

Irmen, Andreas; Klump, Rainer

**Working Paper**

## Factor substitution, income distribution, and growth in a generalized neoclassical model

CESifo Working Paper, No. 2148

**Provided in Cooperation with:**

Ifo Institute – Leibniz Institute for Economic Research at the University of Munich

*Suggested Citation:* Irmen, Andreas; Klump, Rainer (2007) : Factor substitution, income distribution, and growth in a generalized neoclassical model, CESifo Working Paper, No. 2148, Center for Economic Studies and ifo Institute (CESifo), Munich

This Version is available at:

<https://hdl.handle.net/10419/26192>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

FACTOR SUBSTITUTION, INCOME DISTRIBUTION,  
AND GROWTH IN A GENERALIZED  
NEOCLASSICAL MODEL

ANDREAS IRMEN  
RAINER KLUMP

CESIFO WORKING PAPER NO. 2148  
CATEGORY 5: FISCAL POLICY, MACROECONOMICS AND GROWTH  
NOVEMBER 2007

*An electronic version of the paper may be downloaded*

- *from the SSRN website:* [www.SSRN.com](http://www.SSRN.com)
- *from the RePEc website:* [www.RePEc.org](http://www.RePEc.org)
- *from the CESifo website:* [www.CESifo-group.org/wp](http://www.CESifo-group.org/wp)

# FACTOR SUBSTITUTION, INCOME DISTRIBUTION, AND GROWTH IN A GENERALIZED NEOCLASSICAL MODEL

## Abstract

We analyze a generalized neoclassical growth model that combines a normalized CES production function and possible asymmetries of savings out of factor incomes. This generalized model helps to shed new light on a recent debate concerning the impact of factor substitution and income distribution on economic growth. We can show that this impact relies on both an efficiency and an acceleration effect, where the latter is caused by the distributional consequences of an increase in the elasticity of substitution. While the efficiency effect is always positive, the direction of the acceleration effect depends on the particular savings hypothesis. However, if savings out of capital income are substantial so that a certain threshold value is surpassed we find that the efficiency effect dominates and higher factor substitution can work as a major engine of growth.

JEL Code: E21, O11, O41.

Keywords: capital accumulation, elasticity of substitution, income distribution, neoclassical growth model.

*Andreas Irmen*  
*University of Heidelberg*  
*Alfred-Weber-Institute*  
*Grabengasse 14*  
*69117 Heidelberg*  
*Germany*  
*airmen@uni-hd.de*

*Rainer Klump*  
*University of Frankfurt*  
*Department of Economics*  
*Schumannstr. 60*  
*60325 Frankfurt / Main*  
*Germany*  
*klump@wiwi.uni-frankfurt.de*

# 1 Introduction

Aggregate models of economic growth are based on two pillars, namely an assumption how to model aggregate production and an assumption how capital accumulation is fueled by aggregate saving decisions. The first pillar centers around the concept of substitutability between factor inputs, whereas the second deals with the distribution of factor incomes. Some prominent examples show that the assumptions on both of these pillars can be either very general or rather narrow. The standard neoclassical growth model by Solow (1956) combines the very general concept of an aggregate production function that allows for substitutability between the factors labor and capital with a constant saving ratio out of total factor income. As a general functional form to model aggregate production with various degrees of factor substitution, Solow invented what later became known as the CES production function (Arrow, Chenery, Minhas, and Solow (1961)). In contrast, the growth model proposed by Kaldor (1956) is based on the narrow Post-Keynesian assumption of a limitational production function, but at the same time assumes quite generally that savings out of labor and capital income differ. As a consequence, aggregate saving is strongly influenced by the distribution of factor incomes. The latter is also central to the overlapping-generations (OLG) version of the neoclassical growth model (Diamond (1965)), where aggregate saving stems solely from wage income. Similarly, it matters under the “classical savings hypothesis,” as used, e. g., by Uzawa (1961), that regards only capital income as the source of aggregate savings.

In this paper, we explore a neoclassical growth model that incorporates the most general assumptions on both pillars, i. e., various degrees of substitutability in the aggregate production function and possible asymmetries of savings out of factor incomes. Our generalized model helps to shed new light on a recent debate concerning the impact of factor substitution and income distribution on economic growth. This debate began with the contributions by de La Grandville (1989) and Klump and de La Grandville (2000) studying the link between the elasticity of substitution, being treated as a parameter of a normalized CES production function, and growth in the Solow model. They come to the conclusion that the degree of factor substitution is a powerful engine of growth in the sense that a higher elasticity of substitution leads to a higher growth rate and a higher steady-state level of per-capita income. The relevance of this conclusion has been challenged when Miyagiwa and Papageorgiou (2003) explored the growth effects of the elasticity of substitution in a discrete-time OLG framework and did not find a monotonic relationship but rather report cases where a higher elasticity of substitution would also have a negative impact on growth. These results have been confirmed by Irmen (2003) in the context of a Diamond-type growth model set out in continuous time. His explanation of the controversial results is based on the distinction between two effects

which are caused by a change in the aggregate elasticity of factor substitution: an *efficiency effect* by which changes in factor substitution influence the productivity of factor inputs and an *acceleration effect* that relates changes in factor substitution to the evolution of the capital intensity.

In our general model, we demonstrate more broadly how these two effects interact. Moreover, we show that a *distribution effect* surfaces in the presence of differing savings rates out of wage and profit/capital incomes. As a consequence the direction of the acceleration effect depends on the particular saving hypothesis. If the distribution effect is negative, the sign of the overall effect results from its strength relative to the efficiency effect. In growing Diamond-type economies, the tension between these two countervailing forces explains the negative acceleration effect reported in Irmen (2003) and Miyagiwa and Papageorgiou (2003). In the general case, where also capital income is a source of aggregate savings, the direction and size of the overall effect is determined not only by the elasticity of substitution and the different saving ratios but also by the baseline values for capital, production per capita, and the income distribution. Moreover, as long as the savings ratio out of profit income is not lower than the savings ratio out of wages, or that it at least surpasses a certain threshold value conditional on the various parameters of the model, the growth effects of higher factor substitution remain positive.

The rest of the paper is organized as follows. Section 2 briefly highlights and recalls some important analytical properties of normalized CES production functions. In particular, we clarify in what sense the normalized CES allows us to isolate the effect of the elasticity of substitution on the growth process. Section 3 introduces our generalized neoclassical growth model and studies the effects of changing the elasticity of substitution on the growth process. We start by looking at local effects and then proceed to a global analysis. Section 4 concludes. All proofs are relegated to an appendix.

