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Central Bank Digital Currency with Heterogeneous Bank Deposits

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Abstract

This paper analyses the effects of an introduction of a retail central bank digital currency (CBDC) on bank intermediation in a tractable general equilibrium model with heterogeneous bank deposits and an imperfectly competitive loan market. The agents in the economy have preferences over holding central bank money or bank deposits that are not solely based on the assets' returns. I find that the impacts of a CBDC strongly differ depending on whether it is used only as a payment vehicle that competes with short-term transaction deposits or also as a saving vehicle that competes with longer-term saving deposits. A calibration of the model to the US economy from 1987-2006 shows that if a CBDC is only used as a payment vehicle, a 10% outflow of agents from transaction deposits to CBDC decreases bank lending by 1.2%-1.3%. The effect is almost three times stronger if a CBDC is also used as a saving vehicle in which case bank lending shrinks by about 3.0%-3.3%.

Keywords: *central bank digital currency, bank lending, new monetarism, overlapping generations*

JEL codes: E42, E50, E58

1 Introduction

The potential introduction of a retail central bank digital currency (CBDC) is currently a widely discussed topic. In many countries there is ongoing research on the topic, pilot projects are conducted and in the case of the Bahamas a retail CBDC even went live in October 2020. One concern upon the introduction of a retail CBDC is the effect it would have on financial stability and disintermediation of the banking sector. The Bank for International Settlements (2021) states in a report that *“the potential for the introduction of a CBDC to affect financial stability risks arises primarily from a significant substitution away from private money”*.

It is crucial to get a good understanding on how an introduction of a CBDC would affect bank intermediation and other macro variables. The effects may strongly differ depending on whether the agents in the economy use a CBDC only as a payment vehicle that competes with short-term transaction deposits or also as a saving vehicle that competes with longer-term saving deposits. The total amount of saving deposits in an economy is typically larger than the amount of transaction deposits used for payments. Thus, it seems crucial to differentiate what kind of deposits are crowding out when analysing the effects of a CBDC.

Furthermore, one might argue that people have heterogeneous preferences over holding central bank money or bank deposits which are not solely based on the assets' returns. There are many examples to this argument. For instance people who have a rather Libertarian mindset might object a central bank all together and prefer to hold money by the private banking sector. More risk averse people might assess the counterparty risk of holding bank deposits as high or do not trust banks in general and thus prefer to hold central bank money. Other reasons might be marketing efforts to attract depositors or the usability of central bank money. All of these examples could be modeled explicitly. However, to keep the analysis tractable and simple, I consider a demand driven coexistence of bank deposits and central bank money without explicitly arguing about the underlying reasons.

Given the two fundamental building blocks described above, the paper in hand addresses the following questions. What are the effects of an introduction of a CBDC on bank intermediation? How do the effects differ depending on whether CBDC is used only as a payment or also as a saving vehicle? How would an interest rate on CBDC affect these outcomes?

I am going to answer these questions with a tractable theoretical model and provide a quantitative assessment. All analyses in the paper are steady state comparisons, ie I do not model any transitory effects. An introduction of a CBDC is modeled as a change in the fraction of agents who prefer to hold central bank money. As long as a CBDC does not pay interest like cash, differentiating the two assets is not of relevance, since it only matters how total asset demand for central bank money or bank deposits is affected.

If there is an outflow of bank deposits due to an introduction of a CBDC, I find that bank lending declines if banks do not hold voluntary reserves. However, the banks will sacrifice some of their profit to increase the interest rates on both transaction and saving deposits. This rises the demand for deposits and counteracts partly the crowding out. The size of the effect differs depending on whether a CBDC is only used as a payment or also as a saving vehicle. If it is only used as a payment vehicle there are two counteracting effects. On the one hand, the total amount of transaction deposits decreases. On the other hand, there is a stronger demand in saving deposits because of the higher interest rate. This mitigates the negative effect. In the calibration I find that if 10% of the agents holding transaction deposits switch to CBDC, bank lending is reduced by 1.2%-1.3%. If, however, also the agents holding saving deposits switch to CBDC, the compensating effect from a higher saving deposit demand fades which intensifies the decline in bank lending. I here find a drop in bank lending of 3.0%-3.3% in the quantitative assessment.

Furthermore, I analyse how the outcome differs if the central bank starts to pay an interest rate on CBDC. Since I look at total central bank money and do not differentiate between cash and CBDC in the model, this implicitly means that this analysis considers a steady state comparison from a situation in which there is only cash to a new steady state in which agents only hold CBDC on which the central bank can pay an interest rate. An interest rate on CBDC will increase the demand for central bank money, ie some agents want to switch from bank deposits to CBDC. To counteract this outflow, the banks again react by sacrificing some of their profit to rise the interest rate on deposits. As a consequence, the negative effect on bank lending is mitigated. In the calibration I find that an interest rate on CBDC set to the Friedman rule decreases bank lending by about 1.6%. However, the effect in that case on total welfare - defined as the sum of the utilities of all agents - is very small (+0.01%)

Model

For the theoretical model, I combine the Lagos and Wright (2005) environment with a overlapping generations model based on Wallace (1980). Since I want to differentiate between assets that are used as a payment or saving vehicle, it seems straightforward to combine these two kind of models.¹ The resulting model environment is similar to Altermatt (2019).

In the model, time periods are divided into a centralized market (CM) and a decentralized market (DM). There are four types of agents - buyers, sellers, bankers and entrepreneurs - and two main asset types: central bank money and bank deposits. Buyers live for three subperiods and can only work when young. After being born, the buyers draw one of two types. One type wants to consume in the DM and the other one in the CM when old. The former one will demand a payment and the latter one a saving vehicle.

Furthermore, some buyers prefer to hold bank deposits over central bank money and vice versa. At first I assume that the asset choice is exogenous which keeps the model tractable. In that part it is always assumed that there is no interest rate paid on CBDC. Afterwards, I endogenise the asset choice with respect to the assets' interest rates. I assume that the agents' preferences are distributed as such that some prefer central bank money and some prefer bank deposits if the interest rates were equalised. However, if the interest rate differential between central bank money and bank deposits changes, some agents might want to switch their asset choice. This change allows to analyse the effects of an interest rate on CBDC, since now agents react to it and switch between central bank money and bank deposits given the size of the CBDC rate.

Entrepreneurs live for one period, have an investment opportunity but cannot work when young. Thus, they need to get a loan from a banker and pay it back when old. In that way, inside money is endogenously created. The banking sector is similar to Chiu et al. (2019). Bankers issue loans and deposits. In my model, they will want to naturally offer two kind of deposits: liquid transaction deposits and illiquid saving deposits. The former ones will be held by the buyers who want to consume in the DM and the latter ones by the buyers who want to consume in the CM when old. Furthermore, bankers are obliged to a minimum

¹Moreover, the OLG structure keeps the model simple and tractable because I do not have the problem as in more standard New Monetarist models that the agents could hold the assets for longer timer periods. In my model, there is only one decision in one time period on what amount of the asset is acquired.

reserve requirement on transaction deposits. The banking sector is perfectly competitive in the deposit market and imperfectly competitive in the loan market, which is modelled as Cournot competition.

As described above, I do not explicitly model CBDC as a distinct kind of central bank money and do not implicitly differentiate it to cash. Instead I argue that an introduction of CBDC will change the fraction of agents that want to hold central bank money. First, the results depend on whether the minimum reserve requirement constraint is binding or not. If it is non-binding and the banks hold excess reserves, then an outflow of deposits has no effect on bank intermediation. If the constraint is binding, an increase in the fraction of people who want to hold CBDC has an adverse effect on bank lending. Second, the results depend on whether CBDC is a substitute only for transaction deposits or also for saving deposits.

Calibration

To quantify the results from the theoretical model I calibrate the model to the US economy from 1987-2006. The time period is chosen because I want to consider a situation without excess reserves to match the model to the binding case. The data on banks (deposit amounts and interest rates) is derived from FDIC call report data. Other data is taken from FRED. I find that the adverse effect on bank lending is stronger for the exogenous asset choice model compared to the endogenous asset choice. In the former case, a 10% outflow in the number of agents holding transaction (and saving) deposits triggers a decline in bank lending by 1.3% (3.3%), in the latter case bank lending decreases by about 1.2% (3.0%).

Literature

The paper relates to a growing literature that examines CBDC and bank intermediation. It is most closely related to Chiu et al. (2019) who also model an imperfectly competitive banking sector. However, they model a imperfectly competitive deposit market in which CBDC serves as an outside option and thus increases competition in the deposit market. As a consequence the banks' markup is reduced and a more efficient allocation for deposits results. Since CBDC only serves as an outside option, but is not held in equilibrium, they find that bank lending can even increase. This is in contrast to my paper in which CBDC is adopted by the agents in the model, which I would argue is the more realistic assumption. Chiu et al. (2019) also model two types of deposits. However, the interest rate on the illiquid

time deposits they model is constant and hence the heterogeneous deposit component in their paper plays a minor role in contrast to my paper. Additionally, I also use a different model set up by combining the New Monetarism set up with a OLG structure.

Keister and Sanches (2022) build a theoretical model based on Lagos and Wright (2005). They model a perfectly competitive banking sector in which the banks are financially constrained (i.e. there are investment frictions). CBDC is introduced in three different ways: (i) cash-like, (ii) bank deposits-like (iii) both. They find that a cash-like CBDC always raises welfare. For a bank deposit-like CBDC the effect depends, since on the one hand output is increased in DM meetings but on the other hand there is a crowd out of bank deposits, i.e. disintermediation. A higher interest rate leads to a higher output in these meetings but decreases investment and output in the CM. If it is both, it can increase the level of exchange in all decentralized meetings. However, the similar trade-off as in the deposit-like case still exist. Welfare can either increase or decrease depending on which effect described above dominates. Furthermore, their results depend on the extent of the banks' frictions. Andolfatto (2018) assesses the impact of an interest-bearing retail CBDC on a monopoly banking sector in an overlapping generations model. He finds that in a situation where banks use their market power to keep the deposit rates depressed, CBDC has the greatest benefit since its competition increases the deposit rates and attracts more deposits. Consequently, financial disintermediation does not necessarily result, but the bank's profits might decrease. Assenmacher et al. (2021) builds a general equilibrium model with frictions such that agents demand bank deposits and CBDC. Both assets are used to pay for different kind of substitutable input goods to produce. They find that bank lending can increase if there is a high interest rate spread between CBDC and the deposit rate, a tight collateral constraint and high substitutability between the two assets. If the substitutability is low, bank lending decreases. Garratt and Zhu (2021) introduce banks with heterogeneous size. A large bank has a higher convenience value to consumers and has a result a higher market power as a small bank. Similar to Chiu et al. (2019), CBDC increases competition in the deposit market and raises interest rates especially for the large bank with the higher market power. Furthermore, Barrdear and Kumhof (2016) study the macroeconomic consequences of issuing an interest-bearing, retail CBDC which competes with endogenously created private bank-issued money in a rich DSGE model with four sectors, four lending markets and sev-

eral real and nominal rigidities. The key mechanism is that CBDC can only be bought against government bonds. They calibrate their model to pre-crisis US data and find that an issuance of 30% of GDP (against government bonds) could permanently raise GDP by 3%, lead to a consumption gain of 2.23% and an investment gain of 5.28% in steady state. Agur, Ari and Dell’Ariccia (2019) look at how CBDC affects the use of cash and deposits whose demand crucially depends on heterogeneous preferences over anonymity and security. Parlour, Walden and Rajan (2020) specifically look at the payment processing role of banks, its effect on lending activity and how the introduction of either a wholesale or retail CBDC affects lending. They find that a retail CBDC leads to an increase in lending and makes monetary policy implementation through reserves more effective. A wholesale CBDC in contrast, exacerbates lending inequalities. Lastly, Schilling, Fernández-Villaverde and Uhlig (2020) build a model based on Diamond and Dybvig (1983) with a retail CBDC and consider that a central bank cannot allocate the resources coming from the provision of CBDC as efficiently as the private sector.

The paper also relates to the New Monetarist literature with banking. Berentsen, Camera and Waller (2007) first incorporate banking into the New Monetarist literature. Altermatt and Wang (2021) study the effect of an oligopolistic banking sector in a Lagos and Wright (2005) environment.

The rest of the paper is organized as follows. Section 2 describes the model and section 3 the equilibrium. In section 4 I discuss the introduction of a CBDC in the theoretical model and in section 5 I calibrate the model to the data. In section 6 I present the model with the endogenous asset choice and in section 7 I discuss monetary policy. Finally, in section 8 I conclude the paper

2 Environment

Time is discrete and continues forever.² Each period is divided into two consecutive sub-periods, a frictionless centralized market (CM) and a frictional decentralized market (DM). Agents discount between periods with discount factor $\beta \in (0, 1)$. There is a CM good x and

²In Appendix D.1 I give a more intuitive description of the model environment and depict a timeline.

a DM good y which are both non-storable and cannot be transferred to the next subperiod.

In each period t , a new generation of buyers - a continuum with measure 1 - is born who live for three subperiods, ie they are born in the CM in period t , live throughout the DM and die at the end of the CM⁺ in $t + 1$. There are two types of buyers $\theta \in \{\theta^m, \theta^s\}$. After birth, a buyer draws its type from a Bernoulli distribution where θ equals θ^m (θ^s) with probability γ ($1 - \gamma$). The distribution and realization of θ is common knowledge. Both types can only produce in the CM when young, but do not want to consume in it. I assume that the buyers lack commitment and a credit arrangement is not possible. Thus, the buyers need a payment or saving vehicle to consume in later subperiods of life. Type θ^m wants to consume in the second subperiod after birth, ie the DM in period t , and demands a payment vehicle (medium of exchange) m whereas type θ^s wants to consume in the third subperiod after birth when old, ie the CM⁺ in $t + 1$, and demands a saving vehicle s . Young buyers produce the general good x in the CM at linear disutility h where one unit of effort h translates into one unit of good x . The lifetime utility W of a buyer with type θ is

$$W^\theta(h, x, y) = \begin{cases} -h^{\theta^m} + v(y) & \text{if } \theta = \theta^m \\ -h^{\theta^s} + \beta U(x) & \text{if } \theta = \theta^s \end{cases}$$

I assume that $v'(y) > 0, U'(x) > 0, v''(y) < 0, U''(x) < 0, v'(0) = U'(0) = \infty$ and $v'(\infty) = U'(\infty) = 0$ and $-x \frac{U''(x)}{U'(x)} < 1$ for all $x \geq 0$ and $-y \frac{v''(y)}{v'(y)} < 1$ for all $y \geq 0$.

Furthermore, there is a continuum of infinitely lived sellers with measure γ . The sellers get linear utility from consuming x in the CM but cannot produce in it. Furthermore, they do not want to consume in the DM but can produce the DM good y with linear disutility. Thus, the preference of the sellers is given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [x_t - y_t].$$

In the DM, buyers want to purchase the DM good y from the sellers with the payment vehicle m . They have a bilateral trade with probability one in which the buyer makes a take-it-or-leave-it offer.³

³Assuming a take-it-or-leave-it offer keeps the DM problem very simple which is beneficial for overall tractability. If the seller had some market power, the lifetime buyer problem would be more complicated as

Moreover, there exist two other types of agents: bankers and entrepreneurs. There is a unit mass of one-period lived entrepreneurs born each period with an investment opportunity. They can invest the CM good x and get a return $f(x)$ in the subsequent CM⁺. I assume that the production function is of the form $f(x) = Ax^\eta$ where $A > 0$ and $\eta \in (0, 1)$.⁴ An entrepreneur cannot work and has no endowment. Furthermore, there are $B \in \mathbb{N}$ one-period lived bankers. They own a costless record keeping technology and take deposits and issue loans. A bank can offer heterogeneous deposit accounts in the sense that they have different liquidity characteristics. Specifically, it can offer liquid transaction deposits d that act as a payment vehicle, illiquid saving deposits τ that act as a saving vehicle or only one kind of deposits. Transaction deposits can be transferred to other bankers in the DM and also be carried to the subsequent CM. Saving deposits cannot be transferred in the DM but only be used in the subsequent CM. The nominal net interest rate the bank pays on the transaction deposits d is denoted as i_d , the one it pays on the saving deposits τ as i_τ . It is paid to the agent who holds the deposits between periods. Furthermore, the bank has a handling cost $c \geq 0$ per unit of deposit.⁵

To buy the CM good, a young entrepreneur first gets a loan from a bank and will then buy the good x with the deposits the bank credited to the entrepreneur's account. In this process, bank deposits are created endogenously. I assume that a bank cannot transfer its loans to another bank, bankruptcy yields a utility of minus infinity, bankers compete for loans via Cournot competition and that there is perfect competition in the deposit market⁶.

Furthermore, agents can hold a second asset provided by a monetary authority, ie central well.

⁴Assuming this specific function is necessary to proof certain results, however it does not restrict the model considerably.

⁵Eg provision of the payment infrastructure, interacting with depositors etc. I introduce this cost because in the historical data we see that the interest rate on deposits was often lower than the federal funds rate. The model cannot replicate this property without the handling cost. An alternative would be to introduce imperfect competition in the deposit market. However, this would make the model much less tractable.

⁶The main reason for this modeling choice is tractability. Given the two different types of deposits, the model remains tractable due to the perfect competition in the deposit market. Important for my results is that the bankers have any form of market power. It is not key in which market this is. However, I could also argue why this assumption seems reasonable in the model setup. Buyers receiving the deposits from the firms for selling the CM good x could immediately switch to another bank if it offers a slightly higher return. Since the bank cannot transfer its loan contracts, the bank would thus lose its funding and inevitably go bankrupt which yields a utility of minus infinity. This justifies the assumption of perfect competition in the deposit market. On the other hand, the bankers compete with Cournot competition in the loan market. Not issuing any loan yields a utility of 0. This is clearly different to the competition in the deposit market. I thus assume that the finite number of bankers will use their market power to get a positive profit since they strictly prefer this case to the outside option.

bank money. First, bankers hold reserves at the central bank. The central bank sets a reserve requirement $\omega \in [0, 1]$ on liquid transaction deposits d .⁷ I assume that if a bank does not meet the reserve requirement, the government fines the bank such that it is always more profitable to meet the requirement. The real holdings of reserves a bank b holds are denoted by e_b . The nominal stock in period t of reserves is denoted by E_b . The central bank can set a nominal interest rate on reserves (IOR) denoted as i_{e_b} .⁸ Second, buyers and sellers can decide to hold another form of central bank money. This can either be cash or later CBDC. The central bank can pay a nominal interest rate on CBDC denoted as i_e . The total real holdings of central bank money is denoted by e^m and e^s for the θ^m -type and θ^s -type respectively.⁹ Note that bankers cannot hold cash or CBDC in the model. Relaxing this assumption would, however, not change the results considerably. The total nominal stock in period t of central bank money, including the bankers' holdings, is denoted by E . The stock of central bank money grows each period by $E/E^- = \mu$ through injections of the central bank, ie lump-sum transfers to the sellers at the beginning of the CM. E^- denotes the stock of central bank money in $t - 1$ and E^+ in $t + 1$. The amount of CM goods that one unit of central bank money can buy in period t is denoted by ϕ_e . The inflation rate is denoted as $\phi_e/\phi_e^+ - 1 = \pi^+$. To finance the interest payments, the central bank levies a lump-sum tax on the sellers. The difference between the transfer from money creation and the tax is denoted as T . The monetary authority's budget constraint is

$$\phi_e[E - E^-] + T = \left[\sum_{b=1}^B e_b \right] i_{e_b}/\mu + [e^m + e^s] i_e/\mu. \quad (1)$$

Additionally, buyers have a preference over the payment and saving vehicle. For now, I assume that this materializes as such that an exogenous fraction $\alpha_d \in (0, 1)$ of the θ^m -buyer prefers holding transaction deposits over central bank money as a payment vehicle and that an exogenous fraction $\alpha_\tau \in (0, 1)$ of the θ^s -buyer prefers holding saving deposits over central bank money as a saving vehicle. I will later relax that and will endogenise α_d and α_τ w.r.t. the interest rates of the different assets. However for now, holding the fractions fixed makes

⁷There is no endogenous reason in the model why the central bank implements the reserve requirement. I just assume that the central bank decides to do so.

⁸A bank would build its reserve position by receiving real goods from buyers in the centralized market in return for deposits and then giving them to the central bank in return for reserves.

⁹Agents would acquire central bank money by producing the CM good x and selling it to the central bank in the centralized market in return for the nominal asset.

the model more tractable. The fraction of θ^m (θ^s) buyers that hold central bank money as payment vehicle (saving vehicle) then is $1 - \alpha_d$ ($1 - \alpha_\tau$).

Henceforth, I restrict the analysis to a stationary equilibrium and assume that bankers will always meet their promise to pay out central bank money on demand. This implies that $\phi = \phi_d = \phi_\tau = \phi_e$ and $\mu = \phi/\phi^+$. Furthermore, the Fisher equation defines the real return of the assets, ie $(1 + i) = \mu R$ where μ is the inflation rate and R the gross real interest rate.

2.1 The Bank's Problem

A banker $b \in \{1, 2, \dots, B\}$ is one-period lived, issues deposits and makes loans. Before dying, a banker uses the profit to buy the CM good x and consumes it with linear utility. Furthermore, the banker knows that there are two types of agents with different asset demands. The θ^m -type only wants to have a payment vehicle that can be spent in the DM and the θ^s -type demands a saving vehicle that can be transferred to the next period. Furthermore, the bank has to fulfill the reserve requirement only on the liquid transaction deposits. Thus it has to decide whether it wants to offer only liquid transaction deposits, illiquid saving deposits or both. This is defined in the following proposition.

Proposition 1. *Given the two types of agents $\theta \in \{\theta^m, \theta^s\}$ whose distribution and realisation is common knowledge, a reserve requirement constraint on liquid deposits that is binding and a production function of the form $f(x) = Ax^n$, the bankers will offer two kind of deposits, ie liquid transaction deposits d and illiquid saving deposits τ .*

Proof. A formal proof is available in the appendix A.1. The intuition is as following: A bank offering only transaction deposits holds unnecessary reserves on all deposits of the θ^s -types and can increase its profit by offering saving deposits which it can invest one-to-one into loans.¹⁰ Furthermore, a bank could only offer illiquid saving deposits to the θ^s -types. In this case, however, a bank can increase its profit by offering liquid transaction deposits to the θ^m types and get some profit on the part it invests into loans. ■

Since the realization of type θ is public knowledge, a bank can perfectly discriminate the

¹⁰Note that this problem is not trivial, because when offering more loans, the interest rate on all loans decreases due to Cournot competition in the loan market. Thus a bank can only increase its profit if the gain on the additional loans is higher than the loss on all the existing loans.

θ^m -type into transaction deposits and the θ^s -type into saving deposits.

I now analyse the banker's problem given that it offers two kind of deposits. A bank b wants to maximize its profit Π_b which can be consumed next period before dying.

$$\begin{aligned} \max_{\ell_b, e_b, d_b, \tau_b} \quad & \Pi_b = \ell_b R_\ell(\ell) + e_b R_{e_b} - d_b R_d - d_b c - \tau_b R_\tau - \tau_b c \\ \text{s.t.} \quad & e_b \geq \omega d_b \\ & \ell_b + e_b = d_b + \tau_b. \end{aligned}$$

where $\ell = \ell_b + \sum_{b \neq b'} \ell_{b'}$. On the asset side, the bank issues loans ℓ_b with gross real interest rate $R_\ell(\ell)$ and holds reserves that pays gross real interest R_{e_b} . On the liability side, the banker issues transaction deposits d_b on which it pays a gross real interest rate R_d and saving deposits τ_b on which it pays a gross real interest rate R_τ . Furthermore, the banker subtracts the deposit handling cost c per unit of deposits. The first constraint describes the reserve requirement, ie the bank has to hold a minimum amount of the liquid transaction deposits as reserves. The second constraint covers the balance sheet identity.

