,  $p_h = 0.80$ ,  $\xi = 0.70$ , and  $\hat{\theta} = (\hat{\theta}_h, \hat{\theta}_1) = (1, 1)$ .



Figures 2a,b): The figures show  $H_h$  and  $H_l$  in (19) as a function of the mass of the distrustful agents ( $v$ ). A positive (negative) value means that social welfare is increasing (decreasing) in the probability that the government agency forwards its private signal. We have assumed the following parameter values:  $p_h^d = 0.60$ ,  $p_h = 0.80$ ,  $\xi = 0.70$ , and  $\hat{\theta} = (\hat{\theta}_h, \hat{\theta}_l) = (0, 0)$ .



Figures 3a,b): The figures show  $H_h$  and  $H_1$  in (19) as a function of the mass of the distrustful agents ( $v$ ). A positive (negative) value means that social welfare is increasing (decreasing) in the probability that the government agency forwards its private signal. We have assumed the following parameter values:  $p_h^d = 0.60$ ,  $p_h = 0.80$ ,  $\xi = 0.70$ , and  $\hat{\theta} = (\hat{\theta}_h, \hat{\theta}_1) = (1, 0)$ .



Figures 4a,b): The figures show the differences in ex-post social welfare in the cases where  $s_R = g_h$  (top) and  $s_R = g_l$  (bottom) between the strategies  $\theta = (1, 1)$  and  $\theta = (1, 0)$  as a function of the mass of the distrustful agents ( $v$ ), provided that the agents beliefs are  $\hat{\theta} = (1, 1)$  and  $\hat{\theta} = (1, 0)$ , respectively. We have assumed the following parameter values:  $p_h^d = 0.60$ ,  $p_h = 0.80$ ,  $\xi = 0.70$ ,  $D_2 = 1$ ,  $g_h = 1.2$ ,  $g_l = 0.8$ , and  $\beta = 0.95$ .