## 2 Normalized CES Production Function and Per-Capita Output

We consider the following CES per-capita production function

$$y = f(k) = A [ak^\psi + (1 - a)]^{1/\psi}, \quad (1)$$

with  $A > 0$ ,  $1 > a > 0$ ,  $1 > \psi > -\infty$ , and  $k$  denoting the capital-labor ratio. The parameter  $\sigma = 1/(1 - \psi)$  is the elasticity of substitution. Following Klump and de La Grandville (2000), we normalize (1) by choosing some baseline capital-labor

ratio  $\bar{k} > 0$ , some level of per-capita output  $\bar{y}$ , and a marginal rate of substitution  $\bar{m} > 0$ . The normalized CES production function that satisfies these criteria can then be computed to equal

$$y = f_{\sigma}(k) = A(\sigma) [a(\sigma)k^{\psi} + (1 - a(\sigma))]^{1/\psi} \quad (2)$$

with

$$A(\sigma) \equiv \bar{y} \left( \frac{\bar{k}^{1-\psi} + \bar{m}}{\bar{k} + \bar{m}} \right)^{1/\psi} \quad \text{and} \quad a(\sigma) \equiv \frac{\bar{k}^{1-\psi}}{\bar{k}^{1-\psi} + \bar{m}}. \quad (3)$$

The normalization implies a capital share  $\pi_{\sigma}(k)$  with a baseline value  $\pi_{\sigma}(\bar{k}) \equiv \bar{\pi}$ , such that (2) may also be written as

$$y = f_{\sigma}(k) = \frac{\bar{y}}{\bar{k}} \left( \frac{\bar{\pi}}{\pi_{\sigma}} \right)^{1/\psi} k, \quad (4)$$

where

$$\bar{\pi} \equiv \frac{\bar{k}}{\bar{k} + \bar{m}} \quad \text{and} \quad \pi_{\sigma}(k) \equiv \frac{k^{\psi} \bar{k}^{1-\psi}}{k^{\psi} \bar{k}^{1-\psi} + \bar{m}}. \quad (5)$$

This alternative representation of the normalized CES function emphasizes that the initial functional income distribution and its evolution play a central role for the evolution of the economy.

In what follows, we denote partial derivatives with respect to  $k$  by a prime so that  $f'_{\sigma} := \partial f_{\sigma} / \partial k$  and  $\partial f''_{\sigma} / \partial \sigma := \partial^3 f_{\sigma} / \partial^2 k \partial \sigma$ . If not indicated otherwise, the argument of  $f_{\sigma}$  is  $k$ .

The interpretation that we can attach to changes of  $\sigma$  is based on the following implication of the above normalization.

**Lemma 1** The normalized CES production function  $f_{\sigma}(k)$  as given by (2) satisfies

$$\frac{\partial f''_{\sigma}(\bar{k})}{\partial \sigma} > 0. \quad (6)$$

Lemma 1 provides the first key to the understanding of the growth effects of the elasticity of substitution: at  $\bar{k}$  there is an inverse relationship between the elasticity of substitution and the curvature of the normalized CES production function. This relationship has an interpretation in terms of the degree of complementarity of both

input factors. Let  $Y_\sigma = F_\sigma(K, L)$  be the CES production function underlying (2) with  $K$  and  $L$  denoting aggregate capital and labor inputs. Then,

$$F_{\sigma KL} \equiv \frac{\partial^2 F_\sigma(K, L)}{\partial K \partial L} = -k f_\sigma''(k)$$

for all admissible  $(K, L)$  and  $k \equiv K/L$ . Moreover,

$$\left. \frac{\partial F_{\sigma KL}}{\partial \sigma} \right|_{(K,L)=(\bar{K}, \bar{L})} = -\bar{k} \frac{\partial f_\sigma''(\bar{k})}{\partial \sigma} < 0. \quad (7)$$

Therefore, at  $\bar{k}$  a higher elasticity of substitution implies a *lower degree of complementarity between capital and labor*. We show below that this property of the normalized CES, in conjunction with the savings hypothesis, drives the dynamics of capital accumulation and per-capita income in the neighborhood of the baseline value  $\bar{k}$ .

Klump and de La Grandville (2000), Theorem 1, establishes a key global property of (2), namely,

$$\frac{\partial f_\sigma(k)}{\partial \sigma} > 0 \text{ for } k \neq \bar{k}. \quad (8)$$

Hence, the elasticity of substitution has an interpretation as “a measure of the efficiency of the productive system” (de La Grandville (1989), p. 479) in the sense that the higher  $\sigma$ , the higher is per-capita output for any capital-labor ratio other than  $\bar{k}$ . We shall refer to (8) as the *efficiency effect*.

### 3 The Generalized Neoclassical Growth Model

We consider a competitive economy in continuous time, i.e.,  $t \in [0, \infty)$ . If not indicated otherwise the baseline values  $\bar{k}$ ,  $\bar{y}$ , and  $\bar{m}$  can be viewed as initial values of the economies under scrutiny. Marginal cost pricing implies a real wage and a real rate of return on capital equal to

$$w_\sigma(k) = f_\sigma(k) - k f_\sigma'(k), \quad (9)$$

$$r_\sigma(k) = f_\sigma'(k). \quad (10)$$

#### 3.1 Factor Substitution, Income Distribution, and Capital Accumulation

In our generalized model of growth which combines factor substitution à la Solow with possible asymmetries in savings out of factor incomes à la Kaldor, the speed

of capital accumulation is determined by the following equation of motion

$$\begin{aligned}
 \dot{k} &= s^w [f_\sigma(k) - k f'_\sigma(k)] + s^r [k f'_\sigma(k)] - nk, \\
 &= [s^w (1 - \pi_\sigma) + s^r \pi_\sigma] f_\sigma(k) - nk, \\
 &= [s^w (1 - \pi_\sigma) + s^r \pi_\sigma] \frac{\bar{y}}{\bar{k}} \left( \frac{\bar{\pi}}{\pi_\sigma} \right)^{1/\psi} k - nk.
 \end{aligned} \tag{11}$$

Here,  $s^w, s^r \in (0, 1)$  denote the marginal and average savings rates out of wage and capital/profit income, respectively, and  $n$  is the growth rate of the employed labor force. Moreover, equations (9), (10), and (4) were used to derive the expressions.