By assumption there is always some demand for deposits and hence we have that $d_b > 0$ and $\tau_b > 0$. By using the balance sheet identity to replace e_b in the maximization problem, I can formulate a Lagrangian.

$$\begin{aligned} \mathcal{L}(\ell_b, d_b, \tau_b) = & (R_\ell(\ell) - R_{e_b})\ell_b - (R_d + c - R_{e_b})d_b \\ & - (R_\tau + c - R_{e_b})\tau_b + \lambda(d_b(1 - \omega) + \tau_b - \ell_b) \end{aligned}$$

This yields the first order conditions in equation (2)-(4) and the complementary slackness condition in (5).

$$\frac{\partial R_\ell(\ell)}{\partial \ell_b} \ell_b + R_\ell(\ell) - R_{e_b} = \lambda \quad (2)$$

$$R_{e_b} + \lambda(1 - \omega) - c = R_d \quad (3)$$

$$R_{e_b} + \lambda - c = R_\tau \quad (4)$$

$$\lambda(d_b(1 - \omega) + \tau_b - \ell_b) = 0 \quad (5)$$

Due to the properties of the Lagrangian we have $\lambda \geq 0$. The realization of λ can be

interpreted as the additional profit a bank would make if the minimum reserve requirement constraint is relaxed. If $\omega = 0$, this can be seen as the additional profit a bank would make if it could borrow from the monetary authority at rate R_{e_b} .¹¹ Whenever $\lambda > 0$, the bank issued a loan amount that is below the bank's profit-maximizing amount.

From equations (2)-(4) the following result can be derived.

Proposition 2. *Given an imperfectly competitive loan market, ie. $B < \infty$, the following holds. For $c = 0$, a binding reserve requirement constraint, ie $\lambda > 0$, yields $R_\ell > R_\tau > R_d > R_{e_b}$. A non-binding reserve requirement constraint, ie $\lambda = 0$, yields $R_\ell > R_\tau = R_d = R_{e_b}$ and the bank pays the same return on liquid transaction and illiquid saving deposits. For $c > 0$ and $\lambda > 0$, it follows that $R_\ell > R_{e_b}$ and $R_\ell > R_\tau > R_d$. Depending on the size of c , it is possible that either (i) $R_\tau > R_d \geq R_{e_b}$, (ii) $R_\tau \geq R_{e_b} > R_d$, (iii) $R_\tau > R_{e_b} \geq R_d$ or (iv) $R_{e_b} \geq R_\tau > R_d$. For $c > 0$ and $\lambda = 0$, it follows that either $R_\ell > R_\tau = R_d \geq R_{e_b}$ or $R_\ell > R_{e_b} \geq R_\tau = R_d$. The bank always makes a profit $\Pi_b > 0$ since there always is a positive demand for deposits by assumption.*

Proof. A proof can be found in Appendix A.2. ■

Using the Fisher equation and the assumption that the price of deposits is the same as the price for central bank money, proposition 2 implies in turn that the therein mentioned relationships hold for nominal interest rates i as well. In the non-constraint case, the banker holds excess reserves and the deposits become basically identical. Nevertheless, I will also in that case differentiate the deposits for the sake of clarity. Lastly I assume a symmetric equilibrium in the banking sector, ie $\ell = B\ell_b$, $d = Bd_b$ and $\tau = B\tau_b$.

2.2 The Buyer's Problem

Next I analyse a buyer's maximization problem. A θ^m (θ^s) type buyer has to decide how much to work in the CM when young, in order to acquire either central bank money or transaction (saving) deposits. The value function when young of buyer j that has type

¹¹To continue this line of argument, $\omega < 0$ would mean that the bank is allowed to borrow from the monetary authority up to ωd_b .

$\theta \in \{\theta^m, \theta^s\}$ is

$$\begin{aligned}
W_j^{\theta^m} &= \max_{h_j^{\theta^m}, m_j} \{-h_j^{\theta^m} + v \circ y_j(m_j, i_m, \phi_m)\} \\
&\text{s.t. } m_j = h_j^{\theta^m} \\
W_j^{\theta^s} &= \max_{h_j^{\theta^s}, s_j} \{-h_j^{\theta^s} + \beta U \circ x_j^+(s_j, i_s, \phi_s)\} \\
&\text{s.t. } s_j = h_j^{\theta^s}.
\end{aligned}$$

h_j^θ denotes how much a young buyer works or equally how much goods they produce in the first CM. $m_j \in \{d_j, e_j^m\}$ is the amount of the payment vehicle a θ^m -type chooses and $s_j \in \{\tau_j, e_j^s\}$ is the saving vehicle a θ^s type chooses in real terms. ϕ_m (ϕ_s) and i_m (i_s) is the price and the nominal interest rate of the payment vehicle (saving vehicle) respectively.

In the DM, a θ^m type buyer meets a seller with probability one and makes a take-it-or-leave-it-offer to the seller. Hence, the seller's problem is trivial and the buyer just ensures when choosing an offer that the seller's participation constraint holds with equality. The seller can work in the DM with linear disutility to produce good y and gets the payment vehicle m when selling it. The seller gets interest i_m on his received holdings of m in the subsequent CM in which m buys ϕ_m^+ units of the CM good which the seller then consumes with linear utility. Furthermore, the tax/transfer T is always levied on the seller. Hence, the seller's participation constraint is $-y_j + \beta\phi_m^+(1 + i_m)m_j/\phi_m - T \geq -T$. A θ^s -type buyer on the other hand just holds onto his saving vehicle s , gets interest i_s on it in the subsequent CM where s buys ϕ^+ units of the CM good. This yields the following value functions

$$\begin{aligned}
W_j^{\theta^m} &= \max_{h_j^{\theta^m}, m_j} -h_j^{\theta^m} + v(\beta\phi_m^+(1 + i_m)m_j/\phi_m) \\
&\text{s.t. } m_j = h_j^{\theta^m} \\
W_j^{\theta^s} &= \max_{h_j^{\theta^s}, s_j} -h_j^{\theta^s} + \beta U(\phi_s^+(1 + i_s)s_j/\phi_s) \\
&\text{s.t. } s_j = h_j^{\theta^s}.
\end{aligned}$$

The following first order conditions for each type result

$$\begin{aligned}\theta^m : \quad & v' \left(\beta \phi_m^+ (1 + i_m) m_j / \phi_m \right) \beta (1 + i_m) = \phi_m / \phi_m^+ \\ \theta^s : \quad & \beta U' \left(\phi_s^+ (1 + i_s) s_j / \phi_s \right) (1 + i_s) = \phi_s / \phi_s^+.\end{aligned}$$

Rearranging the FOCs yields explicit demand functions for m_j and s_j

$$\begin{aligned}m_j &= v'^{-1} \left(\frac{\phi_m / \phi_m^+}{\beta (1 + i_m)} \right) \frac{\phi_m / \phi_m^+}{\beta (1 + i_m)} \\ s_j &= U'^{-1} \left(\frac{\phi_s / \phi_s^+}{\beta (1 + i_s)} \right) \frac{\phi_s / \phi_s^+}{1 + i_s}.\end{aligned}$$

To get the aggregate demand functions I need to take into account with which probability the buyer ends up as a (i) θ^m -type buyer holding transaction deposits d , (ii) θ^m -type buyer holding central bank money e^m , (iii) θ^s -type buyer holding saving deposits τ , (iv) θ^s -type buyer holding central bank money e^s . The respective probabilities are (i) $\gamma \alpha_d$ (ii) $\gamma(1 - \alpha_d)$, (iii) $(1 - \gamma) \alpha_\tau$ and (iv) $(1 - \gamma)(1 - \alpha_\tau)$. Furthermore, I apply the Fisher equation as defined above. Therefore, I get the following demand functions for the different assets.

$$d = \gamma \alpha_d v'^{-1} \left(\frac{1}{\beta R_d} \right) \frac{1}{\beta R_d} \quad (6)$$

$$e^m = \gamma(1 - \alpha_d) v'^{-1} \left(\frac{1}{\beta R_e} \right) \frac{1}{\beta R_e} \quad (7)$$

$$\tau = (1 - \gamma) \alpha_\tau U'^{-1} \left(\frac{1}{\beta R_\tau} \right) \frac{1}{R_\tau} \quad (8)$$

$$e^s = (1 - \gamma)(1 - \alpha_\tau) U'^{-1} \left(\frac{1}{\beta R_e} \right) \frac{1}{R_e} \quad (9)$$

d denotes the real amount of transaction deposits and e^m the real amount of central bank money that is demanded by θ^m -type buyers whereas τ denotes the real amount of saving deposits and e^s the real amount of central bank money that is demanded by θ^s -type buyers

2.3 The Entrepreneur's Problem

Since entrepreneurs can neither work nor do they have an endowment, they have to borrow from a bank to purchase the CM good x . An entrepreneur demands a nominal loan of size ℓ_n / ϕ with an interest i_ℓ in the loan market. ℓ_n denotes the size of the loan in real terms. The

entrepreneur buys the CM good in period t in the centralized market and invests it. After one period, the investment returns $f(x)$. A part of the return is sold in the subsequent CM to acquire bank deposits and pay back the loan. The entrepreneur consumes the remains with linear utility and then dies. Thus, an entrepreneur wants to maximize the amount of goods consumed in the CM in $t + 1$ and the maximization problem is

$$\max_{\ell_n} \beta f(\ell_n) - \beta \phi^+ \ell_n (1 + i_\ell) / \phi$$

which yields

$$f'(\ell_n) = \frac{1 + i_\ell}{\phi / \phi^+}.$$

I assume a symmetric equilibrium, hence the size of the loan is the same for all entrepreneurs and consequently $\ell = \ell_n$. Furthermore, applying the Fisher equation yields

$$R_\ell(\ell) = f'(\ell) \tag{10}$$

where $R_\ell(\ell)$ is the gross real return on loans. (10) determines the demand for loans.

3 Equilibrium

3.1 Social Planner's Problem

Along the lines of Altermatt and Wipf (2020), I focus on maximizing the utility of a representative generation for the planner's problem. I hereby ignore the initial old. Furthermore, I do not include the bankers in the analysis, even though they get some utility from the profit they make. This is because bankers only provide intermediation with nominal assets which is not crucial to the planner's problem. I assume that the planner weights the utilities of the different agents equally.

The planner maximizes the utility of a representative generation g that is born in period t .

$$V_t^g = \gamma[-h_{j,t}^{\theta^m} + v(y_{j,t}^{\theta^m})] + (1 - \gamma)[-h_{j,t}^{\theta^s} + \beta U(x_{j,t+1}^{\theta^s})] + \gamma[x_{j,t}^s - y_{j,t}^s] + \beta x_{j,t+1}^c \tag{11}$$

The first term reflects the utility of the θ^m buyers with mass γ , who are working when young in the CM of period t and consume good y in the DM of period t . The second term is the utility of the θ^s buyers with mass $(1 - \gamma)$ who are working when young in the CM of period t and consume when old in period $t + 1$. The third term is the utility of the seller who consumes x in the CM and works in the DM to produce y . The last term reflects the consumption of the entrepreneurs who consume x^c with linear utility in period $t + 1$.

The DM consumption of the buyer has to be financed by direct transfers from the sellers. The CM good x is produced by young buyers and by the entrepreneurs. It is consumed by the old θ^s buyers, sellers, old entrepreneurs and used by entrepreneurs for the investment denoted by x^e . This yields the following market clearing conditions.

$$\begin{aligned}\gamma y_{j,t}^s &\geq \gamma y_{j,t}^{\theta^m} & (\mu_{1,t}) \\ \gamma h_{j,t}^{\theta^m} + (1 - \gamma)h_{j,t}^{\theta^s} + f(x_{j,t-1}^e) &\geq (1 - \gamma)x_{j,t}^{\theta^s} + x_{j,t}^e + \gamma x_{j,t}^s + x_{j,t}^c & (\mu_{2,t}).\end{aligned}$$

The social planner's problem can be defined by the following Lagrangian

$$\begin{aligned}\mathcal{L}(h_{j,t}^{\theta^m}, y_{j,t}^{\theta^m}, h_{j,t}^{\theta^s}, x_{j,t}^s, y_{j,t}^s, x_{j,t+1}^{\theta^s}, x_{j,t+1}^p, x_{j,t}^e) = & \\ h_{j,t}^{\theta^m} + v(y_{j,t}^{\theta^m}) - h_{j,t}^{\theta^s} + x_{j,t}^s - y_{j,t}^s + \beta U(x_{j,t+1}^{\theta^s}) + \beta x_{j,t+1}^c & \\ + \mu_{1,t}[y_{j,t}^s - y_{j,t}^{\theta^m}] & \\ + \mu_{2,t}[\gamma h_{j,t}^{\theta^m} + (1 - \gamma)h_{j,t}^{\theta^s} + f(x_{j,t-1}^e) - (1 - \gamma)x_{j,t}^{\theta^s} - x_{j,t}^e - \gamma x_{j,t}^s - x_{j,t}^c] & \\ + \mu_{2,t+1}[\gamma h_{j,t+1}^{\theta^m} + (1 - \gamma)h_{j,t+1}^{\theta^s} + f(x_{j,t}^e) - (1 - \gamma)x_{j,t+1}^{\theta^s} - x_{j,t+1}^e - \gamma x_{j,t+1}^s - x_{j,t+1}^c] &\end{aligned}$$

which yields the optimality conditions

$$v'(y_j^{\theta^m}) = 1 \tag{12}$$

$$U'(x_j^{\theta^s}) = 1 \tag{13}$$

$$f'(x^e) = 1/\beta. \tag{14}$$

This defines $y_j^{\theta^m}$, $x_j^{\theta^s}$, x^e and y_j^s using the market clearing condition for the DM. The other variables $h_j^{\theta^m}$, $h_j^{\theta^s}$, x_j^s and x_j^c are not distinctly determined by the planner's problem. Since

the seller and the old entrepreneur consume x with linear utility and the young buyers can produce x with linear cost, these variables have no direct effect on welfare.

Using the specific functional form of the production function $f'(x) = \eta Ax^{\eta-1}$ and plugging it into the optimality condition of the entrepreneur yields

$$\begin{aligned} 1/\beta &= \eta A(x^e)^{\eta-1}, \\ x^e &= (\beta\eta A)^{\frac{1}{1-\eta}} \end{aligned}$$

and

$$f(x^e) = A(\beta\eta A)^{\frac{\eta}{1-\eta}}.$$

It is unclear who produces how much of x in the CM. As said welfare relevant is only x^e and x^{θ^s} . If $f(x^e) \geq x^e + x^{\theta^s}$, then the entrepreneur produces everything. Again using the specific functional form of the production function we get

$$\begin{aligned} f(x^e) &\geq x^e + (1 - \gamma)x_j^{\theta^s} \\ A(\beta\eta A)^{\frac{\eta}{1-\eta}} &\geq (\beta\eta A)^{\frac{1}{1-\eta}} + (1 - \gamma)U'^{-1}(1) \\ A(\beta\eta A)^{\frac{\eta}{1-\eta}} - (\beta\eta A)^{\frac{1}{1-\eta}} &\geq (1 - \gamma)U'^{-1}(1) \end{aligned}$$

This is more likely to hold if A is large (the entrepreneur is very productive) or γ is large (there are only few θ^s types who want to consume x). If the inequality above does not hold, then also the young buyers need to produce good x in the CM. The total labour supply $H = \gamma h_j^{\theta^m} + (1 - \gamma)h_j^{\theta^s}$ of young buyers in the socially optimal allocation is thus defined by

$$\begin{aligned} H &= \max\{0, (1 - \gamma)x_j^{\theta^s} + x^e - f(x^e)\} \\ &= \max\{0, (1 - \gamma)U'^{-1}(1) + (\beta\eta A)^{\frac{1}{1-\eta}} - A(\beta\eta A)^{\frac{\eta}{1-\eta}}\} \end{aligned}$$

3.2 Market Outcomes

I restrict the results to a stationary equilibrium.

Definition 1. A stationary and symmetric equilibrium is defined such that the quantity of transaction deposits $d = Bd_b$, the quantity of saving deposits $\tau = B\tau_b$, the quantity of bank loans $\ell = B\ell_b$, the quantity of reserves held by bankers Be_b , the quantities of CBDC held by the households e^m and e^s , a transaction deposit rate R_d , a saving deposit rate R_τ and a loan rate R_ℓ solve

- (1) the asset demand equations (6)-(9),
- (2) the entrepreneurs' loan demand (10),
- (3) the bankers' first order conditions (2)-(5).

The equilibrium outcomes crucially depend on whether the reserve requirement constraint is binding or not. Thus the first goal is to get some insight over the parameter space in which the constraint is more likely to bind. Note that the bank's first FOC, ie equation (2), can be rearranged by using the functional form of the production function (see Appendix B.1 for a derivation).

$$\lambda = \max \left\{ 0, \eta A \ell^{\eta-1} \left(\frac{\eta-1}{B} + 1 \right) - R_{e_b} \right\} \quad (15)$$

Furthermore, using the optimality conditions found above, we get equation (16) that implicitly defines λ (see Appendix B.2 for a derivation).

$$\lambda = \max \left\{ 0, \eta A \left[\gamma \alpha_d v'^{-1} \left(\frac{1}{\beta(R_{e_b} + \lambda(1-\omega) - c)} \right) \frac{1}{\beta(R_{e_b} + \lambda(1-\omega) - c)} (1-\omega) + (1-\gamma) \alpha_\tau U'^{-1} \left(\frac{1}{\beta(R_{e_b} + \lambda - c)} \right) \frac{1}{(R_{e_b} + \lambda - c)} \right]^{\eta-1} \left(\frac{\eta-1}{B} + 1 \right) - R_{e_b} \right\} \quad (16)$$

Since λ is the only endogenous variable, it is possible to describe the effects of the exogenous variables on λ . This is formulated in proposition 3.

Proposition 3. *The reserve requirement constraint is more likely to be binding (ie $\lambda > 0$) in the productivity parameter A , the number of bankers B and the reserve requirement rate ω (ie $\frac{\partial \lambda}{\partial A} > 0$, $\frac{\partial \lambda}{\partial B} > 0$, $\frac{\partial \lambda}{\partial \omega} > 0$). It is more likely to be non-binding (ie $\lambda = 0$) in the discount factor β , the shares of buyers holding deposits α_d and α_τ and in the interest rate on reserves R_{e_b} (ie $\frac{\partial \lambda}{\partial \beta} < 0$, $\frac{\partial \lambda}{\partial \alpha_d} < 0$, $\frac{\partial \lambda}{\partial \alpha_\tau} < 0$, $\frac{\partial \lambda}{\partial R_{e_b}} < 0$). The effect of the concavity parameter η in the production function and share of θ^m -types γ is ambiguous (ie $\frac{\partial \lambda}{\partial \eta} \leq 0$, $\frac{\partial \lambda}{\partial \gamma} \leq 0$).*

Proof. A proof can be found in Appendix A.3. ■

A higher A makes the firms more productive. Thus, the resulting interest rate on bank loans will be higher such that the bankers have an incentive to hand out relatively more loans than holding reserves. Therefore it is more likely that the reserve requirement constraint is binding. For B assume that reserve requirement constraint is non-binding. If the number of bankers is increased, the total loan amount will increase due to stronger competition. As a consequence, a bank will reduce its excess reserves and the probability that the constraint becomes binding increases. Furthermore, it is obvious that the reserve requirement constraint is more likely to bind if the minimum reserve requirement ratio is higher. If the discount factor β is higher, agents are more patient and want to save more. The demand for deposits increases which in turn increases ℓ and decreases R_ℓ . A lower interest rate on loans makes it relatively more attractive to hold reserves and thus the constraint is less likely to bind. The same is true for α_d and α_τ since a higher share of deposit holders means more deposits. Lastly, a higher interest rate on reserves R_{e_b} makes it more attractive for a bank to hold reserves compared to loans which in turn means that it is more likely that a bank will hold voluntary reserves and the constraint will not bind.

Given the implicit solution for λ in equation (16), the remaining parts of the model can be solved analytically in closed form solutions. The equilibrium is defined by the following equations (see Appendix B.3 for a derivation of equations 23 and 24).

$$R_d = R_{e_b} + \lambda(1 - \omega) - c \quad (17)$$

$$R_\tau = R_{e_b} + \lambda - c \quad (18)$$

$$d = \gamma \alpha_d v'^{-1} \left(\frac{1}{\beta R_d} \right) \frac{1}{\beta R_d} \quad (19)$$

$$\tau = (1 - \gamma) \alpha_\tau U'^{-1} \left(\frac{1}{\beta R_\tau} \right) \frac{1}{R_\tau} \quad (20)$$

$$e^m = \gamma(1 - \alpha_d) v'^{-1} \left(\frac{1}{\beta R_e} \right) \frac{1}{\beta R_e} \quad (21)$$

$$e^s = (1 - \gamma)(1 - \alpha_\tau) U'^{-1} \left(\frac{1}{\beta R_e} \right) \frac{1}{R_e} \quad (22)$$

$$\ell = \left[\frac{A\eta(1 - \frac{1-\eta}{B})}{R_{e_b} + \lambda} \right]^{1/(1-\eta)} \quad (23)$$

$$R_\ell = \frac{R_{e_b} + \lambda}{1 - \frac{1-\eta}{B}} \quad (24)$$

Note that if the reserve requirement constraint is non-binding ($\lambda = 0$), the total loan amount ℓ and the interest rate on loans R_ℓ do not depend on the demand for deposits, because the bank has enough funds to reach the optimal loan amount and puts additional funding into excess reserves. Furthermore, in this case the interest rate on reserves is one-to-one transmitted to the deposit interest rates. Thus the deposit demand directly depends on the IOR.

3.3 Market Outcomes and Optimality

Let's compare the market outcome to the optimal allocation from the planner's problem. For the θ^m -type we want $v'(y_j^{\theta^m}) = 1$ to hold and we know that $y_j^{\theta^m} = \beta R_m m_j$. This translates to

$$v'(\beta R_m m_j) = v' \left(\beta R_m v'^{-1} \left(\frac{1}{\beta R_m} \right) \frac{1}{\beta R_m} \right) = 1$$

and only holds if $R_d = R_e = 1/\beta$. Analogously, for the θ^s -type who holds deposits we want $U'(x_j^{\theta^s}) = 1$ to hold and we know that $x_j^{\theta^s} = R_s s_j$ which yields

$$U'(R_s s_j) = U' \left(R_s U'^{-1} \left(\frac{1}{\beta R_s} \right) \frac{1}{R_s} \right) = 1$$

and again only holds if $R_\tau = R_e = 1/\beta$.

In the loan market, we know from the planner's problem that optimality requires $f'(x) = 1/\beta$ and since $f'(x) = R_\ell$ this means $R_\ell = 1/\beta$. Thus, the socially optimal allocation can only be reached at the Friedman rule, ie

$$R_e = R_d = R_\tau = R_\ell = 1/\beta$$

Is it possible to get this in a market outcome? Obviously not if there is imperfect competition in the loan market, ie $B < \infty$, because it is always true that $R_\ell > R_\tau \geq R_d$ (see proposition 2). However, the monetary authority can either reach the optimal allocation in the loan market or in the deposit demand.

In the loan market, it can set¹²

$$R_{e_b} = \left(1 - \frac{1 - \eta}{B}\right) / \beta - \lambda. \quad (25)$$

In that case, it follows that $R_d < 1/\beta$ and $R_\tau < 1/\beta$ and thus there is not enough deposit demand.

The monetary authority can reach the optimal allocation for deposit demand in the unconstrained case and in the constrained case if it abolishes the minimum reserve requirement constraint^{13, 14}.

$$R_{e_b} = \begin{cases} 1/\beta + c & \text{if } \lambda = 0, \\ 1/\beta + c - \lambda \text{ and } \omega = 0 & \text{if } \lambda > 0 \end{cases}$$

For central bank money demand, the monetary authority can always reach the optimal allocation by applying the Friedman rule, ie

$$R_e = 1/\beta. \quad (26)$$

If the choice of the agent over holding central bank money or deposits is exogenous, then there is no negative effect for the central bank of setting the optimal rate. However, if the asset choice is endogenous (see section 6), a higher interest rate on central bank money would trigger some agents to switch from deposits to CBDC which would in turn reduce bank lending.