From (11), it is straightforward to derive the *acceleration effect* of the elasticity of substitution, i. e., its influence on the speed of capital accumulation, as

$$\frac{\partial \dot{k}}{\partial \sigma} = [s^w (1 - \pi_\sigma) + s^r \pi_\sigma] \frac{\partial f_\sigma(k)}{\partial \sigma} + f_\sigma(k) (s^r - s^w) \frac{\partial \pi_\sigma}{\partial \sigma}. \tag{12}$$

Equation (12) allows for a basic insight into the mechanics of our generalized neo-classical growth model. According to (8), the first term on the right-hand side is always positive and reflects the *efficiency effect* of a higher degree of factor substitution. A higher elasticity of substitution increases per-capita income which, for a given income distribution, raises savings. The second term on the right-hand side of (12) reflects the *distribution effect*,  $\partial \pi_\sigma / \partial \sigma$ . For a given level of per-capita output, this term captures the impact of the elasticity of substitution on the functional income distribution, and, in turn, on aggregate savings. We know from Klump and de La Grandville (2000) (see, their equation 10) that

$$\frac{\partial \pi_\sigma}{\partial \sigma} = \frac{\pi_\sigma (1 - \pi_\sigma)}{\sigma^2} \ln \left( \frac{k}{\bar{k}} \right). \tag{13}$$

Klump and Saam (2008) propose that the baseline capital intensity corresponds to the capital intensity that would be efficient if the economy's elasticity of substitution were zero. For  $k > \bar{k}$  the economy's relative bottleneck resides in its capacity to make productive use of additional capital. Relaxing this bottleneck by allowing for higher factor substitution (or lower complementarity) would then increase the capital income share. For  $k < \bar{k}$  the same would be true for labor and its income share.

Hence, for  $k > \bar{k}$  a rise in the elasticity of substitution raises the capital share. If, in addition,  $s^r > s^w$ , then such a rise shifts the income distribution in favor of capital income out of which a larger fraction is saved. Then, the channel via the efficiency



effect and the one via the distribution effect are positive.<sup>1</sup> As a result, aggregate saving increases and  $\partial \dot{k} / \partial \sigma > 0$ , i. e., the *acceleration effect* of the elasticity of substitution is positive.

### 3.2 Local Effects of Higher Factor Substitution

Clearly, the acceleration effect need neither be positive nor monotonic for all  $k > 0$  and  $\sigma > 0$ . However, the following proposition establishes that in a small neighborhood of  $\bar{k}$ , the acceleration effect is indeed monotonic for all admissible values of  $k$ . Moreover, its driving force is the change in the degree of complementarity identified in Lemma 1.

**Proposition 1** Let  $k$  belong to a sufficiently small neighborhood of  $\bar{k}$  and define  $\dot{k} = s^w w_\sigma(k) + s^r k r_\sigma(k) - nk \equiv \dot{k}_\sigma(k)$ .

1. If  $s^w \neq s^r$ , then

$$\frac{\partial \dot{k}_\sigma(k)}{\partial \sigma} \simeq \frac{\partial f''_\sigma(\bar{k})}{\partial \sigma} \bar{k} (s^r - s^w) (k - \bar{k}). \quad (14)$$

2. If  $s^w = s^r = s$ , then

$$\frac{\partial \dot{k}_\sigma(k)}{\partial \sigma} \simeq \frac{s}{2} \frac{\partial f''_\sigma(\bar{k})}{\partial \sigma} (k - \bar{k})^2 > 0. \quad (15)$$

According to Proposition 1, the acceleration effect is monotonic in the neighborhood of  $\bar{k}$ , i. e., either positive or negative for all admissible  $k$ . In the general case, if  $s^w \neq s^r$ , what matters is how the savings rates relate to the change in the relative scarcity of capital. More precisely, we have

$$\frac{\partial \dot{k}_\sigma(k)}{\partial \sigma} \geq 0 \quad \Leftrightarrow \quad (s^r - s^w) (k - \bar{k}) \geq 0. \quad (16)$$

This result generalizes previous findings derived for  $s^r = 0$  to the case where  $s^r > 0$  (see, e. g., Irmen (2003), Proposition 1). The presence of the term  $s^r - s^w$  suggests that the distribution effect drives the sign of the acceleration effect. Indeed, for a growing economy where  $k > \bar{k}$ , we learn from the proof of Proposition 1 that a

---

<sup>1</sup>For the same reasons, the channel via the distribution effect is also positive if  $k < \bar{k}$  and  $s^r < s^w$ .

rise in  $\sigma$  increases the rate of return on capital and lowers the wage. Since there is no first-order effect of  $\sigma$  on aggregate income at  $\bar{k}$ , the wage income falls whereas capital income increases. As a consequence, the acceleration effect is positive for  $s^r > s^w$  and negative for  $s^w > s^r$ .

In the usual neoclassical (Solow) case, where  $s^w = s^r = s$ , the distribution effect has no bite. For this case, Proposition 1 provides a new (local) rationale for Theorem 1 in Klump and de La Grandville (2000): the comparative static of Lemma 1 has initially a positive second-order effect on the speed of capital accumulation. Therefore, the sign of the acceleration effect is positive for growing and shrinking economies, i. e., it is independent of  $k \gtrless \bar{k}$ .

We can use these findings to determine the local effect of the elasticity of substitution on the evolution of per-capita income. From  $y(t) = f_\sigma(k(t))$ , we have  $\dot{y}(t) = f'_\sigma(k(t)) \dot{k}(t)$  such that

$$\frac{\partial \dot{y}}{\partial \sigma} = \frac{\partial f'_\sigma}{\partial \sigma} \dot{k} + f'_\sigma \frac{\partial \dot{k}}{\partial \sigma}. \quad (17)$$

The right-hand side shows two channels. First, each unit of capital accumulated between today and tomorrow may have a higher or a lower marginal product depending on whether the *marginal productivity effect*  $\partial f'_\sigma / \partial \sigma$  is positive or not. Second, for a given marginal product of capital, the amount of capital accumulated between today and tomorrow changes in accordance with the *acceleration effect*.

**Proposition 2** Let  $k$  belong to a sufficiently small neighborhood of  $\bar{k}$  and define  $\dot{y} = f'_\sigma(k) \dot{k}_\sigma(k) \equiv \dot{y}_\sigma(k)$ . Then,

$$\frac{\partial \dot{y}_\sigma(k)}{\partial \sigma} \gtrless 0 \quad \Leftrightarrow \quad \left( 2\bar{\pi} \left( \frac{s^r}{s^w} - 1 \right) + \frac{1}{s^w \bar{y}} (s^w \bar{y} - n\bar{k}) \right) (k - \bar{k}) \gtrless 0 \quad (18)$$

Proposition 2 encompasses several interesting cases. For instance, in a growing economy, where  $s^r > s^w$ , the expression (18) is strictly positive. Hence, economies with a higher elasticity of substitution have a higher per-capita income as long as  $k$  remains in the admissible neighborhood. This finding is quite intuitive since in this scenario the acceleration effect is positive by Proposition 1, and a higher factor substitution increases the marginal product of capital (see, equation (34) in the proof of Proposition 2).

Moreover, Proposition 2 may be used to determine a critical savings rate,  $s_c^r$ , such that  $\partial \dot{y}_\sigma(k) / \partial \sigma > 0$  in a growing economy. One finds that

$$s^r > s_c^r \equiv \max \left[ s^w \left( 1 - \frac{1}{\bar{\pi}} \right) + \frac{n\bar{k}}{\bar{\pi}\bar{y}}, s^w \left( 1 - \frac{1}{2\bar{\pi}} \right) + \frac{n\bar{k}}{2\bar{\pi}\bar{y}}, 0 \right] \quad (19)$$

is sufficient for this. Here, the first term in brackets makes sure that  $\dot{k}(\bar{k}) > 0$ , and the second assures that the effect of (18) is positive for  $k > \bar{k}$ . The critical savings rate depends, inter alia, on the chosen baseline values and may fall short of  $s^w$ . If we conclude, invoking the empirical findings of e.g. Bernanke and Gürkaynak (2001), that  $\bar{\pi} \approx 1/3$ , and take  $n \approx 0$  as an approximation for many industrialized countries, condition (19) is satisfied whenever  $s^r > 0$ .

If  $s^r = 0$ , Proposition 2 predicts that economies with a higher elasticity of substitution may have a lower per-capita income. For instance, in a growing economy with  $\bar{\pi} < 1/2$ , the precise condition for  $\partial \dot{y}_\sigma(k) / \partial \sigma < 0$  is  $n\bar{k}/\bar{y}(1 - 2\bar{\pi})^{(-1)} > s^w > n\bar{k}/\bar{y}$ . The second inequality assures that the economy initially grows. The first makes sure that the effect in (18) is strictly negative for  $k > \bar{k}$ . Intuitively, in a growing economy this possibility arises since the acceleration effect of Proposition 1 becomes negative for  $k > \bar{k}$ . This finding confirms results found by Irmen (2003) and Miyagiwa and Papageorgiou (2003) for Diamond-like economies.

For a Solow economy, Proposition 2 is consistent with the findings of Klump and de La Grandville (2000). Indeed, for  $s^r = s^w = s$ , (18) reduces to

$$\frac{\partial \dot{y}_\sigma(k)}{\partial \sigma} > 0 \quad \Leftrightarrow \quad \dot{k}_\sigma(\bar{k})(k - \bar{k}) > 0, \quad (20)$$

i. e., a higher elasticity of substitution means a higher per-capita income independent of whether the economy grows or shrinks.

The local analysis of this section supports the conclusion that the impact of a higher elasticity of substitution on the evolution of per-capita income is positive even if the saving rate  $s^r$  and  $s^w$  differ. While a negative acceleration effect can occur in Diamond-like economies and a negative total effect can therefore not be excluded theoretically, it seems that the empirically relevant case is the one where savings out of capital income are so important that the savings rate out of capital income exceeds the critical threshold value. Moreover in a growing economy, it is sufficient for a positive total effect that the savings rate out of profit income is not lower than the savings rate out of wage income.

It is worth noting that the local analysis of Propositions 1 and 2 may capture the properties of an economy's global dynamics. For instance, this is the case if the economy converges to a steady-state,  $k^*$ , that is part of the admissible neighborhood of  $\bar{k}$ . Much of the trust that growth economist have when they study the local dynamics of a steady state rests on this assumption. Of course, the analysis also applies to the extreme case where  $\bar{k} = k^*$ . However, then by definition the steady state can no longer depend on the elasticity of substitution.

### 3.3 Global Effects

We are now able to proceed to an explicit analysis of global effects of a higher elasticity of substitution on growth given possible asymmetries in the saving ratios. Our results can be regarded as generalizations of the two basic theorems that appear in Klump and de La Grandville (2000).

**Proposition 3** Consider two economies that initially differ only with respect to their elasticity of substitution. Moreover, assume that  $\dot{k}_{\sigma_1}(\bar{k}) = \dot{k}_{\sigma_2}(\bar{k}) > 0$ , where  $\sigma_2 > \sigma_1$ .

If  $s^r \geq s^w$ , then the economy with the higher elasticity of substitution has a larger capital stock and a higher per-capita income for all  $t > 0$ .

Again we see here how the interplay between the efficiency effect, the distribution effect and capital accumulation works. A higher elasticity of substitution leads to a higher efficiency of total factor inputs and also (for  $k > \bar{k}$ ) to an increase in the profit share. If savings stemmed from wage incomes only as it is the case in Diamond-like economies, this redistribution would slow down capital accumulation and could, in the worst case, make the capital intensity decline. According to (12),  $s^r \geq s^w$  is sufficient for a positive acceleration effect.

For a clear-cut global result concerning the evolution of per-capita income in a growing economy, we need more than a positive acceleration effect. In accordance with (17), what matters in addition is how the marginal product of capital responds to a rise in the elasticity of substitution. The proof of Proposition 3 establishes that this effect is indeed strictly positive, i. e.,  $\partial f'_\sigma / \partial \sigma > 0$  for all  $k > \bar{k}$ . Hence the intuition associated with the efficiency effect of (8) extends to the marginal product of capital when  $k > \bar{k}$ .

In the Solow economy underlying Theorem 1 of Klump and de La Grandville (2000), the redistribution of incomes has no effect on total savings. An important implication of our Proposition 3 is that the qualitative results of this theorem survive in an environment with differing saving rates as long as empirically plausible values are employed, i. e., if  $s^r > s^w$ .

Next, we turn to the analysis of the effect of the elasticity of substitution on the steady-state per-capita income. Let  $k^*$  denote a steady state capital intensity and  $\pi_\sigma^* \equiv \pi_\sigma^*(k^*) \in (0, 1)$  the corresponding capital share. From (11), a steady state must satisfy

$$\dot{k} = 0 \quad \Leftrightarrow \quad \frac{f_\sigma(k^*)}{k^*} [s^w (1 - \pi_\sigma^*) + s^r \pi_\sigma^*] = n. \quad (21)$$

To study the effect of the elasticity of substitution on  $k^*$ , we totally differentiate (21). This gives

$$\frac{dk^*}{d\sigma} = \frac{\frac{s^w}{k^*} \frac{\partial f_\sigma(k^*)}{\partial \sigma} + (s^r - s^w) \frac{\partial f'_\sigma(k^*)}{\partial \sigma}}{\frac{s^w f_\sigma(k^*)}{(k^*)^2} (1 - \pi_\sigma^*) - (s^r - s^w) f''_\sigma(k^*)} \quad (22)$$

and leads to the following results.

**Proposition 4** Consider two economies that initially differ only with respect to their elasticity of substitution. Moreover, assume that a steady state for both economies exists and that  $\dot{k}_{\sigma_1}(\bar{k}) = \dot{k}_{\sigma_2}(\bar{k}) > 0$ , where  $\sigma_2 > \sigma_1$ .

If  $s^r \geq s^w$ , then the economy with the higher elasticity of substitution has a larger steady-state capital stock and a higher steady-state per-capita income.