Lastly, if there was perfect competition in the loan market, the loan interest rate would become

$$R_\ell = \lim_{B \rightarrow \infty} \frac{R_{e_b} + \lambda}{1 - \frac{1 - \eta}{B}} = R_{e_b} + \lambda.$$

Thus, the monetary authority could reach optimality by abolishing the minimum reserve

¹²Plug $R_\ell = 1/\beta$ into equation (24) and rearrange for R_{e_b} .

¹³Note that my model abstracts from many reasons why a monetary authority would want to introduce a minimum reserve requirement, as for example risky assets. Thus this result should be taken with caution.

¹⁴This follows from equations (17) and (18).

requirement, setting $R_e = 1/\beta$ and

$$R_{e_b} = \begin{cases} 1/\beta & \text{if } \lambda = 0 \text{ and } c = 0, \\ 1/\beta - \lambda \text{ and } \omega = 0 & \text{if } \lambda > 0 \text{ and } c = 0 \end{cases}$$

4 Introducing CBDC

Next I model an introduction of CBDC. I assume that the introduction of CBDC will change the fraction of people that hold central bank money. For example more people may want to hold central bank money because they want to pay digitally which was not possible so far with central bank money. Thus, I am interested in analysing what effect an increase in α_d or α_τ has on bank lending.

4.1 A Slack Minimum Reserve Requirement Constraint

For the non-binding case ($\lambda = 0$) it is straightforward.

Proposition 4. *If the reserve requirement constraint is non binding ($\lambda = 0$), an increase in the fraction of buyers who hold CBDC (i) has no effect on interest rates R_d and R_τ , (ii) decreases the amount of deposits held (iii) increases CBDC holdings, (iv) has neither an effect on loans ℓ nor on the loan interest rate R_ℓ (v) reduces the excess reserves held by bankers by the same amount as deposits are withdrawn (vi) increases total asset demand and GDP if the interest rate on deposit is smaller than the interest rate on CBDC and (vii) has no effect on the entrepreneurs' and bankers' profit.*

Proof. A proof can be found in Appendix A.4. ■

Let's discuss the intuition behind proposition 4. The line of argument is qualitatively the same for a decrease in α_d or α_τ , thus I choose to mainly discuss a shift in α_d here. However, note that quantitatively the results might vary quite a lot, which is what I consider in section 5.

If the minimum reserve requirement constraint is non-binding, then the bankers hold excess reserves. They only do so, because there is enough demand for deposits and they can thus

reach the optimal loan amount. Thus, as long as a bank holds excess reserves, an outflow of deposits will trigger the bank to reduce its excess reserves by the same amount. Note that from proposition 2 it follows that in the non-binding case we have $R_{e_b} = R_d + c = R_r + c$. Therefore, the interest costs on reserves and the total costs on issuing deposits are equalized and a reduction in both by the same amount does not affect the bankers' profit. As a consequence the bank has no incentive to change interest rates to attract more depositors. Furthermore, the bank would only change the optimal loan amount when the reserve requirement constraint becomes binding and it needs to shift some funds from loans to reserves. Consequently, there neither is an effect on the entrepreneur's profit.

Concerning total asset demand, it is clear that an outflow of transaction deposits to CBDC will decrease d and increase e^m . Which change is stronger depends on the relative size of R_d to R_e . If $R_d > R_e$ then a buyer who switches will hold less CBDC than he held deposits before due the lower interest rate and thus total asset demand will decrease.

With respect to GDP, which is denoted as Y and consists of total CM and DM production, the line of argument is similar. If $R_d > R_e$, a buyer switching from deposits to CBDC will produce less in the CM, since his asset demand is lower. This effect translates to DM production in which the seller produces less in the match because the buyer arrives with less assets. Output from the entrepreneurs is unchanged, since the total loan amount is not affected by a shift in α_d . This yields

$$\frac{\partial(d + \tau + e^m + e^s)}{\partial\alpha_d}, \frac{\partial Y}{\partial\alpha_d} \begin{cases} > 0 & \text{if } R_d > R_e \\ = 0 & \text{if } R_d = R_e \\ < 0 & \text{if } R_d < R_e \end{cases}$$

Directly related to asset demand is consumption of an individual buyer. Consumption C of a buyer j that holds either transaction deposits d or CBDC as a payment instrument e^m is defined by

$$C_j^d = \beta R_d d_j = \beta R_d (\beta R_d)^{\frac{1-\sigma}{\sigma}} = (\beta R_d)^{\frac{1}{\sigma}}$$

$$C_j^{e^m} = \beta R_e e_j^m = \beta R_e (\beta R_e)^{\frac{1-\sigma}{\sigma}} = (\beta R_e)^{\frac{1}{\sigma}}$$

It is straightforward to see that if a buyer switches from deposits to CBDC, his consumption increases if $R_e > R_d$.

4.2 A Binding Minimum Reserve Requirement Constraint

For the binding case ($\lambda > 0$) things become a bit more interesting. The results are summarized in proposition 5.

Proposition 5. *If the reserve requirement constraint is binding ($\lambda > 0$), an increase in the fraction of buyers who hold CBDC as a payment (saving) instrument (i) increases the interest rates on transaction deposits R_d and on saving deposits R_τ where R_τ increases by more than R_d , (ii) decreases the demand for transaction (saving) deposits on the extensive margin, (iii) increases the demand for transaction and saving deposits on the intensive margin, (iv) increases the demand for CBDC, (v) decreases the total loan amount and increases the interest rate on loans R_ℓ , (vi) has no clear effect on the total asset demand and GDP (vii) decreases the profit of the entrepreneurs and the bankers (viii) increases the consumption of buyers holding deposits who do not switch to CBDC.*

Proof. A proof can be found in Appendix A.5. ■

Below I discuss proposition 5 intuitively, mostly for a decrease in α_d . A decrease in α_τ is qualitatively analogous. If $\lambda > 0$, the total loan amount is below the bankers' profit maximizing level. If there is an outflow of deposits to CBDC (ie total CBDC demand increases), there is less funds and the bankers can provide even less loans to entrepreneurs. However, since there is imperfect competition in the loan market the bankers have some margin to mitigate the outflow of deposits. Particularly, they reduce the mark up between the interest rate on loans and the interest rate on deposits which also reduces the profit of a bank. This results in higher interest rates for deposits. Note that a banker lifts both interest rates, the one on transaction and the one on saving deposits. In doing so he can attract more deposits from both buyer types who are still holding deposits. The banker will increase R_τ by more than R_d because for each additional unit of transaction deposits, he needs to divert ω units into reserves, whereas each additional unit of saving deposits can be put directly into loans.

This means that deposit holdings decrease in the extensive margin but increase in the intensive margin. For reasonable parameter values, the effect on the extensive margin is stronger than the effect on the intensive margin. Thus, it follows that total loan supply is reduced if α_d decreases. Given that ℓ decreases, the interest rate on R_ℓ increases because the marginal product of investment becomes higher for a lower loan amount.

Whether total demand for both assets in- or decreases is ambiguous and depends mainly on the relative size of the interest rate on deposits and the interest rate on cash/CBDC (IOC). To illustrate this, assume for now that the interest rate on deposits and the interest rate on CBDC is equalized, ie $R_e = R_d$. In that case, the buyers who switch to CBDC will hold the same amount of CBDC as they held deposits before. However, the buyers who continue to hold deposits will increase their deposit holdings because the bankers increase the interest rate on deposits. What follows is an increase in total asset demand. If $R_e > R_d$, total asset demand will rise even more, because also the buyers switching from deposits to CBDC will demand more deposits on the intensive margin. On the other hand, assume that the interest rate on deposits is higher than the interest rate on CBDC, ie $R_d > R_e$. The buyers switching from deposits to CBDC will have a lower asset demand. However the ones who continue to hold deposits will increase their deposit demand. Whether total asset demand in- or decreases depends on which of the two effects is stronger. If $R_d \gg R_e$ the intensive margin effect becomes larger than the extensive margin effect and total asset demand will decrease if α_d decreases.

For GDP, which consists of CM and DM production, there are two non-linear effects due to the non-linearity in the utility and in the production function. First, if α_d is reduced, production from the entrepreneurs is lower because the total amount of loans is reduced. Second, total asset demand can in- or decrease. If it decreases, which as seen above is the case if $R_d \gg R_e$, GDP will certainly shrink. However, if total asset demand increases, the young buyers in the CM and the sellers in the DM produce more which counteracts the lower production of the entrepreneurs. This makes the effect on GDP ambiguous.

Lastly, let's consider what happens to consumption C of an individual buyer j . The consumption of a switching buyer increases if $R_e > R_d$ or $R_\tau > R_e$. For the buyers who do not switch and continue to hold deposits, their consumption increases because the bankers

increase the interest rates on deposits. These results are summarized in table 1.

	R_d	R_τ	d (ext.)	d (int.)	τ (ext.)	τ (int.)	ℓ	$e^m + e^s$	C_j^d	C_j^τ
$\alpha_d \downarrow$	\uparrow	\uparrow	\Downarrow	\uparrow	$-$	\uparrow	\downarrow	\Uparrow	\uparrow	\uparrow
$\alpha_\tau \downarrow$	\uparrow	\uparrow	$-$	\uparrow	\Downarrow	\uparrow	\downarrow	\Uparrow	\uparrow	\uparrow

Table 1: Effect of an increase in CBDC demand. (ext. = Extensive margin, int. = Intensive margin, C_d : consumption of the buyer holding transaction deposits d , C_τ consumption of the buyer holding saving deposits τ .)

5 Calibration of the Exogenous Asset Choice Model

To quantify the results and especially to get a quantitative idea of the different effects w.r.t. an outflow of transaction deposits or saving deposits, I calibrate the model to the US economy from 1987-2006. The time period is chosen because I want to consider a situation without excess reserves to match the model to the binding case.¹⁵ Note that throughout chapter 5 I assume that a CBDC is non interest bearing, ie $i_e = 0$.

I make the following assumption on the buyers' utility function:

$$v(y) = \frac{y^{1-\sigma}}{1-\sigma} \quad (27)$$

$$U(x) = \frac{x^{1-\sigma}}{1-\sigma} \quad (28)$$

There are a total of 13 parameters to calibrate: the discount factor (β), the gross real interest rate on cash (R_e), the gross real interest rate on reserves (R_{e_b}), the minimum reserve requirement ratio (ω), the fraction of θ^m -buyers (γ), the fraction of θ^m -buyers holding deposits (α_d), the fraction of θ^s -buyers holding deposits (α_τ), the total factor productivity (A), the ‘‘concavity’’ parameter in the production function (η), the utility parameter (σ) and the number of bankers (B). Additionally and in contrast to the model I split the handling cost parameter c into two. One that matches the transaction deposit handling cost c_d and one that matches the saving deposit handling cost c_τ .

The first four parameters are externally calibrated using data from the Federal Reserve

¹⁵Even though since the financial crisis 2007-2008 the banks hold excess reserves and this analysis is therefore not accurate to the current situation, I would argue that it is still relevant to analyse a situation in which balance sheets might normalize.

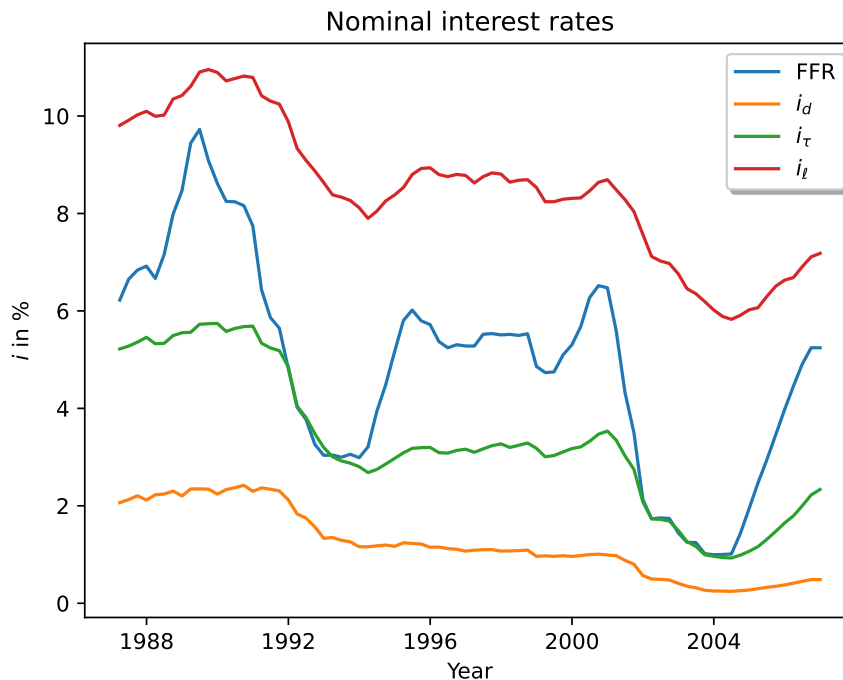


Figure 1: Quarterly interest rate data using FDIC call report data. FFR: Federal funds rate; i_d : interest rate on transaction deposits; i_τ : interest rate on saving deposits; i_ℓ : interest rate on loans.

Economic Data (FRED). For the real interest rate on reserves I use the average Federal Funds Rate (FFR) over the time period adjusted for inflation as an approximation. For the interest rate on cash (IOC) I subtract the average inflation over the time period from the nominal interest rate of zero for cash. β is picked to be consistent with the literature. The minimum reserve requirement rate is set to 10% which was the requirement by US regulation over this time period. Table 2 illustrates the externally calibrated parameters.

Parameter	Notation	Value	Notes
Discount factor	β	0.98	Consistent with literature
Net real IOR	r_{eb}	1.8%	FFR - inflation rate
Net real IOC	r_e	-3.6%	0 - inflation rate
Min. reserve requirement	ω	10%	US regulation

Table 2: Externally calibrated parameters.

Next, I want to match the deposit and loan interest rates. To get data on interest rates, I follow Drechsler, Savov and Schnabl (2017) and Chiu et al. (2019) and use FDIC call report data which contains bank level balance sheets and income statements. The data contains information on interest expenses and interest income for transaction deposits, saving deposits and loans and the corresponding deposit and loan amounts. To get the interest rates, I divide the interest expense by the corresponding deposit amounts. See appendix C for more details.

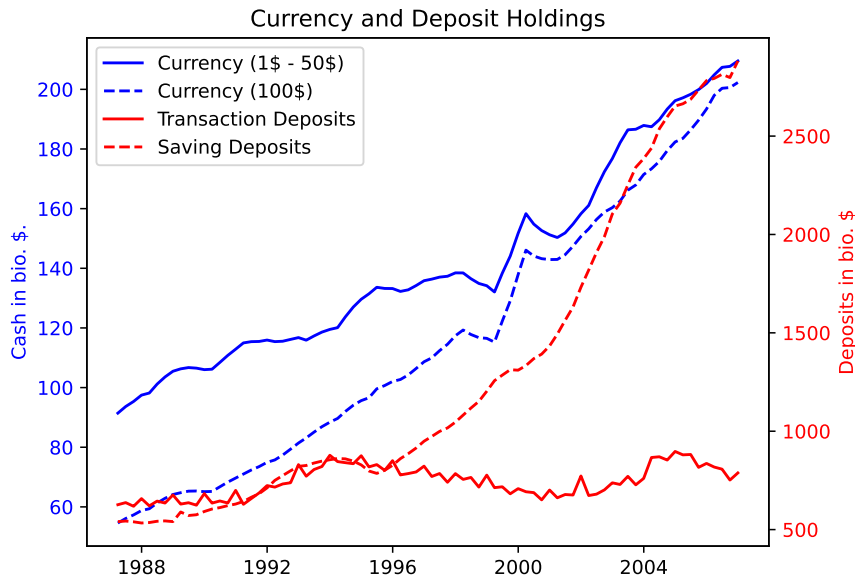


Figure 2: Quarterly data on currency and deposit holdings.

The resulting net nominal interest rates are depicted in figure 1. The model, however, is calibrated using gross real interest rates. I adjust the nominal interest rate for inflation by using FRED inflation data. This results in a net real interest rate on transaction deposits r_d of -1.7% , on saving deposit r_τ of 0.28% and on loans r_ℓ of 5.33% . I directly match these parameters in the calibration.

Lastly, I jointly calibrate the remaining nine parameters $\gamma, \alpha_d, \alpha_\tau, A, \eta, \sigma, B, c_d, c_\tau$ such that the fraction of cash held for transaction purposes to transaction deposits e^m/d , the fraction of cash held for saving purposes to saving deposits e^s/τ , the fraction of transaction deposits to saving deposits d/τ , the currency demand to GDP ratio $L_e = (e^m + e^s)/Y$ and the deposit demand to GDP ratio $L_{d\tau} = (d + \tau)/Y$ match their empirical counterparts.

For data on deposits I use the FDIC data and aggregate the deposit holdings of all banks. For currency holdings, I assume that all 100\$-bills are used as savings whereas all smaller denominations are used for payments. Furthermore, it is well known that a lot of US-currency is held abroad. I want to restrict the analysis to currency that is held within the US and use estimates from Judson (2017) of currency holdings abroad to adjust for that. Figure 2 illustrates the holdings of currency and deposits. According to Judson (2017), mostly 100\$-bills are held abroad which is why the estimate of currency in the US is higher for 1\$-50\$ bills relative to 100\$-bills.

A GDP time series is taken from FRED. GDP in the model is defined by total CM and DM

Parameter	Notation	Value
Fraction of θ^m -types	γ	0.46
Fraction θ^m -type holding d	α_d	0.83
Fraction θ^s -type holding τ	α_τ	0.90
Total factor productivity	A	2.37
Production function “concavity”	η	0.37
Utility parameter	σ	0.23
Number of bankers	B	25
Transaction deposit handling cost	c_d	0.04
Saving deposit handling cost	c_τ	0.02

Table 3: Internally calibrated parameters.

production (output produced by buyers and entrepreneurs in the CM and output produced by the sellers in the DM). The values of the internally calibrated parameters are depicted in table 3.¹⁶

Lastly I illustrate the model fit with respect to the empirical moments in table 4. Furthermore we have that $\lambda = 0.009$ which is quite small and suggests that the bankers would not have been much better off if the constraint was released. Or in other words, the bankers were close to voluntarily holding excess reserves according to the calibrated model.

Target	Notation	Data	Model
Transaction cash-to-transaction deposits ratio	e^m/d	0.19	0.19
Savings cash-to-saving deposits ratio	e^s/τ	0.10	0.10
transaction deposits-to-saving deposits ratio	d/τ	0.74	0.74
Money demand-to-GDP ratio	$L_e = (e^m + e^s)/Y$	0.03	0.03
Deposit demand-to-GDP ratio	$L_{d\tau} = (d + \tau)/Y$	0.23	0.23

Table 4: Model fit.

5.1 Introducing CBDC

Given the calibrated model I want to analyse how a change in the demand for central bank money influences other variables. Especially, how the effects of an outflow in transaction deposits compares to an outflow in saving deposits.

Figure 3 illustrates this for nominal interest rates.¹⁷ The left figure shows a change in the

¹⁶I here only match first order moments. In section 6, when I endogenise the asset choice, I also match the model to an elasticity of money demand. The reason for not doing this here is because the agents’ choice over bank deposits or central bank money does not depend on interest rates which makes matching the model to a money demand elasticity not really feasible.

¹⁷I report nominal interest rate for the sake of intuition. I assume that the inflation rate is constant and

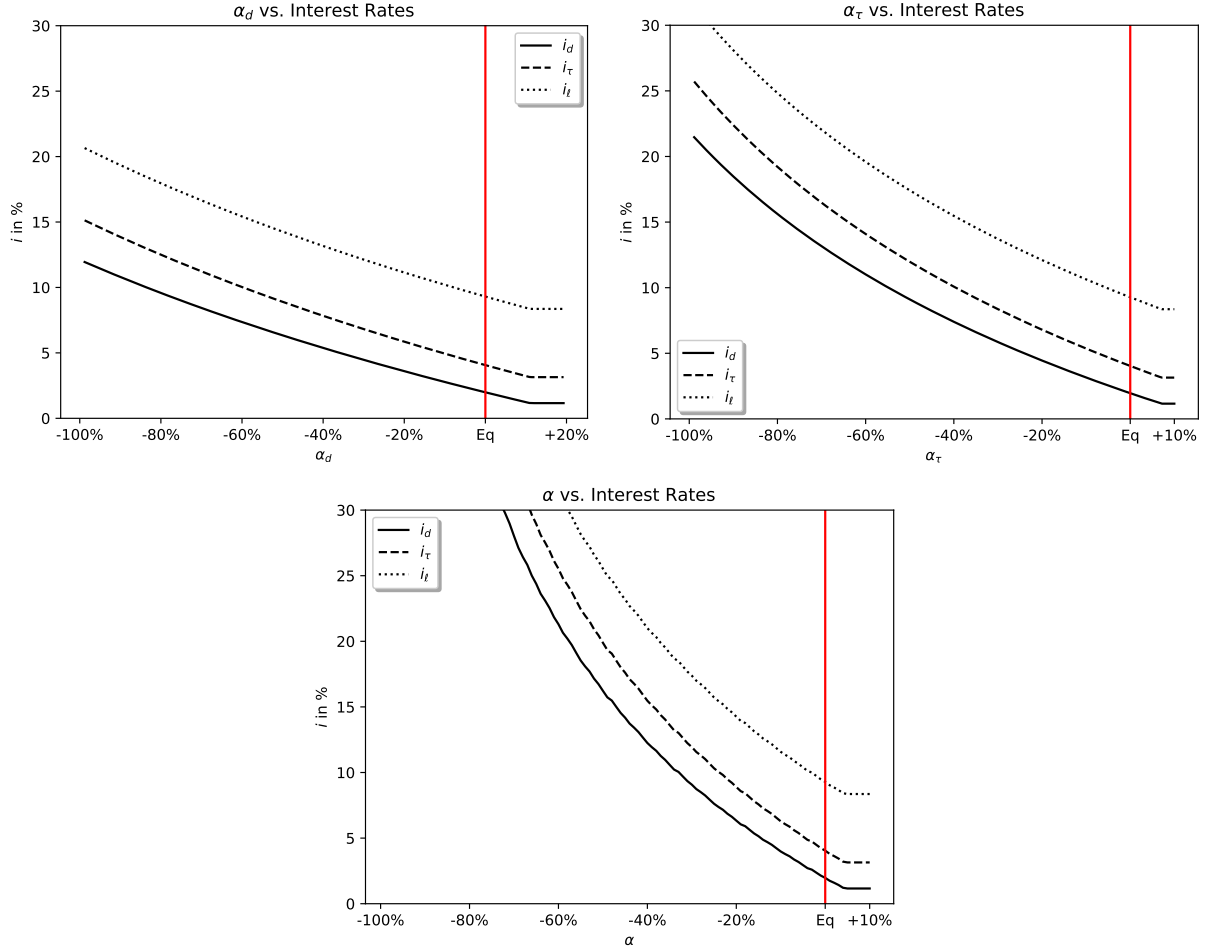


Figure 3: Effect of a change in the fraction of buyers holding deposits on nominal interest rates of transaction deposits (i_d), saving deposits (i_τ) and loans (i_ℓ). Eq is the calibrated equilibrium.

fraction of buyers holding transaction deposits, the right figure a change in the fraction of buyers holding saving deposits and the bottom figure a change in both the fraction of agents holding transaction and saving deposits. The latter is defined by an equal percentage decline. The effects are qualitatively as predicted in section 4.2. Given an outflow from deposits to CBDC, the nominal interest rate on transaction deposits (i_d), on saving deposit (i_τ) and on loans (i_ℓ) all increase.

The effect is larger for a change in α_τ . A 10% decrease in α_d (α_τ) increases the net nominal rates i_d from 2% to 2.8% (3.2%), i_τ from 4.1% to 4.9% (5.4%) and i_ℓ from 9.3% to 10.2% (10.6%). Obviously, the effect of an equal percentage outflow of both transaction and saving deposits the effect is even much stronger. In that case a 10% decrease in α increases the net nominal rates i_d from 2% to 4%, i_τ from 4.1% to 6.3% and i_ℓ from 9.3% to 11.6%.

the Fisher equation holds among all parameter choices.