As long as the savings ratio out of capital income is large enough to overcome possible negative distributional effects of a higher elasticity of substitution on aggregate savings, higher factor substitution induces higher steady-state values of the economy. Again, it is sufficient that both savings ratios are equal as in the Solow model. This is the point of Theorem 2 in Klump and de La Grandville (2000). Our Proposition 4 shows that the qualitative results of this theorem extend to economies where  $s^r \geq s^w$  and  $k^* > \bar{k}$ .

## 4 Concluding Remarks

Since all models of economic growth combine assumptions about the substitutability between factors of production with a hypothesis about savings from factor incomes, the interaction between factor substitution and capital accumulation is the basic engine of growth. The standard neoclassical growth model, working typically with a Cobb-Douglas production function (and thus an elasticity of substitution equal to one) and a constant savings ratio of total factor income, does not allow for an in-depth analysis of this interaction. We therefore propose a generalized neoclassical growth model, in which a normalized CES production function identifies the effect of a variation in the elasticity of substitution between capital and labor, and where the savings hypothesis explicitly includes the possibility of asymmetries in savings out of capital and labor incomes. This general framework then encompasses neoclassical, classical, Post-Keynesian, and OLG-like settings as special cases.

Our results show that the impact of a higher degree of factor substitution on capital accumulation and growth depends on two separate effects. While the efficiency effect

is always positive and independent of any savings hypothesis, the accumulation effect can be positive or negative depending on the distributional consequences of higher factor substitution and on the assumed sources of savings. In the special case of a growing Diamond economy, where all savings come out of labor income, a higher elasticity of substitution squeezes the total rate of capital accumulation by reducing the labor share in total income. If this effect dominates the increase in total income resulting from the efficiency effect, then the overall effect on growth would be negative. We are able to show, however, that this constellation is rather unlikely to occur. As long as the savings ratio out of profits is not lower than the savings ratio out of wages or that it at least surpasses a certain lower threshold value, the growth effects of higher factor substitution remain positive as pointed out by Klump and de La Grandville (2000).

Miyagiwa and Papageorgiou (2003), p. 161, concluded from their analysis of the OLG-model that “. . . whether the elasticity of substitution has a positive or negative effect on economic growth depends on our view of the world, that is, on the particular framework (Solow vs. Diamond) we believe to be a better representation of the world.” Our analysis leads now to a more precise conclusion. As long as in the real world we find significant savings out of capital income the interaction between factor substitution, capital accumulation and growth is much better approximated by the Solow framework than by the Diamond setting. Moreover, our generalized growth model can help to reveal the complex mechanics that make the elasticity of substitution a powerful engine of growth.

## 5 Appendix

### 5.1 Proof of Lemma 1

Due to the normalization we have at  $\bar{k}$  :

$$\bar{y} = f_\sigma(\bar{k}) \text{ and } \bar{m} = m_\sigma(\bar{k}) = (f_\sigma(\bar{k}) - \bar{k}f'_\sigma(\bar{k})) / f'_\sigma(\bar{k})$$

so that the slope of  $f_\sigma$  at  $\bar{k}$  is

$$f'_\sigma(\bar{k}) = \frac{f_\sigma(\bar{k})}{\bar{k} + \bar{m}}. \quad (23)$$

The elasticity of substitution is defined as

$$\sigma \equiv -\frac{f'_\sigma(f_\sigma - kf'_\sigma)}{kf_\sigma f''_\sigma}. \quad (24)$$

Hence,

$$f''_\sigma = -\frac{f'_\sigma(f_\sigma - kf'_\sigma)}{kf_\sigma \sigma}. \quad (25)$$

and

$$\frac{\partial f''_\sigma(\bar{k})}{\partial \sigma} = \frac{f'_\sigma(f_\sigma - kf'_\sigma)}{kf_\sigma \sigma^2} = \frac{-f''_\sigma(\bar{k})}{\sigma} > 0. \quad (26)$$

From the concavity of  $f_\sigma$  we have  $f''_\sigma(\bar{k}) < 0$  which proves (6).

QED.

### 5.2 Proof of Proposition 1

The components of  $\dot{k}_\sigma(k)$  stem from equations (9), (10), and (11). Then, the acceleration effect of the elasticity of substitution, is

$$\frac{\partial \dot{k}_\sigma}{\partial \sigma} = s^w \frac{\partial w_\sigma(k)}{\partial \sigma} + s^r k \frac{\partial r_\sigma(k)}{\partial \sigma} \equiv \frac{\partial \dot{k}_\sigma(k)}{\partial \sigma}. \quad (27)$$

Suppose  $s^w \neq s^r$  and consider a first-order Taylor approximation of  $\partial \dot{k}_\sigma(k) / \partial \sigma$  about  $\bar{k}$ . As to the wage, we obtain the approximation

$$\frac{\partial w_\sigma(k)}{\partial \sigma} = \frac{\partial w_\sigma(\bar{k})}{\partial \sigma} + \frac{\partial w'_\sigma(\bar{k})}{\partial \sigma}(k - \bar{k}) = 0 - \bar{k} \frac{\partial f''_\sigma(\bar{k})}{\partial \sigma}(k - \bar{k}). \quad (28)$$

Here, we use (9) and  $w'_\sigma(k) = -kf''_\sigma(\bar{k})$ .

As to the real rate of return on capital, the same approximation gives

$$\frac{\partial r_\sigma(k)}{\partial \sigma} = \frac{\partial r_\sigma(\bar{k})}{\partial \sigma} + \frac{\partial r'_\sigma(\bar{k})}{\partial \sigma}(k - \bar{k}) = 0 + \frac{\partial f''_\sigma(\bar{k})}{\partial \sigma}(k - \bar{k}). \quad (29)$$

The derivation uses from (10),  $r'_\sigma(k) = f''_\sigma(\bar{k})$ .

Upon substitution of (28) and (29) in (27) gives the desired Taylor-approximation as

$$\begin{aligned} \frac{\partial \dot{k}_\sigma(k)}{\partial \sigma} &= s^w(-\bar{k}) \frac{\partial f''_\sigma(\bar{k})}{\partial \sigma}(k - \bar{k}) + s^r \bar{k} \frac{\partial f''_\sigma(\bar{k})}{\partial \sigma}(k - \bar{k}), \\ &= \bar{k} \frac{\partial f''_\sigma(\bar{k})}{\partial \sigma} (s^r - s^w)(k - \bar{k}). \end{aligned} \quad (30)$$

If  $s^w = s^r = s$ , then the first-order approximation of (14) vanishes. However, a second-order Taylor approximation delivers the result. To see this, observe that (12) becomes

$$\frac{\partial \dot{k}}{\partial \sigma} = \frac{\partial \dot{k}_\sigma(k)}{\partial \sigma} = s \frac{\partial f_\sigma(k)}{\partial \sigma}. \quad (31)$$

Consider the approximation

$$\frac{\partial \dot{k}_\sigma(k)}{\partial \sigma} \simeq s \frac{\partial f_\sigma(\bar{k})}{\partial \sigma} + s \frac{\partial f'_\sigma(\bar{k})}{\partial \sigma}(k - \bar{k}) + \frac{s}{2} \frac{\partial f''_\sigma(\bar{k})}{\partial \sigma}(k - \bar{k})^2. \quad (32)$$

Due to the normalization of the CES, the first two terms vanish and (15) obtains.