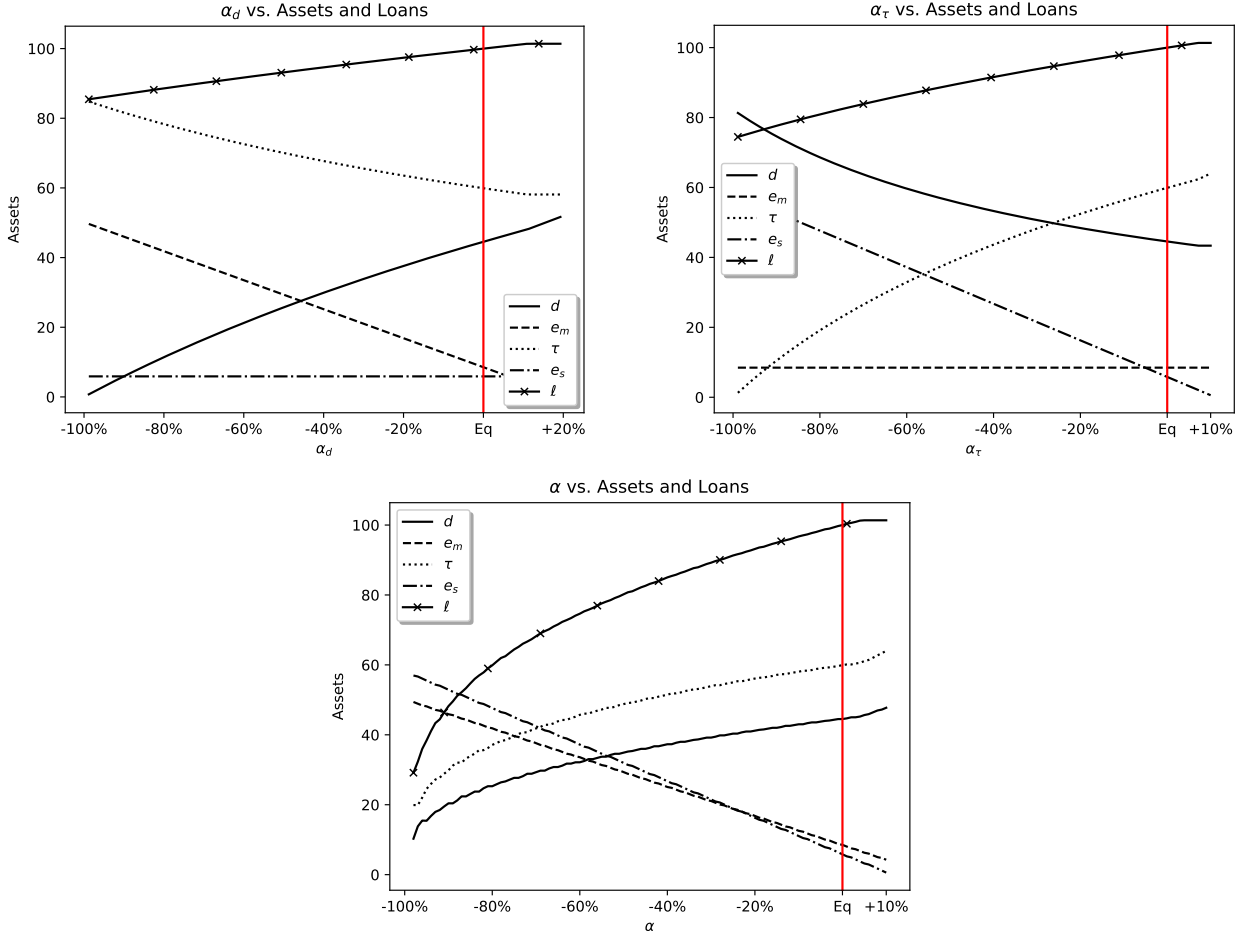


Figure 4: Effect on money (e^m , e^s), deposits (d , τ) and loans (l) given a change in fraction of buyers holding deposits. The curves are indexed such that the loan amount in the calibrated equilibrium is equal to 100.

The marginal effect on the rates becomes larger if α shrinks. This is because the fewer deposits there are, the less loans a banker can grant and the higher the marginal product off the marginal loan which transmits to the deposit rates. In reality, one could say that if the whole banking sector has only very limited funding and can thus grant only very few loans, it would choose to lend money to the most promising projects with the highest returns.¹⁸

If there was an inflow of deposits from CBDC - ie α_d , α_τ or α would increase - the minimum reserve requirement constraint becomes loose and the bank would invest all additional funds into excess reserves. In that case the bank has no incentive anymore to adjust its interest rates.

Figure 4 illustrates the effects on payment and saving instruments and loans. If there is an outflow from transaction (saving) deposits to CBDC, ie α_d (α_τ) decreases, the total amount

¹⁸Obviously this abstracts from uncertainty regarding the asset returns.

of transaction (saving) deposits decreases on the extensive margin, the total amount of cash held for transaction (saving) purposes increases and saving (transaction) deposits increase because the interest rate will be higher. The total loan amount ℓ is decreasing in both cases. The effects for α_d are less pronounced because agents hold less transaction deposits than saving deposits. An outflow of saving deposits on the other hand affects the bankers stronger. Due to the considerably higher interest rates more transaction deposits are attracted on the intensive margin to compensate for the outflow in saving deposits. The effect on loans is even much stronger if agents crowd out from both transactions and saving deposits, which is depicted in the bottom figure. The exact effects given a 10% shift from either transaction deposits, saving deposits or both to CBDC is illustrated in table 5. Furthermore note that we have $d > \ell$ for a very large decline in α_τ because agents need to hold a fraction of d as reserves.

	ℓ	d	τ	e^m	e^s
$\alpha_d : -10\%$	-1.3%	-7.3%	+2.8%	+47.1%	-
$\alpha_\tau : -10\%$	-2.0%	+4.1%	-6.0%	-	+90.0%
$\alpha : -10\%$	-3.3%	-3.4%	-3.2%	+47.1%	+90.0%

Table 5: Effect size on loans, deposits and cash holdings given a decrease in the fraction of buyers holding deposits.

Again, if there is an inflow into deposits from CBDC, the minimum reserve requirement constraint becomes loose. In that case the total loan amount is constant and equal to the bankers' profit maximizing amount. As a further observation note that the deposit holdings increase stronger for a certain percentage outflow if the constraint is loose. This is because the interest rate on deposits is constant in the non-binding case. As a consequence there is no intensive margin effect. In the binding case, for an inflow of saving deposits, the bankers will reduce the interest rates. Thus on the intensive margin deposits decrease which counteracts the effect of more depositors. Since in the non-binding case the interest rate do not adjust, this intensive margin effect is nonexistent which makes the total effect stronger.

The profits of the bankers and the entrepreneurs also declines if deposits are crowding out. For both the profit is reduced by 0.47% (0.73%,1.22%) if α_d (α_τ, α) goes down by 10%.

6 Endogenising Asset Choice

So far I stated that buyers exogenously decide whether they want to hold central bank money or bank deposits. I now endogenise the choice over the two assets with respect to the interest rates. I argue that some buyers prefer central bank money over bank deposits unless the interest rate differential between the assets becomes too large. I claim that for each of the agents there is at some point an interest rate differential that makes him switch the asset. I do not model why the agents prefer one asset over the other. Rather I assume that there is a distribution over how strongly an agent prefers one asset over the other, or more specifically, over how large the interest rate differential must be for a specific agent to choose the other asset.

I model this as a premium ε on the bank deposit interest rate over the interest rate on central bank money that a buyer demands to hold deposits. After being born, a buyer of type θ^m draws a ε from the distribution $G^m(\varepsilon)$ and a a buyer of type θ^s draws a ε from the distribution $G^s(\varepsilon)$ both with support $[-\infty, \infty]$. Hence, there are agents who prefer holding central bank money over deposits and vice versa. Whether an ε -buyer with type θ chooses central bank money or deposits is defined as

$$\theta_\varepsilon^m : m = \begin{cases} d & \text{if } (1 + \varepsilon)(1 + i_e) \leq (1 + i_d) \\ e^m & \text{else} \end{cases}$$

$$\theta_\varepsilon^s : s = \begin{cases} \tau & \text{if } (1 + \varepsilon)(1 + i_e) \leq (1 + i_\tau) \\ e^s & \text{else} \end{cases}$$

where i_d (i_τ) is the nominal net interest rate on transaction (saving) deposits and i_e the net nominal interest rate on central bank money.

For type θ^m (θ^s), there is an agent who is just indifferent between holding central bank money or transaction (saving) deposits, where the threshold value is denoted as $\tilde{\varepsilon}^m$ ($\tilde{\varepsilon}^s$).

$$\tilde{\varepsilon}^m = \frac{(1 + i_d)}{(1 + i_e)} - 1$$

$$\tilde{\varepsilon}^s = \frac{(1 + i_\tau)}{(1 + i_e)} - 1$$

which is equivalent to

$$\tilde{\varepsilon}^m = R_d/R_e - 1 \quad (29)$$

$$\tilde{\varepsilon}^s = R_\tau/R_e - 1. \quad (30)$$

Given the threshold value, I define the fraction of θ^m -buyers that hold transaction deposits as payment vehicle as

$$\alpha_d(R_d, R_e) = \int_{-\infty}^{\tilde{\varepsilon}^m} dG^m(\varepsilon) \quad (31)$$

and the fraction of θ^s -buyers that hold saving deposits as a saving vehicle as

$$\alpha_\tau(R_\tau, R_e) = \int_{-\infty}^{\tilde{\varepsilon}^s} dG^s(\varepsilon). \quad (32)$$

Note that the values of α depend on the interest rates and hence are endogenously determined. The fraction of θ^m (θ^s) buyers that hold CBDC as payment vehicle (saving vehicle) then is $1 - \alpha_d$ ($1 - \alpha_\tau$).

With this change the model becomes hard to solve analytically. Thus, I do another calibration for the adjusted model below.

6.1 Calibration of the Endogenised Asset Choice Model

Again I calibrate the model to the US economy from 1987-2006. The utility functions are defined as above. Additionally, I need to make a distributional assumption on $G^m(\varepsilon)$ and $G^s(\varepsilon)$. I decide to use a normal distribution with mean μ^M (μ^S) and standard deviation σ^M (σ^S) for the payment vehicle (savings) type.

The calibration specification remains mostly as in section 5 with the exception that α_d and α_τ are now endogenous variables and μ^M , μ^S , σ^M , σ^S are additionally used to match the moments from table 4. Additionally, I match not only first order moments but also a second order moment, namely the elasticity of central bank money demand by the public with respect to the interest rate on reserves denoted as ϵ . The idea behind this is as following: In the model an increase in the interest rate on reserves increases the interest rate on deposits.

Calibrated Parameters	Notation	Value	
Fraction of θ^m -types	γ	0.46	
Total factor productivity	A	2.37	
Production function “concavity”	η	0.37	
Utility parameter	σ	0.23	
Number of bankers	B	25	
Transaction deposit handling cost	c_d	0.04	
Saving deposit handling cost	c_τ	0.02	
Mean of normal distribution θ^M -type	μ^M	-0.14	
Standard deviation of normal distribution θ^M -type	σ^M	0.17	
Mean of normal distribution θ^S -type	μ^S	-0.71	
Standard deviation of normal distribution θ^S -type	σ^S	0.59	

Model Fit	Notation	Data	Model
Transaction cash-to-transaction deposits ratio	e^m/d	0.19	0.19
Savings cash-to-saving deposits ratio	e^s/τ	0.10	0.10
transaction deposits-to-saving deposits ratio	d/τ	0.74	0.74
Money demand-to-GDP ratio	$L_e = (e^m + e^s)/Y$	0.03	0.03
Deposit demand-to-GDP ratio	$L_{d\tau} = (d + \tau)/Y$	0.23	0.23
Elasticity of money demand	ϵ	-0.0036	-0.0036

Table 6: Jointly calibrated parameters and model fit of calibration with endogenised asset choice.

This in turn will lead to an inflow to deposits from central bank money since the asset choice is now endogenised. Thus, in the model a higher interest rate on reserves has a negative effect on central bank money demand held by the public. I argue that it is reasonable that this relationship is prevailing in the data as well. I measure the empirical counterpart by the following regression

$$\log(L_e) = \log\left(\frac{Pe_m + Pe_s}{PY}\right) = \beta_0 + \beta_1 r_{e_b} + v_t \quad (33)$$

where P is the price level. The left hand side of the equation is the demand for real balances of central bank money relative to GDP. Note that I measure the semi-elasticity of money demand with respect to the net real interest rate on reserves, since also the other parts of the model are calibrated to real values. I use the OLS estimate of β_1 as a target for the model based semi-elasticity, ie $\partial L_e(r_{e_b})/\partial r_{e_b}$. As expected I find a negative effect of $\epsilon = -0.36\%$, ie a one percentage point increase in the net real rate r_{e_b} decreases the money demand for central bank money by the public by 0.36%. I report the calibrated parameters and the model fit in table 6. In appendix D.2, I illustrate the distributions $G^m(\varepsilon)$ and $G^s(\varepsilon)$. The corresponding values for the share of buyers holding deposits are $\alpha_d = 0.83$ and $\alpha_\tau = 0.90$.

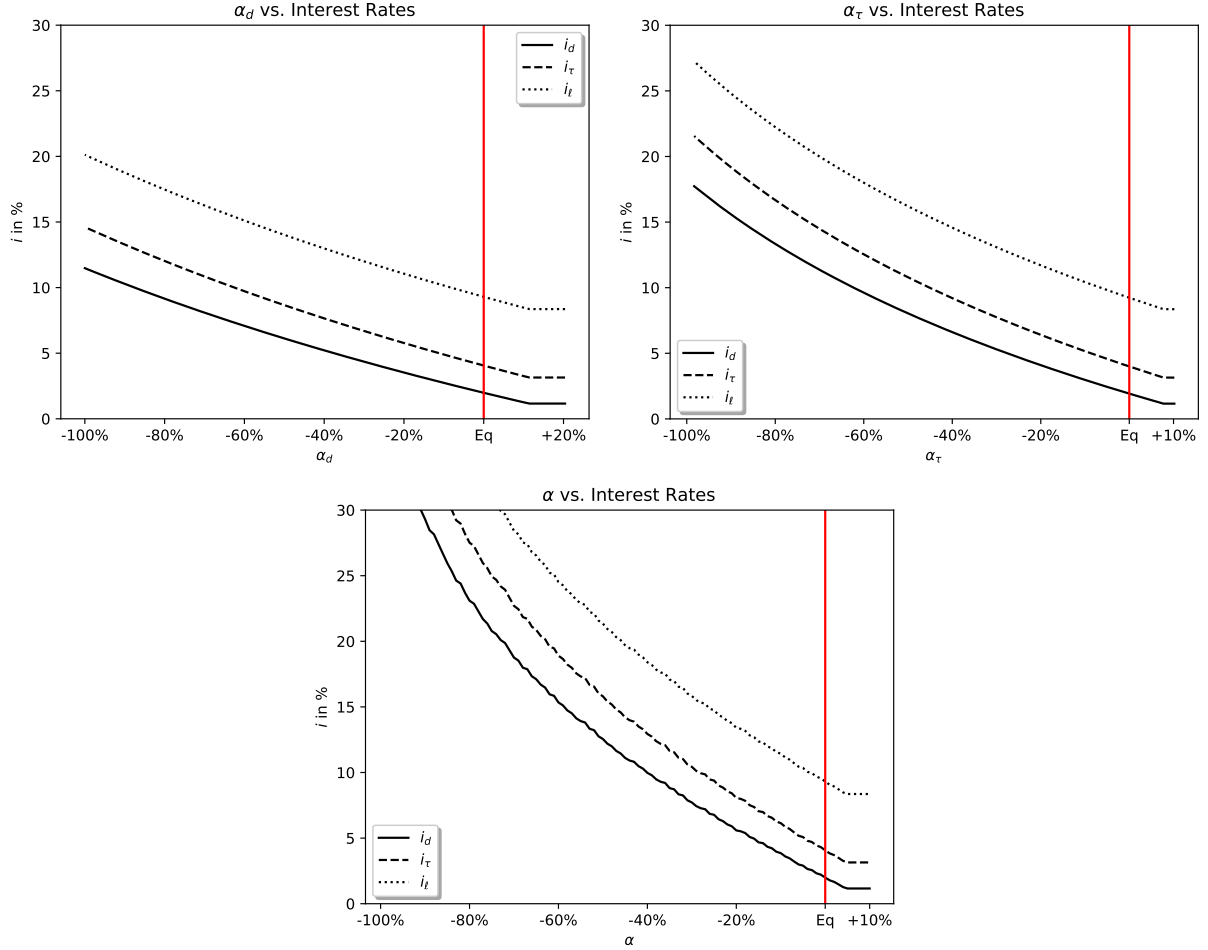


Figure 5: Effect of preference shift over central bank money and bank deposits on nominal interest rates of transaction deposits (i_d), saving deposits (i_τ) and loans (i_ℓ).

I first want to compare the results from section 5.1 to the adjusted model. To do so I estimate the new model in which α_d and α_τ are endogenous. I then alter the preferences of the agents over bank deposits and central bank money by varying the exogenous variables μ^M (μ^S) which translates in a shift in α_d (α_τ). I then report the change in α_d and α_τ in comparison with the variables of interest. Note that I assume that the standard deviations of the normal distributions σ^M and σ^S are constant for a shift in the means. For a shift in the preferences of both payment and saving types, I vary both μ^M and μ^S simultaneously and again match the resulting shift in α to the variables of interest.

Figure 5 depicts a shift in the fraction of agents holding deposits with respect to the interest rates. Comparing this to figure 3 where the asset choice was exogenous, the effect is qualitatively the same but quantitatively weaker. This is because higher interest rates now attract some additional buyers who switch from CBDC to bank deposits. Thus, there is also an extensive margin effect back to deposits such that interest rates have to increase by less to

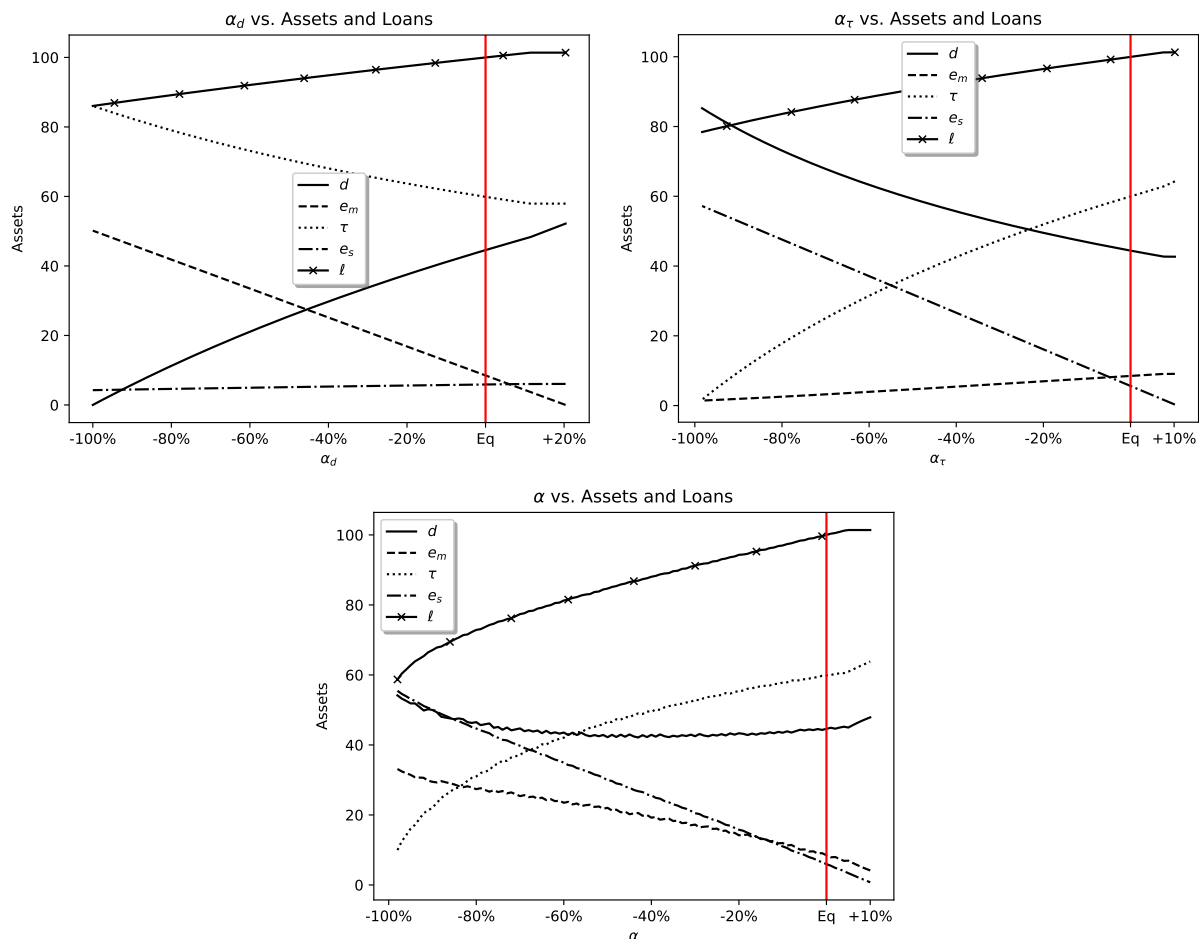


Figure 6: Effect of preference shift over central bank money and bank deposits on the amounts of transaction deposits (d), saving deposits (τ), central bank money held as payment vehicle (e^m), as saving vehicle (e^s) and loans (ℓ). The curves are indexed such that the loan amount in the calibrated equilibrium is equal to 100.

compensate for the outflow of deposits.

Figure 6 illustrates the effects of the preference shift on the asset holdings. Again, the effects are qualitatively mostly the same as in section 5.1, figure 4. However, as above, they are quantitatively weaker. If there is a preference shift from deposits to central bank money, the fraction of agents holding transaction (saving) deposits α_d (α_τ) decreases. As a consequence, the amount of transaction (saving) deposits shrinks and the amount of central bank money that is held as payment (saving) vehicle rises. As before, the amount of saving (transaction) deposits held increases because the interest rates rise which boosts the demand for saving (transaction) deposits on the intensive margin. In addition, under the endogenised asset model, the higher interest rates will trigger some agents to switch from central bank money to deposits. This on the one hand strengthens the heightened demand for saving (transaction) deposits and on the other hand diminishes the demand for e_s (e_m).

Under the exogenous asset choice model, a 10% decrease in α_d (α_τ) dampened total bank lending by 1.3% (2.0%) whereas now the effect is slightly weaker with 1.2% (1.7%). The detailed numbers for a 10% decrease in the fractions of agents holding deposits is illustrated in table 7 which replicates table 5 for the endogenised asset choice model. The dampened effect is again attributable to the endogenised asset choice which triggers some agents to switch from central bank money to deposits due to the higher interest rates.

Moreover, I look at the effects if the preferences are altered as such that both fractions of agents holding deposits as payment and as saving vehicle shift by the same percentage amounts.¹⁹ Comparing the bottom figure to figure 4, it becomes clear that the reduction in bank lending is much less pronounced, especially if only few agents hold deposits. Lastly note that the total transaction deposit demand d increases even if only very few people hold deposits. This is because the interest rates become very high for these parameter values which causes the individual demand for transaction deposits to become very high. However, this is mainly a result of the specific calibrated parameter values and should not be overemphasized.

	ℓ	d	τ	e^m	e^s
$\alpha_d : -10\%$	-1.2%	-8.3%	+3.3%	+53.0%	-
$\alpha_\tau : -10\%$	-1.7%	+5.5%	-6.6%	-	+88.7%
$\alpha : -10\%$	-3.0%	-2.5%	-3.4%	+41.2%	+84.7%

Table 7: Effect of preference shift over central bank money and bank deposits

7 Monetary Policy

Lastly, I consider how monetary policy affects outcomes. The monetary authority has three policy tools: the interest rate on reserves i_{e_b} , the interest rate on central bank money i_e and the minimum reserve requirement rate ω .

So far I always assumed that the interest rate on CBDC is equal to the interest rate on cash, ie $i_e = 0$. I now relax that and assume that the central bank can pay an interest rate on CBDC. Since I look at total central bank money and do not differentiate between cash and CBDC in the model, this implicitly means that this analysis considers a steady state

¹⁹In appendix D.4 I report more detailed figures that show the shift in preferences by a change in μ^M and μ^S on some variables of interest.

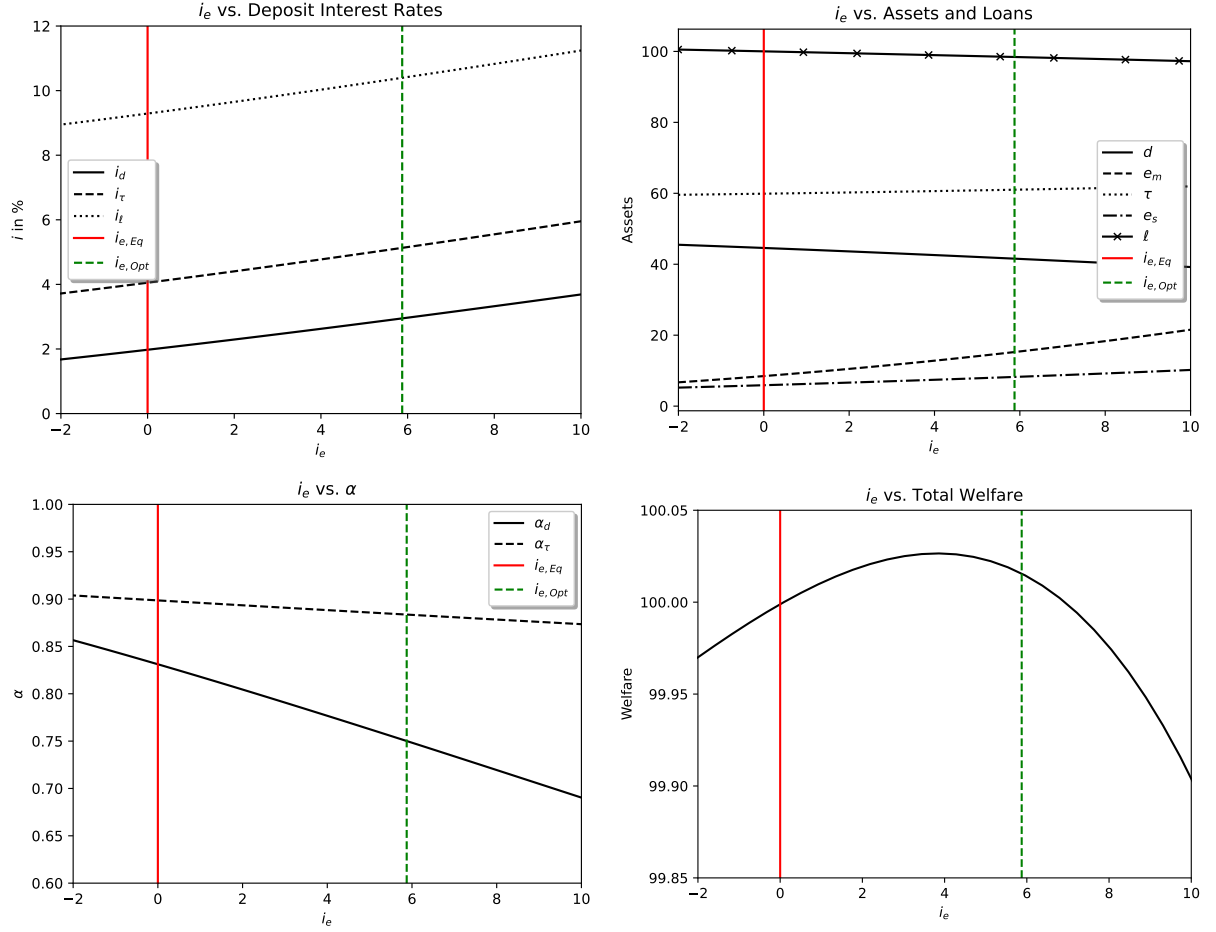


Figure 7: Effect of a change in the interest rate of CBDC on the deposit interest rates, assets, loans, the fraction of agents holding deposits and total welfare. The curves for the asset figure are indexed such that the loan amount in the calibrated equilibrium is equal to 100. Also total welfare is indexed to 100. ($i_{e,Eq}$ donates the interest rate at the calibrated equilibrium. $i_{e,Opt}$ is the optimal interest rate on CBDC defined in equation (26), ie $R_{e,Opt} = 1/\beta$)

comparison from a situation in which there is only cash to a new steady state in which agents only hold CBDC on which the central bank can pay an interest rate.

To analyse the effects I consider the model with the endogenised asset choice, since here the change in the interest rates triggers agents to switch between central bank money and deposits. Figure 7 shows the effect of a change in the interest rate of CBDC on the deposit interest rates, assets, loans, the fraction of agents holding deposits α_d and α_τ and welfare. The top left figure shows that an increase in the interest on CBDC leads to an increase in the deposit and loan rates. This is because a higher CBDC rate makes CBDC more attractive compared to deposits which triggers some buyers to switch from deposits to CBDC. The bankers can mitigate this outflow by sacrificing some of their profit and increasing the interest rates on deposits. As seen in the top right figure, the higher interest rates can compensate

almost entirely for the outflow of deposits such that the total loan amount does not decrease considerably.

Analysing the composition of the effect on loans in more detail, we see that the effect of a change in i_e is different for transaction and saving deposits. Transaction deposits seem to decrease, saving deposits to increase in i_e . To understand that, consider the bottom left figure. It shows that given an increase in the interest rate on CBDC, the number of buyers holding transaction deposits who switch to CBDC is much higher than the number of buyers holding saving deposits. This is because of the shape of the distributions on the premium ε . As reported in table 6 (and illustrated in appendix D.2), the variance on $G^s(\varepsilon)$ is larger than the variance on $G^m(\varepsilon)$ which is why α_d reacts more sensitively than α_τ . So on the one hand, there is the above described crowding out of buyers switching to CBDC for a higher i_e , on the other hand the higher deposit rates increases deposit holdings on the intensive margin. For transaction deposits the former effect dominates, whereas for saving deposits the latter one does. Consequently, the total effect on loans is relatively small. The demand for CBDC increases on the intensive and on the extensive margin since on the one hand buyers holding CBDC demand a higher amount due to the higher interest rate and on the other hand some buyers switch from deposits to CBDC as described above.

Lastly, I illustrate total welfare denoted as W_{Tot} in the bottom right figure. Total welfare is defined as the sum of the utilities of all agents. Note that the overall effect is very small. W_{Tot} is bell shaped with a maximum at $i_e = 3.86\%$ ($R_e = 1.0$) in which welfare is by 0.03% higher than in the calibrated equilibrium. There are several effects at play. Since the loan amount is relatively constant, the utilities of the bankers and the entrepreneurs decrease only very slightly in i_e . The utilities of the buyers and sellers, however, are much stronger affected. On the one hand, buyers will benefit from a higher interest rate on CBDC (and on deposits) which increases their welfare. On the other hand, the higher interest rate is financed by the tax which is levied on the seller. The tax amount T is a convex function in i_e and hence for high values of i_e , the disutility of the seller from working to pay the tax becomes higher than the added utility to the buyer.

To quantify these effects, I want to compare the calibrated equilibrium - ie $i_{e,Eq} = 0\%$ - with the Friedman rule, which is the optimal interest rate on CBDC found in section 3.3

(equation 26), ie $R_{e,Opt} = 1/\beta$ or $i_{e,Opt} = 100(\mu/\beta - 1) = 5.88\%$. Setting the CBDC interest rate to the optimal amount rises the net nominal interest rate on transaction deposits from 2.0% to 3.0%, on saving deposits from 4.1% to 5.2% and on loans from 9.3% to 10.4%. The effects on assets, loans and the share of buyers holding deposits is illustrated in table 8. The entrepreneurs' and the bankers' profit would both decrease by 0.63%.

	ℓ	d	τ	e^m	e^s	α_d	α_τ	W_{Tot}
$i_{e,Eq} \rightarrow i_{e,Opt}$	-1.6%	-6.8%	+1.9%	+80.0%	+39.5%	-9.8%	-1.7%	+0.01%

Table 8: Effect size on assets, loans and the fraction of buyers holding deposits given a rise in the interest rate on CBDC to the optimal rate.

Furthermore I illustrate the effect of a change in the minimum reserve requirement ω in figure 8.²⁰ The top left figure shows that a higher ω on the one hand increases the interest rates on saving deposits and on loans, however, the effect on transaction deposits is bell shaped. On the one hand, a higher reserve requirement means that a higher fraction of deposits needs to be held as reserves which in turns means that bank lending is reduced. To counteract the downward pressure on bank lending, the bankers sacrifice some of their profit to increase interest rates on deposits which then attracts more deposits. However, if ω becomes too big (which happens at $\omega = 0.55$), it is optimal for a banker to attract more funds via saving deposits than via transaction deposits such that the interest rate curve on saving deposits becomes steeper and the interest rate on transaction deposits decreases in ω for $\omega > 0.55$.

The top right figure shows that the banker attracts more deposits due to the higher interest rates. Even though the balance sheet of a banker expands, bank lending decreases because a higher fraction of deposits has to be held as reserves. The bottom left figure illustrates that a higher fraction of agents wants to hold deposits for higher ω , because interest rates increase. Again, for buyers holding transaction deposits this only holds up to $\omega \leq 0.55$ above which the interest rate decreases and hence the fraction of agents who demands transaction deposits diminishes as well. The bottom right figure again shows total welfare. There are two main effects. On the one hand, buyers benefit from a higher ω , because bankers react to an increased ω with higher interest rates. On the other hand, the utilities of the bankers and the entrepreneurs are reduced. The latter effect dominates the former which is why W_{Tot} is decreasing in ω .

²⁰Appendix D.3 depicts a figure for the effect of the interest rates on reserves i_{eb} as well.

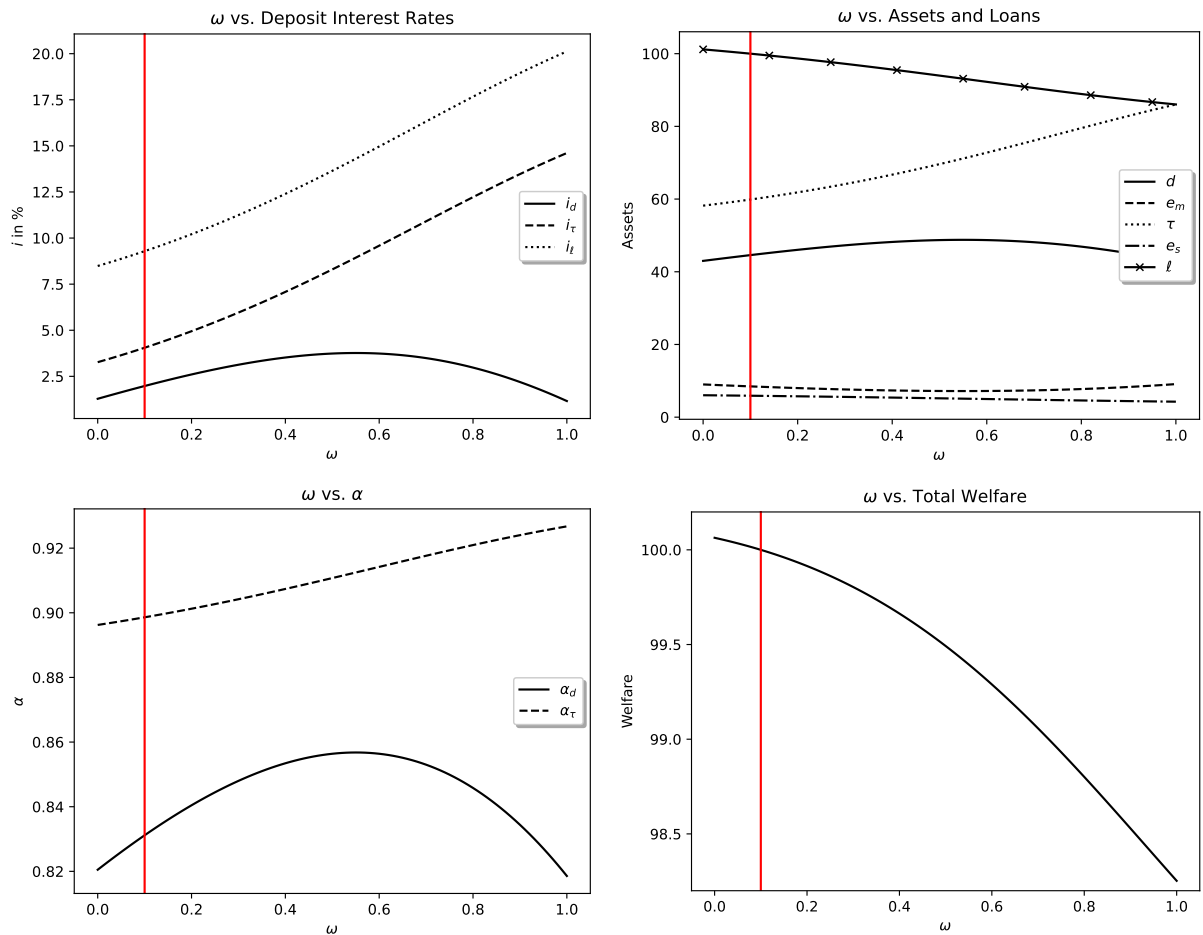


Figure 8: Effect of a change in the minimum reserve requirement ω on the deposit interest rates, assets, loans, the fraction of agents holding deposits and total welfare. The curves for the asset figure are indexed such that the loan amount in the calibrated equilibrium is equal to 100. Also total welfare is indexed to 100.

Lastly, I want to discuss what monetary policy implications my findings would have. First of all and the most important one might be that a central bank should be very careful to design a CBDC only as a payment vehicle if it wants to mitigate the effects on bank lending. Second, the negative effects of an interest rate on CBDC seem to be rather small, because bankers will sacrifice some of their profit to increase rates and attract more depositors. Third, if a central bank wants to mitigate the crowding out of deposits upon an introduction of a CBDC, it could decrease the minimum reserve requirement and in that way boost bank lending - this, however, abstracts from the many important reasons why a minimum reserve requirement exists in the first place, like eg risk.

8 Conclusion

This paper analyses the effects of an introduction of a central bank digital currency (CBDC) on bank intermediation in a tractable general equilibrium model with heterogeneous bank deposits and an imperfectly competitive loan market. The impact of a CBDC may differ depending on whether it is used as a payment or as a saving vehicle. In the model, the banks naturally provide two types of deposits, liquid transaction deposits and illiquid saving deposits. Different types of agents in the model demand either a payment or a saving vehicle and have a preference over holding central bank money or bank deposits. First, I examine the effects of an introduction of a non interest bearing CBDC that increases the fraction of people who want to hold central bank money.

I find that bank lending declines if banks do not hold voluntary reserves. However, the banks will sacrifice some of their profit to increase the interest rates on both transaction and saving deposits. This rises the demand for deposits and counteracts partly the crowding out. The size of the effect differs depending on whether a CBDC is only used as a payment or also as a saving vehicle. If it is only used as a payment vehicle there are two counteracting effects. On the one hand, the total amount of transaction deposits decreases. On the other hand, there is a stronger demand in saving deposits because of the higher interest rate. This mitigates the negative effect. A calibration to the US economy from 1987-2006 shows that if 10% of the agents holding transaction deposits switch to CBDC, bank lending is reduced by 1.2%-1.3%. If, however, also the agents holding saving deposits switch to CBDC, then the compensating effect from a higher saving deposit demand fades which intensifies the decline in bank lending. I find a drop in bank lending of 3.0%-3.3% in the quantitative assessment.

Furthermore, I analyse how the outcome differs if the central bank starts to pay an interest rate on CBDC. An interest rate on CBDC will increase the demand for central bank money, ie some agents want to switch from bank deposits to CBDC. To counteract this outflow, the banks again react by sacrificing some of their profit to rise the interest rate on deposits. As a consequence, the negative effect on bank lending is mitigated. In the calibration I find that an interest rate on CBDC set to the Friedman rule decreases bank lending by about 1.6%.

My findings would have several policy implications. To mention two, a central bank should be careful to design a CBDC only as a payment vehicle if it wants to mitigate the effects on

bank lending and the negative effects of an interest rate on CBDC seem to be rather small.

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Appendix A Proofs

A.1 Proposition 1

Proof. First assume that a bank offers only transaction deposits d , ie it offers only one kind of deposit that can be transferred in the DM. I will show that a bank can increase its profit by offering saving deposits τ as well and hence the aforementioned case cannot be an equilibrium. In this case, the θ^s -buyers who prefer deposits over CBDC will also hold transaction deposits. A bank b's profit function when only offering transaction deposits is $\ell_b^* R_\ell(\ell) + e_b^* R_{e_b} - d_b^* R_d - d_b^* c$ where we have that $e_b^* = \omega d_b^*$, $\ell_b^* + e_b^* = d_b^*$ and $R = 1 + r$ is the gross real interest rate. Given that both constraints hold with equality, I can replace them into the profit function and get a single expression for the bank's profit Π .

$$\Pi_b^* = \ell_b^* \left(R_\ell(\ell^*) + \frac{\omega}{1-\omega} R_{e_b} - \frac{1}{1-\omega} (R_d + c) \right)$$

I now want to analyse whether this can be an equilibrium or whether a bank has an incentive to deviate from it by offering a marginal amount of saving deposits denoted as $\tilde{\tau}$. Since there is perfect competition in the deposit market, a bank takes the interest rates R_d and R_τ as given. I here assume that the bank offers a θ^s -buyer depositor a saving deposit contract with $R_\tau > R_d$. Hence the total amount of deposits does not change. The new profit function is $\tilde{\ell}_b R_\ell(\ell) + \tilde{e}_b R_{e_b} - \tilde{d}_b (R_d + c) - \tilde{\tau}_b (R_\tau + c)$. The bank now holds $\tilde{d}_b = d_b^* - \tilde{\tau}_b$ of transaction deposits. On the asset side, the required amount of reserves the bank has to hold is reduced to $\tilde{e}_b = \omega(d_b^* - \tilde{\tau}_b)$ and the amount of loans it can issue is increased to $\tilde{\ell}_b = \ell_b^* + \omega\tilde{\tau}_b$. Furthermore the balance sheet equality holds, ie $\tilde{\ell}_b + \tilde{e}_b = \tilde{d}_b + \tilde{\tau}_b$. This yields the following profit function

$$\begin{aligned} \tilde{\Pi}_b = & \ell_b^* \left(R_\ell(\ell^* + \omega\tilde{\tau}_b) + \frac{\omega}{1-\omega} R_{e_b} - \frac{1}{1-\omega} (R_d + c) \right) \\ & + \tilde{\tau}_b (\omega R_\ell(\ell^* + \omega\tilde{\tau}_b) - \omega R_{e_b} + (R_d + c) - (R_\tau + c)) \end{aligned}$$

Next, I conjecture and verify that $\tilde{\Pi}_b > \Pi_b^*$, ie a bank can increase its profit by offering

saving deposits as well.

$$\begin{aligned}\tilde{\Pi}_b = & \ell_b^* \left(R_\ell(\ell^* + \omega\tilde{\tau}_b) + \frac{\omega}{1-\omega} R_{e_b} - \frac{1}{1-\omega} (R_d + c) \right) \\ & + \tilde{\tau}_b (\omega R_\ell(\ell^* + \omega\tilde{\tau}_b) - \omega R_{e_b} + R_d - R_\tau) \\ & - \ell_b^* \left(R_\ell(\ell^*) + \frac{\omega}{1-\omega} R_{e_b} - \frac{1}{1-\omega} (R_d + c) \right) > 0\end{aligned}$$

After rearranging I get

$$\ell_b^* (R_\ell(\ell^* + \omega\tilde{\tau}_b) - R_\ell(\ell^*)) + \tilde{\tau}_b (\omega R_\ell(\ell^* + \omega\tilde{\tau}_b) - \omega R_{e_b} + R_d - R_\tau) > 0$$

and then plug in the FOCs (3) for R_d and (4) for R_τ .

$$\ell_b^* (R_\ell(\ell^* + \omega\tilde{\tau}_b) - R_\ell(\ell^*)) + \omega\tilde{\tau}_b (R_\ell(\ell^* + \omega\tilde{\tau}_b) - R_\tau) > 0$$

Note that because we have $\frac{\partial R_\ell(\ell)}{\partial \ell} < 0$ the first term is negative. Furthermore, combining equation (2) and (4) yields $R_\ell(\ell) = R_\tau - \frac{\partial R_\ell(\ell)}{\partial \ell_b} \ell_b$. Again since $\frac{\partial R_\ell(\ell)}{\partial \ell} < 0$, it is obvious that $R_\ell(\ell) > R_\tau$ and the second term is positive. The first term depicts the loss in earnings due to a lower interest rate on all existing loans ℓ_b^* . The second term depicts the additional earnings on the spread between R_ℓ and R_τ that yields from the new loans $\omega\tilde{\tau}_b$. If the additional earnings of the new loans are greater than the loss on all existing loans, the firm can increase its profit by offering saving deposits as well.

To see whether the inequality holds, I replace R_τ by (2) at $\tilde{\ell} = \ell^* + \omega\tilde{\tau}_b$.

$$\begin{aligned}\ell_b^* (R_\ell(\ell^* + \omega\tilde{\tau}_b) - R_\ell(\ell^*)) + \omega\tilde{\tau}_b \left(-\frac{\partial R_\ell(\ell^* + \omega\tilde{\tau}_b)}{\partial \ell_b} (\ell_b^* + \omega\tilde{\tau}_b) \right) > 0 \\ \frac{R_\ell(\ell^* + \omega\tilde{\tau}_b) - R_\ell(\ell^*)}{\omega\tilde{\tau}_b} \ell_b^* > \frac{\partial R_\ell(\ell^* + \omega\tilde{\tau}_b)}{\partial \ell_b} (\ell_b^* + \omega\tilde{\tau}_b)\end{aligned}$$

Both of these terms describe slopes. Given the functional form $f(x) = Ax^\eta$ which yields $R_\ell(\ell) = A\eta(\ell)^{\eta-1}$ and using Bernoulli's inequality, I can verify that the above expression holds. Without loss of generality, I assume that there is only one bank $B = 1$ and hence $\ell^* = \ell_b^*$. For the sake of clarity, I denote $a = \ell_b^*$, $b = \ell_b^* + \omega\tilde{\tau}_b$ and do a proof by contradiction.

Assume that

$$\begin{aligned} a \frac{A\eta(b)^{\eta-1} - A\eta(a)^{\eta-1}}{b-a} &\leq A\eta(\eta-1)b^{\eta-2}b \\ \frac{a}{b-a}(1 - (a/b)^{\eta-1}) &\leq \eta - 1 \\ 1 - \left(\frac{b}{a}\right)^{1-\eta} &\leq (\eta - 1) \left(\frac{b}{a} - 1\right) \end{aligned}$$

Denote $x = b/a > 1$ and $\alpha = 1 - \eta \in (0, 1)$.

$$\begin{aligned} 1 - x^\alpha &\leq \alpha(1 - x) \\ \frac{1 - x^\alpha}{1 - x} &\geq \alpha \end{aligned}$$

By Bernoulli's inequality we have that for $\alpha \in (0, 1)$ and $x > 1$

$$\begin{aligned} x^\alpha &< 1 + (x - 1)\alpha \\ -(x - 1)\alpha &< 1 - x^\alpha \\ \alpha &> \frac{1 - x^\alpha}{1 - x} \end{aligned}$$

And hence we have

$$\begin{aligned} \alpha &\leq \frac{1 - x^\alpha}{1 - x} < \alpha \\ \alpha &< \alpha \end{aligned}$$

which is a contradiction. Hence, bankers offering only transaction deposits is not an equilibrium because a bank can increase its profit when deviating to offering saving deposits as well.