QED.

### 5.3 Proof of Proposition 2

To study the sign of (17) expand  $\partial \dot{y}_\sigma(k) / \partial \sigma$  about  $\bar{k}$ . Due to our normalization  $\partial \dot{y}_\sigma(\bar{k}) / \partial \sigma = 0$ , hence

$$\frac{\partial \dot{y}_\sigma(k)}{\partial \sigma} \simeq \frac{\partial \dot{y}'_\sigma(\bar{k})}{\partial \sigma} (k - \bar{k}). \quad (33)$$

Since  $\dot{y}'_\sigma(k) = f''_\sigma(k) \dot{k}_\sigma(k) + f'_\sigma(k) \partial \dot{k}_\sigma(k) / \partial k$ , we have

$$\frac{\partial \dot{y}'_\sigma(\bar{k})}{\partial \sigma} = \left( \frac{\partial f''_\sigma}{\partial \sigma} \dot{k}_\sigma + \frac{\partial f'_\sigma}{\partial \sigma} \frac{\partial \dot{k}_\sigma}{\partial k} + f''_\sigma \frac{\partial \dot{k}_\sigma}{\partial \sigma} + f'_\sigma \frac{\partial \dot{k}'_\sigma}{\partial \sigma} \right) \Bigg|_{k=\bar{k}}. \quad (34)$$



One readily verifies that the second and the third term in (34) are nil. Then, we obtain

$$\begin{aligned} \frac{\partial y'_\sigma(\bar{k})}{\partial \sigma} &= \frac{\partial f''_\sigma(\bar{k})}{\partial \sigma} (s^w w_\sigma(\bar{k}) + s^r \bar{k} r_\sigma(\bar{k}) - nk) + f'_\sigma \left[ s^w \frac{\partial w_\sigma(\bar{k})}{\partial \sigma} + s^r \bar{k} \frac{\partial r_\sigma(\bar{k})}{\partial \sigma} \right], \\ &= \frac{\partial f''_\sigma(\bar{k})}{\partial \sigma} [s^w \bar{y} (1 - 2\bar{\pi}) + s^r \bar{y} 2\bar{\pi} - n\bar{k}], \\ &= \frac{\partial f''_\sigma(\bar{k})}{\partial \sigma} [\dot{k}_\sigma(\bar{k}) + \bar{y} \bar{\pi} (s^r - s^w)], \end{aligned}$$

where we use (9), (10), and the definition of  $\dot{k}(\bar{k})$ . From Lemma 1 we know that  $\partial f''_\sigma(\bar{k})/\partial \sigma > 0$ . Hence, we find

$$\frac{\partial y'_\sigma(\bar{k})}{\partial \sigma} \gtrless 0 \quad \Leftrightarrow \quad (\dot{k}_\sigma(\bar{k}) + \bar{y} \bar{\pi} (s^r - s^w)) \gtrless 0. \quad (35)$$

Replacing  $\dot{k}_\sigma(\bar{k})$ , we find for the right-hand side of (33)

$$\frac{\partial y'_\sigma(\bar{k})}{\partial \sigma} (k - \bar{k}) \gtrless 0 \quad \Leftrightarrow \quad \left( 2\bar{\pi} \left( \frac{s^r}{s^w} - 1 \right) + \frac{1}{s^w \bar{y}} (s^w \bar{y} - n\bar{k}) \right) (k - \bar{k}) \gtrless 0. \quad (36)$$

Hence, (18) holds.

QED.

## 5.4 Proof of Proposition 3

Consider the case where  $s^r > s^w$ . We show that  $\partial \dot{k}(k)/\partial \sigma > 0$  for all  $k > \bar{k}$ . In view of (12), this requires

$$s^r \left[ \pi_\sigma \frac{\partial f_\sigma(k)}{\partial \sigma} + f_\sigma(k) \frac{\partial \pi_\sigma}{\partial \sigma} \right] > s^w \left[ \frac{\partial f_\sigma(k)}{\partial \sigma} (\pi_\sigma - 1) + f_\sigma(k) \frac{\partial \pi_\sigma}{\partial \sigma} \right] \quad (37)$$

or

$$s^r > s_c \equiv \max \left[ s^w \left( 1 - \frac{\frac{\partial f_\sigma(k)}{\partial \sigma}}{\pi_\sigma \frac{\partial f_\sigma(k)}{\partial \sigma} + f_\sigma(k) \frac{\partial \pi_\sigma}{\partial \sigma}} \right), 0 \right]. \quad (38)$$

According to (8) the efficiency effect is strictly positive for  $k \neq \bar{k}$ . Moreover, in accordance with (13) the distribution effect is positive whenever  $k > \bar{k}$ . Therefore, the term in parentheses on the right-hand side of (38) is strictly smaller than one for all  $k > \bar{k}$ . Hence,  $s_c < s^w$  such that  $s^r > s^w$  is sufficient for  $\partial \dot{k}(k)/\partial \sigma > 0$  to hold for all  $k > \bar{k}$ .

Turning to the evolution of per-capita income, we first state and prove the following claim.

**Claim 1** Let

$$\Phi := \pi_\sigma \ln \left( \frac{\bar{\pi}}{\pi_\sigma} \right) + (1 - \pi_\sigma) \ln \left( \frac{1 - \bar{\pi}}{1 - \pi_\sigma} \right) \quad (39)$$

where  $\bar{\pi} = \pi_\sigma (\bar{k})$ . It holds that

$$\frac{\partial f'_\sigma(k)}{\partial \sigma} = \frac{f'_\sigma}{\sigma^2 \psi^2} \left[ \psi^2 (1 - \pi_\sigma) \ln \left( \frac{k}{\bar{k}} \right) - \Phi \right] \quad (40)$$

and

$$\frac{\partial f'_\sigma(k)}{\partial \sigma} > 0 \text{ for } k > \bar{k}. \quad (41)$$

**Proof of Claim 1**

From  $\pi_\sigma \equiv k f'_\sigma(k) / f_\sigma(k)$  it follows that

$$\frac{\partial \pi_\sigma}{\partial \sigma} = \frac{k}{(f_\sigma)^2} \left( \frac{\partial f'_\sigma}{\partial \sigma} f_\sigma - \frac{\partial f_\sigma}{\partial \sigma} f'_\sigma \right)$$

or

$$\frac{\partial f'_\sigma(k)}{\partial \sigma} = \frac{\partial \pi_\sigma}{\partial \sigma} \frac{f_\sigma}{k} + \frac{\partial f_\sigma}{\partial \sigma} \frac{f'_\sigma}{f_\sigma}. \quad (42)$$

Next, we make use of (13) and of equation 11 in Klump and de La Grandville (2000) stating that

$$\frac{\partial f_\sigma(k)}{\partial \sigma} = -\frac{f_\sigma}{\sigma^2 \psi^2} \Phi. \quad (43)$$

Plugging (13) and (43) into (42) gives after some simple algebraic manipulation (40).