Moreover, a bank could offer only saving deposits τ , ie no θ^m -type will want to hold bank deposits because they cannot be used for payment in the DM. I want to see whether a bank could increase its profit by deviating to offering transaction deposits d as well. A bank b 's profit function when only offering saving deposit τ is

$$\Pi_b^* = \ell_b^* R_\ell(\ell^*) - \ell_b^*(R_\tau + c)$$

using the balance sheet identity $\ell_b^* = \tau_b^*$. A bank that deviates and offers additionally a marginal amount of transaction deposits denoted as \tilde{d}_b at market rate R_d has the profit function $\tilde{\ell}_b R_\ell(\tilde{\ell}) + \tilde{e}_b R_{e_b} - \tau_b^*(R_\tau + c) - \tilde{d}_b(R_d + c)$. Additionally, we have $\tilde{\ell}_b = \ell_b^* + (1 - \omega)\tilde{d}_b$, $\tilde{e}_b = \omega\tilde{d}_b$ and the balance sheet identity $\tilde{\ell}_b + \tilde{e}_b = \tau_b^* + \tilde{d}_b$. The rearranged profit function of the bank then is

$$\tilde{\Pi}_b = \ell_b^* [R_\ell(\ell^* + (1 - \omega)\tilde{d}_b) - (R_\tau + c)] + \tilde{d}_b [(1 - \omega)R_\ell(\ell^* + (1 - \omega)\tilde{d}_b) + \omega R_{e_b} - (R_d + c)].$$

Again I conjecture and verify that $\tilde{\Pi}_b - \Pi_b^* > 0$.

$$\tilde{\Pi}_b - \Pi_b^* > 0$$

$$\ell_b^* [R_\ell(\ell^* + (1 - \omega)\tilde{d}_b) - R_\ell(\ell^*)] + \tilde{d}_b [(1 - \omega)R_\ell(\ell^* + (1 - \omega)\tilde{d}_b) + \omega R_{e_b} - (R_d + c)] > 0.$$

$$\ell_b^* [R_\ell(\ell^* + (1 - \omega)\tilde{d}_b) - R_\ell(\ell^*)] + \tilde{d}_b (1 - \omega) [R_\ell(\ell^* + (1 - \omega)\tilde{d}_b) - (R_\tau + c)] > 0.$$

In the last step I use the FOCs (3) and (4). This is a similar expression as above with the same interpretation. The first term is negative and depicts the loss on all existing loans because the total loan amount increases and the interest rate goes down. The second term is positive and represents the gain on the net revenue on the additionally issued loans. Rearranging and using the FOCs yields

$$\frac{R_\ell(\ell^* + (1 - \omega)\tilde{d}_b) - R_\ell(\ell^*)}{(1 - \omega)\tilde{d}_b} \ell_b^* > \frac{\partial R_\ell(\ell^* + (1 - \omega)\tilde{d}_b)}{\partial \ell_b} (\ell_b^* + (1 - \omega)\tilde{d}_b)$$

This inequality has exactly the same form as above. Again, using Bernoulli's inequality proofs that the inequality holds given the functional form of the production function.

Lastly, assume that the bankers offer both liquid transaction and illiquid saving deposits. Is this an equilibrium or could a bank increase its profit by deviating to offering only one kind of deposits? Inverting the proofs above, it is straightforward that a bank cannot increase its profit by applying a one-deposit strategy. Hence, there is an equilibrium in which the bankers offer two kind of deposits given that the reserve requirement constraint is binding.

■

A.2 Proposition 2

Proof. Independent of λ and c , it can be proven that $R_\ell > R_{e_b}$. From equation (2) we have

$$R_\ell = R_{e_b} + \lambda - \frac{\partial R_\ell(\ell)}{\partial \ell_b} \ell_b$$

Using $f(\ell) = A\ell^\eta/\beta$ and $R_\ell(\ell) = \eta A\ell^{\eta-1}$, the derivative is defined by

$$\frac{\partial R_\ell(\ell)}{\partial \ell_b} = (\eta - 1)\eta A(\ell_b + \sum_{b \neq b'} \ell_{b'})^{\eta-2} < 0$$

since $\eta \in (0, 1)$. Thus, all terms on the right hand side in the equation for R_ℓ defined above are positive and it follows that $R_\ell > R_{e_b}$.

To prove that $R_\ell > R_\tau$ for any $\lambda \geq 0$ and $c \geq 0$, one can combine (2) and (4)

$$R_\ell = R_\tau + c - \frac{\partial R_\ell(\ell)}{\partial \ell_b} \ell_b$$

and since $\frac{\partial R_\ell(\ell)}{\partial \ell_b} < 0$ all terms on the right hand side are positive and it follows that $R_\ell > R_\tau$.

From equations (3) and (4), ie

$$R_{e_b} + \lambda(1 - \omega) - c = R_d$$

$$R_{e_b} + \lambda - c = R_\tau$$

and recalling that $\omega \in (0, 1)$, it is straightforward that $R_\tau > R_d$ if the reserve requirement constraint ($\lambda > 0$) is binding and that $R_\tau = R_d$ if the reserve requirement constraint is non-binding ($\lambda = 0$) for any $c > 0$. Furthermore it follows that $R_d \leq R_{e_b}$ and $R_\tau \leq R_{e_b}$ depending on c .

The firm always makes a profit because of the Cournot competition in the loan market. ■

A.3 Proposition 3

Proof. Without loss of generality, I use the specific utility function

$$u(x) = \frac{x^{1-\sigma}}{1-\sigma}$$

in the proof. This makes the notation more readable. Equation (16) then becomes

$$\lambda = \eta A \left(\gamma \alpha_d \beta^{\frac{1-\sigma}{\sigma}} (R_{e_b} + \lambda(1-\omega) - c)^{\frac{1-\sigma}{\sigma}} (1-\omega) + (1-\gamma) \alpha_\tau \beta^{\frac{1}{\sigma}} (R_{e_b} + \lambda - c)^{\frac{1-\sigma}{\sigma}} \right)^{\eta-1} \cdot \left(\frac{\eta-1}{B} + 1 \right) - R_{e_b}$$

Using implicit differentiation we can find the partial derivatives w.r.t. all exogenous variables.

Productivity parameter A : $\frac{\partial \lambda}{\partial A}$.

$$\begin{aligned} \lambda' = & \underbrace{\eta \left[\gamma \alpha_d \beta^{\frac{1-\sigma}{\sigma}} (R_{e_b} + (1-\omega)\lambda(A) - c)^{\frac{1-\sigma}{\sigma}} (1-\omega) + (1-\gamma) \alpha_\tau \beta^{\frac{1}{\sigma}} (R_{e_b} + \lambda(A) - c)^{\frac{1-\sigma}{\sigma}} \right]^{\eta-1}}_{\Phi_0 > 0} \left(\frac{\eta-1}{B} + 1 \right) \\ & + \underbrace{\eta A (\eta-1) \left[\gamma \alpha_d \beta^{\frac{1-\sigma}{\sigma}} (R_{e_b} + (1-\omega)\lambda(A) - c)^{\frac{1-\sigma}{\sigma}} (1-\omega) + (1-\gamma) \alpha_\tau \beta^{\frac{1}{\sigma}} (R_{e_b} + \lambda(A) - c)^{\frac{1-\sigma}{\sigma}} \right]^{\eta-2}}_{\Phi_1 < 0} \left(\frac{\eta-1}{B} + 1 \right) \\ & \cdot \left[\underbrace{\frac{1-\sigma}{\sigma} \gamma \alpha_d \beta^{\frac{1-\sigma}{\sigma}} (R_{e_b} + (1-\omega)\lambda(A) - c)^{\frac{1-2\sigma}{\sigma}} (1-\omega)^2}_{\Phi_3 > 0} \lambda' + \underbrace{\frac{1-\sigma}{\sigma} (1-\gamma) \alpha_\tau \beta^{\frac{1}{\sigma}} (R_{e_b} + \lambda(A) - c)^{\frac{1-2\sigma}{\sigma}}}_{\Phi_4 > 0} \lambda' \right] \end{aligned}$$

Note that $\eta - 1 < 0$ and $\left(\frac{\eta-1}{B} + 1\right) > 0$.

$$\begin{aligned} \lambda' = & \Phi_0 + \Phi_1 (\Phi_3 \lambda' + \Phi_4 \lambda') \\ \lambda' = & \frac{\overbrace{\Phi_0}^{>0}}{\underbrace{1 - \Phi_1 \Phi_3 - \Phi_1 \Phi_4}_{>0}} \Rightarrow \frac{\partial \lambda}{\partial A} > 0 \end{aligned}$$

Number of bankers B : $\frac{\partial \lambda}{\partial B}$.

$$\begin{aligned} \lambda' = & \underbrace{\eta A \left[\gamma \alpha_d \beta^{\frac{1-\sigma}{\sigma}} (R_{e_b} + (1-\omega)\lambda(B) - c)^{\frac{1-\sigma}{\sigma}} (1-\omega) + (1-\gamma) \alpha_\tau \beta^{\frac{1}{\sigma}} (R_{e_b} + \lambda(B) - c)^{\frac{1-\sigma}{\sigma}} \right]^{\eta-1}}_{\Phi_0 > 0} \left(\frac{1-\eta}{B^2} \right) \\ & + \underbrace{\eta A \left(\frac{\eta-1}{B} + 1 \right) (\eta-1) \left[\gamma \alpha_d \beta^{\frac{1-\sigma}{\sigma}} (R_{e_b} + (1-\omega)\lambda(B) - c)^{\frac{1-\sigma}{\sigma}} (1-\omega) + (1-\gamma) \alpha_\tau \beta^{\frac{1}{\sigma}} (R_{e_b} + \lambda(B) - c)^{\frac{1-\sigma}{\sigma}} \right]^{\eta-2}}_{\Phi_1 < 0} \\ & \cdot \left[\underbrace{\frac{1-\sigma}{\sigma} \gamma \alpha_d \beta^{\frac{1-\sigma}{\sigma}} (R_{e_b} + (1-\omega)\lambda(B) - c)^{\frac{1-2\sigma}{\sigma}} (1-\omega)^2}_{\Phi_3 > 0} \lambda' + \underbrace{\frac{1-\sigma}{\sigma} (1-\gamma) \alpha_\tau \beta^{\frac{1}{\sigma}} (R_{e_b} + \lambda(B) - c)^{\frac{1-2\sigma}{\sigma}}}_{\Phi_4 > 0} \lambda' \right] \end{aligned}$$

Note that $\frac{1-\eta}{B^2} > 0$.

$$\lambda' = \Phi_0 + \Phi_1(\Phi_3\lambda' + \Phi_4\lambda')$$

$$\lambda' = \frac{\overbrace{\Phi_0}^{>0}}{\underbrace{1 - \Phi_1\Phi_3 - \Phi_1\Phi_4}_{>0}} \Rightarrow \frac{\partial \lambda}{\partial B} > 0$$

Discount factor β : $\frac{\partial \lambda}{\partial \beta}$.

$$\lambda' = \eta(\eta-1)A \left(\frac{\eta-1}{B} + 1 \right) \underbrace{\left[\gamma\alpha_d \beta^{\frac{1-\sigma}{\sigma}} (R_{e_b} + (1-\omega)\lambda(\beta) - c)^{\frac{1-\sigma}{\sigma}} (1-\omega) + (1-\gamma)\alpha_\tau \beta^{\frac{1}{\sigma}} (R_{e_b} + \lambda(\beta) - c)^{\frac{1-\sigma}{\sigma}} \right]^{\eta-2}}_{\Phi_0 < 0}$$

$$\cdot \left[\underbrace{\frac{1-\sigma}{\sigma} \gamma\alpha_d \beta^{\frac{1-2\sigma}{\sigma}} (R_{e_b} + (1-\omega)\lambda(\beta) - c)^{\frac{1-\sigma}{\sigma}} (1-\omega)}_{\Phi_1 > 0} + \underbrace{\frac{1-\sigma}{\sigma} \gamma\alpha_d \beta^{\frac{1-\sigma}{\sigma}} (R_{e_b} + (1-\omega)\lambda(\beta) - c)^{\frac{1-2\sigma}{\sigma}} (1-\omega)^2}_{\Phi_2 > 0} \lambda' \right]$$

$$+ \frac{1}{\sigma} \underbrace{(1-\gamma)\alpha_\tau \beta^{\frac{1-\sigma}{\sigma}} (R_{e_b} + \lambda(\beta) - c)^{\frac{1-\sigma}{\sigma}}}_{\Phi_3 > 0} + \frac{1-\sigma}{\sigma} \underbrace{(1-\gamma)\alpha_\tau \beta^{\frac{1}{\sigma}} (R_{e_b} + \lambda(\beta) - c)^{\frac{1-2\sigma}{\sigma}} \lambda'}_{\Phi_4 > 0}$$

$$\lambda' = \Phi_0(\Phi_1 + \Phi_2\lambda' + \Phi_3 + \Phi_4\lambda')$$

$$\lambda' = \frac{\overbrace{\Phi_0\Phi_1 + \Phi_0\Phi_3}^{<0}}{\underbrace{1 - \Phi_0\Phi_2 - \Phi_0\Phi_4}_{>0}} \Rightarrow \frac{\partial \lambda}{\partial \beta} < 0$$

Concavity parameter in production function η : $\frac{\partial \lambda}{\partial \eta}$.

$$\lambda' = A \frac{d}{d\eta} \left(\frac{\eta^2}{B} - \frac{\eta}{B} + \eta \right) \left[\gamma\alpha_d \beta^{\frac{1-\sigma}{\sigma}} (R_{e_b} + (1-\omega)\lambda(\eta) - c)^{\frac{1-\sigma}{\sigma}} (1-\omega) + (1-\gamma)\alpha_\tau \beta^{\frac{1}{\sigma}} (R_{e_b} + \lambda(\eta) - c)^{\frac{1-\sigma}{\sigma}} \right]^{\eta-1}$$

$$\lambda' = \left(\frac{2\eta}{B} - \frac{1}{B} + 1 \right) \underbrace{A \left[\gamma\alpha_d \beta^{\frac{1-\sigma}{\sigma}} (R_{e_b} + (1-\omega)\lambda(\eta) - c)^{\frac{1-\sigma}{\sigma}} (1-\omega) + (1-\gamma)\alpha_\tau \beta^{\frac{1}{\sigma}} (R_{e_b} + \lambda(\eta) - c)^{\frac{1-\sigma}{\sigma}} \right]^{\eta-1}}_{\Phi_0 > 0}$$

$$+ A \left(\frac{\eta^2}{B} - \frac{\eta}{B} + \eta \right) \underbrace{\frac{d}{d\eta} e^{(\eta-1)\log(\gamma\alpha_d \beta^{\frac{1-\sigma}{\sigma}} (R_{e_b} + (1-\omega)\lambda(\eta) - c)^{\frac{1-\sigma}{\sigma}} (1-\omega) + (1-\gamma)\alpha_\tau \beta^{\frac{1}{\sigma}} (R_{e_b} + \lambda(\eta) - c)^{\frac{1-\sigma}{\sigma}})}}_{\Phi_1}$$

Only last part:

$$\begin{aligned}
\Phi_1 = & \underbrace{\left(\gamma \alpha_d \beta^{\frac{1-\sigma}{\sigma}} (R_{e_b} + (1-\omega)\lambda(\eta) - c)^{\frac{1-\sigma}{\sigma}} (1-\omega) + (1-\gamma) \alpha_\tau \beta^{\frac{1}{\sigma}} (R_{e_b} + \lambda(\eta) - c)^{\frac{1-\sigma}{\sigma}} \right)^{\eta-1}}_{\Psi_0 > 0} \\
& \cdot \underbrace{\left[\log \left(\gamma \alpha_d \beta^{\frac{1-\sigma}{\sigma}} (R_{e_b} + (1-\omega)\lambda(\eta) - c)^{\frac{1-\sigma}{\sigma}} (1-\omega) + (1-\gamma) \alpha_\tau \beta^{\frac{1}{\sigma}} (R_{e_b} + \lambda(\eta) - c)^{\frac{1-\sigma}{\sigma}} \right) \right]}_{\Psi_1 \leq 0} \\
& \cdot (\eta-1) \frac{1}{\underbrace{\gamma \alpha_d \beta^{\frac{1-\sigma}{\sigma}} (R_{e_b} + (1-\omega)\lambda(\eta) - c)^{\frac{1-\sigma}{\sigma}} (1-\omega) + (1-\gamma) \alpha_\tau \beta^{\frac{1}{\sigma}} (R_{e_b} + \lambda(\eta) - c)^{\frac{1-\sigma}{\sigma}}}_{\Psi_2 < 0}} \\
& \cdot \lambda' \left(\underbrace{\frac{1-\sigma}{\sigma} \gamma \alpha_d \beta^{\frac{1-\sigma}{\sigma}} (R_{e_b} + (1-\omega)\lambda(\eta) - c)^{\frac{1-2\sigma}{\sigma}} (1-\omega)^2 + \frac{1-\sigma}{\sigma} (1-\gamma) \alpha_\tau \beta^{\frac{1}{\sigma}} (R_{e_b} + \lambda(\eta) - c)^{\frac{1-2\sigma}{\sigma}}}_{\Psi_3 > 0} \right)
\end{aligned}$$

Combine everything to get

$$\begin{aligned}
\lambda' &= \left(\frac{2\eta}{B} - \frac{1}{B} + 1 \right) \Phi_0 + A \left(\frac{\eta^2}{B} - \frac{\eta}{B} + \eta \right) \lambda' \Psi_0 \Psi_1 \Psi_2 \Psi_3 \\
\lambda' &= \frac{\overbrace{\left(\frac{2\eta}{B} - \frac{1}{B} + 1 \right)}^{>0} \overbrace{\Phi_0}^{>0}}{1 - A \underbrace{\left(\frac{\eta^2}{B} - \frac{\eta}{B} + \eta \right)}_{>0} \underbrace{\Psi_0}_{>0} \underbrace{\Psi_1}_{\leq 0} \underbrace{\Psi_2}_{<0} \underbrace{\Psi_3}_{>0}} \Rightarrow \frac{\partial \lambda}{\partial \eta} \begin{matrix} \leq \\ \geq \end{matrix} 0
\end{aligned}$$

Share of θ^m -types γ : $\frac{\partial \lambda}{\partial \gamma}$.

$$\begin{aligned}
\lambda' &= \underbrace{\left(\frac{\eta-1}{B} + 1 \right) (\eta-1) \eta A \left[\gamma \alpha_d \beta^{\frac{1-\sigma}{\sigma}} (R_{e_b} + (1-\omega)\lambda(\gamma) - c)^{\frac{1-\sigma}{\sigma}} (1-\omega) + (1-\gamma) \alpha_\tau \beta^{\frac{1}{\sigma}} (R_{e_b} + \lambda(\gamma) - c)^{\frac{1-\sigma}{\sigma}} \right]^{\eta-2}}_{\Phi_0 < 0} \\
& \cdot \left[\underbrace{\alpha_d \beta^{\frac{1-\sigma}{\sigma}} (R_{e_b} + (1-\omega)\lambda(\gamma) - c)^{\frac{1-\sigma}{\sigma}} (1-\omega)}_{\Phi_1 > 0} + \underbrace{\gamma \alpha_d \beta^{\frac{1-\sigma}{\sigma}} (R_{e_b} + (1-\omega)\lambda(\gamma) - c)^{\frac{1-\sigma}{\sigma}} (1-\omega)^2}_{\Phi_2 > 0} \lambda' \right. \\
& \left. - \underbrace{\alpha_\tau \beta^{\frac{1}{\sigma}} (R_{e_b} + \lambda(\gamma) - c)^{\frac{1-\sigma}{\sigma}}}_{\Phi_3 > 0} + \underbrace{(1-\gamma) \alpha_\tau \beta^{\frac{1}{\sigma}} (R_{e_b} + \lambda(\gamma) - c)^{\frac{1-\sigma}{\sigma}}}_{\Phi_4 > 0} \lambda' \right]
\end{aligned}$$

$$\begin{aligned}
\lambda' &= \Phi_0 [\Phi_1 + \Phi_2 \lambda' - \Phi_3 + \Phi_4 \lambda'] \\
\lambda' &= \frac{\overbrace{\Phi_0}^{<0} \left[\overbrace{\Phi_1}^{>0} - \overbrace{\Phi_3}^{>0} \right]}{1 - \underbrace{\Phi_0}_{<0} \left[\underbrace{\Phi_2}_{>0} + \underbrace{\Phi_4}_{>0} \right]} \Rightarrow \frac{\partial \lambda}{\partial \gamma} \begin{matrix} \leq \\ \geq \end{matrix} 0
\end{aligned}$$

Reserve requirement ω : $\frac{\partial \lambda}{\partial \omega}$.

$$\lambda' = \underbrace{(\eta - 1) \left(\frac{\eta - 1}{B} + 1 \right) \eta A \left[\gamma \alpha_d \beta^{\frac{1-\sigma}{\sigma}} (R_{e_b} + (1 - \omega)\lambda(\omega) - c)^{\frac{1-\sigma}{\sigma}} (1 - \omega) + (1 - \gamma) \alpha_\tau \beta^{\frac{1}{\sigma}} (R_{e_b} + \lambda(\omega) - c)^{\frac{1-\sigma}{\sigma}} \right]^{\eta-2}}_{\Phi_0 < 0} \cdot \left[\underbrace{\frac{1-\sigma}{\sigma} \gamma \alpha_d \beta^{\frac{1-\sigma}{\sigma}} (R_{e_b} + (1 - \omega)\lambda(\omega) - c)^{\frac{1-2\sigma}{\sigma}}}_{\Phi_1 > 0} \left[(1 - \omega)\lambda' - \lambda \right] (1 - \omega) - \underbrace{\gamma \alpha_d \beta^{\frac{1-\sigma}{\sigma}} (R_{e_b} + (1 - \omega)\lambda(\omega) - c)^{\frac{1-\sigma}{\sigma}}}_{\Phi_2 > 0} \right. \\ \left. + \underbrace{\frac{1-\sigma}{\sigma} (1 - \gamma) \alpha_\tau \beta^{\frac{1}{\sigma}} (R_{e_b} + \lambda(\omega) - c)^{\frac{1-2\sigma}{\sigma}}}_{\Phi_3 > 0} \lambda' \right]$$

$$\lambda' = \Phi_0 (\Phi_1 [(1 - \omega)\lambda' - \lambda] (1 - \omega) - \Phi_2 + \Phi_3 \lambda')$$

$$\lambda' = \underbrace{\Phi_0 \Phi_1 (1 - \omega)^2}_{\Phi_4 < 0} \lambda' - \underbrace{\Phi_0 \Phi_1 (1 - \omega) \lambda}_{\Phi_5 < 0} - \underbrace{\Phi_0 \Phi_2}_{\Phi_6 < 0} + \underbrace{\Phi_0 \Phi_3}_{\Phi_7 < 0} \lambda'$$

$$\lambda' = (\Phi_4 \lambda' - \Phi_5 - \Phi_6 + \Phi_7 \lambda')$$

$$\lambda' = \frac{\overbrace{-\Phi_5 - \Phi_6}^{> 0}}{\underbrace{1 - \Phi_4 - \Phi_7}_{> 0}} \Rightarrow \frac{\partial \lambda}{\partial \omega} > 0$$

Share of θ^m -types holding deposits α_d : $\frac{\partial \lambda}{\partial \alpha_d}$.

$$\lambda' = \underbrace{\eta A \left(\frac{\eta - 1}{B} + 1 \right) (\eta - 1) \left[\gamma \alpha_d \beta^{\frac{1-\sigma}{\sigma}} (R_{e_b} + (1 - \omega)\lambda(\alpha_d) - c)^{\frac{1-\sigma}{\sigma}} (1 - \omega) + (1 - \gamma) \alpha_\tau \beta^{\frac{1}{\sigma}} (R_{e_b} + \lambda(\alpha_d) - c)^{\frac{1-\sigma}{\sigma}} \right]^{\eta-2}}_{\Phi_0 < 0} \cdot \left[\underbrace{\gamma \beta^{\frac{1-\sigma}{\sigma}} (1 - \omega) (R_{e_b} + (1 - \omega)\lambda(\alpha_d) - c)^{\frac{1-\sigma}{\sigma}}}_{\Phi_1 > 0} + \underbrace{\gamma \beta^{\frac{1-\sigma}{\sigma}} (1 - \omega) \alpha_d \frac{1-\sigma}{\sigma} (R_{e_b} + (1 - \omega)\lambda(\alpha_d) - c)^{\frac{1-2\sigma}{\sigma}} (1 - \omega) \lambda'}_{\Phi_2 > 0} \right. \\ \left. + \underbrace{\frac{1-\sigma}{\sigma} (1 - \gamma) \alpha_\tau \beta^{\frac{1}{\sigma}} (R_{e_b} + \lambda(\alpha_d) - c)^{\frac{1-2\sigma}{\sigma}}}_{\Phi_3 > 0} \lambda' \right]$$

$$\lambda' = \Phi_0 (\Phi_1 + \Phi_2 \lambda' + \Phi_3 \lambda')$$

$$\lambda' = \frac{\overbrace{\Phi_0 \Phi_1}^{< 0}}{\underbrace{1 - \Phi_0 (\Phi_2 + \Phi_3)}_{> 0}} \Rightarrow \frac{\partial \lambda}{\partial \alpha_d} < 0$$

Share of θ^s -types holding deposits α_τ : $\frac{\partial \lambda}{\partial \alpha_\tau}$.

$$\lambda' = \eta A \left(\frac{\eta-1}{B} + 1 \right) (\eta-1) \underbrace{\left[\gamma \alpha_d \beta^{\frac{1-\sigma}{\sigma}} (R_{e_b} + (1-\omega)\lambda(\alpha_\tau) - c)^{\frac{1-\sigma}{\sigma}} (1-\omega) + (1-\gamma)\alpha_\tau \beta^{\frac{1}{\sigma}} (R_{e_b} + \lambda(\alpha_\tau) - c)^{\frac{1-\sigma}{\sigma}} \right]^{\eta-2}}_{\Phi_0 < 0} \cdot \left[\underbrace{(1-\gamma)\beta^{\frac{1}{\sigma}} (R_{e_b} + \lambda(\alpha_\tau) - c)^{\frac{1-\sigma}{\sigma}}}_{\Phi_1 > 0} + \underbrace{(1-\gamma)\beta^{\frac{1}{\sigma}} \alpha_\tau \frac{1-\sigma}{\sigma} (R_{e_b} + \lambda(\alpha_\tau) - c)^{\frac{1-2\sigma}{\sigma}} \lambda'}_{\Phi_2 > 0} + \underbrace{\frac{1-\sigma}{\sigma} \gamma \alpha_d \beta^{\frac{1-\sigma}{\sigma}} (R_{e_b} + (1-\omega)\lambda(\alpha_\tau) - c)^{\frac{1-2\sigma}{\sigma}} (1-\omega) \lambda'}_{\Phi_3 > 0} \right]$$

$$\lambda' = \Phi_0(\Phi_1 + \Phi_2 \lambda' + \Phi_3 \lambda')$$

$$\lambda' = \frac{\overbrace{\Phi_0 \Phi_1}^{< 0}}{\underbrace{1 - \Phi_0(\Phi_2 + \Phi_3)}_{> 0}} \Rightarrow \frac{\partial \lambda}{\partial \alpha_\tau} < 0$$

Interest rate on reserves R_{e_b} : $\frac{\partial \lambda}{\partial R_{e_b}}$.

$$\lambda' = \eta A \left(\frac{\eta-1}{B} + 1 \right) (\eta-1) \underbrace{\left[\gamma \alpha_d \beta^{\frac{1-\sigma}{\sigma}} (R_{e_b} + (1-\omega)\lambda(R_{e_b}) - c)^{\frac{1-\sigma}{\sigma}} (1-\omega) + (1-\gamma)\alpha_\tau \beta^{\frac{1}{\sigma}} (R_{e_b} + \lambda(R_{e_b}) - c)^{\frac{1-\sigma}{\sigma}} \right]^{\eta-2}}_{\Phi_0 < 0} \cdot \left[\underbrace{\frac{1-\sigma}{\sigma} \gamma \alpha_d \beta^{\frac{1-\sigma}{\sigma}} (R_{e_b} + (1-\omega)\lambda(R_{e_b}) - c)^{\frac{1-2\sigma}{\sigma}} (1-\omega)(1 + (1-\omega)\lambda')}_{\Phi_1 > 0} + \underbrace{\frac{1-\sigma}{\sigma} (1-\gamma)\alpha_\tau \beta^{\frac{1}{\sigma}} (R_{e_b} + \lambda(R_{e_b}) - c)^{\frac{1-2\sigma}{\sigma}} (1 + \lambda')}_{\Phi_2 > 0} \right] - 1$$

$$\lambda' = \Phi_0[\Phi_1(1 + (1-\omega)\lambda') + \Phi_2(1 + \lambda')] - 1$$

$$\lambda' = \Phi_0 \Phi_1 + \Phi_0 \Phi_1 (1-\omega)\lambda' + \Phi_0 \Phi_2 + \Phi_0 \Phi_2 \lambda' - 1$$

$$\lambda' = -\frac{\overbrace{1 - \Phi_0 \Phi_1 - \Phi_0 \Phi_2}^{> 0}}{\underbrace{1 - \Phi_0 \Phi_1 (1-\omega) - \Phi_0 \Phi_2}_{> 0}} \Rightarrow \frac{\partial \lambda}{\partial R_{e_b}} < -1$$

■

A.4 Proposition 4

Proof. An increase in the fraction of buyers who hold CBDC is represented by a decrease in α_d or α_τ . I here present the prove for a change in α_d , the proof for α_τ is analogous.

(i) Using equations (17) and (18)

$$\frac{\partial R_d}{\partial \alpha_d} = \frac{\partial R_\tau}{\partial \alpha_d} = \frac{\partial (R_{e_b} - c)}{\partial \alpha_d} = 0$$

we see that there is no effect on interest rates if α_d changes.

(ii)-(iii) From equations (19)-(22) I can derive

$$\begin{aligned} \frac{\partial d}{\partial \alpha_d} &= \gamma v'^{-1} \left(\frac{1}{\beta R_d} \right) \frac{1}{\beta R_d} > 0 \\ \frac{\partial e^m}{\partial \alpha_d} &= -\gamma v'^{-1} \left(\frac{1}{\beta R_e} \right) \frac{1}{\beta R_e} < 0 \\ \frac{\partial \tau}{\partial \alpha_d} &= \frac{\partial e^s}{\partial \alpha_d} = 0. \end{aligned}$$

which shows that d decreases and e^m increases upon a decrease of α_d .

(iv) A change in α_d has neither an effect on ℓ nor on R_ℓ which can be shown using equations (23) and (24)

$$\begin{aligned} \frac{\partial \ell}{\partial \alpha_d} &= \frac{\partial \left(\left[\frac{A\eta(1-\frac{1-\eta}{B})}{R_{e_b}} \right]^{1/(1-\eta)} \right)}{\partial \alpha_d} = 0 \\ \frac{\partial R_\ell}{\partial \alpha_d} &= \frac{\partial \left(\frac{R_{e_b}}{1-\frac{1-\eta}{B}} \right)}{\partial \alpha_d} = 0. \end{aligned}$$

(v) In the unconstrained case, total reserves are defined by $Be_b = d(\alpha_d) + \tau - \ell$. Upon a change in α_d , only d is affected, τ and ℓ are unaffected. Hence, an outflow of CBDC reduces reserves one-by-one.

(vi) Total asset demand is defined by $d + \tau + e^m + e^s$. We know from above that a change in α_d only affects d and e^m . The total effect is

$$\frac{\partial d}{\partial \alpha_d} + \frac{\partial e^m}{\partial \alpha_d} = \gamma v'^{-1} \left(\frac{1}{\beta R_d} \right) \frac{1}{\beta R_d} - \gamma v'^{-1} \left(\frac{1}{\beta R_e} \right) \frac{1}{\beta R_e} \lesseqgtr 0$$

and it follows that

$$\frac{\partial d}{\partial \alpha_d} + \frac{\partial e^m}{\partial \alpha_d} \begin{cases} > 0 & \text{if } R_d > R_e \\ = 0 & \text{if } R_d = R_e \\ < 0 & \text{if } R_d < R_e \end{cases}$$

Thus, upon a decrease in α_d total asset demand increases if $R_e > R_d$.

GDP is defined by the sum of CM and DM production.

$$Y = A\ell^\eta + d + e^m + \tau + e^s + \beta(dR_d + e^m R_e)$$

It follows that

$$\frac{\partial Y}{\partial \alpha_d} = \gamma v'^{-1} \left(\frac{1}{\beta R_d} \right) \frac{1}{\beta R_d} - \gamma v'^{-1} \left(\frac{1}{\beta R_e} \right) \frac{1}{\beta R_e} + \beta \left[\gamma v'^{-1} \left(\frac{1}{\beta R_d} \right) \frac{1}{\beta} - \gamma v'^{-1} \left(\frac{1}{\beta R_e} \right) \frac{1}{\beta} \right]$$

and thus

$$\frac{\partial Y}{\partial \alpha_d} \begin{cases} > 0 & \text{if } R_d > R_e \\ = 0 & \text{if } R_d = R_e \\ < 0 & \text{if } R_d < R_e \end{cases}$$

(vii) The entrepreneurs' profit is defined by

$$\Pi^e = A\ell^\eta - \ell R_\ell$$

and using result (iv) it is straightforward that $\frac{\partial \Pi^e}{\partial \alpha_d} = 0$. The bankers' profit is defined by

$$\Pi^b = \ell R_\ell + B e_b R_{e_b} - d(R_d + c) - \tau(R_\tau + c)$$

and remembering that the change in d translates one-to-one to reserves and that $R_{e_b} = R_d + c$

yields

$$\begin{aligned}
\frac{\partial \Pi^b}{\partial \alpha_d} &= \left[\gamma v'^{-1} \left(\frac{1}{\beta R_d} \right) \frac{1}{\beta R_d} \right] R_{eb} - \left[\gamma v'^{-1} \left(\frac{1}{\beta R_d} \right) \frac{1}{\beta R_d} \right] (R_d + c) \\
&= - \left[\gamma v'^{-1} \left(\frac{1}{\beta R_d} \right) \frac{1}{\beta R_d} \right] (R_{eb} - (R_d + c)) \\
&= - \left[\gamma v'^{-1} \left(\frac{1}{\beta R_d} \right) \frac{1}{\beta R_d} \right] ((R_d + c) - (R_d + c)) = 0.
\end{aligned}$$

■

A.5 Proposition 5

Proof. I here show the prove for α_d , the prove for α_τ is analogous. Assume that α_d decreases, ie more θ^m -buyers want to hold CBDC as a payment instrument. From proposition 3 it follows that $\partial \lambda / \partial \alpha_d < 0$.

(i) The effect on R_d and R_τ can be found by using equations (17) and (18).

$$\begin{aligned}
\frac{\partial R_d}{\partial \alpha_d} &= \frac{\partial (R_{eb} + (1 - \omega)\lambda(\alpha_d) - c)}{\partial \alpha_d} = (1 - \omega) \frac{\lambda(\alpha_d)}{\partial \alpha_d} < 0 \\
\frac{\partial R_\tau}{\partial \alpha_d} &= \frac{\partial (R_{eb} + \lambda(\alpha_d) - c)}{\partial \alpha_d} = \frac{\lambda(\alpha_d)}{\partial \alpha_d} < 0
\end{aligned}$$

Note that the effect on R_τ is stronger than on R_d because of the $(1 - \omega)$ term.

(ii) Next consider the extensive margin effect on transaction deposits d using equation (19)

...

$$\frac{\partial d}{\partial \alpha_d} = \frac{\partial (\gamma \alpha_d d_j)}{\partial \alpha_d} = \gamma d_j > 0$$

(iii) ... and the intensive margin effects on d and τ considering the individual deposit demand for a single buyer j using equations (19) and (20). Without loss of generalize I assume

$v(x) = U(x) = \frac{x^{1-\sigma}}{1-\sigma}$ for the prove.

$$\begin{aligned}\frac{\partial d_j}{\partial \alpha_d} &= \frac{\partial \left((\beta R_d(\alpha_d))^{\frac{1-\sigma}{\sigma}} \right)}{\partial \alpha_d} = \frac{1-\sigma}{\sigma} (\beta R_d)^{\frac{1-2\sigma}{\sigma}} \frac{\partial R_d}{\partial \alpha_d} < 0 \\ \frac{\partial \tau_j}{\partial \alpha_d} &= \frac{\partial \left(\beta^{\frac{1}{\sigma}} R_\tau(\alpha_d)^{\frac{1-\sigma}{\sigma}} \right)}{\partial \alpha_d} = \frac{1-\sigma}{\sigma} \beta^{\frac{1}{\sigma}} R_\tau^{\frac{1-2\sigma}{\sigma}} \frac{\partial R_d}{\partial \alpha_d} < 0\end{aligned}$$

This is true because of the assumption that $-x \frac{U''(x)}{U'(x)} < 1$ for all $x \geq 0$ and $-y \frac{v''(y)}{v'(y)} < 1$ for all $y \geq 0$ which translates to $\sigma < 1$ for this specific utility function.

(iv) The demand for CBDC increases given equations (21) and (22).

$$\begin{aligned}\frac{\partial e^m}{\partial \alpha_d} &= -\gamma v'^{-1} \left(\frac{1}{\beta R_e} \right) \frac{1}{\beta R_e} < 0 \\ \frac{\partial e^s}{\partial \alpha_d} &= 0\end{aligned}$$

(v) The total loan amount decreases given equation (23)...

$$\begin{aligned}\ell(\alpha_d) &= \left[\frac{R_{e_b} + \lambda(\alpha_d)}{A\eta \left(1 - \frac{1-\eta}{B}\right)} \right]^{-1/(1-\eta)} \\ \frac{\partial \ell(\alpha_d)}{\partial \alpha_d} &= -1/(1-\eta) \left[\frac{R_{e_b} + \lambda(\alpha_d)}{A\eta \left(1 - \frac{1-\eta}{B}\right)} \right]^{-\eta/(1-\eta)} \frac{1}{A\eta \left(1 - \frac{1-\eta}{B}\right)} \frac{\partial \lambda}{\partial \alpha_d} > 0\end{aligned}$$

... and the interest rate on loans decreases given equation (24)

$$\begin{aligned}R_\ell &= \frac{R_{e_b} + \lambda(\alpha_d)}{1 - \frac{1-\eta}{B}} \\ \frac{\partial R_\ell(\alpha_d)}{\partial \alpha_d} &= \frac{1}{1 - \frac{1-\eta}{B}} \frac{\partial \lambda}{\partial \alpha_d} < 0\end{aligned}$$

(vi) The effect on total asset demand $d + \tau + e_m + e_s$ consists of the following individual

effects.

$$\begin{aligned}\frac{\partial d}{\partial \alpha_d} &= \underbrace{\gamma (\beta R_d)^{\frac{1-\sigma}{\sigma}}}_{>0} + \underbrace{\frac{1-\sigma}{\sigma} \gamma \alpha_d (\beta R_d)^{\frac{1-2\sigma}{\sigma}} \beta \frac{\partial R_d}{\partial \alpha_d}}_{<0} \geq 0 \\ \frac{\partial \tau}{\partial \alpha_d} &= \frac{1-\sigma}{\sigma} (1-\gamma) \alpha_\tau \beta^{\frac{1}{\sigma}} R_\tau^{\frac{1-2\sigma}{\sigma}} \frac{\partial R_\tau}{\partial \alpha_d} < 0 \\ \frac{\partial e_m}{\partial \alpha_d} &= -\gamma (\beta R_e)^{\frac{1-\sigma}{\sigma}} < 0 \\ \frac{\partial e^s}{\partial \alpha_d} &= 0\end{aligned}$$

Thus, the effect of α_d on total asset demand depends on parameter values.

As defined above GDP is

$$Y = A\ell^n + d + e^m + \tau + e^s + \beta(dR_d + e^m R_e)$$

and thus

$$\frac{\partial Y}{\partial \alpha_d} = \eta A \ell^{n-1} \underbrace{\frac{\partial \ell}{\partial \alpha_d}}_{>0} + \underbrace{\frac{\partial d}{\partial \alpha_d}}_{\geq 0} + \underbrace{\frac{\partial e^m}{\partial \alpha_d}}_{<0} + \underbrace{\frac{\partial \tau}{\partial \alpha_d}}_{<0} + \beta \left[\underbrace{\frac{\partial d}{\partial \alpha_d}}_{\geq 0} R_d + d \underbrace{\frac{\partial R_d}{\partial \alpha_d}}_{<0} + \underbrace{\frac{\partial e^m}{\partial \alpha_d}}_{<0} R_e \right] \geq 0$$

such that also the effect of α_d on GDP depends on parameter values.

(vii) For the entrepreneurs' profit Π^e we have

$$\begin{aligned}\Pi^e &= A\ell^n - \ell R_\ell \\ \frac{\partial \Pi^e}{\partial \alpha_d} &= \eta A \ell^{n-1} \frac{\partial \ell}{\partial \alpha_d} - \frac{\partial \ell}{\partial \alpha_d} R_\ell - \ell \frac{\partial R_\ell}{\partial \alpha_d} \\ &= \frac{\partial \ell}{\partial \alpha_d} \underbrace{(\eta A \ell^{n-1} - R_\ell)}_{=0} - \ell \underbrace{\frac{\partial R_\ell}{\partial \alpha_d}}_{<0} > 0.\end{aligned}$$

Due to the Cournot competition in the loan market in which the entrepreneurs and the bankers “share” the profit, it follows that an increase in the entrepreneurs' and the bankers' profits cooccurs.

(viii) The individual consumption of the respective buyer types is defined by

$$\begin{aligned}
C_d &= \beta R_d d_j = \beta R_d (\beta R_d)^{\frac{1-\sigma}{\sigma}} = (\beta R_d)^{\frac{1}{\sigma}} \\
C_{e^m} &= \beta R_e e_j^m = \beta R_e (\beta R_e)^{\frac{1-\sigma}{\sigma}} = (\beta R_e)^{\frac{1}{\sigma}} \\
C_\tau &= \beta R_d d_j = R_\tau \beta^{\frac{1}{\sigma}} R_\tau^{\frac{1-\sigma}{\sigma}} = (\beta R_\tau)^{\frac{1}{\sigma}} \\
C_{e^s} &= \beta R_e e_j^s = R_e \beta^{\frac{1}{\sigma}} R_e^{\frac{1-\sigma}{\sigma}} = (\beta R_e)^{\frac{1}{\sigma}}
\end{aligned}$$

It follows

$$\begin{aligned}
\frac{\partial C_d}{\partial \alpha_d} &= \frac{1}{\sigma} (\beta R_d)^{\frac{1-\sigma}{\sigma}} \frac{\partial R_d}{\partial \alpha_d} < 0 \\
\frac{\partial C_\tau}{\partial \alpha_d} &= \frac{1}{\sigma} (\beta R_\tau)^{\frac{1-\sigma}{\sigma}} \frac{\partial R_\tau}{\partial \alpha_d} < 0 \\
\frac{\partial C_{e^m}}{\partial \alpha_d} &= \frac{\partial C_{e^s}}{\partial \alpha_d} = 0
\end{aligned}$$

■

Appendix B Derivations

B.1 Derivation of equation (15)

From equation (10) I have $R_\ell(\ell) = f'(\ell)$ and since $f(\ell) = A\ell^\eta$ I get $R_\ell(\ell) = \eta A\ell^{\eta-1}$.

Furthermore, $\ell = \ell_b + \sum \ell'_b$. I can use this to rearrange the bank's first FOC, ie (2),

$$\begin{aligned}
\lambda &= \frac{\partial R_\ell(\ell)}{\partial \ell_b} \ell_b + R_\ell(\ell) - R_{e_b} \\
&= \frac{\partial \eta A (\ell_b + \sum \ell'_b)^{\eta-1}}{\partial \ell_b} \ell_b + \eta A \ell^{\eta-1} - R_{e_b} \\
&= (\eta - 1) \eta A (\ell_b + \sum \ell'_b)^{\eta-2} \ell_b + \eta A \ell^{\eta-1} - R_{e_b} \\
&= (\eta - 1) \eta A \ell^{\eta-2} \ell_b + \eta A \ell^{\eta-1} - R_{e_b}
\end{aligned}$$

In equilibrium we assume $\ell = B\ell_b$ such that:

$$\begin{aligned}\lambda &= (\eta - 1)\eta A\ell^{\eta-2}\ell/B + \eta A\ell^{\eta-1} - R_{e_b} \\ \lambda &= \eta A\ell^{\eta-1} \left(\frac{\eta - 1}{B} + 1 \right) - R_{e_b}\end{aligned}$$

B.2 Derivation of equation (16)

First I rearrange equation (5) assuming that $\lambda > 0$ and using the symmetric equilibrium assumption

$$\begin{aligned}0 &= \lambda(d_b(1 - \omega) + \tau_b - \ell_b) \\ \ell_b &= d_b(1 - \omega) + \tau_b \\ \ell &= d(1 - \omega) + \tau.\end{aligned}$$

Then I use the deposit demand equations (6) and (8) to get

$$\ell = \gamma\alpha_d v'^{-1} \left(\frac{1}{\beta R_d} \right) \frac{1}{\beta R_d} (1 - \omega) + (1 - \gamma)\alpha_\tau U'^{-1} \left(\frac{1}{\beta R_\tau} \right) \frac{1}{R_\tau}$$

and using the bank's FOCs (3) and (4) yields

$$\begin{aligned}\ell &= \gamma\alpha_d v'^{-1} \left(\frac{1}{\beta(R_{e_b} + \lambda(1 - \omega) - c)} \right) \frac{1}{\beta(R_{e_b} + \lambda(1 - \omega) - c)} (1 - \omega) \\ &+ (1 - \gamma)\alpha_\tau U'^{-1} \left(\frac{1}{\beta(R_{e_b} + \lambda - c)} \right) \frac{1}{(R_{e_b} + \lambda - c)}\end{aligned}$$

Lastly, I plug this into equation (15)

$$\begin{aligned}\lambda &= \eta A \left[\gamma\alpha_d v'^{-1} \left(\frac{1}{\beta(R_{e_b} + \lambda(1 - \omega) - c)} \right) \frac{1}{\beta(R_{e_b} + \lambda(1 - \omega) - c)} (1 - \omega) \right. \\ &\left. + (1 - \gamma)\alpha_\tau U'^{-1} \left(\frac{1}{\beta(R_{e_b} + \lambda - c)} \right) \frac{1}{(R_{e_b} + \lambda - c)} \right]^{\eta-1} \left(\frac{\eta - 1}{B} + 1 \right) - R_{e_b}\end{aligned}$$

B.3 Derivation of Equilibrium Equations

Take equation (15), ie

$$R_{e_b} + \lambda = \eta A \ell^{\eta-1} \left(\frac{\eta-1}{B} + 1 \right).$$

To get equation (23), solve for ℓ

$$\begin{aligned} R_{e_b} + \lambda &= \eta A \ell^{\eta-1} \left(\frac{\eta-1}{B} + 1 \right) \\ \ell^{1-\eta} &= \frac{\eta A \left(\frac{\eta-1}{B} + 1 \right)}{R_{e_b} + \lambda} \\ \ell &= \left(\frac{\eta A \left(1 - \frac{1-\eta}{B} \right)}{R_{e_b} + \lambda} \right)^{\frac{1}{1-\eta}}. \end{aligned}$$

To get equation (24), use that $R_\ell = \eta A \ell^{\eta-1}$, plug it into equation (15) above and solve for R_ℓ

$$\begin{aligned} R_{e_b} + \lambda &= R_\ell \left(\frac{\eta-1}{B} + 1 \right) \\ R_\ell &= \frac{R_{e_b} + \lambda}{1 - \frac{1-\eta}{B}} \end{aligned}$$

Appendix C Data

To calculate the historical interest rates on deposits and loans I use FDIC call report data which contains bank level balance sheets and income statements. Quarterly historical data for all banks is available for bulk download here: <https://www.fdic.gov/foia/ris/>. The data contains information on interest expenses for transaction and saving deposits and interest income on loans. The interest expense and income is required to be reported in year-to-date form. Thus, I first recalculated quarterly expenses and earnings for all banks over the time period.