Equation (41) follows from the facts that for  $k > \bar{k}$ ,  $\ln(k/\bar{k}) > 0$  and  $\Phi < 0$  (see, equation 13 in Klump and de La Grandville (2000)).

QED.

To show that  $\partial \dot{y}(k) / \partial \sigma > 0$  for all  $k > \bar{k}$  consider the terms on the right-hand side of (17). In view of Claim 2 and the fact that the economy grows, the first term is strictly positive for  $k > \bar{k}$ . As shown above, the same is true for  $\partial \dot{k}(k) / \partial \sigma$ .

The results for  $s^r = s^w$  follow immediately from Klump and de La Grandville (2000), Theorem 1.

QED.

## 5.5 Proof of Proposition 4

Since  $\dot{k}_{\sigma_1}(\bar{k}) = \dot{k}_{\sigma_2}(\bar{k}) > 0$  it follows that  $k^* > \bar{k}$ . Therefore, all derivatives with respect to  $\sigma$  that appear on the right-hand side of (22) are strictly positive. Since  $f''_{\sigma} < 0$ , we have  $dk^*/d\sigma > 0$ . As to the steady-state per-capita income, we have  $y^* = f_{\sigma}(k^*)$  such that  $\partial y^*/\partial \sigma = \partial f_{\sigma}/\partial \sigma + f'_{\sigma} dk^*/d\sigma > 0$ .

QED.

## References

- ARROW, K. J., H. B. CHENERY, B. S. MINHAS, AND R. M. SOLOW (1961): "Capital-Labor Substitution and Economic Efficiency," *Review of Economics and Statistics*, 43(3), 225–250.
- BERNANKE, B. S., AND R. S. GÜRKAYNAK (2001): "Is Growth Exogenous? Taking Mankiw, Romer, and Weil Seriously," in *NBER Macroeconomics Annual 2001*, ed. by B. S. Bernanke, and K. S. Rogoff. MIT Press, Cambridge, MA.
- DE LA GRANDVILLE, O. (1989): "In Quest of the Slutsky Diamond," *American Economic Review*, 79(3), 468–481.
- DIAMOND, P. A. (1965): "National Debt in a Neoclassical Growth Model," *American Economic Review*, 55(5), 1126–1150.
- IRMEN, A. (2003): "On the Growth Effects of the Elasticity of Substitution," *mimeo*, University of Mannheim.
- KALDOR, N. (1956): "Alternative Theories of Distribution," *Review of Economic Studies*, 23(2), 83–100.
- KLUMP, R., AND O. DE LA GRANDVILLE (2000): "Economic Growth and the Elasticity of Substitution: Two Theorems and Some Suggestions," *American Economic Review*, 90(1), 282–291.
- KLUMP, R., AND M. SAAM (2008): "Calibration of Normalized CES Production Functions in Dynamic Models," *Economics Letters*, *forthcoming*.
- MIYAGIWA, K., AND C. PAPAGEORGIOU (2003): "Elasticity of Substitution and Growth: Normalized CES in the Diamond Model," *Economic Theory*, 21(1), 155–165.
- SOLOW, R. M. (1956): "A Contribution to the Theory of Economic Growth," *Quarterly Journal of Economics*, 70(1), 65–94.
- UZAWA, H. (1961): "On a Two-Sector Model of Economic Growth I," *Review of Economic Studies*, 29(1), 40–47.

# CESifo Working Paper Series

for full list see [www.cesifo-group.org/wp](http://www.cesifo-group.org/wp)

(address: Poschingerstr. 5, 81679 Munich, Germany, [office@cesifo.de](mailto:office@cesifo.de))

---

- 2084 Robert Fenge, Maximilian von Ehrlich and Matthias Wrede, Fiscal Competition, Convergence and Agglomeration, August 2007
- 2085 Volker Nitsch, Die Another Day: Duration in German Import Trade, August 2007
- 2086 Kam Ki Tang and Jie Zhang, Morbidity, Mortality, Health Expenditures and Annuitization, August 2007
- 2087 Hans-Werner Sinn, Public Policies against Global Warming, August 2007
- 2088 Arti Grover, International Outsourcing and the Supply Side Productivity Determinants, September 2007
- 2089 M. Alejandra Cattaneo and Stefan C. Wolter, Are the Elderly a Threat to Educational Expenditures?, September 2007
- 2090 Ted Bergstrom, Rod Garratt and Damien Sheehan-Connor, One Chance in a Million: Altruism and the Bone Marrow Registry, September 2007
- 2091 Geraldo Cerqueiro, Hans Degryse and Steven Ongena, Rules versus Discretion in Loan Rate Setting, September 2007
- 2092 Henrik Jacobsen Kleven, Claus Thustrup Kreiner and Emmanuel Saez, The Optimal Income Taxation of Couples as a Multi-Dimensional Screening Problem, September 2007
- 2093 Michael Rauber and Heinrich W. Ursprung, Life Cycle and Cohort Productivity in Economic Research: The Case of Germany, September 2007
- 2094 David B. Audretsch, Oliver Falck and Stephan Heblich, It's All in Marshall: The Impact of External Economies on Regional Dynamics, September 2007
- 2095 Michael Binder and Christian J. Offermanns, International Investment Positions and Exchange Rate Dynamics: A Dynamic Panel Analysis, September 2007
- 2096 Louis N. Christofides and Amy Chen Peng, Real Wage Chronologies, September 2007
- 2097 Martin Kolmar and Andreas Wagener, Tax Competition with Formula Apportionment: The Interaction between Tax Base and Sharing Mechanism, September 2007
- 2098 Daniela Treutlein, What actually Happens to EU Directives in the Member States? – A Cross-Country Cross-Sector View on National Transposition Instruments, September 2007