For the transaction deposit amount I use total transaction deposits. Saving deposits include regular saving deposits and money market deposit accounts (MMDAs). Next, I divided the

expenses by the corresponding deposit and loan amounts to get an estimate of the quarterly interest rates. There are outliers in the data on the upside and on the downside which seem to be inconsistent. Thus I exclude the top 10% and bottom 10% of the corresponding interest rates. I then take the sample average of the remaining observations to get an estimate of the deposit interest rates.

For the loan amount, I consider total loans in domestic offices. Furthermore, I follow Chiu et al. (2019) and only consider the bottom 25% of the calculated bank level loan interest rates since I do not model risky loans. Hence it seems reasonable to exclude the higher interest rates which might result from riskier loans. Furthermore, due to inconsistent outliers I again exclude the bottom 10% of observations. I then take the sample average to get an estimate for the interest rates on loans.

The model is calibrated using real interest rates. Hence, the interest rates are adjusted for inflation using price level data from FRED. Lastly, the Federal Funds Rate is taken from FRED as well. The data used for interest rates is illustrated in table 9.

Variable	Data Source	Mnemonic
Transaction Deposits Expense	FDIC	ETRANDP; RIAD4508
Transaction Deposit Amount	FDIC	TRN; RCON2215
Saving Deposits Expense	FDIC	ESAVDP; RIAD0093
Saving Deposits Amount	FDIC	AVSAVDP; RCONB563
Loans Expense	FDIC	ILN; RIAD4010
Loan Amount	FDIC	AVLN; RCON3360
Federal Funds Rate	FRED	DFF
Price Level	FRED	CPILFESL

Table 9: Data sources used to calculate interest rates. The first FDIC mnemonics (eg. ETRANDP) are the once used in the bulk download data. The second FDIC mnemonics are the once that are used in the call reports (eg. RIAD4508). A mapping can be found here: <https://www7.fdic.gov/DICT/app/templates/Index.html#!/Main>

Furthermore I need data to match the relationship between deposits, currency and money demand. I here do not use FDIC data but aggregated data series from FRED.

For cash holdings, I assume that all 100\$-bills are used as savings whereas all smaller denominations are used for payments. FRED provides series for the different denominations, which I use to calculate the relationship between e^m and e^s . However this series is only available back to 1993. On FRED there is a series on the currency component of M1 which is available

Variable	Data Source	Mnemonic
Transaction Deposit Amount	FRED	WDDNS
Saving Deposits Amount	FRED	WSAVNS
Currency component of M1	FRED	WCURRNS
Currency denominations ($\{X\}$ \$)	FRED	CURRVAL $\{X\}$
GDP	FRED	GDP

Table 10: Other data sources

back to 1975. I combine this series with data from Feige (2012) (figure 2) which shows the percentage of currency by denomination back to 1964. Thus I can extend the cash series backwards. Furthermore, it is well known that a lot of US-currency is held abroad. I want to restrict the analysis to currency that is held within the US and use estimates from Judson (2017) of currency holdings abroad to adjust for that. Table 10 summarizes the remaining data sources.

Appendix D Misc

D.1 Model Timeline

Below I give a more intuitive explanation of the model. Figure D.1 shows the basic timeline. The model has discrete time periods $(\dots, t-1, t, t+1, \dots)$ and it continues forever. Agents discount between time periods with the discount factor β . In each time period there are two subperiods, a centralized market (CM) and a decentralized market (DM), ie the part of the model in line with Lagos and Wright (2005). The CM is a centralized, Walrasian market in which all agents interact and trade with each other. The DM is a decentralized market in which only bilateral trades take place. Moreover, there are generations in the model in line with the overlapping generation model. Generation t is born at the beginning of time period t and lives through time period t until it dies in the middle of time period $t+1$ at the end of the first subperiod. There are two goods in the economy, the CM good x and the DM good y . Both are perishable which means that they cannot be stored and consumed in a later period or subperiod.

There are four types of agents in the model which is illustrated in figure D.2. Buyers are

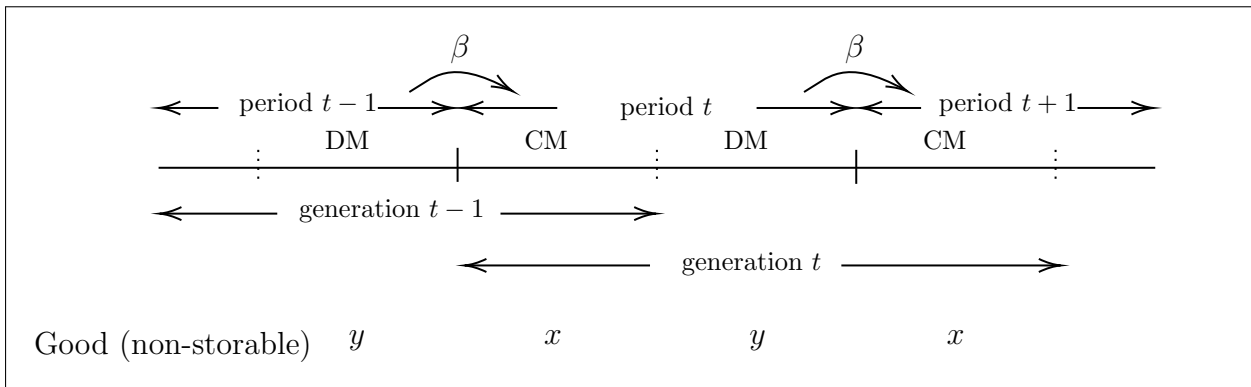


Figure D.1: Basic Timeline.

split in medium of exchange buyers (MoE) and saving buyers. Both types can only work and produce good x when they are young in the CM. However, they want to consume later in life. The medium of exchange buyer wants to consume one subperiod later and the saving buyer in the next period. Since the goods are non-storable the buyers need to transfer consumption in another way to the later periods. Thus, the MoE-buyer works, sells x and receives a medium of exchange (central bank money or liquid transaction deposits) and the savings-buyer receives a saving vehicle (central bank money or illiquid saving deposits). The MoE (saving) buyer can use the medium of exchange (saving vehicle) to acquire goods in the DM (subsequent CM).

Sellers are infinitely lived. They can only work in the DM and produce good y , but want to consume in the CM. Since y cannot be stored, they demand a medium of exchange which can be acquired in the DM and used in exchange for good x in the next subperiod. Thus, MoE-buyers will pay the sellers with their medium of exchange holdings and get good y in return in the DM.

Bankers live for one period. The banker issues loans when young whereby deposits are created endogenously. The bankers pay an interest on deposits and charge an interest rate on loans. The bankers offer two kind of deposits, liquid transaction deposits and illiquid saving deposits. A minimum reserve requirement applies on liquid transaction deposits. This means that a bank needs to hold a certain exogenous fraction as reserves for each unit of transaction deposit it offers. An interest rate is paid on the reserves held. Saving deposits are not allowed to be spent in the DM by assumption. Furthermore, I assume that there is perfect competition in the deposit market and imperfect competition in the loan market.

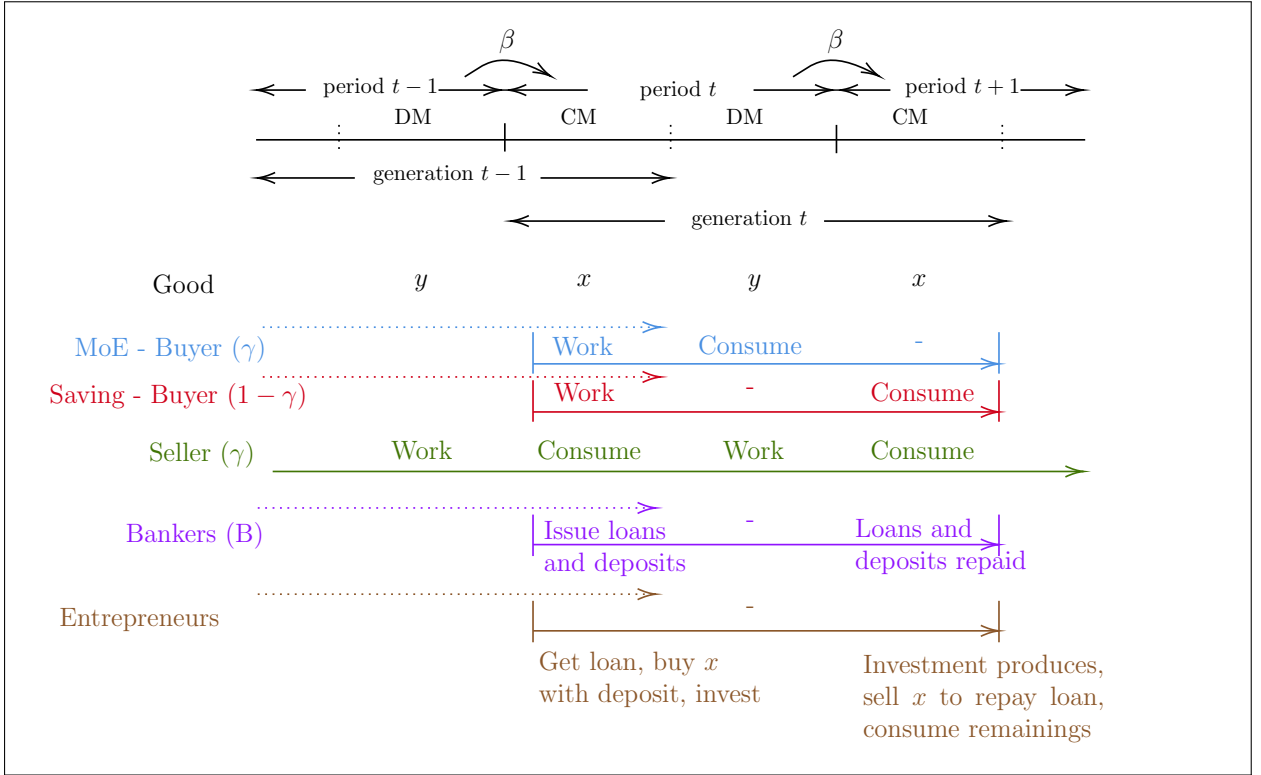


Figure D.2: Timeline with an overview of the different agents.

Entrepreneurs are one period lived as well. They have an investment opportunity but they do not have an endowment and cannot work. Thus, they need to get a loan from a bank. If the bank hands out a loan to the entrepreneur, it credits the amount to the entrepreneur's bank account. The entrepreneur can then use the money on the bank account to buy the good x on the centralized market. After one period the investment produces. The entrepreneur can then sell some of the return in the subsequent centralized market to get bank deposits again to pay back the loan. The remainings of the return is consumed.

D.2 Distributions over the Premium

Given equal interest rates on deposits and CBDC, a draw of $\varepsilon > 0$ implies that an agent prefers CBDC over deposits. $G^m(\varepsilon)$ is the resulting normal distribution for the medium of exchange types where $\mu^M = -0.14$ and $\sigma^M = 0.17$. $G^s(\varepsilon)$ is the normal distribution for the saving types where $\mu^S = -0.71$ and $\sigma^S = 0.59$. The hatched areas represent the corresponding share values, ie $\alpha_d = \int_{-\infty}^{R_d/R_e-1} dG^m(\varepsilon) = 0.83$ and $\alpha_\tau = \int_{-\infty}^{R_\tau/R_e-1} dG^s(\varepsilon) = 0.90$ in the calibrate equilibrium.

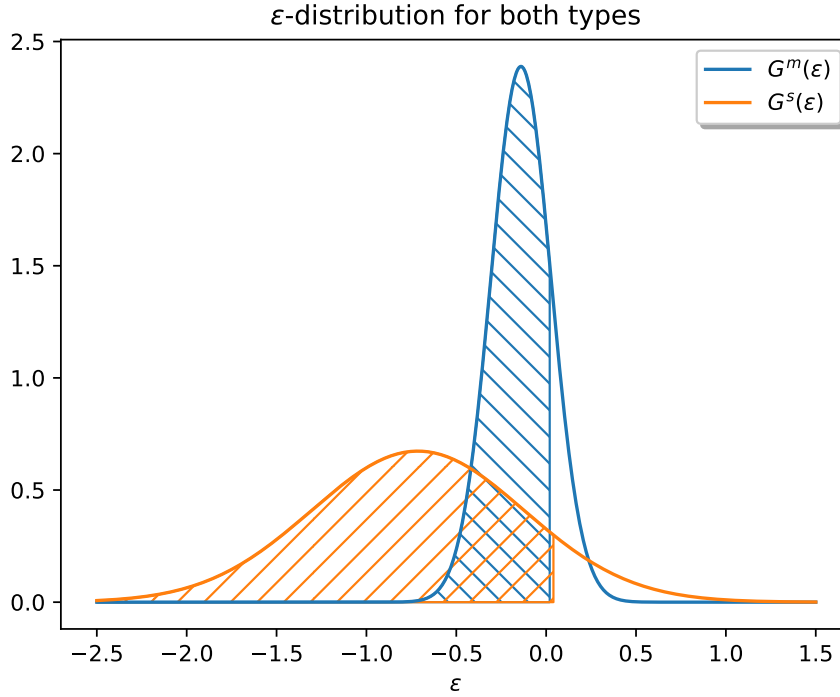


Figure D.3: Distribution of the premium ε

D.3 Effect of Change in the Interest Rate on Reserves

The effect of a change in the interest rate on reserves i_{e_b} is illustrated in figure D.4. A decrease in i_{e_b} compared to the calibrated equilibrium has almost no effect on the interest rate on saving deposits and loans. This is because a lower i_{e_b} increases the Lagrangian parameter λ on the reserve requirement constraint by about the same amount as i_{e_b} is decreased. From equations (2-4) it follows that a change in i_{e_b} affects the interest rate on transaction deposits only by $\lambda(1 - \omega)$. Thus the effect of a lower i_{e_b} is in that case not fully compensated which leads to i_d decreasing.

If the interest rate on reserves i_{e_b} is increased compared to the calibrated equilibrium, the minimum reserve requirement constraint becomes loose (ie $\lambda = 0$) at around $i_{e_b} = 6.59\%$. For an interest rate on reserves higher than this, an increase in i_{e_b} transmits one-to-one to an increase in the deposit and loan rates. The constraint becoming loose means that the bankers hold the profit-maximizing loan amount. If i_{e_b} is further increased, bank lending will decrease because it is optimal for the bankers to shift some of their assets from loans to reserves.

Even though an increase of i_{e_b} above the level where the constraint becomes loose increases

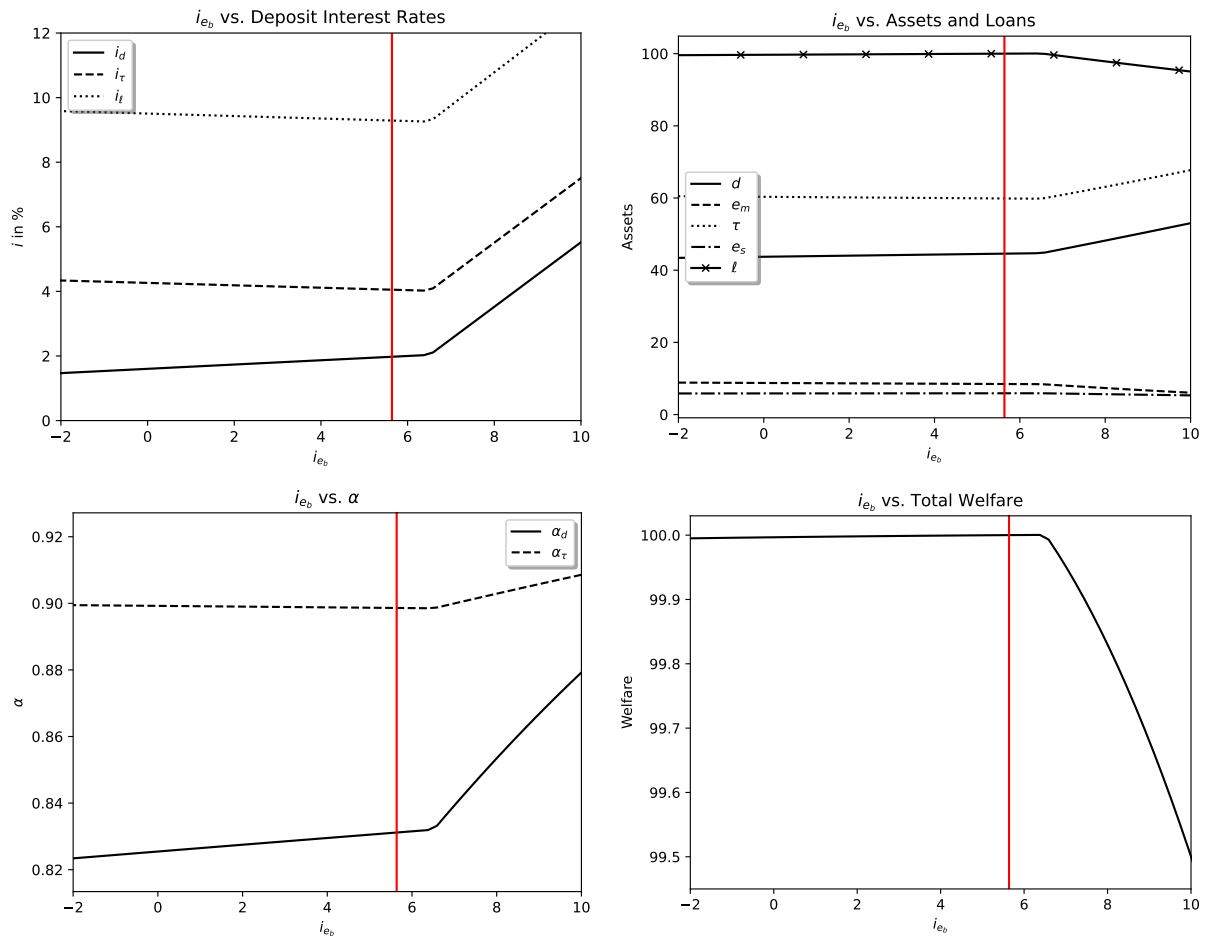


Figure D.4: Effect of change in the interest rate on reserves on deposit interest rates, assets, loans, the fraction of agents holding deposits and welfare.

deposits on the intensive margin (higher interest rates) and on the extensive margin (higher α_d and α_τ), the overall effect is negative because bankers hold too much reserves. This becomes very clear when looking at the bottom right figure which shows total welfare. Besides a decline in bank lending upon increasing i_{e_b} too much, the higher interest rate expense is levied on the seller through the tax T which leads to a higher disutility of the seller. Even though the buyers benefit due to higher interest rates, the other effects dominate and what follows is a negative effect on total welfare.

The monetary policy implication would be that the central bank should not pay a interest rate on reserves that is too high and incentivises bankers to hold levels of excess reserves which lead to a welfare-wise suboptimal asset allocation.

D.4 3d-Figures Endogenous Asset Choice

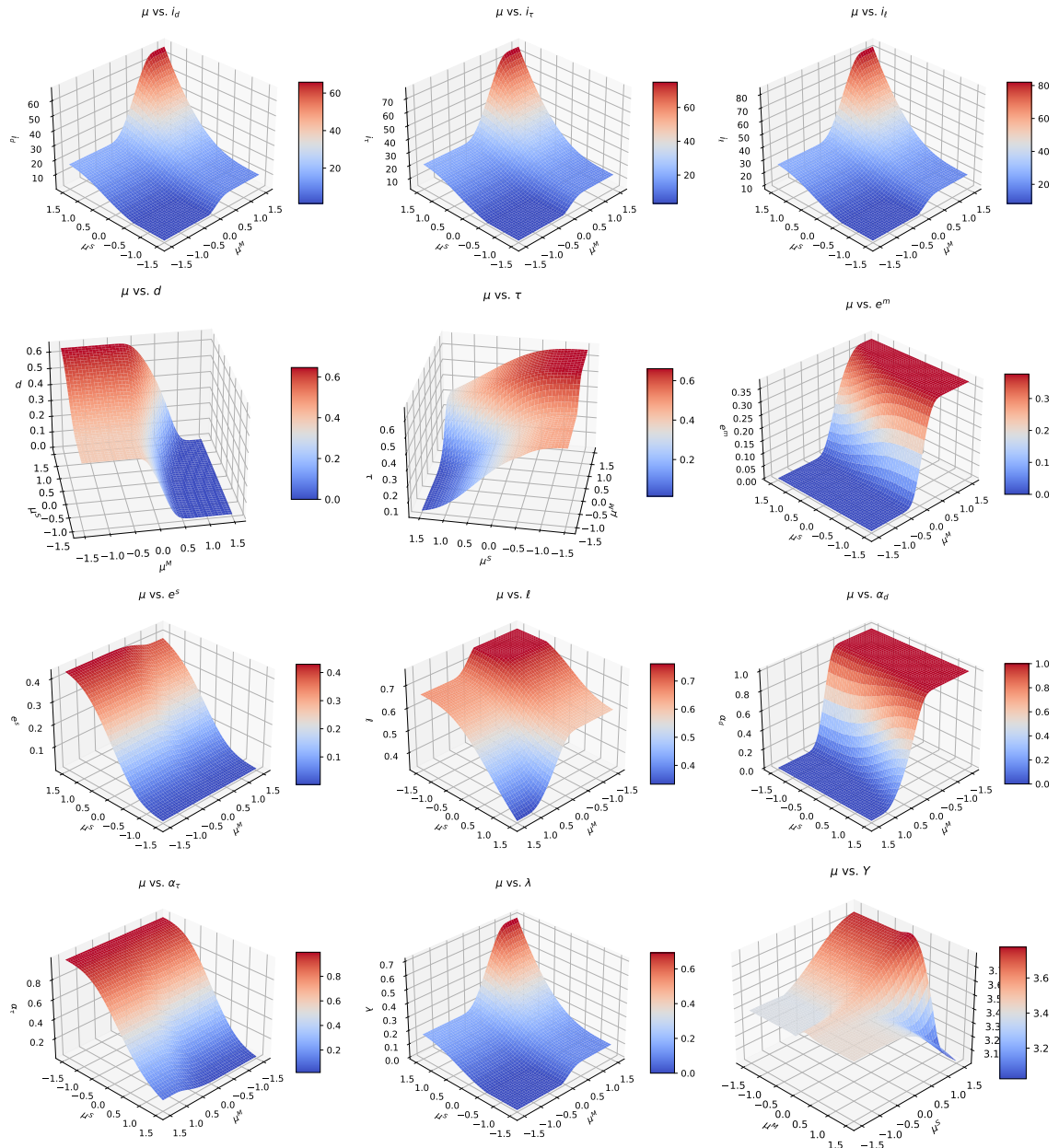


Figure D.5: Effect of preference shift over central bank money and bank deposits on nominal interest rates of transaction deposits (i_d), saving deposits (i_τ) loans (i_ℓ), total amount of transaction deposits (d), saving deposits (τ), central bank money held as payment vehicle (e^m), central bank money held as saving vehicle (e^s), loans (ℓ), the fraction of agents holding deposits as payment vehicle (α_d), as saving vehicle (α_τ), the Lagrange multiplier (λ) and GDP (Y). Note that the axis differ from figure to figure. Remember that a higher μ means that there is a preference shift from deposits to central bank money.