- 2099 Emmanuel C. Mamatzakis, An Analysis of the Impact of Public Infrastructure on Productivity Performance of Mexican Industry, September 2007
- 2100 Gunther Schnabl and Andreas Hoffmann, Monetary Policy, Vagabonding Liquidity and Bursting Bubbles in New and Emerging Markets – An Overinvestment View, September 2007
- 2101 Panu Poutvaara, The Expansion of Higher Education and Time-Consistent Taxation, September 2007
- 2102 Marko Koethenbueger and Ben Lockwood, Does Tax Competition Really Promote Growth?, September 2007
- 2103 M. Hashem Pesaran and Elisa Tosetti, Large Panels with Common Factors and Spatial Correlations, September 2007
- 2104 Laszlo Goerke and Marco Runkel, Tax Evasion and Competition, September 2007
- 2105 Scott Alan Carson, Slave Prices, Geography and Insolation in 19<sup>th</sup> Century African-American Stature, September 2007
- 2106 Wolfram F. Richter, Efficient Tax Policy Ranks Education Higher than Saving, October 2007
- 2107 Jarko Fidrmuc and Roman Horváth, Volatility of Exchange Rates in Selected New EU Members: Evidence from Daily Data, October 2007
- 2108 Torben M. Andersen and Michael Svarer, Flexicurity – Labour Market Performance in Denmark, October 2007
- 2109 Jonathan P. Thomas and Tim Worrall, Limited Commitment Models of the Labor Market, October 2007
- 2110 Carlos Pestana Barros, Guglielmo Maria Caporale and Luis A. Gil-Alana, Identification of Segments of European Banks with a Latent Class Frontier Model, October 2007
- 2111 Felicitas Nowak-Lehmann D., Sebastian Vollmer and Immaculada Martínez-Zarzoso, Competitiveness – A Comparison of China and Mexico, October 2007
- 2112 Mark Mink, Jan P.A.M. Jacobs and Jakob de Haan, Measuring Synchronicity and Co-movement of Business Cycles with an Application to the Euro Area, October 2007
- 2113 Ossip Hühnerbein and Tobias Seidel, Intra-regional Tax Competition and Economic Geography, October 2007
- 2114 Christian Keuschnigg, Exports, Foreign Direct Investment and the Costs of Corporate Taxation, October 2007
- 2115 Werner Bönnte, Oliver Falck and Stephan Heblich, Demography and Innovative Entrepreneurship, October 2007

- 2116 Katrin Assenmacher-Wesche and M. Hashem Pesaran, Assessing Forecast Uncertainties in a VECX Model for Switzerland: An Exercise in Forecast Combination across Models and Observation Windows, October 2007
- 2117 Ben Lockwood, Voting, Lobbying, and the Decentralization Theorem, October 2007
- 2118 Andrea Ichino, Guido Schwerdt, Rudolf Winter-Ebmer and Josef Zweimüller, Too Old to Work, too Young to Retire?, October 2007
- 2119 Wolfgang Eggert, Tim Krieger and Volker Meier, Education, Unemployment and Migration, October 2007
- 2120 Stefan Napel and Mika Widgrén, The European Commission – Appointment, Preferences, and Institutional Relations, October 2007
- 2121 Bertil Holmlund and Martin Söderström, Estimating Income Responses to Tax Changes: A Dynamic Panel Data Approach, October 2007
- 2122 Doina Maria Radulescu, From Separate Accounting to Formula Apportionment: Analysis in a Dynamic Framework, October 2007
- 2123 Jelle Brouwer, Richard Paap and Jean-Marie Viaene, The Trade and FDI Effects of EMU Enlargement, October 2007
- 2124 Kurt R. Brekke, Luigi Siciliani and Odd Rune Straume, Competition and Waiting Times in Hospital Markets, October 2007
- 2125 Alexis Direr, Flexible Life Annuities, October 2007
- 2126 Johannes Becker and Clemens Fuest, Quality versus Quantity – The Composition Effect of Corporate Taxation on Foreign Direct Investment, October 2007
- 2127 Balázs Égert, Real Convergence, Price Level Convergence and Inflation Differentials in Europe, October 2007
- 2128 Marko Koethenbueger, Revisiting the “Decentralization Theorem” – On the Role of Externalities, October 2007
- 2129 Axel Dreher, Silvia Marchesi and James Raymond Vreeland, The Politics of IMF Forecasts, October 2007
- 2130 Andreas Knabe and Ronnie Schöb, Subsidizing Extra Jobs: Promoting Employment by Taming the Unions, October 2007
- 2131 Michel Beine and Bertrand Candelon, Liberalization and Stock Market Co-Movement between Emerging Economies, October 2007
- 2132 Dieter M. Urban, FDI Technology Spillovers and Wages, October 2007

- 2133 Valentina Bosetti, Carlo Carraro, Emanuele Massetti and Massimo Tavoni, Optimal Energy Investment and R&D Strategies to Stabilise Greenhouse Gas Atmospheric Concentrations, October 2007
- 2134 David-Jan Jansen and Jakob de Haan, The Importance of Being Vigilant: Has ECB Communication Influenced Euro Area Inflation Expectations?, October 2007
- 2135 Oliver Falck, Heavyweights – The Impact of Large Businesses on Productivity Growth, October 2007
- 2136 Xavier Freixas and Bruno M. Parigi, Banking Regulation and Prompt Corrective Action, November 2007
- 2137 Jan K. Brueckner, Partial Fiscal Decentralization, November 2007
- 2138 Silvia Console Battilana, Uncovered Power: External Agenda Setting, Sophisticated Voting, and Transnational Lobbying, November 2007
- 2139 Alan J. Auerbach, Michael P. Devereux and Helen Simpson, Taxing Corporate Income, November 2007
- 2140 Lorenzo Cappellari, Paolo Ghinetti and Gilberto Turati, On Time and Money Donations, November 2007
- 2141 Roel Beetsma and Heikki Oksanen, Pension Systems, Ageing and the Stability and Growth Pact, November 2007
- 2142 Hikaru Ogawa and David E. Wildasin, Think Locally, Act Locally: Spillovers, Spillbacks, and Efficient Decentralized Policymaking, November 2007
- 2143 Alessandro Cigno, A Theoretical Analysis of the Effects of Legislation on Marriage, Fertility, Domestic Division of Labour, and the Education of Children, November 2007
- 2144 Kai A. Konrad, Mobile Tax Base as a Global Common, November 2007
- 2145 Ola Kvaløy and Trond E. Olsen, The Rise of Individual Performance Pay, November 2007
- 2146 Guglielmo Maria Caporale, Yannis Georgellis, Nicholas Tsitsianis and Ya Ping Yin, Income and Happiness across Europe: Do Reference Values Matter?, November 2007
- 2147 Dan Anderberg, Tax Credits, Income Support and Partnership Decisions, November 2007
- 2148 Andreas Irmen and Rainer Klump, Factor Substitution, Income Distribution, and Growth in a Generalized Neoclassical Model, November 2007